Power Self-Regulation in Double-Pass High-Gain Laser Amplifiers

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Double-pass laser amplifiers can provide automatic passive regulation of the power in an optical signal. This regulation can significantly reduce the amplitude noise on a laser beam that is intended as a continuous wave light source. Analytic expressions are derived to describe the optical-noise-reduction region of double-pass amplifier operation and the dependence of the self-regulation properties on gain, saturation, and mirror reflectivity. © 2000 American Institute of Physics. [S0021-8979(00)01405-5]

I. INTRODUCTION

Double-pass laser amplifiers have several advantages over more standard single-pass designs. The best known of these advantages are their gain, compactness, and efficiency. While high-gain double-pass amplifiers have been employed using only their own spontaneous emission as input, the majority of applications involve more conventional input/output amplification configurations, and only such configurations are considered here. In a recent study, double-pass amplifier applications and models were reviewed, and a general analytical model was developed for a homogeneously broadened amplifying medium having a single feedback mirror. That model is the starting point for the present analysis.

A double-pass amplifier can be considered to be a specific example of an opto-optical system. Such a device is one in which one optical signal affects the propagation characteristics of another, and opto-optical techniques have been explored for a wide range of physical systems. Thus the underlying optical interactions have been considered to occur in dyes and polymers, liquid crystals, inorganic nonlinear optical media, and semiconductors. Besides their basic transmission interactions, some opto-optical systems have included gratings, interferometers, and lasers. These systems have served to modulate the amplitude, phase, and direction of a propagating beam; and they have also been employed for a variety of optical switching and logic functions.

An important property of double-pass amplifiers is that saturation by a beam traveling in one direction affects the gain of the beam traveling in the other direction. In that respect the double-pass amplifier is similar to other amplifier-based opto-optic devices in which the saturating beams are distinguished by their polarization, path length, or mode profile rather than their propagation direction. As in such systems, the nonlinear interaction of the beams leads to complex behavior that might be the basis for new and practical applications.

The simplest system configuration for a double-pass amplifier includes a single mirror at one end, so that a portion of one transmitted beam constitutes the input for the other. An important behavior of such a system is that under some conditions the net reflected output may decrease as the input is increased. Thus, a plot of output intensity versus input can be analogous to the $I-V$ characteristic of a tunnel diode, and a double-pass amplifier could be an active component in optical circuit design. Under other conditions the output is almost independent of the input, and our main interest here is in exploring the conditions under which this might occur. In these circumstances the double-pass amplifier may be regarded as a power regulator, and such a regulator could find use in optical noise reduction. A possible circuit analog in this case would be a zener diode, where for a wide range of currents the voltage is almost independent of current.

The conditions under which the incremental gain of a double-pass laser amplifier may be zero or negative are developed in Sec. II. The basis for this analysis is a set of saturation equations derived in Ref. 1. The practical implications of these results are discussed in Sec. III, and it is found that the corresponding experimental conditions should be readily achievable in practice.

II. ANALYSIS

The starting point for this analysis is the model developed in Ref. 1. That model ultimately consisted of a pair of differential equations governing the right and left propagating intensities in a one-dimensional bidirectional homogeneously broadened laser amplifier. To be as brief as possible, we start here where that earlier analysis ended. Specifically, in a double-pass amplifier having one mirror and negligible distributed losses, the input intensity $I_i$ and output intensity $I_o$ are related by Eq. (21) of Ref. 1:

$$2s'[\{(1-R)(I_iI_o/R)^{1/2}+I_o-I_i\}+\ln[I_o/(RI_i)]=2gL.$$  

(1)

The laser configuration considered here is indicated schematically in Fig. 1. This figure shows a double-pass amplifier in which the input signal comes from the left and the reflector at the right-hand end has a reflection coefficient of $R$. The amplifier has a saturation parameter $s'$ (inverse of the satura-
tion intensity), and the gain-length product is $gL$. This result is more general than it might appear, as the saturation parameter and gain include a frequency dependence for arbitrary tuning from line center.

Equation (1) is a closed form implicit analytic formula relating the input and output intensities. This equation simplifies a little if $I_i$ and $I_o$ are replaced by the normalized intensities $J_i = s' I_i$ and $J_o = s' I_o$, and the result of these substitutions is

$$2[(1-R)(J_i J_o/R)^{1/2} + J_o - J_i] + \ln[J_0/(R J_i)] = 2 gL.$$  

(2)

As already remarked, for some values of reflectivity $R$ and gain-length product $gL$, the output intensity $J_o$ decreases for increasing values of the input intensity $J_i$, and this behavior has the potential for leading to practical applications.

The significance of Eq. (2) can best be illustrated by means of numerical solutions, and examples for three different values of the mirror reflectivity are given in Figs. 2(a)--2(c). Figure 2(a) is a plot of the normalized output intensity $J_o$ as a function of the normalized input intensity $J_i$ for several different values of the gain length product $gL$ and for a mirror reflection coefficient of $R = 1$. The intensity formulas in this case have been known for many years, and the graphical results in the figure are rather unremarkable. Thus, for any value of gain the output is seen to increase with the input, and, except for the logarithmic scale on the horizontal axis, one would observe the decreasing slope for larger input values that is an indication of gain saturation.

The laser behavior becomes more interesting for lower values of the mirror reflectivity. Figure 2(b) is a plot (solid lines) of the output intensity as a function of the input intensity for several values of the gain length product and a mirror reflection coefficient of $R = 0.1$. It is clear from these graphs that for the larger values of the gain the curves flatten out and ultimately exhibit a region of negative slope. The flattening of the curves means that the output is becoming insensitive to the input, and this behavior can be the basis for power regulation or noise filtering of a laser signal. The negative slope regions can lead to the possibility of active optical circuits. Similar results are obtained for other values of the mirror reflectivity, and the corresponding results for $R = 0.01$ are given in Fig. 2(c).

Given the existence of the negative slope behavior, it is of interest to try to describe in as much detail as possible the parameter space within which this slope occurs. A more detailed characterization of this effect will facilitate its use in practical laser systems. For this purpose we begin by looking for those points at which the output intensity $J_o$ is independent of the input intensity $J_i$. It is near these local maximum and minimum points that the slope of $J_o$ vs $J_i$ may be just starting to become negative. Thus we first differentiate Eq. (2) with respect to $J_i$, while assuming that the reflection coefficient $R$ and gain-length product $gL$ are constant. The result of this differentiation is

$$2 \left[ (1-R) \left( \frac{J_i J_o}{R} \right)^{1/2} + J_o - J_i \right] + \ln[J_0/(R J_i)] + \ln[J_o/(R J_i)] = 2 gL.$$  

(3)
To find the points at which the output intensity \( J_o \) is independent of the input intensity \( J_i \), we may set the derivative \( dJ_o/dJ_i \) in Eq. (3) equal to zero:

\[
2((1-R)(1/2)(J_o/J_i/R)^{-1/2}(J_o/R)-1]-1/J_i=0. \tag{4}
\]

This equation may be solved explicitly for \( J_o \) and the result is

\[
J_o=(R/J_i)[(2J_i+1)/(1-R)]^2. \tag{5}
\]

Equation (5) may be used to express the gain-length value at the local maxima and minima in terms of the input intensity. Thus, if this equation is substituted into Eq. (2), one obtains the result

\[
2gL=2\left(1-R\right)\frac{2J_i+1}{1-R}+\frac{R}{J_i}\left(\frac{2J_i+1}{1-R}\right)^2-J_i
+\ln\left[\frac{1}{J_i}\left(\frac{2J_i+1}{1-R}\right)^2\right]. \tag{6}
\]

Dividing by two and simplifying, the gain length at these special points is given by

\[
gL=J_i+1+\frac{R}{J_i}\left(\frac{2J_i+1}{1-R}\right)^2+\ln\left[\frac{1}{J_i}\left(\frac{2J_i+1}{1-R}\right)^2\right]. \tag{7}
\]

Equation (5) is plotted as dashed lines in Figs. 2(b) and 2(c). It may be seen from the plots that the curve governed by Eq. (5) always passes through those points on the output versus input curves at which the slope is zero. It is also clear from the plots that the interesting region of negative slope is largest for the larger values of the gain-length product. Since high values of gain are not readily obtained in all lasers, it is of interest to determine the smallest value of the gain for which the slope can be zero. It is clear from the graphs that this minimum gain length occurs when the local maxima and minima of the intensity curves coalesce, and this coalescence point is at the minimum of the function given in Eq. (5).

To minimize Eq. (5), we set its derivative to zero:

\[
\frac{dJ_o}{dJ_i}=-\frac{R}{J_i}\left(\frac{2J_i+1}{1-R}\right)^2+\frac{2R}{J_i}\frac{2J_i+1}{1-R}=0. \tag{8}
\]

Equation (8) may be solved for \( J_i \), and the result is \( J_i=1/2 \), independent of the mirror reflectivity \( R \). Thus, the lowest value of gain-length product for obtaining good power regulation always occurs where the input intensity is equal to one half of the saturation intensity.

It is also possible to determine analytically the actual value of the output intensity and the gain length at the minimum gain-length point. If the value \( J_i=1/2 \) is substituted into Eqs. (5) and (7), one obtains

\[
J_o=8R/(1-R)^2, \tag{9}
\]

\[
gL=\frac{1}{2}+1+2R\left(\frac{2}{1-R}\right)^2+\ln\left[\frac{4}{1-R}\right]
=(1+3R)(3+R)
=\frac{2}{2(1-R)^2}+\ln\left(\frac{4}{1-R}\right). \tag{10}
\]

The output intensity and the gain-length product of Eqs. (9) and (10) are plotted as functions of the mirror reflection coefficient in Fig. 3. With these results we have a fairly complete analytical description of the conditions under which the output intensity of a double-pass laser amplifier will decrease when the input intensity increases. Further more detailed results can be obtained from these as necessary.

**III. DISCUSSION**

The analysis in the preceding section has focused on the important result that it is possible in a double-pass amplifier with a single mirror for the output intensity to be nearly constant or to decrease for increasing values of the input intensity. The next step is to determine whether the conditions necessary for this effect can actually be obtained in practice. As a starting point, one might enquire as to the value of mirror reflectivity that would lead to the smallest gain-length product for an input-independent output. From the form of Eq. (10) one finds that the gain-length product \( gL \) is a monotonic function of \( R \), and thus the smallest \( gL \) occurs with \( R=0 \). This value is \( gL=1.5+\ln 4=2.89 \). This is an achievable value of gain for many types of laser amplifier, but unfortunately from Eq. (9) the corresponding output intensity is \( J_o=0 \). Thus power regulation in double-pass lasers requires a gain-length product greater than 2.89.

It is clear from Fig. 2 that when a double-pass amplifier is used as a power regulator the output intensity can never be dramatically larger than the input for any reasonable value of \( gL \). On the other hand, the value \( J_o=0 \) is obviously not acceptable. Therefore, to be specific, one might explore the situation where the derivative of the output with respect to the input equals zero when the output is just equal to the input. As noted above the lowest gain-length value for this condition occurs when the input is \( J_i=1/2 \) and the output is given by Eq. (9). If \( J_o \) is also set equal to 1/2 in Eq. (9), one obtains the quadratic equation

\[
R^2-8R+1=0. \tag{11}
\]

The solution of this equation is \( R=9-4(5)^{1/2}=0.0557 \). Thus, a mirror reflectivity of about 6% leads to a very reasonable power regulator which provides no net gain or loss (near its regulation range). From Eq. (10) the gain-length
product for this gain-neutral regulator is \( gL = 2 - \ln[(5)^{1/2} - 2] = 3.44 \). This result is only a little higher than the useless no-output value \( gL = 2.89 \) found above.

It is now helpful to consider what values of \( gL \) are attainable in practical laser systems. Gain values are sometimes reported as the ratio of output intensity to input intensity, and the conversion to gain-length product is \( gL = \ln(I_o/I_i) \). Also, when gain is reported in dB, the conversion is \( gL = 0.1 \ln(10G_{dB}) \). At the upper end of gain values, several familiar high-gain lasers can achieve sufficient gain that their outputs are dominated by amplified spontaneous emission. While such emission has important applications, it is unnecessary and undesirable for the power-regulating and optical circuit applications envisioned here. In a typical mirrorless double-pass amplifier, amplified spontaneous emission typically becomes substantial enough to cause saturation for a gain-length product of roughly \( gL = 10 \) (or \( gL = 5 \) if the laser has one highly reflecting mirror).\(^{37} \) Such emission has long been seen in high-gain solid,\(^{38} \) gas,\(^{39} \) and liquid lasers,\(^{40} \) and in some cases saturation effects have also been observed. Thus, lasers based on such high-gain media can easily achieve the conditions necessary to observe the double-pass power-limiting effects described here, and gain-length values below \( gL = 10 \) would actually be preferred.

Not all high-gain lasers are of interest for practical applications, and, for example, some of them have very low saturation intensities or other complicating nonlinear properties. Semiconductor lasers are of particular interest for possible active optical circuit applications, and in double-pass semiconductor laser studies single-pass gain values of at least 25 dB (\( gL = 5.8 \)) have been obtained.\(^{41} \) Issues of noise limitation and optical signal processing are especially important in the various communications applications of optical fibers, and fiber amplifiers can also readily satisfy the conditions derived here. Thus, erbium-doped fiber amplifiers have achieved gain values of at least 51 dB (\( gL = 11.7 \)),\(^{42} \) at which point significant saturation by amplified spontaneous emission is also occurring. Fiber amplifiers are already widely studied in double-pass configurations\(^{43}-^{45} \) as are waveguide Er:LiNbO\(_3\),\(^{53}-^{55} \) and Nd:Y\(_3\)Al\(_5\)O\(_{12}\) lasers.\(^{56} \) Thus, no new technology is required for the employment of any of these lasers in noise limiting applications.

It is important to note that sufficient gain for reaching the power-regulating region of a double-pass amplifier is not just restricted to the very small diode and fiber systems that might be of special interest for communications and signal processing. The appropriate conditions are also readily met in dye lasers, where gain values of at least \( 2 \times 10^3 \) (\( gL = 9.9 \)) have long been available.\(^{57} \) Diode-pumped yttrium–aluminum–garnet (YAG) lasers can also readily reach the operating region of interest here. Thus, YAG master-oscillator-power-amplifier (MOPA) systems of the type to be used in the Laser Interferometer Gravitational Wave Observatory (LIGO) consist of four Nd:YAG rods, each of which is side-pumped by two 20 W diode bars.\(^{58} \) It has been reported that with a pumping power of only 16 W per diode bar the single-pass small-signal gain is measured at 2.4 dB per bar. It follows that single-pass gains greater than 8 \( \times 2.4 = 19.2 \) dB (\( gL = 4.4 \)) can be readily achieved. This is well in excess of the requirements described above, and noise control in the LIGO lasers may be of particular concern. In short, though, any laser with even moderate gain is a potential candidate for such double-pass applications.