2-3-2020

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(Dated: February 4, 2020)

Abstract

We present predictions of the distribution of groomed heavy jet mass in electron-positron collisions at the next-to-next-to-leading order accuracy matched with the resummation of large logarithms to next-to-next-to-next-to-leading logarithmic accuracy. Resummation at this accuracy is possible through extraction of necessary two-loop constants and three-loop anomalous dimensions from fixed-order codes.

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High-energy electron-positron collisions are considered as ideal tools for precision studies of particle interactions. The initial state of the hard scattering event is colorless and known precisely, which eliminates significant sources of uncertainties that are ubiquitous at hadron colliders such as the LHC. For instance, the study of hadronic final states at the Large Electron-Positron collider (LEP) was used extensively to study the dynamics of strong interactions [1–11] and especially to determine the strong coupling α_s. Yet, the current state of the art does not support these expectations. Hence, it is somewhat disappointing that presently the second largest spread and uncertainty of determination of α_s among seven sub-fields is found in the group of results based on jets and event shapes of hadronic final states in electron-positron annihilation [12]. This failure of fulfilling expectations calls for an investigation of the possible sources.

The comparison of event shape distributions obtained from data collected by the LEP experiments and from theoretical predictions obtained in QCD perturbation theory reveal the possible causes of such a failure [13, 14]: (i) the QCD radiative corrections are large, (ii) the hadronization corrections are not well understood from first principles, (iii) the two-types of corrections are strongly anti-correlated for analytic models of hadronization. As a result the systematic theoretical uncertainties are large. In order to decrease these corrections, one has to select the observables used for α_s-extraction carefully. For instance, jet rates are expected to be less sensitive to hadronization corrections than event shapes [15], which is supported by a recent Monte Carlo evaluation, resulting in a competitive value for α_s [16]. The latter study is based on the highest perturbative order available for two-jet rates: next-to-next-to-next-to-leading order (N^{3}LO) matched with the resummation of the first three largest logarithms at all orders (N^{2}LL) in perturbation theory.

For precision extraction of the strong coupling the logarithmic accuracy should extend to next-to-next-to-next-to-leading logarithmic order (N^{3}LL) that allows for simple additive matching to fixed-order at N^{2}LO. Such matched predictions are available for thrust [17] and C-parameter [18], and were used for the extraction of α_s from LEP data [19, 20]. However, even so high perturbative accuracy does not guarantee small uncertainty for the determination of α_s due to lack of good control over the hadronization. One way out is to reduce the latter effect. The analysis techniques broadly referred to as jet grooming have been introduced to mitigate contamination radiation in jets from outside of the jet. Jet groomers identify such emissions in the jet and remove them from consideration. The

modified mass-drop tagger (mMDT) [21, 22] and soft drop [23] algorithms are the best understood groomers, due to their unique feature of elimination of non-global logarithms (NGLs) [24] that are the leading correlations between in-jet and out-of-jet scales. Soft drop was indeed found to reduce the hadronization corrections for event shapes in electron-positron annihilation [25].

In this Letter, we present theoretical predictions for the mMDT groomed jet mass in e^+e^- collisions at N^{2}LO matched with N^{3}LL accuracy in perturbation theory. Resummation at this accuracy is made possible by the factorization theorem for jet grooming from Ref. [26] and recent extraction of necessary constants and anomalous dimensions at two- and three-loop order [27–29]. A demonstration of reduction of scale uncertainties and good convergence of the perturbation series will be presented here, but we leave a detailed study of scale variations and inclusion of non-perturbative corrections to groomed jets established in Ref. [30] for future work.

The modified mass-drop tagger groomer (mMDT) [21], or soft drop with angular exponent $\beta = 0$ [23], proceeds as follows:

1. Divide the final state of an $e^+e^- \to$ hadrons event into two hemispheres in any infrared and collinear safe way.

2. Define a clustering metric d_{ij} between particles i and j in the same hemisphere. The metric appropriate for e^+e^- collisions is

$$d_{ij} = 1 - \cos\theta_{ij} \,, \tag{1}$$

with θ_{ij} being the angle between the trajectory of the particles.

3. In each hemisphere, apply the Cambridge/Aachen jet algorithm [31, 32] to produce an angular-ordered pairwise clustering history of particles.

4. Starting with one of the hemispheres (say left) and at widest angle, step through the Cambridge/Aachen particle branching tree. At each branching in the tree, test if

$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\text{cut}} \tag{2}$$

is satisfied, where i and j are the daughter particles at that branching and z_{cut} is some fixed numerical value where $0 \leq z_{\text{cut}} < 0.5$. If the condition (2) is true, then stop and return all particles that remain in the left hemisphere. If it is false, remove the

lower energy branch, and continue to the next branching at smaller angle. Repeat the procedure for the other hemisphere.

5. Once the groomer has terminated, any observable can be measured on the particles that remain in the two hemispheres.

In Ref. [26] a factorization theorem was derived for the cross section differential in the groomed hemisphere masses

$$\tau_i = \frac{m_i^2}{E_i^2}\,, \quad i = \text{L or R} \tag{3}$$

for mass m_i and energy E_i of hemisphere i. For $\tau_i \ll z_{\text{cut}} \ll 1$, the cross section factorizes at all orders in perturbation theory as follows:

$$\frac{1}{\sigma_0} \frac{\text{d}^2\sigma}{\text{d}\tau_\text{L}\, \text{d}\tau_\text{R}} = H(Q^2) S(z_{\text{cut}}) \left[J(\tau_\text{L}) \otimes S_c(\tau_\text{L}, z_{\text{cut}}) \right] \tag{4}$$

$$\times \left[J(\tau_\text{R}) \otimes S_c(\tau_\text{R}, z_{\text{cut}}) \right],$$

where σ_0 is the leading-order cross section for $e^+e^- \to q\bar{q}$, $H(Q^2)$ is the hard function for quark–antiquark production in e^+e^- collisions, $S(z_{\text{cut}})$ is the global soft function for mMDT grooming, $J(\tau_i)$ is the quark jet function for hemisphere mass τ_i, and $S_c(\tau_i, z_{\text{cut}})$ is the collinear-soft function for hemisphere mass τ_i with mMDT grooming. The symbol $\otimes$ denotes convolution over the hemisphere mass τ_i. In the functions we suppressed the dependence on the renormalization scale μ.

Transforming into Laplace space, the cross section assumes a genuine factorized form,

$$\frac{\sigma(\nu_\text{L}, \nu_\text{R})}{\sigma_0} \tag{5}$$

$$= H(Q^2) S(z_{\text{cut}}) \tilde{J}(\nu_L) \tilde{S}_c(\nu_\text{L}, z_{\text{cut}}) \tilde{J}(\nu_R) \tilde{S}_c(\nu_\text{R}, z_{\text{cut}})\,,$$

where ν_L (ν_R) is the Laplace conjugate of τ_L (τ_R). In this product form, each function in the factorization theorem satisfies a simple renormalization group equation (RGE),

$$\mu \frac{\partial \tilde{F}}{\partial \mu} = \left(d_F \Gamma_{\text{cusp}} \log \frac{\mu^2}{\mu_F^2} + \gamma_F \right) \tilde{F}\,, \quad \tilde{F} = H\,, S\,, \tilde{J}\,, \tilde{S}_c \tag{6}$$

where d_F is a constant, μ_F is the canonical scale, and γ_F is the non-cusp anomalous dimension, all depending on the function $\tilde{F}$. Γ_{cusp} is the cusp anomalous dimension for back-to-back light-like Wilson lines in the fundamental representation of color SU(3). Large logarithms of hemisphere masses can be resummed to all orders in α_s using this renormalization group

order	Γ_{cusp}	γ_F	β	c_F	matching
$n = 0$	α_{s}	-	α_{s}	-	-
$n > 0$	α_{s}^{n+1}	α_{s}^n	α_{s}^{n+1}	α_{s}^{n-1}	α_{s}^n

TABLE I. α_{s}-order of ingredients needed for resummation to the logarithmic accuracy given by logarithmic order N^{n}LL. Γ_{cusp} is the cusp anomalous dimension, γ_F is the non-cusp anomalous dimension for function $\tilde{F}$, β is the QCD β-function, and c_F are the low-scale constants for function $\tilde{F}$. The final column shows the relative order to which the resummed cross section can be additively matched to fixed-order.

equation, whose exact solution is presented explicitly including $\mathcal{O}(\alpha_{\text{s}}^3)$ terms in Ref. [19]. The order to which logarithms can be resummed using the RGE (6) depends on the accuracy to which its components are calculated. For the canonical definition of logarithmic accuracy [33], Tab. I shows the order in α_{s} to which the components of the RGE are needed. The two-loop soft function constants were calculated by the SOFTSERVE collaboration [27, 28]. In Ref. [29] we computed the last missing pieces needed for N^{3}LL resummation of the distribution of jet masses with mMDT, namely the two-loop constants $c_{S_c}^{\text{mMDT}}$ of the collinear-soft function and the three-loop anomalous dimension of the global soft function γ_S^{mMDT} (in Laplace conjugate space),

$$c_{S_c}^{\text{mMDT}} = \left(\frac{\alpha_{\text{s}}}{4\pi}\right)^2 \left[C_F^2 \left(22 \pm 4\right) + C_F C_A \left(41 \pm 1\right) \right. \tag{7}$$
$$\left. + C_F T_R n_f \left(14.4 \pm 0.1\right)\right] ,$$

with $C_F = 4/3$, $C_A = 3$, and $T_R = 1/2$ in QCD, n_f is the number of active quark flavors, and

$$\gamma_S^{\text{mMDT}} = \left(\frac{\alpha_{\text{s}}}{4\pi}\right)^3 \left[-11600 \pm 2000\right] \qquad (n_f = 5) . \tag{8}$$

These results enable resummation to N^{3}LL accuracy for jet substructure observables that we present here for the first time.

We present predictions in perturbation theory for the single-differential cross section of the groomed heavy hemisphere mass $\frac{\rho}{\sigma_0} \frac{\text{d}\sigma_g}{\text{d}\rho}$, defined as

$$\frac{\text{d}\sigma_{\text{g}}}{\text{d}\rho} = \int \text{d}\tau_{\text{L}} \, \text{d}\tau_{\text{R}} \frac{\text{d}^2\sigma}{\text{d}\tau_{\text{L}} \, \text{d}\tau_{\text{R}}} \left[\Theta(\tau_{\text{L}} - \tau_{\text{R}}) \, \delta(\rho - \tau_{\text{L}}) \right. \tag{9}$$
$$\left. + \Theta(\tau_{\text{R}} - \tau_{\text{L}}) \, \delta(\rho - \tau_{\text{R}})\right] ,$$

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where the subscript g on the cross section indicates that it is groomed. This definition of the heavy hemisphere mass differs from the standard definition of the ungroomed case when the heavy hemisphere mass is defined as: $\rho = \frac{\max(m_{\mathrm{L}}^2, m_{\mathrm{R}}^2)}{Q^2}$, with Q being the center-of-mass energy. When hemispheres are groomed, the grooming eliminates their dominant correlations, and so it is more natural to define the groomed mass with respect to the hemisphere energy, and not the center-of-mass energy.

The CoLoRFulNNLO subtraction method was developed to compute QCD jet cross sections at the N²LO accuracy. Currently it is completed for processes without colored particles in the initial states, and it is implemented in the MCCSM code (Monte Carlo for the CoLoRFulNNLO Subtraction Method) [34–38]. This program can be used to compute the differential cross section of the mMDT groomed heavy hemisphere mass at fixed order in perturbation theory. MCCSM calculates directly the ρ-dependent coefficients A, B, and C (times their respective coupling factors) in the differential distribution

$$\rho \frac{\mathrm{d}\sigma_{\mathrm{g,NNLO}}}{\mathrm{d}\rho} = \frac{\alpha_s}{2\pi} A_{\mathrm{g}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[B_{\mathrm{g}} + A_{\mathrm{g}}\beta_0 \ln \xi\right]$$
$$+ \left(\frac{\alpha_s}{2\pi}\right)^3 \left[C_{\mathrm{g}} + 2B_{\mathrm{g}}\beta_0 \ln \xi + A_{\mathrm{g}}\left(\frac{\beta_1}{2}\ln \xi + \beta_0^2 \ln^2 \xi\right)\right], \tag{10}$$

where $\alpha_{\mathrm{s}} = \alpha_{\mathrm{s}}(\mu)$ is the strong coupling evaluated at the renormalization scale $\mu = \xi Q$, β_0 and β_1 are the first two coefficients in the perturbative expansion of the QCD β-function and Q is the center-of-mass collision energy. We present the predictions of MCCSM for the normalized cross section $\frac{\rho}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{g}}}{\mathrm{d}\rho}$ at the first three orders in perturbation theory (LO, NLO and N²LO) in the top panel of Fig. 1. The lower panels exhibit the K-factors defined as

$$K_{\mathrm{FO/LO}}(\xi) = \frac{(\mathrm{d}\sigma_{\mathrm{g,FO}}(\mu = \xi Q)/\mathrm{d}\rho)}{(\mathrm{d}\sigma_{\mathrm{g,LO}}(\mu = Q)/\mathrm{d}\rho)}, \tag{11}$$

and the ratio $K_{\mathrm{NNLO/NLO}}$. We see that the $\mathcal{O}(\alpha_{\mathrm{s}}^3)$ corrections stabilize the dependence on the renormalization scale for large values of ρ ($\rho > 0.1$) as expected, while the predictions are clearly not reliable for $\rho \ll 0.1$. To stabilize the latter we need to resum the large logarithmic contributions.

All functions that appear in the factorization formula Eq. 5 can also be found explicitly in Ref. [26], including their matrix-element definitions. Due to the factorized form of the cross section, each function in the factorization theorem has its own natural scale at which it is defined, and they can be varied independently to provide some estimate of residual scale uncertainties. We leave a detailed scale variation study to future work, and here we just vary
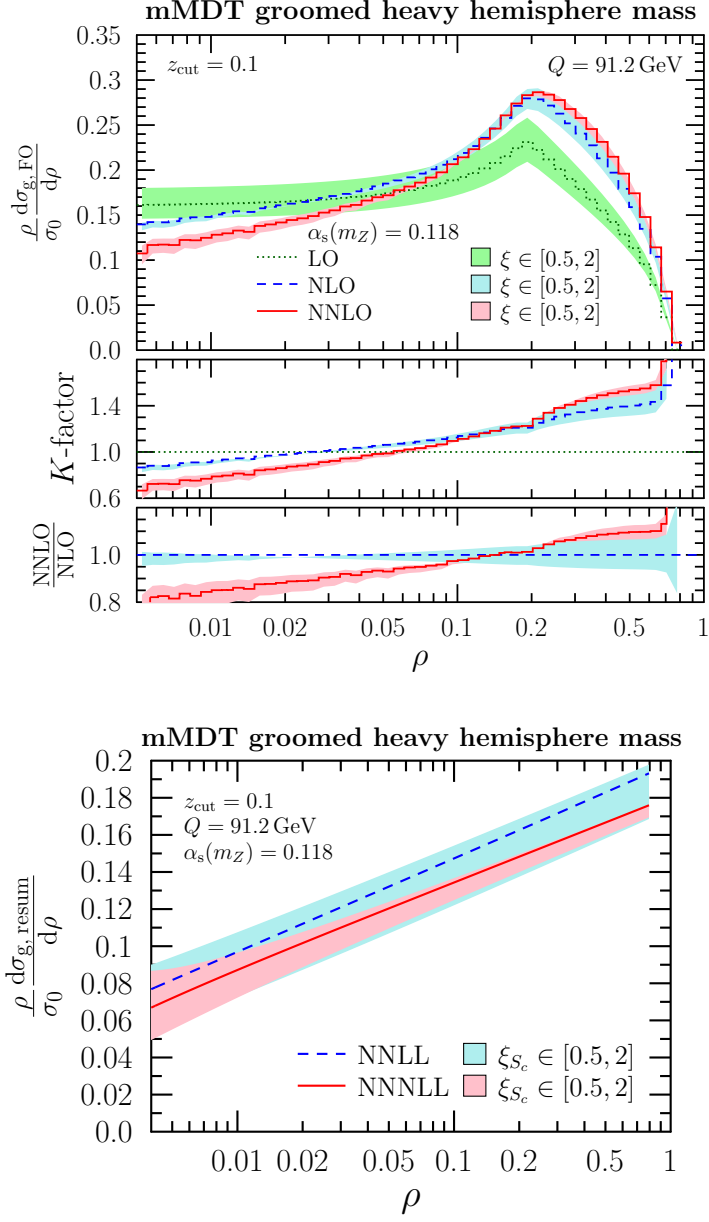
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FIG. 1. Predictions for the groomed heavy jet mass in perturbation theory with $z_{\text{cut}} = 0.1$. Top: at LO, NLO and NNLO accuracies, and their ratios. The bands represent the uncertainties due to the variation of the renormalization scale $\mu = \xi Q$ in the range $\xi \in [1/2, 2]$. Bottom: N^2LL and N^3LL accurate distributions. The bands represent the uncertainties due to the variation of the collinear-soft scale $\mu_{S_c} = \xi_{S_c} 2 e^{-\gamma_E} \sqrt{z_{\text{cut}}\rho}\, Q$ in the range $\xi_{S_c} \in [1/2, 2]$.

the scale of the collinear-soft function $\mu_{S_c} = \xi_{S_c} 2 e^{-\gamma_E} \sqrt{z_{\text{cut}}\rho}\, Q$ in the range $\xi_{S_c} \in [1/2, 2]$. The collinear-soft function is the lowest scale function in the factorization theorem, so varia-

tions of its scale will at least be representative of a more complete analysis. Additionally, we just use the central values of the two-loop constant and three-loop anomalous dimension of Eqs. 7 and 8, with no inclusion of their uncertainty. We present the resummed predictions at N^2LL and N^3LL accuracies for the normalized cross section $\frac{\rho}{\sigma_0}\frac{\mathrm{d}\sigma_\mathrm{g}}{\mathrm{d}\rho}$ in the bottom panel of Fig. 1. We see that these predictions are stable against the variation of the collinear-soft scale, but the range of validity is confined to $\rho \ll z_\mathrm{cut} \ll 1$.

The regions of validity of the predictions at N^2LO and at N^3LL are complementary, the former gives a good description for large, while the latter for small values of ρ. In order to extend the precise description over the full phase space, the fixed-order and resummed predictions have to be matched. The additive matching requires the elimination of the logarithmic terms that are present in both predictions. The coefficients in the expansion of the resummed prediction in α_s,

$$\frac{\mathrm{d}\sigma_{\mathrm{g,LP}}}{\mathrm{d}\rho} = \delta(\rho)D_{\delta,\mathrm{g}} + \frac{\alpha_\mathrm{s}}{2\pi}(D_{A,\mathrm{g}}(\rho))_+ + \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 (D_{B,\mathrm{g}}(\rho))_+$$
$$+ \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^3 (D_{C,\mathrm{g}}(\rho))_+ \,, \tag{12}$$

can be found in Ref. [29] including the $\mathcal{O}(\alpha_\mathrm{s}^3)$ coefficient. For $\rho > 0$ the δ-functions can be ignored and +-distributions reduce to simple functions of ρ. We compare the $D_{C,\mathrm{g}}(\rho)$ function to the $C_\mathrm{g}(\rho)$ coefficient in the fixed-order expansion in the top panel of Fig. 2 where we show the logarithmic expansion with two assumed values of the three-loop non-cusp anomalous dimension $\gamma_S^{(2)}$: 0 and our extracted value with uncertainties from Eq. 8. As the value of z_cut is decreased, improved agreement between the MCCSM results and the singular distribution is observed at small ρ, down to about $\rho \sim 10^{-4}$ where numerical instabilities in MCCSM become significant.

Subtracting this singular distribution from the sum of the N^2LO and N^3LL, we obtain a prediction in perturbation theory with highest available accuracy:

$$\frac{\rho}{\sigma_0}\frac{\mathrm{d}\sigma_\mathrm{g}}{\mathrm{d}\rho} = \frac{\rho}{\sigma_0}\left(\frac{\mathrm{d}\sigma_{\mathrm{g,N^3LL}}}{\mathrm{d}\rho} + \frac{\mathrm{d}\sigma_{\mathrm{g,N^2LO}}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\mathrm{g,LP}}}{\mathrm{d}\rho}\right) \,, \tag{13}$$

which we present in the bottom panel of Fig. 2. Good convergence of the matched predictions is observed for all values of ρ, with the results at $\text{N}^2\text{LO}+\text{N}^3\text{LL}$ lying within the scale variation bands of the $\text{NLO}+\text{N}^2\text{LL}$ prediction. We have truncated this perturbative prediction at a value of ρ that lies above the region in which non-perturbative physics dominates the distribution.
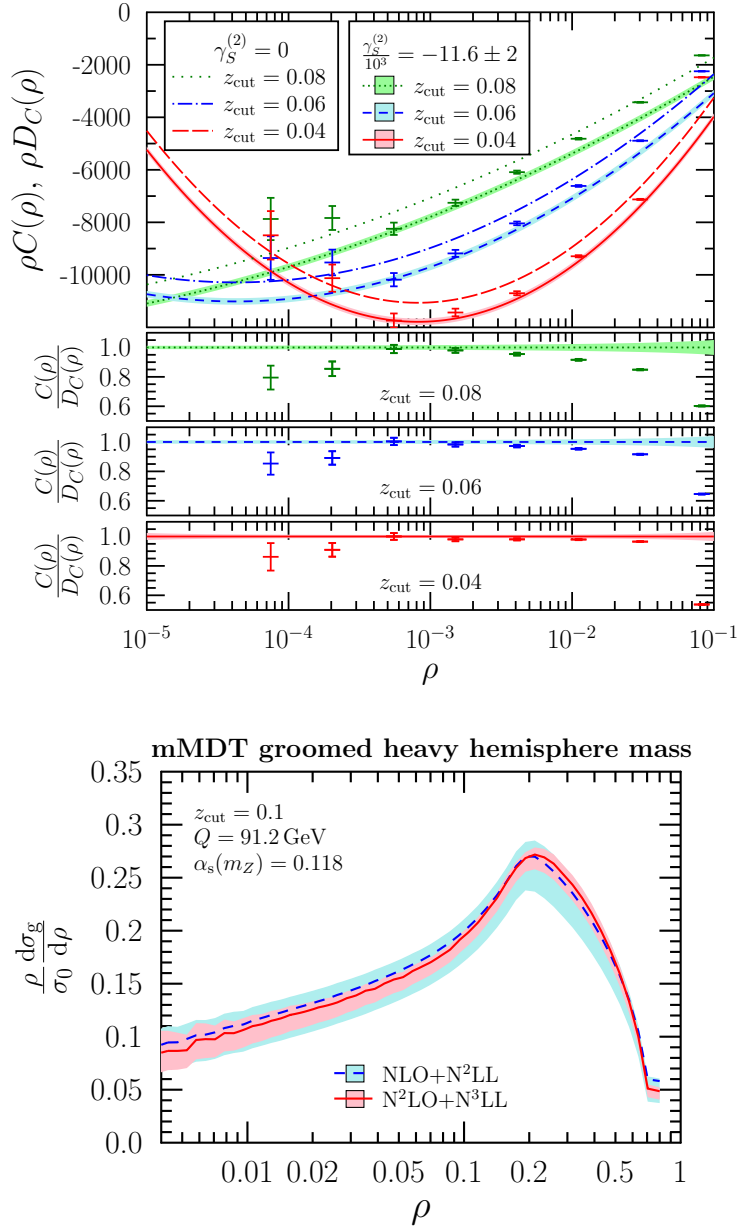
FIG. 2. Predictions for the groomed heavy jet mass in perturbation theory. Top: Comparison of the $\mathcal{O}(\alpha_s^3)$ coefficients at full fixed order and at leading power in ρ. Bottom: Predictions at matched NLO+N^{2}LL and N^{2}LO+N^{3}LL accuracy with $z_{\text{cut}} = 0.1$. The bands represent the uncertainties due to the variation of the renormalization and collinear-soft scales in the range $[1/2, 2]$ times their respective default scales.

We have demonstrated the highest precision perturbative predictions for groomed jets in e^+e^- collisions. These results are sufficiently accurate to enable extraction of α_s, when com-

bined with leading corrections due to non-perturbative physics. While there is no currently-running e^+e^- collider, analyses of archived LEP data have been completed [39], and the results presented here motivate further measurements on these archived data. Due to the elimination of soft radiation with mMDT grooming, the collinear-soft and jet functions in the factorization theorem are identical to that for corresponding measurements at hadron colliders. Thus, we anticipate these results can be used to further improve the theory-data comparisons of groomed jet masses measured at ATLAS and CMS [40–42], and, along with continual advances in fixed-order predictions, enable precision extractions of fundamental constants at the LHC.

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