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## ENTROPY MEASURES IN INPUT-OUTPUT ANALYSIS

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## ABSTRACT

Applications of Shannon's entropy measure to the matrices of technical and interdependence coefficients, to the final demand vector, and to other aspects of input-output tables are proposed. These entropy measures serve as indices of different types of economic diversity. The relevance of such indices for economic planning and for analyses of economic structural complexity and development is discussed.

Shannon's entropy,  $H = -\sum p_i \log p_i$ , can be used, in conjunction with input-output analysis, as a measure of various kinds of economic diversity.\* We here propose three possible applications of the entropy concept: (a) Application to the

matrix of interdependence coefficients. This gives, for each sector, a measure of the diversity of impact on the economic system of a unit change in demand for the sector input. (b) Application to the matrix of technical coefficients. This describes the technological pattern of connectedness between sectors by assigning to each sector measures of input and output diversity. (c) Application to economy-wide vectors of the input-output table, such as total final demand and primary input, and their constituents, and total output (input). These measures assess the diversity of various aspects of the economic system taken as a whole.

The structure of an economy can be represented by an input-output table as shown in the table below (O'Connor and Henry, 1975).

INPUTS	INTERMEDIATE DEMAND				TOTAL FINAL DEMAND	TOTAL OUTPUT
	(1)	(2)	(3)	(N)		
Sector 1	$x_{11}$	$x_{12}$	$x_{13}$	$\dots x_{1n}$	$Y_1$	$v_1$
Sector 2	$x_{21}$	$x_{22}$	$x_{23}$	$\dots x_{2n}$	$Y_2$	$x_2$
Sector 3	$x_{31}$	$x_{32}$	$x_{33}$	$\dots x_{3n}$	$Y_3$	$x_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sector n	$x_{n1}$	$x_{n2}$	$x_{n3}$	$\dots x_{nn}$	$Y_n$	$v_n$
Primary Inputs	$Z_1$	$Z_2$	$Z_3$	$\dots Z_n$		
Total Inputs	$X_1$	$X_2$	$X_3$	$\dots X_n$		

\* Application of the entropy measure to input-output tables for the purpose of studying aggregation problems has been explored by Skolka (1965) and Theil (1967). A general and critical review of the use of entropy measures in business and economics has been offered by Horowitz and Horowitz (1976).

Technical coefficients are calculated by:

$$a_{ij} = \frac{x_{ij}}{x_j} \quad i, j = 1, 2, \dots, n \quad (1)$$

Given a final demand vector  $Y$ , the required outputs are calculated by the following equation:

$$X = (I - A)^{-1} Y \quad (2)$$

where:

$A = n \times n$  matrix of technical coefficients  
 $X = n \times 1$  vector of total output  
 $Y = n \times 1$  vector of total final demand  
 $I = n \times n$  identity matrix

For convenience of discussion, we define matrix  $D = (I - A)^{-1}$ . The elements of matrix  $D$  are known as "interdependence coefficients."

Equation (2) expresses the output of each sector as a function of final demand for the output of all of the sectors. Each column of Matrix  $D$  specifies the effect on all sectors of a unit change in the final demand of a particular sector. Normalization of each column, so that its elements sum to 1, yields a new matrix,  $B$ , from which an entropy for each sector can be defined, as follows:

$$H_j = -\sum_i b_{ij} \log b_{ij} \quad (3)$$

where

$$b_{ij} = d_{ij} / \sum_i d_{ij}$$

$H_j$  will be greater the more non-zero  $b_{ij}$ 's there are in any column of  $B$ , and the closer in magnitude these elements are to one another.  $H_j$  is a measure of the "impact diversity" of demand for the output of the  $j$ th sector. This measure might, for example, be positively correlated with the amount of information required to adjust production processes to fluctuations in demand for a sector's output. (On the relationship between economic complexity and such adjustment processes, see, for example, Robinson and Markandya, 1973.) These calculated entropies could be used for comparing different sectors within an economy. (One might even define a higher-order entropy for the economy as a whole, as  $H = -\sum_j H_j \log H_j$ , where the  $H_j$ 's have been appropriately normalized.) A sector's impact diversity could also be studied over time, or compared from one economy to another.

To study the technological pattern of interconnectedness between sectors, Shan-

non's entropy measure may be applied directly to the matrix of technical coefficients,  $A$ . For each sector, one can define both an input and an output entropy from the columns and rows of  $A$ , respectively, as follows:

$$H_j^{\text{input}} = -\sum_k a_{kj} \log a_{kj} \quad (4)$$

$$H_j^{\text{output}} = -\sum_k a_{jk} \log a_{jk}$$

A sector with high output entropy makes more diverse contributions to the economy than a sector with low output entropy.

$H_j^{\text{output}}$  might thus be used as an index of the "basic-ness" of a sector, a property of some importance for investment planning, though obviously many other measures of this attribute are possible (Chakaravarty, 1969). Input and output entropies may be correlated with other properties of the various sectors. For example, the relationship between the diversity and the stability of sectors might be interesting to examine. (This issue and the relation between diversity and "maturity" has received extensive consideration in the ecological literature. See, for example, May 1974.) As was suggested for the demand-impact entropy defined earlier, input and output diversity measures for the different sectors can be used in static and dynamic intra- and inter-economic analyses.

The entropy measures discussed so far have been defined for individual sectors, but such measures can also be generated for aspects of the economy as a whole. Diversity can be calculated for the final demand or primary input vectors, or for their constituents, such as the consumption and employment vectors, or for the total output (input) vector. Viewing the economic system from the outside as a black box, diversities can be calculated for import and export vectors (Horowitz and Horowitz, 1976). Some of these diversity measures might be useful, for example, in the analysis of economic development. The hypothesis that developed economies have high export and low import diversities, while in less developed economies, these are reversed (Leontief, 1963) could be quantitatively assessed.

In summary, the Shannon entropy expression can be applied to input-output data to generate a variety of diversity measures for individual sectors and for the economy as a whole. These measures may be useful for economic planning and for analyses of economic structural complexity and economic development.

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