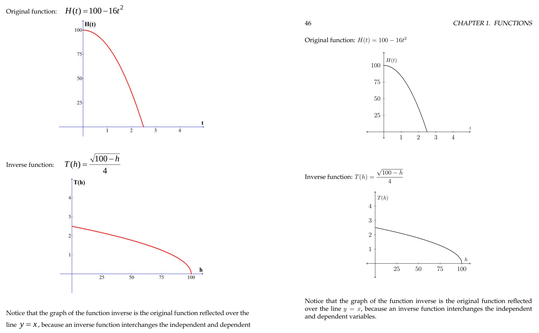
Creating Mathematics OER with LaTeX

Richard W. Beveridge

Clatsop Community College

First effort - Pre-Calculus

My first effort at an OER was a PreCalculus textbook written in MS Word using GRAPH graphing software (www.padowan.dk) for graphics. I've since translated this into LaTeX. Sample pages are shown below.



Original function: $H(t) = 100 - 16t^2$

Inverse function: $T(h) = \frac{\sqrt{100-h}}{4}$

Notice that the graph of the function inverse is the original function reflected over the line $y = x$, because an inverse function interchanges the independent and dependent variables.

39

Finding a formula for an inverse function can be more confusing when we consider a standard function $y = f(x)$. In our standard notation, x is **always** considered to be the independent variable and y is **always** considered to be the dependent variable.

Notice in the example above how when we graphed the function and its inverse, the label on the x axis changed from t to h. In a standard function, the x axis will always be the x axis and the y axis will always be the y axis. To compensate for this, when we find a function inverse for a function stated in terms of x and y, we generally interchange the x and y terms so that x remains the independent variable.

In our example, we had $H(t) = 100 - 16t^2$ and found the inverse to be $T(h) = \frac{\sqrt{100-h}}{4}$. If the original function had been stated in terms of x and y, then the process would have looked like this.

$$f(x) = 100 - 16x^2 \text{ or } y = 100 - 16x^2$$
$$16x^2 = 100 - y$$
$$x^2 = \frac{100-y}{16}$$
$$x = \sqrt{\frac{100-y}{16}} = \frac{\sqrt{100-y}}{4}$$

, then we switch the x and y variables to keep x as the independent variable.

$$y = f^{-1}(x) = \frac{\sqrt{100-x}}{4}$$

So, $f(x) = 100 - 16x^2$ and $f^{-1}(x) = \frac{\sqrt{100-x}}{4}$

40

46 CHAPTER 1. FUNCTIONS

Original function: $H(t) = 100 - 16t^2$

Inverse function: $T(h) = \frac{\sqrt{100-h}}{4}$

Notice that the graph of the function inverse is the original function reflected over the line $y = x$, because an inverse function interchanges the independent and dependent variables.

1.8. INVERSE FUNCTIONS 47

Finding a formula for an inverse function can be more confusing when we consider a standard function $y = f(x)$. In our standard notation, x is *always* considered to be the independent variable and y is *always* considered to be the dependent variable.

Notice in the example above that when we graphed the function and its inverse, the label on the x axis changed from t to h. In a standard function, the x axis will always be the x axis and the y axis will always be the y axis. To compensate for this, when we find a function inverse for a function stated in terms of x and y, we generally interchange the x and y terms so that x remains the independent variable.

In our example, we had

$$H(t) = 100 - 16t^2$$

and found the inverse to be

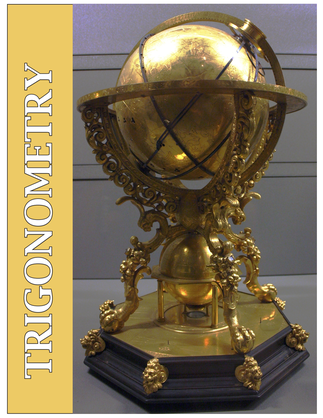
$$T(h) = \frac{\sqrt{100-h}}{4}$$

If the original function had been stated in terms of x and y, then the process would have looked like this:

$$f(x) = 100 - 16x^2$$
$$y = 100 - 16x^2$$
$$16x^2 = 100 - y$$
$$x^2 = \frac{100-y}{16}$$
$$x = \sqrt{\frac{100-y}{16}}$$
$$x = \frac{\sqrt{100-y}}{4}$$

Second Effort - Trigonometry

My next effort was a Trigonometry textbook written in LaTeX. There was a significant learning curve, however LaTeX is a powerful open source publishing software package. The cover was done using an open source photo from Wikimedia and our graphic designer on campus put it together using InDesign.



Third Effort - College Algebra

The cover for the College Algebra text I did myself, again using a Wikimedia image and Adobe Acrobat together with LaTeX. Downloading and setting up LaTeX can be daunting, but I believe that it is worth the effort. I had the opportunity at ORMATYC a few years ago to attend a session on LaTeX with Alex Jordan from Portland CC, which was very helpful.

College Algebra

LaTeX code

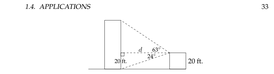
Some important parts of LaTeX code involve the "tabular" command, which allows you to manually establish tabs in a page, and the "tikzpicture" and "pgfplots" commands which control graphing.

```
\textbf{\Large Exercises 2.4}

Solve each inequality.

\begin{tabular}{l @{\hspace{1.5in}} l}
1)\SP $\D \frac{x+4}{x^2-8x+12} > 0$ & 2)\SP $\D
\frac{2x+3}{x^2-2x-35} < 0$\\
& \\
& \\
3)\SP $\D \frac{x^2-5x-14}{x^2+3x-10} < 0$ & 4)\SP
$\D \frac{2x^2-x-3}{x^2+10x+16} > 0$\\
& \\
& \\
5)\SP $\D \frac{3x+2}{x^2+x-5} < 0$ & 6)\SP
$\D\frac{x^2+2x+5}{x^2-3x-7} > 0$\\
& \\
& \\
7)\SP $\D \frac{x^3+9}{x^2+x-1} > 0$ & 8)\SP $\D
\frac{x^3+9}{x^2+x+1} > 0$\\
\end{tabular}

\vspace{0.5in}

Solve each inequality.

\begin{tabular}{l @{\hspace{1.5in}} l}
9)\SP $\D \frac{x^2-2x-9}{3x+11} > 0$ & 10)\SP $\D
\frac{x^2+4x+3}{2x+1} < 0$\\
& \\
& \\
11)\SP $\D \frac{x^2+x-5}{x^2-x-6} > 0$ & 12)\SP
$\D \frac{x^3+2}{x^2-2} > 0$\\
& \\
& \\
13)\SP $\D \frac{x^2+2x-7}{x^2+3x-6} < 0$ & 14)\SP
$\D\frac{2x-x^2}{x^2-4x+6} < 0$\\
& \\
& \\
15)\SP $\D \frac{x^2-7}{x^2+5x-1} < 0$ & 16)\SP $\D
\frac{x-5}{3x^2-2x-3} > 0$\\
\end{tabular}
```

```
\textbf{Example}

$\D \frac{x^2+2x-1}{x^2+7x+5} \leq 0$

\begin{centering}

\begin{tikzpicture}
\begin{axis}[axis lines=middle, xmin=-8, xmax=8,
ymin=-8, ymax=8, samples=200]
\addplot[domain=-8:-6.3,thick, <->] { (x^2 + 2*x -
1) / (x^2 + 7*x + 5)};
\addplot[domain=-6:-0.85,thick, <->] { (x^2 + 2*x -
1) / (x^2 + 7*x + 5)};
\addplot[domain=-0.75:8,thick,<->] { (x^2 + 2*x -
1) / (x^2 + 7*x + 5)};
\draw[dashed,<->] (axis cs:-6.2,-8)--(axis cs:-
6.2,8);
\draw[dashed,<->] (axis cs:-0.807,-8)--(axis cs:-
0.807,8);
\end{axis}
\end{tikzpicture}

\end{centering}

\underline{\textbf{Roots}}

$x^2+2x-1=0$

$x \approx -2.414, 0.414$

\underline{\textbf{Asymptotes}}

$x^2+7x+5=0$

$x \approx -6.193, -0.807$

We can see that the dividing points important to
the solution of the inequality are $x \approx -
6.193, -2.414, -0.807, 0.414$. The intervals where
the $y$ values are less than or equal to zero are
$-6.193 \leq x \leq -2.414$ OR $-0.807 \leq x \leq
0.414$.

\newpage
```

Sample pages from the Trigonometry and College Algebra texts

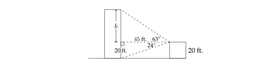
1.4. APPLICATIONS 33



If we work on the bottom triangle first, then we know that the height of the triangle is 20 ft. and the angle opposite this side is 24°. So, we can say that:

$$\tan 24° = \frac{20}{d}$$
$$d = \frac{20}{\tan 24°}$$
$$d \approx 45 ft.$$

Now that we know that the apartment building is 45 feet away, we can use the upper triangle to determine the height othe building.



$$\tan 65° = \frac{b}{45}$$
$$45 * \tan 65° = b$$
$$88.3 ft. \approx b$$
$$20 + 88.3 = 108.3 ft.$$

Since the variable b only represents the part of the building that is in the second triangle, we need to add 20 feet to b to find the actual height of the building.

2.2. GRAPHING TRIGONOMETRIC FUNCTIONS 67

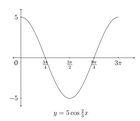
Example 2

Graph one full period of the function $y = 5\cos\frac{1}{2}x$.

The amplitude of the function is 5 because $A = 5$, so the maximum y-value will be 5 and the minimum y-value will be -5. The period of the graph will be $\frac{2\pi}{B}$, which in this case is $\frac{2\pi}{\frac{1}{2}} = 2\pi * \frac{2}{1} = 3\pi$. So the period is 3π. The critical values along the x-axis will start at 0 and be separated by "jumps" of $3\pi * \frac{1}{4} = \frac{3\pi}{4}$. So the critical values along the x-axis will be:

$$0, \frac{3\pi}{4}, \frac{6\pi}{4}, \frac{9\pi}{4}, \text{ and } \frac{12\pi}{4}$$

We want to express these in lowest terms so we would label them as $\frac{0\pi}{4}, \frac{3\pi}{4}, \frac{6\pi}{4}, \frac{9\pi}{4}$, and 3π. The graph will start at the maximum y-value of 5 at $x = 0$, then it will go to zero at $x = \frac{3\pi}{4}$, down to the minimum y-value of -5 at $x = \frac{6\pi}{4}$, back through 0 at $x = \frac{9\pi}{4}$, and then up to the maximum y-value of 5 at $x = 3\pi$ to complete one full period of the graph. The graph of $y = 5\cos\frac{1}{2}x$, is shown below.



$y = 5\cos\frac{1}{2}x$

2.7. SYNTHETIC DIVISION 121

$$\begin{array}{r} 2x^3 + 4x^2 \boxed{-3x} \\ x-5 \overline{)\ 2x^4 - 6x^3 - 23x^2 + 16x - 5} \\ \underline{-2x^4 + 10x^3} \\ 4x^3 - 23x^2 \\ \underline{-4x^3 + 20x^2} \\ \boxed{-3x^2} + 16x \\ \underline{3x^2 - 15x} \\ \boxed{1x} - 5 \\ \underline{-1x + 5} \\ 0 \end{array}$$

The last part of our answer will come from multiplying the -3 in the answer times the 5 in the divisor (making -15) and combining this with the $+16$ in the polynomial we're dividing into:

$$\begin{array}{r} 2x^3 + 4x^2 - 3x \boxed{+1} \\ x-5 \overline{)\ 2x^4 - 6x^3 - 23x^2 + 16x - 5} \\ \underline{-2x^4 + 10x^3} \\ 4x^3 - 23x^2 \\ \underline{-4x^3 + 20x^2} \\ -3x^2 + 16x \\ \underline{3x^2 - 15x} \\ \boxed{1x} - 5 \\ \underline{-1x + 5} \\ 0 \end{array}$$

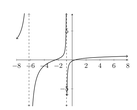
The process of Synthetic Division uses these relationships as a shortcut to finding the answer. The set-up for a Synthetic Division problem is shown below:

$$5 \;|\; 2 \quad -6 \quad -23 \quad 16 \quad -5$$

106 CHAPTER 2. POLYNOMIAL AND RATIONAL FUNCTIONS

Example

$\frac{x^2+2x-1}{x^2+7x+5} \leq 0$



Roots

$x^2 + 2x - 1 = 0$

$x \approx -2.414, 0.414$

Asymptotes

$x^2 + 7x + 5 = 0$

$x \approx -6.193, -0.807$

We can see that the dividing points important to the solution of the inequality are $x \approx -6.193, -2.414, -0.807, 0.414$. The intervals where the y values are less than or equal to zero are $-6.193 \leq x \leq -2.414$ OR $-0.807 \leq x \leq 0.414$.

I've put together all three of these texts into a single 500 page "College Algebra and Trigonometry" text, which I have available on a flash drive, or you can send me an email and I'll send you a copy.

College Algebra & Trigonometry

Contact Information

- Email: rbeveridge@clatsopcc.edu