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# Analytic modeling of gain-switched lasers. I. Laser oscillators

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# Analytic modeling of gain-switched lasers. I. Laser oscillators

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Detailed mathematical models are derived for the evolution of light pulses in gain-switched laser oscillators. Unlike previous instantaneous-switching models, arbitrary pump and spontaneous relaxation rates are considered. Explicit expressions are obtained for the gain and pulse characteristics in several practical situations, and both homogeneous and inhomogeneous limits are considered. By proper adjustment of the laser pump any output-pulse shape can be obtained. The results are especially relevant for recent short-pulse ultraviolet lasers and also for more conventional devices such as TEA $CO_2$ lasers.

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## I. INTRODUCTION

An ever-widening range of laser applications requires the formation of short optical pulses having high intensity and possibly prespecified temporal characteristics. Besides the various radar and related applications, pulses of this type are essential for most laser-fusion schemes. In pellet-compression laser fusion, for example, maximizing the intensity of the laser pulse is a basic requirement. Nearly as obvious is the requirement that the pulse have a precisely specified time shape. Many industrial, medical, and metrological applications also require well-controlled optical pulses. Therefore, it is important that the characteristics of pulsed laser oscillators and amplifiers be understood in the greatest possible detail.

There have previously been several investigations of the output properties of pulsed lasers.[1,2] Most of these treatments have assumed as their starting point that the gain switching or $Q$ switching is instantaneous and that the optical field consists of a single monochromatic frequency at the gain center in a homogeneously broadened medium. After the gain is switched on (or the loss is switched off) in such a model, the optical pulse rapidly builds up from spontaneous emission. When the gain has been depleted, the pulse decays away due to cavity losses. Optimum coupling conditions have also been calculated.[3] Recently the spectral characteristics of such gain-switched pulses have been described in detail including the gradual resolution of the longitudinal modes from the broad-band spontaneous emission and the narrowing of the over-all spectrum.[4] The effects of inhomogeneous broadening have been considered too. For many practical lasers, however, the instantaneous-switching model provides a somewhat inadequate description of the actual gain characteristics. Often the gain rises gradually due to continued pumping and spontaneous relaxation may be significant during the development of the optical pulse. The purpose of the present work is to develop new analytic models for the laser output including a time-dependent pump rate and spontaneous relaxation. These solutions are reduced to the simplest possible mathematical form and then applied to specific laser systems. The results are relevant for high-pressure ultraviolet (uv) xenon lasers, because in these systems a substantial decay of the inversion may take place during the rise time of the optical pulse. With TEA lasers, on the other hand, the output pulse normally occurs during the rise time of the gain. For all such lasers and other similar devices the techniques developed here provide a much more reasonable description of the output characteristics than is possible with the instantaneous-switching approximation. In addition it becomes possible to choose a pump function which will yield any desired output-pulse shape.

In a related study the properties of gain-switched laser amplifiers have been considered in detail.[5] For very-high-power applications it is always simplest and most effective to add a pulse amplifier chain after the laser-oscillator source. Exact analytical models have been developed for the pulse transfer characteristics of typical amplifier chains including for the first time a completely arbitrary space- and time-dependent pumping function. The oscillator pulse shapes derived here are used as input pulses for the amplifiers considered in the following paper.

The basic pulse-evolution formulas are derived in Sec. II. With an exponential pump characteristic, the predicted gain behavior is found to be in excellent agreement with previously obtained data involving 1700-Å xenon lasers and 10.6-$\mu$ $CO_2$ lasers. In Sec. III the special case of instantaneous pumping with gradual spontaneous decay is considered in greater detail, and the complementary case of slow pumping with no decay is treated in Sec. IV. In each of these limits the saturation behavior is governed by a simple differential equation. For the easiest case of instantaneous pumping and no relaxation, approximate closed-form solutions for the output pulse can be obtained, and these are described in Sec. V. In Sec. VI formulas are derived which indicate the necessary pumping function for achieving any desired gain or output-pulse shape, and some effects of inhomogeneous broadening are discussed briefly in Sec. VII.

## II. DERIVATION OF INTENSITY EQUATIONS

There are various ways that one can approach the problem of light propagation in laser systems and the details depend on the type of approximations that can be made. In the majority of practical applications the coherence time $T_2$ is short compared to any other time of interest and the behavior is governed by familiar rate equations.[1-4] Most previous treatments have employed this simplification and it is the starting point for the present analysis as well. Thus the coupled pair of rate equations governing the frequency-dependent population densities of the upper and lower laser states in a medium with inhomogeneous broadening are[4]

$$\frac{\partial n_2(y,z,t)}{\partial t} = S_2(y,z,t) - n_2(y,z,t)\left(A_2 + \frac{2B_0}{\pi\Delta\nu_h}\int_{-\infty}^{\infty}\frac{I(y_l,z,t)}{1+(y-y_l)^2}dy_l\right) + n_1(y,z,t)\frac{2B_0}{\pi\Delta\nu_h}\int_{-\infty}^{\infty}\frac{I(y_l,z,t)}{1+(y-y_l)^2}dy_l, \tag{1}$$

$$\frac{\partial n_1(y,z,t)}{\partial t} = S_1(y,z,t) + n_2(y,z,t)\left(A_{21} + \frac{2B_0}{\pi\Delta\nu_h}\int_{-\infty}^{\infty}\frac{I(y_l,z,t)}{1+(y-y_l)^2}dy_l\right) - n_1(y,z,t)\left(A_1 + \frac{2B_0}{\pi\Delta\nu_h}\int_{-\infty}^{\infty}\frac{I(y_l,z,t)}{1+(y-y_l)^2}dy_l\right). \tag{2}$$

Here $S_1(y,z,t)$ and $S_2(y,z,t)$ are space- and time-dependent pumping rates, $y_l = 2(\nu_l - \nu_0)/\Delta\nu_h$ is a frequency parameter normalized in units of the homogeneous linewidth $\Delta\nu_h$, $I(y_l,z,t)$ is the average spectral density at the frequency $y_l$, and the significance of the Einstein $A$ and $B$ coefficients is apparent. If the lower state decays quickly compared to the pulse length, it follows from Eqs. (1) and (2) that the inversion density $n = n_2 - n_1$ is governed by an equation of the form

$$\frac{\partial n(y,z,t)}{\partial t} = S(y,z,t) - n(y,z,t)\frac{2B_0}{\pi\Delta\nu_h}\times\int_{-\infty}^{\infty}\frac{I(y_l,z,t)}{1+(y-y_l)^2}dy_l - \frac{n(y,z,t)}{\tau_2}. \tag{3}$$

Except for the spontaneous decay terms, the same basic form is also obtained in the limit of negligible lower-state decay.

In a laser medium of the type considered here the growth of intensity is proportional to the population inversion, and the governing equation is

$$\frac{\partial I(y_l,z,t)}{\partial z} + \frac{1}{v_g}\frac{\partial I(y_l,z,t)}{\partial t} = h\nu_l\frac{2B_0}{\pi\Delta\nu_h}[I(y_l,z,t)+\eta]\times\int_{-\infty}^{\infty}\frac{n(y,z,t)}{1+(y-y_l)^2}dy - \gamma I(y_l,z,t), \tag{4}$$

where $v_g$ is the group velocity and $\gamma$ represents distributed losses. The noise parameter is $\eta = h\nu_l\Delta\nu_h/2A$, where $A$ is the mode area. Equations (3) and (4), which couple the intensity to the population inversion, provide the basis for most of the following analysis. They give a very general description of the development of an intensity spectrum in a saturating laser amplifier having an arbitrary amount of inhomogeneous broadening and an arbitrary pump distribution. The equations are, of course, highly nonlinear and they cannot be solved in any general sense. However, for most practical problems enormous simplifications are possible and often analytic solutions can be obtained. In the remainder of this paper we are concerned with deriving some of these solutions and investigating their implications for practical laser systems.

An important special case of the previous results includes the homogeneously broadened lasers, and this limit is appropriate for most practical short-pulse devices. In a homogeneously broadened laser it is generally the case that both the intensity spectrum $I(y_l,z,t)$ and the inversion spectrum $n(y,z,t)$ are narrow compared to the Lorentzian homogeneous line shape. Then Eqs. (3) and (4) can be integrated over frequency to obtain

$$\frac{\partial N(z,t)}{\partial t} = S(z,t) - sI(z,t)N(z,t) - \frac{N(z,t)}{\tau_2}, \tag{5}$$

$$\frac{\partial I(z,t)}{\partial t} + \frac{1}{v_g}\frac{\partial I(z,t)}{\partial t} = h\nu_l s[I(z,t)+I_0]N(z,t) - \gamma I(z,t), \tag{6}$$

where a new inversion density $N(z,t) = \int_{-\infty}^{\infty} n(y,z,t)\,dy$, intensity $I(z,t) = \int_{-\infty}^{\infty} I(y_l,z,t)\,dy_l$, pump $S(z,t) = \int_{-\infty}^{\infty} S(y,z,t)\,dy$, saturation parameter $s = 2B_0/\pi\Delta\nu_h$, and input noise $I_0 = \pi\eta$ have been introduced. Close to threshold the intensity spectrum is actually not narrow compared to the homogeneous width, but Eqs. (5) and (6) apply under typical operating conditions.

Equation (5) is a linear first-order differential equation with nonconstant coefficients. The solution of such equations is well known and the result is[6]

$$N(z,t) = \exp\left[-s\int_{-\infty}^{t} I(z,t')\,dt' - t/\tau_2\right] \times \int_{-\infty}^{t} S(z,t')\exp\left[s\int_{-\infty}^{t'} I(z,t'')\,dt'' + t'/\tau_2\right]dt', \tag{7}$$

where it has been assumed that the inversion at $t = -\infty$ is zero. With Eq. (6) the intensity is therefore governed by

$$\frac{\partial I(z,t)}{\partial t} + \frac{1}{v_g}\frac{\partial I(z,t)}{\partial t} = h\nu_l s[I(z,t)+I_0]\exp\left(-s\int_{-\infty}^{t} I(z,t')\,dt' - \frac{t}{\tau_2}\right) \times\int_{-\infty}^{t} S(z,t')\exp\left(s\int_{-\infty}^{t'} I(z,t'')\,dt'' + \frac{t'}{\tau_2}\right)dt' - \gamma I(z,t). \tag{8}$$

Equation (8) is the basic working equation for homogeneously broadened lasers. This result is an appropriate starting point for a variety of problems involving transient effects in laser amplifiers and oscillators.

As a first check on the practical applicability of Eq. (8), one may consider the determination of the unsaturated gain. The single most important pumping function in practice is the $z$-independent exponential

$$S(z,t) = S_0\exp(-t/\tau_3), \quad t > 0$$
$$= 0, \quad t < 0. \tag{9}$$

This corresponds to an exponentially decaying transfer of excitation into the upper laser state from some other state (level three) of the system. Then Eq. (8) takes the form

$$\frac{\partial I(z,t)}{\partial z} + \frac{1}{v_g}\frac{\partial I(z,t)}{\partial t} = \frac{g_0}{\tau_3}[I(z,t)+I_0]\exp\left(-s\int_0^{t'} I(z,t')\,dt' - \frac{t}{\tau_2}\right) \times\int_0^{t}\exp\left(s\int_0^{t'} I(z,t'')\,dt'' + \frac{t'}{\tau_2} - \frac{t'}{\tau_3}\right)dt' - \gamma I(z,t), \tag{10}$$

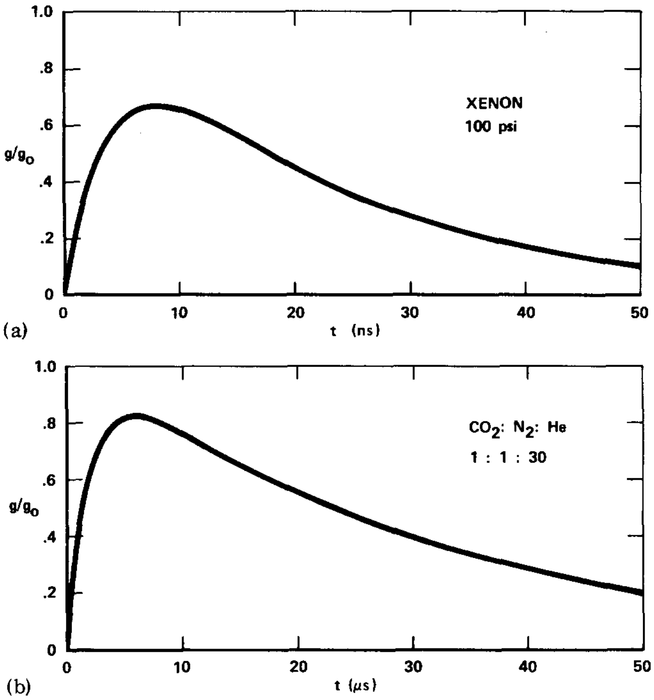


FIG. 1. Time-dependent gain functions for (a) a high-pressure electron-beam-excited xenon laser and (b) a resistive-pin TEA $CO_2$ laser. The xenon data is from Ref. 7, and the $CO_2$ curve agrees with the experimental results of Ref. 8.

where the gain parameter $g_0 = \tau_3 h\nu_l s S_0$ has been introduced. In the absence of the saturation exponentials it follows from Eq. (10) that the instantaneous unsaturated gain coefficient is

$$g(t) = \frac{g_0}{\tau_3}\exp\left(-\frac{t}{\tau_2}\right)\int_0^t \exp\left(\frac{t'}{\tau_2}-\frac{t'}{\tau_3}\right)dt'$$
$$= \frac{g_0}{1-\tau_3/\tau_2}\left[\exp\left(-\frac{t}{\tau_2}\right)-\exp\left(-\frac{t}{\tau_3}\right)\right]. \quad (11)$$

Differentiation of this equation shows that the peak value of the gain occurs at the time

$$t_{\max} = \frac{\ln(\tau_2/\tau_3)}{\tau_3^{-1}-\tau_2^{-1}}. \quad (12)$$

Equations (11) and (12) are in good agreement with measurements of the unsaturated gain or fluorescence in practical gain-switched lasers. In short-pulse electron-beam-excited uv xenon lasers at 1700 Å, for example, the fluorescence is known to be well represented by a sum of exponentials.[7] The spontaneous decay time is $\tau_2 = 20$ ns, and at a pressure of 100 psi the pumping time is about 4 ns. A plot of the corresponding gain function from Eq. (11) is given in Fig. 1(a). From Eq. (12) the gain maximum occurs about 8 ns after the excitation current.

These results are also in agreement with gain measurements made with many types of TE $CO_2$ lasers. In resistive-pin devices, for example, the discharge is usually completed in a few hundred ns.[8] Then the upper laser state is pumped by the exponentially decaying transfer of excitation from the vibrationally excited $N_2$ molecules. The relevant lifetimes are governed by the equations[9,10]

$$\tau_3 = (6\times10^{-13}N_{N_2})^{-1}, \quad (13)$$

$$\tau_2 = (5.0\times10^{-15}N_{CO_2} + 1.6\times10^{-15}N_{N_2} + 1.1\times10^{-15}N_{He})^{-1}, \quad (14)$$

where $N_x$ is the number of molecules of component $x$ per $cm^3$. But the density of molecules at standard temperature and pressure is $N_{total} = 2.687\times10^{19}$ $cm^{-3}$. Therefore, in a TEA laser having a 1 : 1 : 30 mixture of $CO_2:N_2:He$, the lifetimes are $\tau_3 = 2.0$ $\mu$s and $\tau_2 = 30.1$ $\mu$s. With these values the gain is plotted in Fig. 1(b) using Eq. (11). The resulting curve is in excellent agreement with the corresponding experimental data over the entire time history of the gain.[8] From Eq. (12) the gain maximum should occur at the time $t_{\max} = 5.8$ $\mu$s, while the reported value is 5.5 $\mu$s. Thus the analytical model described here exhibits the essential gain characteristics of practical gain-switched lasers, at least in the limit of negligible saturation. The detailed validity of these results will be explored more fully in Secs. III–V which deal with saturating laser oscillators.

## III. SATURATING OSCILLATOR: FAST PUMP, SLOW DECAY

In Sec. II general analytical gain saturation equations have been developed. These equations may now be applied to determine the output intensity of specific laser systems. The intensity equation simplifies greatly in certain special cases. In this section we consider situations in which the gain rises rapidly to its maximum value and then decays slowly during the development of the saturating optical pulse. This limit is appropriate for many practical lasers including the nanosecond pumped 1700-Å uv xenon lasers. The opposite limit of a slowly rising gain is considered in the Sec. IV.

From Eq. (8) the general equation for the intensity in a gain-switched laser oscillator is

$$\frac{dI(t)}{dt} = \frac{2l}{T}h\nu_l s[I(t)+I_0]\exp\left(-2s\int_{-\infty}^{t}I(t')\,dt' - \frac{t}{\tau_2}\right)$$
$$\times\int_{-\infty}^{t}S(t')\exp\left(2s\int_{-\infty}^{t'}I(t'')\,dt'' + \frac{t'}{\tau_2}\right)dt' - \gamma_c I(t), \quad (15)$$

where $I(t)$ is the one-way intensity, $l$ is the length of the laser medium, and $T$ is the round-trip time. The factor of 2 in the saturation exponentials results from the fact that with homogeneous broadening the population inversion is depleted by the radiation propagating to the right and to the left. The cavity losses are lumped into the parameter $\gamma_c = (2\gamma l - \ln R_1R_2)/T$, where $R_1$ and $R_2$ are the mirror reflectivities. Equation (15) may be used for many types of problems involving optimum coupling and transients in laser oscillators. $Q$ switching and more complicated output-coupling schemes could be treated by introducing the time-dependent loss parameter $\gamma_c(t)$.

With the exponential gain function of Eq. (9), Eq. (15) is

$$\frac{dI(t)}{dt} = \frac{g_c}{\tau_3}[I(t)+I_0]\exp\left(-2s\int_0^t I(t')\,dt' - \frac{t}{\tau_2}\right)$$
$$\times\int_0^t \exp\left(2s\int_0^{t'}I(t'')\,dt'' + \frac{t'}{\tau_2} - \frac{t'}{\tau_3}\right)dt' - \gamma_c I(t), \quad (16)$$

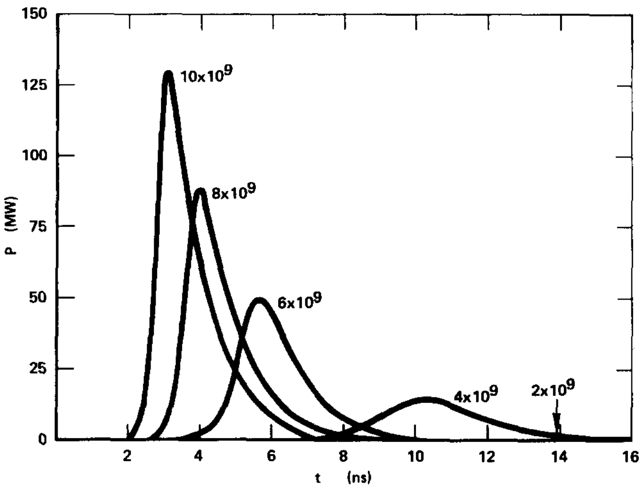


FIG. 2. Output power characteristics for a 1700-Å xenon laser as a function of initial unsaturated gain. The arrow marks the location of the small power maximum for $g_c \approx 2 \times 10^9$ s$^{-1}$. Experimental results of Ref. 7 correspond approximately to the curve $g_c \approx 4 \times 10^9$ s$^{-1}$.

where the gain is represented by the coefficient $g_c = 2lh\nu_l s S_0 \tau_3/T$. In the limit $\tau_3 \to 0$ the exponential $\exp(-t'/\tau_3)$ in Eq. (16) acts like a $\delta$ function and the equation reduces to

$$\frac{dI(t)}{dt} = g_c[I(t) + I_0] \exp\left(-2s\int_0^t I(t')\,dt' - \frac{t}{\tau_2}\right) - \gamma_c I(t). \qquad (17)$$

This equation can be readily solved numerically, since the derivative at any instant of time predicts the intensity at a later time according to $I(t+\Delta t) = I(t) + (dI/dt)\Delta t$.

Plots of Eq. (17) are given in Fig. 2 using numbers appropriate to the xenon laser mentioned previously. In particular, with a cavity length $L = 0.1$ m and mirror reflectivities of 70% the loss coefficient is approximately $\gamma_c = 10^9$ s$^{-1}$. With reasonable 1-m mirror curvatures the mode area is about $A = 3.7 \times 10^{-8}$ m$^2$.[11] Since the homogeneous linewidth is $\Delta\lambda_h = 150$ Å,[7] it follows that the spontaneous-emission noise intensity is $I_0 = 7515$ W/m$^2$. The saturation parameter is approximately.

$$s = \frac{2B_0}{\pi\Delta\nu_h} = \frac{c^2}{4\pi^2 h\nu^3\tau_2\Delta\nu_h} = 2.1\times 10^{-4}\ \frac{\mathrm{m}^2}{\mathrm{J}} \qquad (18)$$

The vertical scale in Fig. 2 is obtained by assuming somewhat arbitrarily that the total beam area is 0.1 cm$^2$ and that the mirror transmission is 10%. From the curves in Fig. 2 it is evident that the stimulated emission takes the form of a short pulse when the gain $g_c$ exceeds the loss $\gamma_c$. When the gain is about four times the loss it follows from the figure that the pulse width is about 3 ns, which is the same as the experimentally obtained value.[7] The output power level of about 15 MW is also comparable to typical experimental results.[12]

It may be emphasized that the existence of a narrow peak in the laser output is not conclusive evidence that the laser medium has been saturated by the optical field. In the absence of the saturation integrals Eq. (16) reduces to

$$\frac{dI(t)}{dt} = \frac{g_c}{1-\tau_3/\tau_2}[I(t)+I_0]\left[\exp\left(-\frac{t}{\tau_2}\right) - \exp\left(-\frac{t}{\tau_3}\right)\right] - \gamma_c I(t). \qquad (19)$$

This is a linear first-order differential equation and the solution is[6]

$$I(t) = \frac{g_c\tau_2 I_0}{\tau_2-\tau_3}\exp\left\{\frac{g_c\tau_2}{\tau_2-\tau_3}\left[-\tau_2\exp\left(-\frac{t}{\tau_2}\right) + \tau_3\exp\left(-\frac{t}{\tau_3}\right)\right] + g_c\tau_2 - \gamma_c t\right\}\int_0^t\left[\exp\left(-\frac{t'}{\tau_2}\right) - \exp\left(-\frac{t'}{\tau_3}\right)\right]$$
$$\times\exp\left\{\frac{g_c\tau_2}{\tau_2-\tau_3}\left[\tau_2\exp\left(-\frac{t'}{\tau_2}\right) - \tau_3\exp\left(-\frac{t'}{\tau_3}\right)\right] - g_c\tau_2 + \gamma_c t'\right\}dt', \qquad (20)$$

where the initial intensity has been set equal to zero. If $\tau_3$ is also set equal to zero, Eq. (20) simplifies to

$$I(t) = g_c I_0 \exp\{g_c\tau_2[1-\exp(-t/\tau_2)] - \gamma_c t\} \times \int_0^t \exp\{-g_c\tau_2[1-\exp(-t'/\tau_2)] - t'/\tau_2 + \gamma_c t'\}\,dt'. \qquad (21)$$

Equation (21) is plotted in Fig. 3 for various values of the gain $g_c$. The numerical constants used in obtaining these results are the same as those employed previously in analyzing the 1700-Å xenon laser. It is evident from the figure that even in an unsaturated laser oscillator the output takes the form of a narrow pulse of radiation. For the curves shown the pulse width is about 10 ns, which is much greater than the corresponding experimental values. One can conclude, therefore, that the experimental stimulated-emission intensities are sufficient to saturate the 1700-Å transition.

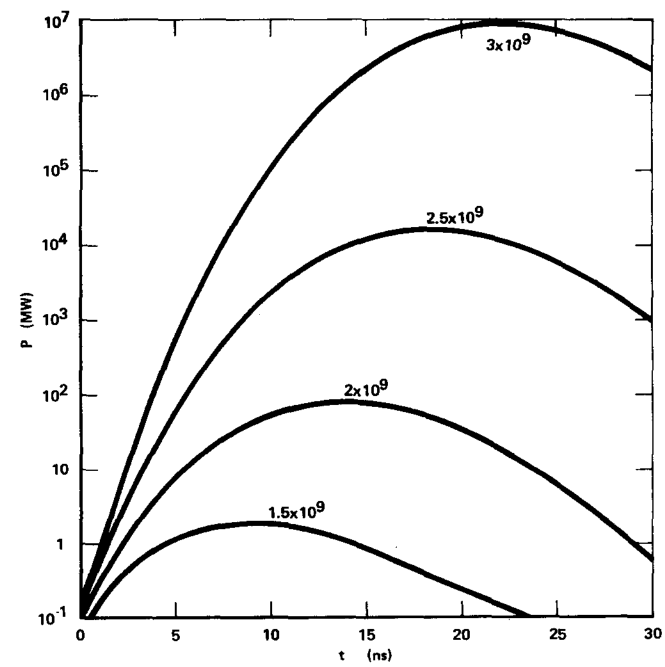


FIG. 3. Power characteristics as a function of the gain $g_c$ for the xenon laser operating close to threshold where saturation is negligible.

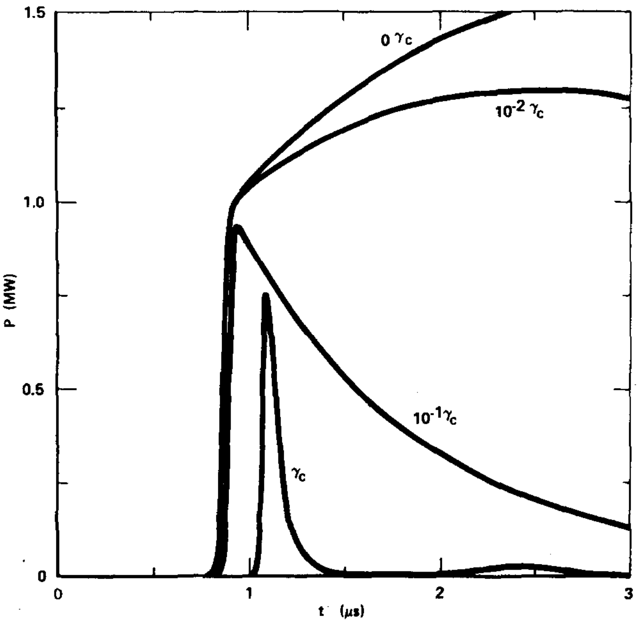


FIG. 4. Output power of a $CO_2$ laser with a 5:5:90 gas mixture of $CO_2$:$N_2$:He. The gain and loss coefficients are $g_c = 1.5\times10^8$ $s^{-1}$ and $\gamma_c = 1.7\times10^7$ $s^{-1}$. As the loss is reduced the tail of the pulse changes, but the important leading edge and pulse delay are only slightly affected.

## IV. SATURATING OSCILLATOR: SLOW PUMP, NO DECAY

Another important class of gain-switched laser oscillators occurs when longer-lived molecules are present. It then may happen that the stimulated-emission pulse is over before the pumping is complete, and spontaneous decay of the upper laser level may be entirely negligible. This model is approximately valid for many TE $CO_2$ lasers, for example, and we use $CO_2$ as our principal illustration. A complicating feature of $CO_2$ lasers is that the lower-state relaxation time is not always negligible as we have assumed. In fact the lower state decays via complicated resonance processes, and the relevant mechanisms and lifetimes are still the subjects of investigations.[13] Far above threshold the lower-level saturation effects can lead to a slight reduction in pulse height accompanied by an increase of energy in the pulse tail. The important leading edge of the pulse is unaffected.

In the limit $\tau_2\to\infty$ it follows from Eq. (16) that the intensity in an exponentially pumped laser oscillator is governed by

$$\frac{dI(t)}{dt}=\frac{g_c}{\tau_3}[I(t)+I_0]\exp\left(-2s\int_0^t I(t')\,dt'\right)\times\int_0^t \exp\left(2s\int_0^{t'} I(t'')\,dt''-\frac{t'}{\tau_3}\right)dt'-\gamma_c I(t). \quad (22)$$

If the spontaneous-emission input is neglected for the moment, Eq. (22) can be written

$$\frac{dI(t)}{dt}=-\frac{g_c}{2s\tau_3}\frac{d}{dt}\left[\exp\left(-2s\int_0^t I(t')\,dt'\right)\times\int_0^t \exp\left(2s\int_0^{t'} I(t'')\,dt''-\frac{t'}{\tau_3}\right)dt'-\gamma_c I(t). \quad (23)$$

The integral of this equation is

$$I(t)=-\frac{g_c}{2s\tau_3}\exp\left(-2s\int_0^t I(t')\,dt'\right)\times\int_0^t \exp\left(2s\int_0^{t'} I(t'')\,dt''-\frac{t'}{\tau_3}\right)dt' -\frac{g_c}{2s}\left[\exp\left(-\frac{t}{\tau_3}\right)-1\right]-\gamma_c\int_0^t I(t')\,dt'+I_0, \quad (24)$$

where the effective initial intensity has been set equal to the spontaneous emission input $I_0$. This definition is valid as long as the unsaturated gain $g_c$ is much greater than the loss. In terms of the new parameter $C(t)=2s\int_0^t I(t')\,dt'$, Eq. (24) becomes

$$\frac{dC(t)}{dt}=-\frac{g_c}{\tau_3}\exp[-C(t)]\int_0^t \exp\left(C(t')-\frac{t'}{\tau_3}\right)dt' -g_c\left[\exp\left(-\frac{t}{\tau_3}\right)-1\right]-\gamma_c C(t)+2sI_0 \quad (25)$$

This equation is of lower order than Eq. (22) and numerical solutions are readily obtained.

The laser output intensity from Eq. (25) is plotted in Fig. 4 using numbers appropriate to $CO_2$ lasers. For a reasonable 0.5 : 0.5 : 9 gas mixture of $CO_2$ : $N_2$ :He, Eqs. (13) and (14) yield the lifetimes $\tau_3 = 1.24$ μs and $\tau_2 = 28.2$ μs. It will turn out that this value of $\tau_2$ is much larger than any other time of interest, so the previous neglect of the spontaneous relaxation term is justified. If the cavity length is $L=2$ m and the reflectivity of the coupling mirror is 80%, it follows that the loss parameter is $\gamma_c=-0.75\times10^8\ln(0.8)=1.7\times10^7$ $s^{-1}$. The homogeneous pressure-broadened linewidth of the $\lambda=10.6$ μ transition at a temperature of 300 °K is given by[13]

$$\Delta\nu_h=7.58\times10^6(\psi_{CO_2}+0.73\psi_{N_2}+0.6\psi_{He})P, \quad (26)$$

where $\psi_x$ is the partial fraction of gas $x$ and $P$ is the pressure in Torr. For the gas mixture mentioned previously the linewidth at atmospheric pressure is 3.61 GHz. Therefore, a mode area of 1 $cm^2$ implies an input noise intensity $I_0 = 1.06\times10^{-6}$ $W/m^2$. A reasonable saturation energy for a resistive-pin laser is 0.1 J and thus the saturation parameter is $s=10^{-3}$ $m^2/J$.

Several features of the solutions of Eq. (25) are significant. With increased pumping (larger $g_c$) the output pulse becomes higher and narrower as observed in practice. Also, the delay between the discharge and the pulse maximum is reduced as the gain is increased. The secondary pulse seen in Fig. 4 is due to continued pumping from the excited nitrogen molecules, and this pulse is also observed with practical TEA $CO_2$ lasers. The gain used in the plots is $g_c = 1.5\times10^8$ $s^{-1}$, which corresponds to the reasonable value $g_0 = 1.0$ $m^{-1}$ and an amplifier length of $l = 1$ m. The loss has been varied for the different curves shown in the figure instead of the gain, and it can be seen that the leading edge of the pulses are nearly independent of the loss $\gamma_c$. For very low losses useful explicit expressions for the intensity can be obtained.

Multiplying Eq. (25) by $\exp C(t)$ and differentiating yields

$$\frac{d^2 \exp C(t)}{dt^2} = \left\{ g_c \left[ 1 - \exp\left( -\frac{t}{\tau_3} \right) \right] + 2sI_0 \right\} \frac{d \exp C(t)}{dt}, \quad (27)$$

where it has been assumed that cavity losses are small ($\gamma_c \to 0$). This equation can be integrated to obtain

$$\frac{d \exp C(t)}{dt} = 2sI_0 \exp\left\{ g_c t - g_c \tau_3 \left[ 1 - \exp\left( -\frac{t}{\tau_3} \right) \right] + 2sI_0 t \right\}. \quad (28)$$

A second integration gives

$$\exp C(t) = 2sI_0 \int_0^t \exp\{ g_c t' - g_c \tau_3 [1 - \exp(-t'/\tau_3)] + 2sI_0 t' \} dt' + 1. \quad (29)$$

The final expression for the intensity in a low-loss gain-switched laser oscillator is

$$I(t) = \frac{1}{2sC(t)} \frac{dC(t)}{dt} = I_0 \exp\left\{ g_c t - g_c \tau_3 \left[ 1 - \exp\left( -\frac{t}{\tau_3} \right) \right] + 2sI_0 t \right\} \times \left( 2sI_0 \int_0^t \exp\left\{ g_c t' - g_c \tau_3 \left[ 1 - \exp\left( -\frac{t'}{\tau_3} \right) \right] + 2sI_0 t' \right\} dt' + 1 \right)^{-1} \quad (30)$$

This result is useful when the main interest is in the leading edge of the laser output pulse. In the limit of instantaneous pumping ($\tau_3 \to 0$) Eq. (30) simplifies to

$$I(t) = I_0 \frac{g_c + 2sI_0}{g_c \exp(-g_c t - 2sI_0 t) + 2sI_0}. \quad (31)$$

These formulas are used in a related work as input pulse shapes for the analysis of pulse propagation in saturating laser amplifiers.[5] It will be shown that only the leading edge of the pulse is important in determining the transfer characteristics of such amplifiers.

## V. SATURATING OSCILLATOR: FAST PUMP, NO DECAY

As a final example of a saturating gain-switched laser oscillator, we consider the simplest possible case where the gain is switched on instantaneously and no spontaneous relaxation occurs. In the limit $\tau_3 \to 0$, $\tau_2 \to \infty$, Eq. (16) reduces to

$$\frac{dI(t)}{dt} = g_c [I(t) + I_0] \exp\left( -2s \int_0^t I(t')\, dt' \right) - \gamma_c I(t) \quad (32)$$

and numerical solutions are elementary. One can also obtain approximate analytical solutions which include all of the significant features of Eq. (32). The general form of these solutions is suggested by an examination of the low-loss expression given in Eq. (31). In that result saturation occurs abruptly when the exponential term in the denominator ceases to be much larger than $2sI_0$. Therefore, we are led to try the pulse shape

$$I(t) = \frac{I_p g_c \exp[-\gamma_c (t - t_p)]}{\gamma_c \{ \exp[-g_c (t - t_p)] + g_c/\gamma_c - 1 \}}. \quad (33)$$

This is an exponential pulse having a peak intensity $I_p$ occurring at the time $t_p$.

The usefulness of Eq. (33) can readily be demonstrated. Early in the pulse both Eqs. (32) and (33) imply that the intensity grows exponentially like $\exp[(g_c - \gamma_c)t]$. Late in the pulse after heavy saturation has occurred both equations imply that the intensity decays like $\exp(-\gamma_c t)$. Equation (33) can also be matched to the exact solution of Eq. (32) with respect to peak intensity and pulse delay by proper choice of the parameters $I_p$ and $t_p$.

For the limit under consideration Eqs. (5) and (6) governing the intensity and population inversion reduce to

$$\frac{dI(t)}{dt} = \frac{g_c}{N_0} I(t) N(t) - \gamma_c I(t), \quad (34)$$

$$\frac{dN(t)}{dt} = -2sI(t)N(t), \quad (35)$$

where $N_0$ is the initial population inversion. If Eq. (34) is divided by Eq. (35), one obtains

$$\frac{dI}{dN} = -\frac{g_c}{2sN_0} + \frac{\gamma_c}{2sN}. \quad (36)$$

The integral of this equation is

$$I = -\frac{g_c (N - N_0)}{2sN_0} + \frac{\gamma_c}{2s} \ln\left( \frac{N}{N_0} \right), \quad (37)$$

where the initial intensity is assumed to be small. From Eq. (34) the peak intensity occurs when the inversion is $N_p = N_0 \gamma_c / g_c$. With this value of the inversion it follows from Eq. (37) that the peak intensity is[1]

$$I_p = \frac{\gamma_c}{2s} \left[ \frac{g_c}{\gamma_c} - \ln\left( \frac{g_c}{\gamma_c} \right) - 1 \right]. \quad (38)$$

An expression for the pulse delay $t_p$ can also be obtained. From Eq. (32) the pulse in its initial stages is described by

$$I(t) = \frac{I_0 g_c}{g_c - \gamma_c} \exp[(g_c - \gamma_c)t]. \quad (39)$$

On the other hand, Eq. (33) implies that the initial intensity variation is

$$I(t) = (I_p g_c / \gamma_c) \exp[(g_c - \gamma_c)(t - t_p)]. \quad (40)$$

From a comparison of these expressions one finds the relationship

$$\frac{I_0 g_c}{g_c - \gamma_c} = \frac{I_p g_c}{\gamma_c} \exp[-(g_c - \gamma_c) t_p]. \quad (41)$$

Thus the pulse delay is approximately

$$t_p \simeq \frac{-1}{g_c - \gamma_c} \ln\left( \frac{\gamma_c}{g_c - \gamma_c} \frac{I_0}{I_p} \right). \quad (42)$$

With Eq. (38) this is

$$t_p \simeq -\frac{1}{g_c - \gamma_c} \ln\left( \frac{2sI_0}{(g_0 - \gamma_c)[g_c/\gamma_c - \ln(g_c/\gamma_c) - 1]} \right). \quad (43)$$

Equation (33) is plotted in Fig. 5 using Eqs. (38) and (43) together with the numerical constants $g_c = 1.5 \times 10^8$ $s^{-1}$, $\gamma_c = 1.7 \times 10^7\ s^{-1}$, $s = 10^{-3}\ m^2/J$, $I_0 = 10^{-6}\ W/m^2$, an output coupling of 20%, and a mode area of 1 $cm^2$. The peak power in this example is $I_p = 0.96$ MW, and the pulse delay is $t_p = 0.30\ \mu s$. The corresponding numerical solution of Eq. (32) has also been obtained, and within the accuracy that can be represented in the figure

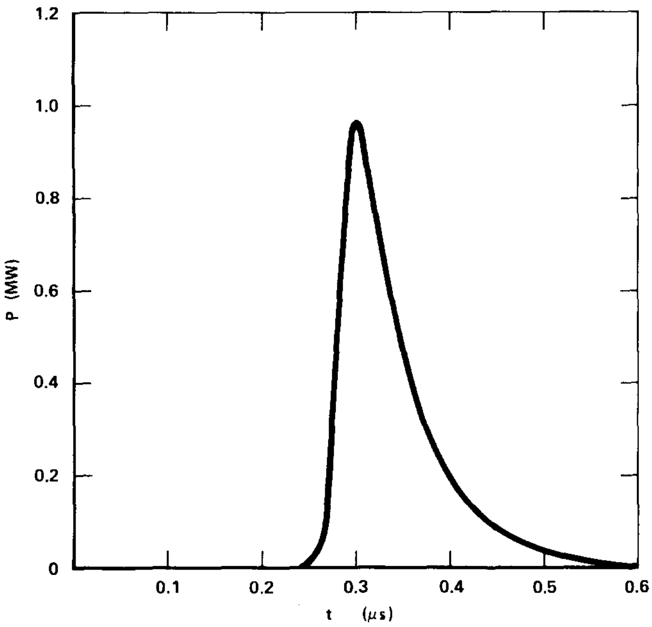


FIG. 5. Output power for a laser with instantaneous gain switching and no decay of the population inversion. This is a plot of the explicit formula given in Eq. (33).

the two solutions are identical. The only possible discrepancy with this model is a slightly incorrect decay of the pulse tail for operation close to threshold where saturation is incomplete. For most purposes, though, Eq. (33) provides a highly accurate and useful description of the output pulses from gain-switched lasers.

## VI. OUTPUT-PULSE SHAPING

For some applications of pulsed laser oscillators it is desirable to generate an optical pulse having a prespecified temporal shape. An important example would be pellet-compression laser fusion, where optimum heating and compression are only achieved with a specific shape of the incident laser pulse.[14] The pulse shape, however, depends directly on the time-dependent pump function $S(t)$. Therefore, one might expect that any output-pulse shape could be achieved if $S(t)$ were chosen properly. In practice considerable freedom is usually available in choosing the electrical and geometrical factors which govern the rate of pumping. The purpose of this section is to derive relationships between the pump $S(t)$ and the desired gain or power characteristics of the laser.

Choosing a pump function $S(t)$ to achieve a particular gain $g(t)$ is straightforward. From Eq. (8) the net unsaturated gain is

$$g(t)=h\nu_l s \exp(-t/\tau_2)\int_{-\infty}^{t} S(t')\exp(t'/\tau_2)\,dt'-\gamma; \qquad (44)$$

multiplying by $\exp(t/\tau_2)$ and differentiating yields immediately

$$S(t)=\frac{1}{h\nu_l s}\left(\frac{dg(t)}{dt}+\frac{g(t)+\gamma}{\tau_2}\right). \qquad (45)$$

For any desired unsaturated gain $g(t)$ Eq. (45) gives the required pump function $S(t)$.

A somewhat more practical problem is to determine the pump function $S(t)$ that will yield a particular intensity $I(t)$ in a laser oscillator. Equation (15) can be written

$$\int_{-\infty}^{t} S(t')\exp\left(2s\int_{-\infty}^{t'} I(t'')\,dt''+\frac{t'}{\tau_2}\right)dt'$$
$$=\frac{T}{2lh\nu_l s[I(t)+I_0]}\left(\frac{dI(t)}{dt}+\gamma_c I(t)\right)\exp\left(2s\int_{-\infty}^{t} I(t')\,dt'+\frac{t}{\tau_2}\right) \qquad (46)$$

and differentiation yields

$$S(t)=\frac{T}{2lh\nu_l s}\frac{d}{dt}\left[\frac{1}{I(t)+I_0}\left(\frac{dI(t)}{dt}+\gamma_c I(t)\right)\right]$$
$$+\frac{T}{2lh\nu_l s}\,\frac{2sI(t)+\tau_2^{-1}}{I(t)+I_0}\left(\frac{dI(t)}{dt}+\gamma_c I(t)\right). \qquad (47)$$

This is an explicit expression for the pumping needed to achieve an arbitrary intensity $I(t)$ in a homogeneously broadened laser oscillator. The function $S(t)$ is not necessarily positive, and if very rapid decay of the intensity is needed one finds that a negative pump is required.

When the noise input $I_0$ is small compared to $I(t)$ Eq. (47) simplifies to

$$S(t)=\frac{T}{2lh\nu_l s}\left[\frac{d^2}{dt^2}\ln I(t)+\left(2sI(t)+\frac{1}{\tau_2}\right)\left(\frac{d}{dt}\ln I(t)+\gamma_c\right)\right]. \qquad (48)$$

Now suppose, for example, that a Gaussian intensity pulse is required in the form

$$I(t)=I_1\exp(-t^2/t_0^2). \qquad (49)$$

Therefore, the laser oscillator must be pumped with the function

$$S(t)=\frac{T}{2lh\nu_l s}\left\{-\frac{2}{t_0^2}+\left[2sI_1\exp\left(-\frac{t^2}{t_0^2}\right)+\frac{1}{\tau_2}\right]\left[-\frac{2t}{t_0^2}+\gamma_c\right]\right\}. \qquad (50)$$

Equation (50) is plotted in Fig. 6 using the values $sI_1=10^9\ \mathrm{s}^{-1}$, $\tau_2=20$ ns, and $\gamma_c=10^9\ \mathrm{s}^{-1}$ corresponding to the xenon laser described in Sec. III. A value of

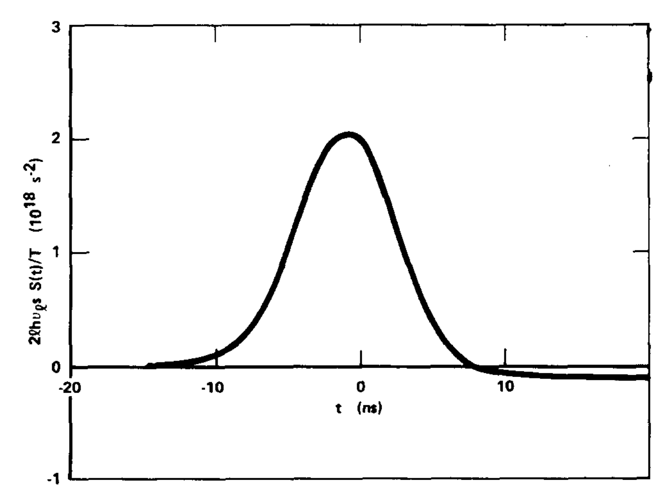


FIG. 6. Pump function required to obtain a Gaussian intensity pulse of maximum amplitude $sI_1=10^9\ \mathrm{s}^{-1}$ and width $\Delta t=8.3$ ns.

$t_0 = 5$ ns is also chosen corresponding to a desired pulse width of $\Delta t = 2t_0(\ln 2)^{1/2} = 8.3$ ns. It is apparent from the figure that the pump pulse is similar in shape to the Gaussian intensity pulse. In order to achieve the rapid decay of the Gaussian function, the theoretical pump function becomes negative after about 8 ns. In practice negative pumping would ordinarily not be feasible, so the Gaussian pulse tail could not be obtained. More slowly decaying intensity functions do not require negative pumping.

A complementary application of this formalism is in the diagnosis of laser amplifiers and oscillators. For example if gain or fluorescence measurements on a particular pulsed laser amplifier implied a gain function $g(t)$, one could from Eq. (45) immediately deduce the time dependence of the pumping. Similarly, if the output-intensity-pulse shape from a laser oscillator is known, the pump function follows from Eq. (47). A knowledge of the pump $S(t)$ should provide information regarding the processes which excite the upper laser level. With very-short-pulse oscillators an analysis of this type would often provide the most direct measurement of the time-dependent pumping.

## VII. DISCUSSION

A straightforward mathematical formalism has been developed for analyzing the properties of practical gain-switched laser oscillators. The initial equations include arbitrary pumping rates and spontaneous relaxation. A very common pumping characteristic in practice corresponds to an exponentially decaying transfer of excitation into the upper laser level from some other state of the system. Using this particular pumping characteristic, several specific analytical models have been developed. In some lasers, such as electron-beam-pumped devices, the pumping time may be short compared to the rise time of the optical fields. In this case the mathematics simplifies greatly, and the results have been illustrated using the 1700-Å xenon laser. Under typical conditions the gain pulse is about 25 ns wide, while the output laser pulse is about 3 ns wide. In the opposite limit of very slow pumping the analysis again simplifies and this case was illustrated with the TEA $CO_2$ laser. Here the gain pulse is typically 25 $\mu$s wide, while the laser pulse is about 100 ns wide. The procedure may also be reversed so that one can predict the pump function needed to produce a particular output pulse.

All of the examples that have been considered have involved the limit of homogeneous broadening, and in most practical short-pulse laser oscillators homogeneous broadening is indeed the dominant mechanism. For completeness, however, it is appropriate to mention also the limit in which inhomogeneous broadening is much greater than the homogeneous contribution. Neodymium-glass lasers, for example, fall in this category. Equations (3) and (4) include an arbitrary level of inhomogeneous broadening, so they are an appropriate starting point for this limit as well. A reasonable assumption is that the population inversion is uniform over the width $\Delta\nu_h$, and often the mode spacing is small compared to $\Delta\nu_h$. Then Eqs. (3) and (4) reduce to

$$\frac{\partial n(y,z,t)}{\partial t} = S(y,z,t) - \pi s I(y,z,t) n(y,z,t) - \frac{n(y,z,t)}{\tau_2}, \quad (51)$$

$$\frac{\partial I(y,z,t)}{\partial t} + \frac{1}{v_g}\frac{\partial I(y,z,t)}{\partial t} = \pi h \nu_l s [I(y_l,z,t) + \eta] n(y,z,t).$$
$$= \pi h \nu_l s [I(y,z,t) + \eta] n(y,z,t) - \gamma I(y,z,t). \quad (52)$$

But these equations are identical in form to Eqs. (5) and (6) if one makes the changes $s \to \pi s$ and $I_0 \to \eta$. Therefore, the previously given solutions for the homogeneous limit apply to each small spectral region in such an inhomogeneously broadened laser. Each spectral region produces an output pulse which depends only on its own local values of gain and loss and is independent of the other portions of the spectrum. Since the previous analysis applies, this limit is not considered further. Its primary practical consequence is to cause a rebroadening of the output spectrum when line-center saturation occurs.


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