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Mathematics Orality and Literacy

Abstract

To become a mathematician, a student must learn how to "do" mathematics and also how to communicate with other mathematicians. Through a special language, both oral and written, mathematicians share a discourse community. This community extends not only across the boundaries of natural language, but also across centuries. My paper explores the following question: How does a person enter the discourse community of mathematicians?

My research shows that learning mathematics parallels the learning of natural language reading and writing. Much like learning a foreign language, learning mathematics has been based mostly on oral tradition. The orality of mathematics learning is confirmed by my interviews with three college math instructors. A key difference is that orality alone is not sufficient to "do" mathematics. Literacy in the language of mathematics is necessary both to "do" mathematics and to tell mathematical "stories." Orality is, however, crucial to the student's learning process.

What Is the Language of Mathematics?

What did an early 20th century Englishman, a mid-century Hungarian, and a poor Brahmin Indian who died in 1920 at age 32 have in common? They all spoke and wrote in the language of mathematics. Englishman G. H. Hardy wrote one of the seminal textbooks on number theory, Hungarian Paul Erdös published more papers than any other mathematician, and young Indian Srinivasa Ramanujan, largely self-taught, produced highly unconventional results with no exposure to the mathematical mainstream in Europe.

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Almost by chance, in 1913 Ramanujan sent Hardy some manuscripts about his mathematical “discoveries.” Hardy very nearly dismissed these manuscripts because Ramanujan was not fluent in the language of mathematics. Fortunately for both men, Hardy was able to recognize Ramanujan’s genius and invited him to Cambridge University, where they formed a successful collaboration until Ramanujan’s untimely death from tuberculosis. In spite of Ramanujan’s deep mathematical insight, Hardy had to teach him “formal” mathematics, the proper language of mathematicians (Snow 36).

Mathematicians throughout the world refer to their “Erdös number.” Having an “Erdös number” of one means that person has co-authored a mathematical paper with Paul Erdös. An “Erdös number” of two means co-authoring a paper with someone else who has co-authored a paper with Erdös, and so on. The lower one’s “Erdös number” is, the more prestige one has in the mathematical community.

To enter the discourse community of mathematicians, students must learn both a written and a spoken language, a very specialized language that is based on many assumptions and conventions. Robert E. Jamison explains that teaching mathematics means disclosing the rules of the game: “the assumptions upon which the mathematical community bases its discourse” (47). As students gain proficiency and become able to “do” mathematics, gradually they learn to tell mathematical “stories,” that is, to communicate their results to other mathematicians (du Sautoy 21-23). At first students tell these stories to their teachers and classmates through oral discourse. If they are successful enough, they eventually will tell their stories through publication of their research results. Mathematical communications thus include both orality and literacy.

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To answer the question, “how does a person enter the discourse community of mathematician?” we first need to determine whether mathematics truly has the characteristics of a language. Most linguists believe that mathematics is a form of notation and not a language (Wagner 451). Even Jamison, who champions the use of language as a mathematics teaching tool, observes that mathematics differs from natural language in three fundamental respects (457):

(i) It is more precise than natural language

(ii) It lacks emotional content

(iii) It is nontemporal

David Wagner says that mathematics is a special kind of natural language, which linguists call the “mathematics register” (451). The mathematics register encompasses both the written and spoken language used in communicating mathematics.

In his comprehensive essay on mathematical musings, F. David Peat suggests that mathematics is both *more*, and *less*, than a natural language (sec. 4). *More*, in the sense that it involves nonverbal thinking:

> [A] particular kind of visual and sensory motor thinking that does not seem to be characteristic of ordinary language …. This “non-verbal” thinking … appears to involve a form of mental activity that goes beyond anything in the domain of spoken or written language. It could be that, at such times, mathematical thought has direct access to a form of thinking that is much deeper and more primitive than anything available in any natural language. This pre-linguistic mental activity may be the common source from which both mathematics and ordinary language emerge (Sec. 4).

I question whether it could be a “common” source because few people seem to have this type of mathematical insight. Or, perhaps, many people have it but fail to recognize it because, as Jamison puts it, they don’t know the “rules of the game.”

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Assuming there is a form of nonverbal thinking involved in mathematical thought (at least for some people), how does one communicate such thoughts to other mathematicians? I think there is an analogy to emotions, which start out as feelings (e.g., apprehension, fear, anger) rather than mental words. Sometimes we translate the feelings into words in order to communicate them to others, but often we don't or we can't. Mathematicians, as a rule, are motivated to share their mathematical thoughts with other mathematicians and will attempt to convert the nonverbal thoughts into mathematical expression. Sometimes a mathematician will expand upon or adapt existing notation or terms. If notation or words are lacking, mathematicians will invent new terminology or symbols. For example, Ramanujan invented his own mathematical terminology which he used in writing to Hardy. Marcus du Sautoy provides an example of his own creative process, describing the "birth" of a mathematical theorem as a "legacy" – a "bit of immortality" (24-25).

Peat says that mathematics is also *less* than a natural language because it lacks richness, nuance, and ambiguity: "mathematics is a limited, technical language in which much that is of deep human value cannot be expressed" (sec. 4). While mathematics could very well describe a rainbow, for example, it could not describe how I feel when I see a rainbow or how the rainbow might inspire a song or a poem. Yet, contrast Peat's assertion that mathematics lacks ambiguity with Mohan Ganesalingam's observation:

> Ambiguity is absolutely pervasive in both symbolic and textual mathematics, and (as with other phenomena) there are enough examples of ambiguity which intertwine symbolic and textual information that a unified account of textual and symbolic aspects is necessary. The proper description of ambiguity requires so much space that we will dedicate Chapter 4 to it, and will not discuss it further in this section, except to clear up one possible source of misunderstanding. To wit: most ambiguity in mathematics is *not* noticed by mathematicians, just as the extensive ambiguity in natural languages is "simply not noticed by the majority of language users and this testifies to the efficiency and robustness of their mechanisms for disambiguation" (Briscoe 22). Mathematicians'

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> mechanisms for disambiguating mathematical language are comparably efficient and robust (38).

Despite the pervasiveness of ambiguity in mathematical language, Ganesalingam claims that it contains no "vagueness":

> [W]hile mathematical language can contain (extensive) ambiguity, it is completely free from vagueness. All mathematical assertions are either true or false, with no middle ground. Similarly, mathematical language is not used metaphorically, ironically or in any similar way; the meaning of mathematical language is always its literal meaning. We can sum up effects of this kind by saying that all of the meaning of mathematics can be captured by (nearly) standard semantic representations to a much greater degree than is the case with general natural language (31-32).

The key to resolving ambiguities is through the combination of the textual material with the symbolic, "unlike anything in linguistics or computer science" (Ganesalingam 271). Ganesalingam, a mathematician and computer scientist, describes mathematics as a "single seamless language of which text and symbol are superficial facets" (271). This description reminds me of W. J. T. Mitchell's discussion of the relationship between words and images: "a complex one of mutual translation, interpretation, illustration, and embellishment" (530).

Douglas Hofstadter makes some interesting comments about "thought" that correlate well with Peat's observation about how mathematics involves nonverbal thinking. In his iconic book, *Goëdel, Escher, Bach: An Eternal Golden Braid*, Hofstadter describes thought as "images and intuitions and motivations" that "lie mingled close in the mind" (623). The concept of "thought" involving images rather than words is similar to Peat's description of mathematical mental activity being a kind of visual and sensory motor thinking. Some areas of mathematics such as geometry, integral calculus, and topology definitely have visual counterparts where images are useful. Other areas, however, such as number theory and abstract algebra, generally don't have associated imagery. Perhaps these areas involve a type of nonverbal thinking that is also

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nonvisual. Mitchell's linkage of words and images seems incomplete or inadequate in these complex areas.

Going well beyond the linguists' position that mathematics is a limited purpose language, Sundar Sarukkai argues that mathematics is the closest to being a "pure" language (3669). What does he mean by this statement? He means that it is not restricted by the physical realities of our real world:

> Mathematics is a "free" language, probably the freest language, for the simple reason that it is not moderated by the demands of the real world. Mathematics does not have to answer to the limitations of this world. Neither does it have the complete platonic world before its eyes which can limit its capacity to grow. Mathematics in this sense comes closest to a "pure" language, a language for its own sake. Applying mathematics is to make something of it, to bring it into the orbit of other verbal languages (3669).

It is the *application* of mathematics to the real world that makes it resemble natural language, which is rooted in the real world. Sarukkai also says that mathematics is really a family of languages, rather than a single language, because there are many areas of mathematical thinking. Some terminology and symbols are shared across many or all areas; others are more specialized.

> Mathematics is not one discipline although it masquerades as one. It is composed of many other disciplines such as real analysis, complex analysis, functional analysis, number theory, set theory, group theory, topology and so on. Strictly speaking, mathematics is a *family* of languages. Each of the languages corresponds to each of its many disciplines …. In this, it is quite opposite to English or other verbal languages (3668).

He expands upon this "family of languages" concept, positing that these related languages can create richer narratives than natural language:

> To me, the essence of mathematics as a language lies in its ability to *proliferate narratives* in a way that verbal languages cannot …. Mathematics, seen as a collection of many sub-languages, each with its own semantic universe, succeeds in creating very rich narratives about mathematical entities as well as about the world whenever correspondence between the world and mathematics is made …. (3668).

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Sarukkai's "narratives" are like du Sautoy's "stories," a form of mathematical literature. If "doing" mathematics is telling stories or narratives about mathematics, how does a person learn to tell such stories? That is, how does he or she become literate in the language of mathematics?

Learning the Language of Mathematics

We have seen that mathematicians and linguists agree that the language of mathematics shares many characteristics of a natural language, and, in some respects, goes beyond natural language in its ability to communicate narratives. Not surprisingly, the process of learning mathematics parallels the process of learning a natural language. David Moursund relates how a written mathematical language started with the development of natural written languages, or what he calls "general purpose" written languages. He likens learning mathematics to learning a foreign language (7-8). Like teaching a foreign language, math is taught using the methods of oral tradition:

> Students learn to recognize some math symbols and math words. However, many do not learn to read math at a level that allows them to learn math by reading math. It is only when students reach the more advanced high school math courses that there is a significant emphasis on learn the math by reading the math book and other resource materials (13).

Extending the analogy to learning a foreign language, Moursund says to learn math is to become a "native math language speaker," that is, someone who has a "high level of fluency in reading, writing, speaking, listening, thinking, and creative problem solving in the discipline of mathematics. A native math language speaker knows the culture of mathematicians. In short, a native math language speaker is a mathematician" (14). We might question whether the term "native math language speaker" is exactly appropriate, because it implies that mathematics is the

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person's first language. "Fluency" in the language of mathematics better fits the analogy to learning a foreign language.

Jamison, too, outlines the parallels to learning a foreign language:

> It requires painstaking study of details that, once grasped, pass naturally into the routine, just as a foreign language student must give meticulous attention to the declensions and conjugations so that he can use them later without consciously thinking of them. The learning tools are the same as those in a language class: writing, speaking, listening, memorizing models and learning the history and culture (47).

People outside the math community often describe mathematics as learning how to manipulate numbers and symbols. But symbolic expressions of mathematics are not what mathematics is; they are shorthand for the mathematician's thoughts. "Formulae are not ends in themselves but derive their real importance only as vehicles for expression of deeper mathematical thoughts" (Jamison 45).

To gain access to the discourse community of mathematicians, then, the student must learn both the written language – a symbolic language, a special vocabulary, and special rules of grammar – and a spoken language, which includes "math vernacular" as well as informal jargon. The informal jargon of mathematicians "consists of expressions such as 'conceptual proof' and 'intuitive.' These communicate something about the process of doing mathematics, but do not themselves communicate mathematics" (Wells 2). Informal jargon includes attitudes, behaviors, myths and cognitive phenomena connected with learning and doing mathematics (Wells 4-5).

What is "math vernacular"? Jamison describes it as the "rhetorical and syntactical structure of mathematical discourse" (47). As one example, he relates the tendency of students to use the active voice, as in "a relates to b," rather than standard mathematical usage of the passive voice, as in "a is related to b."

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> Attention to this single, simple linguistic detail seems to heighten the focus on *listening* [emphasis added] for proper usage and as a consequence proper understanding …. Shallow listening leads to shallow understanding (51).

Thus, math vernacular is important not just in communicating with other mathematicians but in understanding the mathematical concepts themselves. Note, too, the special role that "listening" plays. Listening to how the teacher "says mathematics" is just as important as learning the mathematical rules. The orality of mathematics, while seemingly subtle at times, is crucial to the student's learning process.

James W. Stigler and Ruth Baranes observe that math in the schoolroom is practiced very differently from other subjects: math is learned by listening to the teacher "tell" it (289). They relate a study that compared the teaching of social studies to the teaching of mathematics:

> Whereas social studies in the elementary school is learned through a combination of independent reading and group discussion, mathematics is learned by listening to the teacher "tell" it from the front of the room. Thus, children come to believe that although knowledge about social studies could be attained through their own reading or thinking, knowledge of mathematics can only be received from experts, not learned from a book or from thinking and discovery. And while statements about social studies can be evaluated by students and teachers alike on the basis of whether or not they make sense, solutions to mathematics problems are evaluated by the experts, who are assumed to have privileged knowledge of what the right answer is (289).

Their observations imply that orality is more important in teaching math than other school subjects. In fact, this conclusion is confirmed by my interviews with three college level mathematics instructors.

I interviewed the three instructors on October 12, 2012. Dan Kalman teaches advanced college mathematics at American University in Washington DC. Rich Zucker teaches at Irvine Valley College, a community college in California. Peter Taborek is chair of the physics department at University of California, Irvine, and teaches advanced mathematics to graduate level physics students. When I asked how they teach mathematics and whether in-person

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lectures are essential, they all said that college level mathematics generally is still taught through lectures. There was some disagreement, however, about whether the lectures are necessary.

Kalman stated that “there will always be a need for interaction in small numbers with an expert.” While there are many opportunities to take the live teacher out of the equation, through online delivery of lectures, the psychological element – what he calls “regimentation” – is still very important. The teacher provides both positive and negative reinforcement. I think he is talking about teaching math vernacular as well as the more formal elements of math literacy. However, he did admit that “one size does not fit all,” meaning that some students learn better by reading the text or by doing problems online, others learn better by listening to a lecture, and some need multiple channels.

Zucker believes that the teacher is essential to keep the students engaged and motivated – they need the “structure” of the classroom setting, similar to Kalman’s comment about “regimentation.” Zucker has started using the “flipped” classroom approach in many of his courses. The students view a videotaped lecture on their own, then do the homework problems in the classroom with the instructor walking around to observe and assist. The lecture is still an “oral” delivery even though it is electronic. What differs is that the students have the opportunity to ask questions during the homework session rather than during the lecture. Zucker’s view is that online learning, in general, won’t replace the mathematics classroom.

Taborek has a somewhat different opinion about the orality of learning mathematics. His view is that in-person lectures are not really needed. However, the students like the lectures and the social interaction of the classroom. Very possibly his opinion differs because his students are studying graduate level mathematics with specific applications to physics, rather than more basic college level mathematics.

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Despite the importance of orality in teaching mathematics, there is evidence that orality alone is not sufficient to "do" mathematics. Kalman said, "To the extent that mathematics is a language, math utterances are orthogonal to the issues of solving [math] problems." By this he means that what you say about a math problem is not the same as "doing" the math problem. You can't "do" mathematics through talking. Math literacy is needed to "do" mathematics.

In some respects, this distinction parallels the shift from an oral culture to a literate culture that Walter Ong discusses, but to me seems more profoundly key to becoming a mathematician. One can learn a great deal about history, for example, without reading about it. However, Ong stresses that learning history in the pre-literate world is based on memorization and formulaic repetition of oral stories (33-36). Learning history in a literate world operates very differently, through the analysis of written texts, records, and archives. Moreover, Ong claims that "literacy … is absolutely necessary for the development not only of science but also of history, philosophy, explicative understanding of literature and of any art, and … for the explanation of language (including oral speech) itself" (14-15). Maybe it is a matter of degree. One can become an excellent cook without being literate, one can become a good musician without being literate, but one cannot go beyond even basic arithmetic (counting on fingers and toes) without being math literate.

Arthur N. Applebee elaborates on the distinction between orality and literacy, which he calls "utterance" and "text." The utterance is a predominantly oral version of language and is highly context bound, while text is a predominantly written version of language that is independent of context.

> Meaning in utterances derives from context and background knowledge, while meaning in text derives from the premises in the text itself. Truth in utterances stems from consistency with "the wisdom of the elders," while truth in text is more scientific, a product of correspondence between statements and observations, whether or not the

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> conclusions make “common sense.” Finally, the functions of utterance are primarily interpersonal, while those of text are primarily logical or ideational (579).

He could well be describing mathematics utterances versus mathematics text. The “wisdom of the elders” is the teacher lecturing. Theorems and proofs, textbooks, and the mathematician’s “stories” are the “truth in text.”

Conclusion

G. H. Hardy observed that “the Greeks first spoke a language which modern mathematicians understand” (81). Indeed, the famous theorems of Euclid and Pythagoras are still considered among the most elegant of mathematical expression. The discourse community of mathematicians is not confined to the present – it extends across the centuries.

The language of mathematics bears similarities to natural language but does have some fundamental differences. Its precision and lack of vagueness are distinguishing features. Surprisingly, mathematics communication is full of ambiguity, but the combination of text with symbolic material completely disambiguates mathematical language.

Teaching math literacy parallels the teaching of natural language literacy. A key distinction, however, is that orality is important in learning mathematics but is not sufficient to “do” mathematics. Literacy is needed to “do” mathematics and to tell mathematical “stories.”

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