When Mathematicians and Mathematics Teachers Come Together

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When Mathematicians and Mathematics Teachers Come Together

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In accordance with recommendations from educators and educational researchers [Wilcox, *et al.*, 1991; Schwab, 1976], recent attempts to integrate the communities of professional mathematicians and high school mathematics teachers in the U.S.A. have resulted in several summer institutes which bring these groups together for up to four weeks. Through living, working, and studying together, the two normally disparate groups are expected to develop a dialogue for the mutual benefit of the participants and for their students. The present study of one such institute was conducted with the use of interviews of the participants, examination of some of the teachers' daily journals, and naturalistic observations of classes, lectures, socializing, and dorm life. In addition to documenting the respective goals and activities of senior research geometers, seeking-to-be researchers, and high school mathematics teachers, this study also documents the types of mathematical discussions which characterize each of these groups. The study concludes that the mathematical discussions of mathematicians differ from the mathematical discussions of mathematics teachers with respect to goals, values, social configurations, and style. Furthermore, there are political ramifications in that the mathematicians are accorded status over the teachers.

Background

This study derives from an evaluation of an NSF-sponsored summer institute on the topic of optimization problems in geometry. The institute was unique in that it was the first NSF-funded mathematics institute which brought together several disparate communities within the larger, international mathematics community: distinguished and internationally renowned mathematics researchers in optimization problems in geometry, university mathematicians currently engaged in research on optimization geometry, graduate students, undergraduate students, and high school teachers. The most apparent goal of the conference was to facilitate geometry research. A secondary goal was to give high school mathematics teachers an opportunity to learn about current work in geometry and to afford both the researchers and the teachers an opportunity to learn how the other group lives and works. This report will focus on each of these communities separately, but with cross-references to the others. The main purpose is to examine (a) the goals of each of the respective participant groups, (b) their mathematical habits, preferences, and activities, (c) the form of their interactions with and perceptions of other attendant groups, and (d) the degree of mutual benefit derived from the cross-communication of the groups. The study concludes with a comparison of the culture of academic mathematicians engaged in mathematics research and the culture of high school mathematics teachers.

The data were collected during the final two weeks of the four-week institute. Participants were observed at talks, classes, workshops, meals, and leisure activities. In addition to naturalistic observations, 40 interviews were conducted, each for approximately one hour. Twenty-one of these interviews were recorded, and an additional 19 non-recorded interviews of subjects (either unwilling to be recorded or, more commonly, of subjects who on discovering my declared role spontaneously "collared" me to share their impressions) were documented with notes written immediately subsequent to the discussions. Finally, several high school teachers generously provided their daily journals for further documentation.

It was clear during the data collection process and upon analysis that there existed five groupings of participants at the institute: active researchers, college mathematics educators who wished to become active researchers, graduate students, undergraduate students, and high school mathematics teachers. Because the graduate students and the undergraduate students closely resembled their research mathematician mentors with respect to their goals and habits, their relatively small numbers may be subsumed in the broader category of "seeking-to-be researchers". These three groups then, active researchers, seeking-to-be researchers, and mathematics teachers, are described with respect to their goals for the conference, their activities, their social organization for work and conversation, and their respective interactions and impressions of the other attendant groups.

Active researchers

The largest community at the summer institute was the community of geometry researchers. With perhaps the exception of the few internationally renowned "experts" who were invited to lecture to the research community and who may be "in a class of their own", the researchers appeared separated into "active researchers" and "seeking-to-be (STB) researchers".

The active researchers described themselves as being engaged in on-going research in one or two current topics in geometry. They have published recent articles, and they reported working on one or more articles at the time of the institute. They also reported previous acquaintance with many of their colleagues from prior conferences, and they were knowledgeable about each other's work. Their most frequently stated goals for the institute and the methods they used to attain them are as follows:

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- Share their own work (usually accomplished through giving talks, distributing papers, and conversations).
- Get help with their current research from researchers expert in specific areas (usually accomplished with one-on-one problem sessions).
- Collaborate with researchers with whom they have a prior working relationship but who live in geographically distant regions (also accomplished with one-on-one problem sessions).

Although many of these researchers report having had conversations with the invited "expert" speakers, they appeared mainly to interact with each other at the conference. Furthermore, while most of them may not be internationally renowned, they nevertheless possess specialized expertise for which they are sought after. Some of the areas of expertise are: computer software for solving optimization problems, particular techniques for solving optimization problems such as slicing or calibrations, expertise in a particular field of geometry such as Lorenzian geometry, and applications of areas of mathematics to the solving of geometry problems such as numerical analysis and partial differential equations. Most, if not all, of these researchers presented talks to their peers, and some gave "general" talks intended for the wider audience f researchers and teachers.

Talks as "advertisements"

Almost all of the researchers reported spending their days attending prepared talks by guest speakers and colleagues. Their reactions to the talks were overwhelmingly positive, although they also reported that most of the talks were beyond their immediate understanding. When asked for their estimates of the percentage of the talks they understood, approximately 50% of the mathematicians wanted to qualify their estimates with statements such as, "well, that depends on what you mean by understand". The researchers generally distinguished between "following a talk" and "understanding a talk". Most of them reported that they could follow most (up to 90%) of a talk, while they understood, on average, about 50% (estimates ranged between 10% and 90% depending on their familiarity with the topics, and also on the intended audience). One researcher observed, "Mathematicians don't particularly want to understand all of the talk ... Talks are mainly advertisements for further discussion." Another researcher described a talk as a "table of contents"—topics and ideas about which the listener makes subsequent inquiry. Most researchers stated that audience questions to the speaker signified a good talk, and that some of the best talks are given by speakers who state "risky and uncertain ideas" so that they may provide "food for discussion".

In addition to working together, the researchers also socialized together. They reported little interaction with the groups at the conference. Although most of them had spoken with the teachers and graduate students on occasion at meals and in the dorms, few reported having attended either the graduate student seminars or the teacher sessions. There was almost no interaction with the undergraduate students.

The researchers generally appreciated having the teachers present at the conference. With few exceptions, the researchers described their interactions with the teachers as pleasant and refreshing. While most of them reported that they offered their tutorial services to the teachers, they also admitted that with the exception of one or two teachers, their offers were generally not taken.

Reactions to the teachers attending

According to the researchers, some of the advantages in having the teachers at the conference were:

- *More women*: More women attended than normally attend mathematics conferences (reported by both men and women researchers).
- *Educational issues*: Educational issues arose which have a bearing on university instruction, both because of the greater understanding the researchers achieved regarding their undergraduate students (from teacher descriptions of high school mathematics instruction) and from the researchers' reflections about their own teaching and about teaching in general.
- *Simpler talks*: Several researchers reported that they believe that some of the talks were qualitatively and positively affected by the presence of the teachers. The talks which were recommended for a general audience were thought to be more intelligible to the research community. Although the researchers acknowledged that the same talks may not have been accessible to the teachers, they were certainly more accessible to themselves. As one researcher expressed it, "When mathematicians pretend to give a talk to teachers, they generally give a talk that a mathematician can understand." Another researcher observed that the talks for the general audience were frequently too elementary and obvious to be of mathematical interest. But this same researcher observed that "the part of my brain which I don't need for understanding the talk I divert to paying attention to how the speaker presents the talk." This emphasis on the effectiveness of communication and teaching was attributed to the presence of the teachers, if not to specific conversations with the teachers
- *"Normal" society*: The teachers provided a social outlet for some of the researchers who tired of constant mathematical discussion and who said they appreciated the company of "normal people".
- *"Lowest common denominator"*: There were also some negative responses to the presence of the teachers. Several researchers reported that although they appreciated sharing their mealtimes with the teachers in the first two weeks of the institute, they felt that the novelty wore off and they resented the presence of a teacher at their table when they wished to discuss mathematics. "Discussion is reduced to the lowest common denominator," said one researcher. This same researcher, who insisted on speaking anonymously because his/her ideas were not "politic", expressed resentment that the teachers were using NSF funds which would more appropriately be spent in bringing more mathematics faculty to the conference, "There are brilliant young mathematicians who are teaching summer school who should be here with us. Let the teachers teach summer school. They are far better teachers than we are. Let the researchers do what they do best—research."

• *Helping the teachers:* While some of the researchers appeared to ignore the teachers, there was no open hostility, and, in fact, several of the researchers attended the teachers' sessions, and some of them organized sessions for the teachers. For example, several researchers convened an evening meeting for the teachers where they discussed "What a mathematician does". Even the researcher who expressed the negative comments above also organized discussion sections for the teachers as well as for the graduate students. This researcher deliberately sought different groups to engage in conversation: "One does the best one can."

Seeking-to-be researchers

Another group of mathematics faculty are those who work in colleges and universities where there is an emphasis on teaching rather than research. Many of these faculty reported that the demands of teaching three and four courses a term restrict their research and scholarly activities severely. These faculty also stated that in addition to their teaching, they are expected to produce published research for promotion and tenure. They attended the conference with the following goals:

- to learn about optimization geometry;
- to learn about current research being conducted in the field;
- to find a researchable problem or topic;
- to locate a mentor or a co-author;
- to invigorate their appreciation of mathematics

Their main activities appeared to be attending talks and listening as non-active participants to one-on-one problem solving sessions between active researchers. They occasionally engaged in discussion with the active researchers where the exchange appeared as teacher/student. Although the graduate students closely resembled this group in their goals and activities, the "seeking-to-be" [STB] researchers reported little or no interaction with the graduate students and few of them attended any of the graduate student seminars. None of the STB researchers reported any interaction with the undergraduate students. Most of them interacted with the teachers in a social setting and engaged in discussions of education, but rarely about mathematics, and none of them appeared to have attended any of the teachers' classroom sessions. Like the active researchers, the STB researchers also appreciated the presence of the teachers, mainly for the reason that those talks which were intended for a general audience were more intelligible to them.

Although the STB researchers relied mostly on the talks for their learning experiences at the institute, they admitted that in many of the talks they were "lost after the first five minutes, and sometimes after the first thirty seconds." The STB researchers stated that the chief advantage in attending the talks was that they were exposed to vocabulary, the problems, and some of the techniques and characteristics of current research. All of the researchers believed that they learned a lot of mathematics during the four weeks, and even though they may not have identified a research problem (and none reported having done so), most stated that they were "on the way to finding one."

High school teachers

While only twenty-four high school teachers of mathematics were present at the institute, they appeared to have influenced the conference significantly and perhaps disproportionately to their numbers. All of the participants at the conference noted how distinct a group the teachers constituted. There was seldom any confusion over who was a teacher and who was a researcher. The teachers reported speaking with some of the researchers and graduate students. They did not communicate with the undergraduates. Their interaction with a mathematically-gifted high school student who described his research was specially enjoyable for the teachers.

There was unity among the teachers, so that they were seen eating together, socializing among themselves, and sitting together in talks. Few of the teachers deliberately joined the researchers at "their" tables during meals, although they welcomed the presence of the researchers at the "teachers' tables". The teachers were not expected to understand the talks intended for the mathematics researchers so other "joint talks" were organized and posted on the schedule board in a color ink which signified such talks. In fact, the weekly schedules were divided into hourly blocks which were also divided into a teachers' side and a researchers' side. While everyone was free to attend any of the talks and classes they chose, each event was advertised for an intended audience.

At the end of the second week, several of the teachers voiced their dissatisfaction with the lack of organization of the institute. Their complaints focused on what they perceived to be a lack of structure and a lack of purpose. For most of the teachers, their stated goals for attending the conference were:

- to learn how to teach geometry using computers in the classroom;
- to learn more geometry;
- to learn how to teach specific geometry concepts to students in their schools;
- to identify and/or develop curricular materials for classroom use.

However, all of the teachers reported that the first two weeks of the institute seemed to urge them to attend talks which confused and intimidated them. Furthermore, they felt compelled to endure the talks even when they knew within the first five to ten minutes that they would understand next to nothing of the remaining hour or so of presentation and questions. The initial conflict between the teachers' goals and expectations and these first activities created much dissatisfaction and resentment from the teachers toward the institute's organizers—a conflict which required several attempts at mediation.

The talks overwhelm

During the first three days of the institute, the teachers attended a number of talks which, for most of the teachers, were too advanced. They felt intimidated by the mathematics, and they were confused about what they would accomplish during the four weeks. Later in the first week, their instructor arrived. He too admits that he was unprepared to deal with the teachers' expectations, given the fact that the talks were accorded so much priority (the mornings were reserved for teacher sessions while the remainder of each day and the evening were devoted to attending and discussing talks). In an attempt to help the teachers understand the talks, their teacher valiantly worked to prepare them in the morning for the afternoon talk, and debrief them in the evening after listening to the talk. Unfortunately the task proved impossible.

Many of the teachers felt that their instructor's lessons were either too advanced or too simple. The teachers seemed also to be divided into two groups: those who had completed courses in higher mathematics, and those who had not. Most of the teachers with a strong math background found the talks stimulating, although they admitted being confused by much of them. These mathematically strong teachers also found that their instructor's lectures were too simplistic and too unfocused. They complained that they rarely achieved closure on the problems presented. The teachers with weak mathematics backgrounds found the talks simply confusing and frustrating. Learning higher mathematics was not perceived by this group to be relevant to their teaching and they preferred to be given instructional advice and materials. One of the teachers who said that he had seldom taught geometry stated the following suggestion: "The teachers' institute should be organized like a writers' workshop where we could all go off for a part of the day in groups to develop curriculum, perhaps with advice from a mentor-geometer, and then we would return for a few hours to present our ideas."

A glad shift: from learning to teaching

In the middle of the third week the first instructor departed the institute and a second instructor arrived. For the remainder of the institute a focus on learning advanced geometry shifted to an emphasis on using and developing instructional programs and materials. The teachers spent a portion of each morning describing how they taught geometry in their classrooms. They attended fewer talks given by researchers and they concentrated on learning geometry from one another. Most teachers felt that the few researchers who attended their morning sessions gave outstanding lectures to the teachers. Several of the teachers described two of the presenters as being "master teachers". They were very appreciative of these speakers and of their new instructor.

The shift in emphasis from learning geometry to teaching geometry appeared to help the teachers regain their confidence as mathematics learners and as mathematics teachers. Several of the teachers who were exceedingly critical of the institute in the second week had much more positive reappraisals of the institute by the fourth week. By the end of the program all of the teachers felt that they had indeed learned more geometry than they initially realized, and they suspected that with time even more understanding would become apparent to them. They also reevaluated their previous instructor more positively in retrospect. Finally, while most of the teachers still felt that the institute was indeed without a clear purpose, they felt that they had developed their own sense of purpose which aligned well with their stated goals. Furthermore, they felt that their tribulations united them as a group to an extent that each of them reported liking and respect for one another. They were also very appreciative of the work of the coordinator of their program, himself a teacher.

Summary: two cultures

Mathematicians: the lone learners

The culture of the mathematics research community is a culture of intellectual fixation, private cogitation, individual accomplishment, and readily recognizable success. The highest value is knowledge, which is also its own highest reward. While the community makes available other incentives and rewards (for example, one researcher at the institute offered a one-hundred dollar prize to any individual who could solve a problem he proposed; another offered a champagne dinner), these incentives are more for fun than for serious consideration. The main satisfaction lies in solving the problem. "After you solve an important problem, it's nice to gloat a little bit," said one researcher. Two other researchers, European and American, compared their educational establishments and noted that highly productive American mathematicians are given substantial economic rewards while their European counterparts are not given any salary differential in their own countries. However, both researchers acknowledged that, regardless of economics, they would work with fervor to produce mathematics.

All of the researchers described their learning processes as individual. When they listen to talks, when they prepare talks, when they read, when they solve problems and when they write, they prefer to function individually. Only when the produce a solution or a question do they generally seek discussion. Their discussions are usually one-on-one with another mathematician. Three-way conversations were not deemed useful. One researcher commented, "It's difficult enough conducting a conversation with one other mathematician; it's almost impossible with more." When asked if, during the conference, they had listened in on any two-way conversations between mathematicians, another researcher responded, "What for?" Another researcher indicated that mathematicians seldom publish articles with more than one author, and very seldom with more than two authors: "You may see an Einstein-Bose paper, but you never see an Einstein-Bose-Dirac-Fermi paper—the ideas are too subtle for that many authors to contribute significantly."

One senior researcher described himself as exceptional in that he preferred to work with larger groups of authors. He attributed his interest in cooperative authorship to his age and reputation; "I find that I know more and more people who work in my field, and I like working with them, it's fun. After a while it seems like a mini-marriage where I can anticipate the other person's thoughts. Sometimes I can finish their sentences for them." This researcher acknowledged that he is exceptional in this respect to most mathematicians, especially young mathematicians. "When I was younger, I preferred to work alone. First, there is the problem of learning the mathematics, and this usually happens alone. Then there is the problem of getting hired. Certain "nasty" people may object to a job candidate if all their articles are co-authored. Young faculty are generally advised to prove themselves individually before they get into shared authorship."

The difficulty of the intellectual task of creating new mathematics is apparent to all of the researchers. They do not pretend to understand much of the mathematics which their colleagues describe to them, nor are they troubled by this lack of understanding. "I am confused at least 50% of the time," said one senior researcher. "If the public saw mathematicians talking together, they would be shocked. They'd think we were idiots. We're constantly telling each other how little we understand. We say things like, "My head is a rock today"." Alternatively, mathematical discussions can be very brief and powerful. "Sometimes the best mathematical discussions I've had are one sentence long. Once when I finished a talk, someone in the audience said that something I said reminded him of an article published years before. When I read the article, I realized that it contained just what I needed to continue my work. That one sentence shaped the direction of my life for six months. Really, it happens all the time."

Not surprisingly, the researchers show great tolerance for any lack of organization or sudden changes in their schedules or work environment. All they generally require is that they have a place, any place, to work on their mathematics.

Mathematics teachers: student knowledge first

Contrasting with the individualistic culture of the mathematicians is the "other-directedness" of the high school teachers. While the researchers are concerned first of all with their own problems and puzzles, the teachers see themselves as servants aiding the intellectual development of their students. Moreover, some of the teachers appeared to resent the single-mindedness of the mathematicians. As one teacher stated,

> The researchers are totally self-centered. All they really care about is their own task or problem. I know where they're coming from. I've been there. When I got my doctorate in math education I was just like that. Everything and everyone came second to my work. But now, my main concern is helping my students to learn. They come first now. I don't think the mathematicians understand this. I understand them, but I don't think they understand us [teachers].

The confusion and frustration the teachers experienced in their attempts to understand the talks given at the beginning of the institute only reinforced their belief that lectures are ineffective for instruction. The teachers stated that dialogue and whole-class discussion was necessary to investigate and assess student understanding—simply talking at the students was unanimously regarded as inadequate. In their micro-teaching of instructional units in geometry, the teachers regularly employed a version of "discovery learning". They typically used a math-lab, problem-solving set-up to create an environment of mathematical investigation and discovery. All of the lessons appeared to try to recreate for the students the excitement of the mathematician's quest and delight upon discovering mathematical notions.

"Discovery" is an apt term for the teachers' view of the acquisition of the content of instruction. For them the content is known; the problems have known solutions, correct answers, and established terminology. While they recognized that research mathematicians work to create new mathematics, the teachers did not view this as their vocation. For the teachers, learning new mathematics is far less important than learning new methods of teaching old mathematics.

The learning milieu for teachers is almost never one-on-one. The classroom necessitates social engagement, and while mathematics content is central to the educational agenda, the teachers accept that it is not the only agenda of concern. Furthermore, the teachers echoed the recent NCTM *Standards* [1989] which advocate communication and cooperation among students learning mathematics. During the summer institute the teachers sat themselves at tables in groups of five or six. They frequently worked together and felt that sharing their work in progress was educationally more effective and more ethical than not sharing.

The teachers imagined that researchers also learn mathematics in groups. In spite of the fact that the mathematicians described their work to the teachers as solitary and obsessive problem solving, and disregarding the observation that the mathematicians all around them were working either singly or in pairs, the teachers insisted that mathematicians must solve problems as they do—in groups

Cultures together?

Clearly, the chief concern of the summer institute was the geometry. To the extent that one individual or group was more expert in this field than others, he, she, or it was accorded more status. The presentation of the groups in this paper recreates some of the shared perception of that status. Since the researchers are the experts in this field, they represent the "upper class" of the mathematics community. They give talks to everyone, and they invite questions from any interested parties. Following them are mathematics faculty with doctorates who are not yet regarded as experts, although they aspire to that status. they seldom give talks, since they recognize that they do not yet have a substantial contribution. The graduate students hold the active researchers in awe. They seek their help and mentorship only when they feel confident that they can engage in meaningful dialogue. Mainly they are focused on their rite of passage into the community—the attainment of the Ph D. They talk almost exclusively to each other. Like the graduate students, the undergraduates researchers devote themselves to learning and to impressing their mentors that they "have what it takes" to become mathematicians. In the bottom stratum are the high school teachers. They give talks to no one outside their group but everyone with anything to say about mathematics gives talks to them: researchers, graduate students, undergraduate students, even a high school student with a geometry project, made presentations to the teachers. All of the prestige and intellectual kudos belong to the mathematicians. No casual or institutional recognition of the achievements of the teachers was observed. The teachers gave no public talks; they distributed no publications; nor were they asked to share their expertise in any organized fashion.

Furthermore, each group recapitulated the status hierarchy of knowledge. The researchers distinguished who the "real mathematician" are. One researcher stated that there were only a handful or "real mathematicians" in the world today. The graduate students were aware of whose dissertations were "important" and of the significance of producing one themselves. Even the undergraduates with specialized knowledge wanted status recognition from their less knowledgeable peers: they insisted that they be asked for their help, even though they were supposed to be working cooperatively on their projects. Finally, even the teachers were cognizant of their differences in mathematics training and perceived abilities. Those with more mathematics background complained of being held back by the teachers with weaker background. Finally, one of the institute's organizers indicated at the end of the conference that those teachers who attend such intensive institutes often return to their schools of invigorated with their mathematics experiences that they intimidate their colleagues who spent their summer at home. And when these teachers return to their classrooms, what messages will they convey to their students? Will the knowledge hierarchy filter down into the high school mathematics classes? Middle school? Elementary school?

In the larger mathematics community, the culture of teaching is subordinate to the culture of creating knowledge. Bringing mathematicians and mathematics teachers together only serves to reinforce this status quo. For the two cultures to genuinely interact there will have to be some mutual acknowledgement of the values of each and the differences between them.

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Mathematics, as we know it, appears to us to be one of the necessary forms of our thought. The archaeologist and the historian have shown us civilizations from which mathematics were absent. It is indeed doubtful whether they would ever have become more than a technique, at the service of technologies, if it had not been for the Greeks; and it is possible that, under our very eyes, a type of human society is being evolved in which they will be nothing but that. But for us, whose shoulders sag with the weight of the heritage of Greek thought and who walk in the paths traced out by the heroes of the Renaissance, a civilization without mathematics is unthinkable. Like the parallel postulate, the postulate that mathematics will survive has been stripped of its "evidence"; but, while the former is no longer necessary, we would not be able to get on without the latter.

André Weil
