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CORRECTION TO "STOCHASTIC COMPARISONS OF PARALLEL SYSTEMS WHEN COMPONENTS HAVE PROPORTIONAL HAZARD RATES"

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In our article [3] we have found a gap in the middle of the proof of Theorem 3.2. Therefore, we do not know whether Theorem 3.2 is true for the reverse hazard rate order. However, we could prove the following weaker result for the stochastic order.

Theorem: *Let $X_1, \ldots, X_n$ be independent random variables with X_i having survival function $\bar{F}^{\lambda_i}$, $i = 1, \ldots, n$. Let $Y_1, \ldots, Y_n$ be a random sample with common population survival distribution $\bar{F}^{\bar{\lambda}}$, where $\bar{\lambda} = \sum_{i=1}^n \lambda_i/n$, then*

$$Y_{n:n} - Y_{1:n} \leq_{\rm st} X_{n:n} - X_{1:n}.$$

Proof: From David and Nagaraja [1, p. 26], the distribution function of $R_X = X_{n:n} - X_{1:n}$ is, for $x \geq 0$,

$$F_{R_X}(x) = \sum_{i=1}^n \int_0^\infty \lambda_i \bar{F}^{\lambda_i - 1}(u) f(u) \prod_{j=1,\, j\neq i}^n [\bar{F}^{\lambda_j}(u) - \bar{F}^{\lambda_j}(u+x)]du.$$

Similarly, the distribution function of $R_Y = Y_{n:n} - Y_{1:n}$ is, for $x \geq 0$,

$$F_{R_Y}(x) = n \int_0^\infty \bar{\lambda} \bar{F}^{\bar{\lambda}-1}(u) f(u) [\bar{F}^{\bar{\lambda}}(u) - \bar{F}^{\bar{\lambda}}(u+x)]^{n-1} du.$$

Hence, it is enough to prove, for $u \geq 0, x \geq 0$,

$$\sum_{i=1}^{n} \lambda_i \bar{F}^{\lambda_i - 1}(u) f(u) \prod_{j=1,\, j\neq i}^{n} [\bar{F}^{\lambda_j}(u) - \bar{F}^{\lambda_j}(u+x)] \\ \leq n\bar{\lambda}\bar{F}^{\bar{\lambda}-1}(u) f(u) [\bar{F}^{\bar{\lambda}}(u) - \bar{F}^{\bar{\lambda}}(u+x)]^{n-1};$$

that is,

$$\sum_{i=1}^{n} \frac{\lambda_i}{1 - \bar{F}_u^{\lambda_i}(x)} \prod_{j=1}^{n} [1 - \bar{F}_u^{\lambda_j}(x)] \leq n\bar{\lambda}[1 - \bar{F}_u^{\bar{\lambda}}(x)]^{n-1}, \tag{0.1}$$

where

$$\bar{F}_u(x) = \frac{\bar{F}(u+x)}{\bar{F}(u)},$$

which is the survival function of $X_u = X - u | X > u$, the residual life of X at time $u \geq 0$. Now, using the transform

$$H(x) = -\log \bar{F}_u(x), \qquad u \geq 0,$$

it follows that that inequality (0.1) becomes

$$\sum_{i=1}^{n} \frac{\lambda_i}{1 - e^{-\lambda_i H(x)}} \prod_{j=1}^{n} [1 - e^{-\lambda_j H(x)}] \leq n\bar{\lambda}[1 - e^{-\bar{\lambda} H(x)}]^{n-1}.$$

Theorem 3.1 in Kochar and Rojo [2] proves the following inequality:

$$\sum_{i=1}^{n} \frac{\lambda_i}{1 - e^{-\lambda_i x}} \prod_{j=1}^{n} [1 - e^{-\lambda_j x}] \leq n\bar{\lambda}[1 - e^{-\bar{\lambda} x}]^{n-1}.$$

Replacing x with $H(x)$, the required result follows immediately. ∎

References

1. David, H.A. & Nagaraja, H.N. (2003). *Order statistics*, 3rd ed. New York: Wiley.
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3. Kochar, S. & Xu, M. (2007). Stochastic comparisons of parallel systems when components have proportional hazard rates. *Probability in the Engineering and Informational Sciences* 21: 597–609.