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Control Uniqueness in Reconstructability Analysis

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When the reconstructability analysis of a directed system yields a structure in which a generated variable appears in more than one subsystem, information from all of the subsystems can be used in modeling the relationship between generating and generated variables. The conceptualization and procedure proposed here is discussed in relation to Klir's concept of control uniqueness.

INDEX TERMS: reconstructability analysis, directed systems, control uniqueness, structural modeling, log-linear modeling, mask analysis

1. INTRODUCTION
This paper addresses the following question in the reconstructability analysis [Klir, 1985; Krippendorff, 1986] of directed systems: if in this analysis, one obtains, as the simplest structure which fits the data (or as a structure which, for whatever reason, one wishes to consider), a model having two or more subsystems which overlap in the generated variables they contain, how should one interpret this multiple determination of generated variables and how should one proceed with the analysis?

This question is relevant whether the directedness of a system is synchronic or diachronic, i.e., whether the generated variables are causally influenced by "environmental" variables operating at the same moment in time, and/or, in dynamic systems, by prior states of (some or all of) the generated variables. It is also relevant both to "selection systems" ("crisp possibilistic" systems), defined set-theoretically in terms of relations, and to "probabilistic systems," defined information-theoretically (or in the essentially equivalent log-linear modeling framework) in terms of multivariate probability distributions.

This note offers an approach different from the one proposed by Klir [1985] involving "control uniqueness." It is structured as follows. In Section 2, the essence of the proposal is outlined. Section 3 follows with a detailed exposition, applied to an example of a three variable probabilistic system, the simplest multivariate system in which this issue can arise. The conceptualization of the problem of "control uniqueness" by Klir is reviewed and his proposed solution summarized. Then, the alternative proposal which advocates using the reconstructed distribution is presented. In Section 4, this proposal is applied to an example of a selection system, more specifically to the reconstructability analysis of a mapping of three variables onto a fourth. While in Section 3, the structure obtained from the analysis gives a distribution very close to the original data and statistically indistinguishable from it, in Section 4, the structure obtained from the analysis is identical to the original mapping. Finally, in Section 5, a discussion of the
realms of applicability of the two methods is offered, and the relevance of this proposal to the analysis of dynamic systems is noted.

2. ESSENCE OF THE PROPOSAL
The essence of the proposal, summarized now for probabilistic systems, is to use fully the structural model obtained from reconstructability analysis to obtain the relationship between generating and generated variables, rather than using only a subsystem of the model. The model subsystems are not viewed as directed systems themselves, independently producing the output variable(s); conditional probabilities linking generating and generated variables which these subsystems offer are not utilized. Only the overall system is regarded as a directed system, and only the overall reconstructed probability distribution is used to obtain conditional probabilities. Thus, when generated variables are shared by these subsystems, one is not forced to choose one subsystem to derive the generating relationship. Instead, one obtains the generating conditional probabilities from the calculated multivariate distribution, this distribution having been derived by applying the maximum uncertainty condition to the subsystem distributions as constraints. One uses the information in all of the subsystems, not just one.

3. EXAMPLE-1: A THREE-VARIABLE PROBABILISTIC SYSTEM

3.1 Approach Based Upon "Control Uniqueness"
Consider the directed system where A and B are the generating variables and C is the generated variable. The levels of the lattice of structures (including variable permutations, but not showing linkages of descent between the models) are shown in Table-1.

Table-1. Levels of Structures for 3-Variable System

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ABC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. AB:AC:BC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. AB:AC</td>
<td>AB:BC</td>
<td>AC:BC</td>
</tr>
<tr>
<td>4. A:BC</td>
<td>B:AC</td>
<td>C:AB</td>
</tr>
<tr>
<td>5. A:B:C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Consider AC:BC, the third structure at level 3, which might be graphically represented by Figure-1. (An alternative representation will be proposed later in Section 3.2.) The interpretation of the structure reflected in the figure is problematic. What does it mean to say that C depends separately on A and on B? If the relationship between generating and generated variables were reversed, i.e., if C generated A and B (if the arrows in Figure-1 had their directions reversed), there would be no problem, as there is no reason why one variable cannot determine two others. But, with A and B as the generating variables and C as the generated variable, interpretation is not straightforward. How can A and B independently and separately determine, i.e., control, C?
This is the issue raised by Klir of "control uniqueness." His solution, for the present example, would be to choose one of the two subsystems as the controlling one. In effect this eliminates the AC:BC model and replaces it with either AC or BC, with the choice between the two governed by the minimum uncertainty principle, i.e., by whichever of $H(C|A)$ or $H(C|B)$ is smaller. If A reduces the uncertainty of C more than B does, the subsystem AC is chosen to represent the system. (If, alternatively, one were not committed to A and B as the generating and C as the generated variables, but yet wanted the system to be directed, one might choose between $A \rightarrow C \rightarrow B$ and $B \rightarrow C \rightarrow A$ by comparing $H(C|A) + H(B|C)$ with $H(C|B) + H(A|C)$.)

In effect this solution eliminates from consideration, among the set of structures considered in reconstructability analysis, all structures with subsystems sharing generated variables. The analysis is somewhat more complicated if there is more than one generated variable, but the principle is the same, and so for reasons of simplicity, the two examples discussed in this paper involve systems with only one generated variable.

The procedure for dealing with control uniqueness can be illustrated with an example of a three-variable contingency table (ABC) given in Table-2a with its two-variable subtables. This example is borrowed from the log-linear analysis of Knoke and Burke [1980] who derived their data from the 1977 General Social Survey. A, B, and C are age (A0 = young, A1 = old), geographic region (B0 = South, B1 = non-South), and education level (C0 = no college, C1 = college).
Table-2. Example-1: A Three-Variable Probabilistic System

2a. Three-variable contingency table, ABC, and its subtables, AC, BC, and AB [Knoke & Burke, 1980, p.44]. (Entries are counts; total count = 1478.)

\[
\begin{array}{c|cc|cc}
\text{C0} & \text{B0} & \text{B1} & \text{B0} & \text{B1} \\
\hline
\text{A0} & 143 & 253 & 77 & 182 \\
\text{A1} & 227 & 411 & 46 & 139 \\
\end{array}
\]

ABC table

\[
\begin{array}{c|cc|cc}
\text{C0} & \text{C1} & \text{B0} & \text{C0} & \text{C1} \\
\hline
\text{A0} & 396 & 259 & 370 & 123 \\
\text{A1} & 638 & 185 & 664 & 321 \\
\end{array}
\]

AC subtable

\[
\begin{array}{c|cc|cc}
\text{C0} & \text{C1} & \text{B0} & \text{B1} \\
\hline
\text{A0} & 220 & 435 \\
\text{A1} & 273 & 550 \\
\end{array}
\]

BC subtable

\[
\begin{array}{c|cc|cc}
\text{C0} & \text{C1} & \text{B0} & \text{B1} \\
\hline
\text{A0} & 142 & 254 & 72 & 187 \\
\text{A1} & 228 & 410 & 51 & 134 \\
\end{array}
\]

AB subtable

The results of reconstructability analysis, given in Table-3, show that the ABC table (the data) has the property that there is no significant loss of information in reconstructing the system from its AC and BC subsystems, i.e., that the AC:BC model with its degree of freedom of five is equivalent to the more complex (seven parameter) ABC model. This is seen from values of a, the probability of making an error in rejecting the null hypothesis that a specific model is statistically indistinguishable from the data. One wants a model, simpler than the data, with high a, i.e., a simple model where one is very likely to be in error if one rejects the (statistical) identity of the model to the data. If one is to depart at all from ABC, this model is AC:BC and not AB:AC:BC. (Note that one does not seek a small a; see the discussion of Knoke and Burke of the differences between this type of analysis and the more conventional use of Likelihood-Ratio statistics.)

The reconstructed ABC distribution is given in Table-2b. If one had started from this distribution, the AC:BC model would not merely be statistically indistinguishable from ABC, but as numerically identical with it as discrete counts permit.
Table-3. Reconstructability analysis of the ABC distribution of Example-1. $L^2$, df, & a, the Likelihood Ratio Chi-Square and the degrees of freedom from the data and alpha, the probability of a Type I error are given, as is, for convenience, the df for each model. The analysis was done by the program, AMLAT, by Doug Anderson.

<table>
<thead>
<tr>
<th>Model</th>
<th>From the data (ABC)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L^2$</td>
<td>df</td>
</tr>
<tr>
<td>ABC</td>
<td>00.00</td>
<td>0</td>
</tr>
<tr>
<td>AB:AC:BC</td>
<td>00.76</td>
<td>1</td>
</tr>
<tr>
<td>AB:AC</td>
<td>10.58</td>
<td>2</td>
</tr>
<tr>
<td>AB:BC</td>
<td>51.71</td>
<td>2</td>
</tr>
<tr>
<td>AC:BC</td>
<td><strong>01.31</strong></td>
<td>2</td>
</tr>
<tr>
<td>AB:C</td>
<td>61.00</td>
<td>3</td>
</tr>
<tr>
<td>AC:B</td>
<td>10.61</td>
<td>3</td>
</tr>
<tr>
<td>BC:A</td>
<td>51.74</td>
<td>3</td>
</tr>
<tr>
<td>A:B:C</td>
<td>61.03</td>
<td>4</td>
</tr>
</tbody>
</table>

To proceed with the control uniqueness analysis: given the model AC:BC, a single subsystem must be chosen to control C. The uncertainty of C is $H(C) = 0.8818$, and generative (conditional) uncertainties are as follows: $H(C|A) = 0.8572$ and $H(C|B) = 0.8772$; so one chooses subsystem AC to model the overall system (i.e., one chooses model AC:B). From this subsystem, one obtains the conditional probabilities, $p(C|A)$, given in Table-4. These are the probabilities one would use to predict C from A. If one were given also B, one would simply ignore this information.

Table-4. Conditional Probabilities from the AC Subsystem

| $p(C0|A0)$ | $p(C1|A0)$ | $p(C0|A1)$ | $p(C1|A1)$ |
|------------|------------|------------|------------|
| .605       | .395       | .775       | .225       |

Note that by choosing AC:B, one is choosing a model whose a values (see Table-3) indicate that the model is clearly different from the data, i.e., the probability of making an error in denying the statistical identity of AC:B and ABC is very small (0.014).

3.2 An Alternative: Using the Reconstructed Distribution

The alternative procedure recommended here is to retain the information in both the AC and BC subsystems, that is, to retain the interaction of both A and B with C. This can be done if one does not regard AC and BC as directed subsystems themselves, i.e., if one rejects an interpretation such as is given in Figure-1, where the boxes are considered to contain the $p(C|A)$ and $p(C|B)$ information, which do indeed conflict.

Instead, one regards the AC and BC subsystems as constraints to be satisfied in a calculated maximum-uncertainty ABC distribution. It is only this overall three-variable distribution which one interprets as a directed system, and from it one calculates the necessary conditional
Control Uniqueness in Reconstructability Analysis

probabilities for C. This follows closely what one is actually doing in reconstruction; this interpretation of the AC:BC model is represented in Figure-2.

2a. Generating the calculated ABC probability distribution (not a model of a physical process) and, from it, the C|AB conditional probability distribution

2b. Generating the probabilities of C values via the calculated conditional distribution; a model of the directed system based on the AC:BC structure

Figure-2. Alternative view of the AC:BC model

The calculated overall distribution and probabilities derived from it are represented by the letter q to distinguish them from the original ("observed") data and probabilities derived from it, referred to by the letter p. The calculated three-variable distribution is

\[ q_{AC:BC}(A,B,C) = p(A,C) p(B|C) = p(B,C) p(A|C) = p(A,C) p(B,C) / p(C) \]

This is the maximum uncertainty solution for the ABC distribution, i.e., the distribution which maximizes

\[ H(A,B,C) = - \sum q_{AC:BC}(A,B,C) \log_2 q_{AC:BC}(A,B,C) \]

where the AC:BC subscript on q indicates that the AC and BC observed distributions are treated as constraints on q, i.e.,

\[ q_{AC:BC}(A,C) = \sum_{B} q_{AC:BC}(A,B,C) = p(A,C) = \sum_{B} p(A,B,C) \]

\[ q_{AC:BC}(B,C) = \sum_{A} q_{AC:BC}(A,B,C) = p(B,C) = \sum_{A} p(A,B,C) \]

From this solution, one obtains the conditional probabilities,

\[ q_{AC:BC}(C|A,B) = q_{AC:BC}(A,B,C) / q_{AC:BC}(A,B) \]
where

\[ q_{AC:BC}(A,B) = \sum_C q_{AC:BC}(A,B,C) \]

Note that the above equations for the reconstructed ABC distribution and for the conditional probabilities it determines are based upon both AC and BC subsystems and indeed are completely symmetric with respect to these subsystems. The values of the conditional probabilities are given in Table-5.

Table-5. \( q_{AC:BC}(C|A,B) \): Conditional Probabilities, from the Reconstructed Distribution based on the AC:BC Model

| C          | q(C|A0,B0) = .664 | q(C|A0,B1) = .576 | q(C|A1,B0) = .817 | q(C|A1,B1) = .754 |
|------------|-------------------|-------------------|-------------------|-------------------|
| C0         |                   |                   |                   |                   |
| C1         |                   |                   |                   |                   |
| C0         |                   |                   |                   |                   |
| C1         |                   |                   |                   |                   |

Clearly, the description of the system given in Table-5 is different from the description given before in Table-4. Table-5 is preferred since it preserves information about the dependency of C also upon B (as noted above, it treats the two subsystems symmetrically), while Table-4 omits this information. Since the original ABC table was selected so that an AC:BC model suffers no major loss of information, predictions of C based on Table-5 are nearly the same predictions which would be made from the original data table. This would be less true for predictions made from Table-4.

The information in the two tables can also be compared by considering the conditional uncertainties of the generated variable, C, given the generating variables, either A alone in the approach of Section 3.1 or A and B in the approach of this section. From the original (or reconstructed) ABC table, \( H(C|AB) = 0.8520 \), to be compared with \( H(C|A) = 0.8572 \), noted before. Using knowledge of both A and B gives slightly lower uncertainty about C than knowledge of A alone. The difference is admittedly small, but this is specific to this data set; data sets could easily be constructed where this difference was much larger. More importantly, Table-3 shows that AC:B is not a good approximation to ABC, while AC:BC is.

4. EXAMPLE-2: A FOUR-VARIABLE SELECTION SYSTEM

4.1 General Approach

The same issues arise for selection systems defined set-theoretically ("crisp possibilistic" systems). This section gives a general discussion of this fact, and the following section offers an illustrative example. Consider, for example, a directed system defined by a mapping from generating variables A, B, and C, onto the generated variable, D, i.e.,
Because the system maps the cartesian product of A, B, and C onto D, there are no constraining relations between A, B, and C; we can thus ignore such relations. The possible dependencies of D define a lattice of structures whose levels are shown in Table-6.

<table>
<thead>
<tr>
<th></th>
<th>Possible dependencies of D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ABCD</td>
</tr>
<tr>
<td>2</td>
<td>ABD:ACD:BCD</td>
</tr>
<tr>
<td>3</td>
<td>ABD:ACD (or ABD:BCD or ACD:BCD)</td>
</tr>
<tr>
<td>4</td>
<td>ABD:CD (or ACD:BD or BCD:AD)</td>
</tr>
<tr>
<td>5</td>
<td>ABD (or ACD or BCD)</td>
</tr>
<tr>
<td>6</td>
<td>AD:BD:CD</td>
</tr>
<tr>
<td>7</td>
<td>AD:BD (or AD:CD or BD:CD)</td>
</tr>
<tr>
<td>8</td>
<td>AD (or BD or CD)</td>
</tr>
<tr>
<td>9</td>
<td>D</td>
</tr>
</tbody>
</table>

Reconstructability analysis is done by finding the simplest structure, whose subsystems, treated as constraints, coupled with the maximum uncertainty condition, yield exactly the original mapping (ABCD). For probabilistic systems, uncertainty was defined by the familiar Shannon entropy expression; here, for selection systems involving crisp possibilities, uncertainty is defined as the log of the cardinality of the set of possible values [Hartley, 1928]. Maximum uncertainty in this context means the least constrained relation, i.e., the largest cardinality possible (allowed by the constraints) for the set defining the relation.

Operationally, a reconstructed system can be obtained by first "expanding" each subsystem relation by calculating its cartesian product with sets of values of the variables absent in the relation, and then taking the intersection of these expanded relations. Each expanded relation is the result of maximum uncertainty operating on one subsystem; it joins to the restricted set of values for the subsystem variables, which defines the subsystem relation, all possible values of variables omitted in the subsystem. The intersection of the expanded relations reflects the joint imposition of all the constraints reflected in the model under consideration.

4.2 A Specific Case
Consider a specific mapping: the ABCD table, and those of its immediate subtables which involve the generated variable, D, shown in Table-7.
Table-7. Example-2: A 3-variable to 1-variable mapping (ABCD) and its immediate subtables involving D. Values of variables omitted in the subtable are denoted by ".". Values of the generated variable, D, which are both 0 and 1 are denoted by "+", e.g., (A,B,,D) = (0,1,*) means that both (0,1,0) and (0,1,1) tuples occur in the AB.D relation. (The "*" notation gives the subtables a compact form.) Note that the ABCD table is a mapping, but the three subtables are relations.

<table>
<thead>
<tr>
<th>A B C</th>
<th>D</th>
<th>A B C</th>
<th>D</th>
<th>A B C</th>
<th>D</th>
<th>A B C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>1</td>
<td>0 0 1</td>
<td>0</td>
<td>0 0 1</td>
<td>0</td>
<td>0 0 1</td>
<td>0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>1</td>
<td>0 1 *</td>
<td>1</td>
<td>0 1</td>
<td>0</td>
<td>0 1 *</td>
<td>0</td>
</tr>
<tr>
<td>0 1 0</td>
<td>0</td>
<td>1 0 *</td>
<td>0</td>
<td>1 0</td>
<td>0</td>
<td>1 0 0</td>
<td>1</td>
</tr>
<tr>
<td>0 1 1</td>
<td>1</td>
<td>1 0 0</td>
<td>1</td>
<td>1 0</td>
<td>0</td>
<td>1 1 *</td>
<td>1</td>
</tr>
<tr>
<td>1 0 0</td>
<td>0</td>
<td>1 0 1</td>
<td>1</td>
<td>BCD subtable</td>
<td>ACD subtable</td>
<td>ABD subtable</td>
<td>1</td>
</tr>
<tr>
<td>1 1 0</td>
<td>0</td>
<td>1 1 0</td>
<td>0</td>
<td>1 1 0</td>
<td>1</td>
<td>1 1 0</td>
<td>1</td>
</tr>
</tbody>
</table>

ABCD table

First consider the structure, ABD:ACD:BCD, at level #2 of Table-6. Its reconstructed relation, R', is given by

\[ R' = (\text{ABD} \otimes C) \equiv (\text{ACD} \otimes B) \equiv (\text{BCD} \otimes A), \]

where, since ABD (see Table-7) is \{00.1, 01.*, 10.*, 11.0\},

\[ \text{ABD} \otimes C = \{0001,0011,0100,0101,0110,0111,1000,1001,1010,1011,1100,1110\}, \]

and similarly for the other two terms on the right side of the equation. R' turns out to be a mapping identical to the original mapping, m, and hence level #2 provides an acceptable simplification of the original data. Note that each of the subtables is a relation, i.e., is stochastic, since the dependent variable D is not uniquely specified by the independent variables of the subsystem. However, the intersection of the expanded subtables is a mapping, i.e., is deterministic, as D is uniquely specified given A, B, and C.

We now descend to level #3, and consider whether our original mapping can be decomposed into two of the three triadic relations. It indeed can, namely with the ACD and BCD relations, i.e.,

\[ R'' = (\text{ACD} \otimes B) \equiv (\text{BCD} \otimes A) \]

turns out also to be identical to m. However, descending to the next level by eliminating either ACD or BCD produces a relation, and not a mapping, which is necessarily different from m. Thus, the simplest model which fully specifies the original mapping is ACD:BCD.
Note that the subsystems in levels #2 and #3 in Table-6 can be (and, as shown in Table-7, are in fact) relations, which by composition yield mappings; in contrast, structures on levels #1, 5, and 8 have only a single component which, to agree with the data, must be a mapping. (In level #9, D is a constant.)

4.3 The Issue of Control Uniqueness

The same question arises here as was discussed for the probabilistic system: how should one interpret the subsystems ACD and BCD? If each of these is regarded as a directed system as in Figure-3 (analogous to Figure-1), how can AC and BC separately determine, i.e., control, D?

The answer is the same. One should not interpret the subsystems as directed components of the system, sufficient in themselves to represent the system. As with the probabilistic system example discussed earlier, one obtains the reconstructed higher-order relation (in this case, actually a mapping) by using the maximum uncertainty principle, with the subsystems treated as constraints (Figure-4, analogous to Figure-2).

4a. Generating the calculated ABCD relation

4b. Generating the possible values of D from the calculated relation; a model of the directed system based on the ACD:BCD structure

In this case, invoking control uniqueness to select one subsystem as the controller of D would be especially unsatisfactory since it would alter the character of the system. Each of the level #2
subsystems is stochastic (i.e., is a relation), while the original system is deterministic (i.e., is a mapping). If this is expressed in terms of uncertainties, the result is similar to that found in the previous probabilistic example: using the full distribution calculated from the model gives lower uncertainty than using the distribution for a single model subsystem, selected because of control uniqueness. Specifically, \( H(D|A,B,C) = 0 \) if the reconstructed mapping is used to generate \( D \), but since \( H(A,B,D) \) and \( H(A,B) \) are log2 6 and log2 4, respectively,\[ H(D|A,B) = H(A,B,D) - H(A,B) = \log_2 1.5 = H(D|A,C) = H(D|B,C). \]

Thus there is nonzero uncertainty if either the ABD or the ACD or the BCD relation is used exclusively to generate \( D \). This supports the proposition that the structural model should be thought of as providing constraints for a maximum uncertainty reconstruction, and not as a description of the system as composed of three directed subsystems from which one must choose only one subsystem to control the generated variable.

In this selection systems modeling exercise, the simplified ACD:BCD model exactly reproduces the original mapping, as compared to the earlier probabilistic example where the simplified AC:BC model is only indistinguishable statistically from the original distribution. One could, however, here too allow the reconstructed relation to deviate from the original mapping as long as it met some criterion of acceptance, e.g., an acceptable distance to the original data).

It is interesting to note, that while "control uniqueness" need not be considered in interpreting a multi-subsystem structure, and should not in fact be used to reduce this structure to a single one of its subsystems, there still occurs in this particular example (that is, in the set of all 2^8 possible mappings of three binary variables onto a fourth) a phenomenon bearing a strong resemblance to the control uniqueness problem. It turns out that only 6 of the dependencies listed in Table-6 need be considered; levels #4, 6, and 7 do not occur, leaving only the six structural types of #1, 2, 3, 5, 8, 9 (Zwick & Shu, 1993). Structures on these missing levels appear to have the common property of overlapping in the generated variable, \( D \), but not overlapping in any of the generating variables, \( A \), \( B \), and \( C \). Whether this finding is specific to the case considered here or is more general is being investigated; this may yield further insight into the issue of "control uniqueness."

5. DISCUSSION

The central issue here is the interpretation and use of the simplified models obtained by the reconstructability analysis. If these models are to be used in the conceptualization of physical systems built by linking directed components (e.g., in the decomposition of Boolean functions in terms of primitive operations for logic design), then confrontation with the problem of control uniqueness is necessary, as two or more physical components indeed cannot independently control the same output variable.

However, if the object of reconstructability analysis is to obtain a mathematical model simpler than but adequate to the original data, then the control uniqueness principle unnecessarily sacrifices information contained in other subsystems, and the procedure proposed in this paper should instead be adopted. Subsystems are not regarded as physical black boxes which output the generated variables, because in the reconstruction procedure they do not individually generate
variable values, but only do so collectively and only in conjunction with the additional principle of maximum uncertainty. This is illustrated strikingly in the selection system example, wherein the individual subsystems are stochastic, and only collectively are deterministic, as is necessary to model a mapping.

Structural models in reconstructability analysis do not constitute a complete description of how generated variables are to be obtained. That is, the models do not offer any indication, in either their graphical or symbolic form, that their subsystems are to be used in conjunction with a maximum uncertainty (or other) principle. Nor do they indicate the precise nature of the interconnection between the subsystems. The object of reconstructability analysis is not the derivation of a set of simpler distributions (or relations or mappings) to replace the original higher order distribution, but rather the derivation of a distribution of the same order as the original, agreeing satisfactorily with the original distribution, yet based on fewer parameters.

One final qualification: while the structural model is not intended to provide a black box specification for the physical construction of a system, one can, by using additional intermediate, variables, use it to guide such purposes. Specifically, the mapping discussed in Section 4.2 could be implemented as shown in Figure-5.

Figure-5. A possible design for the mapping described in Section 4.2 using intermediate variables, D' and D''. The two boxes on the left implement the expanded ABD and BCD relations (the subtables of Table-7), while the box on the right implements the intersection operation.

Note that the argument in this paper applies also to the "identification problem" [Klir, 1985] in directed systems, in which one does not have an original higher order distribution, but in which one also calculates a maximum uncertainty distribution subject to the known (subtable) constraints (assuming the subtables are mutually consistent). In this situation as well, one can make use of multiple subtables involving the same generated variable(s).

In summary, in the reconstructability analysis of directed systems, when the purpose is mathematical modeling and not physical design, generated variables need not be defined by a unique subsystem but rather should be defined by the reconstructed probability distribution (or relation). Even where the purpose of the analysis is physical design, use of all of the subsystems is possible if one uses Figure-2 or Figure-4 (or, in the latter case, alternatively, Figure-5) as a basis for design, i.e., if one includes in one's physical design the implementation of maximum uncertainty and the satisfaction of the constraints.
Another most important implication of this proposal is for the study of dynamic systems, in which one typically first determines, using mask analysis, which lagged variables are most predictive of the state variables, and then proceeds to a simplification of the model defined by sampling variables using reconstructability analysis. The imposition of control uniqueness has been proposed as part of the reconstructability analysis in such applications [Klir, 1990]. With the present proposal one would alternatively retain the full information about the generated variables from all of the subsystems in the model obtained from the reconstructability analysis.

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REFERENCES


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