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Examining Differences in Student Achievments in Differential Equations

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This paper presents the results of an in-depth analysis of student responses to a differential equations modeling problem administered as part of an international comparison study. The international study compared students' skills and understandings in an inquiry-oriented approach to the teaching and learning of differential equations (project classes) to other traditional approaches (comparison classes). The guiding question of the research was to identify why United States comparison students fared better overall than project students on a routine modeling problem. To answer the research question a tripartite coding scheme was developed. The coding scheme illustrated that project students were failing to: (1) initiate a correct model of the problem, (2) understand conceptually the presence of time within a differential equation, and (3) appropriately interpret and apply the initial condition of the given modeling problem. Suggestions for improvements to the differential equations curriculum are included.

INTRODUCTION

Investigations of reform curricula on undergraduate students' mathematics achievement and attitudes have become increasingly predominant in mathematics education literature (Schoenfeld, Kaput, & Dubinsky, 1998; Dubinsky, Schoenfeld, & Kaput, 2000). This study contributes to this growing body of research in undergraduate mathematics education by presenting a comparative analysis of student achievement for modeling differential equations in an Inquiry-Oriented Differential Equations (IODE) classroom (project) versus other traditional (comparison) approaches. We explore student understanding and conceptual learning of modeling techniques within differential equations by examining student's written work on final exams. In our analysis we hope to contribute to current IODE research present within undergraduate education and provide ideas of student learning of differential equations. We begin with a description of the larger, international study from which data for the present study was drawn.

Background: An International Comparison Study

In order to investigate students' beliefs, skills, and understandings in IODE as compared to other approaches to differential equations, data was gathered from four international sites. Two instruments were developed to reflect the goals of possible differences in student skills and understandings. The first instrument, referred to as the routine assessment, consisted of eight items that covered a range of problems that reflect more analytic methods of solving differential equations and other topics typically emphasized in traditional approaches. The second instrument, referred to as the conceptual assessment, also consisted of eight items designed to reflect relational understandings (Skemp, 1987) and ways of conceptualizing the subject that are more consistent with a dynamical systems point of view.

Both the routine and conceptual assessments were developed so that they would be fair for all students. In keeping with this spirit, several mathematicians whose area of expertise is related to differential equations were asked to review the two assessments. These reviewers informed us that the items developed represented an important collection of skills and understandings for students in both traditional and reform-oriented approaches. The routine and conceptual assessments were organized around four themes centrally important to the study of differential equations: (a) Predicting and Structuring Solutions; (b) Modeling; (c) Parametric Thinking; and (d) Solving Analytic Problems. The work described in this paper focuses on the Modeling Theme.

Data Collection

In fall 2002, data was collected on IODE project and comparison students' beliefs, skills, and understandings in differential equations at four different locations, three of which were in the United States (henceforth referred to as the Midwest, Northwest, and Southwest sites) and at one international location in South Korea. Students at all sites were primarily engineering or mathematics majors (including prospective secondary mathematics teachers). This paper presents an analysis of data from the three United States sites only.

The routine assessment items (or a subset thereof, as determined by site instructors) were administered as part of students' final examinations. Table 1 summarizes the way that the final exams (and hence the routine assessment) were administered. As events turned out, the overwhelming majority of differences would more likely benefit the comparison students rather than the IODE project students.

Site	Routine Assessment Administration
Midwest Project	Class A: Closed book, no notes, calculators permitted Class B: Closed book, no notes, calculators permitted
Midwest Comparison	Class A: Closed book, no notes, calculators permitted, Class B: Take home final exam, Closed book, no notes, calculators

	permitted
Northwest Project	Closed book, no notes, calculators permitted
Northwest Comparison	One-page, double-sided sheet of notes and calculators permitted
Southwest Project	Closed book, no notes, no calculators
Southwest Comparison	Open book, no notes, calculators and computer algebra system Maple permitted

Table 1. Description of routine assessment by site

The conceptual assessment was administered to volunteers after the final exam. At all sites the conceptual assessment lasted 60 minutes. Students were permitted to use calculators, but none of the problems required or benefited from the use of a calculator.

After students completed the assessments each paper was coded and student names removed so that scoring of the papers was blind. Rubrics for scoring the routine and conceptual assessments were developed and each paper was graded by two project team members. A third project team member resolved differences that could not be resolved by the two graders. All results were entered into a database for subsequent analysis.

Site Descriptions

Comparison classes at all sites typically followed a lecture-style format whereas IODE project classes at all sites typically followed an inquiry-oriented format where students cycled between small group work on instructional tasks and whole class discussion of their ideas in order to foster progressive mathematization. Use of technology in the form of graphing calculators and Java applets were routinely integrated into instruction in the IODE project classes. The following paragraphs provide further details about each site, including institution description, teacher background, and comparison course texts and use of technology.

At the Midwest site students in the IODE project attended a mid-sized public institution with an open admission policy. There were two IODE project teachers at the Midwest site. One of teachers regularly taught differential equations for more than 10 years, but this was his first time teaching with the IODE project materials. Prior to this course all of his teaching had been conducted using a traditional lecture-style format. The other IODE project teacher was a recent PhD in mathematics with post-doctoral work in mathematics education and this was his third time teaching differential equations with IODE project materials. Since there were no other sections of differential equations at this site, the Midwest location recruited comparison students at a nearby private university with considerably more stringent entrance requirements. The two teachers at the Midwest comparison site also routinely taught differential equations. Unlike the project class that was a 3 credit hour course, the comparison class was a 4 credit hour course that included treatment of linear algebra. The textbook used in the comparison class was *Differential Equations & Linear Algebra (2E)* by Farlow, Hall, McDill, and West (2000). Approximately 2/3 of the course was devoted to differential equations, which is roughly equivalent to the amount of time the IODE project students spent studying differential equations. Students in both comparison classes were allowed to use calculators for all work outside of class and they often used the computer algebra system Derive was often used in class.

At the Northwest site students in both the comparison class and the IODE project class attended a large state university. The IODE project teacher is a recent Ph.D. in mathematics education and this was her second time teaching differential equations with the IODE instructional materials. A teacher with more than 10 years of experience teaching differential equations taught the comparison class and the course used the text, *Differential Equations*, 2nd edition, by Blanchard, Devaney, and Hall (2002), which treats the subject of differential equations from a dynamical systems point of view. Students in the comparison class were assigned three projects during the term and used ODE Architect (2001) to complete the lab projects. ODE Architect was not available for students to use during exams.

At the Southwest site students in both the IODE project class and the comparison class attended a two year community college. The IODE project teacher had less than 2 years of experience teaching differential equations and this was the first time that he taught differential equations with the IODE project materials. The comparison teacher had more than 10 years experience teaching differential equations and the course used the text, *Elementary Differential Equations*, 7th edition, by Boyce and DiPrima (2001). Students in the comparison class were assigned three projects during the semester using the computer algebra system Maple.

Early Results

Although results of the international comparison study showed no significant difference between the IODE project and comparison classes when all eight routine problems were combined, an item-by-item analysis of the problems with the United States students revealed a significant difference in favor of the comparison students on the routine modeling problem. Using this result as a starting point, we set out to answer the question: Why was there a significant difference in favor of United States comparison students on the routine modeling problem? In this paper we analyze student solutions to the modeling problem and identify where project students had difficulty to answer our primary research question. We also provide suggestions for improved student achievement of modeling differential equations.

METHOD

To address the Modeling theme, we agreed with Rasmussen et al. (2004) that the salty tank problem given to students on their final exam was an illustration of student's conceptual understanding of the relation between a differential equation and the model which represents it.

All students were administered a final exam comprised of eight routine items of assessment. Of all the routine items, comparison students fared significantly better than project students for only one problem. The problem of project student errors characterized a model of a salty tank with an inflow and outflow. Students were asked to develop a differential equation to represent the model and identify the initial condition. The actual problem given to students was:

> A large tank initially contains 60 pounds of salt dissolved into 90 gallons of water. Salt water flows in at a rate of 4 gallons per minute, with a salt density of 2 pounds per gallon. The incoming water is mixed in with the contents of the tank and flows out at the same rate. Develop a differential equation and an initial condition which predicts the amount of salt in the tank as a function of time. You do not have to solve the equation.

Student's responded to the given routine question in a variety of ways. It was determined by an initial review of student solutions that student approaches to the problem were not a proper indicator of a successful completion of the problem. For instance, although a large portion of students approached the problem by drawing a picture of the activity and/or identifying that a differential equation representing the rate of change of salt with respect to time would be set up as inflow rate of a salt solution minus outflow rate of a salt solution, these did not guarantee the successful development of a final and correct differential equation for the problem. This becomes quite clear in our analysis. To provide an explicit example of an appropriate method for solving the salty tank modeling problem see figure 1.

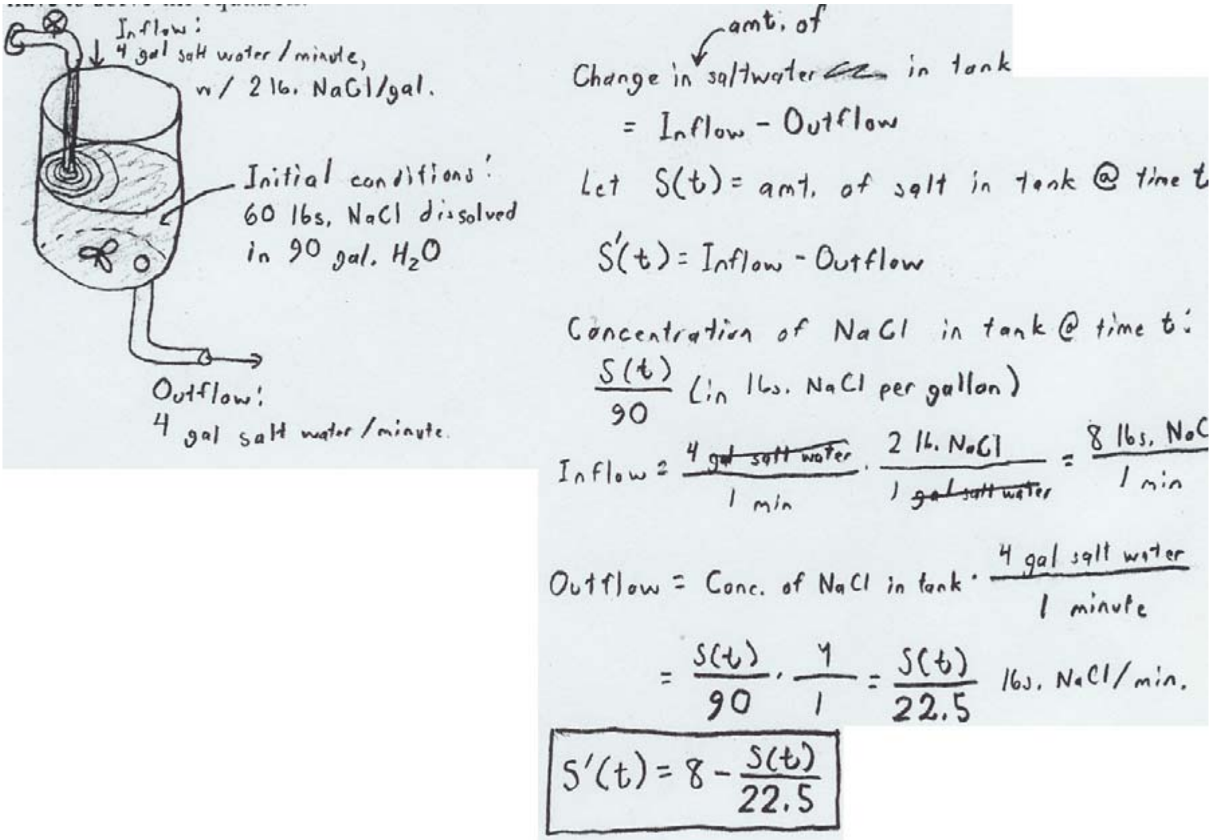

Figure 1. Example of a correct student solution to the routine salty tank problem.

Figure 1 illustrates a mapping of steps to a solution for the differential equation. As shown in the figure, the student properly (1) constructed a picture representing the activity, (2) clearly identified the initial condition and its relationship to the activity, and (3) recognized the differential equation as flow rate in minus flow rate out by stating "change in amt. of saltwater in tank = inflow – outflow". Figure 1 visibly shows an appropriate method for solving the salty tank modeling problem.

In order to quantify student solution methods it was concluded that an explicit analysis of student final solutions was necessary. The final solutions considered for analysis were solutions clearly marked as a final solution (i.e. circled), or at the end of a series of mathematical steps.

In reviewing student's final solutions to the salty tank problem on their final exam we observed reoccurring errors were being made. These errors included student's setting up the solution incorrectly (unable to identify "flow rate in – flow rate out"), including a dependent variable with their final solution, and including the initial condition within their final solution. Note that this is not an exhaustive list of all errors, but the three most commonly performed errors by students. Additionally, students may have made more than one error within their solution, which is reflected within the data analysis.

To fully analyze student solutions it was necessary to construct an error coding system, in addition to the grading rubric previously established by Rasmussen et al. (2004). The preliminary tripartite coding scheme was conjointly developed by both authors, but refined by Horst to aid in further explicit analysis.

The preliminary coding scheme consisted of three layers simulating steps to a correct different equation that represented the model. The preliminary coding scheme consisted of a numerical value of whether the student successfully achieved each layer. Once the preliminary coding was completed it was necessary to further analyze where student errors existed within the solution which is represented by the refined error coding scheme. Next we describe the three layers of coding that emerged from our data analysis.

A basic understanding of the salty tank problem involves recognizing that the rate change of salt with respect to time (dS/dt) can be expressed as "flow rate in – flow rate out". We see this as similar to Sherin's (2000) symbolic forms, in that we have the basic structure "?" = "?" – "?". Thus, the first layer of coding focuses on the ways in which students set up the differential equation. If students were unable to set up the problem correctly, they were unable to conclude a final, correct solution. To aid in analysis an explicit coding scheme was established to provide explanations why students did not complete the first layer. Within the coding scheme, a numerical value was given for the error or errors students made. Of all the errors student's made, six were popular among the majority of students. These errors were: (1) no attempt at the problem, (2) setting up the differential equation as dS/dt = "?", (3) setting up the differential equation as dS/dt = "?" – "?" +/- "?", (4) including the initial condition within the differential equation, (5) setting up the differential equation as dS/dt = "?" + "?", and (6) including the dependent variable t within the differential equation.

An illustrative example of a student who did not successfully complete the first layer is shown in figure 2.

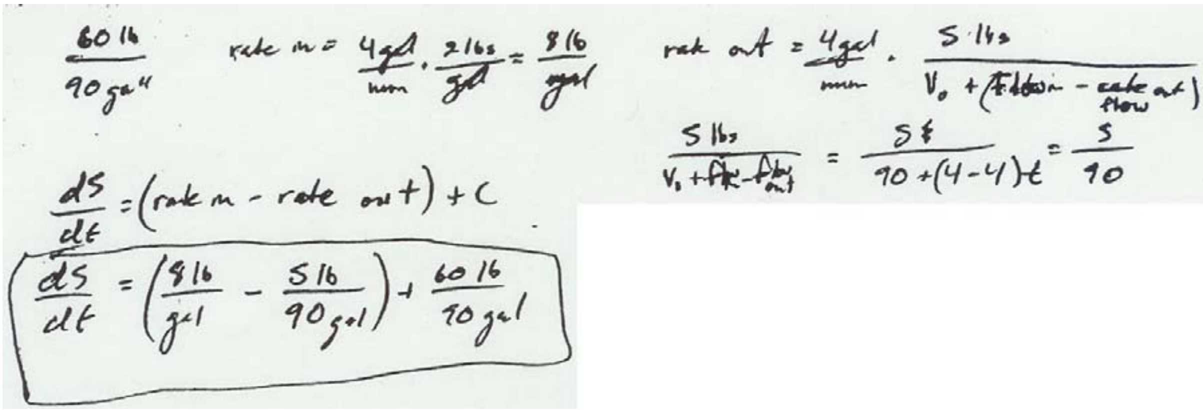

Figure 2. Example of a student who did not identify dS/dt = “?” – “?”.

Figure 2 illustrates that the student was unable to set up the differential equation correctly. The student identified the differential equation as equal to “(rate in – rate out) + C”. Although the student did identify “rate in – rate out”, he/she also added a variable “C” which we consider to be a constant term the student believes would influence the differential equation.

The second layer represents an identification of the appropriate rate in for the differential equation dS/dt = flow rate in – flow rate out. Both authors determined a portion of comparison and project students, 9 and 9 respectively, were setting up the problem correctly, but determining the flow rate in incorrectly, and felt due to the number of student’s doing this, it should be considered as the second layer. For all but 4 of the 45 comparison students, the students who did not identify the proper flow rate in also did not identify the proper flow rate out. As with the first layer, the second layer was also given a numerical error coding scheme for student error or errors. Of all the errors student’s made, six were popular among the majority of students. These errors were: (1) including the dependent variable t within first term of the differential equation (i.e. dS/dt = 8t – 4S/90), (2) a basic arithmetic error (i.e. 2*4 = 6), (3) dividing the entire right-hand side of the equation by 90 (i.e. dS/dt = (8-4S)/90), (4) including the initial condition within first term of the differential equation, (5) placing the flow rate in as the flow rate out (i.e. dS/dt = "?" – 8), and (6) determining the flow rate in as 2/4 (i.e. dS/dt = 2/4 - "?").

An illustrative example of a student who did not successfully complete the second layer is shown in figure 3.

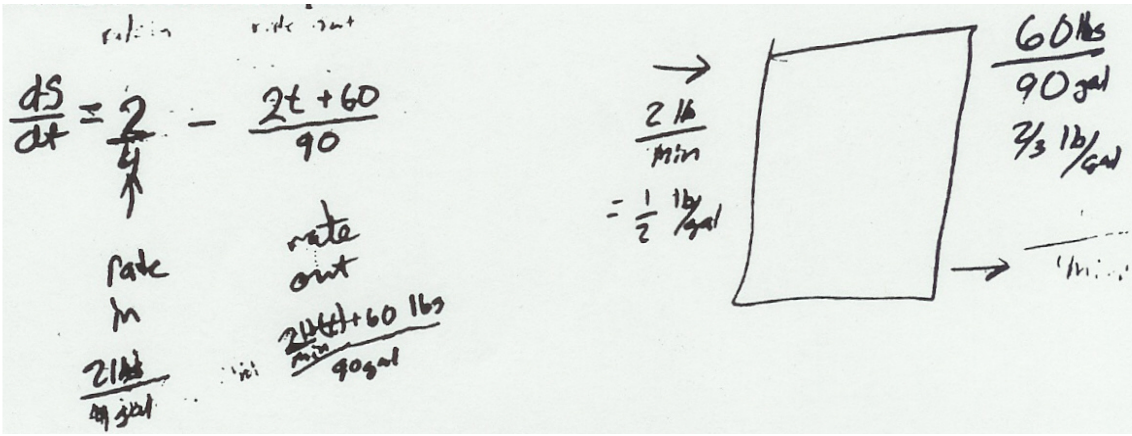

Figure 3. Example of a student who did not identify the correct flow rate in.

Figure 3 clearly demonstrates that the student was unable to determine the correct flow rate in for the differential equation of the rate of change of salt with respect to time. We hypothesize the student did not interpret the inflow and outflow as rates, but as amounts. Although the student did provide a picture of the activity and explicitly states "dS/dt = rate in – rate out", this was not enough to determine a correct solution.

The third layer represents a final, correct differential equation as a solution to the model of the salty tank. Both authors agree that this should be the final layer within the error coding scheme. The last layer identifies that the student was able to properly identify flow rate out, in addition to the previous requirements of setting up the problem correctly and determining the appropriate flow rate in. Clearly, this layer is the last and final step for solving a differential equation for the salty tank model. Similarly to both the first and second layers, the third also has a numerical error coding scheme to provide explicit information where students went awry. Of all the errors student's made, five were popular among the majority of students. These errors were: (1) including the dependent variable t within the denominator of the differential equation (i.e. $dS/dt = 8 – 4S/(90 + t)$), (2) including the dependent variable t within the numerator of the differential equation (i.e. $dS/dt = 8 – (60 + 8t)/90$), (3) including the initial condition present within the second term of the differential equation (i.e. $dS/dt = 8 – 4*(60 + S)/90$), (4) for the incorrect solution $dS/dt= 8 - 4S$, and (5) for the incorrect solution $dS/dt= 8 - S/90$.

An illustrative example of a student who did not successfully complete the third layer is shown in figure 4.

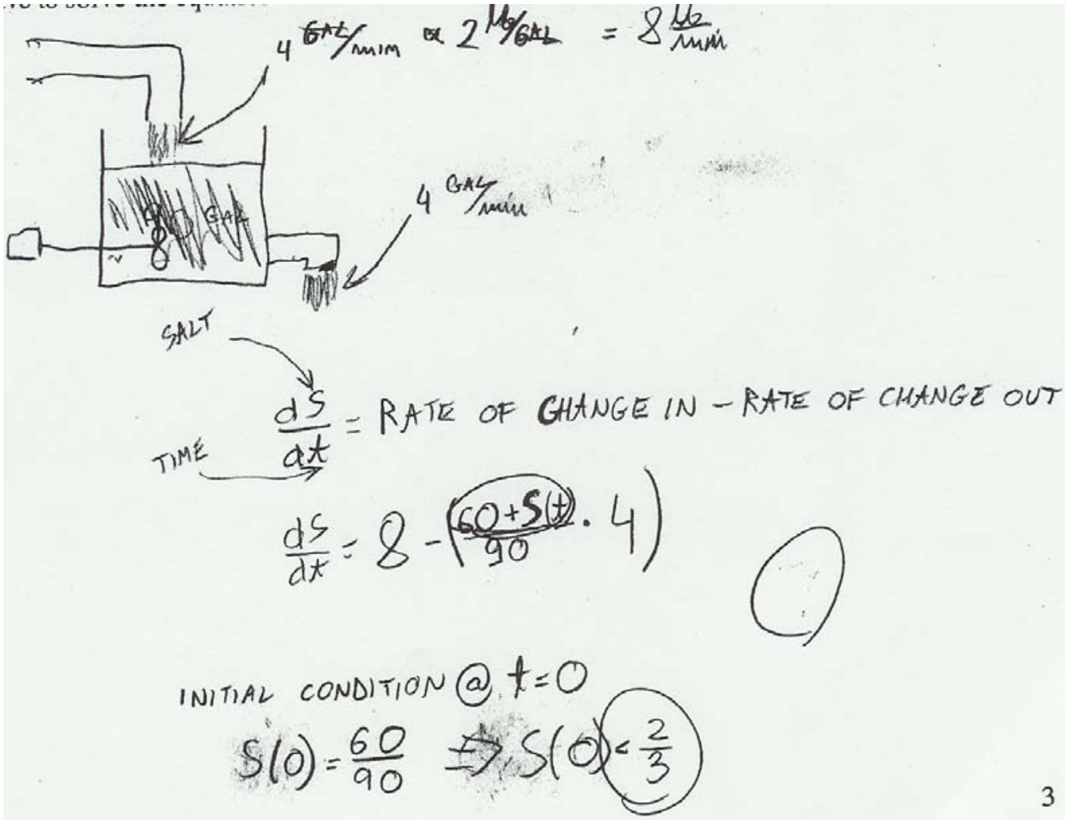

Figure 4. Example of a student who did not identify the correct flow rate out.

Figure 4 depicts a student who was unable to determine a correct flow rate out for the differential equation of the rate of change of salt with respect to time. We hypothesize the student attempted to include the initial condition within the differential equation by including the values 60 and 90, components of the given initial condition, within calculated flow rate out. Although the student provided a picture of the activity and identified that dS/dt = "rate of change in – rate of change out", this was not sufficient to led to a correct solution.

In the next section, we further analyze the implications of student errors when solving the routine salty tank modeling problem.

RESULTS

The successful completion of each layer leads to a final and correct solution for the salty tank routine modeling problem. Although a large portion of students were able to correctly construct a picture representing the activity and/or properly identify the rate of change of salt with respect to time would equal flow rate in minus flow rate out, this did not ensure a correct final solution.

As we analyzed student's final solutions and their approaches to these solutions it was apparent that three components were continually rising to the surface, which is illustrated within the tripartite coding scheme. Each layer coincides with an error or errors that students made in the process of solving the modeling problem. Note that if students did not successfully complete the previous layer, they were unable to progress, which is represented by the decreasing population of students within each layer. To clearly see the quantity of students who did and did not progress through each layer three tables were constructed, tables 2, 3, and 4.

Layer 1: dS/dt = "?" – "?"	
Comparison (n=45)	87%
Project (n=61)	69%

Table 2. Quantity of students whom successfully identified dS/dt = "?" – "?" .

Table 2 illustrates the percentage of students, comparison and project, whom successfully set up the differential equation correctly. As exhibited by table 2, 87% of the comparison students and 69% of the project students correctly set up the differential equation by identifying that dS/dt = flow rate in – flow rate out. The quantity of project

students who incorrectly set up the differential equation is significantly greater than the quantity of comparison students, implying project students do not understand how a differential equation representing the rate of change of salt with respect to time would be initially set up.

Layer 2: dS/dt = 8 – "?"	
Comparison (n=39)	77%
Project (n=42)	79%

Table 3: Quantity of students whom successfully identified dS/dt = 8 – "?".

Table 3 presents the percentage of students, comparison and project, whom successfully identified the correct flow rate in for the differential equation. As shown in table 3, 77% of comparison students and 79% of project students correctly identified dS/dt = flow rate in – flow rate out and calculated flow rate in as 8. The percentages of students from both groups are quite similar, implying that improperly identifying rate in was an error demonstrated equally by both groups.

Layer 3: dS/dt = 8 – (4S)/90	
Comparison (n=30)	80%
Project (n=34)	44%

Table 4: Quantity of students whom successfully identified dS/dt = 8 – (4S)/90**.**

Table 4 exhibits the percentage of students, comparison and project, whom successfully identified the correct flow rate out for the differential equation. In turn, these students also identified the correct differential equation representing the activity. As shown in table 4, 80% of comparison students and 44% of project students correctly

identified dS/dt = 8 – (4S)/90. The quantity of project students who were unable to correctly identify flow rate out is significantly greater than the quantity of comparison students, implying project students are unable to identify and compute a flow rate out for a differential equation representing the rate of change of salt with respect to time.

It is evident within tables 2, 3, and 4, that the majority of project students were unable to successfully reach a correct final solution. Only 25% of the total project students actually reached a correct final solution, whereas 53% of the total comparison students were able to successfully complete the problem. Project students, and few comparison students, had trouble (1) initiating a proper model of the activity by determining that the rate of change of salt with respect to time would equal flow rate in minus flow rate out, (2) identifying whether time was an independent or dependent variable, and (3) the relationship of the initial condition to the differential equation representing the model. It appears obvious from the data that project students do not fully understand modeling a differential equation in greater numbers than comparison students. The next section will focus on suggestions to improve overall student achievement, leading to the development of correct solutions for modeling differential equations.

SUGGESTIONS FOR IMPROVED STUDENT ACHIEVEMENT

To prevent future project and comparison students from making the same reoccurring errors as our study students did, we provide some suggestions for improved student achievement.

After concluding what and where students were making their errors, we revisited the format of the class which covered modeling of differential equations. Within the specific class that addresses modeling, students are instructed to discuss and solve a problem similar to the routing salty tank problem. Unlike the routine question asked on the final exam, students are presented with a modeling problem where time is an independent variable. We postulate that students do not fully understand the relationship of time within the differential, and the process of identifying whether time is an independent or dependent variable. To counteract students' misunderstanding we propose three solutions for improved achievement.

To address students' overall understanding of independent and dependent variables within differential equations we suggest providing homework problems similar to the routine salty tank problem, that probe students thinking of variables within differential equations. We additionally propose students reflect on their learning by completing a journal entry concentrating on independent and dependent variables within differential equations. Some questions that will require students to examine their understanding of variables are:

(1) Why does the rate of change with respect to time equal flow rate in minus flow rate out?
(2) When and why are independent and dependent variables represented within a differential equation?
(3) Is it possible to have a differential equation where the only variable present is the independent variable?
(4) Is it possible to have a differential equation where the only variable present is the dependent variable?

To exemplify the importance of modeling differential equations, we find it essential to focus only on modeling for the duration of the class session where modeling is first introduced. The project students were subjected to modeling differential equations in addition to the introduction and application of the Reverse-Product Rule. The two topics addressed are seemingly disjoint to students first experiencing them. It appears that student's focus primarily on the Reverse-Product Rule, paying little or no attention to the modeling technique of the differential equation presented in the first half of the class session. We postulate that students will gain a better conceptual understanding of the modeling technique of differential equations by devoting an entire class session.

Lastly, and most obvious, we suggest reexamining the modeling technique prior to the conclusion of the term and final exam. Reiterating the key concepts of modeling differential equations should clear any confusion students may still possess at the close of the term.

If these three suggestions for improved student achievement are applied, we hypothesize that student's conceptual understanding of modeling differential equations will greatly improve, in turn leading to the development of correct differential equations and their respective models.

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