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Congestion Modeling and Mitigation in the National Airspace System

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Congestion modeling and mitigation in the National Airspace System

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Presented at

Portland State University

10.24.13

Outline

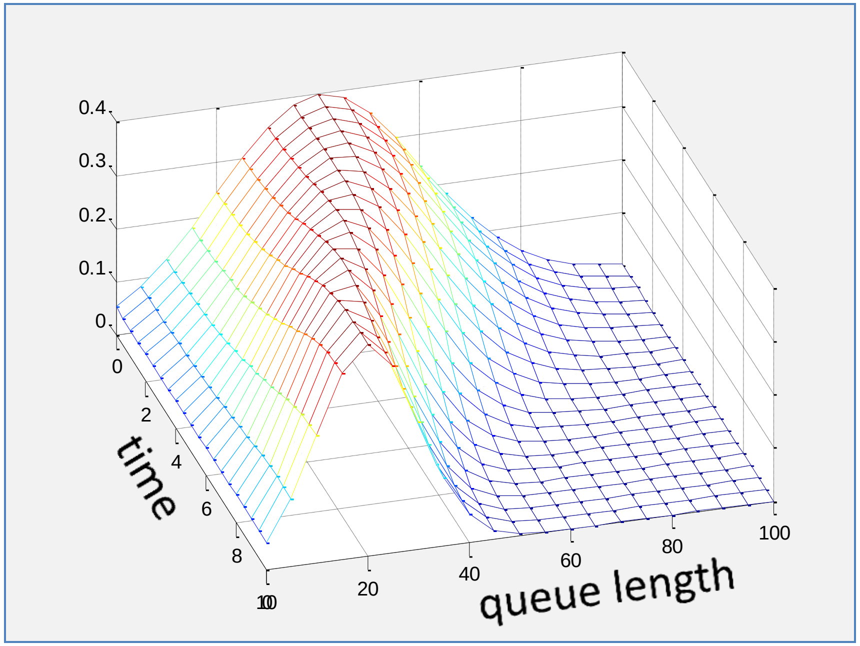
- Diffusion models of queueing delays at individual airports (NASA)
- Equitable resource allocation methods for airspace flow program planning (FAA)

DIFFUSION MODELS OF QUEUEING DELAYS AT INDIVIDUAL AIRPORTS

Project Sponsor: NASA

Single airport queue formulation

$f_i(x;t) =$ density of length of queue i at time t



Assumptions

- Continuity
- Markov
- 2nd order approximatable

$$\frac{\partial f(x;t)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2} V(x;t) f(x;t) - \frac{\partial}{\partial x} M(x;t) f(x;t)$$

Fokker-Plank equation

The Fokker-Plank equation and boundary conditions

PDE:

$$\frac{\partial f_i(x;t)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2} V_i(x;t)\, f_i(x;t) - \frac{\partial}{\partial x} M_i(x;t)\, f_i(x;t)$$

Boundary Conditions:

$$f(0;t)M(0;t) - \frac{\partial}{\partial x}\left(f(x;t)V(x;t)\right)\Big|_{x=0} = 0 \quad t > 0$$

Reflecting barrier to prevent negative queue length

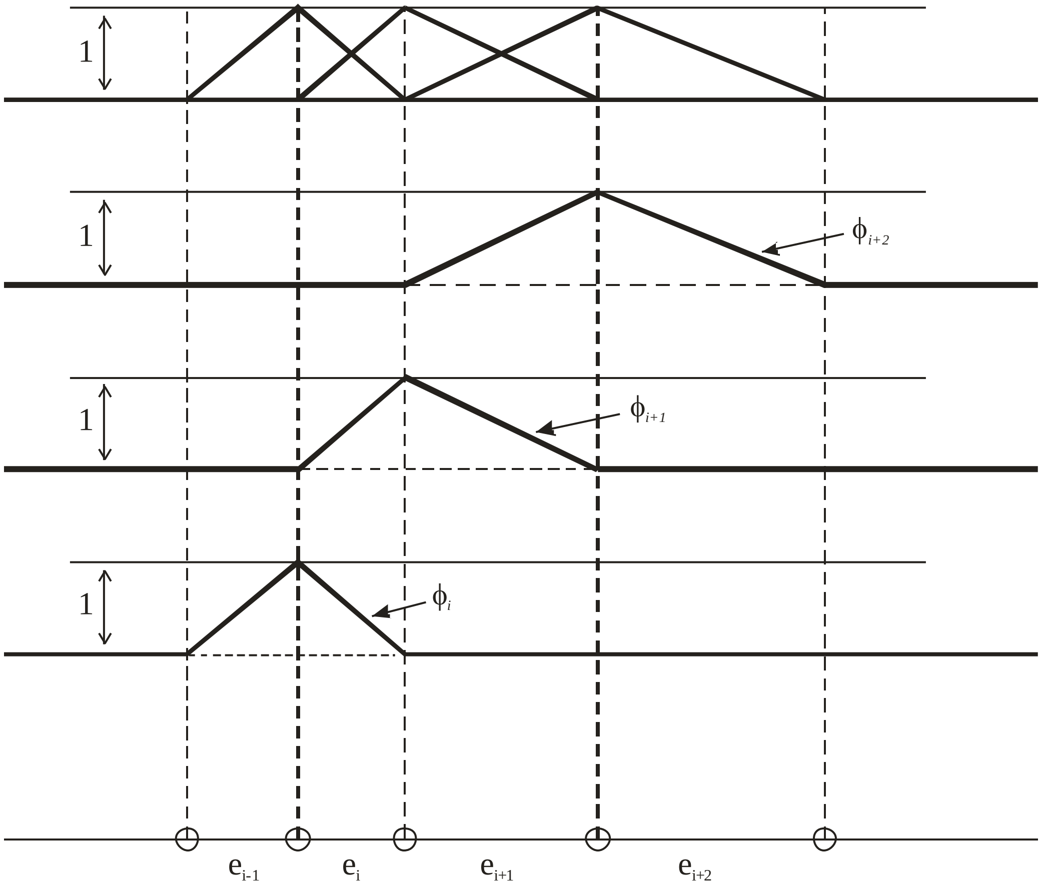
Initial conditions:

$$f(x;0) = \delta(x)$$

Queue empty at the beginning of the day

Mesh generation for the finite element method

- Allow for non-uniform finite element widths
- Standard FEM implementations might assume uniform element widths when computing stiffness matrix and load vector



Global stiffness matrix assembly

- Goal: solve the linear system

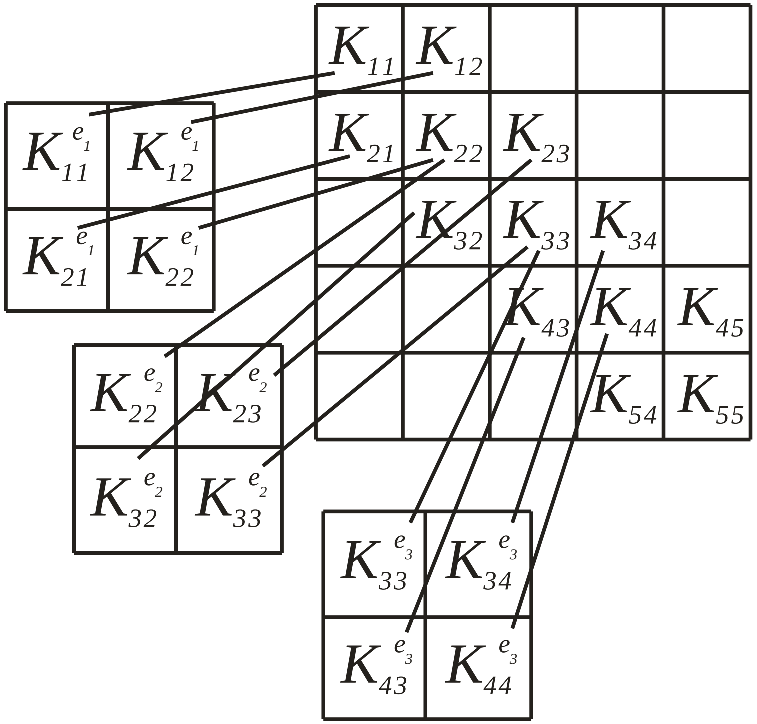
$$\sum_{j=1}^{N} a_j^{L+1} K_{ij} = R_i$$

where

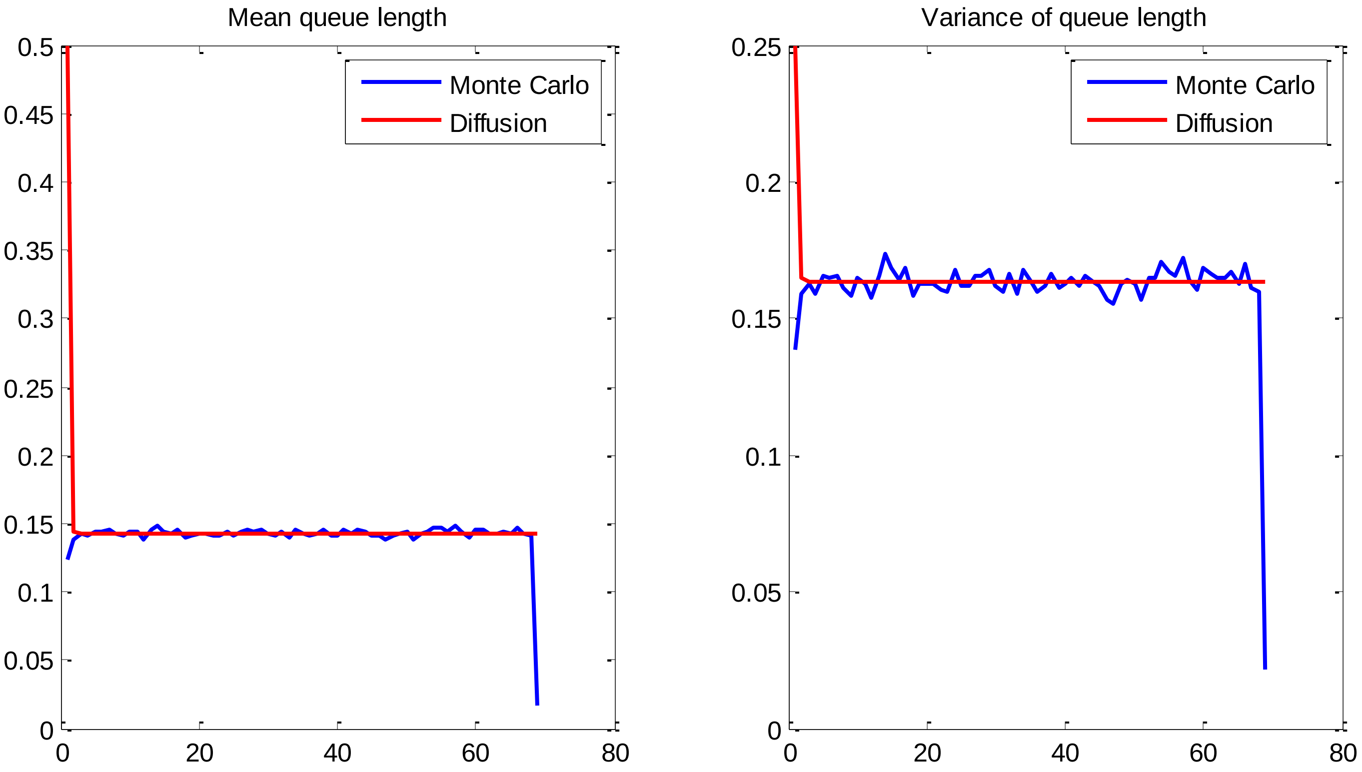
$$K_{ij} = \frac{1}{2}\int_{\Omega} V^{L+1}\phi_j'(x)\phi_i'(x)\,dx - \int_{\Omega} M^{L+1}\phi_j(x)\phi_i'(x)\,dx$$

$$+\int_{\Omega}\frac{1}{\Delta t}\phi_j(x)\phi_i(x)\,dx$$

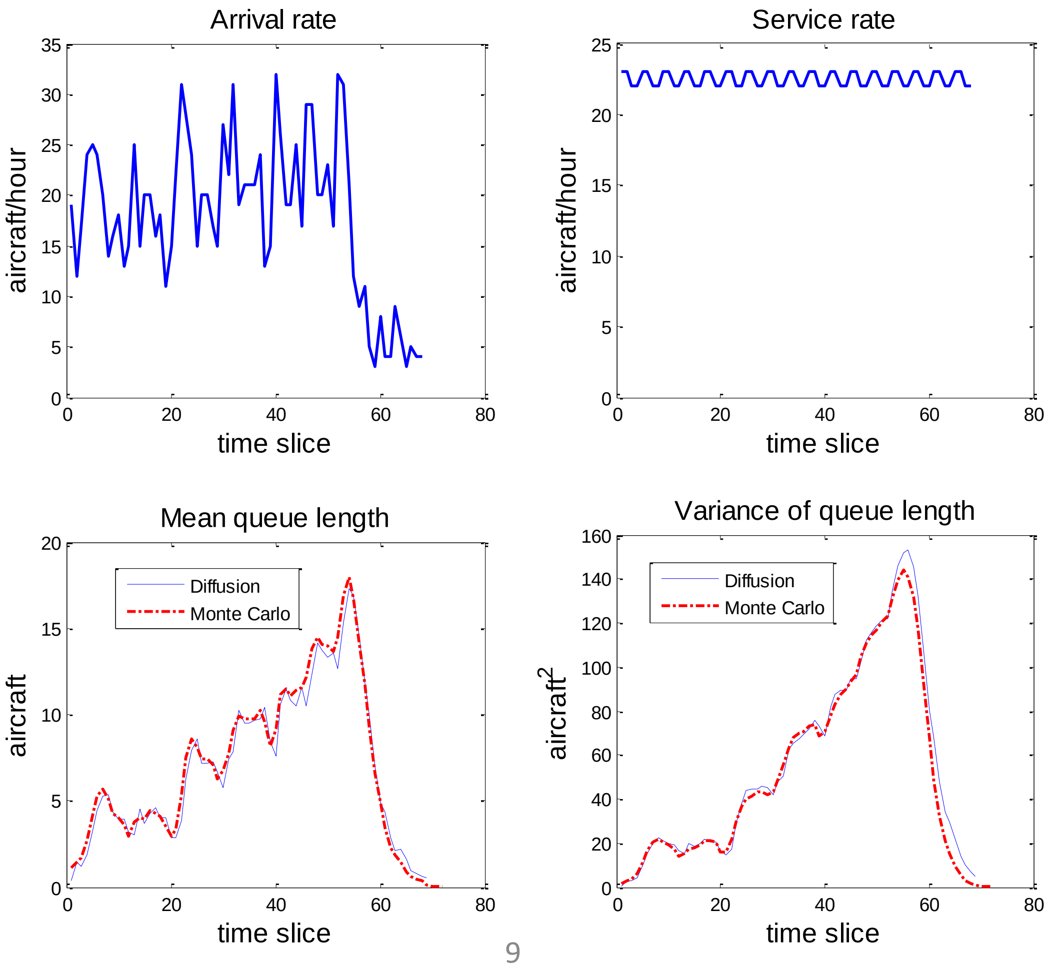
- The products $\phi_j'\phi_i'$, $\phi_j\phi_i'$, and $\phi_j\phi_i$ are only non-zero for $|i-j|\le 1$
- Thus, the matrix $\{K_{ij}\}$ is tridiagonal
- One option is to assemble the matrix from 2x2 element-wise contributions; however, they are NOT symmetric



M/M/1, λ = 5, μ = 40, n = 10,000
MC time = 106.9 sec, diff time = 8.17 sec



Results from Chicago O’Hare Airport



Contributions

- Less distribution dependence:
  - Can specify distributions only up to 1st and 2nd moments
- Independent mean and variance:
  - Important stochastic properties can be evaluated, and can propagate if these models are chained together to form a network
- Solution time
  - A complete stochastic profile of the solution can be generated in a single run of the model (a few seconds) rather than having to run Monte Carlo thousands of times

Final results

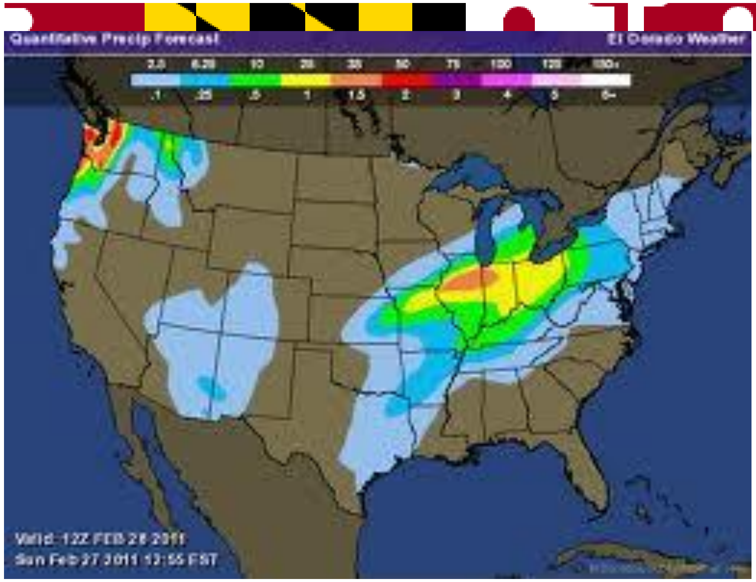
- Lovell, David J., Kleoniki Vlachou, Tarek Rabbani, and Alexandre Bayen (2013). A diffusion approximation to a single airport queue. *Transportation Research Part C: Emerging Technologies*, vol. 33, pp. 227-237.

EQUITABLE RESOURCE ALLOCATION METHODS FOR AIRSPACE FLOW PROGRAM PLANNING

Project Sponsor: FAA

Problem Description

- During adverse weather conditions, reduced en-route capacity leads to reduction in the number of flights that can pass the impacted area
- The available slots at the boundary of the constrained area are less than the flights scheduled to pass that portion of the airspace

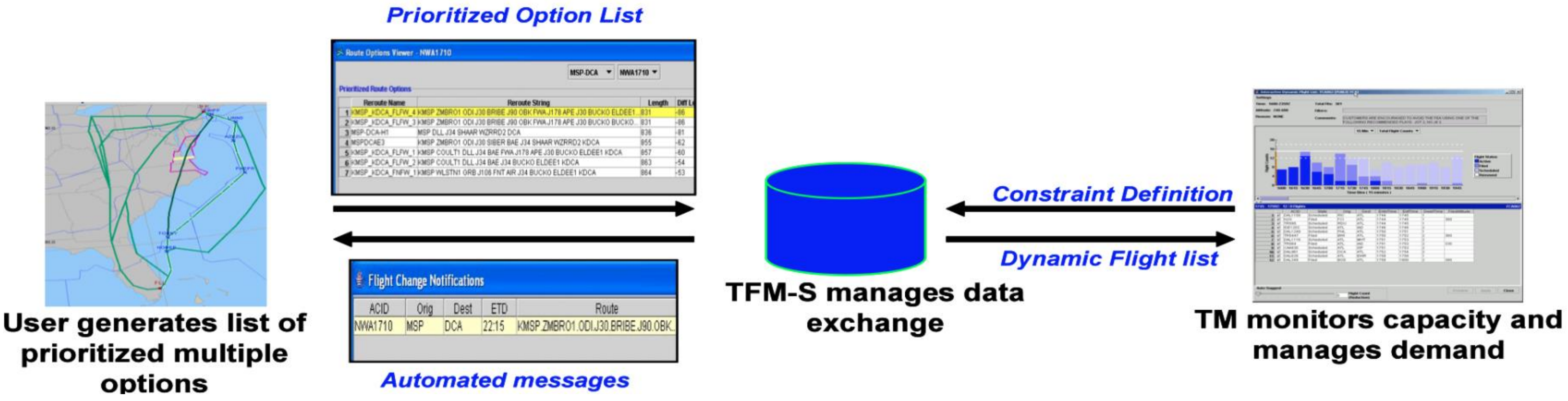


Traffic Flow Management (TFM) Tools

- Ground Delay Programs (GDP’s)
  - A GDP issues departure delay to aircraft expected to arrive at a constrained airport. These ground delays are less costly and safer than the airborne delays that would result without such actions.
  - Ration-By-Schedule
- Flow Constrained Areas (FCA’s)
  - FCAs are used to show areas where the traffic flow should be evaluated or where initiatives should be taken due to severe weather or volume constraints.
- Airspace Flow Program (AFP)
  - AFP combines the power of GDP’s and FCAs to allow more efficient, effective, equitable, and predictable management of airborne traffic in congested airspace.

Collaborative Trajectory Options Program (CTOP)

- Two key enabling ideas
  - NAS customers submit cost weight sets of trajectory options to the traffic management system
  - Traffic managers manage demand on resources by setting capacities on those resources then running allocation algorithms that adjust demand to meet those capacities



Prioritized Option List

Route Options Viewer - NWA1710

MSP-DCA ▾ NWA1710 ▾

Prioritized Route Options

	Reroute Name	Reroute String	Length	Diff L
1	KMSP_KDCA_FLFW_4	KMSP ZMBRO1 ODI J30 BRIBE J90 OBK FWA J178 APE J30 BUCKO ELDEE1...	831	-86
2	KMSP_KDCA_FLFW_3	KMSP ZMBRO1 ODI J30 BRIBE J90 OBK FWA J178 APE J30 BUCKO BUCKO...	831	-86
3	MSP-DCA-H1	MSP DLL J34 SHAAR WZRRD2 DCA	836	-81
4	MSPDCAE3	KMSP ZMBRO1 ODI J30 SIBER BAE J34 SHAAR WZRRD2 KDCA	855	-62
5	KMSP_KDCA_FLFW_1	KMSP COULT1 DLL J34 BAE FWA J178 APE J30 BUCKO ELDEE1 KDCA	857	-60
6	KMSP_KDCA_FLFW_2	KMSP COULT1 DLL J34 BAE J34 BUCKO ELDEE1 KDCA	863	-54
7	KMSP_KDCA_FNFW_1	KMSP WLSTN1 GRB J106 FNT AIR J34 BUCKO ELDEE1 KDCA	864	-53

Flight Change Notifications

ACID	Orig	Dest	ETD	Route
NWA1710	MSP	DCA	22:15	KMSP ZMBRO1 ODI J30 BRIBE J90 OBK...

Automated messages

User generates list of prioritized multiple options

TFM-S manages data exchange

Constraint Definition

Dynamic Flight list

TM monitors capacity and manages demand

Address Specific Problems

- Characterize preference and cost information provided by flight operators
- Explicit consideration of three performance metrics
  - System efficiency (performance criteria such as throughput and flight delay)
  - Equity (flight operators are treated fairly)
  - User cost (internal flight operator cost function)

Allocation Procedure

- Allocate fairly the reduced number of slots to airlines
- “Proportional random allocation” is used to estimate the fair share of each flight and each airline for each of the slots

Fair Share Computation

Definitions

- Time of flights: the time each flight (f_i) is scheduled to arrive at the boundary of the FCA
  $i = 1, 2, 3, 4, 5, 6$ *for our example*
- Time of slots (s_j)
  $j = 1, 2, 3, 4$ *for our example*
- Index of which flight corresponds to which airline (Airlines: A_1=1, A_2=2, A_3=3)

Example

$$\begin{array}{cccccc} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \\ \left[358 \right. & 400 & 402 & 403 & 405 & \left. 406 \right] \end{array}$$

$$\begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \\ \left[400 \right. & 402 & 404 & \left. 406 \right] \end{array}$$

$$\begin{array}{cccccc} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \\ \left[1 \right. & 2 & 1 & 2 & 3 & \left. 3 \right] \end{array}$$

Fair Share Computation

- Find the earliest slot that each flight can be assigned to (Slots: $S_1=1$, $S_2=2$, $S_3=3$, $S_4=4$)
- Find the total number of flights that can be assigned to each slot

$$\begin{array}{cccccc} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \\ \left[1 \right. & 1 & 2 & 3 & 4 & \left. 4 \right] \end{array}$$

$$N_{i,j} = \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$N_{i,j} = \begin{cases} 1, & \textit{if flight i can be assigned to slot j} \\ 0, & \textit{otherwise} \end{cases}$$

$$n_m = \sum_i N_{i,m}$$

n_m is the the number of flights that can be assigned to the respective slot

Fair Share Computation

- Find the share of each flight for each slot, where share given by

$$Share^{f_i}_{sj} = N_{i,j} * \frac{1}{\prod_{m=k}^{j}(n_m - (m-1))}$$

where k is the earliest slot that flight f_i can be assigned to and n_m is the number of flights that can be assigned to the respective slot

$$m = 1,2,3,4 \; for\, our\, example$$

Example,

$$Share^{f_1}_{s_2} = \frac{1}{(2-(1-1))\times(3-(2-1))} = \frac{1}{4}$$

$$\begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{24} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{24} \\ 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{12} \\ 0 & 0 & \frac{1}{2} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix} \end{array}$$

Fair Share Computation

- Find the total share of each flight for all slots

$$\begin{array}{c} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{array} \begin{bmatrix} \frac{22}{24} \\ \frac{22}{24} \\ \frac{10}{12} \\ \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

Fair Share Computation

- Find the fair share of each airline for all available slots

$$\begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} \frac{42}{24} & \frac{38}{24} & \frac{2}{3} \end{bmatrix} \end{array}$$

Airline preference information

- Priority number and maximum delay (before cancellation or re-route) for each flight:

flight	f_1	f_2	f_3	f_4	f_5	f_6
carrier	1	2	1	2	3	3
priority	2	3	4	4	1	1
max delay	35	25	23	32	50	33

Preference Based Proportional Random Allocation (PBPRA)

- Start by considering only fractional shares
  - For carriers with large shares, this should be approximately uniformly distributed
  - For small carriers with only a fractional share, this allows them not to be systematically disadvantaged
- Once fractional shares are exhausted, revert to integer shares

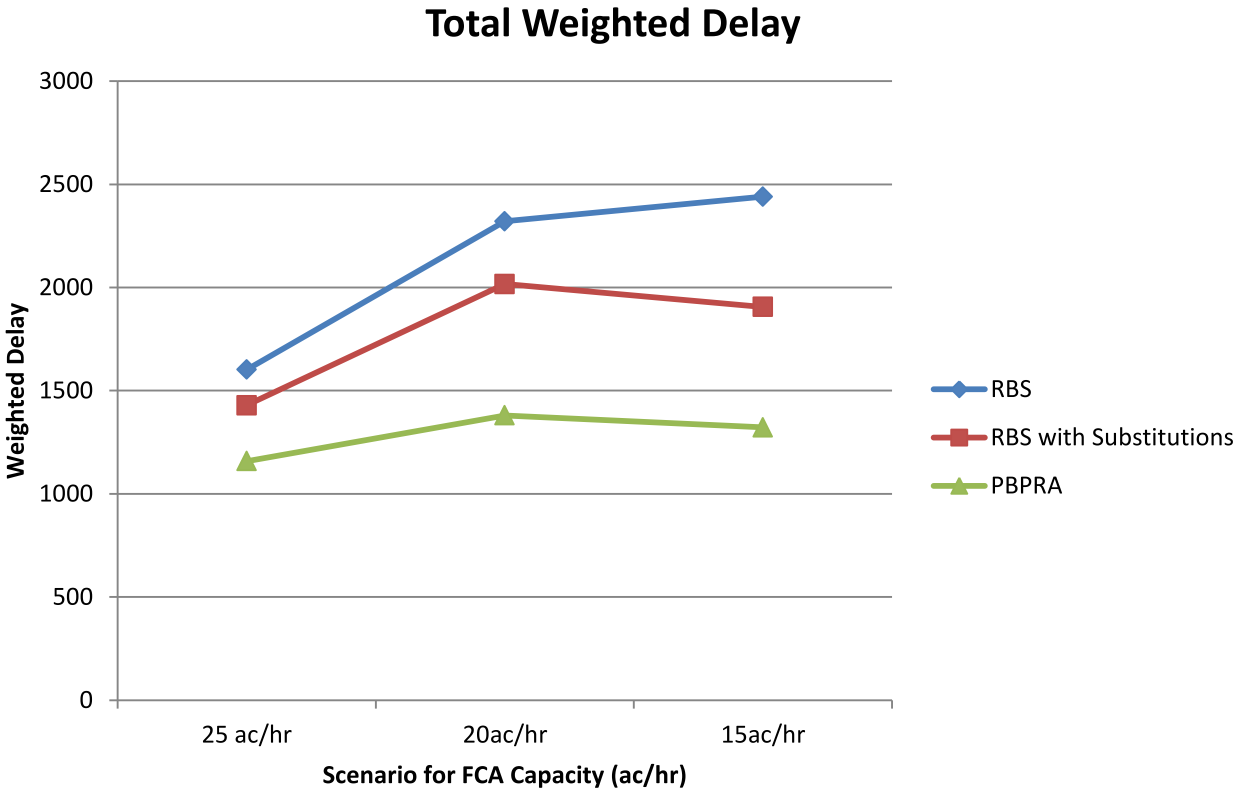
PBPRA 2

- For each slot, determine the carriers that have a claim on that slot
  - Enforce maximum delay constraints
- Allocate the slot randomly, but with probabilities proportional to the magnitude of the claims
- Assign the slot to the flight of the winning carrier with the highest priority number
- Reduce the winning carrier’s claims to subsequent slots where that flight contributed to its fair share
- Repeat until all slots/flights are either assigned or rejected (cancelled or re-routed)

Variance in Slot Allocation

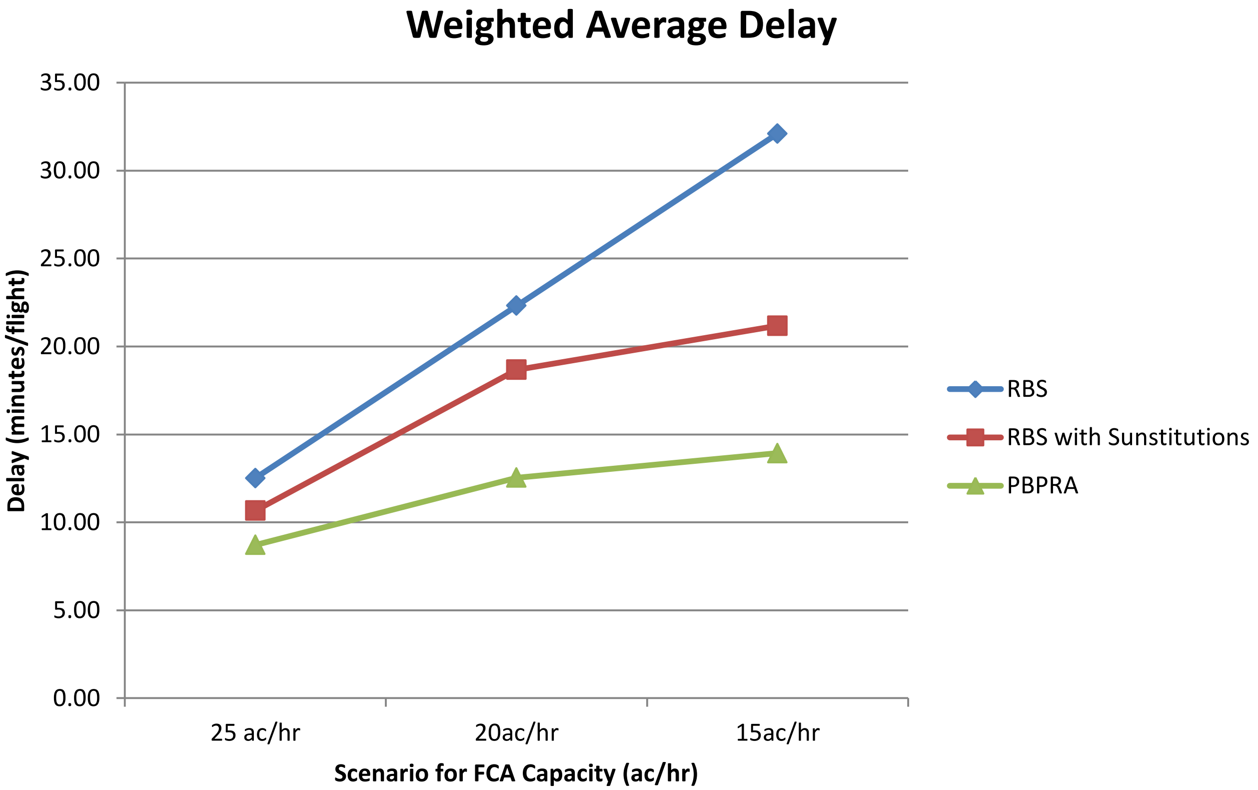
- In a given day the slots allocated to an airline won’t match exactly its fair share
- Over a large number of days the airlines will get what they want on average
- Fair Share – Actual Allocation = Error

PBPRA results



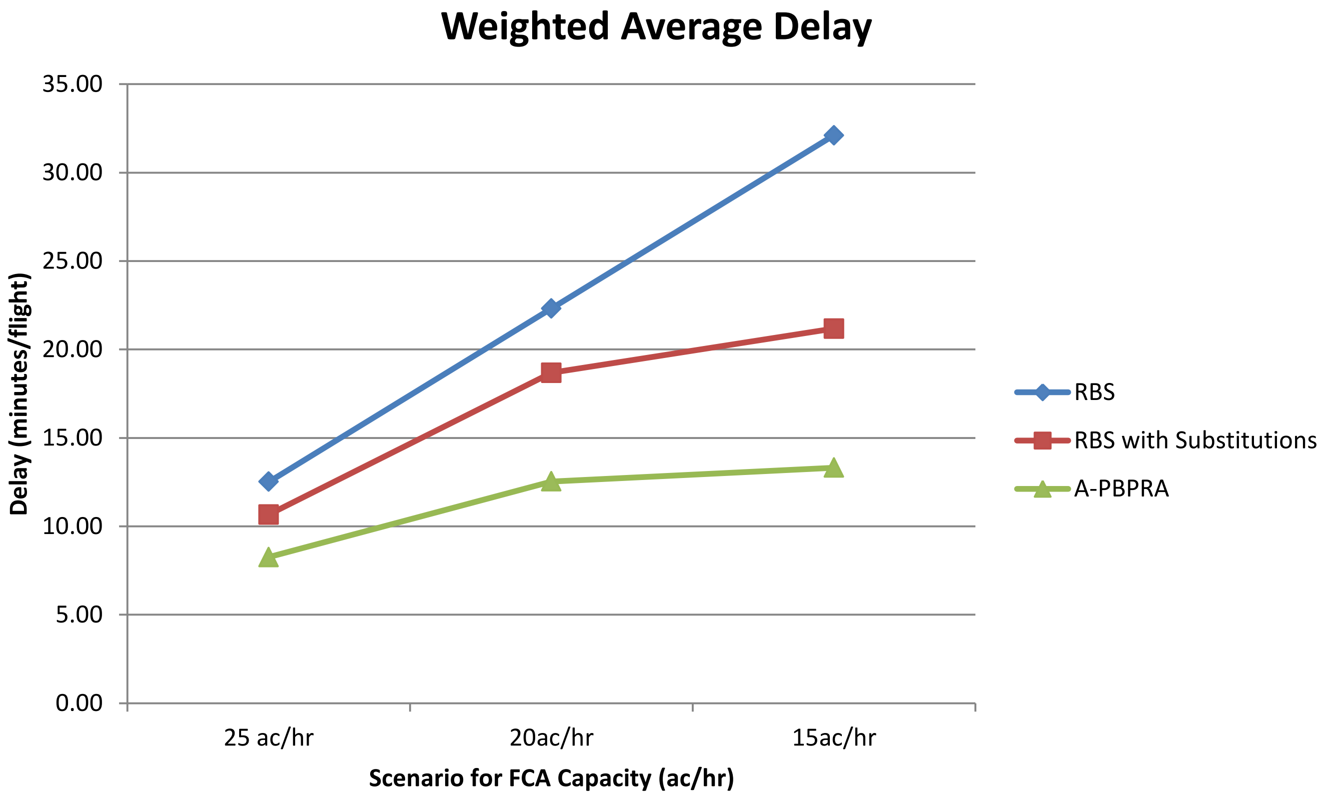
- Total delay can decrease at high levels of congestion because flights are cancelled

PBPRA results 2



- This is weighted average delay amongst only those flights that were assigned slots (delays)

A-PBPRA results



- Two types of airlines:
  - A) prefer earlier (fewer) slots
  - B) prefer more (later) slots

Final results

- Vlachou, K. and David J. Lovell (2013). Mechanisms for equitable resource allocation when airspace capacity is reduced. *Transportation Research Record* 2325, pp. 97-102.

Thank you!