1982

Entropy Measure in Input-Output Analysis

Martin Zwick
Portland State University, zwick@pdx.edu

Abbas Heiat
Portland State University

Citation Details
Zwick, M., Heiat, A. (1982) "Entropy Measures in Input-Output Analysis." Systems Science and Science:
Proceedings of the Twenty-Sixth Annual Meeting of the Society for General Systems Research, ed. by L.
Troncale, SGSR, Louisville, KY, 1982.

ENTROPY MEASURES IN INPUT-OUTPUT ANALYSIS

Martin Zwick
and
Abbas Heiat

Systems Science PhD Program
Portland State University
Portland, Oregon   97207

ABSTRACT

Applications of Shannon's entropy measure to the matrices of technical and interdependence coefficients, to the final demand vector, and to other aspects of input-output tables are proposed. These entropy measures serve as indices of different types of economic diversity. The relevance of such indices for economic planning and for analyses of economic structural complexity and development is discussed.

Shannon's entropy, $H = -\Sigma\, p_i \log p_i$, can be used, in conjunction with input-output analysis, as a measure of various kinds of economic diversity.* We here propose three possible applications of the entropy concept: (a)   Application to the matrix of interdependence coefficients. This gives, for each sector, a measure of the diversity of impact on the economic system of a unit change in demand for the sector input.   (b)   Application to the matrix of technical coefficients. This describes the technological pattern of connectedness between sectors by assigning to each sector measures of input and output diversity.   (c) Application to economy-wide vectors of the input-output table, such as total final demand and primary input, and their constituents, and total output (input). These measures assess the diversity of various aspects of the economic system taken as a whole.

The structure of an economy can be represented by an input-output table as shown in the table below (O'Connor and Henry, 1975).

INPUTS	INTERMEDIATE DEMAND					TOTAL FINAL DEMAND	TOTAL OUTPUT
	(1)	(2)	(3)		(N)		
Sector 1	x_{11}	x_{12}	x_{13}	. . . .	x_{1n}	Y_1	X_1
Sector 2	x_{21}	x_{22}	x_{23}	. . . .	x_{2n}	Y_2	X_2
Sector 3	x_{31}	x_{32}	x_{33}	. . . .	x_{3n}	Y_3	X_3
.	.	.	.		.	.	.
.	.	.	.		.	.	.
.	.	.	.		.	.	.
Sector n	x_{n1}	x_{n2}	x_{n3}	. . . .	x_{nn}	Y_n	X_n
Primary Inputs	z_1	z_2	z_3	. . . .	z_n		
Total Inputs	X_1	X_2	X_3	. . . .	X_n		

---

* Application of the entropy measure to input-output tables for the purpose of studying
   aggregation problems has been explored by Skolka (1965) and Theil (1967). A general
   and critical review of the use of entropy measures in business and economics has been
   offered by Horowitz and Horowitz (1976).

Technical coefficients are calculated by:

$$a_{ij} = \frac{x_{ij}}{x_j} \qquad i, j = 1, 2, \ldots n \qquad (1)$$

Given a final demand vector Y, the required outputs are calculated by the following equation:

$$X = (I - A)^{-1} Y \qquad (2)$$

where:

A=n x n matrix of technical
    coefficients
X=n x 1 vector of total output
Y=n x 1 vector of total final demand
I=n x n identity matrix

For convenience of discussion, we define matrix $D = (I - A)^{-1}$. The elements of matrix D are known as "interdependence coefficients."

Equation (2) expresses the output of each sector as a function of final demand for the output of all of the sectors. Each column of Matrix D specifies the effect on all sectors of a unit change in the final demand of a particular sector. Normalization of each column, so that its elements sum to 1, yields a new matrix, B, from which an entropy for each sector can be defined, as follows:

$$H_j = -\sum_i b_{ij} \log b_{ij} \qquad (3)$$

where

$$b_{ij} = d_{ij} / \sum_i d_{ij}.$$

H_j will be greater the more non-zero b_{ij}'s there are in any column of B, and the closer in magnitude these elements are to one another. H_j is a measure of the "impact diversity" of demand for the output of the jth sector. This measure might, for example, be positively correlated with the amount of information required to adjust production processes to fluctuations in demand for a sector's output. (On the relationship between economic complexity and such adjustment processes, see, for example, Robinson and Markandya, 1973.) These calculated entropies could be used for comparing different sectors within an economy. (One might even define a higher-order entropy for the economy as a whole, as $H = -\sum_j H_j \log H_j$, where the H_j's have been appropriately normalized.) A sector's impact diversity could also be studied over time, or compared from one economy to another.

To study the technological pattern of interconnectedness between sectors, Shan-

non's entropy measure may be applied directly to the matrix of technical coefficients, A. For each sector, one can define both an input and an output entropy from the columns and rows of A, respectively, as follows:

$$H_j^{input} = -\sum_k a_{kj} \log a_{kj} \qquad (4)$$

$$H_j^{output} = -\sum_k a_{jk} \log a_{jk}$$

A sector with high output entropy makes more diverse contributions to the economy than a sector with low output entropy.

H_j^{output} might thus be used as an index of the "basic-ness" of a sector, a property of some importance for investment planning, though obviously many other measures of this attribute are possible (Chakaravaryt, 1969). Input and output entropies may be correlated with other properties of the various sectors. For example, the relationship between the diversity and the stability of sectors might be interesting to examine. (This issue and the relation between diversity and "maturity" has received extensive consideration in the ecological literature. See, for example, May 1974.) As was suggested for the demand-impact entropy defined earlier, input and output diversity measures for the different sectors can be used in static and dynamic intra- and inter-economic analyses.

The entropy measures discussed so far have been defined for individual sectors, but such measures can also be generated for aspects of the economy as a whole. Diversity can be calculated for the final demand or primary input vectors, or for their constituents, such as the consumption and employment vectors, or for the total output (input) vector. Viewing the economic system from the outside as a black box, diversities can be calculated for import and export vectors (Horowitz and Horowitz, 1976). Some of these diversity measures might be useful, for example, in the analysis of economic development. The hypothesis that developed economies have high export and low import diversities, while in less developed economies, these are reversed (Leontief, 1963) could be quantitatively assessed.

In summary, the Shannon entropy expression can be applied to input-output data to generate a variety of diversity measures for individual sectors and for the economy as a whole. These measures may be useful for economic planning and for analyses of economic structural complexity and economic development.

REFERENCES

Chakaravaryt, S
   1969   Capital and Development Planning.
          Cambridge, Mass: The MIT Press.

Horowitz, Ann R and Horowitz, Ira
   1976   "The Real and Illusory Virtues of
          Entropy-Based Measures for Busi-
          ness and Economic Analysis."
          Decision Science. 7: 121-136.

Leontief, Wassily
   1963   "The Structure of Development."
          Scientific American. September:
          164.

May, Robert
   1974   Stability and Complexity in Model
          Ecosystems. Princeton, NJ: The
          Princeton University Press.

O'Connor, R and Henry, E W
   1975   Input-Output Analysis and Its Appli-
          cations. New York: Hafner Press.

Robinson, S and Markandya, A
   1973   "Complexity and Adjustment in Input-
          Output System." Oxford Bulletin of
          Economic Statistics. May: 119-34.

Shannon, Claude and Weaver, Warren
   1975   The Mathematical Theory of Communi-
          cation. Urbana, Ill: The Univer-
          sity of Illinois Press.

Skolka, Jiri
   1965   "The Use of the Quantity of Infor-
          mation Measure in the Aggregation
          of Input-Output Tables."
          Kybernetica. 1: 33-49.

Theil, Henri
   1967   Economics and Information Theory.
          Amsterdam: North-Holland Publish-
          ing Company, chapter 9.

Author:
   Martin Zwick
   Systems Science PhD Program
   Portland State University
   Portland, Oregon 97207

Biography: PhD, Biophysics, MIT, 1968;
Assistant Professor, Biophysics and Theo-
retical Biology, U Chicago; research on
mathematical methods of structure analysis.
Shift to systems theory and cybernetics;
now professor in interdisciplinary systems
program at PSU.