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Fluttering Fountains: Annular Geometry

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Under certain conditions of flow rate, height, and feedback, periodic or chaotic fluttering oscillations can be observed as the sheet of water from a dam or waterfall fountain descends through the air. Numerical and analytical interpretations of this phenomenon have recently been reported. The extension of these results to other fountain geometries is discussed here together with experimental observations on an annular waterfall fountain. © *1996 American Institute of Physics.* [S0021-8979(96)01502-5]

I. INTRODUCTION

The feature of a thin uniform sheet of water falling through the air is most widely observed in waterfall fountains. Under some conditions the water sheets in such fountains exhibit a wavelike fluttering motion at a frequency in the range of about 2.5 to 25 Hz, and numerical models of this effect provide good agreement with experimental observations.[1] Approximate analytic solutions[2] and stability criteria[3] for this phenomenon have also been reported. The fluttering oscillations are often highly periodic and represent laserlike feedback oscillations of the air–water system. Small displacements of the water sheet are amplified by the Helmholtz mechanism as the sheet moves downward through the surrounding air.[1] The large amplitude wave motions at the bottom tend to compress or expand the air trapped behind the sheet, and this air in turn pulls in or pushes out on the water surface at the top.

All previous reports and experiments relating to this fluttering instability have concerned dams or waterfall fountains that are basically linear in design. While systems that clearly exhibit the instability are not particularly common, the author has now collected videos and photographs of about a dozen good examples. All of these examples are linear in the sense that the crest of the weir is very nearly a horizontal straight line. While this linearity is a natural aspect of dam development, it is by no means an appropriate restriction in the design of ornamental fountains. For example, circular fountains have been popular since the Renaissance,[4] and it would seem possible that the annular water sheets in such systems might also exhibit related instability effects. One purpose of this study has been to investigate this question of fluttering instabilities in circular fountains.

It may be noted that annular liquid flow has long been studied in other contexts. In 1833, Savart showed that a bell-like sheet of water could be produced by placing a disk-shaped obstruction in the path of a vertical cylindrical jet of water.[5] Analyses of such flows by Boussinesq, Taylor, and others have emphasized systems in which some of the forces such as gravity, surface tension, pressure, and air resistance may be important, but Helmholtz amplification is negligible.[6,7] Small diameter annular liquid jets exhibit an instability in which a periodic train of liquid shells or bubbles is formed.[8,9] A more recent study has included other periodic oscillations and possible applications of annular flows to chemical and nuclear reactors.[10] Our results here show that in larger annular flows Helmholtz amplification and associated instabilities may have a dominant effect on the flow dynamics.

Several possible circular fountain configurations are discussed in Sec. II, and their anticipated oscillation characteristics are also considered. A specially designed circular fountain has been constructed, and observations of fluttering oscillations in this system are reported in Sec. III.

II. ANNULAR GEOMETRY

There are two basic categories of circular fountain, and these are illustrated in schematic cross section in Fig. 1. The interior design in Fig. 1(a) might also be called an inverted fountain, because in one realization the water seems to simply disappear into a hole in the ground rather than, as more usual, flowing outward from some localized source. The exterior design in Fig. 1(b) represents a more traditional circular fountain in which the water flows over the edge of a raised circular basin. This second design is the one studied in the experiments reported below.

At first glance, one might imagine that the theoretical analysis of the dynamics of the water sheet would have to be reexamined for the circular geometry. Thus, the relationship between displacement and air pressure should not be the same for the linear and circular geometries. However, most analyses of linear fountain behavior assume that the surface displacements are very small. In this limit the displacement-pressure relationships are independent of curvature of the water sheet in a horizontal plane. Thus, for our present needs most of the results of preceding analyses can be adopted directly.

One important result of the theory is an approximate analytic formula for the frequency of the oscillation modes of the fluttering water sheet.[2] This formula is

$$f=\frac{m+1/4}{(v_0/g)[(1+2gy_0/v_0^2)^{1/2}-1]}, \tag{1}$$

where v_0 is the downward velocity of the water as it separates from the weir, g is the acceleration of gravity, y_0 is the

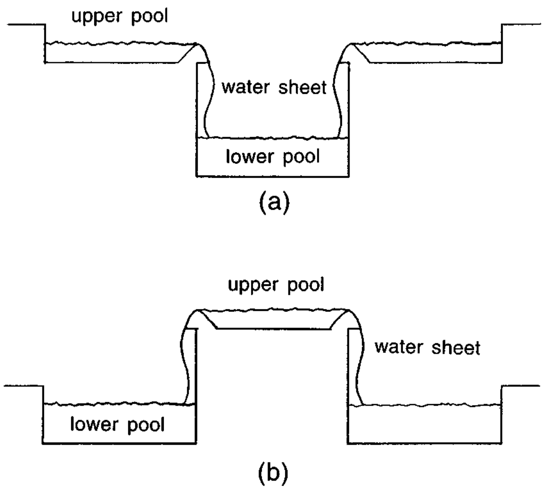

FIG. 1. Schematic representation of circular fountains including (a) internal and (b) external geometries.

height of the fall, and m is an integer indicating the order of the mode. The lowest order or fundamental mode in this notation corresponds to $m=1$.

For systems in which the weir height is very small, Eq. (1) may sometimes be simplified further. If viscosity of the water (or other liquid) is neglected, the initial downward velocity v_0 is related approximately to the weir height y_w by the familiar expression

$$v_0=(2gy_w)^{1/2}. \tag{2}$$

With this substitution Eq. (1) becomes

$$f=\frac{m+1/4}{(2y_w/g)^{1/2}[(1+y_0/y_w)^{1/2}-1]}. \tag{3}$$

It is now clear that if the fountain height is large compared to the weir height ($y_0 \gg y_w$) Eq. (3) reduces to

$$f=\frac{m+1/4}{(2y_w/g)^{1/2}(y_0/y_w)^{1/2}}=\left(m+\frac{1}{4}\right)\left(\frac{g}{2y_0}\right)^{1/2}. \tag{4}$$

With $g=9.8\ \mathrm{m\ s^{-2}}$, the frequency of the fundamental $m=1$ mode varies with height according to the simple approximate formula

$$f=2.77y_0^{-1/2}. \tag{5}$$

This formula is compared with experimental data for our circular fountain system.

III. EXPERIMENT

In order to observe the fluttering oscillations of interest here, it is necessary that the falling water sheet be highly uniform. In typical circular fountains the weir is not smooth enough, and excessive waves and turbulence in the pool behind the weir also seem to be common. As is noted below, these fountains are also much more susceptible to perturbations from the wind than are the more protected linear designs. Accordingly, it was considered important to build our own fountain and verify that the theoretical models can be extended to the circular geometry. Because of time and budget constraints, we determined to build the simplest and smallest system that was likely to exhibit the instabilities.

The first and most obvious decision was to build an exterior system like that shown in Fig. 1(b). Interior designs would seem to be much more difficult to construct. Our experience with linear systems suggested that the minimum fountain height to obtain adequate Helmholtz gain would probably be in the range of 0.5–1.0 m. This height also implies a minimum fountain diameter of almost the same magnitude to prevent the bottom of the water sheet from being drawn in excessively by the flow-generated air pressure difference. The falling water sheet tends to entrain air and pump it out of the enclosed space, and ultimately the resulting pressure difference can pull the sheet into the back or central surface before it reaches the lower pool. In the circular geometry this effect is already known in the context of water bells.[10]

For our minimal fountain design, we chose to have a variable height and a diameter of 37.5 cm. The column supporting the upper pool is a section of standard 3 in. (7.62 cm) i.d. PVC drain pipe through which the input water is pumped upward. The upper pool is a circular plastic planter tray, which was strengthened and modified for the central water input. The column and basin setup was positioned in a spa, which served as the lower pool. The fountain height could be easily varied by pumping the water in the spa back and forth to an adjoining swimming pool using the existing pool plumbing. The water flow to the fountain itself was provided by a small auxiliary swimming pool pump and the flow rate could be varied between 1 and 2 liters per meter of fountain circumference per second.

Fluttering oscillations were readily observed with the fountain system just described. In these oscillations circularly symmetric waves grow as they fall toward the surface of the lower pool. Video pictures of these oscillations are quite striking and are reminiscent of the undulations of a hula skirt. A photograph of an oscillation mode is given in Fig. 2. At the instant of this photograph the oscillating water sheet is bulging outward at the top and inward near the bottom. The concentric wire gauze screens visible in the upper pool are introduced to reduce turbulence in the flow, and such screens have been employed for a similar purpose in earlier flow studies.[7] The water pump for the fountain is visible in the background, and the white post tied to the black column is for support only.

For this relatively small diameter system the height range between initial instability and water bell conditions is quite narrow, with distinct periodic motion being observed for heights between about 50 and 62 cm. For slightly greater heights the motion becomes irregular with some hints of the next higher-order mode. If the fountain height is increased to about 70 cm, the water sheet contracts to form a water bell as discussed above. A photograph of such a contracted sheet is given in Fig. 3. A summary of the experimental periodic data is given in Fig. 4 together with a plot of the simplest frequency model of Eq. (5). Within the experimental uncertainties of about 0.2 Hz, the frequencies are in good agreement with the simple model.

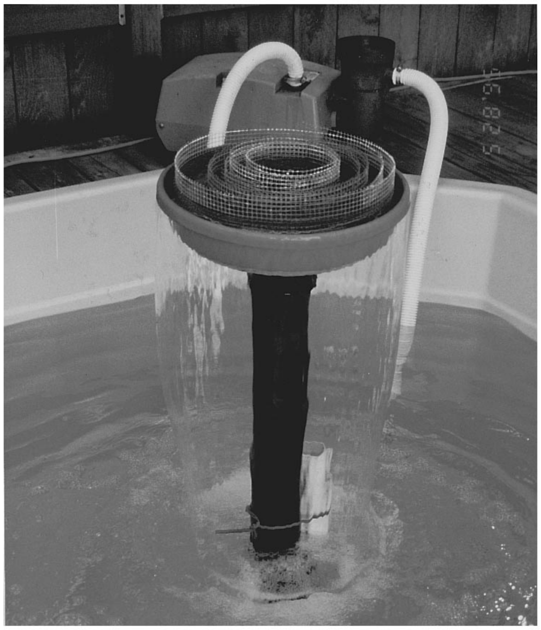

FIG. 2. Experimental circular fountain showing the fundamental oscillation mode as observed for heights between about 50 and 62 cm. Contrast in the photograph is limited by the necessary uniformity of the water sheet.

In a not so well characterized experiment, we have slightly warped the tray containing the upper pool so that the effective weir height was slightly higher on two opposite sides (N and S, for example) and slightly lower on the in-between sides (E and W). This had the effect of making the flow rate higher on the two lower sides and lower on the higher sides. The result was that the low-flow (high gain) sides fluctuated strongly toward and away from each other while the high-flow (low gain) sides moved very little.

FIG. 3. Experimental fountain showing contraction of the water sheet with a height of about 70 cm.

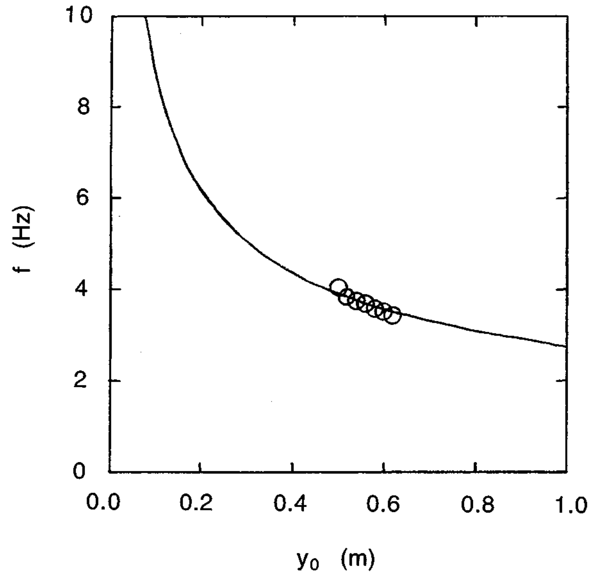

FIG. 4. Oscillation frequency as a function of fountain height y_0. Experimental data points are represented by circles, and the line is a plot of Eq. (5).

Another observation is that this external circular configuration is extremely sensitive to wind. It is by definition exposed on all sides and lacks the partial protection provided by the back wall in a typical linear fluttering fountain. For an ideal experiment it would be better to operate the fountain indoors. The internal fluttering fountain shown in Fig. 1(a) provides good protection from the wind but would probably be more difficult to construct. We have not seriously considered constructing an interior fountain and, among other things, consider such systems to be relatively lacking in esthetic appeal.

IV. CONCLUSION

It has been shown that the same fluttering instability that has been observed and studied in linear waterfall fountains also exists in fountains having a circular geometry. Experiments have been conducted with the external circular configuration, and the frequencies and stability threshold for this system are in good agreement with theoretical models. The previously noted tendency of the water sheet in a linear fountain to be drawn against the back wall[3] translates in the circular fountain into a tendency for the water sheet to form a water bell.

The design we chose is really a minimal system, and a wider range of oscillation behaviors would be possible in a fountain having a larger diameter. Another type of periodic mode should be possible in very large circular systems in which the central column is large enough to prevent direct acoustic communication between opposite sides of the fountain. In this case helical modes could occur in which the phase of the waves for a given time and height on the water sheet varies linearly with angular position around the fountain perimeter. The total phase shift of one of these modes for one loop around the fountain would be an integer multiple of 2π, and the modes are similar in that respect to electromagnetic-beam modes having helical phase fronts. A transverse wind across the face of a linear fountain seems to cause a transverse phase delay,[1] and it is possible that the helical modes in a circular fountain could be encouraged by an appropriate wind behind or in front of the water sheet.

While the emphasis here has been on the simplest possible circular generalization of the fluttering oscillations of more familiar linear waterfall fountains, many other geometries should also be possible. One can imagine systems in which the curvature, flow rates, or heights of the upper or lower pools vary with position along the fountain boundary. Nonvertical flows should exhibit related behaviors in systems using jets to provide the initial fluid flow.[7] Previous treatments of annular flows have always neglected Helmholtz amplification, and as shown here this mechanism must be included to obtain a valid description of the flow dynamics for larger annular systems.

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