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Modes of a laser resonator with a retroreflecting roof mirror

Guosheng Zhou and Lee W. Casperson

The self-consistent integral equation for the field distribution of the resonant modes in a resonator with a tilted retroreflecting roof mirror is solved. The field distribution in the direction of the roof can be described in terms of Hermite-Gaussian functions. The beam matrix for a retroreflecting roof is found, and a new type of resonator which does not need precise alignment is proposed.

I. Introduction

The flat-roof laser resonator has been investigated by several authors. Di Francia's¹ analysis was based on the (closed) diamond-shaped cavity, Kahn² solved this problem by ray theory, while Checcacci *et al.* evaluated the Fresnel integral equation using an iterative Fox and Li method.³⁻⁵ Experimental investigations have been carried out at microwave frequencies.^{4,5} All these studies have dealt with only the flat-roof resonator, in which the angles of the roofs are extremely small (of the order of 10^{-3} rad) and the flat roof is perpendicular to the optical axis. Bergstein *et al.*⁶ proposed a symmetric resonator with two right-angle roof mirrors, but mathematical details were not included.

The important advantage of the roof mirror in laser resonators is that within certain approximations it returns a bundle of rays parallel to its original path. Thus, in some respects the device acts as a flat mirror perpendicular to the beam regardless of the rotational angle around the roof line. In principle, at least, the resulting resonators should be relatively insensitive to misalignment in one direction. For this reason the retroreflecting roof has been used to terminate ruby rods, and rotating roof mirrors have been used as laser Q-switches. However, a detailed theoretical analysis of lasers with retroreflecting roof mirrors has not yet been given. In particular, it is not known what the effect

might be of the transverse ray displacement at the roof, and the amplitude and phase distributions of the electromagnetic modes are unknown.

This paper presents a detailed diffraction integral calculation of the modes of a retroreflecting roof resonator with a roof angle of 90° and arbitrary roof misalignments. Surprisingly, for an appropriate choice of coordinates the ray properties of a retroreflecting roof mirror can be described with a simple unimodular 2×2 ray matrix. The modes of a resonator consisting of a roof mirror facing a spherical mirror are almost the same as the Hermite-Gaussian distributions obtained in conventional spherical mirror resonators. One important difference is that an additional mode dependent phase shift occurs in lasers with retroreflecting mirrors. Thus the mode frequencies of lasers with retroreflecting mirrors are different from the frequencies of conventional resonators, even though the amplitude distributions are similar.

Starting from the results given here one could estimate the mode distortions that would occur for excessive misalignment of a retroreflecting resonator mirror or imperfections in the mirror surfaces. The stability of these modes with respect to perturbations in the resonator can also be determined. In regard to ease of fabrication, the right-angle roof reflectors are probably less convenient than spherical mirrors in the optical region, but the resulting insensitivity to misalignment may be an important advantage. These mirrors can also compensate for the distortion caused by a prism, and they might be useful for compensating 1-D distortion in chemical lasers or large volume gas lasers.

The basic self-consistent integral equation is derived in Sec. II, and the solutions for resonators with retroreflecting roof and spherical mirrors are given in Sec. III. The ray matrix for a retroreflecting roof is given in Sec. IV, and a resonator consisting of two retroreflecting roof mirrors and a cylindrical lens is also described. The advantage of this resonator is that it supports simple Hermite-Gaussian modes and is extremely insensitive to angular alignment.

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Fig. 1. Schematic representation of a light ray in a resonator with a retroreflecting roof.

II. Self-Consistent Integral Equation

The retroreflecting roof resonator is represented in Fig. 1. It consists of a roof mirror facing a spherical mirror. The reflectivities of both roof and spherical mirror are assumed to be uniform. The spherical mirror has a radius of curvature R_2 and is square in cross section. The optical axis (z axis) passes through the center of the spherical mirror and the apex of the retroreflecting roof mirror O and is perpendicular to the roof line. The y axis is parallel to the roof line. The coordinate origin is the apex of the roof, and the distance between the center of the spherical mirror and the apex O is represented by L . The angle between the x axis and the retroreflective roof surface is θ , which is positive when the surface of the roof is turned counterclockwise from the x axis.

An incident ray from the point $P_2(x_2, y_2)$ passes through the point P'_1 on the surface of the roof mirror and impinges at the point A'_1 . This ray then experiences two reflections at A'_1 and A_1 and goes to P_1 at the surface of the roof mirror. The reflected ray is parallel to the incident ray $P_2P'_1$. It is shown in the Appendix that the path length difference Δ from the point P'_1 through A'_1 and A_1 to P_1 can be written in the form

$$\Delta = d \sec \theta - d \tan \theta \sec \theta (x_1 + x_2)/L, \quad (1)$$

where $d = AB$ is the width of the roof. The path length difference from the point $P_2(x_2, y_2)$ through P'_1 and A'_1 to $Q'_1(x'_1, y'_1)$ can be expressed in the form⁷

$$\rho'(x'_1, y'_1; x_2, y_2) = \rho'(x'_1, x_2) + \rho(y'_1, y_2), \quad (2)$$

where

$$\rho'(x'_1, x_2) \approx L/2 + x'_1 \tan \theta + (x_1^2 + g_2 x_2^2 - 2x'_1 x_2)/2L, \quad (3)$$

$$\rho(y'_1, y_2) \approx L/2 + (y_1^2 + g_2 y_2^2 - 2y'_1 y_2)/2L, \quad (4)$$

$$g_2 = 1 - L/R_2. \quad (5)$$

The path length difference from the point $Q_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is given by

$$\rho(x_1, y_1; x_2, y_2) = \rho''(x_1, x_2) + \rho(y_1, y_2), \quad (6)$$

where

$$\rho''(x_1, x_2) = L/2 + x_1 \tan \theta + (x_1^2 + g_2 x_2^2 - 2x_1 x_2)/2L. \quad (7)$$

The self-consistent Fresnel-Kirchhoff integral equation can be expressed as follows:

$$\gamma E^{(2)}(x_2, y_2) = \iint dx'_2 dy'_2 K(x_2, y_2; x'_2, y'_2) E^{(2)}(x'_2, y'_2), \quad (8)$$

$$K(x_2, y_2; x'_2, y'_2) = (i/\lambda)^2 \iint dx'_1 dy'_1 \exp\{[-ik\rho'(-x'_1, y'_1; x_2, y_2) - ik\rho(x'_1, y'_1; x_2, y_2)]\} [\rho'(-x'_1, y'_1; x_2, y_2) + \rho(x'_1, y'_1; x_2, y_2)]^{-1}, \quad (9)$$

where the double primes denote integration variables corresponding to x_1, y_1, x_2 , and y_2 . The field on the surface of the spherical mirror is represented by $E^{(2)}(x_2, y_2)$ and γ is the corresponding eigenvalue. K is the kernel of the integral equation. The wave number is $k = 2\pi/\lambda$, and λ is wavelength in the resonator. Here the relationship $x_1 = -x'_1$ has been used (see Appendix).

Since the kernel of the integral equation $K(x_2, y_2; x'_2, y'_2)$ can be separated in the x and y directions, the field expression splits into two independent integral equations, one for each lateral coordinate. These can be written as follows (see Appendix):

$$\gamma_x E_x^{(2)}(x_2) = (i/\lambda L) \int dx'_2 E_x^{(2)}(x'_2) \int dx'_1 (1 - x'_1 \tan \theta/L)^{-1} \cdot (1 + x'_1 \tan \theta/L)^{-1} \exp\{-ik[\rho'(-x'_1, x'_2) + \rho''(x'_1, x_2)]\}, \quad (10)$$

$$\gamma_y E_y^{(2)}(y_2) = (i/\lambda L) \int dy'_2 E_y^{(2)}(y'_2) \cdot \int dy'_1 \exp\{-ik[\rho(y'_1, y'_2) + \rho(y_1, y_2)]\}, \quad (11)$$

$$E^{(2)}(x_2, y_2) = E_x^{(2)}(x_2) E_y^{(2)}(y_2), \quad (12)$$

$$\gamma = \gamma_x \gamma_y, \quad (13)$$

where $E_x^{(2)}(x_2)$ and $E_y^{(2)}(y_2)$ illustrate, respectively, the field variations in the x and y directions on the surface of the spherical mirror, with γ_x and γ_y as the corresponding eigenvalues. According to the approximate rules used in the Huygens-Kirchhoff equation, in the integrand function small quantities to first order are kept and in the exponential small quantities to second order are kept. The factor $(1 - x'_1 \tan \theta/L)(1 + x'_1 \tan \theta/L)$ comes from the distance $P_1 P_2$ in the denominator. The limits of the second integration with respect to x'_1 and y'_1 are assumed to approach infinity, which can be interpreted to mean that the modes of the resonator will be sufficiently confined about the axis and the roof mirror be sufficiently large that the contribution to the integral from points other than those close to the axis may be neglected.

By substituting Eqs. (3) and (7) into Eq. (10) and integrating with respect to x'_1 , we can simplify Eq. (10) as follows:

$$\gamma_x E_x^{(2)}(x_2) = \exp(i\pi/4 - ikL)(2\lambda L)^{-1/2} \cdot \int_{-\infty}^{\infty} dx'_2 E_x^{(2)}(x'_2) \exp\left\{-\frac{ik}{4L} [(2g_2 - 1)(x_2^2 + x_2'^2) + 2x_2 x_2']\right\}. \quad (14)$$

III. Modes of a Resonator with Retroreflective Roof and Spherical Mirrors

The integral equation Eq. (11) has been solved in the limit that the Fresnel number $a^2/(L\lambda)$ approaches infinity, where $2a$ is the width of the spherical mirror and the confinement criterion $0 < g_2 < 1$ is assumed to be satisfied. The solution of Eq. (11) in this case can be written in the form

$$E_y^{(2)}(y_2) = \phi_n(y_2/w_2), \quad (15)$$

where ϕ_n is the normalized Hermite-Gaussian function.⁸

$$\phi_n(y_2/w_2) = N_n^{(2)} H_n(\sqrt{2}y_2/w_2) \exp(-y_2^2/w_2^2), \quad (16)$$

$$N_n^{(2)} = (\sqrt{2}/w_2)^{1/2} (2^n n! \sqrt{\pi})^{-1/2}, \quad (17)$$

$$w_2 = (\pi/\lambda L)^{-1/2} [g_2(1 - g_2)]^{-1/4}. \quad (18)$$

Here H_n is a Hermite polynomial of order n , $N_n^{(2)}$ is a normalization factor, and w_2 is the beam spot size on the spherical mirror. The corresponding eigenvalue can be expressed as

$$\gamma_{yn} = \sigma_n, \quad (19)$$

where

$$\sigma_n = \exp[-ikL + i(n + 1/2)(\pi/2 + \tan^{-1} \{(1 - 2g_2)[1 - (1 - 2g_2)^2]^{-1/2}\})]. \quad (20)$$

To solve integral equation Eq. (14), part of the kernel can be expanded in terms of the products of eigenfunctions in both variables according to

$$(2\lambda L)^{-1} \exp[-i\pi/4 - ikL - ik[(2g_2 - 1)(x_2^2 + x_2'^2) - 2x_2x_2']/4L] \\ = \sum_m \sigma_m \phi_m(x_2/w_2) \phi_m(x_2'/w_2). \quad (21)$$

Then Eq. (14) can be expressed in the form

$$\gamma_x E_x^{(2)}(x_2) = \sum_m \sigma_m C_m \phi_m(x_2/w_2), \quad (22)$$

where

$$C_m = \int_{-\infty}^{\infty} dx_2 E^{(2)}(-x_2') \phi_m(x_2'/w_2) \\ = \int_{-\infty}^{\infty} dx_2 E^{(2)}(x_2') \phi_m(-x_2'/w_2). \quad (23)$$

Multiplying both sides of Eq. (22) by $\phi_n(-x_2/w_2)$ and integrating, one obtains

$$E_{xm}^{(2)}(x_2) = \phi_m(x_2/w_2), \quad (24)$$

$$\gamma_x = (-1)^m \sigma_m, \quad (25)$$

where we have employed the orthogonality relation

$$\int_{-\infty}^{\infty} \phi_m(x_2/w_2) \phi_n(-x_2/w_2) dx_2 = (-1)^m \delta_{mn}. \quad (26)$$

It is interesting that even for large inclination angles of the roof θ , the field distribution can still be expressed in terms of an ordinary Hermite-Gaussian function, and the propagation constant γ_{xm} is the same as for a common spherical resonator except for a multiplier $(-1)^m$. This multiplier means that the odd symmetrical modes have an additional phase shift of π . These results are consequences of the retroreflecting property of the right-angle roof mirror. The resonant frequency ν_{mnq} can be found from Eqs. (13), (19), and (25):

$$\nu_{mnq} = (c/2L)[q - m/2 + (m + n + 1)(\pi)^{-1} \cos^{-1} g_2^{1/2}]. \quad (27)$$

After the field distribution on the spherical mirror has been determined the field variation in the x direction on the imaginary plane Q_1Q_1' of the retroreflecting roof mirror can be derived by Fresnel integration

$$E_{xm}^{(1)} = \chi'_m (1 - x_1 \tan\theta/L) \exp(-ikx_1 \tan\theta) \phi_m^{(1)}(x_1/w_1), \quad (28)$$

where

$$\chi'_m = \exp(-ikL/2 + i(m + 1/2)\{\pi/2 - \tan^{-1} [g_2/(1 - g_2)]^{1/2}\}), \quad (29)$$

$$\Phi_m^{(1)}(x_1/w_1) = N_m^{(1)} H_m(\sqrt{2}x_1/w_1) \exp(-x_1^2/w_1^2), \quad (30)$$

$$N_m^{(1)} = (\sqrt{2}/w_1)^{1/2} (2^m m! \pi^{1/2})^{-1/2}, \quad (31)$$

$$w_1 = (\pi/\lambda L)^{-1/2} [g_2^{-1}(1 - g_2)]^{-1/4}. \quad (32)$$

Here the phase factor $\exp(-ikx_1 \tan\theta)$ comes from the inclination of the roof, and $(1 - x_1 \tan\theta/L)$ comes from the denominator in the integrand. Thus, the field distribution on the surface Q_1Q_1' is also very similar to that on the plane mirror of a conventional resonator.

IV. Beam Matrix for a Retroreflecting Roof

Consider that an arbitrary Hermite-Gaussian beam expressed by $F_m(x_2)$ at z propagates a distance z and impinges on the tilted retroreflecting roof in such a way that a component ray travels through points P_1' and A_1 to Q_1' (see Fig. 1). Then the field variations in the x direction on the surface of Q_1Q_1' can be derived by Fresnel-Kirchhoff integration:

$$F_m^{(1)}(x_1) = (i/\lambda z)^{1/2} (1 - x_1 \tan\theta/z) \exp(-ikz/2 + ikx_1 \tan\theta) \\ \cdot \int dx_2 F_m(x_2) \exp[-ik(x_1^2 + x_2^2 + 2x_1x_2)/(2z)], \quad (33)$$

where we have assumed that the optical axis of this beam passes through the apex of the roof. After reflection the beam propagates a distance z to its starting point. The field variation in the x direction can be found as follows:

$$F_m^{(3)}(x_2) = (i/\lambda z)^{1/2} \int dx_1 F_m^{(1)}(x_1) (1 + x_1 \tan\theta/z)^{-1} \\ \cdot \exp[-ikz/2 - ikx_1 \tan\theta - ik(x_1^2 + x_2^2 - 2x_1x_2)/2z]. \quad (34)$$

Substituting Eq. (33) into Eq. (34) and using the same procedure as in Sec. IV, one gets

$$F_m^{(3)}(x_2) = (-1)^m (2\lambda z)^{-1/2} \exp(-ikz) \\ \cdot \int_{-\infty}^{\infty} dx_2' F_m(x_2') \exp[-ik(x_2^2 + x_2'^2 - 2x_2x_2')/(4z)]. \quad (35)$$

On the other hand, we may also imagine that the same Hermite-Gaussian beam $F_m(x_2)$ simply travels a distance $2z$. The field variation in the x direction then can be written as

$$F_m'(x_3) = (2\lambda z)^{-1/2} \exp(-ikz) \int_{-\infty}^{\infty} dx_2' F_m(x_2') \\ \cdot \exp[-ik(x_2^2 + x_3^2 - 2x_2x_3)/(4z)]. \quad (36)$$

Comparing Eq. (36) with Eq. (35) one finds that they are almost the same apart from an additional constant factor $(-1)^m$ in Eq. (35). Obviously this additional factor does not affect the spot size or the phase of the beam except for an additional phase shift of π . Using this information, together with Kogelnik's ABCD transformation,⁹ it follows that the amplitude and phase distribution for the retroreflecting roof mirror can be characterized by the transformation matrix

Fig. 2. Low loss resonator which does not need fine angular alignment.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (37)$$

When substituted into the appropriate phase transformation^{10,11} this matrix leads to the $(-1)^m$ factor noted above, and when applied as a ray matrix Eq. (37) also gives the expected transformation of position and slope. Thus this matrix can be used to calculate the modes of a complicated resonator which contains a retroreflecting roof.

As an example, consider a resonator consisting of two retroreflecting roofs and a thin cylindrical lens as shown in Fig. 2. The optical axis passes through the apexes of both roofs, and if the lens is rigidly attached to a reflector as suggested in the figure, the axis would be slightly bent at the lens. The cylindrical lens has a long focal length f , $f \gg w$ (w is the resulting spot size), and thus the beam matrix for a round trip starting just before the right-hand mirror can be calculated as

$$\begin{aligned} \begin{pmatrix} A & B \\ C & D \end{pmatrix} &= \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{\cos\theta}{f} & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{\cos\theta}{f} & 1 \end{pmatrix} \\ &\times \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \\ &= \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2\frac{\cos\theta}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \end{aligned} \quad (38)$$

where the $\cos\theta$ factor is the first correction that occurs whenever a lens is tilted. From this equation, one finds that the resonator is equivalent to an ordinary resonator with a plane mirror and a cylindrical mirror with radius of curvature

$$R = f/\cos\theta. \quad (39)$$

This resonator has some obvious advantages. First, the roofs do not need alignment in the x direction except perhaps to keep the optical axis in line with the amplifying medium. Second, Eq. (39) suggests that in this configuration the effective radius of curvature R may be changed by rotation of the mirror. It would, of course, also be possible to place a second cylindrical lens at the right-hand end of the cavity, and the desired output coupling could be obtained with suitable mirror coatings.

V. Conclusion

We have solved the self-consistent Fresnel-Kirchhoff integral equation for the field distribution of the resonant modes of a resonator with a tilted retroreflecting roof mirror and a spherical mirror with infinite Fresnel numbers. Even when the inclination angle of the roof mirror is quite large, low loss modes still exist, and the amplitude distributions of these modes can be expressed in terms of familiar Hermite-Gaussian functions. In hindsight, such modes might have been expected from the analogy of a retroreflecting mirror and an aligned flat mirror. The phase of the modes is, however, different from the phase in a conventional laser resonator, and hence the mode frequencies are different too.

The beam matrix for the retroreflecting roof mirror has also been discussed, and it may be written as simply minus one times the identity matrix. A new type of resonator has been proposed, and the unusual feature of this resonator is that it does not need fine angular alignment. This resonator might find applications in adverse environments where maintenance of laser alignment would otherwise be difficult. We are presently investigating the extension of these results to other types of retroreflecting laser systems.

Guosheng Zhou is a visiting scholar at UCLA from the Department of Physics of Shanxi University, China.

Appendix

The diagram used for calculation of path length differences is represented in Fig. 3. Two triangles BOB_1 and B_1OB_2 are congruent to the roof triangle AOB . The line Q_1Q_1 passes through the apex of the roof and is parallel to the roof surface AB . This line intersects the line $P_1'S$ at Q_1' and intersects the line $P_1A_1Q_1$ at Q_1 . An incident ray from P_1' experiences two reflections at A_1' and A_1 and goes to P_1 at the surface of the roof mirror. By simple geometry one can find that the ray $P_1'A_1'A_1P_1$ is equivalent to the ray $P_1'Q_1'S$ except that these rays propagate in different directions. Therefore the path length difference $\Delta = P_1'A_1' + A_1'A_1 + A_1P_1$ is equal to $P_1'S$, which is given by

$$\Delta = P_1'S = d/\cos(\theta - \beta), \quad (A1)$$

Fig. 3. Schematic representation for calculating path length differences.

where θ is the angle between the x axis and the roof surface, and β is the angle between the incident ray and the z axis. The latter angle may be approximately expressed as $\beta \simeq (x_2 - x'_1)/L$. Generally speaking, β is of the order of $w/L \simeq 10^{-3}$ rad, while θ may be much larger. Therefore, $\cos(\theta - \beta)$ is approximately equal to $\cos\theta + \beta \sin\theta$, and

$$\Delta \simeq d \sec\theta - d(x_2 - x'_1) \tan\theta \sec\theta/L. \quad (\text{A2})$$

Obviously a ray from P'_1 through A'_1 and A_1 to P_1 can also be equivalent to a ray which propagates from P'_1 through A'_1 to Q'_1 , experiences a displacement Q'_1Q_1 , which does not contribute to any path length difference, and then travels from Q_1 through A_1 to P_1 . Therefore, the plane $Q_1Q'_1$ can be regarded as a reference plane for calculating the field.

Since $A_1O = OA''_1$ and A_1Q_1 is parallel to $Q'_1A''_1$, the triangles OQ_1A_1 and $OQ'_1A''_1$ are congruent. Therefore, $Q_1O = OQ'_1$, or

$$x_1 = -x'_1. \quad (\text{A3})$$

The path length difference in Eq. (3) can be expressed in the form

$$\begin{aligned} \rho'(x'_1, x_2) &= L/2 + x'_1 \tan\theta + (x'^2_1 + g_2 x^2_2 - 2x'_1 x_2)/2L, \quad (\text{A4}) \\ &= L/2 - x_1 \tan\theta + (x^2_1 + g_2 x^2_2 + 2x_1 x_2)/2L, \end{aligned}$$

and this result is used in reducing Eqs. (8) and (9).

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Simple solar tracking device activates motor to focus collector

A patent on a simple solar collector tracking apparatus has been awarded to the Department of Energy in the name of its inventor, Robert P. Stromberg of Sandia National Laboratories. Patent number is 4,274,394.

The device, which activates a motor to drive a collector into focus, consists basically of two parallel bimetallic strips separated by a shade plate. The strips, which are about two inches by one-half inch and similar to those found in many thermostats, are attached at one end to a horizontal shaft. One strip is also anchored at the other end, but the end of the second strip is an electrical contact suspended midway between two fixed contacts.

Equally exposed to sun

When the two strips are equally exposed to the sun, the collector is in focus and the electrical circuit from the sensor is open. However, when the strips are unequally illuminated, indicating the collector is not in focus, the circuit is closed by the flexing action of one of the strips. Closure of the overhead contact drives the motor in one direction, while closure of the bottom contact drives the motor in the other direction.

The shade plate is sufficient for coarse tracking; for fine tracking, a shadow bar is located along the collector axis about 12 inches above the centerline of the two strips.

Thermal lag in the strips permits the collector to be driven squarely into focus before the circuit is broken, stopping the tracking motion. Tracking also stops during periods of cloudiness because insufficient heat is created to flex the strips.

SOLAR COLLECTOR TRACKING DEVICE