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Complexity and Decomposability of Relations

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COMPLEXITY AND DECOMPOSABILITY OF RELATIONS

ABSTRACT

IS THERE A GENERAL DEFINITION OF STRUCTURE?

WHAT DOES KNOWING STRUCTURAL COMPLEXITY GIVE YOU?

OPEN QUESTIONS, CURRENT RESEARCH, WHERE LEADING

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ABSTRACT

A discrete multivariate relation, defined set-theoretically, is a subset of a cartesian product of sets which specify the possible values for two or more variables. Where three or more variables are involved, the highest order relation, namely the relation between all of the variables, may or may not be decomposable without loss into sets of lower order relations which involve subsets of the variables. In a completely parallel manner, a relation defined information-theoretically, namely a joint probability distribution involving all the variables, may or may not be decomposed without loss into lower-order distributions involving subsets of the variables. Decomposability analysis, also called "reconstructability analysis," is the specification of the losses suffered by all possible decompositions.

The decomposability of relations, defined either set- or information-theoretically, offers a fundamental approach to the idea of "complexity" and bears on all of the themes prominent in both the new and the old "sciences of complexity."

Decomposability analysis gives precise meaning to the idea of "structure," i.e., to the interrelationship between a whole and its parts, where these are conceived either statically or dynamically. It specifies the structuring and distribution and the amount of information needed to describe complex systems. It sheds specific light on chaotic versus non-chaotic dynamics in discrete dynamic systems. It provides a framework for characterizing the dual processes of integration and differentiation which govern the diachronics of self-organization.

IS THERE A GENERAL DEFINITION OF STRUCTURE?

ANSWER: Yes, a **structure** is a set of **relations**.

1. **Relation** = a *constraint* linking entities, e.g., variables.
2. Variables can be *nominal* \Rightarrow discipline-general; can be *dynamic*.
3. Constraint defined, e.g., (a) set- or (b) info.-theoretically, i.e.,
(a) subset of cartesian product or (b) multivariate probability distribution.
4. **Projections** of relation define **lattice of relations (LOR)**.
5. **Structure** = *cut* through LOR = **decomposition** of a relation.
6. **Complexity**₁*(structure) = # degrees of freedom (info.-theor.)
= # parameters needed to specify it
7. Represents *topology*, not *strength*, of constraints.
8. *Resolution*-dependent; data-*independent*.
9. **Lattice of structures (LOS)** = all possible decompositions.

*Other definitions possible *in this framework*.

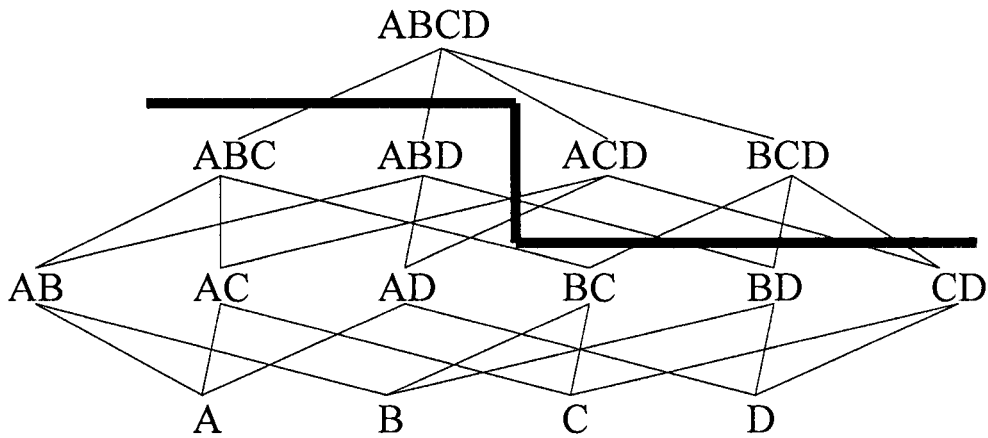
RELATION:

$$R = \{ (a_i, b_j, c_k, d_l) \} \subseteq A \otimes B \otimes C \otimes D \quad \text{SET-THEOR.}$$

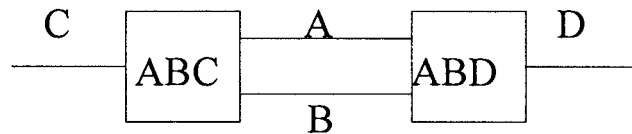
$$R = \{ p(a_i, b_j, c_k, d_l) \} \quad \text{INFO.-THEOR.}$$

PROJECTION: $R \equiv R_{ABCD} \Rightarrow R_{ABC}$ call R_X simply X

LATTICE OF RELATIONS (LOR)



STRUCTURE = CUT thru LOR, e.g., ABC:ABD



COMPLEXITY₁ (info.-theor.) = DEGREES OF FREEDOM

$$\begin{aligned} \text{df}(ABC : ABD) &= \text{df}(ABC) + \text{df}(ABD) - \text{df}(AB) \\ &= N_{ABC} - 1 + N_{ABD} - 1 - N_{AB} + 1 \end{aligned}$$

WHAT DOES KNOWING STRUCTURAL COMPLEXITY GIVE YOU?

Descr. length/incompressibility/randomness of constraint

related to dimension. INTEGRALITY.

CONTEXT: RECONSTRUCTABILITY (log-linear) MODELING

1. **MAX ENTROPY** distrib. given **CONSTRAINTS (model)**.

e.g., $\max H_{\text{Shannon}}(q_{ABCD})$ given p_{ABC}, p_{ABD}

(Equivalent $\max H_{\text{Hartley}}$ for *set-theor.* reconstruction)

2. Descend LOS to **simplest lossless structure**.

3. *Further* decomposition \Rightarrow **loss**. (Some might be *acceptable*.)

USE: TRADE OFF **LOSS vs. COMPLEXITY**

e.g., $LR-\chi^2 + df \Rightarrow p(\text{error});$ like RISSANEN descr. length

EX. of PREDICTIVE USE:

df (SIMPLEST LOSSLESS structure), LOSSES (df)

improves CHAOS prediction in Elem. Cellular Automata.

PREDICTING CHAOS IN ELEM. CELLULAR AUTOMATA
(REDUCTION OF ATTRACTOR UNCERTAINTY)

q = rule attribute

	q	H(a q)	% ΔH	Pred.Power
	-	.679		
Walker-Ashby, Langton	λ	.600	11.6	.044
COMPLEXITY ₁	σ	.553	18.6	.069
LOSS SPECTRUM	τ	.263	61.3	.102
Wuensche	Z	.458	32.6	.114

$$H(\lambda | \tau) = H(\sigma | \tau) = H(Z | \tau) = 0$$

CONCLUSIONS

τ vector is the **BEST OVERALL (% ΔH) PREDICTOR**

τ **SUBSUMES** ALL OTHER MEASURES

EVEN THOUGH:

SIMPLE LAWS ⇒ COMPLEX DYNAMICS

STILL:

MORE COMPLEX_{NON-DECOMPOSABLE} LAWS ⇒

MORE COMPLEX_{CHAOTIC} DYNAMICS

OPEN QUESTIONS, CURRENT RESEARCH, WHERE LEADING

THEORY/METHOD

1. Generalization of

- complexity₁ to set-theoretic relations
- reconstructability to fuzzy-distributions, non-max-H criteria

2. Improved algorithms for

- searches through big lattices
- optimal binning of quant. variables
- optimal complexity/loss tradeoff

3. COMPLEXITY₂ : maximized **between** top and bottom of LOS

- topological complexity
- #structures(df)
- sensitivity of complexity/loss tradeoff

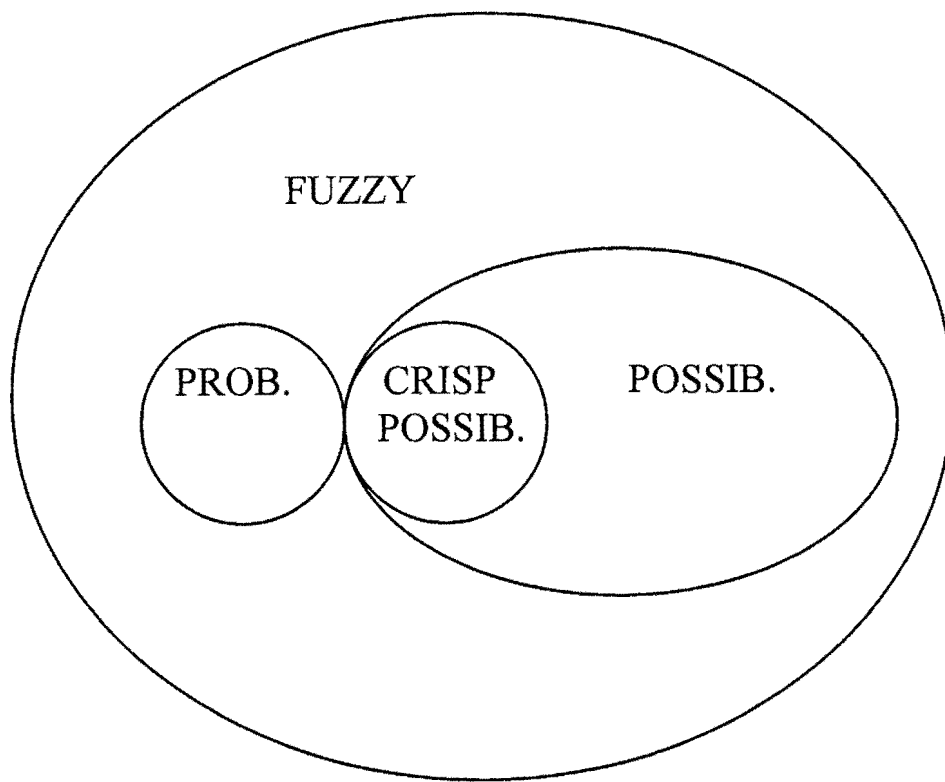
SOME APPLICATIONS (reconstructability, complexity measure)

4. Applications of reconstructability to DATA-MINING.

5. Meta-dynamics of *differentiation/integration* (wholes \Leftrightarrow parts).

6. Extend CA work

- more complex CAs
- boolean nets
- relate to continuous systems



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