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A Resource Constrained Shortest Paths Approach to Reducing Personal Pollution Exposure

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Abstract—As wildfires surge in frequency and impact in the Pacific Northwest, in tandem with increasingly traffic-choked roads, personal exposure to harmful airborne pollutants is a rising concern. Particularly at risk are school-age children, especially those living in disadvantaged communities near major motorways and industrial centers. Many of these children must walk to school, and the choice of route can effect exposure. Route-planning applications and frameworks utilizing computational shortest paths methods have been proposed which consider personal exposure with reasonable success, but few have focused on pollution exposure, and all have been limited in scalability or geographic scope. This paper addresses the lack of studies on this subject in Portland, OR. An application of the *A*Prune* algorithm is proposed for the purpose of reducing personal exposure for children attending Harriet Tubman Middle School in NE Portland, one area of Portland where residents are disproportionately affected by pollution. This method can sometimes identify alternative routes which reduce pollution exposure without significantly increasing travel distance over the shortest route.

I. INTRODUCTION

Pollutants commonly encountered in urban centers, such as nitrogen dioxide (NO₂) and particulate matter (PM), have been linked with negative health consequences, and exposure over a short time can sometimes be significant [1] [2] [3]. In recent decades, several factors have led to increasing danger from personal pollution exposure in the Pacific Northwest (PNW). As early as the 1990, both Portland and Seattle experienced increases in population as well as a marked increase in per-capita travel distance by car [4]. Traffic is one of the main factors contributing to PM in the air [5]. In Portland, OR, high concentrations of NO₂, which is often identified as a marker of PM [5] [6], have also been shown to closely coincide with major roadways and their vicinity [7], as has Diesel PM [8]. Meanwhile, with the recent increase in wildfire frequency and impact in the Northwestern U.S., the region has seen an increase in the concentration of particulate matter of less than 2.5 μ m (PM_{2.5}) in the atmosphere [9]. Further, with rapid development and industrialization in East Asia, it has been shown that some PM from Asian sources can travel far enough to effect background PM levels in the PNW, though this effect is not as relevant as a percentage of overall PM in urban areas [10]. These facts motivate a concern for the potential harm caused by increased personal exposure to pollutants, particularly in urban settings. Previous studies have

also suggested that harm is not equitably distributed, but that those living in lower income areas, or near major motorways and industrial sources of pollution shoulder a disproportionate amount of the costs of pollution [11] [12]. At even greater risk are the children living in these areas, who are at greater risk from pollution than adults [13]. Our study focuses on reducing pollution exposure for students from Harriet Tubman Middle School who walk to school. Harriet Tubman Middle School is located near the I-5 freeway and Moda Center, and as such is in an area with comparatively high concentrations of NO₂ [7] and PM. It has been noted that journey-time exposure to pollution may be only a small part of the picture, with time spent in the home and at school playing a major role [14]. However, as it has been demonstrated that short term exposures can have negative health consequences, a part of the problem which is readily solved by existing shortest paths methods is the focus of this paper, which shows that for some pollution surfaces and routes, alternative healthier routes can sometimes be found.

II. PROBLEM FORMULATION

In searching for a computational approach to reducing students' personal exposure levels, we are motivated by an understanding that finding least-polluted walking paths can be readily formulated as a shortest paths problem, in which the quantity to minimize is pollution. However, because it is likely that students will be unwilling to increase the distance they travel over the minimum by more than a threshold, as has been seen among bikers [15], an additional constraint on the maximum distance of viable walking routes is added. Then the problem becomes finding the least-polluted path in a graph, subject to a constraint on the maximum distance.

A. Mathematical Formulation

Consider a graph $G = (V, E)$, with V the set of nodes in G and E the set of edges. An edge $e \in E$ is defined as (i, j) , where $i, j \in V$ and are, of course, connected by an edge. Each edge e has weights $w_{exp}(i, j)$ and $w_{len}(i, j)$ corresponding to a pollution exposure and distance measure, respectively. We define a cost $C(i, j)$ for e , which is a function of w_{exp} and w_{len} . Let $P(u, v)$ be the set of all possible simple paths from u to v for $u, v \in V$. Here a simple path $p(u, v)$ is defined as a sequence of adjacent nodes $(n_0, n_1, n_2, \dots, n_k)$ such that no node is visited twice, $n_0 \equiv u$, $n_k \equiv v$, and k is the number of

nodes in the path. Then we define the total pollution exposure, distance, and cost, respectively, along a path p as follows:

$$\begin{aligned} w_{exp}(p) &= \sum_{m=0}^{m=k-1} w_{exp}(n_m, n_{m+1}) \\ w_{len}(p) &= \sum_{m=0}^{m=k-1} w_{len}(n_m, n_{m+1}) \\ C(p) &= \sum_{m=0}^{m=k-1} C(n_m, n_{m+1}) \end{aligned}$$

Given starting node u and target node v , the shortest path problem without constraints would be to find a path $r(u, v)$ such that

$$C(r(u, v)) = \min_{p(u, v) \in P(u, v)} C(p(u, v))$$

Now consider a constraint D_{max} , which we interpret here as the largest total path distance to be allowed in a valid path. The new shortest path problem with the constraint D_{max} is the same as above, except the the set of allowable paths is smaller. Let $P_D \subseteq P$ be the set of all paths that satisfy the constraint, that is

$$\forall p_D \in P_D, w_{len}(p_D) \leq D_{max}$$

Then the shortest path problem with this constraint is to find a path $r(u, v)$ such that

$$C(r(u, v)) = \min_{p_D(u, v) \in P_D(u, v)} C(p_D(u, v))$$

B. Literature Review

Both the shortest path problem and the shortest path with resource constraints problem have been extensively studied. There are many well-known and effective solutions to the shortest path problem, including the Bellman-Ford, Dijkstra, and A^* algorithms. A^* was proposed as a class of algorithms which were known to find the shortest path while searching the fewest possible paths [16]. For a problem closely related to finding the shortest path with a resource constraint, a modification of A^* has been proposed. A^*Prune has proven to be an effective algorithm for finding the K shortest paths with multiple resource constraints [17]. Several past studies have looked at shortest paths methods and route finding applications which consider personal pollution exposure. Some of the most recent examples are web-based route finders developed for cyclists in Montreal [6] and Vancouver [18], Canada. These applications had success in finding least polluted paths which differ from either the shortest path or some representative path. However, both are limited to their respective cities, and only the Montreal study was focused on reducing personal pollution exposure; it finds the minimum exposure path, regardless of distance. While application built for Vancouver is able to take into account a preference for a shorter versus less polluted route, the user must choose only one of distance or exposure as the factor to minimize. Both of these tools are designed with cyclist, rather than pedestrians, in mind.

Socharoentum and Karimi proposed a route planning framework for multi-modal, walking-inclusive routes which takes into account multiple factors including distance and pollution exposure along the walking segment [19], but their application was not focused on pollution exposure and used a coarse, categorical measure for pollution levels. Additionally, two studies have been conducted in the UK which look at reducing personal pollution exposure during transit for school children through shortest paths methods, and which show a potential for reducing exposure in this way [20] [21]. However, their methods only considered minimization of pollution exposure, and were not readily scalable to other cities due in part to necessary manual data manipulation [20]. To the author's knowledge, there are no applications of shortest path methods for walking which take into account both minimizing exposure and satisfying a maximum distance constraint. Further, the author is not aware of any studies which consider reducing personal pollution exposure through shortest paths methods in Portland, OR. This study shares many of the limitations of previous studies, including geography, and it should be noted that a complete framework for routing, comprising an accurate pollution model, routing tools, and a user interface, is beyond the scope of this paper. What this paper proposes is a novel application and implementation of the A^*Prune algorithm to reducing personal pollution exposure in Portland.

III. METHODOLOGY

A. Path Finding Algorithm

This algorithm is based on the A^*Prune algorithm proposed by Liu and Ramakrishnan [17]. The algorithm conducts an A^* search from s to t but at each step extends only those paths which could possibly satisfy the constraint based on a look-ahead procedure. The look-ahead heuristic used to help decide priority for path extension is the Dijkstra minimum cost from the last node in the path to the target. The look-ahead heuristic used to eliminate paths that should not be extended is the Dijkstra minimum distance from the last node in the path to the target.

B. Important Notation for Procedure 1

Let $h_c(i, j)$ for $i, j \in V$ be the cost of the simple least-cost path from i to j . Let $h_d(i, j)$ for $i, j \in V$ be the distance of the simple shortest-distance path from i to j . For paths $p_1(i, j)$ and $p_2(j, k)$ which share an end node, we write the combination of the two paths, which is a path from i to k , as

$$p_1(i, j) * p_2(j, k) = p(i, k)$$

We call p_1 a head path of p and p_2 a tail path of p . The projected cost of a path $p(s, j)$ given target t is defined as $C(p(s, j)) + h_c(j, t)$.

C. Implementation

The A^*Prune algorithm was implemented for finding a single path and considering a single constraint, the maximum allowable distance. For ordering paths to search in the A^* search, a priority queue which prioritizes the path with

Procedure 1 A*Prune for a Single Path with One Constraint

Inputs: The graph G , a source node s , a target node t , a cost function $C(\cdot)$, and a constraint D_{max}

Outputs: The path $p^*(s, t)$ satisfying

$$C(p^*) = \min_{p \in P_D} C(p)$$

if it exists, along with $C(p^*)$, $w_{exp}(p^*)$, and $w_{len}(P^*)$

```
1: Initialize the trivial path  $p(s, s)$ 
2:  $w_{exp}(p(s, s)) \leftarrow 0.0$ 
3:  $w_{len}(p(s, s)) \leftarrow 0.0$ 
4:  $C(p(s, s)) \leftarrow 0.0$ 
5:  $Viable\_Paths \leftarrow [p(s, s)]$ 
6: while  $Viable\_Paths$  is not empty do
7:    $cp(s, u) \leftarrow$  the path in  $Viable\_Paths$  with least projected cost
8:   Remove  $cp(s, u)$  from  $Viable\_Paths$ 
9:    $u \leftarrow$  the last node in  $cp(s, u)$ 
10:  if  $u \equiv t$  then
11:    return  $cp(s, t)$ ,  $C(cp)$ ,  $w_{exp}(cp)$ , and  $w_{len}(cp)$ 
12:  end if
13:   $Successors \leftarrow \{\text{successor nodes of } u\}$ 
14:  while  $Successors$  is not empty do
15:     $nbr \leftarrow$  a node in  $Successors$ 
16:    Remove  $nbr$  from  $Successors$ 
17:    if  $nbr \in cp(s, u)$  then
18:      Goto 14
19:    end if
20:     $np(s, nbr) \leftarrow cp(s, u) * (u, nbr)$ 
21:     $C(np(s, nbr)) \leftarrow C(cp(s, u)) + C(u, nbr)$ 
22:     $w_{exp}(np(s, nbr)) \leftarrow w_{exp}(cp(s, u)) + w_{exp}(u, nbr)$ 
23:     $w_{len}(np(s, nbr)) \leftarrow w_{len}(cp(s, u)) + w_{exp}(u, nbr)$ 
24:    if  $w_{len}(np(s, nbr)) + h_d(nbr, t) > D_{max}$  then
25:      Goto 14
26:    end if
27:    Insert  $np(s, nbr)$  into  $Viable\_Paths$ 
28:  end while
29: end while
30: return No Path Found
```

minimum projected cost was used. The projected cost for a path is the sum of the actual path cost so far and the cost of the Dijkstra least-cost path from the termination of the path. In general, Dijkstra's algorithm was used for computing cost-to-go and distance-to-go heuristics. Since it was likely not necessary to compute heuristics for all possible nodes in the graph, and computation of each heuristic was sufficiently computationally cheap, heuristics were computed on-line and as needed. A heap was used to store viable paths to search which prioritized the path with least projected cost. The cost function was defined as

$$C(p) = (1 - \lambda)exp(p) + \lambda len(p)$$

Here $exp(p)$ and $len(p)$ are respectively the total exposure and length of the path or edge. The program was implemented in

Python [22] utilizing the OSMnx package [23] for extracting street data from Open Streets Maps [24] and constructing the walking network. The Scikit-Learn package [25] was used for interpolating and predicting spatial pollution concentration functions based on available data. Matplotlib [26] and the matplotlib basemap toolkit were used for plotting. The implementation was also built upon NumPy [27] [28], Pandas [29], and NetworkX [30].

D. Pollution Exposure Model and Walking Network

Gaussian process regression was used via Scikit-Learn for producing plausible pollution maps based on available and constructed data. The use of Gaussian processes and kriging for interpolating and predicting air quality is well documented and reasonably effective [7] [31]. Several data sets were used to fit different regression models. These included data from sparse, stationary AirNow sensors [32], sparsely reconstructed data from a land use regression (LUR) model for Portland [7], as well as purely contrived data with no relation to real data. The accuracy of the testing model pollution surfaces was not verified, is likely poor, and is beyond the scope of this paper. Once a model was fitted, it was sampled in a regular grid to produce a discrete pollution surface. The network obtained via OSMnx was comprised of nodes representing intersections as well as numerous intermediary nodes along the edges, or streets, between intersections. Nodes were assigned the pollution concentration value corresponding to their location on the discrete pollution surface. Because edges were generally short, fairly straight segments of road between nodes, edges were assigned a pollution concentration value that was the arithmetic mean of the values at the two end nodes. Exposure was then defined as the product of the pollution concentration and the length of the segment in meters, as a proxy for the time a pedestrian would spend along the segment.

IV. RESULTS

A. Existence of Less Polluted Paths

Given a contrived pollution surface with a single, centered pollution spike, the program found paths which avoided the worst of the pollution, rather than routing directly through the pollution hot spot, as it would if finding the least distance path. The degree to which selected routes may differ from the shortest route depended on the extremity of the spatial differences in pollution. There existed at least some pollution surfaces and routes for which the potential exposure reduction from the least distance to least exposure path was large (-84.95%), and was larger than the increase in distance (+50%), as in 1. In fact, for many of the pollution surfaces that were tested, including those based on AirNow and LUR data, there were some shortest routes for which alternative, less polluted routes could be identified, though the reductions were often small, as seen in 2.

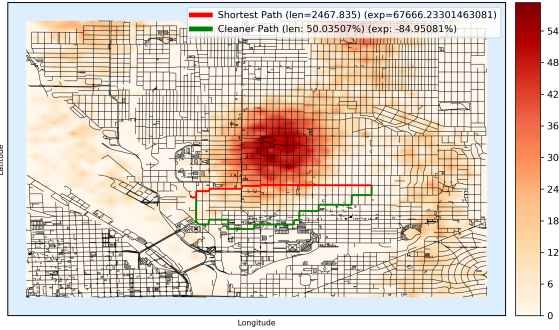


Fig. 1. An example least distance path, shown in red, which traverses through a pollution spike, and an alternative path with less journey-time pollution exposure, shown in green, given a contrived pollution surface.

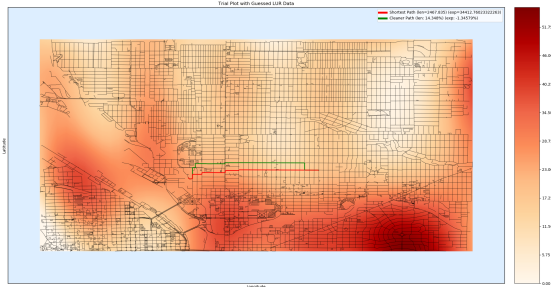


Fig. 2. A least distance path (shown in red as the lower path) and an less polluted path (shown in green as the upper path) given sparsely reconstructed LUR data. The alternative path is 14.348% longer with a total exposure reduction of 1.34579%.

B. Effects of Varying Cost-Function Parameters

Varying the cost parameter λ controlled the degree to which distance is included in the cost function, which is not equivalent to setting a maximum distance threshold. While distance was already to some degree included in the measure of exposure, this was useful for controlling the degree to which the least cost path differed from the least distance path. With $\lambda = 1$, the least cost path is the least distance path. With $\lambda = 0$, the least cost path is the least exposed path. For some pollution surfaces and routes, as λ is increased to 1, relatively large reductions in pollution could be achieved with relatively small increases in distance. A reduction in exposure almost as great as the maximum reduction could be obtained with an increase in distance much smaller than the maximum increase for the routes represented in 3.

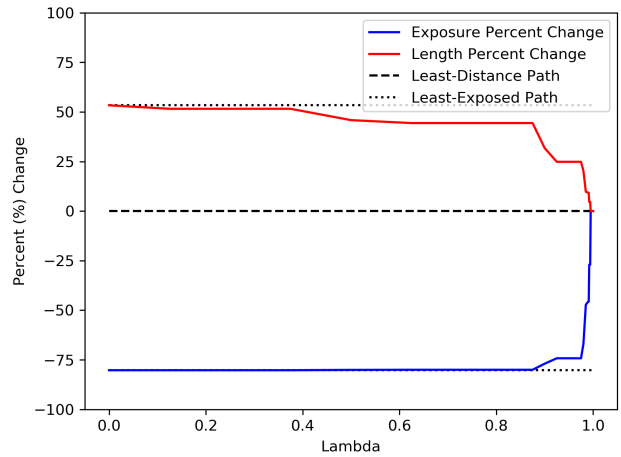


Fig. 3. λ versus the percent changes in distance and exposure for the least cost path given a pollution surface and fixed source and target nodes. The functions are discrete because the changes in the path are discrete.

V. DISCUSSION AND CONCLUSION

The shortest paths approach shows potential for reducing personal exposure in Portland, especially in areas where there may be large differences in the concentrations of pollutants, such as near major roadways. For at least some pollution surfaces and routes, an alternative route was found with significant reductions in pollution exposure. For some routes, a relatively large reduction in pollution could be achieved with relatively small increases in distance. Sometimes, a route could be found which reduced exposure almost as much as the least exposed route but which had a much smaller increase in distance; this suggests that sometimes including a maximum distance threshold could be useful, and result in a much shorter route which has almost equal benefit compared with the least polluted path.

A. Future Work

The application of shortest paths methods with resource constraints for reducing personal pollution exposure warrants more research in Portland. Future work could focus on producing a complete and useful framework. This would involve producing and validating an accurate pollution surface for Portland based on real data, as well as producing a user interface and potentially a web application so that the methods are available for use and their efficacy can be assessed. Given such a structure, future research could then better determine how well these methods perform in aggregate over various conditions and neighborhoods. Such work has the potential to reduce pollution-related inequality, as well as to help mitigate the pollution-related health, economic, and social costs increasingly faced by urban centers in the PNW.

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