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A New Measure of Baseball Batters Using DEA

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Data Envelopment Analysis (DEA) is used to create an alternative to traditional batting statistics called the Composite Batter Index (CBI). Advantages of CBI over traditional statistics include the fact that players are judged on the basis of what they accomplish relative to other players and that it automatically accounts for changing conditions of the game that raise or lower batting statistics. Historical results are examined to show how the industry of baseball batting has matured and potential uses of CBI are discussed.

The application of baseball suggests that random variation may have an effect on CBI. We investigated this effect by creating noisy data sets based on actual data sets and then compared the results, which revealed a negative bias in the majority of cases. We then present and test an extension to DEA for mitigating this effect of noise in evaluating a batter's true "skill."

Keywords: DEA, noise, baseball

0. Introduction

This paper starts by describing an interesting application of data envelopment analysis (DEA) for evaluating baseball batters. First, a historical background on baseball statistics is provided. Next, the DEA-based measure called the composite batter index (CBI) is explained. Since CBI is based on DEA, it is an inherently relative measure and has several advantages over traditional baseball statistics.

Next, CBI scores are computed historically for the years 1901 to 1993 in both the American and National Leagues, (AL, NL), resulting in 186 different analyses. The changing distributions of the scores are related to changes in the game. In particular, a case is made that the "industry of baseball batting" can be said to be "maturing." In later years there are more CBI league leaders and a higher percentage of league leaders. The same sort of historical analysis could be used to evaluate the "maturity" of other industries such as automobile manufacturing by examining the number and distribution of efficient manufacturers.

The last issue raised is that of the effect of noise. CBI is designed to measure productivity or "how much a player produced in a year." Another item of interest is measuring the "skill" of a batter. First, simulation results demonstrate that using CBI to estimate skill yields biased results. Since baseball batting is a stochastic process, the number of hits a batter gets is not necessarily a perfect reflection of his skill. This differentiation between estimating "productivity" and "skill" is important. More important than the fact that a batter's productivity score may be subject to this random variation or "noise" is the fact that he is being compared on a relative measure to his peers, all of whom also are subject to this noise. Once the importance of this noise problem is established, a new procedure for reducing the effect of noise in DEA (and therefore CBI) is described. Simulation results verify that this noise correction extension to DEA can significantly reduce bias and mean squared error in estimating a batter's skill via CBI.

1. Background on Baseball Statistics

1.1. Fixed Weight Statistics

Baseball batting statistics can generally be categorized as one of two types, traditional fixed weights and derived fixed weights. Examples of traditional fixed weight statistics include the number of home runs (HR), batting average (BA), and slugging average (SLG). In each case, a simple formula is used to calculate the statistic. These statistics are usually the most popular with fans because they are the easiest to calculate and understand. It is widely accepted though that these simple statistics do not provide an accurate measure of player.

Operations researchers and baseball researchers have shown that more accurate formulas can be determined in a variety of ways such as historical records and simulation. These alternative measures are classified as derived fixed weight measures. During the preceding thirty years, a variety of approaches have been used to analyze the problem of determining weights. One method was to examine how a player's probability distribution of hits affected the number of runs scored by a team in an inning. Examples of these and other approaches can be found (Lindsey, 1963) (Cover and Keilers, 1977) (Thorn and Palmer, 1984).

1.2. Variable Weight Statistics

A third type of baseball batting statistic, variable weight, was first developed using DEA by (Mazur, 1995). Mazur used a zero dimensional input, three output model which consisted of just batting average, home runs, and runs-batted-in (RBI). Mazur's goal was to demonstrate the use of DEA for evaluating sports and not to provide a new method for the baseball community to adopt. He justifies these outputs as being the triple crown criteria and that they are three of the most commonly cited statistics by baseball fans and writers. He goes on to state that other inputs and outputs could be used.

Mazur achieves his goal but the model would not be accepted by baseball statisticians. Runs-batted-in is largely a function of the number of players on base when the player bats and whether or not the pitcher decides to pitch around the hitter. Therefore, it may depend more on a hitter's place in the batting order and the skills of his teammates than on his own skills. Home runs are related in part to the number of at-bats which is omitted from Mazur's model. Batting average does not reflect the value of walks but this could be easily fixed by replacing this measure with on-base-percentage (OBP). Despite questions about the model, Mazur accomplished his goal of demonstrating the use of DEA in sports.

Another interesting application of DEA to baseball was in analyzing salary equity (Howard and Miller, 1993). The authors used a 29-input, single-output model of salary. As inputs, the model included official at-bats, runs-batted-in, runs scored, and fielding average among others on both a single season and career total basis. The number of seasons played was included as an input while walks were not. Although the choice of inputs might be questioned in an evaluation of player skill or productivity, the inputs used are often perceived as positives by the public and team management and a case could therefore be made to include them in the author's model with regards to evaluating salary equity. This model would not be an appropriate basis upon which to evaluate overall player productivity though.

2. Advantages of DEA Based Approaches

A DEA based approach (or other variable weight method) has several advantages over other techniques. One advantage is that it can provide a relative measure of a player's skill compared to players throughout the league. For example, when home runs became easier to hit, traditional fixed weight statistics make it appear as if many players have suddenly become much more skilled.

The classic example of this was the introduction of the so called lively baseball along with rule changes affecting spitballs and scuffed balls in the late 1910s. The home run totals immediately increased dramatically. It would be a mistake to attribute this increase entirely to an increase in player skill. If it became "twice as easy" to hit a home run in 1920, then a batter should hit twice as many home runs or else make up for it in other areas. Other situations where this is useful would be when league expansion dilutes pitching quality, when park sizes change, and when rule changes or reinterpretations occur.

A numerical example illustrates the tendency of fixed weight measures to be deceptively applied to cross era comparisons. It might be tempting to say that Bill Terry was a much better hitter in 1930 when he hit 0.401 than was Carl Yastrzemski who hit 0.301 in 1968, even though they both led their respective leagues. This overlooks the fact that they played in different eras under vastly different circumstances. In fact, the league-wide batting average in 1930 was 0.312 while the AL average in 1968 was only

0.238. This means that Yastrzemski hit almost the same percentage above the league-wide average as Bill Terry did (Gould, 1986). This led to extensions of the traditional fixed weight methods such as relative batting average.

3. The CBI Model

3.1. DEA Formulation

First it is necessary to explain the model. The CBI model consists of one input and five outputs. The input X is plate appearances, which is the number of official at-bats plus the number of walks (also known as a base-on-balls or BB). Sacrifice flies and sacrifice bunts are excluded from consideration at this time, but they could easily be added for a variation of CBI. Hit-by-pitch values were excluded because the batter usually has no intention getting hit by a hard object traveling at 90 miles/hour. The outputs Y are the number of walks, singles, doubles, triples and home runs.

An input oriented CCR primal formulation was used for determining DEA efficiency scores (Charnes, Cooper, and Rhodes, 1978). The standard CCR formulation assumes constant returns to scale. This is a reasonable assumption in this application because doubling the number of inputs (plate appearances) should result in double the number of outputs (hits and walks.) A standard linear programming formulation of DEA is given in Eq. 1 and is explained in more detail in a variety of sources such as (Seiford and Thrall, 1990).

$$
\begin{aligned}
&\min \quad \Theta, \\
&s.t. \quad Y\lambda \geq Y_0, \\
&\qquad \Theta X_0 - X'\lambda \geq 0, \\
&\qquad \Theta \quad free, \quad \lambda \geq 0.
\end{aligned}
\tag{1}
$$

DEA determines a score Θ for each player ranging between 0 and 1.0 to indicate his productivity relative to the rest of the league. For example, if a player receives a score of Θ = 0.8, this means that some other hitter or combination of hitters in the league could have produced at least as many of every type of hit in 20% fewer plate appearances. Players receiving a score of 1.0 are considered "league-leaders" since they could not be surpassed by any combination of other players in fewer opportunities or plate appearances. The vector λ is a set of virtual multipliers. It describes the combination of league leaders equals or exceeds the player studied.

3.2. Dominance Transformation

Initial results showed that some players were classified as league-leaders strictly on the basis of an ability to obtain singles or walks. Although this could be quite acceptable, further examination of these results showed that occasionally some of these player evaluations were unreasonable since certain players were able to hit enough longer hits to overcome a deficit in shorter hits and surpass these shorter hitting specialists.

A good illustration of this was in the National League in 1992. Andy van Slyke and Tony Gwynn were both found to be efficient by DEA:

Player (1992)	PA	1b	2b	3b	HR	BB

Tony Gwynn	566	129	27	3	6	46
Andy van Slyke	672	128	45	12	14	58

Basically, Tony Gwynn had a higher number of singles per plate appearance (129/566) than any other player. On the other hand, Andy van Slyke had more walks, doubles, triples, and home runs per plate appearance than Gwynn. In fact, Gwynn's superiority at hitting singles was smaller than van Slyke's cumulative advantage at hitting doubles, triples, and home runs:

$$\frac{129}{566}-\frac{128}{672}-\left(\frac{45+12+14}{672}-\frac{27+3+6}{566}\right)=-0.0046$$

Since van Slyke could have chosen to stop at first base instead of continuing on to second on those hits, it is reasonable to conclude that longer hits require more skill than singles. From a manager’s point of view, they are also more valuable. Therefore, it is reasonable to conclude that van Slyke was actually more productive than Tony Gwynn. This suggests a dominance relationship among the types of hits.

The dominance transformation of hits was performed as a preprocessing step by simply adding each type of output (walk or hit) to the number of longer hits by that player. Home runs were therefore left unchanged. The output of triples became the sum of triples and home runs; doubles became the sum of doubles, triples, and home runs; singles became the sum of singles, doubles, triples, and home runs. A walk was treated as a type of hit inferior to a single, since it has less opportunity for runner advancement. This meant that the walks output became the number of walks plus singles, doubles, triples, and home runs.

The connection between the principle of dominance and the aggregation of outputs is fully described in (Ali, Cook, and Seiford, 1991). It should be noted that this aggregation technique has been shown to be valid only in the case of strict ordinal relations. This means that hitting a home run is more difficult than a triple, rather than "at least as difficult" as a triple.

The dominance relationship generally reduced the number of league leaders by about fifty percent. All later analyses include this dominance relationship. The resulting model and results are referred to as the Composite Batter Index or CBI.

4. Analysis Results

Separate CBI analyses were conducted on both the American League (AL) and the National League (NL) for the years 1901 to 1993 (Thorn and Palmer, 1993). These 186 analyses provide interesting trend information and an ability to check the reasonableness of the results. Historical results are shown for the AL in figures 1, 2, and 3. Only players with 350 or more at-bats with a single team in a season were included in the analysis so that full time players were not compared against people that played the equivalent of a half season or less. Hitters that played significantly less than a full season such as platoon players, unknown rookies, and pitchers trying to hit could make comparisons with full-time players unrealistic and were therefore excluded.

In 1993 the AL had five CBI league leaders: Frank Thomas, Juan Gonzalez, John Olerud, Carlos Baerga, and Ken Griffey Jr. Meanwhile, the NL had only two league leaders, due to the unusually dominating seasons of Barry Bonds and Andres Galarraga.

4.1. Apparent Trends

Several trends seem to appear in the historical results, although no statistical significance tests have been performed. First, the trend toward higher league-wide CBI scores implies batting skill is becoming more uniformly distributed. Second, there has been a general increase in the number of league leaders in each year in terms of raw numbers but this can be partially explained by the increasing number of players. For example, there were six occurrences when the CBI league leaders in a league included ten or more player, but only one of these occurrences was before 1975. This is an indication that it is becoming more difficult to dominate as thoroughly as in the past, like Rogers Hornsby during the 1920s in the NL. A third trend is the decrease in the proportion of players with low CBI scores and an increase in the proportion of players with high CBI scores.

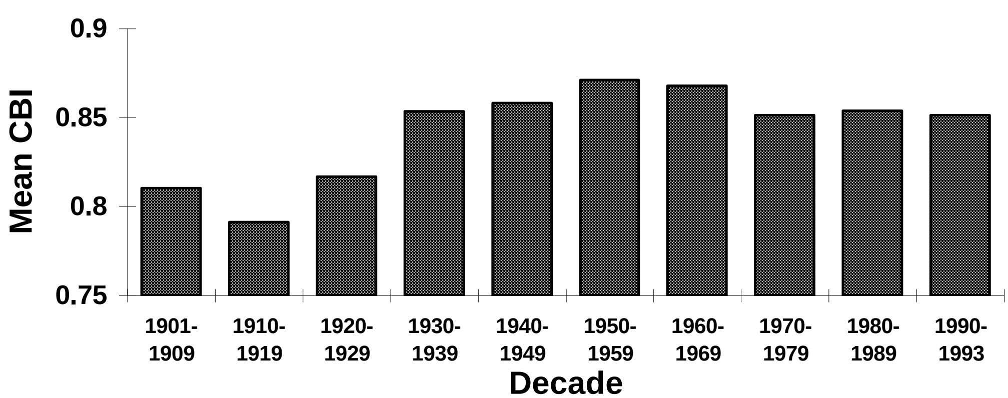

Figure 1 - American League Mean CBI.

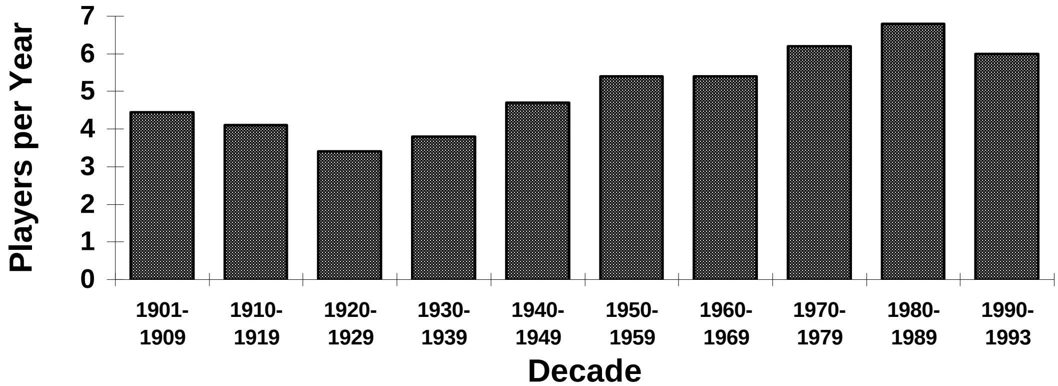

Figure 2 - Average Number of CBI League Leaders (AL) per Year.

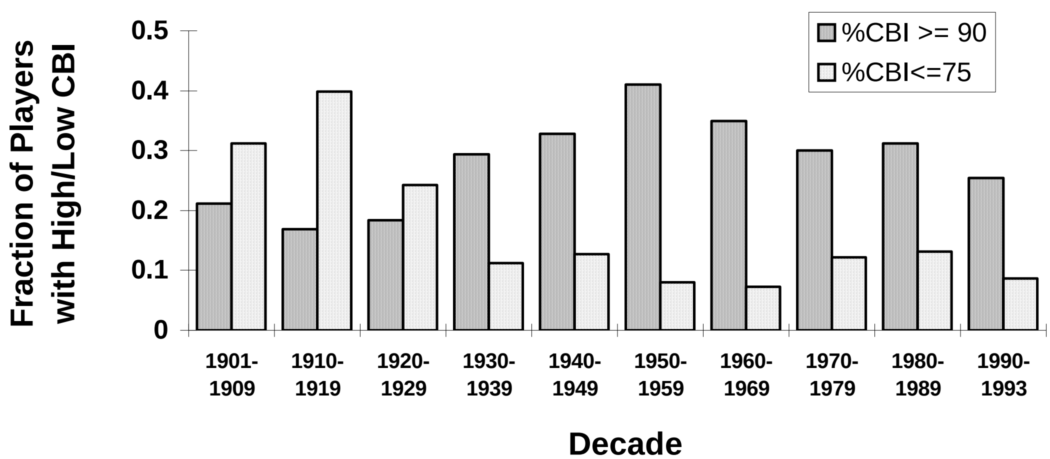

Figure 3 - Changing Distribution of CBI Scores over Time.

These results agree with observations by various researchers using more traditional approaches (Gould, 1986). Gould found that the league standard deviation in batting averages has dramatically and consistently declined since the turn of the century. This helps explain why it has been so long since there has been a 0.400 hitter. As variation decreases, a player must be a higher number of standard deviations above the mean to reach the elusive 0.400 figure since rules have been changed to keep the league-wide mean batting average near 0.270.

Only two players were ever determined to be the only league leader in a given year: Rogers Hornsby (1922 and 1925) and Carl Yastrzemski (1967). Not surprisingly, both players also received triple crowns in those years and would therefore have been the only leaders based on Mazur's measure. More surprisingly, other triple crown recipients, such as Lou Gehrig, Ted Williams, and Mickey Mantle, were never the sole league leader on the basis of CBI.

4.2. Potential Application to Other Industries

It may be possible to generalize such historical trends and apply them to other areas. In the early stage of an industry the efficiency scores of the entire industry might be determined by only a few companies with technologies that work much better than those of their competitors. Over time the technological advantage of the original companies may dissipate as their competitors improve their operations. Graphically, this could be visualized as other companies approaching or helping to form the efficiency frontier. Figures 4 and 5 illustrate these general trends in relation to efficiency frontiers.

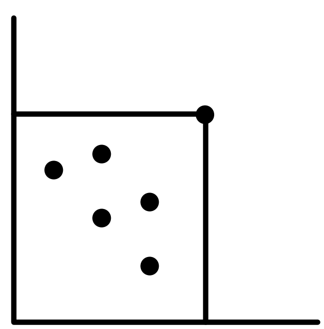

Figure 4 - Early Stage of an Industry.

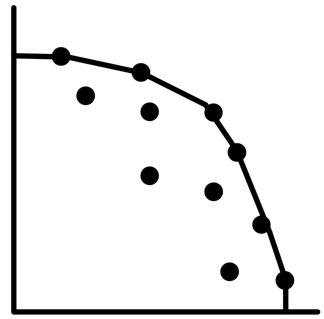

Figure 5 - Mature Stage of an Industry.

The world automobile industry could be a good example of this industrial cycle. In the 1940s and 1950s some of the most efficient car producers in world were in Detroit. By the 1970s Japanese companies, such as Toyota, Honda, and Nissan had, not only caught up with the American companies but also surpassed them in many ways by use of techniques such as just-in-time production. The 1980s and 1990s have seen a frantic race by the American companies to catch up. The result is that Ford and Chrysler may once again lie on the efficiency frontier. Similar examples could be determined for other industries such as ship building.

This type of historical analysis could be extended to indicate the maturity of an industry or the diffusion of technologies. Applications could then be found in the area of business and technology forecasting.

5. Effects of Noise

5.1. Motivation

A player receiving a low CBI score may try to rationalize his score by saying that the frontier players had "lucky" seasons. The difference between a league leader and another player with just a high CBI could be a single swing of the bat.

This can also be seen by thinking of a league with only ten identically skilled players and a simplified CBI model that only used home runs as the only output. Through simple random variation, there would typically be one player with more home runs per plate appearance than any other player. The other nine players would have lower CBI scores despite having identical skill. If we tried to use CBI to estimate the relative skill of the ten players, the ten player league would have a mean CBI < 1, and this value would therefore be a biased estimator of skill. This differentiation between measuring productivity and underlying skill is important: in many applications skill would be of more interest than relative production during a period of time. For example, a baseball general manager may want a measure of skill in determining trade possibilities to improve her team for the next season.

5.2. Simulating Noise

The effect of noise on DEA measures has not been fully addressed and has been typically discussed with regards to strictly theoretical data sets. The application of baseball batting provides a rich area in which to examine the effects of simple random variation. Towards this end, the effects of noise on CBI were investigated. First, we wanted to estimate how much of an effect noise could have on the data. This was done by taking an actual data set, perturbing it with noise, and then comparing the CBI results between the actual original data set and the noisy data set. Since baseball batting can be reasonably approximated as a multinomial process, the noisy data set was generated as a multinomial random variable by using event probabilities from the original data set.

5.3. Results

Analyses were conducted using nine different data sets, with the results illustrated in figure 6. The mean CBI bias induced by noise was negative in the majority of the cases. The typical bias was approximately -3% for the years examined. The average mean squared error was about 0.004.

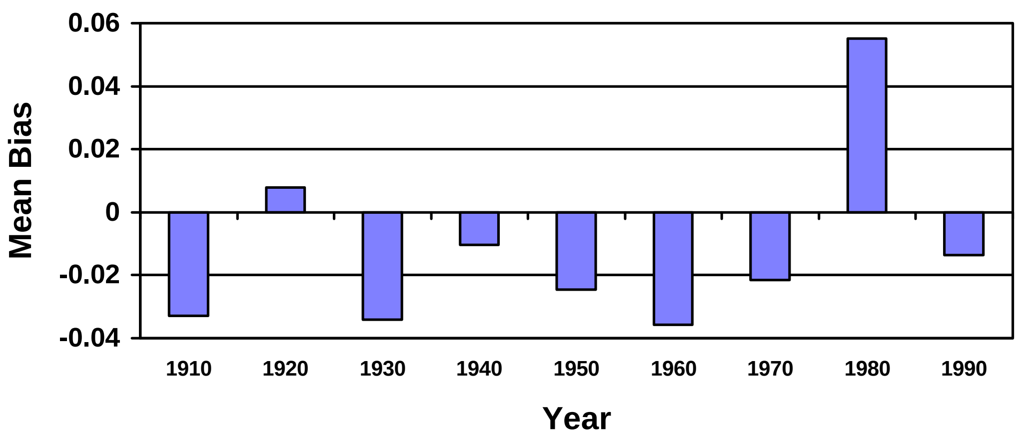

Figure 6 - Effect of Artificial Noise on Actual Data Sets. Mean bias of CBI scores.

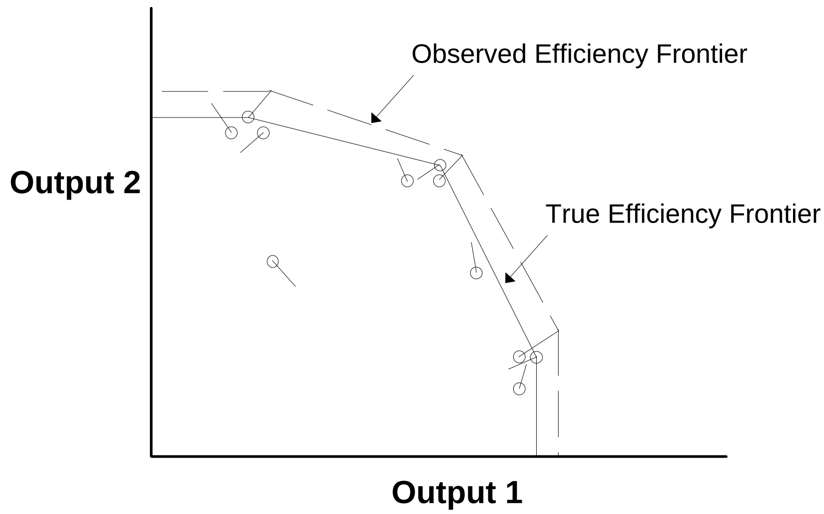

Figure 7 - Example of Noise Biasing the Efficiency Frontier. The circle represents the player's (or firm's) "true" skill. The lines represent the effect of noise and the end of the line is then the observed data point.

Since the perturbed data set is multinomial with large N (at least 350), the change in the data will be approximately symmetrically distributed with a mean of about zero. The tendency of symmetrically distributed noise to lower efficiency scores by raising the frontier is illustrated in figure 7. Clearly, cases can be generated where noise will lower the efficiency frontier too and this occurred in two of the years studied. A positive mean CBI bias may occur more frequently in applications with different noise distributions.

5.4. Noise Correction

A procedure has been developed and tested for reducing the effects of noise as described in figure 8. Basically, the procedure consists of adjusting the observed data set of players (or DMUs) to use in forming the virtual players against which the actual observed player is compared.

1. Read input matrix (vector), X, and output matrix, Y, for all players

$$X = \begin{bmatrix} \text{AB + BB for player 1} & \text{AB + BB for player 2} & \ldots \end{bmatrix}$$

$$Y = \begin{bmatrix} \text{walks for player 1} & \text{walks for player 2} & \ldots \\ \text{singles for player 1} & \text{singles for player 2} & \ldots \\ \text{doubles for player 1} & \text{doubles for player 2} & \ldots \\ \text{triples for player 1} & \text{triples for player 2} & \ldots \\ \text{home runs for player 1} & \text{home runs for player 2} & \ldots \end{bmatrix}$$

2. Set Y_0 equal to column i of Y
3. Derate outputs Y to form Z by use of statistical transformation for all players except player i. For example, for player 1, the transformation would operate as the following:

$$\sigma[player\ 1] = \sqrt{npq} = \begin{bmatrix} \sqrt{\text{X[player 1] (\% walks for player 1) (1 - \% walks for player 1)}} \\ \sqrt{\text{X[player 1] (\% singles for player 1) (1 - \% singles for player 1)}} \\ \sqrt{\text{X[player 1] (\% doubles for player 1) (1 - \% doubles for player 1)}} \\ \sqrt{\text{X[player 1] (\% triples for player 1)(1 - \% triples for player 1)}} \\ \sqrt{\text{X[player 1] (\% home runs for player 1) (1 - \% home runs for player 1)}} \end{bmatrix}$$

$$Z[player\ 1] = Y\ [player\ 1] - c\sigma[player\ 1]$$

4. Aggregate outputs to form the dominance relationship between the hits and walks. For example, in the case of player i, the aggregation would operate in the following manner:

$$Z'\ [player\ 1] = \begin{bmatrix} \text{Z[player 1, walks] + Z[player 1, singles] + Z[player 1, doubles]} \\ \quad \text{+ Z[player 1, triples] + Z[player 1, home runs]} \\ \text{Z[player 1, singles] + Z[player 1, doubles]} \\ \quad \text{+ Z[player 1, triples] + Z[player 1, home runs]} \\ \text{Z[player 1, doubles] + Z[player 1, triples] + Z[player 1, home runs]} \\ \text{Z[player 1, triples] + Z[player 1, home runs]} \\ \text{Z[player 1, home runs]} \end{bmatrix}$$

5. Perform standard DEA with input X and output Z' upon player i.

Figure 8 - Procedure for Computing CBI of Player i with Statistical Extension.

A variety of methods could be used for adjusting the observed data set. We used a simple adjustment method based on derating the observed data set by a fractional multiple (c-value) of each player's standard deviation for each multinomial output. Given the "true" data set, it is possible to find a c-value that will eliminate noise bias and reduce the noise-induced mean squared error. Although these are interesting results, it is an insufficient test since it relies on unobservable information (the "true" player skills.) The optimal c-value is in part a function of the distribution of the data set.

5.5. Evaluation of Noise Correction

To test this procedure of correcting for noise, the nine years of data analyzed earlier to show the effects of noise were examined in an attempt to mitigate the noise influence. Before we could use the statistical extension, we needed to find an appropriate c-value for each year. This required the creation of several extra data sets for each year. The actual data set, designated set A, was assumed to represent the true skill of the players. Data set B was created by adding multinomial noise to data set A, and then it was treated as the observed data set. Data set C was created by adding multinomial noise to data set B and was then used to "tune" the adjustment method by finding a c-value that eliminated the noise bias in C relative to B. The c-values showed considerable variation so 100 different data set C’s were generated and the mean c-value was determined and used in the statistical extension procedure.

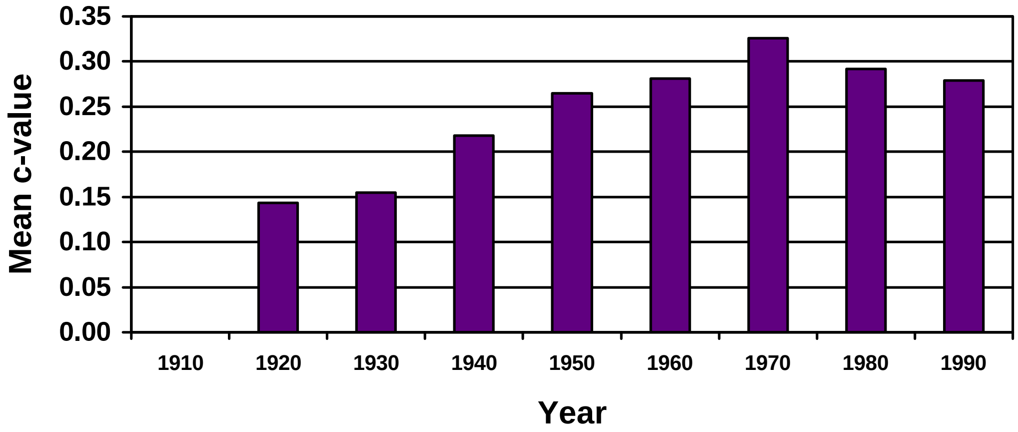

Figure 9 - Mean c-value for Each Year.

The mean c-value (as shown in figure 9) was then used to adjust data set B, since in general the distribution of data set C was similar to B. Note that a negative mean c-value would call for increasing the values in the comparison set. This could be appropriate in certain applications with significant asymmetric noise but since this application had symmetric noise such correction would be inappropriate. Therefore when the mean c-value was negative, a value of zero was used. This occurred once in the nine years examined. The fact the recommended mean c-value increased as the number of players in the league (see table 1) increased highlights the possible effect of noise in large data set DEA problems.

		Before Correction			*After Correction*	
Year	**Players**	**Mean Bias**	**MSE**	**Mean c**	**Mean Bias**	**MSE**
1910	49	-0.0329	0.0056	0.0000	-0.0329	0.0056
1920	56	0.0080	0.0039	0.1437	0.0200	0.0045
1930	49	-0.0342	0.0042	0.1549	-0.0200	0.0025
1940	59	-0.0104	0.0056	0.2176	0.0105	0.0041
1950	58	-0.0245	0.0040	0.2647	0.0012	0.0033
1960	50	-0.0357	0.0045	0.2808	-0.0039	0.0031
1970	76	-0.0216	0.0045	0.3258	0.0078	0.0040
1980	98	0.0551	0.0065	0.2921	0.0850	0.0110
1990	103	-0.0137	0.0032	0.2792	0.0135	0.0031

Table 1 - Results of Statistical Extension Procedure on Actual Data Sets with Artificial Noise.

The mean c-value was then used to derate data set B. As shown in figure 10 and table 1, the statistical correction improved the results in the majority of the years studied on the basis of both mean bias and mean-squared-error. In the one case where a c-value of zero was used, the statistically extended CBI analysis was identical to a regular CBI analysis.

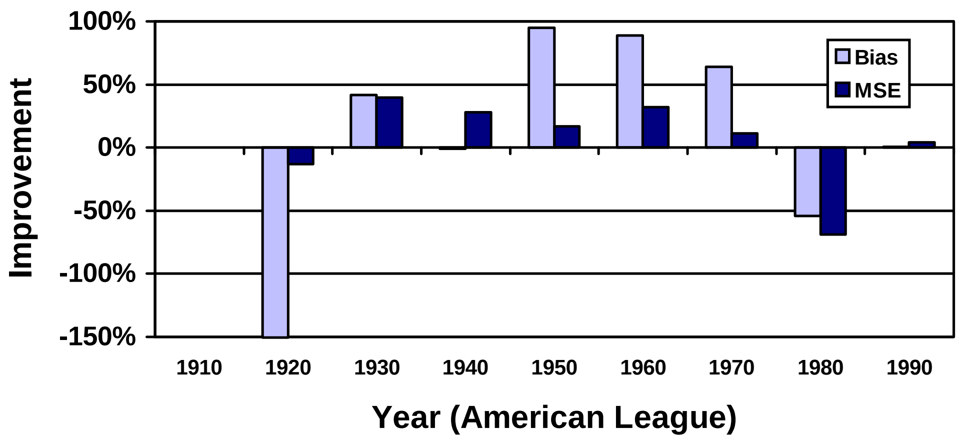

Figure 10 - Improvement Due to Statistical Extension.

It may be possible to improve the CBI noise correction results by applying different statistical transformations (or derating methods) in step 3 of figure 8. The statistical transformation used here was tailored to baseball batting but other applications of DEA may be able to use different methods of derating to reduce noise bias. Although the noise correction procedure may not be possible in all DEA applications, it does illustrate the potential benefits of the statistical correction for noise and the need for further work.

6. Conclusion

DEA was used to develop a new measure of baseball batter performance. This measure, the composite batter index (CBI), is defined as the percentage of plate appearances that would be required for the best virtual player to produce at least as much as the player studied. Since power hitters tend to be compared to other power hitters and contact hitters with other contact hitters, it provides a good measure of how well players fulfill their roles on teams. It automatically adjusts to different conditions such as the dead ball and lively ball eras by being an inherently relative measure. Analyses were performed on actual historic baseball statistics for every year from 1901 to 1993, and trends showing a maturation of the industry were discussed. CBI could be easily extended to incorporate other factors such as home park and position played.

This application of DEA could also be used as an intuitive introduction to DEA modeling. DEA is a powerful tool with many subtleties, but this power and flexibility also bring the potential for modeling errors. For example, this application highlights the importance of carefully examining the initial analysis results. As would be expected, this examination revealed the need for ordinal relations between the outputs in this application.

Next, it was demonstrated that DEA efficiency scores tend to be negatively biased by noise that is approximately symmetrically distributed with a zero mean. This led to the development of a procedure for extending DEA to correct for noise in the data sets. The procedure calls for comparing each player (or DMU) against a derated data set of other DMUs. This derating was performed by a statistical (or stochastic) transformation. Different transformations could be applied in this step depending on the application. The results of the noise mitigation extension to DEA show promise but also indicate that the statistical transformation step needs refinement.

Acknowledgments

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