Taylor Instability in Ryholite Lava Flows

B. A. Baum
Arizona State University

W. B. Krantz
Arizona State University

Jonathan H. Fink
Portland State University, jon.fink@pdx.edu

R. E. Dickinson
Arizona State University

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Abstract. A refined Taylor instability model is developed to describe the surface morphology of rhyolite lava flows. The effect of the downslope flow of the lava on the structures resulting from the Taylor instability mechanism is considered. Squire's (1933) transformation is developed for this flow in order to extend the results to three-dimensional modes. This permits assessing why ridges thought to arise from the Taylor instability mechanism are preferentially oriented transverse to the direction of lava flow.

Measured diapir and ridge spacings for the Little and Big Glass Mountain rhyolite flows in northern California are used in conjunction with the model in order to explore the implications of the Taylor instability for flow emplacement. The model suggests additional lava flow features that can be measured in order to test whether the Taylor instability mechanism has influenced the flow's surface morphology.

Introduction

Estimating the rheology of lava flows is an essential component of volcanic hazard evaluations and also is a tool in the remote determination of flow compositions on other planets. For flows whose emplacement is not observed, indirect methods must be used to assess such physical parameters as viscosity, yield strength, density, and content of volatiles. Most studies of this type have assumed that the geometry of large-scale morphological features on the flow surface reflects the bulk rheology of the active lava.

One class of lava surface structures that is particularly well suited to this sort of interpretation are those with periodic features that result from fluid instabilities. At least two instabilities have been identified and utilized in lava flow studies: surface folding and Taylor or gravity-induced instabilities. Both can lead to the development of regularly spaced structures on the surfaces of lava flows such as shown in Figure 1. This figure shows an aerial photograph of a transverse ridge pattern on the Big Glass Mountain flow in northern California which has a slope of approximately 9°. The flow is nearly 3 km long and has a thickness of approximately 75 m. The corrugations on this flow have a wavelength of approximately 60 m and amplitudes averaging 8 m.

The geometry of surface folds, first analyzed by Fink and Fletcher (1978), has been used by Fink (1980a), Zimbelman (1985), Head and Wilson (1986), and others to estimate the rheology of lava flows on other planets. Fink and Fletcher's (1978) analysis assumes that lava flows have a temperature-dependent Newtonian rheology and that the lava's viscosity decreases exponentially inward from the cooler upper flow surface.

The presence of a Taylor instability was proposed by Fink (1978) to explain certain regularly spaced domal outcrops or diapirs of low-density pumice on the surfaces of silicic lava flows. Subsequent investigations (Fink, 1980c, 1985; Eichelberger and Fink, 1978; Manley and Fink, 1987, 1989; Manley and Fink, 1987) have attempted to relate the density inversion to the distribution and migration of volatiles within actively advancing flows. These models are important for studies of volcanic hazards and eruption processes, since the presence of volatiles in silicic magmas is considered a major factor in the inception of explosive volcanism. Recent drilling investigations in rhyolite flows (Eichelberger et al., 1984, 1985; Goff et al., 1986) have provided new information about their volatile gas distribution and internal arrangement of textures which now permits more accurate specification of boundary conditions for modeling instabilities. The drill cores have shown that low-density, scoriaceous pumice may be present in flow interiors even when it is not exposed on flow surfaces. Hence diapirs may not be the only evidence for Taylor instabilities in rhyolite flows. Although domal outcrops of coarsely vesicular pumice have only limited occurrence on rhyolite flows, regularly spaced surface ridges are nearly ubiquitous. One of the goals of the present paper is to investigate the possibility that these ridges reflect a Taylor instability at an internal interface, rather than being caused by a folding instability at the upper surface.

Fink's (1980c) original linear stability analysis assumes Newtonian rheology, an absence of slope-induced shear stresses, a rigid upper flow surface, and a two-dimensional perturbation of the unstable interface. The new drill core data have provided motivation to develop a refined model for the Taylor instability in silicic lava flows. Our formulation now allows for three-dimensional disturbances, a mean shear flow, a deformable upper free surface, and the inclusion of terms neglected in prior analyses. We use this new model to (1) determine whether the formation of two-dimensional ridges transverse to the flow is favored over longitudinal ridges or three-dimensional modes; (2) compare the deformation of the free surface to that of the unstable internal interface to assess whether ridges might arise from a scoriaceous pumice layer interior to the rhyolite flow; (3) determine the phase velocity of...
the ridges; and (4) use the viscosity ratio to help determine the state of volatiles within the buoyant layer (which has consequences for hazards). In addition we will suggest future observations that could be made of active rhyolite flows or deeply dissected flows in order to test the relevance of the Taylor instability model.

**Field Observations**

Active rhyolite flows have never been the subject of detailed field observation. Hence identification of Taylor instabilities in rhyolite flows has relied on information inferred from the mapping of lava surface structures and textures. Fresh rhyolite flows typically have blocky surfaces with contrasting areas of light- and dark-colored pumice. Fink (1978, 1980c) showed that on several young rhyolite flow lobes, outcrops of darker, less dense, scoriaceous lava have regular spacings in the downstream direction. The outcrops were interpreted to be diapirs that rose to the flow surface in response to a Taylor instability, and the spacings were related analytically to the thicknesses and viscosities of textural layers observed in flow fronts. Most of the scoriaceous outcrops were elongate transverse to the flow direction, which was cited as evidence that they rose and became stretched kinematically while the flows were still advancing.

The first two drill cores of the Inyo Scientific Drilling Project (Eichelberger et al., 1984, 1985) penetrated Obsidian Dome, another young rhyolite flow with a textural stratigraphy and distribution of surface outcrops similar to those described by Fink (1980c) at Medicine Lake Highland. The VC-1 drill hole through the 130,000- to 140,000-year-old Banco Bonito rhyolite flow in the Valles Caldera in New Mexico (Self et al., 1986; Goff et al., 1986) revealed a similar vertical arrangement of lava textures. The drill cores provided accurate thicknesses for the different textural layers and showed that the coarsely vesicular pumice found in the regularly spaced outcrops had up to 5 times higher water content (0.5 versus 0.1 wt %), and significantly

**Table 1. Thickness of Internal Textural Layers in Rhyolite Flows Determined From Drill Cores of Inyo and Valles Drilling Projects**

<table>
<thead>
<tr>
<th>Total Drill Hole</th>
<th>Lava Flow</th>
<th>Thickness, m</th>
<th>$d_1$, m</th>
<th>$d_2$, m</th>
<th>$d_1 + d_2$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDO-2A</td>
<td>Obsidian Dome</td>
<td>55</td>
<td>4</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>RDO-2B</td>
<td>Obsidian Dome</td>
<td>55</td>
<td>12</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>VC-1</td>
<td>Banco Bonito</td>
<td>150</td>
<td>26</td>
<td>3</td>
<td>29</td>
</tr>
</tbody>
</table>

Data from the Inyo Drilling Project are from Eichelberger et al. (1984, 1985), and data from the Valles Drilling Project are from Goff et al. (1986).

- **a** Upper dense layer thickness.
- **b** Lower buoyant layer thickness.
- **c** Portion of flow affected by instability.
characterized by the superposition of two or more fluid layers having different densities. Thus this review focuses specifically on key developments which relate to the Taylor instability mechanism and its application to lava flow morphology.

Taylor (1950) in his original treatment of this instability mechanism considered the unstable stratification of two infinitely thick, initially motionless, fluid layers. His linear stability analysis, which ignored both viscous and surface tension effects but included the unstable state terms, indicated that discontinuous density stratification is always unstable. His analysis also indicated that the growth rate of the unstable modes increases with increased wave number (decreased wavelength) such that there is no most highly amplified or preferred wavelength having finite wave number.

Bellman and Pennington (1954) included both the viscous and surface tension effects ignored in Taylor's (1950) analysis. They found that both viscosity and surface tension reduce the growth rate of the shorter wavelength modes, although the former cannot stabilize any modes. Including either viscosity or surface tension results in a most highly amplified wavelength.

The idea of a buoyant instability mechanism leading to formation of various types of diapirs in salt domes has been investigated by Danes (1964), Selig (1965), Whitehead and Luther (1975), and Marsh (1979), among others. Selig analyzed the situation of an underlying fluid layer penetrating into an infinite fluid. Results were obtained for growth rates of small disturbances for a configuration of fluids in an initially static state.

Ramberg (1967) applied the Taylor instability mechanism to five sets of conditions thought to describe geophysical stratification situations. He included the viscous terms in his linear stability development but ignored the surface tension and unsteady state terms. One consequence of omitting the unsteady state terms is that his model for two infinitely thick fluid layers predicts that the most unstable mode has an infinite wavelength, which contradicts Taylor's original results. In three other models developed by Ramberg, only one of the fluid layers is assumed to be infinitely thick, and the surface of the uppermost layer is assumed to be undeformable and rigid. The

### Table 2. Geometric Parameter for Three Flow Lobes

<table>
<thead>
<tr>
<th>Lobe</th>
<th>Total Thickness, ( \lambda_d ), m</th>
<th>( \lambda_t ), m</th>
<th>( d_1 ), m</th>
<th>( d_2 ), m</th>
<th>( \theta ), degrees</th>
<th>( \lambda_d/d_2 ), m</th>
<th>( \lambda_t/d_2 ), m</th>
<th>( d_1/d_2 ), m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northeast LGM</td>
<td>35</td>
<td>43</td>
<td>15</td>
<td>10-15</td>
<td>4</td>
<td>4</td>
<td>11</td>
<td>3.8</td>
</tr>
<tr>
<td>Northwest LGM</td>
<td>55</td>
<td>70</td>
<td>30</td>
<td>20-65</td>
<td>5</td>
<td>6</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>North BGM</td>
<td>75</td>
<td>60</td>
<td>40</td>
<td>30</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>6.7</td>
</tr>
</tbody>
</table>

Data are from Fink (1979, 1980a,) and Manley and Fink (1987).

a Average diapir spacing.
b Average ridge spacing.
c Measured upper dense layer thickness.
d Lower buoyant layer thickness inferred from Table 1.
e Slope angle relative to horizontal.
f Little Glass Mountain (LGM) flow on Medicine Lake Highland Volcano in northern California.
g Big Glass Mountain (BGM) flow on Medicine Lake Highland Volcano in northern California.

lower density (0.8-1.5 versus 2.25 g/cm^3) than the flows as a whole. These drill core data for the total flow thickness, upper dense obsidian layer thickness \( d_1 \), and buoyant coarse pumice layer thickness \( d_2 \) are summarized in Table 1. Since density increases below the buoyant layer, the Taylor instability can be operative only in the upper portion of the flow; hence the last column in Table 1 gives the thickness of the flow affected by the instability.

Fink and Manley (1989) cite several lines of evidence suggesting that the buoyant pumice layer could increase in both thickness and volatile content as a flow advanced. They also suggest that exposure of this volatile-rich pumice by collapse of a flow front could cause formation of highly destructive pyroclastic flows. The idea that the coarse vesicular pumice is associated with concentrations of volatiles is further supported by the observation that on several flows, scoriaceous pumice occurs near the bases of explosion pits up to 13 m deep that apparently formed by the violent release of volatiles from the flow interior.

One of the goals of this analysis is to estimate the viscosities of rhyolite flows from the geometry of diapirs and ridges and inferred thicknesses of the various textural layers in these flows. Total flow thickness, diapir spacing \( \lambda_t \), ridge spacing \( \lambda_d \), measured upper dense layer thickness \( d_1 \), estimated buoyant layer thickness \( d_2 \), slope \( \theta \), \( \lambda_d/d_2 \), \( \lambda_t/d_2 \), and \( d_1/d_2 \) for three lobes of the Little and Big Glass Mountain rhyolite flows on the Medicine Lake Highland Volcano in northern California are summarized in Table 2. These data, which include those of Fink (1980a, c) along with recent measurements of Manley and Fink (1987), will be used in the Taylor instability model to assess the implications of three flow emplacement hypotheses.

Previous Studies of Taylor Instability

The rhyolite lava flows of interest here are characterized by the superposition of two or more fluid layers having different densities. Thus this review focuses specifically on key developments...
interface between the two fluid layers is assumed to be deformable but laterally immobile. This latter assumption, which Ramberg (1967, p. 181) refers to as the "welded interface condition," is strictly correct only if both fluid layers are infinitely thick. Ramberg also developed a model for two fluid layers having finite thicknesses. The interface between the two fluid layers is again assumed to be "welded"; however, the surface of the upper fluid layer is allowed to be both deformable and laterally mobile. Ramberg included viscous effects in this model but ignored the surface tension and unsteady state terms.

Whitehead and Luther (1975) reported on experiments in which a thin horizontal layer of fluid was perturbed, forming upwelling spouts that rose through a more dense fluid of a different viscosity. It was observed that the structural features were strongly dependent on which fluid was more viscous. The model employed in this study is of an underlying thin, horizontally infinite layer of fluid and an overlying, infinitely deep region of denser fluid. Equations were recovered for the growth rate and most highly amplified wave numbers, both for the case of a free slip boundary below the thin layer and for the case of a no slip boundary below the thin layer. The growth rate was found to be determined mostly by the large-viscosity fluid. A problem not addressed in this study is the effect of fluid flow on the growth rate of disturbances.

Fink (1978) also explored the implications of the Taylor instability mechanism for geophysical flows and developed two new models which include the viscous terms but ignore the surface tension and unsteady state terms. In contrast to Ramberg's (1967) analysis Fink allowed for a fully deformable, laterally mobile interface between the two fluid layers. One of Fink's models allows for two fluid layers having finite thicknesses, but the surface of the upper fluid layer is assumed to be both undeformable and rigid. Fink invoked this latter assumption in order to account for the constraining effect of surface solidification owing to cooling. The wavelength of the most highly amplified mode predicted by this latter model is of the same size as the characteristic diapir spacing observed on three lobes of the Little and Big Glass Mountain rhyolite flows. However, this model cannot explain why diapirs are oriented transverse to the direction of flow.

All of the Taylor instability analyses discussed thus far assume that the unstably stratified fluid layers are initially motionless. However, the lava flows of interest here exhibit a net downslope flow. Determining the influence of a net shear flow on this instability mechanism is one of the primary goals of this study. Kao (1965) investigated the influence of unstable density stratification on the stability of a bilayer film flowing down an inclined plane. The flow is assumed to be unstable primarily because of shear. The scaling employed by Kao precludes applying his model to a flow which is unstable primarily because of unstable density stratification.

The Taylor instability model presented in this paper attempts to address the limitations of the previous studies. We develop a two-layer model in which the underlying fluid is less dense, corresponding to coarsely vesicular pumice and the overlying fluid is more dense, corresponding to obsidian. A surface vesicular layer of fine vesicular pumice is not included. In order to more accurately represent the conditions at the upper surface of a rhyolite flow, the analysis should include a cooled surface layer whose viscosity decreases exponentially with depth. Preliminary results from a model for the cooling of the upper surface of a rhyolite flow by conduction show that the time needed to cool the upper layer to the glass transition temperature is of the order of years to tens of years while the time scale of the growth rates for the diapirs is of the order of days (C.R. Manley, personal communication, 1988). The model becomes intractable with the inclusion of the surface crust and remains outside the scope of this study.

Outline of the Model

The model is of two superimposed laterally unbounded, incompressible, Newtonian fluids having constant physical properties and layer thicknesses, flowing down a gentle slope under the influence of gravity as shown in Figure 2. The basic state flow of the two fluids is steady state and fully developed. As shown in Figure 2a, the coordinate axes \((x, y, z)\) are located at the interface between the two layers with the basic state flow parallel to the \(x\) axis. The gas phase is inviscid, and the solid boundary is impermeable. The upper and lower fluids, denoted by subscripts 1 and 2, respectively, have viscosity \(\mu_1\), thickness \(d_1\), surface tension \(\gamma_1\), and density \(\rho_1\). A mean shear flow which only varies with the vertical coordinate is introduced, and allowance is made for traveling waves which propagate in the direction of net flow. The free upper surface and the interface between the buoyant pumice and more dense obsidian is deformable. Surface tension effects at the upper

![Fig. 2. (a) Schematic of two stratified laterally unbounded fluids having constant layer thicknesses \(d_1\) and \(d_2\) flowing down a plane with slope \(\theta\) under the influence of gravity and (b) schematic of wave flows arising from Taylor Instability.](image-url)

Fig. 2. (a) Schematic of two stratified laterally unbounded fluids having constant layer thicknesses \(d_1\) and \(d_2\) flowing down a plane with slope \(\theta\) under the influence of gravity and (b) schematic of wave flows arising from Taylor Instability.
free surface and at the interior interface are included for completeness, and both fluid layers are considered to be of finite thickness.

The linear stability analysis is derived in detail in the appendix, along with relevant scaling parameters and kinematic considerations. The development of the full three-dimensional set of equations is not central to our discussion, but reference will be made to the final system of equations presented in (A24) through (A37). A fourth-order differential equation is derived for each fluid layer, and the solution of these two equations requires two boundary conditions at the lower surface, two boundary conditions at the free upper surface, and four boundary conditions at the interface between the two liquids. It should be noted that in the final set of differential equations and boundary conditions developed in the appendix, there are two dimensionless wave numbers denoted by $\alpha$ and $\beta$, which are oriented in the $x$ and $z$ directions, respectively.

Squire's Transformation

The advantage of developing (A24) through (A37) for three-dimensional modes is that they include two-dimensional modes as special cases. The equations describing two-dimensional transverse modes corresponding to parallel ridges perpendicular to the downslope flow are obtained by setting $\beta = 0$. The resulting equations then can be recast into a form identical to that describing three-dimensional modes if an effective slope angle $\theta$ is redefined as follows:

$$\theta = \tan^{-1} \beta$$

Since $\theta$ is always greater than $\alpha$, this equation, which constitutes Squire's (1933) transformation for this flow, implies that three-dimensional modes are described by the linear stability solution for two-dimensional transverse modes evaluated at a slope angle $\theta$, less than the true value $\beta$.

The equations describing two-dimensional longitudinal modes corresponding to ridges parallel to the downslope flow are obtained by setting $\alpha = 0$ in (A24)-(A37). The resulting equations do not contain the basic state flow velocity and are identical to those describing the Taylor instability in the absence of a mean shear flow. This then implies that longitudinal modes do not interact or extract energy from the basic state flow.

In a later section the presence of a basic state flow is shown to be destabilizing; that is, a net flow transfers energy to transverse two-dimensional and oblique three-dimensional modes, permitting them to grow more rapidly.

Furthermore, increasing the slope is shown to be destabilizing, since it increases the net flow. From these results we will conclude that two-dimensional transverse ridges are the Taylor instability mode which should be most prominent on rhyolite lava flows.

Taylor Instability Analysis in the Absence of a Net Flow

The equations describing the Taylor instability for zero net flow are obtained by setting the phase angle $\theta = 0$ in (A24)-(A37). The resulting equations are identical to those describing the two-dimensional longitudinal mode in the presence of a shear flow. That is, the mode corresponding to ridges parallel to the direction of flow cannot interact with basic state shear flow.

The solution to be developed here differs from those of other investigators, since it allows for finite thicknesses of both liquid layers and includes the unsteady state terms.

When $\theta = 0$, (A24) and (A25) can be solved exactly in closed form. The resulting solutions for $\psi_1$ and $\psi_2$ when substituted into the boundary conditions yield a set of homogeneous algebraic equations for the integration constants. A nontrivial solution then requires that the determinant of the coefficient matrix of the integration constants be zero. This dictates a unique relationship between the parameters which constitute the temporal growth coefficient $\sigma$, its wave number $\alpha$, and the dimensionless groups containing the layer thicknesses and physical properties of the two liquids. The growth coefficient for specified values of the wave number and dimensionless groups was determined using a matrix decomposition algorithm in conjunction with a complex equation root solver as described in the thesis of Baum (1985).

A linear stability analysis for Taylor instability seeks to determine how the physical properties and layer thicknesses influence the growth coefficients of the unstable modes and the wave number of the most unstable mode. The model results are evaluated for properties characteristic of rhyolite flows. Since the results are presented in dimensionless form to consolidate the effect of many parameters, the influence of parameters contained in the scale factors must be allowed for when inferring the parametric dependence of a dimensional quantity from Figures 3-7.
Figure 3 shows the effect of the dimensionless fluid layer thicknesses $\delta_1$ and $\delta_2$ on the dimensionless growth coefficient $\alpha_{\text{max}}$ of the most highly amplified mode as a function of the viscosity ratio $\mu_2/\mu_1$. The upper and lower liquid densities are 2.4 and 1.4 g/cm$^3$, respectively, to ensure that the length scale remains constant. This is to be expected, since Bellman and Pennington's (1954) analysis demonstrates that a most highly amplified wave number results from the viscous effects which are held constant in Figure 6.

Figure 4 shows the effect of $\delta_1$ and $\delta_2$ on the dimensionless most highly amplified wave number $\alpha_{\text{max}}$. Both fluid viscosities are 106 P and again either $\delta_1$ or $\delta_2$ is fixed at 1.3 when the other is varied. The most highly amplified wave number decreases with an increase in either upper or lower liquid density owing to the flow attempting to minimize its viscous dissipation by forming cells whose wavelength is of the same size as the layer thickness. Again the upper and lower boundaries have a negligible effect on the Taylor instability when $\delta_1 > 2$ and $\delta_2 > 6$, respectively.

Figure 5 shows the effect of $\mu_2/\mu_1$ on $\alpha_{\text{max}}$ and $\alpha_{\text{max}}$. The upper and lower liquid densities are 2.4 and 1.4 g/cm$^3$, respectively, and both $\delta_1$ and $\delta_2$ are 1.3. The viscosity ratio is varied while holding $\mu_1 = 106$ P to insure that the length and time scales remain constant. An increase in $\mu_2$ relative to $\mu_1$ causes a decrease in the growth coefficient and an increase in the most highly amplified wave number because it increases the influence of the solid boundary.

Figure 6 plots the dimensionless growth coefficient $\sigma_1$ as a function of a dimensionless wave number $\alpha^*$ for three density ratios, $\rho_1/\rho_2$. Here, $\alpha^*$ is nondimensionalized using the length scale $y = (\mu_1^2/(\rho_2^2gcos#))^{1/2}$ which does not contain the density ratio. Both $\mu_1$ and $\mu_2$ are 106 P, and both $\delta_1$ and $\delta_2$ are 1.3. The density ratio is varied while holding $\rho_2 = 1.4$ g/cm$^3$ to ensure that the length scale remains constant. An increase in $\rho_1$ relative to $\rho_2$ causes an increase in $\sigma_1$ because of an increase in the driving force. However, the most highly amplified wave number remains constant. This is to be expected, since Bellman and Pennington's (1954) analysis demonstrates that a most highly amplified wave number results from the viscous effects which are held constant in Figure 6.
Taylor Instability Analysis in the Presence of a Shear Flow

An exact analytical solution is not possible because of the nonconstant coefficients introduced by the nonuniform velocity profile in \( (A24)-(A37) \). However, the effect of a shear flow on the Taylor instability can be assessed via an asymptotic solution for long waves, or equivalently, a perturbation expansion solution in small wave number.

In the presence of a net flow the Taylor instability will be manifest by traveling rather than stationary modes; that is, the interfacial corrugations will propagate with respect to the two fluid phases at a nonzero phase velocity \( c_r \). This implies that the parameter \( \sigma \) introduced via (A21) will now be a complex number whose real part \( \sigma_r \) is related to the phase velocity of the waves and whose imaginary part \( \sigma_i \) will be the temporal amplification factor. Since both the real and imaginary parts are of the same magnitude as the wave number \( \alpha \), it is convenient to express \( \sigma \) in terms of \( \alpha \) and the complex wave velocity \( c \):

\[ \sigma = \sigma_r + \mathbf{i} \sigma_i = \alpha c - c_r + \mathbf{i} \alpha c_i \]

where \( c_i \) is the imaginary part of \( c \). In this asymptotic analysis for small wavenumber the complex wave velocity is represented as \( c = c_0 + \alpha c_1 + O(\alpha^2) \) in which \( c_0 \) and \( c_1 \) are obtained from the zero- and first-order solutions, respectively.

The zero-order solution is obtained by setting \( \alpha = 0 \) in \( (A24)-(A37) \). When the solutions to the resulting forms of \( (A24) \) and \( (A25) \) are substituted into the boundary conditions, one finds that the dimensional phase velocity \( c^{*}_r \) is equal to twice the surface velocity of the upper liquid layer which can be obtained from (A1); this result is to be expected for very long periodic traveling waves (cf. Lighthill and Whitman, 1955). The imaginary part of \( c, c_{10} \), is identically zero in this zero-order solution, thus implying that infinitely long waves are neutrally stable.

The properties of small nonzero wave numbers can be ascertained by considering the first-order solution in wave number which is obtained by retaining terms proportional to \( \alpha \). The first-order solution can be obtained analytically, although the resulting algebraic equation for the complex wave velocity \( c \) must be solved numerically as described in the thesis of Baum (1985).

The first-order contribution \( c_1 \) to the complex wave velocity \( c \) is found to be imaginary. Figure 8 is a plot of \( c_1 \) versus the slope for three values of \( \mu_2/\mu_1 \). The densities \( \delta_1 \) and \( \delta_2 \) are 2.4 and 1.4 \( \text{g/cm}^3 \), respectively, and \( \delta_1 \) and \( \delta_2 \) are 1.3. Note that the growth coefficient \( c_1 \) is equal to \( \alpha^2 c_1 \). Since this perturbation solution in wave number converges very slowly for very small slopes, the three lines are not drawn continuously to zero slope; however, it can be shown that \( c_1 \) is equal to 0 for \( \alpha = 0 \). Figure 8 indicates that \( c_1 \) and hence the growth coefficient increases monotonically with increasing slope and decreasing \( \mu_2/\mu_1 \) for long waves. Increasing slope is destabilizing because it increases the downslope velocity and also reduces the effective stabilizing force of gravity.

Note that Figure 8 implies that the growth coefficient \( c_1 \) increases monotonically with wave number \( \alpha \), since \( c_1 \) is independent of \( \alpha \), in
flow lobes these outcrops are stretched into ridges oriented perpendicular to the downslope flow. These transverse ridges extend beyond the margins of the coarsely vesicular pumice outcrops into the unbrecciated lava interior. In some places these ridges completely lack coarsely vesicular pumice and seem indistinguishable from surface ridges formed by compression. The Taylor instability model developed here indicates that the preferred mode corresponds to ridges oriented perpendicular to the downslope flow. Moreover, Figure 7 indicates that these transverse ridges can occur in the absence of any outcrops of the less dense material; that is, ridges can develop on the upper free surface in response to ridges formed within the lava flow at the lower interface.

Our theory suggests ways to distinguish between flow structures formed by the Taylor instability as opposed to the surface folding mechanism. Figure 7 indicates that the amplitude of a wave formed at the interior interface should decrease toward the upper free surface in contrast to ridges formed by compressive folding. Hence by comparing the amplitudes of the upper and interior interfaces of deeply dissected flows containing both transverse surface ridges and internal horizontal flow layering, the dominant instability mechanism can be determined.

Another test of the Taylor instability model relies upon future observations of active rhyolite flows. Our analysis for the Taylor instability in the presence of a shear flow predicts that surface waves formed in response to an internal density inversion will have a phase velocity twice that of the upper surface of the flow. In contrast, compressional folds would not travel faster than the mean flow. Although such observations have yet to be made, determination of the phase velocity on an active flow should be relatively straightforward.

Prior to the recent Inyo and Valles drilling programs, no interior views were available of rhyolite flows young enough to have well-preserved upper surfaces. Thus it was not possible to conclusively correlate diapir or ridge spacings and layer thicknesses. Figures 4 and 5 imply that measurements of the diapir or ridge wavelengths will permit estimating either \(d_1/d_2\) or \(\mu_1/\mu_2\), but not both. In his earlier study, Fink (1980c) used his Taylor instability model to estimate the diapir wavelengths for three lobes on the Little and Big Glass Mountain rhyolite flows from thicknesses of the dense and buoyant layers inferred from flow fronts and assumed viscosity ratios \(\mu_1/\mu_2\). These estimates had very large error bars, since it was unclear how far the buoyant layer extended into the flow and whether the thicknesses contained at depth were present in a dissolved or gas bubbles.

The drilling results allow us to more confidently estimate the layer thicknesses and the portion of the flow affected by the Taylor instability. For the three drill holes studied by Eichelberger et al. (1984, 1985) and Goff et al. (1986), the depth to the top of the buoyant layer \(d_1\) is roughly one tenth to one fifth of the total thickness of the flow, and the bottom of the buoyant layer \(d_{1}+d_{2}\) lies about one fifth to one fourth of the way into the flow. The density increases below the buoyant layer, thus implying
that the Taylor instability can be operative only in the upper portion of the flow. The Taylor instability model developed here can be used in conjunction with refined estimates of the dense and buoyant layer thicknesses to determine $\mu_1/\mu_2$ from the measured diapir and ridge wavelengths for the Little and Big Glass Mountain rhyolite flows; these data and estimates are summarized in Table 2. Our model predictions for the dimensionless wavelength $\lambda/d_2$ as a function of the layer thickness ratio $d_1/d_2$ for three viscosity ratios $\mu_1/\mu_2$ are shown in Figure 9. The observed diapir spacings of 43-70 m and buoyant layer thicknesses of 4-6 m in Table 2 suggest a $\mu_1/\mu_2$ ratio between 10 and 100. This implies that the crust would be 100 times more viscous than the buoyant layer, and that the volatiles would probably have been dissolved in the lava rather than present as gas bubbles when the diapirs were forming. If the surface ridges also formed in response to the Taylor instability, then their smaller wavelengths of 15-40 m imply $\mu_1/\mu_2$ is closer to 1, suggesting that while the buoyant layer's density was reduced by volatile enrichment, its viscosity either remained the same or decreased only slightly. This in turn could be explained if the vapor had already begun to evolve as bubbles when the instability operated. Since many diapirs appear to have superimposed ridges, it is possible that the Taylor instability acted in two stages. During the first stage, while volatiles remained dissolved, long-wavelength diapirs formed. As the diapirs rose toward the surface and bubbles began to form, the viscosity of the buoyant layer increased. Consequently, smaller-spaced ridges formed, both at the internal interface and at the overlying free surface.

The Taylor instability model predictions can be used to assess the applicability of three flow emplacement models proposed to explain how a water-rich layer develops within a rhyolite flow. A model advanced by Eichelberger et al. (1986) suggests that rhyolite flows are emplaced as finely vesicular foams that compress during advance owing to their own weight, so that only the uppermost portion retains any vesicularity. Although Eichelberger et al. (1986) do not discriminate among different vesicular lava textures, their model implies that the coarsely vesicular pumice might develop at a depth where retained heat and consequently lowered viscosity would allow small bubbles to coalesce. A second model proposed by Fink and Manley (1987) argues that volatiles released into microcracks in the interior of an advancing flow are able to rise upward until they become trapped beneath the flow's rigid carapace. There they pool to form a volatile-rich layer that becomes thicker and more buoyant as the flow advances. An earlier model contends that inversion of a volatile-stratified magma body would result in the emplacement of volatile-rich tephra layers overlaid by a volatile-poor lava flow. Extending this idea to the flow itself, Fink (1983) argued that earliest-emplaced lava would have the highest volatile content and lowest density. The volatile-rich lava would then form a buoyant basal layer that would be spread out and thinned by later, overriding denser lava.

Since the preferred wavelength of the Taylor instability depends on both the viscosities and thicknesses of the layers, diapir spacings alone cannot be used to distinguish between these three models. However, the Taylor instability model developed here can be helpful in this assessment. For a constant total flow thickness Figure 4 implies that as the buoyant layer thickness $d_2$ increases, the wavelength first will increase to a maximum and then decrease. Thus for cases in which the buoyant layer makes up a small fraction of the total flow, diapir or ridge spacing will increase as the buoyant layer thickness increases. For cases in which the buoyant layer makes up more than half the flow, diapir or ridge spacing will decrease as the buoyant layer thickness increases. These relationships are further complicated because diapir spacing also is inversely proportional to the buoyant layer viscosity as seen in Figure 5. Finally the growth rate of the instability, which depends on the density and thickness of the buoyant layer relative to the overlying dense layer, should determine whether or not diapirs or ridges generated at the interface can penetrate through the dense layer to the surface before it solidifies.

In view of these considerations the first model requires a thin buoyant layer that becomes progressively thinner as the lava moves away from the vent. Furthermore, it indicates a distal decrease in density. These changes would cause the amplitude of the instability and occurrence of diapirs at the surface to increase, but their spacing to decrease.

The second model indicates that both the thickness and the buoyancy of the scoriaceous layer should increase outward. Consequently the occurrence of diapirs near the flow margins should increase. Diapir spacings cannot be used as a diagnostic test for this model, however, since the viscosity could either increase or decrease as the water content increases, depending on whether or not the gases were dissolved at the depth of the layer.

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have shown the third model to be less plausible than the other two because the coarsely vesicular layer does not extend to the base of the flow (Fink and Manley, 1987; Manley and Fink, 1987). Nonetheless, consideration of this model provides a useful illustration of how the Taylor instability analysis can help constrain emplacement conditions. The third model implies a distal decrease in the buoyant layer thickness while the viscosity and density remain constant. This combination should lead to a distal increase in diapir spacing, since the buoyant layer is assumed to make up nearly half of the flow thickness, and to a distal decrease in diapir occurrence, which should be proportional to the growth rate of the instability. In order for diapirs to be visible their growth rate must be large relative to the cooling and solidification rates of the flow as a whole and of the surface crust in particular.

Observations of diapirs tend to support either the first or second models. Diapirs are much more common toward the outer margins of flows, indicating a distal increase in buoyancy. Spacings appear to decrease on some lobes and increase on others. Other field observations tend to favor the second model, however. The increased occurrence of explosion pits toward the flow front and their association with coarsely vesicular diapirs imply an outward thickening and increase in volatile content of the buoyant layer. In addition the nearly continuous presence of coarsely vesicular pumice outcrops in flow fronts would not be expected if the layer were undergoing a radial decrease in thickness.

If the second model does apply to rhyolite flows, it has significant implications for hazard assessments (Fink and Manley, 1987, 1989). The initial appearance of ridges followed by the emergence of diapirs on the surface of an active rhyolite flow would indicate increasing volatile pressure in the interior. Collapse of the front of such a flow could lead to sudden release of this pressure and the generation of highly destructive pyroclastic flows. Hence active rhyolite flows should be monitored for the appearance and spacing of surface ridges and dark diapirs of coarsely vesicular pumice in order to mitigate possible explosive hazards.

Conclusions

1. Recent drill core studies indicate that unstable density stratification exists in rhyolite lava flows thus providing the potential for the Taylor instability.

2. The diapirs and superimposed ridges oriented transverse to the downslope flow are consistent with the Taylor instability model which indicates that a two-dimensional transverse mode can always be found which is more highly amplified than any longitudinal or three-dimensional long wavelength mode.

3. Ridges formed by the Taylor instability can be distinguished from those formed by compressive folding by observing whether they propagate more rapidly than the surface velocity of active rhyolite flows or whether their amplitude is less than the amplitude of the interface between the buoyant and dense layers in dissected older flows.

4. Measured diapir and ridge wavelengths along

with inferred buoyant and dense layer thicknesses can be used in the Taylor instability model to estimate the dense to buoyant layer viscosity ratio.

5. Viscosity ratios estimated from the Taylor instability model can be used to infer the state of the volatiles within a rhyolite flow.

6. The distal dependence of diapir and ridge wavelengths as well as the appearance of outcrops of vesicular pumice permit assessing models proposed for rhyolite flow emplacement.

Appendix

The Stability Analysis

A three-dimensional linear stability analysis is developed here as a foundation for the discussion of wave growth in rhyolitic lava flows. The subscript notation and the coordinates to be used are shown in Figure 2. The development of the basic state velocity profiles are determined from solution of the unperturbed equations of motion and are presented for further reference as

\[ \bar{U}_1 = \frac{\rho_{\text{gas}}} {\mu_1} \left[ \frac{d_1 d_2 g d_1^2} {\mu_2} + \frac{\nu_1 d_2^2} {2 \nu_2} - d_1 y - \frac{y^2} {2} \right] \]

and

\[ \bar{U}_2 = \frac{\rho_{\text{gas}}} {\mu_1} \left[ \frac{d_1 d_2 g d_1^2} {\mu_2} + \frac{\nu_1 d_2^2} {2 \nu_2} - \frac{\mu_1 d_1 y} {\mu_2} - \frac{\nu_1 y^2} {2 \nu_2} \right] \]

with the kinematic viscosity defined as \( \nu_1 = \rho_{\text{gas}} / \rho_1 \).

Linearized Disturbance Equations

We seek to analyze the linear stability of the unstratified flow shown in Figure 2b. It is convenient to express the velocities and pressure, \( u, v, w, \) and \( P \), as the sum of a basic state quantity (denoted by the overbar) and a perturbed quantity (denoted by the accent) as follows:

\[ u = \bar{u} + u' \]

\[ v = \bar{v} + v' \]

\[ w = \bar{w} + w' \]

\[ P = \bar{P} + P' \]

A linear stability analysis ignores quadratic and higher-order terms in the perturbed quantities. The resulting linearized three-dimensional equations of motion for the Taylor instability problem are:

\[ \rho_1 \left[ \frac{\partial u'} {\partial t} + \bar{U}_1 \frac{\partial u'} {\partial x} + v_1 \frac{\partial u'} {\partial y} \right] = - \frac{\partial p'} {\partial x} + \mu_1 \frac{\partial^2 u'} {\partial y^2} \]
At the interface between the two layers we require that the velocity components be continuous (equations (A13), (A14), and (A15)), that both components of the tangential stress be continuous (equations (A16) and (A17)), and that the sum of the normal stress components in each phase balance the surface tension force (equation (A18)):

\[ u_1' + \frac{d\vec{u}_1}{dy} \eta = u_2' + \frac{d\vec{u}_2}{dy} \eta \quad y=0 \quad (A13) \]

\[ v_1 = v_2 \quad y = 0 \quad (A14) \]

\[ w_1 = w_2 \quad y = 0 \quad (A15) \]

At the lower stationary impermeable solid boundary we demand that the velocity components be zero (equations (A19), (A20), and (A21)):

\[ u_2 = 0 \quad y = d_2 \quad (A19) \]

\[ v_2 = 0 \quad y = d_2 \quad (A20) \]

\[ w_2 = 0 \quad y = d_2 \quad (A21) \]

Scaling and Nondimensionalization

In order to obtain a general solution the governing equations and associated kinematic and boundary conditions will be nondimensionalized. The dimensionless variables must be well behaved in the limit of zero net flow which precludes using the average or surface velocity as scale factors. The following scale factors which reflect the highly viscous nature of lava flows then are appropriate:

\[ y_s = \left[ \frac{\mu_1}{\rho_1 (\rho_1 - \rho_2) \eta \cos \theta} \right]^{1/3} \quad (A22a) \]

\[ r_s = \frac{\mu_1}{(\rho_1 - \rho_2) \eta \cos \theta} \quad (A22b) \]
The pressure scale is derived from the normal stress boundary condition (A18) by balancing the pressure and viscous terms. The time scale is derived from the upper surface kinematic condition (A8). The velocity and length scales are developed by balancing the viscous, pressure, and unsteady state terms in the $y$ component of the momentum equation for the upper fluid (A5 with $i = 1$).

The resulting dimensionless forms of (A4) through (A7) can be reduced to a single fourth-order differential equation using the procedure of Squire (1933). The general solution to this equation for three-dimensional disturbances has the following form:

$$a' = a \exp \left\{ i (m s + n z - o t) \right\}$$

(A23)

where $o = o_1 + i o_2$, and $m$ and $n$ are wavenumbers. The linear stability of this flow is described by the following dimensionless differential equations:

$$\frac{\partial \hat{v}_1}{\partial x} + i [\sigma - 2 \alpha^2] \hat{v}_1 + [4 - i \alpha \nu_2] \hat{v}_2 - \delta_1 \hat{v}_2 + [4 - \alpha] \hat{v}_1 = 0$$

(A24)

and

$$\frac{\partial \hat{v}_2}{\partial x} + i [\sigma - 2 \alpha^2] \hat{v}_2 + [4 - i \alpha \nu_2] \hat{v}_1 - \delta_1 \hat{v}_1 + [4 - \alpha] \hat{v}_2 = 0$$

(A25)

in which $o = m y_1$, $\beta = n y_1$, and $\alpha = (\alpha + \beta^2)/2$. The superscripts on $\hat{v}_i$ denote the order of differentiation with respect to $y$.

The kinematic conditions (A8) and (A9) can be used to eliminate the surface displacements from the boundary conditions applied at the free surfaces. The boundary conditions given by (A10) through (A21) then are nondimensionalized using the scale factors defined by (A22) and further simplified using (A23).

The tangential stress conditions at the upper free surface can be combined using the continuity equation (A7) to obtain the following dimensionless form of the tangential stress boundary condition:

$$\left\{ \frac{\hat{v}_2}{\hat{v}_1} \right\} \tan \delta \left\{ \frac{\hat{v}_1}{\hat{v}_2} \tan \delta \right\} = 0$$

(A26)

in which $K_1$ is defined by

$$K_1 = \left[ \frac{\rho_1}{\rho_1 \rho_2} \right] \left[ \frac{\delta_1 \delta_2^2 \mu_1}{\mu_2} + \frac{\nu_1 \delta_2^2}{\nu_2} + \frac{\nu_1 \delta_2^2}{\nu_2} \right]$$

(A27)

The $y$ component of the momentum equation (A5) can be used to eliminate the pressure to obtain the following dimensionless form of the normal stress balance at the upper free surface:

$$\left\{ \frac{1}{\rho_1} + \frac{S_1}{\rho_1 \rho_2} \right\} \left[ \frac{\delta_1 \delta_2^2 \mu_1}{\mu_2} + \frac{\nu_1 \delta_2^2}{\nu_2} \right] \hat{v}_1 + \hat{v}_1 = 0$$

(A28)

in which $S_1$ is the surface tension group for the upper liquid defined by

$$S_1 = \frac{\gamma_1^2}{(\rho_1 \rho_2)^2 \cos \theta \mu_1^{2/3}}$$

(A29)

The continuity equation can be used to obtain the following dimensionless form of the combined continuity of $x$ and $z$ velocity components at the lower free surface:

$$\left\{ \frac{\rho_1}{\rho_1 \rho_2} \right\} \hat{v}_1 + \hat{v}_1 = 0$$

(A30)

in which $K_2$ is defined by

$$K_2 = \left[ \frac{\rho_1}{\rho_1 \rho_2} \right] \left[ \frac{\delta_1 \delta_2^2 \mu_1}{\mu_2} + \frac{\nu_1 \delta_2^2}{\nu_2} \right]$$

(A31)

The companion dimensionless condition at the lower free surface demands continuity of the $y$ velocity component:

$$\hat{v}_1 - \hat{v}_1 = 0$$

(A32)

The tangential stress conditions at the lower free surface again can be combined to obtain the following dimensionless form of the tangential stress conditions at the lower free surface:

$$\hat{v}_1 = 0$$

(A33)
stress boundary condition:

\[
\begin{bmatrix}
    v_1 - \frac{\mu_2}{\mu_1} v_2 \\
    v_1 - \frac{\mu_2}{\mu_1} v_2
\end{bmatrix} + \frac{\partial}{\partial y} \left[ \frac{\tan^2 \theta}{\tan \theta} \right] v_2 = 0 \quad y = 0 \quad (A33)
\]

Again the y component of the momentum equation can be used to eliminate the pressure to obtain the following dimensionless form of the normal stress balance at the lower free surface:

\[
\begin{bmatrix}
    \alpha_2^2 (1 - \alpha^2 S^2) v_2 \right]^T \left[ \frac{3 \alpha_2^2}{\mu_1} \right] v_2^T + 3 \alpha_2^2 \frac{\partial}{\partial y} v_1^T - \frac{\rho_2 \alpha}{\mu_1} v_2^T + \frac{\partial}{\partial y} \left[ \frac{\alpha_2 \alpha_2^2}{\mu_1} \right] v_2 = 0 \quad y = 0 \quad (A34)
\]

in which \( S^2 \) is the surface tension group for the lower liquid defined by

\[
S_2 = \frac{\gamma_{12}^2}{(\rho_1 - \rho_2) \mu_1 \cos \theta} 2^{1/3} \quad (A35)
\]

The continuity equation again can be used to combine the no slip and impermeable boundary conditions to obtain the following two dimensionless boundary conditions at the solid surface:

\[
\begin{bmatrix}
    \lambda_{2}^T v_2 = 0 \\
    \lambda_{1}^T v_2 = 0
\end{bmatrix} \quad y = \delta_2 \quad (A36)
\]

\[
\begin{bmatrix}
    \lambda_{2}^T v_2 = 0 \\
    \lambda_{1}^T v_2 = 0
\end{bmatrix} \quad y = \delta_2 \quad (A37)
\]

In summary, a fourth-order dimensionless differential equation is developed for each fluid layer and is described in (A24) and (A25). The solution of these differential equations require eight boundary conditions, which are described by (A26)-(A37). These equations, however, are three-dimensional. By finding the appropriate transformation, these equations are then cast in two-dimensional form (see text). Further discussion of the solution methods can be found in the work of Baum [1985].

Notation

- \( a \) dimensionless amplitude.
- \( c \) dimensionless complex wave velocity.
- \( c_i \) imaginary part of \( c \), proportional to temporal growth coefficient.
- \( c_r \) real part of \( c \), phase velocity.
- \( c_0 \) zeroth-order solution for \( c \).
- \( c_1 \) first-order solution for \( c \).
- \( d_1 \) dimensional thickness of phase 1.
- \( g \) gravitational acceleration.
- \( K_1 \) constant defined by (A27).
- \( K_2 \) constant defined by (A31).
- \( m \) dimensionless wave number in \( x \) direction.
- \( n \) dimensionless wave number in \( z \) direction.
- \( p_1 \) pressure in phase 1.
- \( S_1 \) surface tension group in phase 1 defined by (A29) and (A35).
- \( t \) time.
- \( u_1 \) velocity component in \( x \) direction in phase 1.
- \( v_1 \) velocity component in \( y \) direction in phase 1.
- \( w_1 \) velocity component in \( z \) direction in phase 1.
- \( x \) coordinate measured in direction of basic state flow.
- \( y \) coordinate measured in direction normal to basic state flow.
- \( z \) coordinate measured in direction lateral to basic state flow.
- \( \alpha \) wavenumber in \( x \) direction nondimensionalized with \( y_\ast \).
- \( \alpha_\ast \) nondimensionalized \( \alpha \).
- \( \beta_\ast \) wave number in \( z \) direction.
- \( \gamma_1 \) surface tension of phase 1.
- \( \delta_1 \) thickness of phase 1 nondimensionalized with \( y_\ast \).
- \( \eta \) local amplitude of interface perturbation.
- \( \theta \) slope angle relative to horizontal.
- \( \theta_e \) effective slope angle for three-dimensional disturbances.
- \( \lambda_d \) diapir spacing or wavelength.
- \( \lambda_r \) ridge spacing or wavelength.
- \( \mu_1 \) shear viscosity of phase 1.
- \( \nu_1 \) kinematic viscosity of phase 1.
- \( \xi \) local amplitude of upper free surface perturbation.
- \( \rho_1 \) mass density of phase 1.
- \( \sigma \) dimensionless temporal growth coefficient.

Subscripts

- \( \max \) property of the most highly amplified mode.
- \( s \) denotes a scale factor.
- \( 1 \) property of upper, more dense phase.
- \( 2 \) property of lower, buoyant phase.

Superscripts

- \( \cdot \) denotes perturbed quantity arising from instability.
- \( \wedge \) denotes one-half, peak-to-peak amplitude.
- \( \ast \) denotes dimensional quantity.
- \( I \) denotes first-derivative with respect to \( y \).
- \( II \) denotes second-derivative with respect to \( y \).
- \( III \) denotes third-derivative with respect to \( y \).
- \( IV \) denotes fourth-derivative with respect to \( y \).

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B. A. Baum, School of Geophysical Sciences, Georgia Institute of Technology, Atlanta, GA 30332.

R. E. Dickinson, National Center for Atmospheric Research, Boulder, CO 80307.

J. H. Fink, Department of Geology, Arizona State University, Tempe, AZ 85287.

W. B. Krantz, Department of Chemical Engineering, University of Colorado, Boulder, CO 80309.

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