3-12-2010

Reconstructability Analysis of Elementary Cellular Automata

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RECONSTRUCTABILITY OF ELEMENTARY CELLULAR AUTOMATA

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Abstract

Reconstructability analysis is a method to determine whether a multivariate relation, defined set- or information-theoretically, is decomposable with or without loss (reduction in constraint) into lower ordinality relations. Set-theoretic reconstructability analysis (SRA) is used to characterize the mappings of elementary cellular automata. The degree of lossless decomposition possible for each mapping is more effective than the λ parameter (Walker & Ashby, Langton) as a predictor of chaotic dynamics.

Complete SRA yields not only the simplest lossless structure but also a vector of losses of all decomposed structures, indexed by parameter, τ. This vector subsumes λ, Wuensche's Z parameter, and Walker & Ashby's "fluency" and "memory" parameters within a single framework, and is a strong but still imperfect predictor of the dynamics: less decomposable mappings more commonly produce chaos. The set-theoretic constraint losses are analogous to information distances in information-theoretic reconstructability analysis (IRA). IRA captures the same information as SRA, but allows λ, fluency, and memory to be explicitly defined.

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April 2005

1. ELEMENTARY CELLULAR AUTOMATA

<u>THE PROBLEM</u>: PREDICTING CHAOTIC DYNAMICS FROM RULE ATTRIBUTES

2. RECONSTRUCTABILITY ANALYSIS: SET & INFO.-THEORETIC

A METHOD FOR DECOMPOSABILITY (SIMPLICITY) ANALYSIS

3. RECONSTRUCTABILITY of ECAs

λ, σ, τ, f, Z *AS PREDICTORS OF CHAOS*

ELEMENTARY CELLULAR AUTOMATA

<u>THE PROBLEM</u>: PREDICTING CHAOTIC DYNAMICS FROM RULE ATTRIBUTES

1. (original motivation) Langton's SELF-REPLICATING LOOP

2. ECA STRUCTURE AND DYNAMICS

3. PREDICTING DYNAMICS FROM STRUCTURE

```
  2 2 2 2 2 2 2 2
2 1 7 0 1 4 0 1 4 2
2 0 2 2 2 2 2 2 0 2
2 7 2         2 1 2
2 1 2         2 1 2
2 0 2         2 1 2
2 7 2         2 1 2
2 1 2 2 2 2 2 2 1 2 2 2 2 2
2 0 7 1 0 7 1 0 7 1 1 1 1 1 2
  2 2 2 2 2 2 2 2 2 2 2 2 2

      TIME = 0
```

Fig. 6. Self-reproducing loop.

Table I
Transition function table for self-reproducing loops

CTRBL->I	CTRBL->I	CTRBL->I	CTRBL->I	CTRBL->I
00000->0	02527->1	11322->1	20242->2	30102->1
00001->2	10001->1	12224->4	20245->2	30122->0
00002->0	10006->1	12227->7	20252->0	30251->1
00003->0	10007->7	12243->4	20255->2	40112->0
00005->0	10011->1	12254->7	20262->2	40122->0
00006->3	10012->1	12324->4	20272->2	40125->0
00007->1	10021->1	12327->7	20312->2	40212->0
00011->2	10024->4	12425->5	20321->6	40222->1
00012->2	10027->7	12426->7	20322->6	40232->6
00013->2	10051->1	12527->5	20342->2	40252->0
00021->2	10101->1	20001->2	20422->2	40322->1
00022->0	10111->1	20002->2	20512->2	50002->2
00023->0	10124->4	20004->2	20521->2	50021->5
00026->2	10127->7	20007->1	20522->2	50022->5
00027->2	10202->6	20012->2	20552->1	50023->2
00032->0	10212->1	20015->2	20572->5	50027->2
00052->5	10221->1	200[illegible]1->2	20622->2	50052->0
00062->2	10224->4	20022->2	20672->2	50202->2
00072->2	10226->3	20023->2	20712->2	50212->2
00102->2	10227->7	20024->2	20722->2	50215->2
00112->0	10232->7	20025->0	20742->2	50222->0
00202->0	10242->4	20026->2	20772->2	50224->4
00203->0	10262->5	20027->2	21122->2	50272->2
00205->0	10264->4	20032->6	21126->1	51212->2
00212->5	10267->7	20042->3	21222->2	51222->0
00222->0	10271->0	2005[illegible]->7	21224->2	51242->2
00232->2	10272->7	20052->2	21226->2	51272->2
00522->2	10542->7	20057->5	21227->2	60001->1
01232->1	11112->1	20072->2	21422->2	60002->1
01242->1	11122->[illegible]	20102->2	21522->2	60212->0
01252->5	111[illegible]4->4	20112->2	21622->2	6121[illegible]->[illegible]
01262->1	11125->1	20122->2	21722->2	6[illegible]21[illegible]->1
01272->1	11126->1	20142->2	22[illegible]27->2	[illegible]122[illegible]->5
01275->1	11127->7	2017[illegible]->2	[illegible]22[illegible]4->2	70007->7
01422->1	11152->2	2020[illegible]->[illegible]	[illegible]2246->2	70112->0
01432->1	11212->1	20203->2	22276->2	70122->0
01442->1	11222->1	20205->2	22277->2	70125->0
01472->1	11224->4	20207->3	30001->3	70212->0
01625->1	112[illegible]->[illegible]	20212->2	30002->2	70222->1
01722->1	112[illegible]7->7	20215->2	30004->1	70225->1
01725->5	112[illegible]2->1	20221->2	30007->6	70232->1
01752->1	11242->4	20222->2	[illegible]0012->[illegible]	70252->5
01762->1	11262->1	20227->2	3[illegible]042->1	70272->0
01772->1	11272->7	20232->1	30062->2	

Neighborhoods are read as follows (rotations are not listed):

```
  T
L C R      ==>      I
  B
```

QUESTIONS

(stimulated by Langton's self-replicating CA):

1. IS THERE A

- MINIMUM COMPLEXITY <u>PATTERN</u>
- MINIMUM COMPLEXITY <u>RULE</u>
- MINIMUM COMPLEXITY <u>PATTERN + RULE</u>

i.e., a *THRESHOLD* of *EMERGENCE*

FOR REPLICATION?

2. PATTERN COMPLEXITY CAN BE QUANTIFIED

BY SHANNON ENTROPY, BUT

HOW SHOULD RULE COMPLEXITY BE DEFINED?

RULE COMPLEXITY =?= <u>ALGORITHMIC</u> INFORMATION

```
 22222222
217 14 142
2 222222 2
272    212
212    212
2 2    212
272    212
21222222122222
2 71 71 7111112
 2222222222222
```

Fig. 1.

2. Langton's automaton

Table I
Transition function table for

```
 222
21412
23 32
21312
 225
```

CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I	
00003	1	10000	0	20000	0	30001	0	40003	5	50001	0
00012	2	10001	0	20015	5	30003	0	40022	5	50022	5
00013	1	10004	0	20022	0	30011	0	40035	2	50032	5
00015	4	10033	0	20035	5	30235	3	40043	4	50122	5
00025	4	10043	1	20202	0	30245	5	40212	4	50222	0
00031	5	10325	5	20215	5	31235	5	40232	4	50244	5
00032	3	10421	4	20235	5	3----	1	40242	4	50322	5
00042	2	10423	4	20252	5			40252	0	50412	4
00121	1	10424	4	2----	2			40325	5	50422	0
00204	2	11142	4					41452	5	5----	2
00324	3	11423	4					4----	1		
00422	2	12234	4								
00532	3	12334	4								
0----	0	12443	4								
		1----	3								

Neighborhoods are read as follows (rotations are not listed):

```
  T
L C R → I
  B
```

Table II
Transition function table for

```
22
2312
2342
25
```

CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I	
00003	1	10003	3	20215	5	31215	1	40242	4
00012	2	10004	0	20235	3	31223	1	40252	0
00013	1	10033	0	20252	5	31233	1	40325	5
00015	2	10043	1	2----	2	31235	5	4----	3
00025	5	10321	3	30001	0	31432	1	50022	5
00031	5	11253	1	30003	0	31452	5	50032	5
00032	3	12453	3	30011	0	3----	3	50212	4
00042	2	1----	4	30012	1			50222	0
0----	0	20000	0	30121	1	40003	5	50322	0
		20015	5	30123	1	40043	4	5----	2
10000	0	20022	0	31122	1	40212	4		
10001	0	20202	0	31123	1	40232	4		

```
 22
2312
2342
  5
```

(a)

With the addition of 2 more rules (for a total of 59) the structure depicted in fig. 5(a) will reproduce itself.

```
 22
 312
2342
  5
```

(c)

If four further transition rules are applied (for a total of 63) then the 10 cell structure of fig. 5(c) will produce 2 exact copies of itself before becoming sterile.

ELEMENTARY CELLULAR AUTOMATA (ECA)

1-d *circular* cell array; k=2 discrete states: S = [0,1]

neighborhood = adjacent single (r=1) cell on left & on right

mapping (rule): f: $S_t(i-1) \otimes S_t(i) \otimes S_t(i+1) \rightarrow S_{t+1}(i)$

an example: ECA RULE # 150 (10010110)

t: i-1 (A)	t: i (B)	t: i+1 (C)	t+1: i (D)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

2^8 = 256 mappings

t	**0**	**1**	**1**		A	B	C
t+1		**0**				D	

DYNAMICS (8 cell circular array):

t	0	1	0	1	1	0	1	0
t+1	1	1	0	0	0	0	1	1
t+2	1	0	1	0	0	1	0	1

1.3 EQUIVALENCE CLASSES of ECA MAPPINGS

$f(s(i-1),s(i),s(i+1))$ EQUIVALENT UNDER

(1) REFLECTION to

$$f_1(s(i-1),s(i),s(i+1)) = f(s(i+1),s(i),s(i-1))$$

(2) COMPLEMENTING to

$$f_2(s(i-1),s(i),s(i+1)) = \neg f(\neg s(i-1),\neg s(i),\neg s(i+1))$$

(3) REFLECTION & COMPLEMENTING to

$$f_3(s(i-1),s(i),s(i+1)) = \neg f(\neg s(i+1),\neg s(i),\neg s(i-1))$$

88 EQUIVALENCE CLASSES (with 1*, 2, or 4 members)

ECA RULES

Equivalence class #	Representative member	Other members		
1		255		
2	1	127		
3	2	16	191	247
4	3	17	63	119
5	4	223		
6	5	95		
7	6	20	159	215
8	7	21	31	87
9	8	64	239	253
10	9	65	111	125
11	10	80	175	245
12	11	47	81	117
13	12	68	207	221
14	13	69	79	93
15	14	84	143	213
16	15	85		
17	18	183		
18	19	55		
19	22	151		
20	23*			
21	24	66	189	231
. . .				
. . .				
. . .				
86	200	236		
87	204			
88	232			

0	255			26	82	167	181	56	98	185	227	136	192	238	252
1	127			27	39	53	83	57	99			137	110	124	193
2	16	191	247	28	70	157	199	58	114	163	177	138	174	208	244
3	17	63	119	29	71			60	102	153	195	140	196	206	220
4	223			30	86	135	149	72	237			142	212		
5	95			32	251			73	109			146	182		
6	20	159	215	33	123			74	88	173	229	150			
7	21	31	87	34	48	187	243	76	205			152	188	194	230
8	64	239	253	35	49	59	115	77				154	166	180	210
9	65	111	125	36	219			78	92	141	197	156	198		
10	80	175	245	37	91			90	165			160	250		
11	47	81	117	38	52	155	211	104	233			161	122		
12	68	207	221	40	96	235	249	105				162	176	186	242
13	69	79	93	41	97	107	121	106	120	169	225	164	218		
14	84	143	213	42	12	171	241	108	201			168	224	234	248
15	85			43	113			128	254			170	240		
18	183			44	100	203	217	129	126			172	202	216	228
19	55			45	75	89	101	130	144	190	246	178			
22	151			46	116	139	209	131	62	118	145	184	226		
23				50	179			132	222			200	236		
24	66	189	231	51				133	94			204			
25	61	67	103	54	147			134	148	158	214	232			

Table-3. The 88 equivalence classes of ECA rules. The first rule number is the "representative member" of the class; following it are the other members (if any) of the class.

1.4 DYNAMICS: CLASSIFICATION OF ATTRACTOR TYPES

WOLFRAM CLASSIFICATION:

I. HOMOGENEOUS
II. FIXED POINT OR PERIODIC
III. CHAOTIC
IV. COMPLEX

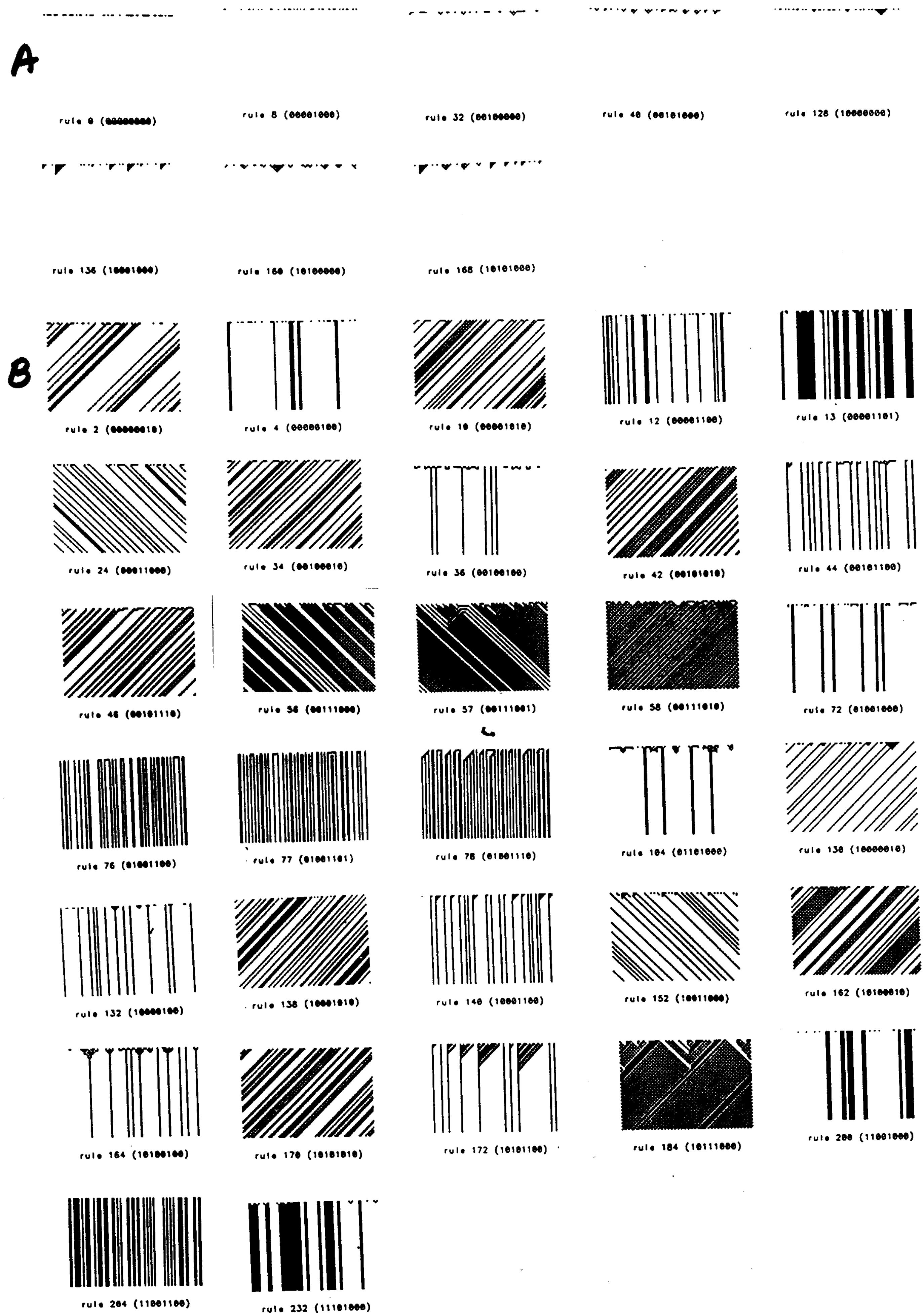
LI & PACKARD CLASSIFICATION:

A. NULL (8) (I)	0	8	32	40	128	136	160	168		
B. FIXED POINT (32) (II)	2	4	10	12	13	24	34	36	42	44
	46	56	57	58	72	76	77	78	104	130
	132	138	140	152	162	164	170	172	184	200
	204	232								
C. PERIODIC (31) (II)	1	3	5	6	7	9	11	14	15	19
	23	25	27	28	29	33	35	37	38	41*
	43	50	51	74	108	131	133	134	142	156
	178									
D. LOCALLY CHAOTIC (3) (II or III)	26	73	154							
E. CHAOTIC (III) (14)	18	22	30	45	54	60	90	105	106	129
	137	146	150	161		(54 137: IV)				

* may be chaotic

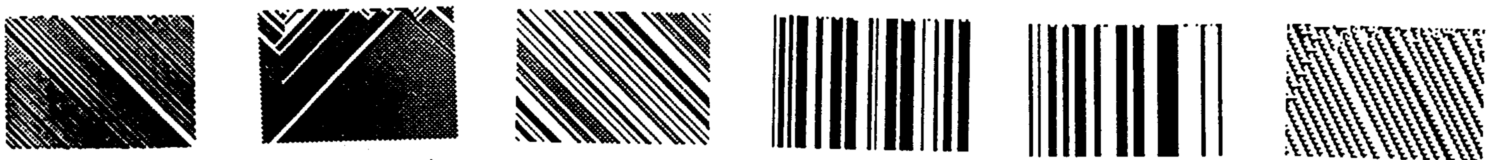
MINIMAL CLASSIFICATION

1. NON-CHAOTIC ([illegible]: null, fixed point, periodic)
2. CHAOTIC ([illegible]: locally chaotic, chaotic)

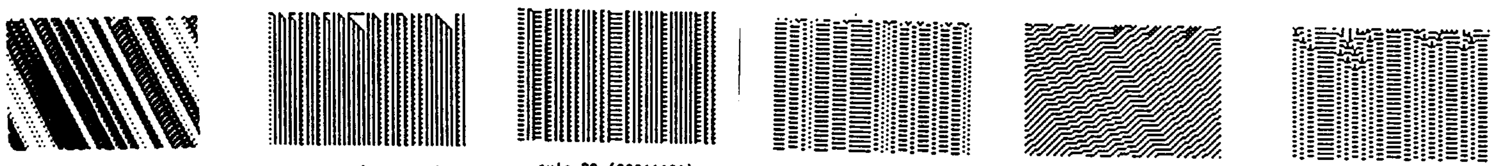

A
rule 0 (00000000)
rule 8 (00001000)
rule 32 (00100000)
rule 40 (00101000)
rule 128 (10000000)
rule 136 (10001000)
rule 160 (10100000)
rule 168 (10101000)
B
rule 2 (00000010)
rule 4 (00000100)
rule 10 (00001010)
rule 12 (00001100)
rule 13 (00001101)
rule 24 (00011000)
rule 34 (00100010)
rule 36 (00100100)
rule 42 (00101010)
rule 44 (00101100)
rule 46 (00101110)
rule 56 (00111000)
rule 57 (00111001)
rule 58 (00111010)
rule 72 (01001000)
rule 76 (01001100)
rule 77 (01001101)
rule 78 (01001110)
rule 104 (01101000)
rule 130 (10000010)
rule 132 (10000100)
rule 138 (10001010)
rule 140 (10001100)
rule 152 (10011000)
rule 162 (10100010)
rule 164 (10100100)
rule 170 (10101010)
rule 172 (10101100)
rule 184 (10111000)
rule 200 (11001000)
rule 204 (11001100)
rule 232 (11101000)

C

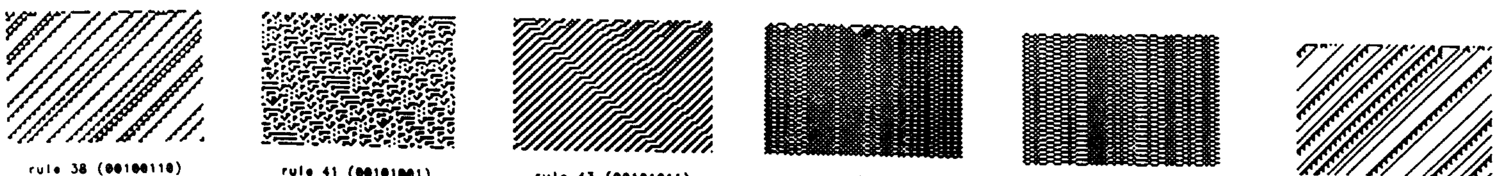
ule 1 (00000001) · rule 3 (00000011) · rule 5 (00000101) · rule 6 (00000110) · rule 7 (00000111) · rule 9 (00001001)



rule 11 (00001011) · rule 14 (00001110) · rule 15 (00001111) · rule 19 (00010011) · rule 23 (00010111) · rule 25 (00011001)

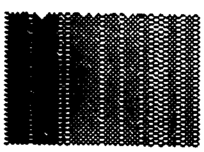


rule 27 (00011011) · rule 28 (00011100) · rule 29 (00011101) · rule 33 (00100001) · rule 35 (00100011) · rule 37 (00100101)



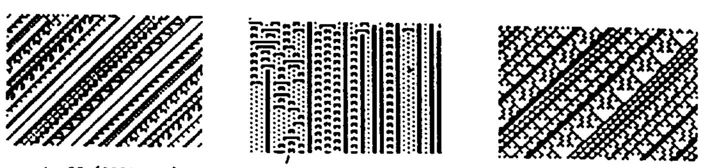
rule 38 (00100110) · rule 41 (00101001) · rule 43 (00101011) · rule 50 (00110010) · rule 51 (00110011) · rule 74 (01001010)

le 108 (01101100) · rule 62 (00111110) /31 · rule 94 (01011110) /33 · rule 134 (10000110) · rule 142 (10001110) · rule 156 (10011100)



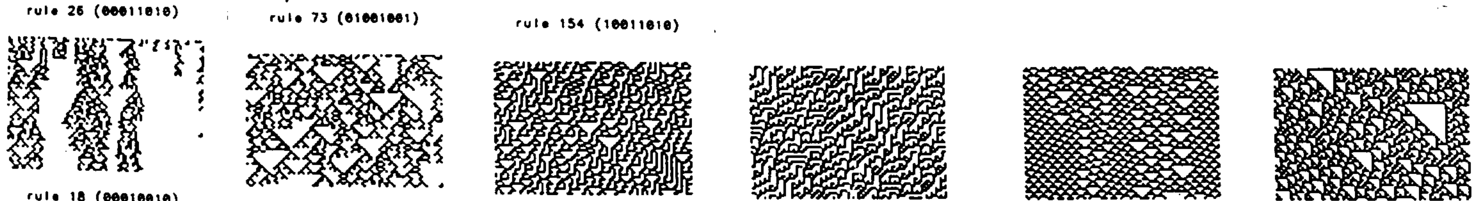
rule 178 (10110010)

D

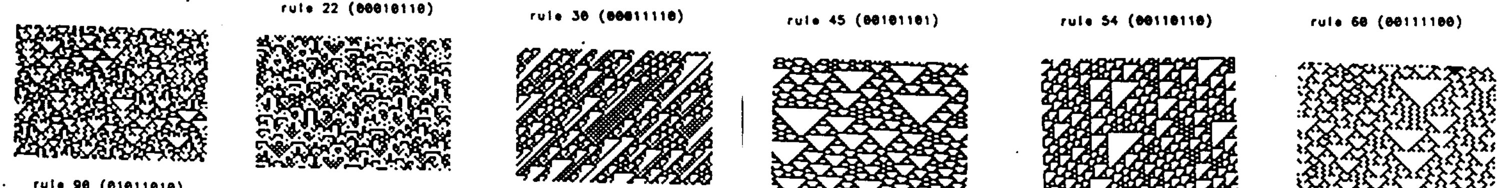


rule 26 (00011010) · rule 73 (01001001) · rule 154 (10011010)

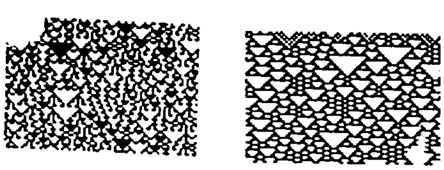
E



rule 18 (00010010) · rule 22 (00010110) · rule 30 (00011110) · rule 45 (00101101) · rule 54 (00110110) · rule 60 (00111100)

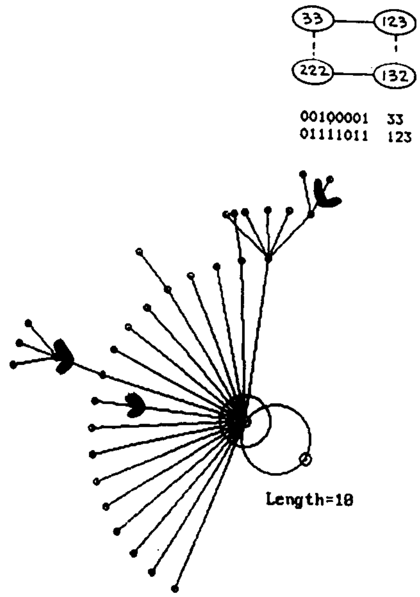


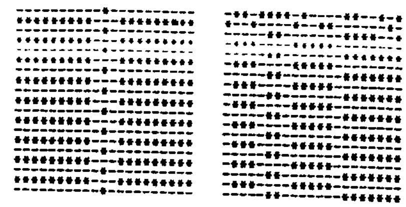
rule 90 (01011010) · rule 105 (01101001) · rule 106 (01101010) · rule 126 (01111110) 129 · rule 110 (01101110) 137 · rule 146 (10010010)

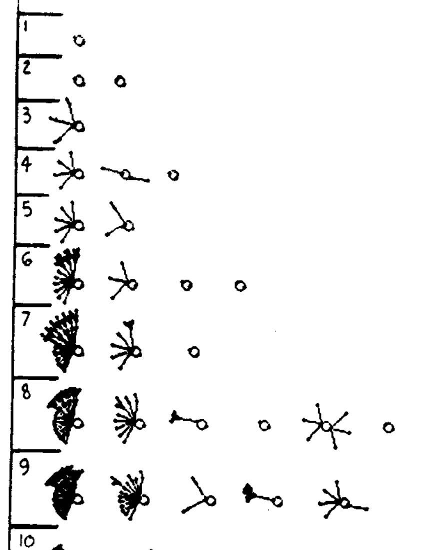


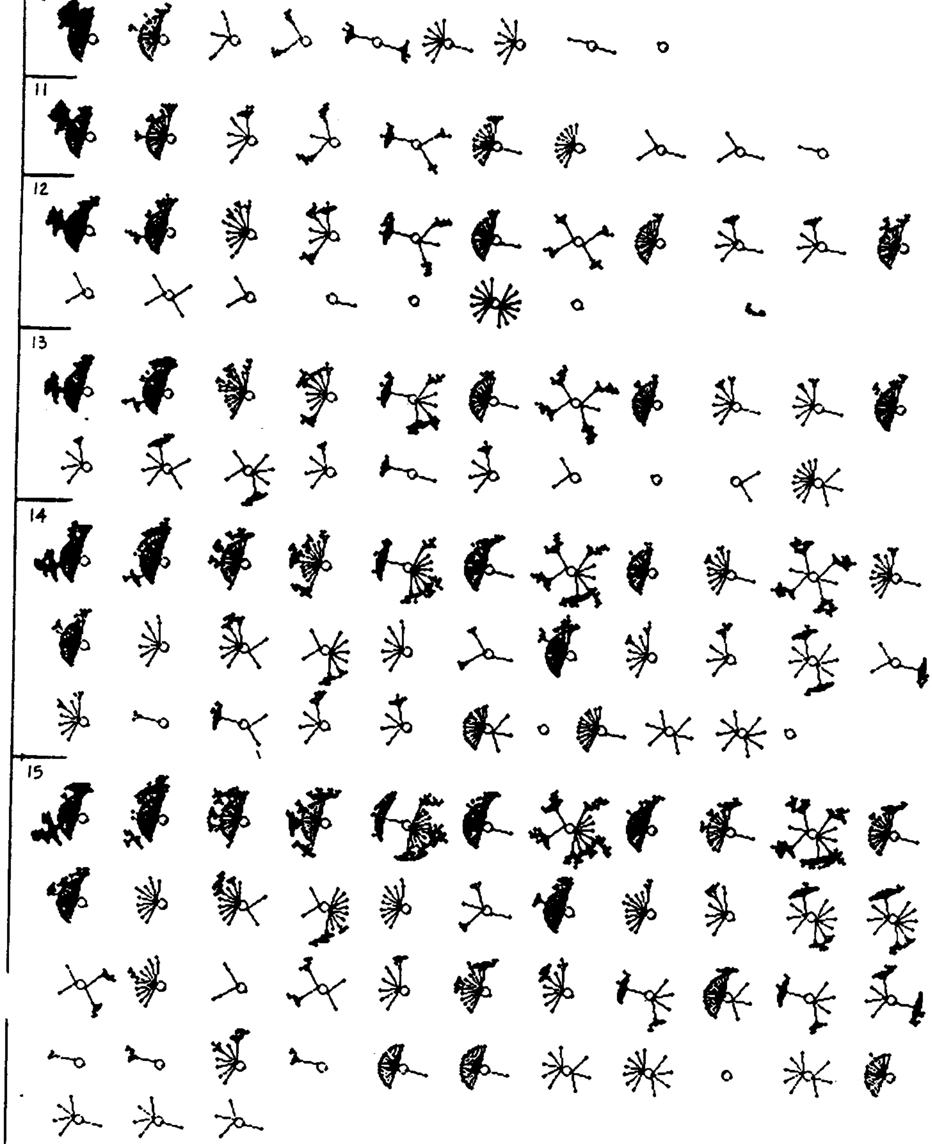
rule 150 (10010110) · rule 122 (01111010) 161 λ ratio = .5 Z = .5

0000110000000011-0000110000000011-rule 201526275
=3-rule 33 -00100001
Length=1 -15



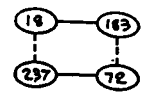


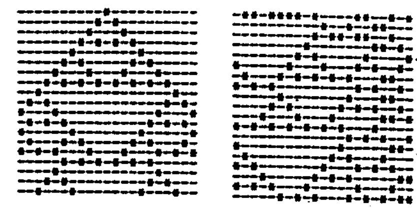


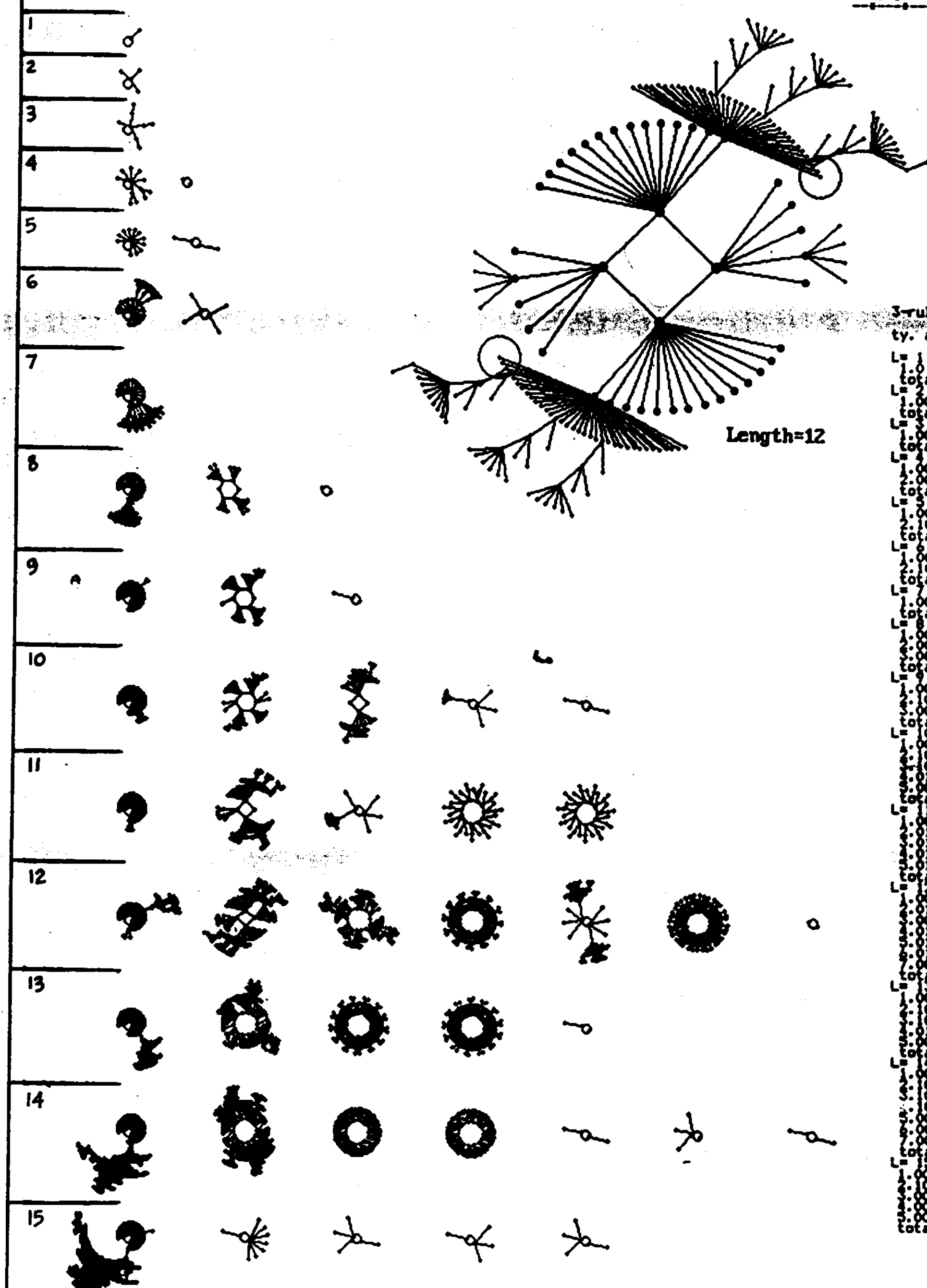



```
3-rule 33 =00100001
ty. at no(p)s g ml mp
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.000 1(2)8 3 2 4
 total NATs = 1
L= 4
 1.0000 1(2)6 4 1 5
 2.0001 2(2)4 2 1 2
 3.0101 1(2)2 0 0 1
 total NATs = 4
L= 5
 1.00000 1(2)7 5 1 6
 2.00001 5(2)5 2 2 3
 total NATs = 6
L= 6
 1.000000 1(2)26 15 3 10
 2.000001 6(2)5 3 1 4
 3.000011 3(2)2 0 0 1
 4.010101 1(2)2 0 0 1
 total NATs = 11
L= 7
 1.0000000 1(2)44 21 4 15
 2.0000001 7(2)10 7 2 6
 3.0000011 7(2)2 0 0 1
 total NATs = 15
L= 8
 1.00000000 1(2)54 36 3 21
 2.00000001 8(2)15 9 3 9
 3.00000011 8(2)7 4 2 4
 4.00000111 4(2)2 0 0 1
 5.00010001 2(2)8 6 1 4
 6.01010101 1(2)2 0 0 1
 total NATs = 24
L= 9
 1.000000000 1(2)89 66 3 31
 2.000000001 9(2)20 13 4 13
 3.000000011 9(2)6 2 2 3
 4.000000111 9(2)14 9 3 7
 5.000010001 9(2)7 5 1 5
 total NATs = 37
L= 10
 1.0000000000 1(2)157 105 5 46
 2.0000000001 10(2)33 25 3 19
 3.0000000011 10(2)7 3 2 4
 4.0000000111 10(2)14 8 3 3
 5.0111100000 5(2)28 20 3 10
 6.0000010001 10(2)10 8 1 8
 7.0000100001 5(2)9 6 2 7
 8.0000100011 10(2)4 2 1 2
 9.0101010101 1(2)2 0 0 1
 total NATs = 62
L= 11
 1.00000000000 1(2)233 165 6 67
 2.00000000001 11(2)50 38 3 28
 3.00000000011 11(2)18 11 3 7
 4.00000000111 11(2)23 15 4 5
 5.01111100000 11(2)29 20 3 14
 6.00000010001 11(2)20 17 2 12
 7.00000100001 11(2)12 10 1 11
 8.00000100011 11(2)5 3 1 3
 9.00000110001 11(2)5 3 1 3
 10.00001000011 11(2)3 1 1 2
 total NATs = 100
L= 12
 1.000000000000 1(2)354 283 5 98
 2.000000000001 12(2)78 60 5 41
 3.000000000011 12(2)22 12 3 9
 4.000000000111 12(2)39 29 4 11
 5.011111100000 12(2)51 38 4 21
 6.000000010001 12(2)23 19 2 17
 7.011111000000 6(2)26 16 3 4
 8.000000100001 12(2)20 17 2 16
 9.000000100011 12(2)15 12 2 8
 10.000000110001 12(2)15 12 2 8
 11.000001000001 6(2)31 25 3 18
 12.000001000011 12(2)4 2 1 3
 13.000001000111 12(2)6 4 1 3
 14.000001100001 12(2)4 2 1 3
 15.000001100011 12(2)3 1 1 2
 16.000011000011 3(2)2 0 0 1
 17.000100010001 2(2)16 14 1 8
 18.010101010101 1(2)2 0 0 1
 total NATs = 163
L= 13
 1.0000000000000 1(2)574 468 5 144
 2.0000000000001 13(2)124 10[illegible]
 3.0000000000011 13(2)31 19 3 13
 4.0000000000111 13(2)47 34 4 10
 5.0111111100000 13(2)79 61 4 31
 6.0000000010001 13(2)31 27 2 25
 7.0111111000000 13(2)59 41 5 6
 8.0000000100001 13(2)28 23 2 23
 9.0000000100011 13(2)14 10 2 7
 10.0000000110001 13(2)14 10 2 7
 11.0000001000001 13(2)40 33 3 26
 12.0000001000011 13(2)9 6 2 3
 13.0000001000111 13(2)25 21 3 13
 14.0111100000100 13(2)25 21 3 13
 15.0000001100001 13(2)9 6 2 3
 16.0000001100011 13(2)15 12 2 11
 17.0000010000011 13(2)12 8 3 5
 18.0000010000111 13(2)4 2 1 3
 19.0000011000011 13(2)2 0 0 1
 20.0000011000111 13(2)4 2 1 3
 21.0000100010001 13(2)14 12 1 10
 total NATs = 261
```

λ ratio = .5 $Z = .5$ 0000001100001100-0000001100001100-rule 51118860
=3-rule 18 -00010010   Length=1 -15








```
3-rule 18 =00010010
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.000 1(1)8 4 2 5
 total NATs = 1
L= 4
 1.0000 1(1)12 9 2 8
 2.0011 2(2)2 0 0 1
 total NATs = 3
L= 5
 1.00000 1(1)12 11 1 12
 2.10010 5(2)4 2 1 2
 total NATs = 6
L= 6
 1.000000 1(1)46 33 4 19
 2.100010 3(2)6 4 1 3
 total NATs = 4
L= 7
 1.0000000 1(1)128 92 5 30
 total NATs = 1
L= 8
 1.00000000 1(1)132 109 5 48
 2.00000011 4(6)30 20 2 8
 3.00110011 2(2)2 0 0 1
 total NATs = 7
L= 9
 1.000000000 1(1)80 76 2 77
 2.100000010 9(6)45 34 3 13
 3.000110011 9(2)3 1 1 2
 total NATs = 19
L= 10
 1.0000000000 1(1)134 123 2 124
 2.1000000010 5(6)62 52 3 14
 3.1000000100 5(4)96 72 5 21
 4.0100101001 5(2)12 6 2 6
 5.0001010011 10(2)4 2 1 2
 total NATs = 26
L= 11
 1.00000000000 1(1)211 199 2 200
 2.01000000100 11(4)136 110 6 34
 3.01000101001 11(2)23 19 3 8
 4.01001000011 11(1)44 22 2 3
 5.01001011000 11(1)44 22 2 3
 total NATs = 25
L= 12
 1.000000000000 1(1)534 459 8 323
 2.010000000100 6(4)198 168 6 35
 3.000100010011 6(14)278 238 5 35
 4.010001010001 1(36)180 120 2 5
 5.010001010001 3(2)114 90 3 13
 6.010001011000 1(36)180 120 2 5
 7.001100110011 2(2)2 0 0 1
 total NATs = 20
L= 13
 1.0000000000000 1(1)2121 1730 8 522
 2.1000000000010 13(14)428 373 6 89
 3.1000001010100 1(39)234 182 2 6
 4.0100010101000 1(39)234 182 2 6
 5.0001100110011 13(2)3 1 1 2
 total NATs = 29
L= 14
 1.00000000000000 1(1)11960 10300 17 844
 2.10000000000010 7(14)582 518 6 90
 3.10000010101000 1(14)98 84 1 7
 4.10000010001010 1(14)98 84 1 7
 5.00010100110011 14(2)4 2 1 2
 6.00011001100011 14(2)5 3 1 4
 7.00011001010011 7(2)4 2 1 2
 total NATs = 45.
L= 15
 1.000000000000000 1(1)32448 28619 19 1365
 2.100101000101010 5(2)10 8 1 8
 3.000101000110011 15(2)6 4 1 4
 4.000101000110011 15(2)6 4 1 4
 5.000101000110011 15(2)6 4 1 4
 total NATs = 51
```

PREDICTION OF DYNAMICS

independent variable		dependent variable
RULE ATTRIBUTE	→	ATTRACTOR TYPE (chaotic or not)

without running the dynamics

RULE ATTRIBUTES:

1. λ (Walker & Ashby, Langton, Li & Packard)
   min{ # 1's in mapping, # of 0's }   0 - 4 (normalize by 4)

2. σ, STRUCTURAL LEVEL   1 - 6 (but 9 different structures)
   from reconstruction analysis

3. τ, VECTOR OF LOSSES: (extra tuples, Transmission) (22 patterns)
   from reconstruction analysis

4. f, FLUENCY, m; MEMORY, h; HESITANCY (Walker & Ashby)
   also information theoretic functions from reconstr. analysis

----

5. Wuensche's Z parameter

Not considered here: Boolean length, Boolean function classifications; mean field analysis, etc.

λ AS PREDICTOR

1. **WEAK PREDICTOR** OF CHAOS FOR ECAs

Higher λ values associated with chaos.

Hence: although simple laws generate complex$_1$ dynamics,

more complex$_1$ laws tend to generate more complex$_1$ dynamics.

2. SHOWS **PHASE TRANSITION** IN ATTRACTOR TYPE

3. REFLECTS (a) **COMPLEXITY$_1$** OF RULE CODING

(b) DISTANCE TO CONSTANT MAP (f = 0 or 1)

WILL SHOW THAT λ IS:

4. DIFFERENT FROM, **LESS PREDICTIVE THAN** σ

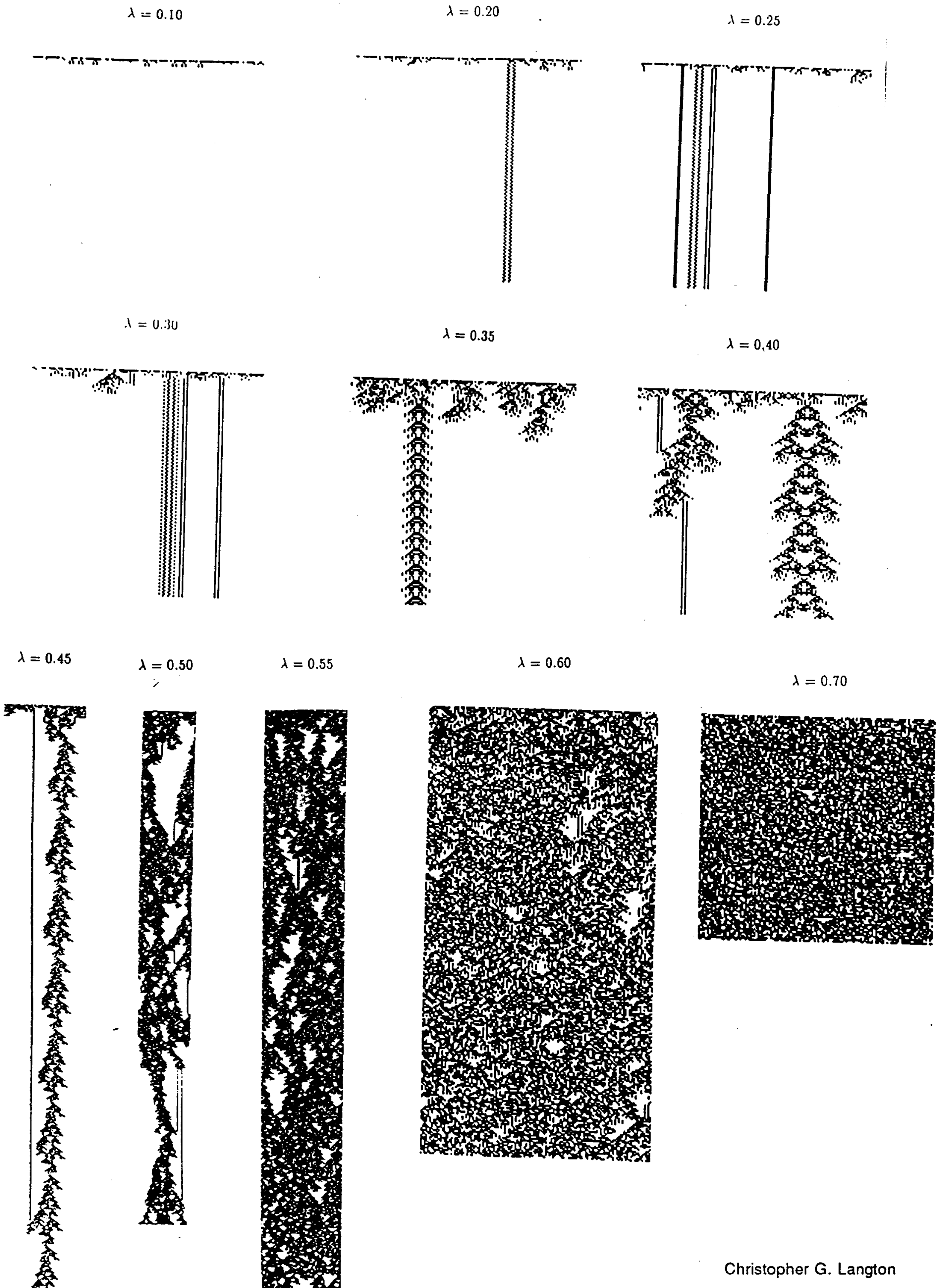
(PARTIAL SET-TH. RECONSTRUCTABILITY ANALYSIS)

5. **SUBSUMED IN** τ

(FULL RECONSTRUCTABILITY ANALYSIS)

Complexity$_1$ = ~~randomness~~ pseudo-chaotic

Complexity$_2$ = Wolfram Class IV sense, "edge of chaos"



Christopher G. Langton

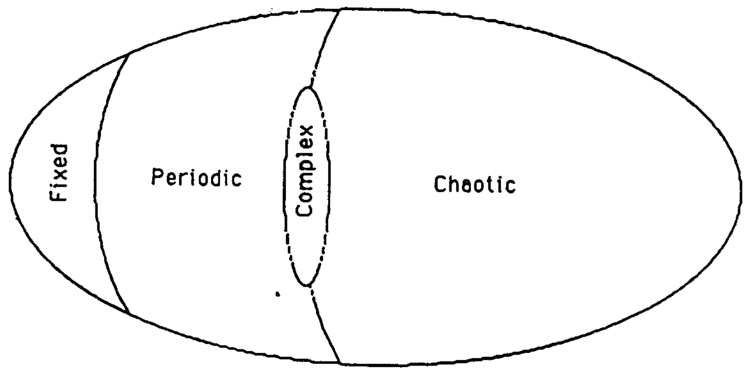


FIGURE 21 Schematic drawing of CA rule space indicating relative location of periodic, chaotic, and complex regimes.

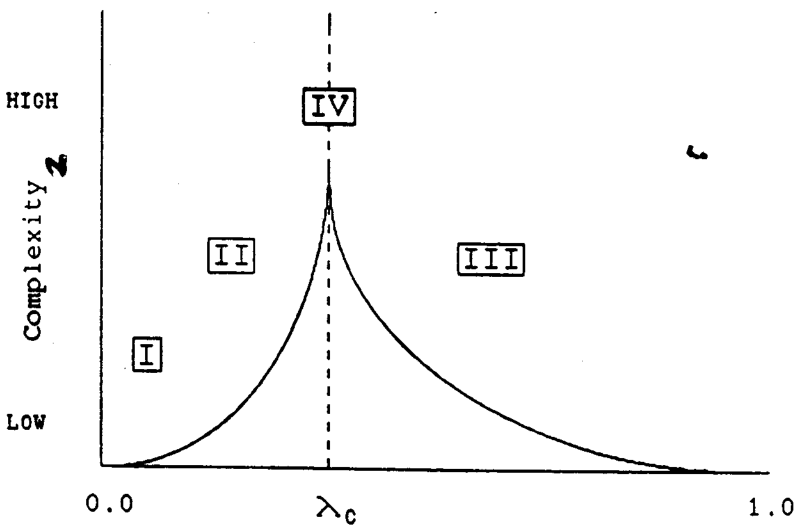


FIGURE 22 Schematic drawing of complexity versus λ over CA rule space, showing the relationship between the Wolfram classes and the underlying phase-transition structure.

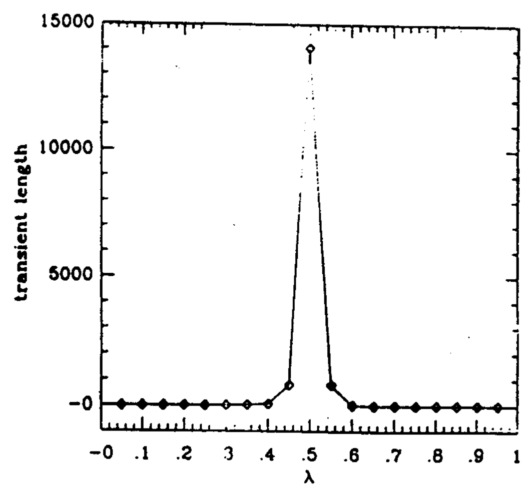


FIGURE 3 Average transient length T versus λ. Transient length apparently diverges rapidly in the vicinity of the transition.

Langton

1. ELEMENTARY CELLULAR AUTOMATA

__PROBLEM__: PREDICTING CHAOTIC DYNAMICS FROM RULE ATTRIBUTES

2. RECONSTRUCTABILITY OF ECAs

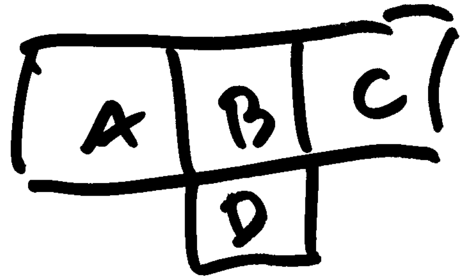
__METHOD__: SET- & INFO.-THEORETIC DECOMPOSABILITY ANALYSIS

- STRUCTURES FOR ECA MAPPINGS
- SET-THEORETIC STRUCTURE ANALYSIS
- STRUCTURAL ANALYSIS OF A RULE (EXAMPLE)
- TABLE OF RULE STRUCTURES
- LOSS VECTORS (τ) and STRUCTURE LEVELS (σ)
- INFORMATION-THEORETIC STRUCTURE ANALYSIS
- WALKER - ASHBY & related INFO.-THEORETIC MEASURES

3. PREDICTING DYNAMICS

__RESULTS__: λ, σ, τ, f, Z AS PREDICTORS OF CHAOS

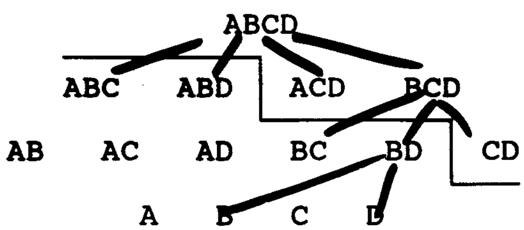
RELATIONS and STRUCTURES



RELATION: ABCD ⊆ A ⊗ B ⊗ C ⊗ D

MAPPING, f: A ⊗ B ⊗ C ---> D

CAs USE MAPPINGS, BUT CONSIDER FIRST **LATTICE OF RELATIONS**



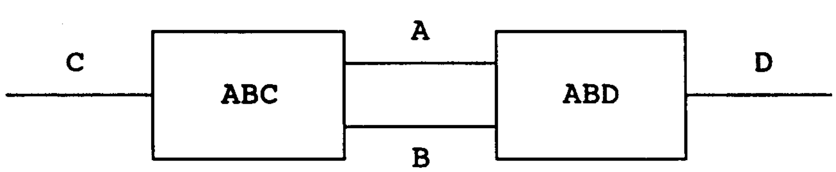
STRUCTURE = SET OF RELATIONS,

NONE A PROJECTION OF ANOTHER,

INVOLVING ALL VARIABLES

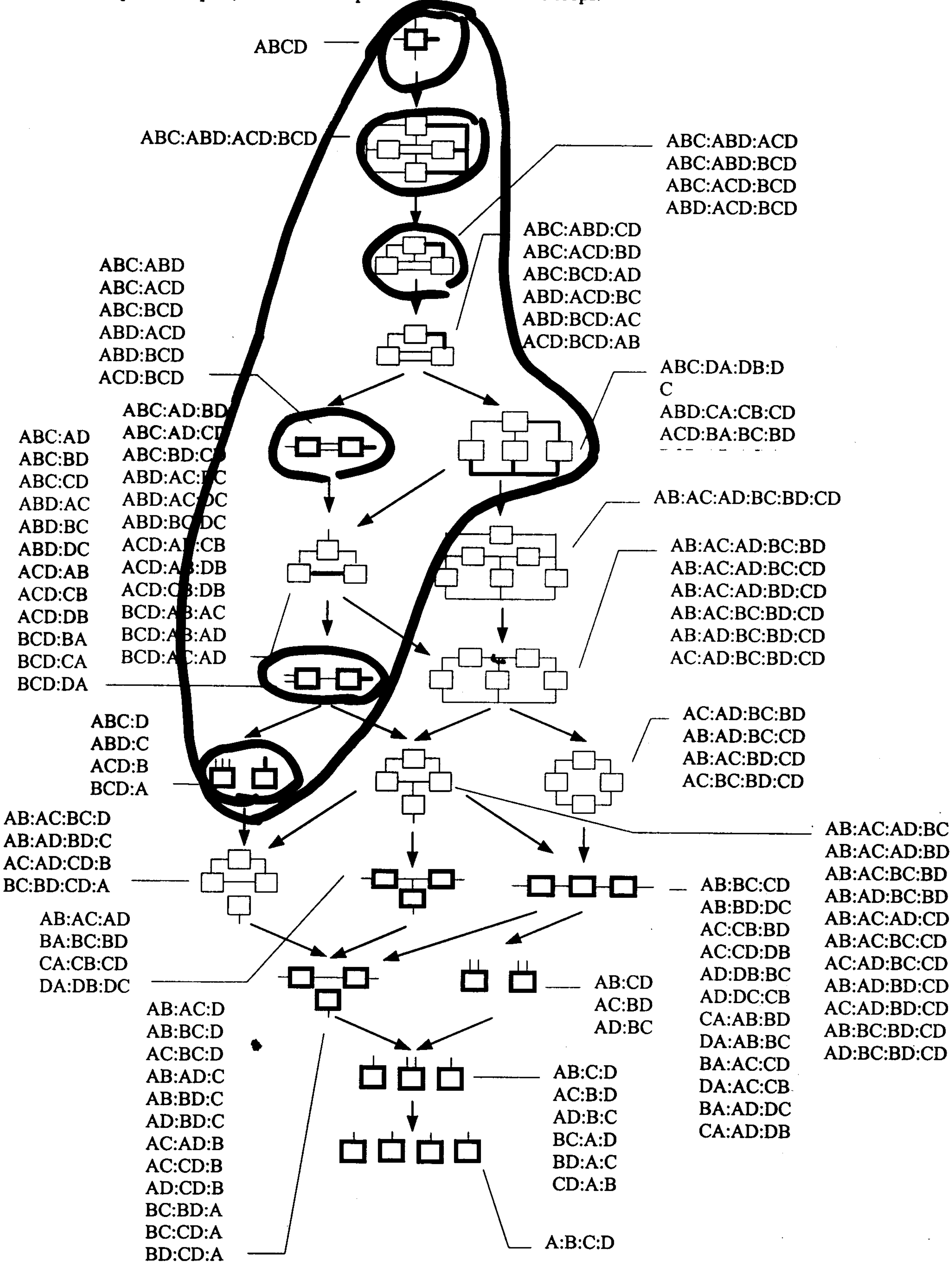
= A CUT THRU LATTICE OF RELATIONS

E.G. (ABOVE) ABC:ABD



4-VARIABLE STRUCTURES (20 general, 114 Specific)

For 1 output & 3 inputs, bold line = output. All boxes bold = no loops.



STRUCTURES FOR ECA MAPPINGS

STRUCTURE	= { RELATIONS }
RELATION (SET-THEOR.)	= SUBSET OF CARTESIAN PRODUCT
	e.g., $R_{ABD} \subseteq A \otimes B \otimes D$
RELATION (INFO.-THEOR.)	= PROBABILITY DISTRIBUTION
	e.g., $R_{ABD} = p(A_j, B_k, D_l)$

σ	STRUCTURES			
6	**ABCD**			mapping
5	**ABD:ACD:BCD**			3 relations (→ mapping)
4	**ABD:BCD**, ABD:ACD, ACD:BCD,			2 relations (→ mapping)
3	**ACD**,	BCD,	ABD	mapping
2	**AD**,	CD,	BD	mapping
1	**D**			constant

6 LEVELS (σ) 12 STRUCTURES 9 STRUCTURE TYPES

FULL NOTATION ADDS ABC TO ALL STRUCTURES (EXCEPT ABCD)

E.G., **ABD** *IS REALLY* ABC:**ABD**.

SET-THEORETIC STRUCTURE ANALYSIS

GIVEN RELATION, R, e.g., **ABCD,**

and **STRUCTURE,** $P_1 : P_2 : \ldots : P_n$, e.g., **ABD:ACD:BCD**, where

P_i = a PROJECTION of R, and

M_i = CARTESIAN PRODUCT of VARIABLES ABSENT in P_i

e.g., $M_1 = C$, $M_2 = B$, $M_3 = A$

RECONSTRUCTED RELATION (MAXIMUM LIKELIHOOD):

$$R' = (P_1 \otimes M_1) \cap (P_2 \otimes M_2) \ldots \cap (P_n \otimes M_n)$$

e.g., $\mathbf{R' = (ABD \otimes C) \cap (ACD \otimes B) \cap (BCD \otimes A)}$

STRUCTURE FITS RELATION **W/O LOSS** iff R' = R

STRUCTURE OF R = SIMPLEST STRUCTURE THAT FITS

CONSTRAINT LOSS (degree of non-fit)

$= |R'| - |R|$ (number of additional tuples), or

$= LOG_2 (|R'| / |R|)$ (like Info.-Th. Transmission)

STRUCTURAL ANALYSIS OF A RULE (EXAMPLE)

Rule # 7:	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>1</u>
	128	64	32	16	8	4	2	1

Rule Mapping A B C	D	Projections A B	D	A C	D	B C	D
0 0 0	1	0 0	1	0 0	1	0 0	1
0 0 1	1			0 1	1	0 1	1
0 1 0	1	0 1	1			1 0	1
0 1 1	0	0 1	0	0 1	0	1 1	0
1 0 0	0	1 0	0	1 0	0	0 0	0
1 0 1	0			1 1	0	0 1	0
1 1 0	0	1 1	0			1 0	0
1 1 1	0						

Reconstruction based on ABD:ACD:BCD

A B C	D	Expanded Relations ABD⊗C	ACD⊗B	BCD⊗A	Reconstructed Relation
0 0 0	0			x	
0 0 0	1	*	*	*	0 0 0 1
0 0 1	0		x	x	
0 0 1	1	*	*	*	0 0 1 1
0 1 0	0	x		x	
0 1 0	1	*	*	*	0 1 0 1
0 1 1	0	*	*	*	0 1 1 0
0 1 1	1	x	x		
1 0 0	0	*	*	*	1 0 0 0
1 0 0	1			x	
1 0 1	0	*	*	*	1 0 1 0
1 0 1	1			x	
1 1 0	0	*	*	*	1 1 0 0
1 1 0	1			x	
1 1 1	0	*	*	*	1 1 1 0
1 1 1	1				

TABLE OF RULE STRUCTURES

STRUCTURAL TYPE	RULES									
6. **ABCD**	1	2	4	6	8	9	18	22	24	25
(47)	26	28	30	32	33	36	37	38	40	41
	44	45	54	56	57	72	73	74	104	105
	106	108	128	129	130	131	132	133	134	137
	146	150	152	154	156	161	164			
5. **ABD:ACD:BCD**	7	11	13	14	19	23	35	42	43	50
(20)	76	77	138	140	142	162	168	178	200	232
4. **ABD:ACD** (4.1)	27	29	46	58	78	172	184			
ABD:BCD (4.2)										
ACD:BCD (4.3)										
(7)										
3. **ABD** (3.1)	3	5	10	12	34	60	90	136	160	
BCD (3.2)										
ACD (3.3)										
(9)										
2. **AD** (2.1)	15	51	170	204						
BD (2.2)										
CD (2.3)										
(4)										
1. **D**	0									
(1)										

LOSS VECTORS (τ) & STRUCTURE LEVEL (σ)
in COMPLETE SET-TH. RECONSTRUCTABILITY ANALYSIS

		Structures											
σ	τ	6	5	4.1	4.2	4.3	3.1	3.2	3.3	2.1	2.2	2.3	1
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	2	0	0	0	0	0	0	0	8	8	8	0	8
	3	0	0	0	0	0	0	8	0	0	8	8	8
3	4	0	0	0	0	2	4	4	0	4	4	8	8
	5	0	0	0	0	8	8	8	0	8	8	8	8
	6	0	0	0	2	0	0	4	4	4	8	4	8
	7	0	0	0	8	0	0	8	8	8	8	8	8
4	8	0	0	0	2	2	4	4	4	8	8	8	8
	9	0	0	2	2	0	4	4	4	8	8	8	8
5	10	0	0	1	1	1	2	2	6	8	8	4	8
	11	0	0	1	1	1	2	6	2	4	8	8	8
	12	0	0	2	2	2	4	4	4	8	8	8	8
6	13	0	1	1	1	1	2	2	2	4	4	4	8
	14	0	1	1	1	5	6	6	2	8	8	8	8
	15	0	1	1	5	1	2	6	6	8	8	8	8
	16	0	2	2	2	2	4	4	4	8	8	8	8
	17	0	2	2	2	4	4	4	4	8	8	4	8
	18	0	2	2	4	2	4	4	4	4	8	8	8
	19	0	2	2	4	4	4	8	4	8	8	8	8
	20	0	2	4	4	2	4	4	8	8	8	8	8
	21	0	4	5	5	5	6	6	6	8	8	8	8
	22	0	8	8	8	8	8	8	8	8	8	8	8

RULE MAPPING (RULE # 150)

A	B	C		D
0	0	0		0
0	0	1		1
0	1	0		1
0	1	1	----->	0
1	0	0		1
1	0	1		0
1	1	0		0
1	1	1		1

EXPRESSED AS PROBABILITY TABLE

		C	*0*		*1*	
A	B	D	*0*	*1*	*0*	*1*
0	*0*		1/8	0	0	1/8
	1		0	1/8	1/8	0
1	*0*		0	1/8	1/8	0
	1		1/8	0	0	1/8

INFORMATION-THEORETIC STRUCTURE ANALYSIS

CALCULATE TRANSMISSIONS (= LOSSES) FOR ALL STRUCTURES

σ(STRUCTURE) = LOWEST STRUCTURE FOR WHICH SOME T=0

σ		
6	T(**ABCD**)	
5	T (ABC: **ABD: ACD: BCD**)	
4	T (ABC: **ABD: ACD**)	and permutations
3	T (ABC: **ABD**)	and permutations
2	T (ABC: **AD**)	and permutations
1	T (ABC: **D**)	

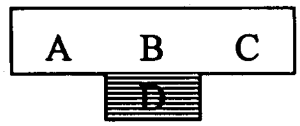
T(ABC: **D**) = H(A, B, C) + H(D) - H (A , B , C , D) = H (D)

H(D) MAPS 1:1 WITH λ.

WALKER - ASHBY & related INFO.-THEORETIC MEASURES

1. **HOMOGENEITY** ≡ λ = *variability of output*

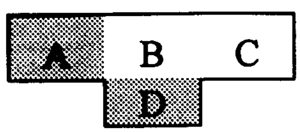
Maps 1:1 with H(**D**)

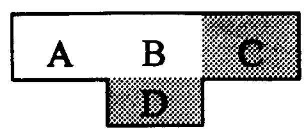


2. **FLUENCY**, f = *throughput from input, A or C, to output, D*

Related to $T_{BC}(A{:}D) = T(ABC{:}\mathbf{BCD})$, $T_{AB}(C{:}D) = T(ABC{:}\mathbf{ABD})$

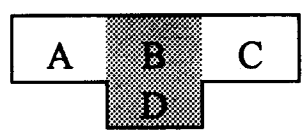
$$f = \{\, T_{BC}(A{:}D) \quad , \quad T_{AB}(C{:}D), \,\}$$
$$f' = T_{BC}(A{:}D) + T_{AB}(C{:}D)$$





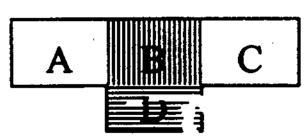
3. **MEMORY, m** = *sensitivity to element's past history*

Maps 1:1 with $T_{AC}(B{:}D) = T(ABC{:}\mathbf{ACD})$



4. **HESITANCY, h** = *stability of the element*

Related to T(ABC: **BD**) = H(D | B)



1. ELEMENTARY CELLULAR AUTOMATA

PROBLEM: PREDICTING CHAOTIC DYNAMICS FROM RULE ATTRIBUTES

2. RECONSTRUCTABILITY OF ECAs

METHOD: SET- & INFO.-THEORETIC DECOMPOSABILITY ANALYSIS

3. PREDICTING DYNAMICS

RESULTS: λ, σ, τ, f, Z AS PREDICTORS OF CHAOS

- CONTINGENCY TABLES: λ, σ versus ATTRACTOR (a)
- UNCERTAINTY CALCULATIONS (TO ASSESS PREDICTION)
- REDUCTION OF ATTRACTOR UNCERTAINTY (SUMMARY)
- UTTERLY UNEXPAINABLE UNCERTAINTY
- STATE SPACE PLOTS OF UNDIFFERENTIATABLE RULES
- CONCLUSIONS

CONTINGENCY TABLES: RULE ATTRIBUTE VS. ATTRACTOR

attractor: N = NONCHAOTIC; C = CHAOTIC

λ	attractor N	attractor C	
0	2	-	2
1	16	-	16
2	52	4	56
3	96	16	112
4	44	26	70
	210	46	256

σ	attractor N	attractor C	
1	2	-	2
2	6	-	6
3	24	6	30
4	24	-	24
5	56	-	56
6	98	40	138
	210	46	256

σ \ λ	0	1	2	3	4	
1	2	-	-	-	-	2
2	-	-	-	-	6	6
3	-	-	24	-	6	30
4	-	-	-	-	24	24
5	-	-	-	48	8	56
6	-	16	32	64	26	138
	2	16	56	112	70	256

UNCERTAINTY CALCULATIONS

MEASURE UNPREDICTABILITY BY UNCERTAINTY (SHANNON ENTROPY)

$$H(Y) = - \Sigma\ P(Y_i)\ LOG_2\ P(Y_i)$$

$$H(Y/X) = \Sigma\ P(X_i)\ H(Y/X_i)$$

$$= - \Sigma\ P(X_i)\ \Sigma\ P(Y_j/X_i)\ LOG_2\ P(Y_j/X_i)$$

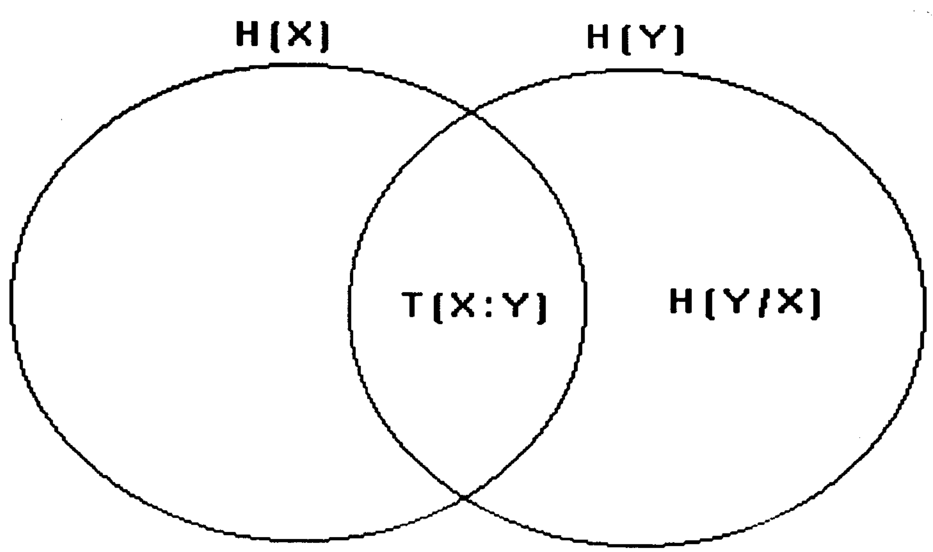
$$= H(X,Y) - H(X)$$

% REDUCTION IN H(Y) BY X = (H(Y) - H(Y/X)) / H(Y)

= T(X:Y) / H(Y)

PREDICTIVE POWER OF X = (H(Y) - H(Y/X)) / H(X)

= T(X:Y) / H(X)



REDUCTION OF ATTRACTOR UNCERTAINTY

SUMMARY, ADDING WUENSCHE'S Z PARAMETER

q = rule attribute

q	H(a | q)	% ΔH	Pred.Power
-	.679		
λ	.600	11.6	.044
σ	.553	18.6	.069
τ	.263	61.3	.102
f	.355	47.7	.124
f′	.447	34.2	.151
Z	.458	32.6	.114

$$H(\lambda \mid \tau) = H(\sigma \mid \tau) = H(f \mid \tau) = H(f' \mid \tau) = H(Z \mid \tau) = 0$$

CONCLUSIONS

1. τ vector IS THE BEST OVERALL (% ΔH) PREDICTOR

2. τ SUBSUMES ALL OTHER MEASURES

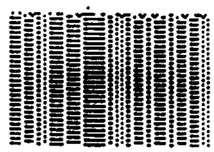
3. f′ HAS THE HIGHEST PREDICTIVE POWER

WHAT ABOUT THE REMAINING UNCERTAINTY?!

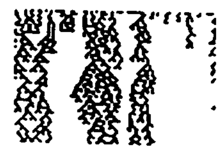
UTTERLY UNEXPLAINABLE UNCERTAINTY σ₂

(ONLY IN STRUCTURE TYPE 6 RULES)

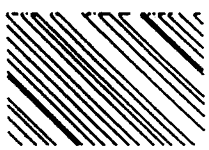
λ	b	f	m	h	τ	NONCHAOTIC		CHAOTIC	
2	5	3	3	3	c	**33**		**18**	<---
2	7	3	3	5	d	**24**	**36**	**129**	<---
3	5	4	2	4	f	**37**		**161**	<---



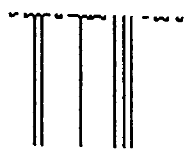
rule 33 (00100001)



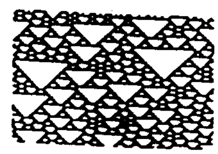
rule 18 (00010010)



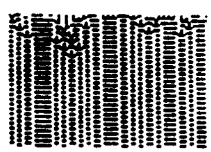
rule 24 (00011000)



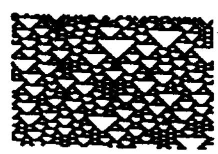
rule 36 (00100100)



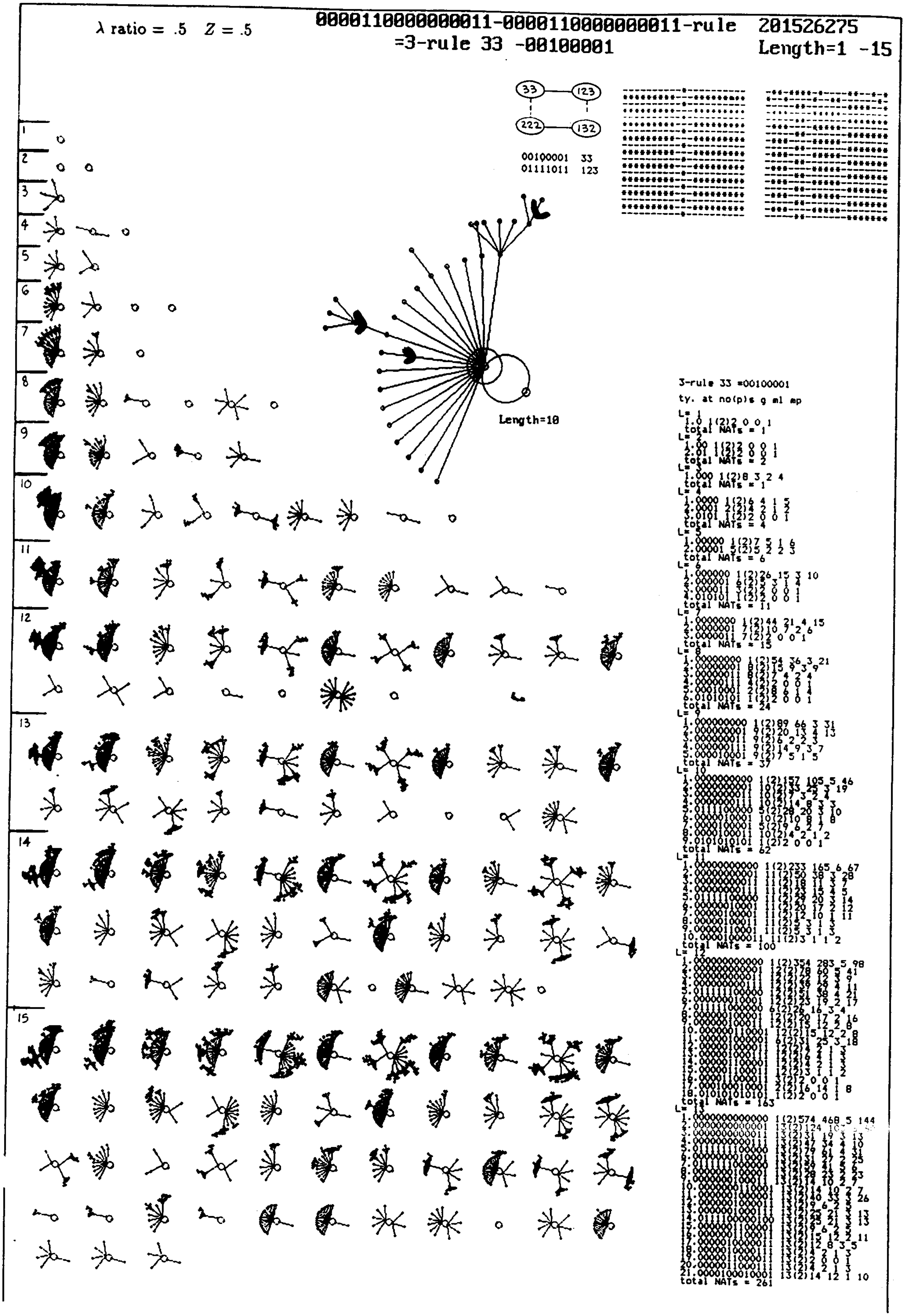
rule 126 (01111110)
/129



rule 37 (00100101)

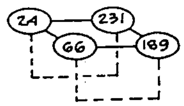


rule 122 (01111010)
/161

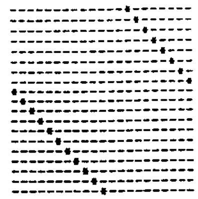

λ ratio = .5 Z = .5
00001100000000011-00001100000000011-rule 201526275
=3-rule 33 -00100001
Length=1 -15
33
123
222
132
00100001 33
01111011 123
Length=10
3-rule 33 =00100001
ty. at no(p)s g ml mp
L= 1
1.0 1(2)2 0 0 1
total NATs = 1
L= 2
1.00 1(2)2 0 0 1
2.01 1(2)2 0 0 1
total NATs = 2
L= 3
1.000 1(2)8 3 2 4
total NATs = 1
L= 4
1.0000 1(2)6 4 1 5
2.0001 2(2)4 2 1 2
3.0101 1(2)2 0 0 1
total NATs = 4
L= 5
1.00000 1(2)7 5 1 6
2.00001 5(2)5 2 2 3
total NATs = 6
L= 6
1.000000 1(2)26 15 3 10
2.000001 6(2)5 3 1 4
3.000011 3(2)2 0 0 1
4.010101 1(2)2 0 0 1
total NATs = 11
L= 7
1.0000000 1(2)44 21 4 15
2.0000001 7(2)10 7 2 6
3.0000011 7(2)2 0 0 1
total NATs = 15
total NATs = 24
total NATs = 37
total NATs = 62
total NATs = 100
total NATs = 163
total NATs = 261

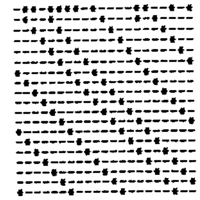
λ ratio = .5 Z = .5   0000001111000000-0000001111000000-rule 62915520
=3-rule 24 -00011000   Length=1 -15

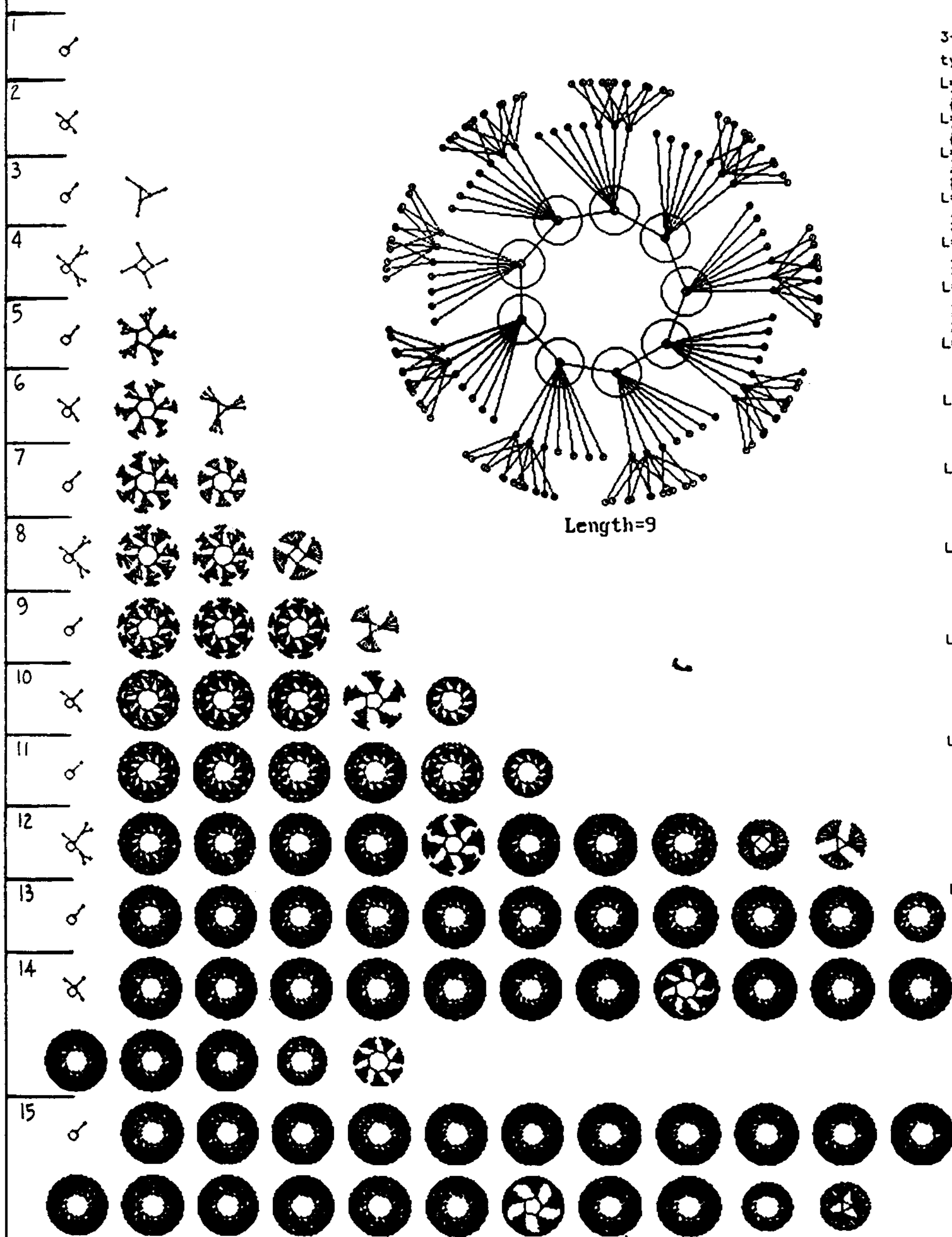
Nonchaotic



00011000 24
11100111 231
10111101 189
01000010 66







Length=9

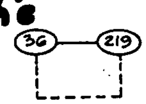
3-rule 24 =00011000

ty. at no(p)s g ml mp

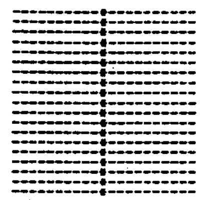
L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)4 3 1 4
total NATs = 1
L= 3
1.000 1(1)2 1 1 2
2.001 1(3)6 3 1 2
total NATs = 2
L= 4
1.0000 1(1)8 5 2 4
2.0001 1(4)8 4 1 2
total NATs = 2
L= 5
1.00000 1(1)2 1 1 2
2.00001 1(5)30 20 2 4
total NATs = 2
L= 6
1.000000 1(1)4 3 1 4
2.000001 1(6)48 36 2 4
3.100100 1(3)12 9 1 4
total NATs = 3
L= 7
1.0000000 1(1)2 1 1 2
2.0000001 1(7)84 63 2 6
3.1000100 1(7)42 35 1 6
total NATs = 3
L= 8
1.00000000 1(1)8 5 2 4
2.00000001 1(8)112 88 2 6
3.10000100 1(8)96 80 2 8
4.10001000 1(4)40 36 1 10
total NATs = 4
L= 9
1.000000000 1(1)2 1 1 2
2.000000001 1(9)180 144 2 8
3.100000100 1(9)144 126 2 10
4.100001000 1(9)162 144 2 12
5.100100100 1(3)24 21 1 8
total NATs = 5
L= 10
1.0000000000 1(1)4 3 1 4
2.0000000001 1(10)240 200 2 8
3.1000000100 1(10)240 210 2 12
4.1000001000 1(10)240 220 2 16
5.0100001000 1(5)180 160 2 16
6.1000100100 1(10)120 110 1 12
total NATs = 6
L= 11
1.00000000000 1(1)2 1 1 2
2.00000000001 1(11)330 275 2 10
3.10000000100 1(11)330 297 2 14
4.10000001000 1(11)396 363 2 18
5.01000001000 1(11)528 484 2 20
6.10000100100 1(11)264 242 2 16
7.10001000100 1(11)198 187 1 18
total NATs = 7
L= 12
1.000000000000 1(1)8 5 2 4
2.000000000001 1(12)408 348 2 10
3.100000000100 1(12)480 432 2 16
4.100000001000 1(12)552 516 2 22
5.010000001000 1(12)864 792 2 24
6.100000100000 1(6)384 360 2 26
7.100000100100 1(12)384 360 2 20
8.100001000100 1(12)432 408 2 24
9.100001001000 1(12)432 408 2 24
10.100010001000 1(4)104 100 1 26
11.100100100100 1(3)48 45 1 16
total NATs = 11
L= 13
1.0000000000000 1(1)2 1 1 2
2.0000000000001 1(13)546 468 2 12
3.1000000000100 1(13)624 572 2 18
4.1000000001000 1(13)780 728 2 24
5.0100000001000 1(13)1170 1092 2 28
6.1000000100000 1(13)1248 1170 2 30
7.1000000100100 1(13)624 585 2 24
8.1000001000100 1(13)624 598 2 30
9.1000001001000 1(13)624 598 2 30
10.1000010001000 1(13)702 676 2 36
11.1000010000100 1(13)936 884 2 32
12.1000100100100 1(13)312 299 1 24
total NATs = 12
L= 14
1.00000000000000 1(1)4 3 1 4
2.00000000000001 1(14)672 588 2 12
3.10000000000100 1(14)840 770 2 20
4.10000000001000 1(14)1008 952 2 28
5.01000000001000 1(14)1680 1568 2 32
6.10000000100000 1(14)1680 1596 2 36
7.10000000100100 1(14)840 798 2 28
8.10000001000100 1(14)1008 966 2 36
9.01000000100000 1(7)1008 945 2 36
10.10000001001000 1(14)1008 966 2 36
11.10000010001000 1(14)1008 980 2 44
12.10000010000100 1(14)1344 1288 2 40
13.01000001001000 1(14)1344 1288 2 40
14.10000100010000 1(14)1512 1456 2 48
15.10000100100100 1(14)672 644 2 32
16.10001000100100 1(14)504 490 1 36
17.10001001000100 1(7)252 245 1 36
total NATs = 17
L= 15
1.000000000000000 1(1)2 1 1 2
2.000000000000001 1(15)840 735 2 14
3.100000000000100 1(15)1050 975 2 22
4.100000000001000 1(15)1350 1275 2 30
5.010000000001000 1(15)2160 2040 2 36
6.100000000100000 1(15)2400 2280 2 40
7.100000000100100 1(15)1200 1140 2 32
8.100000001000100 1(15)1350 1305 2 42
9.010000000100000 1(15)2700 2565 2 42
10.100000001001000 1(15)1350 1305 2 42
11.100000010001000 1(15)1620 1575 2 54
12.100000010000100 1(15)2160 2070 2 48
13.010000001001000 1(15)2160 2070 2 48
14.100000100010000 1(15)2160 2100 2 60
15.010000010000010 1(15)1920 1860 2 50
16.100000100001000 1(15)2160 2100 2 60
17.100000100100100 1(15)960 930 2 40
18.100001000100100 1(15)1080 1050 2 48
19.010000100001000 1(5)1080 1040 2 64
20.100001001000100 1(15)1080 1050 2 48
21.100001001001000 1(15)1080 1050 2 48
22.100010001000100 1(15)810 795 1 54
23.100100100100100 1(3)96 93 1 32
total NATs = 23

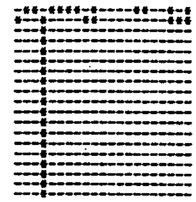
λ ratio = .5 Z = .5 0000110000110000-0000110000110000-rule 204475440
=3-rule 36 -00100100 Length=1 -15

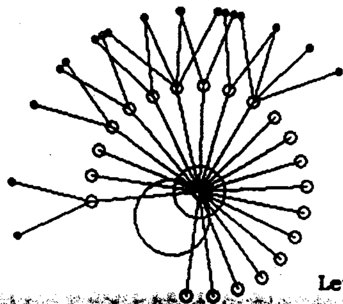
Non chaotic



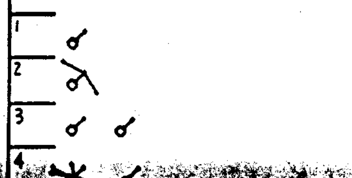
00100100 36
11011011 219

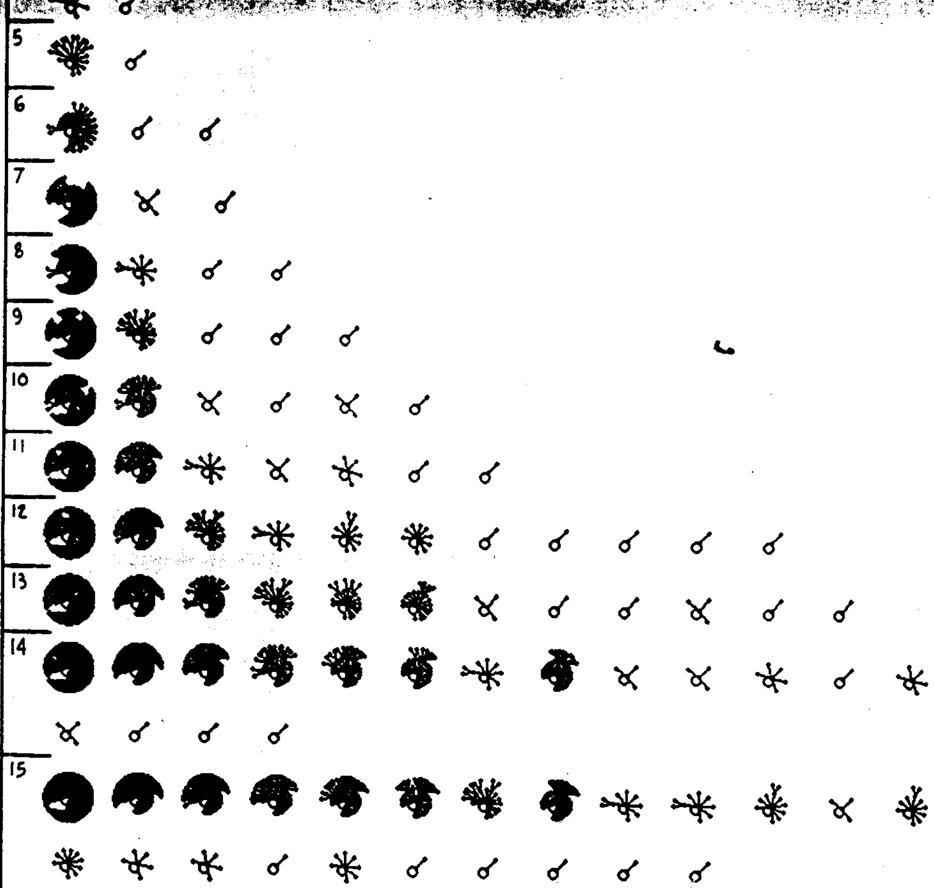






Length=10





3-rule 36 =00100100

ty. at no(p)s g al mp

L= 1
1.0 1(1)2 1 1 2
total NATs = 1

L= 2
1.00 1(1)4 2 2 2
total NATs = 1

L= 3
1.000 1(1)2 1 1 2
2.001 3(1)2 1 1 2
total NATs = 4

L= 4
1.0000 1(1)8 6 2 6
2.0001 4(1)2 1 1 2
total NATs = 5

L= 5
1.00000 1(1)22 16 2 12
2.00001 5(1)2 1 1 2
total NATs = 6

L= 6
1.000000 1(1)46 32 2 20
2.000001 6(1)2 1 1 2
3.001001 3(1)2 1 1 2
total NATs = 10

L= 7
1.0000000 1(1)86 64 2 30
2.0000001 7(1)4 3 1 4
3.0001001 7(1)2 1 1 2
total NATs = 15

L= 8
1.00000000 1(1)152 114 2 46
2.00000001 8(1)10 8 2 8
3.00001001 8(1)2 1 1 2
4.00010001 4(1)2 1 1 2
total NATs = 21

L= 9
1.000000000 1(1)272 217 2 74
2.000000001 9(1)22 17 2 14
3.000001001 9(1)2 1 1 2
4.000010001 9(1)2 1 1 2
5.001001001 3(1)2 1 1 2
total NATs = 31

L= 10
1.0000000000 1(1)504 412 2 122
2.0000000001 10(1)42 33 2 22
3.0000001001 10(1)4 3 1 4
4.0000010001 10(1)2 1 1 2
5.0000100001 5(1)4 3 1 4
6.0001001001 10(1)2 1 1 2
total NATs = 46

L= 11
1.00000000000 1(1)948 793 2 200
2.00000000001 11(1)76 61 2 34
3.00000001001 11(1)10 8 2 8
4.00000010001 11(1)4 3 1 4
5.00000100001 11(1)6 5 1 6
6.00001001001 11(1)2 1 1 2
7.00010001001 11(1)2 1 1 2
total NATs = 67

L= 12
1.000000000000 1(1)1790 1530 2 324
2.000000000001 12(1)136 113 2 54
3.[illegible] 12(1)22 17 2 14
4.[illegible] 12(1)10 8 2 8
5.[illegible] 12(1)12 10 2 10
6.[illegible] 6(1)10 9 1 10
7.[illegible] 12(1)2 1 1 2
8.[illegible] 12(1)2 1 1 2
9.[illegible] 12(1)2 1 1 2
10.[illegible] 4(1)2 1 1 2
11.001001001001 3(1)2 1 1 2
total NATs = 98

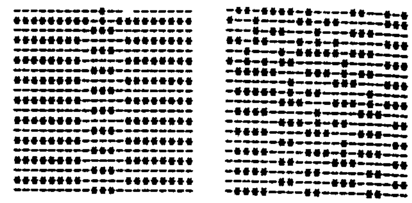
L= 13
1.0000000000000 1(1)3356 2926 2 522
2.[illegible] 13(1)250 214 2 88
3.[illegible] 13(1)42 33 2 22
4.[illegible] 13(1)22 17 2 14
5.[illegible] 13(1)26 18 2 16
6.[illegible] 13(1)20 18 2 16
7.[illegible] 13(1)4 3 1 4
8.[illegible] 13(1)2 1 1 2
9.[illegible] 13(1)2 1 1 2
10.[illegible] 13(1)4 3 1 4
11.[illegible] 13(1)2 1 1 2
12.0001001001001 13(1)2 1 1 2
total NATs = 144

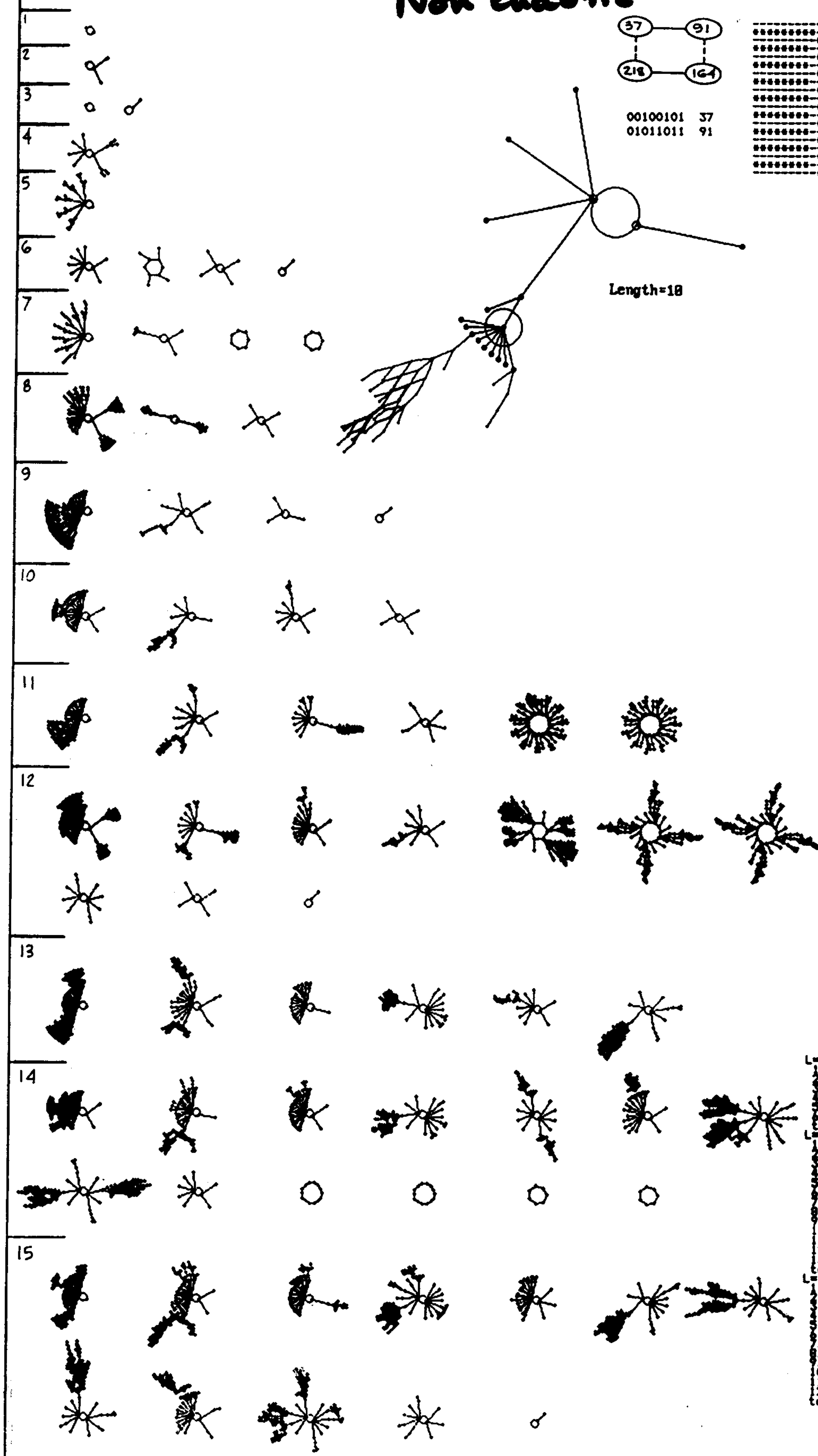
L= 14
1.00000000000000 1(1)6262 5560 2 842
2.[illegible] 14(1)468 408 2 144
3.[illegible] 14(1)76 61 2 34
4.[illegible] 14(1)42 33 2 22
5.[illegible] 14(1)42 33 2 26
6.[illegible] 14(1)34 30 2 24
7.[illegible] 14(1)10 8 2 8
8.[illegible] 7(1)40 36 2 26
9.[illegible] 14(1)4 3 1 4
10.[illegible] 14(1)4 3 1 4
11.[illegible] 14(1)6 5 1 6
12.[illegible] 14(1)2 1 1 2
13.[illegible] 14(1)6 5 1 6
14.[illegible] 14(1)4 3 1 4
15.[illegible] 14(1)2 1 1 2
16.[illegible] 14(1)2 1 1 2
17.[illegible] 7(1)2 1 1 2
total NATs = 211

L= 15
1.000000000000000 1(1)11662 10516 2 1362
2.[illegible]
3.[illegible] 15(1)136 113 2 54
4.[illegible] 15(1)76 61 2 34
5.[illegible] 15(1)78 62 2 42
6.[illegible] 15(1)92 85 2 38
7.[illegible] 15(1)22 17 2 14
8.[illegible] 15(1)66 61 2 40
9.[illegible] 15(1)10 8 2 8
10.[illegible] 15(1)10 8 2 8
11.[illegible] 15(1)12 10 2 10
12.[illegible] 15(1)4 3 1 4
13.[illegible] 15(1)12 10 2 10
14.[illegible] 15(1)10 9 1 10
15.[illegible] 15(1)6 5 1 6
16.[illegible] 15(1)6 5 1 6
17.[illegible] 15(1)2 1 1 2
18.[illegible] 5(1)8 7 1 8
19.[illegible] 15(1)2 1 1 2
20.[illegible] 15(1)2 1 1 2
21.[illegible] 15(1)2 1 1 2
22.[illegible] 15(1)2 1 1 2
23.001001001001001 3(1)2 1 1 2
total NATs = 309

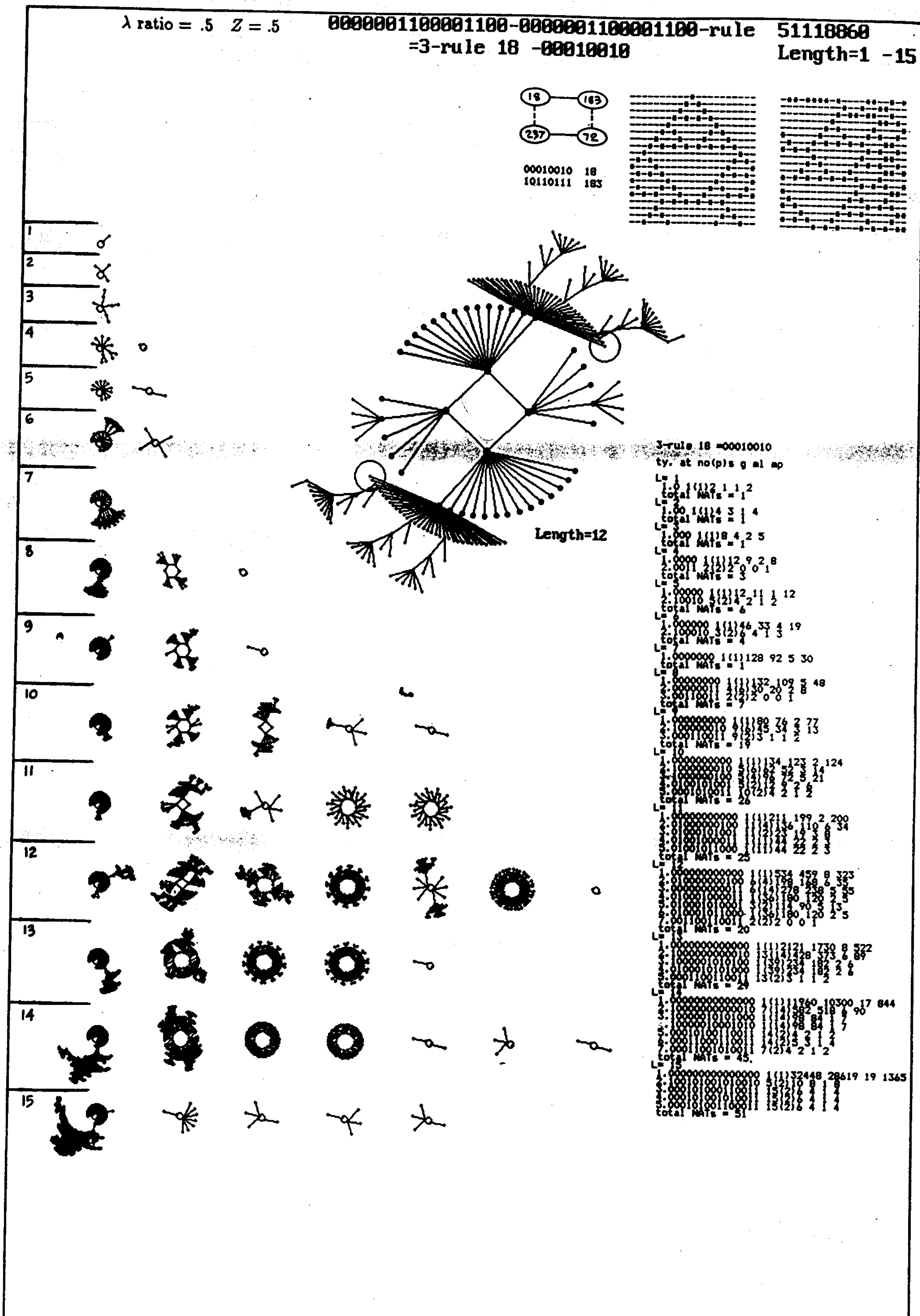
λ ratio = .75 Z = .75   0000110000110011-0000110000110011-rule 204672051
=3-rule 37 -00100101   Length=1 -15

Non chaotic

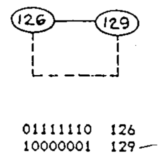


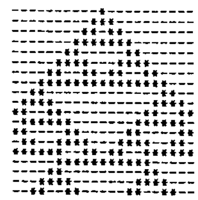


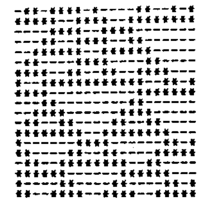

```
3-rule 37 =00100101
ty. at no(p)s g al ap
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)2 0 0 1
 2.001 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(2)16 8 3 5
 total NATs = 1
L= 5
 1.00000 1(2)32 15 4 6
 total NATs = 1
L= 6
 1.000000 1(2)10 8 1 7
 2.000001 3(6)10 4 1 2
 3.000011 3(2)6 4 1 3
 4.001001 3(1)2 1 1 2
 total NATs = 10
L= 7
 1.0000000 1(2)44 14 5 8
 2.1000011 7(2)10 6 3 4
 3.0001011 1(7)7 0 0 1
 4.0001101 1(7)7 0 0 1
 total NATs = 10
L= 8
 1.00000000 1(2)112 48 5 13
 2.10000011 4(2)24 14 4 5
 3.00000011 8(2)6 4 1 3
 total NATs = 13
L= 9
 1.000000000 1(2)227 90 7 19
 2.100000011 9(2)26 12 10 7
 3.000000011 9(2)5 3 1 3
 4.001001001 3(1)2 1 1 2
 total NATs = 22
L= 10
 1.0000000000 1(2)94 67 4 26
 2.1000000011 10(2)75 35 13 9
 3.0000000011 10(2)15 9 5 6
 4.0000000111 5(2)6 4 1 3
 total NATs = 26
L= 11
 1.00000000000 1(2)112 55 6 34
 2.[illegible] 11(2)89 48 12 13
 3.[illegible] 11(2)54 21 11 9
 4.[illegible] 11(2)9 5 2 4
 5.01111011000 1(33)132 44 3 3
 6.01111000011 1(33)132 44 3 3
 total NATs = 36
L= 12
 1.000000000000 1(2)226 116 5 47
 2.100000000011 12(2)86 53 6 18
 3.000000000011 12(2)30 20 7 12
 4.001110000000 12(2)26 13 8 6
 5.000001000001 3(6)362 150 8 8
 6.100001100000 3(12)164 72 11 5
 7.110000100000 3(12)164 72 11 5
 8.000000011111 6(2)12 8 2 5
 9.000011000011 3(2)6 4 1 3
 10.001001001001 3(1)2 1 1 2
 total NATs = 58
L= 13
 1.0000000000000 1(2)366 208 8 66
 2.[illegible] 13(2)141 70 12 25
 3.[illegible] 13(2)25 19 2 15
 4.[illegible] 13(2)87 42 8 10
 5.[illegible] 13(2)31 16 8 7
 6.[illegible] 13(2)318 142 18 9
 total NATs = 66
L= 14
 1.00000000000000 1(2)340 226 5 96
 2.[illegible] 14(2)127 77 11 34
 3.[illegible] 14(2)39 27 4 21
 4.[illegible] 14(2)168 95 11 13
 5.[illegible] 7(2)72 38 10 9
 6.[illegible] 14(2)86 43 9 16
 7.[illegible] 7(2)600 301 16 12
 8.[illegible] 14(2)378 174 18 9
 9.[illegible] 14(2)9 7 1 6
 10.[illegible] 1(14)14 0 0 1
 11.[illegible] 1(14)14 0 0 1
 12.[illegible] 1(7)7 0 0 1
 13.[illegible] 1(7)7 0 0 1
 total NATs = 103
L= 15
 1.000000000000000 1(2)497 270 6 126
 2.[illegible] 15(2)275 151 19 47
 3.[illegible] 15(2)75 53 6 30
 4.[illegible] 15(2)202 100 10 17
 5.[illegible] 15(2)29 21 2 15
 6.[illegible] 15(2)461 221 17 18
 7.[illegible] 15(2)315 148 24 13
 8.[illegible] 15(2)315 148 24 13
 9.[illegible] 15(2)243 125 15 22
 10.[illegible] 15(2)233 127 11 16
 11.[illegible] 15(2)13 8 2 6
 12.001001001001001 3(1)2 1 1 2
 total NATs = 154
```

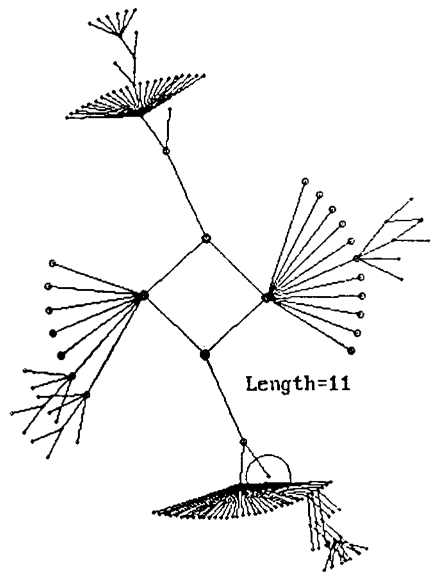

λ ratio = .5 Z = .5
0000001100001100-0000001100001100-rule 51118860
=3-rule 18 -00010010
Length=1 -15
18
183
237
72
00010010 18
10110111 183
Length=12
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
3-rule 18 =00010010
ty. at no(p)s g ml mp
L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)4 3 1 4
total NATs = 1
L= 3
1.000 1(1)8 4 2 5
total NATs = 1
L= 4
1.0000 1(1)12 9 2 8
2.0011 2(2)2 0 0 1
total NATs = 3
L= 5
1.00000 1(1)12 11 1 12
2.10010 5(2)4 2 1 2
total NATs = 6
L= 6
1.000000 1(1)46 33 4 19
2.100010 3(2)6 4 1 3
total NATs = 4
L= 7
1.0000000 1(1)128 92 5 30
total NATs = 1
L= 8
1.00000000 1(1)132 109 5 48
3.00110011 2(2)2 0 0 1
total NATs = 7
L= 9
1.000000000 1(1)80 76 2 77
3.000110011 9(2)3 1 1 2
total NATs = 19
L= 10
1.0000000000 1(1)134 123 2 124
total NATs = 26
L= 11
1.00000000000 1(1)211 199 2 200
total NATs = 25
L= 12
1.000000000000 1(1)534 459 8 323
total NATs = 20
L= 13
1.0000000000000 1(1)2121 1730 8 522
total NATs = 29
L= 14
1.00000000000000 1(1)11960 10300 17 844
7.00110010100011 7(2)4 2 1 2
total NATs = 45.
L= 15
1.000000000000000 1(1)32448 28619 19 1365
total NATs = 51

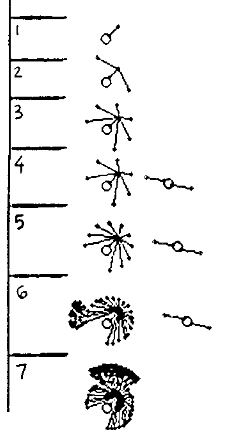
λ ratio = .5   Z = .5   0011111111111100-0011111111111100-rule 1073496060
=3-rule 126 -01111110   Length=1 -15

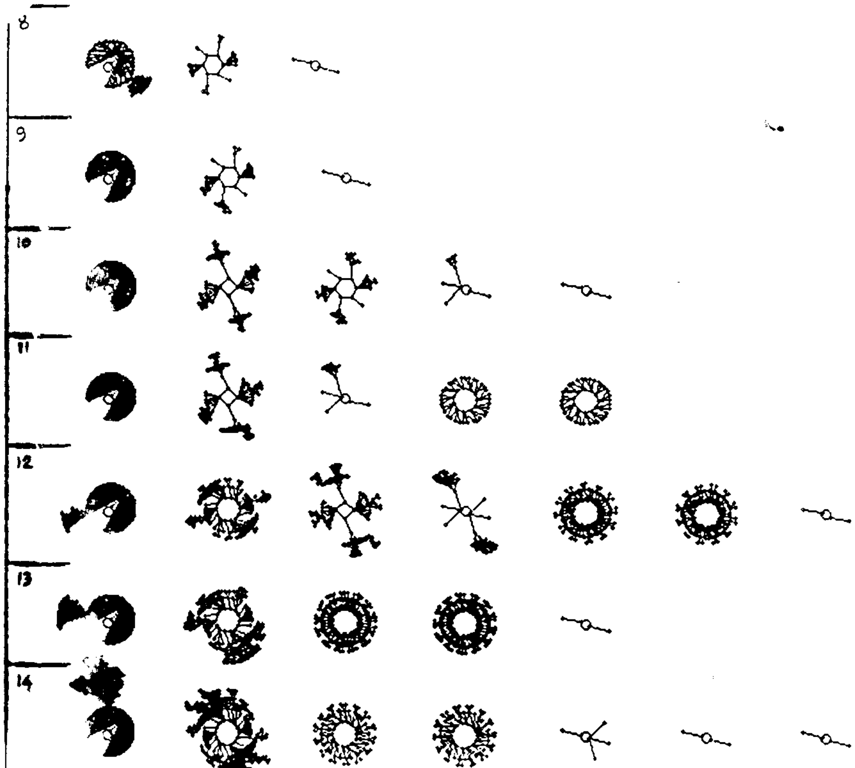




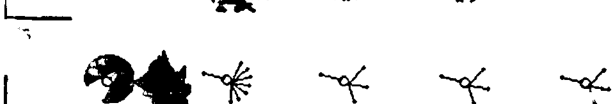






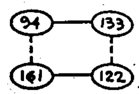



```
3-rule 126 =01111110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)8 6 2 6
 total NATs = 1
L= 4
 1.0000 1(1)8 6 2 6
 2.1011 2(2)4 2 1 2
 total NATs = 3
L= 5
 1.00000 1(1)12 10 2 10
 2.10011 5(2)4 2 1 2
 total NATs = 6
L= 6
 1.000000 1(1)52 35 5 20
 2.100111 3(2)4 2 1 2
 total NATs = 4
L= 7
 1.0000000 1(1)128 91 5 28
 total NATs = 1
L= 8
 1.00000000 1(1)136 106 6 46
 2.10000011 4(6)28 20 2 8
 3.10111011 2(2)4 2 1 2
 total NATs = 7
L= 9
 1.000000000 1(1)80 78 2 78
 2.100000011 9(6)44 33 3 12
 3.100111011 9(2)4 2 1 2
 total NATs = 19
L= 10
 1.0000000000 1(1)124 122 2 122
 2.1100000110 5(4)104 82 6 22
 3.1000000111 5(6)56 44 3 12
 4.1110111101 5(2)12 7 3 4
 5.1001110111 10(2)4 2 1 2
 total NATs = 26
L= 11
 1.00000000000 1(1)200 198 2 198
 2.11000000110 11(4)136 109 6 34
 3.11100111101 11(2)24 18 4 8
 4.10001111101 1(11)44 33 1 4
 5.10001101111 1(11)44 33 1 4
 total NATs = 25
L= 12
 1.000000000000 1(1)548 495 8 324
 2.100000000011 6(14)284 242 5 54
 3.110000001100 6(4)188 154 6 34
 4.111001111001 3(2)124 96 6 14
 5.110001100111 1(36)168 120 2 4
 6.110001111100 1(36)168 120 2 4
 7.101110111011 2(2)4 2 1 2
 total NATs = 20
L= 13
 1.0000000000000 1(1)2160 1820 9 520
 2.1000000000011 13(14)428 371 5 90
 3.1100001100111 1(39)208 143 2 4
 4.1100001111100 1(39)208 143 2 4
 5.1001110111011 13(2)4 2 1 2
 total NATs = 29
L= 14
 1.00000000000000 1(1)11932 10327 18 842
 2.10000000000111 7(14)588 520 7 90
 3.11000011001111 1(14)84 56 2 4
 4.11000011111100 1(14)84 56 2 4
 5.10011100111011 14(2)6 4 1 4
 6.10011101110111 14(2)4 2 1 2
 7.10011101111011 7(2)4 2 1 2
 total NATs = 45
L= 15
 1.000000000000000 1(1)32448 28621 20 1366
 2.100111001110011 5(2)10 8 1 8
 3.100111001110111 15(2)6 4 1 4
 4.100111001111011 15(2)6 4 1 4
 5.100111011100111 15(2)6 4 1 4
 total NATs = 51
```

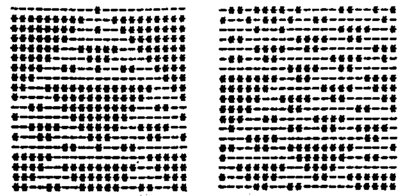


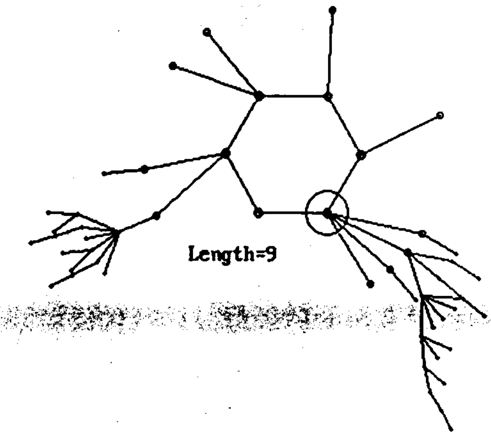
λ ratio = .75 Z = .75 1100110000000011-1100110000000011-rule 3422800899
=3-rule 161 -10100001 Length=1 -15

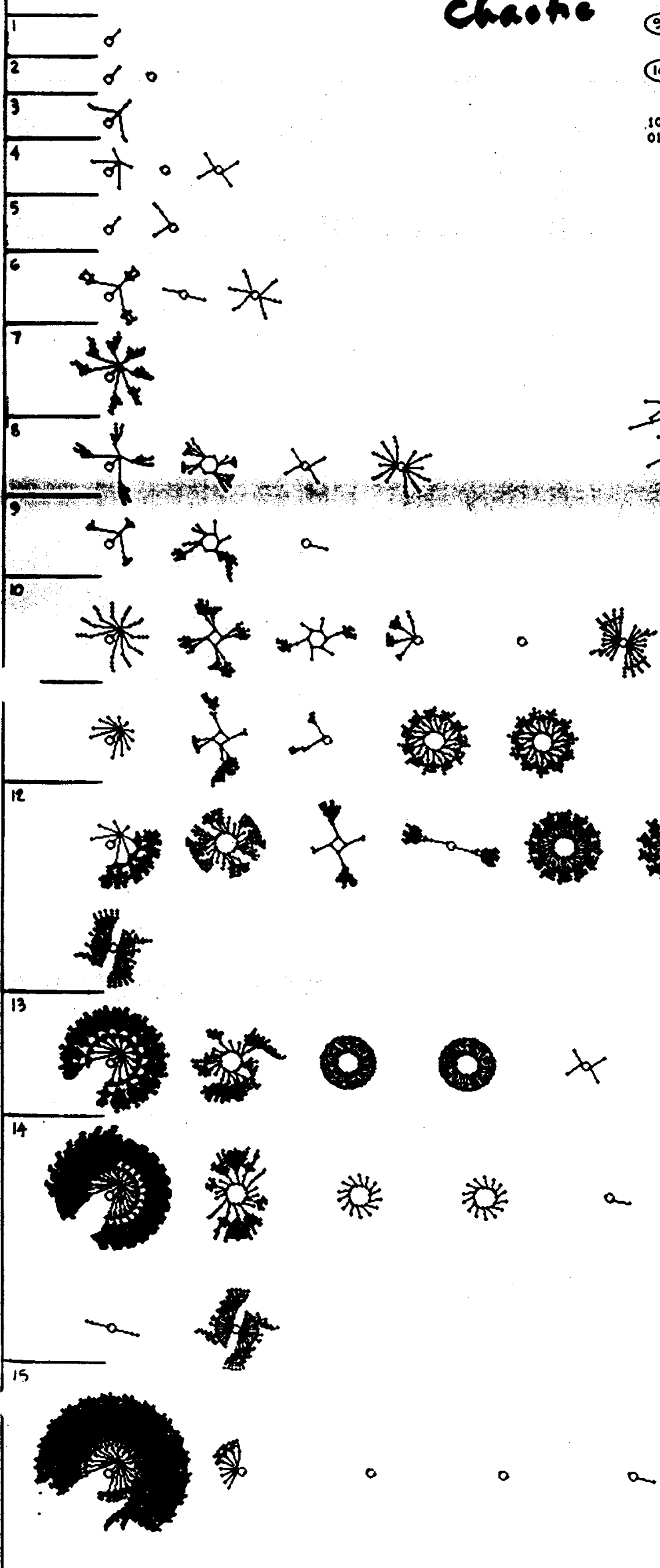
Chaotic



.10100001 161
01111010 122







3-rule 161 =10100001

ty. at no(p)s g al mp

L= 1
1.1 1(1)2 1 1 2
total NATs = 1
L= 2
1.11 1(1)2 1 1 2
2:01 1(2)2 0 0 1
total NATs = 2
L= 3
1.111 1(1)8 3 3 3
total NATs = 1
L= 4
1.1111 1(1)6 4 2 4
2:0001 2(2)2 0 0 1
3.0101 1(2)6 4 1 3
total NATs = 4
L= 5
1.11111 1(1)2 1 1 2
2.00001 5(2)6 2 3 3
total NATs = 6
L= 6
1.111111 1(1)38 21 5 5
2:000011 3(2)4 2 1 2
3.010101 1(2)14 6 2 4
total NATs = 5
L= 7
1.1111111 1(1)128 63 7 7
total NATs = 1
L= 8
1.11111111 1(1)54 16 7 4
2:00000001 4(6)40 22 3 4
3:00010001 2(2)6 4 1 3
4.01010101 1(2)30 12 3 7
total NATs = 8
L= 9
1.111111111 1(1)17 12 3 4
2:000000001 9(6)52 26 7 6
3.000010001 9(2)3 1 1 2
total NATs = 19
L= 10
1.1111111111 1(1)42 10 5 10
2:0011111001 5(4)100 52 6 7
3:0000000011 5(6)48 26 5 7
4:0010000100 5(2)32 19 3 6
5:0000100011 10(2)2 0 0 1
6.0101010101 1(2)62 30 4 11
total NATs = 27
L= 11
1.11111111111 1(1)13 11 2 11
2:00111111001 11(4)134 74 9 8
3:00010000110 11(2)19 9 5 3
4.00111000001 1(11)176 99 4 6
5.00111001000 1(11)176 99 4 6
total NATs = 25
L= 12
1.111111111111 1(1)342 229 9 13
2:000000000001 6(14)282 164 5 11
3:001111110011 6(4)98 50 7 5
4:000100000110 3(2)132 60 8 7
5:001111000001 1(36)468 252 5 6
6:001111001000 1(36)468 252 5 6
7:000100010001 2(2)8 6 1 4
8.010101010101 1(2)126 70 5 18
total NATs = 21
L= 13
1.1111111111111 1(1)2576 1495 12 13
2:0000000000001 13(14)1396 229 11 13
3.0111110000010 1(39)195 130 2 5
4.0111100100000 1(39)195 130 2 5
5.0000100010001 13(2)6 4 1 3
total NATs = 29
L= 14
1.11111111111111 1(1)13246 8127 18 21
2:00000000000011 7(14)384 210 9 10
3:00000110011111 1(14)28 14 1 2
4:00000001111001 1(14)28 14 1 2
5:00010000010001 14(2)3 1 1 2
6:00010000100011 14(2)4 2 1 2
7:00001000110001 7(2)6 2 2 2
8.01010101010101 1(2)254 154 6 29
total NATs = 46
L= 15
1.111111111111111 1(1)32483 20208 23 21
2:001000010000100 5(2)36 26 3 9
3:000010000100011 15(2)2 0 0 1
4:000010000110001 15(2)2 0 0 1
5.000010001000011 15(2)3 1 1 2
total NATs = 51

CONCLUSIONS

1. CHAOTIC DYNAMICS IS **PARTIALLY PREDICTABLE**

2. **NON-DECOMPOSABLE RULES** *TEND* TO **GENERATE CHAOS**

3. **RECONSTRUCTABILITY** IS A **COHERENT FRAMEWORK** FOR CHARACTERIZING ECA MAPPINGS AND PREDICTING CHAOS

QUESTION: *WHAT ACCOUNTS FOR THE RESIDUAL UNCERTAINTY?*