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Bubble mobility in mud and magmatic volcanoes

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Abstract

The rheology of particle-laden fluids with a yield stress, such as mud or crystal-rich magmas, controls the mobility of bubbles, both the size needed to overcome the yield stress and their rise speed. We experimentally measured the velocities of bubbles and rigid spheres in mud sampled from the Davis-Schrimp mud volcanoes adjacent to the Salton Sea, Southern California. Combined with previous measurements in the polymer gel Carbopol, we obtained an empirical model for the drag coefficient and bounded the conditions under which bubbles overcome the yield stress. Yield stresses typical of mud and basaltic magmas with sub-mm particles can immobilize millimeter to centimeter sized bubbles. At Stromboli volcano, Italy, a vertical yield stress gradient in the shallow conduit may immobilize bubbles with diameter $\lesssim 1$ cm and hinder slug coalescence.

Keywords: yield stress, mud volcano, Herschel-Bulkley rheology, bubble, Salton Sea, Stromboli

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1 **1. Introduction**

2 Mud and magmatic volcano eruptions are controlled by the dynamics of
3 gas bubbles growing and ascending within a viscous medium of complicated
4 rheology (Gonnermann and Manga, 2007). Both muds and magmas exhibit
5 a finite yield stress and shear thinning behavior for sufficient particle/crystal
6 volume fractions (O'Brien and Julien, 1988; Philpotts et al., 1998; Saar et al.,
7 2001), which determines whether bubbles can move with respect to the fluid
8 and the speed at which they move. In turn this controls bubble size and
9 whether the ascending bubbly fluid remains homogeneous or whether bubbles
10 can coalesce to form gas slugs that separate from the fluid (e.g., Sherwood
11 and Sáez, 2014). Characterization of bubble motion in yield stress fluids is
12 relevant not only to volcanic eruptions but also to a range of natural and
13 engineered systems, such as diapirs in Earth's mantle and crust (Weinberg
14 and Podladchikov, 1994), bubble column reactors (Godbole et al., 1984), and
15 gas kicks within drilling fluids (Johnson and White, 1993).

16 Bubble motion in fluid suspensions may be roughly categorized into dis-
17 tinct flow regimes. When bubbles are smaller than or comparable in size to
18 surrounding particles or crystals, gas may propagate through pathways be-
19 tween individual particles (Belien et al., 2010). Larger bubbles may interact
20 with the surrounding suspension as a continuum. At low particle volume
21 fraction, these suspensions behave as Newtonian fluids (e.g., Einstein, 1906,
22 1911; Roscoe, 1952) but with increasing particle concentrations, suspensions
23 become shear thinning and possess a yield stress (Costa, 2005; Caricchi et al.,
24 2007; Mueller et al., 2010). Dense suspensions or suspensions of interacting
25 particles can behave like elastic solids. Boudreau et al. (2005) and Algar

26 et al. (2011), in particular, imaged and modeled bubble propagation by lin-
27 ear elastic fracture in natural muddy sediments; Fauria and Rempel (2011)
28 observed gas propagation by both capillary invasion and fracture in uncon-
29 solidated sand, though propagation was initiated by gas overpressure rather
30 than buoyancy.

31 Even if bubbles are large enough that the surrounding suspension may
32 be treated as a continuum, muds and magmas are not simple to character-
33 ize. Magma rheology depends strongly on temperature, composition, and
34 both suspended crystals and bubbles (Mader et al., 2013). Yield stresses
35 in lavas smaller than stresses required for measurable deformation are diffi-
36 cult to constrain (e.g., Lavallée et al., 2007; Castruccio et al., 2014), while
37 yield stresses estimated from the geometry of emplaced lava flows must re-
38 flect “end-stage” rheology of crystallized, nearly halted lavas (Chevrel et al.,
39 2013). Lavas crystallize continuously over a small temperature range (e.g.,
40 Pinkerton and Norton, 1995; Gurioli et al., 2014) and several days may be
41 required to achieve equilibrium deformation in laboratory experiments (Vona
42 et al., 2011). In muds, clay-sized particles interact electrically amongst them-
43 selves and dissolved electrolytes, and may spontaneously flocculate (Luckham
44 and Rossi, 1999; van Olphen, 1964). In both muds and magmas, suspended
45 particles and bubbles may orient or deform under applied shear and bulk
46 compositional differentiation may occur (e.g., particle settling, shear band-
47 ing). These and other processes give rise to time-dependent rheology (e.g.,
48 Bekkour et al., 2005; Tabuteau et al., 2007b) and spatial variation in material
49 properties that must be considered in rheological measurements and in single
50 or multiple phase flow experiments.

51 Although mud and magmatic eruptions differ in viscosity, yield stress,
52 and temperature, both types of eruptions produce morphologically similar
53 features including constructional conical edifices and flow deposits with lo-
54 bate toes and ropy surface textures. Bubbles bursting at the surface of mud
55 volcanoes have diameters $\sim 0.01\text{--}1$ m in several field sites, much larger than
56 suspended particle sizes (e.g., Hovland et al., 1997; Rudolph and Manga,
57 2010; Vanderkluysen et al., 2014). In magmatic volcanoes such as Stromboli
58 or Kilauea, basalts of low viscosity compared to more silicic melts may per-
59 mit buoyant bubble motion with respect to the surrounding melt (Sahagian,
60 1985; Cashman et al., 1994) and preserve distinctive vesicle shapes in lavas
61 (e.g., Walker, 1987). Muds and magmas develop yield stress at comparable
62 particle volume fractions, $\sim 0.1\text{--}0.3$ in muds (O'Brien and Julien, 1988) and
63 $\sim 0.2\text{--}0.5$ in magmas (Saar et al., 2001; Mueller et al., 2010). Muds are more
64 easily studied in the lab and at field sites than magmas, but they may yield
65 insight into eruptive processes of their magmatic cousins as well.

66 We performed laboratory experiments to measure bubble ascent velocities
67 in natural mud, extending prior experiments with the viscoplastic polymer
68 gel Carbopol (see Piau, 2007). Rigid particle drag and immobilization in Car-
69 bopol have been studied experimentally and numerically, but bubble motion
70 is less fully characterized. We first review dimensionless scalings for Newto-
71 nian fluids that have been extended to shear thinning, yield stress viscoplastic
72 fluids. Within this formalism, we verify that our mud may be treated as vis-
73 coplastic by testing a known drag scaling for rigid particles. We use our
74 measurements of bubble velocities in conjunction with available experimen-
75 tal data to obtain an empirical drag scaling for bubbles in viscoplastic fluids

76 and determine a yielding criterion bound consistent with available data. Fi-
 77 nally, we consider bubble mobility in muds and magmas and apply our drag
 78 empiricism to a model of Stromboli volcano's shallow conduit.

79 2. Dimensionless formalism

80 Here we review the Stokes and Hadamard-Rybczynski drag scalings for
 81 rigid particles and bubbles in Newtonian fluids. We introduce the viscoplastic
 82 Herschel-Bulkley rheology used to describe shear thinning and yield stress
 83 fluids such as muds and magmas, then extend the Newtonian drag results
 84 to Herschel-Bulkley fluids. The dimensionless formalism allows us to more
 85 easily compare particle motion across a range of physical parameters spanned
 86 by polymer gels (Carbopol), muds, and magmas.

87 2.1. Newtonian drag

88 Consider a fluid or solid particle moving vertically due to gravitational
 89 acceleration g at terminal velocity u_t within a homogeneous fluid medium.
 90 Let the particle have density ρ_p , volume V , equivalent spherical diameter
 91 $d^* = (6V/\pi)^{1/3}$, and maximum horizontal diameter d_h (i.e., diameter of the
 92 widest horizontal cross-section); d_h is a proxy for particle shape assuming
 93 axisymmetry. Let the surrounding fluid have density ρ_L and surface tension
 94 σ at the bubble-fluid interface. For creeping flow, ρ_p and g only enter our
 95 problem through a buoyancy (weight) per volume $\Delta\rho g$ with $\Delta\rho = \rho_p - \rho_L$.
 96 For solid particles, surface tension σ is unimportant; for spheres, $d_h = d^*$.

97 In steady flow, buoyancy and drag force balance as:

$$\frac{1}{2}C_D\rho_L u_t^2\pi\left(\frac{d_h}{2}\right)^2 = \Delta\rho gV. \quad (1)$$

98 The dimensionless drag coefficient C_D is a scaled drag force. The force bal-
 99 ance merely requires that drag, however generated, balances buoyancy; the
 100 drag coefficient C_D then encapsulates all information about fluid flow and
 101 rheology. In a dimensionless treatment, we determine how C_D scales with
 102 the remaining dimensionless groups (collapsing the relevant physical param-
 103 eters) describing the motion of single particles. From equation (1) we compute
 104 C_D directly from experimental measurements (u_t , $\Delta\rho$, V , d_h) to determine
 105 or test an appropriate scaling.

106 For a rigid sphere in a Newtonian fluid of dynamic viscosity μ , the Stokes
 107 solution for creeping flow may be formulated in terms of 5 physical param-
 108 eters – $\Delta\rho g$, d^* , ρ_L , u_t , and μ – or, by the Buckingham Pi theorem, 2 di-
 109 mensionless groups: the drag coefficient $C_D = (\Delta\rho g V)/(\rho_L u_t^2 \pi (d^*/2)^2/2)$ as
 110 defined above, and the Reynolds number $Re = \rho_L u_t d^*/\mu$. The analytic so-
 111 lution is $C_D = 24/Re$ for $Re \ll 1$. The extension to spherical bubbles is
 112 the Hadamard-Rybczynski solution $C_D = 16/Re$, although bubble drag in
 113 practice is often increased towards that of rigid spheres due to surfactants
 114 (Clift et al., 1978). For small Re , bubble drag is independent of an additional
 115 parameter, surface tension, parameterized by the Bond (or Eötvös) number
 116 $Bo = \Delta\rho g (d^*)^2/\sigma$ (Clift et al., 1978). As inertial effects become important,
 117 surface tension (or lack thereof) influences bubble shape and hence drag will
 118 depend on both Re and Bo (e.g., Clift et al., 1978, Fig 2.5).

119 2.2. Herschel-Bulkley rheology and drag

120 The Herschel-Bulkley model captures time-independent power-law and
 121 yield stress behavior and is often used to describe mud and magma rheology

122 (e.g. Coussot and Piau, 1994; Mueller et al., 2010):

$$\tau = \tau_c + K\dot{\gamma}^n \quad (2)$$

123 where τ is shear stress (Pa), γ is shear strain (-), $\dot{\gamma}$ is shear rate (s^{-1}), τ_c is
 124 the yield stress (Pa), K is the consistency (Pa s^n), and n is the power-law
 125 index (-). For comparison to Newtonian fluids, we also introduce the effective
 126 (or apparent) viscosity, taking characteristic strain rate $\dot{\gamma} = u_t/d^*$:

$$\mu_{\text{eff}} = \frac{\tau}{u_t/d^*} \quad (3)$$

127 The particle motion is now characterized by nine independent parameters:
 128 rheological parameters K , n , τ_c , fluid density ρ_L , particle diameters d^* , d_h ,
 129 terminal velocity u_t , surface tension σ , and particle buoyancy per volume
 130 $\Delta\rho g$. Our system is fully characterized by $9 - 3 = 6$ dimensionless groups per
 131 the Buckingham Pi theorem. We retain C_D and Bo as previously defined, but
 132 we modify Re and introduce the Bingham number (Bi), a ratio of length scales
 133 (d_h/d^*) that characterizes bubble asphericity, and the power-law index (n)
 134 as new dimensionless groups. The named dimensionless groups are defined
 135 as:

$$\begin{aligned} C_D &= \frac{\Delta\rho g V}{\frac{1}{2}\rho_L u_t^2 \pi (d_h/2)^2} \sim \frac{\Delta\rho g d^*}{\rho_L u_t^2} \\ \text{Re} &= \frac{\rho_L u_t^2}{K(u_t/d^*)^n} \\ \text{Bi} &= \frac{\tau_c}{K(u_t/d^*)^n} \\ \text{Bo} &= \frac{\Delta\rho g d^*}{\sigma/d^*}. \end{aligned}$$

136 The drag coefficient remains identical to that of (1) and balances buoyancy
 137 against drag, which is proportional to an inertial stress $\rho_L u_t^2$. The Reynolds

138 number balances inertial stress against the Herschel-Bulkley model's viscous
 139 stress using u_t/d^* as a characteristic shear rate for equation (2). The Bing-
 140 ham number uses the same viscous stress formulation to balance yield stress.

141 Although these 6 groups are sufficient to characterize our system, it will
 142 be convenient to introduce the yield parameter (Y):

$$Y = \frac{2\tau_c\pi(d_h/2)^2}{\Delta\rho gV} = \frac{3\tau_c}{\Delta\rho g d^*} \left(\frac{d_h}{d^*}\right)^2;$$

143 note that $Y \propto \text{Bi}/(\text{Re}C_D)$ is not independent of our chosen dimensionless
 144 groups. The yield parameter balances buoyancy against an immobilizing
 145 yield stress force that should be proportional to τ_c and cross sectional area
 146 $\pi(d_h/2)^2$. The exact form or pre-factor is not obvious, but the choice of
 147 $2\tau_c\pi(d_h/2)^2$ reduces to a common definition of Y for spherical particles ($d_h =$
 148 d^*); this definition of Y is due to Sikorski et al. (2009). Previous studies have
 149 sought a critical value of the yield parameter, Y_c , above which bubbles or rigid
 150 particles should be immobilized (Beris et al., 1985; Dubash and Frigaard,
 151 2004; Tsamopoulos et al., 2008); the critical value $Y_c = 0.143$ is commonly
 152 accepted for rigid spheres (Tabuteau et al., 2007a).

153 From the dimensional analysis, we expect a functional relation of form
 154 $C_D = C_D(\text{Re}, \text{Bi}, \text{Bo}, n, d_h/d^*)$. For rigid spheres (where Bo and d_h/d^* are
 155 not relevant), the relation:

$$C_D = \frac{24X(n)}{\text{Re}} (1 + k\text{Bi}) \quad (4)$$

156 has been verified experimentally for spheres in Carbopol (Atapattu et al.,
 157 1995; Tabuteau et al., 2007a). Here $X(n)$ is an empirical correction factor
 158 for shear-thinning that takes on values of 1–1.5 for $n \in (0, 1)$ and reduces to
 159 $X(n = 1) = 1$ for Newtonian fluids. The constant k describes the relative

160 contributions of yield and viscous stresses; note that the multiplicative term
 161 $(1 + k\text{Bi})$ reduces to 1 for either $k = 0$ or $\text{Bi} \propto \tau_c = 0$. We use $k = 0.823$,
 162 estimated from numerical simulations by Beaulne and Mitsoulis (1997), and
 163 interpolate $X(n)$ using data from numerical simulations by Gu and Tanner
 164 (1985).

165 To better intuit this result, we recast the drag scaling in terms of a gen-
 166 eralized Reynolds number Re^* as:

$$C_D = \frac{24}{\text{Re}^*}$$

167 where Re^* is defined as

$$\text{Re}^* = \frac{\text{Re}}{X(n)(1 + k\text{Bi})} = \frac{\rho_L u_t^2}{X(n) (K(u_t/d^*)^n + k\tau_c)}. \quad (5)$$

168 Whereas Re considers only viscous stress as quantified by K , Re^* accounts
 169 for both yield and viscous stresses in the denominator and incorporates the
 170 correction factor $X(n)$ so that any functional dependence $C_D = C_D(\text{Re}^*)$
 171 is independent of physical parameters. If we take $k = 1$, then $\text{Re}^* =$
 172 $\rho_L u_t^2 / (X(n)\tau) = \rho_L u_t d^* / (X(n)\mu_{\text{eff}})$ where μ_{eff} is given by equation (3). Thus
 173 Re^* balances inertia against the total fluid resistance to motion. We note
 174 that Re^* is derived from a “dynamical parameter” $Q = \text{Re}/(1 + k\text{Bi})$ used in
 175 previous studies (Ansley and Smith, 1967; Atapattu et al., 1995; Tabuteau
 176 et al., 2007a); here we simply fold $X(n)$ into Re^* as well. Ansley and Smith
 177 (1967), in particular, physically motivate the definition of Re^* / Q and thus
 178 derived an estimate of $k = 7\pi/24$.

179 Our drag scalings should reduce to empiricisms for particles in power-
 180 law, Bingham, and Newtonian fluids in the appropriate limiting cases. The
 181 spherical particle drag scaling $C_D = 24/\text{Re}^*$ correctly gives $C_D = 24X(n)/\text{Re}$

182 for $\tau_c = 0$ (Crochet et al., 1984) and the Stokes solution $C_D = 24/\text{Re}$ for
183 $X(n = 1) = 1$.

184 In what follows, we attempt to construct a similar empiricism for bub-
185 bles in Herschel-Bulkley fluids from experimental data. We might expect an
186 analogous correlation $C_D = 16/\text{Re}^*$; indeed, empiricisms of form $C_D = 16/\text{Re}$
187 have been obtained for gas bubbles in shear thinning fluids without a yield
188 stress (Miyahara and Yamanaka, 1993; Dewsbury et al., 1999; Margaritis
189 et al., 1999).

190 **3. Materials and Methods**

191 *3.1. Davis-Schrimpf mud volcanoes*

192 The Davis-Schrimpf mud volcanoes are comprised of extrusional gryphons
193 (conical mud volcanoes $\sim 1\text{--}2$ m tall) and more fluid mud pots (cf. Kopf,
194 2002, on nomenclature) located near the Salton Sea, Southern California
195 (Figure 1). These mud volcanoes are driven by the ascent of CO_2 produced
196 through metamorphic decarbonation reactions in the underlying Salton Sea
197 Geothermal System. Previous studies have characterized the tectonic setting
198 (Lynch and Hudnut, 2008), geomorphology (Onderdonk et al., 2011), gas
199 chemistry (Svensen et al., 2009; Mazzini et al., 2011), and response to earth-
200 quakes (Rudolph and Manga, 2010, 2012). We collected mud from gryphons
201 at the Davis-Schrimpf mud volcanoes on September 15, 2010 for material
202 characterization and bubble ascent experiments.

203 Mud from the Davis-Schrimpf site consists primarily of quartz (41 wt%),
204 feldspar (24%), carbonates (9%), and clay minerals illite, montmorillonite,
205 and kaolinite (27% in total) suspended in water. Mineral abundances are

206 based on our own X-ray diffraction measurements; weight percentages do not
207 sum to 100% due to rounding. We also imaged the mud's 3-D structure using
208 X-ray microtomography at the Advanced Light Source, Lawrence Berkeley
209 National Laboratory, and we measured the mud's particle size distribution
210 (Figure 2) using a LISST-Portable particle sizer. From our measured water
211 contents of ~ 40 wt% (Table 1), the mud density ρ_L is ~ 1.6 g/cm³ and the
212 solid volume fraction is ~ 0.37 assuming particle density 2.6 g/cm³. For
213 comparison, field studies at the Salton Sea between 2003 and 2010 recorded
214 mud densities of 1.4–1.7 g/cm³ for gryphons (the source of the samples used
215 here), and 1.1–1.3 g/cm³ for mud pools (Svensen et al., 2007; Mazzini et al.,
216 2011; Onderdonk et al., 2011).

217 The mud was stored in an airtight container prior to and in between
218 experiments. The same mud was reused between different experiments and
219 rehydrated with distilled water to vary rheology and offset evaporative wa-
220 ter loss. The amount of water added was not controlled, but water content
221 was determined post hoc for each experiment (see Section 3.5). We assume
222 that mud's solid composition (mineralogy, size distribution) does not change
223 throughout our experiments; we also neglect changes in water chemistry,
224 which may differ from field conditions due to our rehydration procedure. The
225 agreement between the mineralogy reported above, measured after the ex-
226 periments were performed (and more than a year after the mud samples were
227 collected), and previously reported mineralogy (Sturz et al., 1992; Rudolph
228 and Manga, 2010) indicates that no measurable change in mineralogy oc-
229 curred.

230 *3.2. Experimental apparatus*

231 We measured the velocities of rigid spheres and air bubbles moving through
232 a standing column of Salton Sea mud (Figure 3) in four distinct experi-
233 ments. The mud was contained in a hollow acrylic tube with inner diameter
234 $D = 13.9$ cm, sealed at the bottom. The maximum ratio of bubble or par-
235 ticle diameter to cylinder diameter in our experiments was 0.35 for bubbles
236 (assumed spherical) and 0.37 for rigid spheres.

237 At the start of each experiment, we mixed the mud using a paint stir-
238 rer and electric drill to a uniform consistency (typically ~ 10 minutes) and
239 loaded the mud into our apparatus. We recorded the mud column’s height
240 (h) with uncertainty ± 0.5 cm due to uneven surface morphology. We then
241 measured particle or bubble velocities as a function of volume and/or buoy-
242 ancy within a 12–24 hour period. As particle velocities depend on the mud
243 column’s deformation history, we limited time-dependent effects by repeat-
244 ing measurements at regular time intervals in well-mixed mud. The repeated
245 particle trajectories apply a consistent deformation and subsequent relax-
246 ation, allowing the mud column to approach some equilibrium state. Each
247 reported particle velocity averages the repeated measurements. To make
248 rheometer and water content measurements, we sampled $\gtrsim 10$ mL of mud
249 from just below the site of bubble bursting after the first ~ 10 particle tran-
250 sits through mud column, to ensure that the mud sample was representative
251 of the sheared mud encountered by our bubbles and rigid spheres. Taken
252 together, the measurements of mud height h , rheology and water content,
253 and particle velocities constitute one “experiment” as referred to in Table 1
254 and subsequent figures.

255 The initially well-mixed mud column develops heterogeneity due to gas
256 flow. At the column's surface, mud outside of the bubble burst zone in
257 Experiment 4 has yield stress ~ 16 Pa (15%) higher than that of mud at
258 the center of the column (Table 1). We suggest that over time, the bubbles
259 generate a conduit of mud that is continuously fluidized and recirculated by
260 bubble passage. Bubbles could ascend more easily along a locally weaker
261 path of bubble ascent, similar to behavior observed in Carbopol (Mougin
262 et al., 2012). This demonstrates the importance of sampling mobilized mud
263 for rheological characterization.

264 3.3. Sphere velocity measurements

265 For rigid particle measurements, we used stainless steel spheres (ABMA
266 grade 100, density 7.95 ± 0.03 g/cm³) attached to a counterweight by braided
267 polyester twine run over two pulleys (Figure 3b). Each sphere was glued to
268 a length of twine long enough that both sphere and counterweight traveled
269 freely when released. We filmed the counterweight's motion at 60 frames
270 per second to determine counterweight displacement as a function of time;
271 the resulting displacement track was fitted to a line or parabola to measure
272 terminal velocity or acceleration respectively.

273 We alternately dropped and raised each sphere at regular temporal inter-
274 vals. Each sphere was placed just below the mud's free surface and released
275 by hand. We recorded the counterweight's motion as the sphere fell. After
276 the sphere came to rest, we waited for a time interval Δt and then manually
277 raised the sphere, at a velocity comparable to its fall velocity, to its initial
278 position below the free surface. We waited for a time Δt and then released
279 the sphere again; $\Delta t \approx 30$ s was the shortest interval we could reasonably

280 achieve by hand.

281 We performed one experiment using 3 sphere sizes (0.041, 0.048, and
282 0.051 m diameter; or, 1 5/8, 1 7/8, and 2 inches), each with 3 differ-
283 ent counterweight masses between 0.01–0.13 kg (see Table 2). For each
284 sphere/counterweight combination considered, we performed at least 12 drops
285 at $\Delta t \approx 30$ s intervals and measured terminal velocities of the last 2 drops;
286 we averaged the two measurements. As noted above, we measured termi-
287 nal velocities from linear least squares fits to displacement tracks. To guide
288 the fitting, we also computed velocities from finite differences of displacement
289 data. We fitted the displacement track over a time interval when velocity was
290 relatively stable (i.e., avoiding acceleration/deceleration) and took velocity
291 error to be the maximum deviation between point-wise calculated velocity
292 and fitted velocity.

293 Each sphere’s effective weight was set by a variable counterweight mass
294 and reduced slightly by pulley friction. To account for friction, we mea-
295 sured the acceleration of various spheres and counterweights in air on our
296 pulley apparatus (Figure 3b, without the mud column), computed frictional
297 force loss by comparison to the expected acceleration for an ideal pulley, and
298 fitted measured force losses as a linear function of total suspended weight.
299 For each sphere/counterweight combination, we then computed an effective
300 density difference $\Delta\rho_{\text{eff}}$ (i.e., an effective weight up to factors of gV) that
301 was substituted for $\Delta\rho$ throughout our calculations. The frictional force loss
302 should not be subtracted from the sphere’s weight as a decrease in weight
303 also decreases drag in our experiments.

304 We also corrected sphere terminal velocities for wall drag using an empir-

305 ical relationship by Atapattu et al. (1990). For a sphere falling at velocity u
 306 in the presence of confining walls, the deviation from the terminal velocity
 307 in an unbounded medium, u_t , may be specified by $u = fu_t$ where $f \leq 1$.
 308 Atapattu et al. (1990) give the correction factor f as:

$$f = \begin{cases} 1, & d^*/D < (d^*/D)_{\text{crit}} \\ 11.7[(d^*/D) - (d^*/D)_{\text{crit}}], & d^*/D > (d^*/D)_{\text{crit}} \end{cases}$$

309 where D is containing cylinder radius and the critical diameter ratio is
 310 $(d^*/D)_{\text{crit}} = 0.055 + 1.114Y$. Although our sphere velocities are higher than
 311 those of Atapattu et al. (1990) ($\text{Re}^* \sim 1\text{--}10$ in mud with uncorrected veloc-
 312 ities $0.2\text{--}0.9$ m/s, vs. $\text{Re}^* \lesssim 0.2$ with $u \leq 0.08$ m/s), a wall-correction is
 313 physically reasonable and does not alter our conclusions.

314 3.4. Bubble velocity measurements

315 As we could not directly image or track bubbles in our mud column, we
 316 measured bubble ascent time t_a and approximated bubble terminal velocity
 317 (u_t) as equal to mean ascent velocity $\bar{u} = h'/t_a$, where $h' = (h - 2r - h_{\text{valve}})$;
 318 h' corrects mud column height h by subtracting check valve height h_{valve} and
 319 bubble radius r at top and bottom (Figure 3). We neglected the effect of
 320 bubble injection on mud height, as the displaced volume was comparable to
 321 or less than our uncertainty in h .

322 We manually injected bubbles into our mud column through a 60 mL
 323 syringe connected to a check valve centered on the bottom of the mud column
 324 tube (Figure 3a). Bubble volumes at 1 atm pressure were read off of syringe
 325 markings; to account for bubble decompression within the column, we used
 326 bubble volume halfway between top/bottom of the column with error one

327 half the difference of top/bottom bubble volumes (error ≤ 3.5 mL). We
328 recorded video (30 frames per second) of the apparatus from above so that the
329 mud column's surface and the injecting syringe below were simultaneously
330 visible. Bubble ascent time t_a was measured by counting frames between
331 complete depression of the syringe plunger and first disruption of the mud's
332 surface. Frame-counting uncertainty was ± 0.2 s, comparable to or smaller
333 than measurement scatter.

334 For each bubble volume considered (25–60 mL in intervals of 5 mL), we
335 injected a set of at least 15 bubbles at a regular time interval $\Delta t \approx 30$ s and
336 recorded ascent times of the last 10 bubbles. We averaged the 10 recorded
337 ascent times to compute velocity; velocity error was linearly propagated from
338 the standard deviation of ascent times. As for spheres, the time interval
339 between injections (previous burst to next bubble injection) was the shortest
340 interval we could reasonably achieve with manual bubble injection. Between
341 each set of $\gtrsim 15$ injections, we mixed the mud in situ using large (~ 60 mL)
342 bubbles and/or a bicycle pump to prevent suspended particles from settling.
343 Bubble volumes below ~ 25 mL did not rise consistently in our mud column
344 (within ~ 100 s, vs. typical ascent times of 1–10 s), so we could not measure
345 the velocities of nearly immobilized bubbles.

346 We performed three different experiments to measure bubble velocities
347 with different mud heights and mud water contents. We varied mud column
348 height in experiments 2 and 3 and performed the experiments within a three-
349 day period to hold mud composition and rheology approximately constant.
350 We varied mud rheology in experiment 4 by increasing the mud's bulk water
351 content.

352 Mean ascent velocity may deviate from terminal velocity due to edge
353 effects arising from 1) acceleration from injection to terminal velocity, 2)
354 bounded flow near the bottom of the mud column, and 3) deceleration and
355 deformation at the mud column's free surface. For a sufficiently tall column
356 (i.e., many bubble radii in height), edge effects should be independent of
357 height h . We could not quantify the effects of bounded flow conditions at top
358 and bottom of the mud column. However, the time taken to accelerate from
359 rest to terminal velocity is negligible and may be estimated from our empirical
360 drag coefficient scaling, which we subsequently present in equation (6), to
361 solve for time-dependent velocity. Bubbles in our experiments should reach
362 99% of terminal velocity within 1–2 milliseconds, ruling out one possible
363 source of error. Container walls may further suppress bubble velocities, but
364 in the absence of continuous bubble tracking we neglected wall effects in our
365 work.

366 The choice of time interval between injections (Δt) may also affect our
367 measured velocities. In Experiment 2, we collected additional data at $\Delta t =$
368 60 s and found a slight decrease ($\lesssim 10\%$) in ascent velocity u_t for 60 mL
369 bubbles injected $\Delta t = 60$ seconds apart; we could not discern any effect
370 within error for 30 mL bubbles.

371 3.5. Salton Sea mud rheology

372 We characterized the rheology of mud in our experiments using a Thermo-
373 Haake RheoScope 1 cone-plate rheometer with cone-plate angle 4° , cone di-
374 ameter 60 mm, and cone-plate gap 0.140 mm; for Experiment 2 only, the
375 cone-plate gap was ~ 0.3 mm. To prevent particle separation from the cone
376 surface, which leads to wall slip-like shear, we attached ANSI 150-grit sand-

377 paper (Orvis and Grissino-Mayer, 2002) to both cone and plate following
378 Magnin and Piau (1990). Measurements were made at 20–25 °C. To pre-
379 shear the sample, we applied a steady rotation (shear rate $\dot{\gamma} = 30 \text{ s}^{-1}$) for
380 10 s. Immediately after pre-shear, we measured shear stress (τ) as a func-
381 tion of shear rate ($\dot{\gamma}$) by stepwise increasing shear rate from 0.01 to 30 s^{-1} ,
382 then stepwise decreasing shear rate to 0.01 s^{-1} ; steps are logarithmically
383 spaced in shear rate. At each step, the mud sample was first sheared for
384 at least 3 seconds; shear stress was measured after either equilibrium was
385 reached ($\dot{\tau}/\tau < 0.05\%$), or 10 seconds elapsed. This procedure gave distinct
386 ascending and descending flow curves, implying some hysteresis despite the
387 pre-shear step.

388 Ascending and descending flow curves were each fitted to the Herschel-
389 Bulkley model with fit parameters τ_c , K , and n for shear rates within 1–
390 30 s^{-1} (Figure 4) using the Levenberg-Marquardt algorithm for nonlinear
391 least squares fitting (Press et al., 1992, chapter 15.5). We present and use
392 rheological parameters from the ascending (higher effective viscosity) curve
393 in subsequent analysis.

394 Water content was measured by drying mud samples in an oven at 93 °C
395 and measuring mass before and after complete drying. The uncertainty on
396 our sample mass measurement was $\sim 0.01 \text{ g}$, yielding a consistent 0.3% un-
397 certainty for our sample sizes of $\sim 5 \text{ g}$.

398 Shear stresses appear to decrease with increasing shear rates between
399 $0.1\text{--}1 \text{ s}^{-1}$ and hence our flow curves exhibit shear-stress minima at shear rate
400 $\sim 1 \text{ s}^{-1}$. Pignon et al. (1996) have shown that flow curve minima for clay
401 suspensions may be associated with incomplete sample deformation (i.e.,

402 shear localization); therefore, data at shear rates below $\sim 1 \text{ s}^{-1}$ may not
403 accurately describe sample deformation.

404 Table 1 summarizes our measurements of mud rheology. The natural
405 Salton Sea muds in our experiments have K ranging from 3 to 5 Pa s n , n
406 from 0.5 to 0.7, and τ_c from 90 to 130 Pa. τ_c is typically well-constrained,
407 whereas K and n are more uncertain and may concomitantly vary without
408 strongly altering the description of flow behavior. We also measured the
409 rheology of mud closer to the column walls (i.e., less mobile mud away from
410 the location of bubble bursting) at the end of Experiment 4 and found an
411 increased yield stress.

412 4. Motion of bubbles and rigid spheres

413 4.1. Velocity results

414 Table 2 presents measured sphere velocities with diameters and effective
415 densities. The measured sphere velocities are generally larger than measured
416 bubble velocities (Figure 5), but the values have comparable order of magni-
417 tude despite much larger density differences and slightly larger sphere sizes.
418 The velocity correction of Atapattu et al. (1990) increases our estimates of
419 terminal velocity in an unbounded medium by $\sim 15\text{--}50\%$.

420 Figure 5 plots measured bubble velocity as a function of volume for our
421 experiments. Experiments 2 and 3 agree within uncertainty, suggesting that
422 mud column height h has negligible effect on our results. Decreasing yield
423 stress qualitatively corresponds to increased velocities as expected; we are
424 unable to remark on trends in K or n .

425 *4.2. Dimensionless parameters*

426 Here we present our measurements in dimensionless form to characterize
 427 particle drag and immobilization in a yield stress fluid. We incorporate data
 428 collected by Sikorski et al. (2009) and Lavrenteva et al. (2009) on particles
 429 in Carbopol. The additional experimental data allow us to consider bubble
 430 motion over a larger parameter space (in particular, $Bi \sim 0.1\text{--}10$ and $Re^* \sim$
 431 $10^{-5}\text{--}10^0$).

432 Sikorski et al. (2009) measured the ascent velocities of air bubbles in Car-
 433 bopol at two different concentrations, with yield stresses of 24 and 34 Pa.
 434 Lavrenteva et al. (2009) measured the fall velocities of liquid tetrachloroethy-
 435 lene (hereafter, PCE) droplets in Carbopol of lower concentration with yield
 436 stresses of $\sim 2\text{--}5$. Sinking liquid droplets could deviate from expected behav-
 437 ior for buoyantly rising gas bubbles; however, the droplets may be treated
 438 as inviscid since the viscosity ratio between PCE and Carbopol is of order
 439 10^{-3} (assuming a typical shear rate of 1 s^{-1} and PCE viscosity 0.9 mPa s).
 440 As inertia is also unimportant ($Re^* < 1$, Figure 6), we conclude that droplet
 441 motion in Carbopol should be comparable to gas bubbles. We also consider
 442 only PCE/Carbopol data collected without a confining tube (labeled $R \rightarrow \infty$
 443 in Lavrenteva et al. (2009)).

444 The relevant dimensionless numbers vary greatly between the datasets
 445 considered. Our measurements in mud span $Re^* \sim 0.1\text{--}1$, whereas air/Carbopol
 446 measurements span $10^{-3}\text{--}1$ and PCE/Carbopol spans $10^{-5}\text{--}0.1$ (Figure 6).
 447 The values of Re span approximately the same range as Re^* in Carbopol
 448 but are $\sim 2\times$ larger for the smallest Re^* values. However, for mud we find
 449 $Re \sim 1\text{--}10$ is an order of magnitude larger. The deviation between Re^* and

450 Re is due to the Bingham number Bi, which is ~ 10 – 20 for air/mud and
451 ~ 0.3 – 3 for both sets of Carbopol measurements. In muds, the yield stress
452 strongly influences bubble ascent in all our measurements, whereas the yield
453 stress only influences air/PCE bubbles in Carbopol at very small Re^* . The
454 yield stress is important near the mobilization threshold for both mud and
455 Carbopol, as expected. We do not consider Re or Bi individually as our
456 expected drag scaling folds both numbers into Re^* .

457 The Bond number Bo ranges between 300–500 for air/mud, 10–200 for
458 air/Carbopol, and 6–30 for PCE/Carbopol. We have assumed surface tension
459 $\sigma = 0.073$ N/m for air/mud, $\sigma = 0.066$ N/m for air/Carbopol (Boujlel and
460 Coussot, 2013), and $\sigma = 0.044$ N/m for PCE/Carbopol (using the value
461 for PCE/water, following Lavrenteva et al. (2009)). In general, we do not
462 expect variation in Bo to greatly affect our results as the large values of Bo
463 imply that surface tension is relatively unimportant, though some bubbles in
464 Carbopol at small Re^* could be influenced by surface tension. Recalling that
465 Bo is relatively unimportant in creeping Newtonian flow, we do not consider
466 Bo variation in our drag analysis. We discuss the possibility of surface tension
467 dependence in our results further below.

468 4.3. Drag behavior

469 We plot C_D against Re^* in Figure 6 for rigid spheres and air bubbles in
470 Salton Sea mud, air bubbles in Carbopol (Sikorski et al., 2009), and tetra-
471 chloroethylene (PCE) drops in Carbopol (Lavrenteva et al., 2009). We apply
472 an empirical fit for shape (d_h) by Sikorski et al. (2009) to air bubbles in Car-
473 bopol when computing C_D ; we assume spherical shapes for all other data.
474 The values of $X(n)$ for all data considered range between 1.3–1.45. For rigid

475 spheres, we substitute an effective weight with corrections for counterweight
 476 mass and apparatus friction in place of $\Delta\rho gV$ throughout our equations (C_D
 477 and Y in particular), as previously noted.

478 Our spherical particle data (Experiment 1) are consistent with the drag
 479 empiricism $C_D = 24/\text{Re}^*$ (equation (2.2)) for rigid spheres in Herschel-
 480 Bulkley fluids (Atapattu et al., 1995; Tabuteau et al., 2007a). The reduced χ^2
 481 is 0.51 (degrees of freedom equal to number of data as there are no model pa-
 482 rameters to vary). Without the velocity correction of Atapattu et al. (1990),
 483 a reduced $\chi^2 = 1.20$ still suggests reasonable agreement. The agreement
 484 between data and empiricism gives us confidence that our procedure cap-
 485 tures mud rheology and particle dynamics appropriately, despite stronger
 486 time-dependent effects as compared to Carbopol.

487 Bubbles in Herschel-Bulkley fluids appear to obey a drag scaling of similar
 488 functional form ($C_D \propto 1/\text{Re}^*$). To account for inertial effects, we introduce
 489 a factor $(1 + 3\text{Re}^*/16)$ imitating the Oseen correction in Newtonian fluids
 490 (Oseen, 1910; Clift et al., 1978). This correction causes the drag curve to
 491 level off at $C_D = 3/16$ for large Re^* ; the constant $3/16$ is derived from the drag
 492 force on a rigid sphere computed with a linearized inertial term in the Navier-
 493 Stokes equations (i.e., Oseen’s equation). Although the Oseen correction
 494 applies to rigid spheres and not bubbles, the additional term is physically
 495 reasonable and accords with similar empiricisms in power-law fluids (e.g.,
 496 Dewsbury et al., 1999). We thus fit our data to the model:

$$C_D = \frac{A}{\text{Re}^*} \left(1 + \frac{3}{16}\text{Re}^* \right) \quad (6)$$

497 where A is a free fitting parameter. For each of our experiments 2–4 and the
 498 two Carbopol datasets of Sikorski et al. (2009), we performed unweighted

499 least squares fits of equation (6) to the individual datasets in log-log space
 500 and obtained five values of A between 4.6 and 6.2. The mean of five fitted
 501 values of A is 5.4; we fit individual experiments' data separately because a
 502 combined fit of all data together would be heavily weighted by the amount
 503 of data for air bubbles in Carbopol. Figure 6 plots equation (6) with $A = 5$.

504 We propose an approximate drag relation $C_D = 5(1 + 3\text{Re}^*/16)/\text{Re}^*$ for
 505 bubbles in the range $\text{Re}^* = 0.001$ –1. The drag relation's coefficient of de-
 506 termination r^2 computed in log-log space is 0.996 for the Carbopol data of
 507 Sikorski et al. (2009) and 0.922 for the mud data of Experiments 2–4. We do
 508 not attempt to capture the PCE droplet drag with this approximate relation.
 509 Fits of equation (6) to PCE droplet data favor $A \sim 10$ and do not capture
 510 the slightly steeper relation between C_D and Re^* visible in the PCE data
 511 (i.e., a relation of form $C_D \propto \text{Re}^{*-s}$ with $s > 1$ may better fit the PCE data).

512 Our drag scalings should reduce to appropriate empiricisms for parti-
 513 cles in power-law, Bingham, and Newtonian fluids in appropriate limiting
 514 cases. The spherical particle drag scaling $C_D = 24/\text{Re}^*$ correctly gives
 515 $C_D = 24X(n)/\text{Re}$ for $\tau_c = 0$ (Crochet et al., 1984) and the Stokes solu-
 516 tion $C_D = 24/\text{Re}$ for $X(n = 1) = 1$. However, our empiricism for bubbles
 517 does not match the drag scaling $C_D = 16/\text{Re}$ for shear-thinning power-law
 518 fluids (Miyahara and Yamanaka, 1993; Dewsbury et al., 1999; Margaritis
 519 et al., 1999) when $\tau_c = 0$. The expected $C_D = 16/\text{Re}^*$ moreover falls outside
 520 of the error bars for all bubble data considered, mud and Carbopol.

521 The mismatch of drag empiricisms, with $A = 5$ rather than $A = 16$,
 522 results in predicted bubble velocities at least ~ 2 – $3\times$ larger than expected
 523 from approximating the fluid as Newtonian with effective viscosity μ_{eff} (equa-

524 tion (3)), with increasing discrepancy as $\text{Re}^* \rightarrow 0$. In a Newtonian approxi-
 525 mation, the Stokes velocity is given implicitly by:

$$u_t = \frac{\Delta\rho g (d^*)^2}{12\mu_{\text{eff}}}. \quad (7)$$

526 This result is equivalent to taking $A = 16$ in the drag empiricism (equa-
 527 tion (6)) and $k = 1$ in the definition of Re^* (equation (5)). Figure 7 illustrates
 528 the discrepancy between predictions using rheological parameters compara-
 529 ble to those appropriate for a basaltic magma. For Salton Sea muds, the
 530 Newtonian approximation predicts that the bubbles considered (Figure 5)
 531 should be immobile.

532 The lack of correspondence between our drag scaling and the accepted
 533 drag scaling for power-law fluids may be associated with the onset of yielding
 534 behavior. The PCE droplets of Lavrenteva et al. (2009) encounter increased
 535 drag compared to bubbles and droplets at larger Re^* ; this effect is clearest
 536 for bubbles in 0.07 wt% concentration Carbopol (black squares), which has
 537 a yield stress almost 2 orders of magnitude smaller than the yield stresses of
 538 our Salton Sea mud. One possible explanation is that in a yield stress fluid,
 539 only material immediately adjacent to an ascending bubble is deformed and
 540 mobilized, permitting more rapid ascent if less energy is dissipated into the
 541 surrounding medium. But, if so, we would expect a similar drag decrease for
 542 solid particles.

543 4.4. *Yielding behavior*

544 With a practical model of yielding (cf. Barnes, 1999), we expect that there
 545 exists a critical value of the yield parameter (Y_c) above which particles will
 546 be immobile. Our drag coefficient scaling, equation (6), naturally yields a

547 yielding criterion. Dropping the Oseen-like correction factor, which becomes
 548 unimportant at vanishing Re^* , we may solve $C_D = A/\text{Re}^*$ to obtain an
 549 expression for terminal velocity:

$$u_t = d_h \left[\frac{1}{K} \left(\frac{\frac{4}{3}\Delta\rho g d^*}{AX(n)} \left(\frac{d^*}{d_h} \right)^2 - k\tau_c \right) \right]^{1/n} \quad (8)$$

550 Requiring that terminal velocity $u_t > 0$ yields the criterion:

$$Y = \frac{3\tau_c}{\Delta\rho g d^*} \left(\frac{d_h}{d^*} \right)^2 < \frac{4}{kAX(n)}; \quad (9)$$

551 particles should then be immobilized for $Y > Y_c = 4/(kAX(n))$. In Figure 8,
 552 we plot Re^* as a function of Y . As in Figure 6, we apply an empirical fit
 553 for shape (d_h) to air bubbles in Carbopol when computing Y (Sikorski et al.,
 554 2009) and assume spherical shapes for all other data.

555 Our rigid sphere data (Figure 8b) are qualitatively consistent with the
 556 commonly cited $Y_c = 0.143$ (Beris et al., 1985; Blackery and Mitsoulis, 1997;
 557 Tabuteau et al., 2007a). We note that $Y_c = 0.143$ is not a strict criterion for
 558 motion, as slow creeping flow may occur for $Y > Y_c$ (Atapattu et al., 1995;
 559 Beaulne and Mitsoulis, 1997).

560 We do not observe a singular yield criterion for bubble motion in Herschel-
 561 Bulkley fluids (Figure 8a). Predictions of Y_c for bubbles have been obtained
 562 from variational analysis (Dubash and Frigaard, 2004) and numerical simula-
 563 tions (Tsamopoulos et al., 2008; Dimakopoulos et al., 2013). But, air bubbles
 564 in mud and Carbopol are mobile for larger values of Y than expected for all
 565 but the most conservative predictions, as previously observed by Sikorski
 566 et al. (2009) (e.g., Dubash and Frigaard (2004) give $Y_c = \sqrt{3}/2 = 0.866$ for
 567 a spherical bubble). Figure 8 plots $Y_c = 0.705$ for bubbles computed from

568 equation (9) with $A = 5$ and $X(n = 0.6) = 1.38$. Although this criterion
569 cannot account for variation in yielding behavior, it may provide a lower
570 bound or constraint for any more general yielding criterion.

571 Our analysis has neglected bubble shape (d_h/d^*) and surface tension (Bo),
572 but we argue that neither parameter can explain the observed spread in drag
573 and yielding behavior. Shape measurements could change the data plotted in
574 Figures 6 and 8. However, Sikorski et al. (2009) observed decreased deforma-
575 tion with larger bubble size and our mud bubbles are mostly larger than those
576 of Sikorski et al. (2009), suggesting a relatively small effect (within a factor
577 of 2). The PCE/Carbopol bubbles were generally teardrop shaped, which
578 would decrease Y (leftward shift on Figure 8) if shape were accounted for.
579 Thus shape cannot explain the varying yield behavior in all three datasets
580 we consider.

581 Surface tension is likely not a primary control on bubble shape at small
582 Re^* . Observed bubble shapes in Carbopol are consistently aspherical and
583 teardrop shaped, and Sikorski et al. (2009) reported increasing deforma-
584 tion with smaller bubble volume and hence smaller Re^* , larger Bo. Cusped
585 bubbles are reproduced in simulations considering viscoelastic effects (Algar
586 et al., 2011) but not in purely viscoplastic simulations (Beris et al., 1985;
587 Tsamopoulos et al., 2008; Dimakopoulos et al., 2013), so both yield stresses
588 and elasticity may be important in setting bubble shape and hence deforma-
589 tion and yielding. More detailed studies with low yield stress fluids may
590 explain our observations and, ideally, collapse the dimensionless drag and
591 yield data to a single scaling.

592 5. Geologic applications

593 Table 3 compiles both in situ and laboratory rheological measurements of
 594 mud and basaltic magma rheology from several active volcanoes. For mag-
 595 matic volcanoes, we assume magma density 2700 kg/m^3 and take $n = 1$ where
 596 the power-law index n is unknown or may not be meaningful. From rheolog-
 597 ical parameters we calculate the minimum diameters of mobile bubbles from
 598 the yield criterion, equation (9); we also calculate the terminal velocities of
 599 mobile bubbles with twice the minimum mobile volume using our drag em-
 600 piricism, equation (6), with $A = 5$. In the low Re^* limit, equation (8) agrees
 601 with equation (6) quite well; only velocities of $\sim 0.1\text{--}1 \text{ m/s}$ in Table 3 are
 602 affected by the Oseen-like correction. Several mobile bubble velocity values
 603 are extremely small, corresponding to materials where $K \gg \tau_c$ or where τ_c
 604 is quite small ($\lesssim 1 \text{ Pa}$). These velocities correspond to comparatively small
 605 Re^* values where our drag empiricism may not be applicable, and should be
 606 taken only as a rough proxy for bubble behavior near the yielding threshold.

607 5.1. Mud volcanoes

608 At the Salton Sea, based on gas flux measurements (Rudolph and Manga,
 609 2012) and observations of the growth of mounds of mud, we infer that com-
 610 parable masses of gas, water and solid particles are discharged at the surface.
 611 These mounds are essentially gas vents and the cones are built largely by the
 612 spatter produced when bubbles burst at the surface of the pools of mud (see
 613 video in the supplement); the processes in the field thus resemble our lab
 614 experiments. Svensen et al. (2009) suggested two conceptual models for gas
 615 ascent, one in which hot gas from depth is the carrier of heat, and one in

616 which hot mud is circulated from depth by ascending exsolved gas (Figure 9).
617 Svensen et al. (2009) favored the second model, on the qualitative basis that
618 mud could transport heat more effectively than gas alone, and the mud at
619 the surface is hot (in excess of 70 °C at some vents). At the surface we
620 observe ~ 10 cm diameter bubbles (e.g., Figure 1). Using the measured rhe-
621 ology, ~ 3 cm diameter bubbles are large enough to be mobilized (Table 3).
622 If we make three assumptions – neglect coalescence, assume constant tem-
623 perature, and ignore growth by gas exsolution (i.e., consider only isothermal
624 expansion) – the 10 cm bubbles at the surface would be ~ 3 cm at depths
625 of about 200 m, consistent with the favored model of Svensen et al. (2009).
626 However, as we will see next, at other mud volcanoes the observed size of
627 bubbles suggests that some combination of these assumptions may not be
628 justified.

629 The Italian mud volcanoes studied by Manga and Bonini (2012) have bub-
630 bles of diameter ~ 2 to 10 cm bursting at their vents and lower yield stresses
631 than Salton Sea mud. Again, accounting for only isothermal expansion of
632 the bubbles, the mobilization depth is at least several km, greater than the
633 inferred source of the mud. Bubble coalescence or growth by exsolution are
634 necessary to create the observed sizes.

635 Large bubbles, up to 3 m in diameter, also burst at Lumpur Sidoarjo
636 (Lusi), an active mud eruption in East Java, Indonesia (Vanderkluyesen et al.,
637 2014). The erupted mud contains kaolinite, smectite, and illite; water content
638 was ~ 70 wt% at the beginning of the eruption but has decreased to ~ 30 wt%
639 (Mazzini et al., 2007; Istadi et al., 2009). The corresponding solid volume
640 fractions are 0.14–0.47. Rifai (2008) measured (effective) Newtonian viscosi-

641 ties ~ 1 Pa s for Lusi mud with 60 wt% water; this suggests that the most fluid
642 erupted muds should have yield stress < 10 – 100 Pa. In Table 3, we estimate
643 end-member rheological properties of erupted mud at Lusi from laboratory
644 rheometry of natural mud suspensions and mudflow deposits (O’Brien and
645 Julien, 1988; Coussot and Piau, 1994), assuming that suspensions of similar
646 solid volume fractions are comparable to Lusi muds. Because the mud origi-
647 nates at depths of ~ 1.5 km (Mazzini et al., 2007), large bubbles such as those
648 seen at the surface require some combination of coalescence and exsolution
649 during ascent.

650 5.2. *Magmatic volcanoes*

651 For the reported yield stresses, basaltic magmas should be able to immo-
652 bilize bubbles with diameters of several mm to several cm. The minimum
653 diameters in Table 3 are obtained with $A = 5$ in our drag and yielding
654 empiricism (equations (6,9)).

655 Crystals are $10\times$ smaller than the typical mobile bubble size of 1 cm. The
656 largest crystals in the tabulated basaltic melts are ~ 1 mm, up to ~ 4 mm
657 for the 1975 Etna lava flow (Pinkerton and Sparks, 1978; Castruccio et al.,
658 2014). Ryerson et al. (1988) did not report crystal sizes in their viscometer
659 experiments, but the relevant crystals should not be larger than a few mm.
660 Although the largest crystals are comparable to the smallest mobile bubbles,
661 bubbles moving appreciably will be larger still and should interact with the
662 surrounding melt as a continuum.

663 Basalt measurements have much larger consistency K compared to muds,
664 both in absolute magnitude and in comparison to the yield stress τ_c (i.e, typ-
665 ical Bi larger). Thus, the yield stress is negligible for magmas in most typical

666 flow settings (say, $Re^* \sim 1$), as evidenced by many rheological measurements
667 of lavas that do not identify a yield stress. But, K does not impact the
668 yielding criterion (derived for vanishing Re^* and hence $Bi \rightarrow 0$).

669 *5.3. Mobility in Stromboli's conduit*

670 Here we apply our drag and yield empiricisms to bubbles in Stromboli's
671 conduit. Strombolian eruptions are commonly modeled as being caused by
672 bursting gas slugs at the surface (Blackburn et al., 1976; Lane et al., 2013),
673 but the nature of slug formation and ascent remains uncertain (see James
674 et al., 2011; Suckale et al., 2011). Two dominant models for slug forma-
675 tion are bubble coalescence in ascending magma (Wilson and Head, 1981;
676 Parfitt and Wilson, 1995) and foam collapse in magma reservoirs (Jaupart
677 and Vergnolle, 1988, 1989); both models are reviewed by Parfitt (2004). An
678 alternative model disfavors slug bursting as the cause of Strombolian ex-
679 plosions, suggesting that gas ascent is hindered by viscous and crystal-rich
680 magma near the conduit surface (Belien, 2011; Gurioli et al., 2014) and that
681 slugs may not rise stably in Stromboli's conduit (Suckale et al., 2010).

682 The mobility of bubbles in magmas with a yield stress may affect whether
683 slugs can coalesce at shallow depths in volcanic conduits. Magma deforma-
684 tion or decompression can cause film rupture and hence coalescence in both
685 mobile and immobile bubbles, particularly near the conduit surface or walls
686 (e.g., Shields et al., 2014; Namiki and Kagoshima, 2014). But, mobile bubbles
687 may also coalesce by hydrodynamic interactions depending on their size and
688 velocity distribution (Manga and Stone, 1995). A yield stress in the upper
689 conduit may affect large slugs as well, but the motion of large conduit-filling
690 slugs falls outside the scope of this work.

691 We compute ascent velocities for bubbles between 0–3 km in a model of
 692 Stromboli’s conduit with depth-dependent rheology. Throughout, we assume
 693 hydrostatic pressure with magma density 2700 kg/m^3 . We assume a Bing-
 694 ham rheology ($n = 1$) and adopt depth-dependent consistency K and crystal
 695 volume fraction ϕ computed by Beckett et al. (2014) for ascending magma
 696 in an exchange flow model of Stromboli’s conduit. To obtain consistency,
 697 crystal fraction, and other rheological parameters, Beckett et al. (2014) com-
 698 puted crystallization and volatile degassing pathways using MELTS (Ghiorso
 699 and Sack, 1995; Asimow and Ghiorso, 1998) and VolatileCalc (Newman and
 700 Lowenstern, 2002) based on the composition of volatile-rich, low phenocryst
 701 (LP) pumice erupted in one of Stromboli’s “paroxysmal” eruptions (Métrich
 702 et al., 2010). For the yield stress, we use an empiricism obtained by Castruc-
 703 cio et al. (2014) for lava flow samples from Mount Etna:

$$\tau_c(\phi) = \begin{cases} (1.4 \times 10^9 \text{ Pa}) (\phi - \phi_c)^8, & \phi > \phi_c \\ 0, & \phi < \phi_c \end{cases} \quad (10)$$

704 The crystal volume fraction at which a yield stress first develops, ϕ_c , de-
 705 pends strongly on crystal size distribution and aspect ratio; we fix $\phi_c = 0.29$,
 706 which falls in the range of typical reported values (~ 0.2 – 0.4) (Garboczi et al.,
 707 1995; Hoover et al., 2001; Saar et al., 2001; Mueller et al., 2010). Conduit
 708 parameters are plotted against depth in Figure 10.

709 Empiricisms and scalings for yield stress as a function of particle volume
 710 fraction, sizes, and shape (Gay et al., 1969; Wildemuth and Williams, 1984;
 711 Ryerson et al., 1988; Zhou et al., 1995; Heymann et al., 2002; Castruccio
 712 et al., 2010; Mueller et al., 2010; Castruccio et al., 2014) give a wide range of
 713 predictions, which differ by up to 1–2 orders of magnitude (e.g., Castruccio

714 et al., 2010, Figure 6). It is also unclear whether a yield stress will form
 715 in the model of Beckett et al. (2014), which fixes vesicularity at 50% above
 716 75 MPa to permit open-system degassing and thus lowers the maximum
 717 crystal fraction to ~ 22 vol.% in ascending melt, inclusive of both bubbles
 718 and crystals. But, magma mingling at the surface and during ascent (Landi
 719 et al., 2004; Lautze and Houghton, 2005, 2007) and segregation of gas slugs
 720 and degassed melt may increase crystal fraction and suppress vesicularity in
 721 melt encountered by the bubbles we consider here. Despite the uncertainty
 722 in conduit conditions, a yield stress can be relevant to Stromboli for the
 723 crystal fractions considered. Our exact results are sensitive to the assumed
 724 value of ϕ_c and the choice of yield stress scaling (equation (10)), but our
 725 qualitative conclusions are reasonable for yield stresses of 10^2 – 10^3 Pa in the
 726 shallow conduit.

727 Figure 11 plots bubble velocities in the conduit for (1) arbitrary bubble
 728 size and depth, and (2) nucleating 1 cm diameter bubbles at each depth,
 729 ascending and expanding isothermally with no overpressure, continued ex-
 730 solution, or wall/bubble interaction. A 1 cm bubble is approximately the
 731 maximum post-nucleation bubble size expected from diffusive growth alone
 732 (Proussevitch and Sahagian, 1996; Suckale et al., 2010). And, bubbles with
 733 $d \gtrsim 1$ cm should interact with the surrounding suspension as a continuum,
 734 as the largest crystals observed in bombs and scoriae are ~ 1 – 10 mm (Lautze
 735 and Houghton, 2007).

736 Velocities of mm to cm diameter bubbles are typically $\lesssim 1\%$ of ascending
 737 magma velocity, ~ 0.1 m/s (Beckett et al., 2014), so bubbles smaller than
 738 cm size are entrained with the melt and immobilized near the conduit sur-

739 face. Although our continuum melt assumption may break down for mm size
 740 bubbles, low bubble velocities are consistent with observations of mm to cm
 741 sized bubbles in Strombolian ejecta. The imposed increases in both yield
 742 stress and consistency (Figure 10) retard bubble motion over a few hundred
 743 meters. A 1 cm diameter bubble nucleated at $\lesssim 0.9$ km depth will decelerate
 744 and be immobilized, but may remobilize due to decompression within tens
 745 of meters of the surface and outgas.

746 A strong rheological gradient in consistency (viscosity) and yield stress
 747 can inhibit coalescence of mm to cm size bubbles in Stromboli's shallow
 748 conduit. To travel through our model rheological gradient without being
 749 immobilized, bubbles must have $d \gtrsim 2$ cm, whether by forming at depths
 750 $\gtrsim 0.9$ km or by coalescence between 1–3 km. In general, yield stresses
 751 $\tau_c \sim 10^2$ to 10^3 Pa in the upper ~ 1 km can strongly slow or immobilize
 752 bubbles with $d < 0.01$ to 0.1 m. For comparison, the same magma without
 753 a yield stress ($\tau_c = 0$ throughout) permits bubbles of mm to cm size to
 754 ascend at $\sim 10^{-4}$ to 10^{-3} m/s through the shallow conduit. These velocities,
 755 although slow, permit appreciable movement over the timescale of magma
 756 ascent. Assuming bulk ascent velocity 0.1 m/s, a bubble of velocity 10^{-4} m/s
 757 may travel 1 m in the time taken for the surrounding magma to ascend 1 km.

758 By inhibiting coalescence, a rheological gradient can naturally give rise
 759 to two regimes of bubble behavior. Large bubbles with $d \sim 0.1$ m at depths
 760 > 0.9 km are mobile with respect to the melt and may penetrate through
 761 the rheological gradient – whether as a coherent slug, a localized packet or
 762 wave of bubbles, or by fracturing through a viscous plug. Smaller bubbles
 763 at depth that could not grow or coalesce above ~ 1 –2 cm will be gradually

764 immobilized as the entraining magma develops a yield stress in the shallow
765 conduit. Bubbles nucleating in the shallow conduit will be immobilized im-
766 mediately and may not coalesce with larger bubbles from depth due to their
767 disparate velocities and sizes (Manga and Stone, 1995), consistent with the
768 inference that gas from normal Strombolian eruptions is likely sourced from
769 $\sim 0.8\text{--}3$ km (Burton et al., 2007a).

770 The population of immobilized bubbles may remobilize within tens of me-
771 ters of the conduit surface and contribute to passive and “puffing” degassing
772 at Stromboli (see Ripepe et al., 2002; Harris and Ripepe, 2007). A model of
773 passive degassing driven by immobilized, melt-entrained bubbles would be
774 inconsistent with open-system degassing in the top 1.8–3.6 km of conduit
775 at Stromboli, suggested by Burton et al. (2007b) on the basis of computed
776 exsolution paths for H_2O and CO_2 . Large bubbles at depth must account
777 for the bulk of the required open-system degassing and volatile loss, and/or
778 bubbles of sub-mm size must percolate efficiently between crystals to degas
779 magma at depths $\gtrsim 2$ km.

780 Although we cannot favor or disfavor a model for Strombolian eruptions,
781 the increase in consistency and yield stress near the surface could stabilize
782 large slugs against breakup (Suckale et al., 2010). The deceleration of mobile
783 bubbles at the base of the rheological gradient may also allow gas accumula-
784 tion or expedite coalescence, which could be released through foam collapse
785 (Jaupart and Vergnolle, 1988) or plug failure (Barberi et al., 1993; Gurioli
786 et al., 2014)

787 6. Conclusion

788 Bubbles in yield stress fluids are more mobile than expected based on
789 previous work on bubbles in power-law fluids and rigid particles in yielding
790 fluids. We propose an empirical drag correlation $C_D = 5(1 + 3\text{Re}^*/16)/\text{Re}^*$
791 for bubbles in yield stress fluids based on our measurements of bubble veloc-
792 ities in mud from mud volcanoes near the Salton Sea in southern California
793 and previous measurements of bubble velocities in the polymer gel Carbopol
794 (Sikorski et al., 2009; Lavrenteva et al., 2009). Using our aggregated datasets,
795 we also bound a potential yielding criterion for bubbles in yield stress fluids,
796 although we do not find consistent dynamical behavior between the data we
797 consider.

798 We employ our drag and yielding empiricism to determine minimum mo-
799 bile bubble sizes in natural muds and basaltic magmas. In particular, using
800 a model for Stromboli's conduit rheology by Beckett et al. (2014), we find
801 that mm to cm size bubbles in Stromboli's shallow conduit could plausibly
802 be immobilized by the onset and increase of a yield stress towards the con-
803 duit surface. The yield stress may thus restrict coalescence at shallow depths
804 ($\lesssim 1$ km).

805 Our results are obtained for well-circulated mud with moderate water
806 content, which we have implicitly assumed to deform plastically in response
807 to bubble ascent. This assumption may not apply to well-settled muds or
808 other materials with significant elasticity (Boudreau et al., 2005; Algar et al.,
809 2011). Other studies have suggested that a standing discrepancy between
810 numerically simulated and observed bubble shapes in Herschel-Bulkley fluids
811 may be due to elastic effects (e.g., Dimakopoulos et al., 2013). We may

812 expect a continuum of behavior in our experiments as water content and mud
813 composition are varied. Further studies of bubble shape, shear localization,
814 and more sophisticated rheological models (Balmforth et al., 2014) in muds
815 and magmas will elucidate the effect of fluid yielding on bubble dynamics.

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826 **References**

- 827 Algar, C.K., Boudreau, B.P., Barry, M.A., 2011. Initial rise of bubbles in
828 cohesive sediments by a process of viscoelastic fracture. *J. Geophys. Res.*
829 116. doi:10.1029/2010JB008133.
- 830 Ansley, R.W., Smith, T.N., 1967. Motion of spherical particles in a Bingham
831 plastic. *AIChE J.* 13, 1193–1196. doi:10.1002/aic.690130629.
- 832 Asimow, P.D., Ghiorso, M.S., 1998. Algorithmic modifications extending
833 MELTS to calculate subsolidus phase relations. *American Mineralogist*
834 83, 1127–1131.

- 835 Atapattu, D.D., Chhabra, R.P., Uhlherr, P.H.T., 1990. Wall effect for spheres
836 falling at small Reynolds number in a viscoplastic medium. *J. Non-Newt.*
837 *Fluid Mech.* 38, 31–42. doi:10.1016/0377-0257(90)85031-S.
- 838 Atapattu, D.D., Chhabra, R.P., Uhlherr, P.H.T., 1995. Creeping sphere
839 motion in Herschel-Bulkley fluids: flow field and drag. *J. Non-Newt. Fluid*
840 *Mech.* 59, 245–265. doi:10.1016/0377-0257(95)01373-4.
- 841 Balmforth, N.J., Frigaard, I.A., Ovarlez, G., 2014. Yielding to stress: Recent
842 developments in viscoplastic fluid mechanics. *Annu. Rev. Fluid Mech.* 46,
843 121–146. doi:10.1146/annurev-fluid-010313-141424.
- 844 Barberi, F., Rosi, M., Sodi, A., 1993. Volcanic hazard assessment at Strom-
845 boli based on review of historical data. *Acta Vulcanol.* 3, 173–187.
- 846 Barnes, H.A., 1999. The yield stress—a review or ‘*παντα ρει*’—everything
847 flows? *J. Non-Newt. Fluid Mech.* 81, 133–178. doi:10.1016/
848 S0377-0257(98)00094-9.
- 849 Beaulne, M., Mitsoulis, E., 1997. Creeping motion of a sphere in tubes
850 filled with Herschel-Bulkley fluids. *J. Non-Newt. Fluid Mech.* 72, 55–71.
851 doi:10.1016/S0377-0257(97)00024-4.
- 852 Beckett, F.M., Burton, M., Mader, H.M., Phillips, J.C., Polacci, M., Rust,
853 A.C., Witham, F., 2014. Conduit convection driving persistent degassing
854 at basaltic volcanoes. *J. Volcanol. Geotherm. Res.* 283, 9–35. doi:10.1016/
855 j.jvolgeores.2014.06.006.

- 856 Bekkour, K., Leyama, M., Benchabane, A., Scrivener, O., 2005. Time-
857 dependent rheological behavior of bentonite suspensions: an experimental
858 study. *J. Rheol.* 49, 1329–1345. doi:10.1122/1.2079267.
- 859 Belien, I.B., Cashman, K.V., Rempel, A.W., 2010. Gas accumulation
860 in particle-rich suspensions and implications for bubble populations in
861 crystal-rich magma. *Earth Planet. Sci. Lett.* 297, 133–140. doi:10.1016/
862 j.epsl.2010.06.014.
- 863 Belien, I.L.M.B., 2011. Gas migration through crystal-rich mafic volcanic
864 systems and application to Stromboli volcano, Aeolian Islands, Italy. Ph.D.
865 thesis. Univ. Oregon. Eugene, Oregon, USA.
- 866 Beris, A.N., Tsamopoulos, J.A., Armstrong, R.C., Brown, R.A., 1985. Creep-
867 ing motion of a sphere through a Bingham plastic. *J. Fluid Mech.* 158,
868 219–244. doi:10.1017/S0022112085002622.
- 869 Blackburn, E.A., Wilson, L., Sparks, R.S.J., 1976. Mechanisms and dynamics
870 of strombolian activity. *J. Geol. Soc.* 132, 429–440. doi:10.1144/gsjgs.
871 132.4.0429.
- 872 Blackery, J., Mitsoulis, E., 1997. Creeping motion of a sphere in tubes filled
873 with a Bingham plastic material. *J. Non-Newt. Fluid Mech.* 70, 59–77.
874 doi:10.1016/S0377-0257(96)01536-4.
- 875 Boudreau, B.P., Algar, C., Johnson, B.D., Croudace, I., Reed, A., Fu-
876 rukawa, Y., Dorgan, K.M., Jumars, P.A., Grader, A.S., Gardiner, B.S.,
877 2005. Bubble growth and rise in soft sediments. *Geology* 33, 517–520.
878 doi:10.1130/G21259.1.

- 879 Boujlel, J., Coussot, P., 2013. Measuring the surface tension of yield stress
880 fluids. *Soft Matter* 9, 5898–5908. doi:10.1039/C3SM50551K.
- 881 Burton, M., Allard, P., Muré, F., La Spina, A., 2007a. Magmatic gas com-
882 position reveals the source depth of slug-driven Strombolian explosive ac-
883 tivity. *Science* 317, 227–230. doi:10.1126/science.1141900.
- 884 Burton, M.R., Mader, H.M., Polacci, M., 2007b. The role of gas percolation in
885 quiescent degassing of persistently active basaltic volcanoes. *Earth Planet.*
886 *Sci. Lett.* 264, 46–60. doi:10.1016/j.epsl.2007.08.028.
- 887 Caricchi, L., Burlini, L., Ulmer, P., Gerya, T., Vassalli, M., Papale, P., 2007.
888 Non-Newtonian rheology of crystal-bearing magmas and implications for
889 magma ascent dynamics. *Earth Planet. Sci. Lett.* 264, 402–419. doi:http:
890 //dx.doi.org/10.1016/j.epsl.2007.09.032.
- 891 Cashman, K.V., Mangan, M.T., Newman, S., 1994. Surface degassing and
892 modifications to vesicle size distributions in active basalt flows. *J. Volcanol.*
893 *Geotherm. Res.* 61, 45–68. doi:10.1016/0377-0273(94)00015-8.
- 894 Castruccio, A., Rust, A., Sparks, R., 2014. Assessing lava flow evolution
895 from post-eruption field data using Herschel-Bulkley rheology. *J. Volcanol.*
896 *Geotherm. Res.* 275, 71–84. doi:10.1016/j.jvolgeores.2014.02.004.
- 897 Castruccio, A., Rust, A.C., Sparks, R.S.J., 2010. Rheology and flow of
898 crystal-bearing lavas: Insights from analogue gravity currents. *Earth*
899 *Planet. Sci. Lett.* 297, 471–480. doi:10.1016/j.epsl.2010.06.051.
- 900 Chevrel, M.O., Platz, T., Hauber, E., Baratoux, D., Lavallée, Y., Ding-
901 well, D.B., 2013. Lava flow rheology: A comparison of morphologi-

- 902 cal and petrological methods. *Earth Planet. Sci. Lett.* 384, 109–120.
903 doi:10.1016/j.epsl.2013.09.022.
- 904 Clift, R., Grace, J.R., Weber, M.E., 1978. *Bubbles, Drops, and Particles*.
905 Academic Press, New York.
- 906 Costa, A., 2005. Viscosity of high crystal content melts: Dependence on solid
907 fraction. *Geophys. Res. Lett.* 32. doi:10.1029/2005GL024303.
- 908 Coussot, P., Piau, J.M., 1994. On the behavior of fine mud suspensions.
909 *Rheol. Acta* 33, 175–184. doi:10.1007/BF00437302.
- 910 Crochet, M.J., Davies, A.R., Walters, K., 1984. *Numerical Simulation of*
911 *Non-Newtonian Flow*. Elsevier, Amsterdam.
- 912 Dewsbury, K., Karamanev, D., Margaritis, A., 1999. Hydrodynamic char-
913 acteristics of free rise of light solid particles and gas bubbles in non-
914 Newtonian liquids. *Chem. Eng. Sci.* 54, 4825–4830. doi:10.1016/
915 S0009-2509(99)00200-6.
- 916 Dimakopoulos, Y., Pavlidis, M., Tsamopoulos, J., 2013. Steady bubble rise in
917 Herschel-Bulkley fluids and comparison of predictions via the Augmented
918 Lagrangian Method with those via the Papanastasiou model. *J. Non-Newt.*
919 *Fluid Mech.* 200, 34–51. doi:10.1016/j.jnnfm.2012.10.012.
- 920 Dubash, N., Frigaard, I., 2004. Conditions for static bubbles in viscoplastic
921 fluids. *Phys. Fluids* 16, 4319–4330. doi:10.1063/1.1803391.
- 922 Einstein, A., 1906. Eine neue Bestimmung der Moleküldimensionen. *Annalen*
923 *der Physik* 19, 289–306. doi:10.1002/andp.19113390313.

- 924 Einstein, A., 1911. Berichtigung zu meiner Arbeit: „Eine neue Bestimmung
925 der Moleküldimensionen“. *Annalen der Physik* 34, 591–592. doi:10.1002/
926 andp.19113390313.
- 927 Fauria, K.E., Rempel, A.W., 2011. Gas invasion into water-saturated, un-
928 consolidated porous media: Implications for gas hydrate reservoirs. *Earth*
929 *Planet. Sci. Lett.* 312, 188–193. doi:10.1016/j.epsl.2011.09.042.
- 930 Garboczi, E.J., Snyder, K.A., Douglas, J.F., Thorpe, M.F., 1995. Geometri-
931 cal percolation threshold of overlapping ellipsoids. *Phys. Rev. E* 52, 819–
932 828. doi:10.1103/PhysRevE.52.819.
- 933 Gay, E.C., Nelson, P.A., Armstrong, W.P., 1969. Flow properties of suspen-
934 sions with high solids concentration. *AIChE J.* 15, 815–822. doi:10.1002/
935 aic.690150606.
- 936 Ghiorso, M.S., Sack, R.O., 1995. Chemical mass transfer in magmatic pro-
937 cesses IV. A revised and internally consistent thermodynamic model for
938 the interpolation and extrapolation of liquid-solid equilibria in magmatic
939 systems at elevated temperatures and pressures. *Contributions to Miner-*
940 *alogy and Petrology* 119, 197–212. doi:10.1007/BF00307281.
- 941 Godbole, S.P., Schumpe, A., Shah, Y.T., Carr, N.L., 1984. Hydrodynamics
942 and mass transfer in non-Newtonian solutions in a bubble column. *AIChE*
943 *J.* 30, 213–220. doi:10.1002/aic.690300207.
- 944 Gonnermann, H.M., Manga, M., 2007. The fluid mechanics inside a volcano.
945 *Annu. Rev. Earth Planet. Sci.* 39, 321–356. doi:10.1146/annurev.fluid.
946 39.050905.110207.

- 947 Gu, D., Tanner, R.I., 1985. The drag on a sphere in a power-law fluid. *J.*
948 *Non-Newt. Fluid Mech.* 17, 1–12. doi:10.1016/0377-0257(85)80001-X.
- 949 Gurioli, L., Colò, L., Bollasina, A.J., Harris, A.J.L., Whittington, A., Ripepe,
950 M., 2014. Dynamics of Strombolian explosions: Inferences from field and
951 laboratory studies of erupted bombs from Stromboli volcano. *J. Geophys.*
952 *Res. Solid Earth* 119, 319–345. doi:10.1002/2013JB010355.
- 953 Harris, A., Ripepe, M., 2007. Temperature and dynamics of degassing at
954 Stromboli. *J. Geophys. Res.* 112. doi:10.1029/2006JB004393.
- 955 Heymann, L., Peukert, S., Aksel, N., 2002. On the solid-liquid transition
956 of concentrated suspensions in transient shear flow. *Rheologica Acta* 41,
957 307–315. doi:10.1007/s00397-002-0227-1.
- 958 Hoover, S., Cashman, K., Manga, M., 2001. The yield strength of subliquidus
959 basalts – experimental results. *J. Volcanol. Geotherm. Res.* 107, 1–18.
960 doi:10.1016/S0377-0273(00)00317-6.
- 961 Hovland, M., Hill, A., Stokes, D., 1997. The structure and geomorphology of
962 the Dashgil mud volcano, Azerbaijan. *Geomorphology* 21, 1–15. doi:10.
963 1016/S0169-555X(97)00034-2.
- 964 Istadi, B.P., Pramono, G.H., Sumintadireja, P., Alam, S., 2009. Modeling
965 study of growth and potential geohazard for LUSI mud volcano: East Java,
966 Indonesia. *Mar. Petrol. Geol* 26, 1724–1739. doi:10.1016/j.marpetgeo.
967 2009.03.006.
- 968 James, M.R., Llewelin, E.W., Lane, S.J., 2011. Comment on “It takes three
969 to tango: 2. Bubble dynamics in basaltic volcanoes and ramifications for

- 970 modeling normal Strombolian activity” by J. Suckale, B. H. Hager, L.
971 T. Elkins-Tanton, and J.-C. Nave. *J. Geophys. Res. Solid Earth* 116.
972 doi:10.1029/2010JB008167.
- 973 Jaupart, C., Vergnolle, S., 1988. Laboratory models of Hawaiian and Strom-
974 bolian eruptions. *Nature* 331, 58–60. doi:10.1038/331058a0.
- 975 Jaupart, C., Vergnolle, S., 1989. The generation and collapse of a foam layer
976 at the roof of a basaltic magma chamber. *J. Fluid Mech.* 203, 347–380.
977 doi:10.1017/S0022112089001497.
- 978 Johnson, A.B., White, D.B., 1993. Experimental determination of gas mi-
979 gration velocities with non-Newtonian fluids. *Int. J. Multiphase Flow* 19,
980 921–941. doi:10.1016/0301-9322(93)90070-B.
- 981 Kopf, A.J., 2002. Significance of mud volcanism. *Rev. Geophys.* 40, 2–1–2–52.
982 doi:10.1029/2000RG000093.
- 983 Landi, P., Métrich, N., Bertagnini, A., Rosi, M., 2004. Dynamics of
984 magma mixing and degassing recorded in plagioclase at Stromboli (Ae-
985 olian Archipelago, Italy). *Contributions to Mineralogy and Petrology* 147,
986 213–227. doi:10.1007/s00410-004-0555-5.
- 987 Lane, S.J., James, M.R., Corder, S.B., 2013. Volcano infrasonic signals and
988 magma degassing: First-order experimental insights and application to
989 stromboli. *Earth Planet. Sci. Lett.* 377–378, 169–179. doi:10.1016/j.
990 epsl.2013.06.048.

- 991 Lautze, N.C., Houghton, B.F., 2005. Physical mingling of magma and com-
992 plex eruption dynamics in the shallow conduit at Stromboli volcano, Italy.
993 *Geology* 33, 425–428. doi:10.1130/G21325.1.
- 994 Lautze, N.C., Houghton, B.F., 2007. Linking variable explosion style and
995 magma textures during 2002 at Stromboli volcano, Italy. *Bulletin of Vol-*
996 *canology* 69, 445–460. doi:10.1007/s00445-006-0086-1.
- 997 Lavallée, Y., Hess, K.U., Cordonnier, B., Dingwell, D.B., 2007. Non-
998 Newtonian rheological law for highly crystalline dome lavas. *Geology* 35,
999 843–846. doi:10.1130/G23594A.1.
- 1000 Lavrenteva, O.M., Holenberg, Y., Nir, A., 2009. Motion of viscous drops
1001 in tubes filled with yield stress fluid. *Chem. Eng. Sci.* 64, 4772–4786.
1002 doi:10.1016/j.ces.2009.06.055.
- 1003 Luckham, P.F., Rossi, S., 1999. The colloidal and rheological properties of
1004 bentonite suspensions. *Adv. Colloid Interface Sci.* 82, 43–92. doi:10.1016/
1005 S0001-8686(99)00005-6.
- 1006 Lynch, D.K., Hudnut, K.W., 2008. The Wister mud pot lineament: South-
1007 eastward extension or abandoned strand of the San Andreas Fault? *Bull.*
1008 *Seismol. Soc. Am.* 98, 1720–1729. doi:10.1785/0120070252.
- 1009 Mader, H.M., Llewelin, E.W., Mueller, S.P., 2013. The rheology of two-
1010 phase magmas: a review and analysis. *J. Volcanol. Geotherm. Res.* 257,
1011 135–158. doi:10.1016/j.jvolgeores.2013.02.014.
- 1012 Magnin, A., Piau, J.M., 1990. Cone-and-plate rheometry of yield stress

- 1013 fluids. Study of an aqueous gel. *J. Non-Newt. Fluid Mech.* 36, 85–108.
1014 doi:10.1016/0377-0257(90)85005-J.
- 1015 Manga, M., Bonini, M., 2012. Large historical eruptions at subaerial mud
1016 volcanoes, Italy. *Nat. Hazards Earth Syst. Sci.* 12, 3377–3386. doi:10.
1017 5194/nhess-12-3377-2012.
- 1018 Manga, M., Stone, H.A., 1995. Collective hydrodynamics of deformable drops
1019 and bubbles in dilute low Reynolds number suspensions. *J. Fluid Mech.*
1020 300, 231–263. doi:10.1017/S0022112095003673.
- 1021 Margaritis, A., te Bokkel, D.W., Karamanev, D.G., 1999. Bub-
1022 ble rise velocities and drag coefficient in non-Newtonian polysaccha-
1023 ride solutions. *Biotechnol. Bioeng.* 64, 257–266. doi:10.1002/(SICI)
1024 1097-0290(19990805)64:3<257::AID-BIT1>3.0.CO;2-F.
- 1025 Mazzini, A., Svensen, H., Akhmanov, G.G., Aloisi, G., Planke, S., Malthe-
1026 Sørenssen, A., Istadi, B., 2007. Triggering and dynamic evolution of the
1027 LUSI mud volcano, Indonesia. *Earth Planet. Sci. Lett.* 261, 375–388.
1028 doi:10.1016/j.epsl.2007.07.001.
- 1029 Mazzini, A., Svensen, H., Etiope, G., Onderdonk, N., Banks, D., 2011. Fluid
1030 origin, gas fluxes and plumbing system in the sediment-hosted Salton Sea
1031 Geothermal System (California, USA). *J. Volcanol. Geotherm. Res.* 205,
1032 67–83. doi:10.1016/j.jvolgeores.2011.05.008.
- 1033 Métrich, N., Bertagnini, A., Di Muro, A., 2010. Conditions of magma storage,
1034 degassing and ascent at Stromboli: New insights into the volcano plumbing

- 1035 system with inferences on the eruptive dynamics. *Journal of Petrology* 51,
1036 603–626. doi:10.1093/petrology/egp083.
- 1037 Miyahara, T., Yamanaka, S., 1993. Mechanics of motion and deformation
1038 of a single bubble rising through quiescent highly viscous Newtonian and
1039 non-Newtonian media. *J. Chem. Eng. Japan* 26, 297–302. doi:10.1252/
1040 jcej.26.297.
- 1041 Mougín, N., Magnin, A., Piau, J.M., 2012. The significant influence of inter-
1042 nal stresses on the dynamics of bubbles in a yield stress fluid. *J. Non-Newt.*
1043 *Fluid Mech.* 171–172, 42–55. doi:10.1016/j.jnnfm.2012.01.003.
- 1044 Mueller, S., Llewellyn, E.W., Mader, H.M., 2010. The rheology of suspensions
1045 of solid particles. *Proc. R. Soc. A* 466, 1201–1228. doi:10.1098/rspa.
1046 2009.0445.
- 1047 Namiki, A., Kagoshima, T., 2014. Intermittent and efficient outgassing by
1048 the upward propagation of film ruptures in a bubbly magma. *J. Geophys.*
1049 *Res. Solid Earth* 119, 919–935. doi:10.1002/2013JB010576.
- 1050 Newman, S., Lowenstern, J.B., 2002. VolatileCalc: a silicate melt–H₂O–CO₂
1051 solution model written in Visual Basic for excel. *Computers & Geosciences*
1052 28, 597–604. doi:10.1016/S0098-3004(01)00081-4.
- 1053 O’Brien, J.S., Julien, P.Y., 1988. Laboratory analysis of mudflow properties.
1054 *J. Hydraul. Eng.* 114, 877–887. doi:10.1061/(ASCE)0733-9429(1988)
1055 114:8(877).
- 1056 van Olphen, H., 1964. Internal mutual flocculation in clay suspensions. *J.*
1057 *Colloid Sci.* 19, 313–322. doi:10.1016/0095-8522(64)90033-9.

- 1058 Onderdonk, N., Mazzini, A., Shafer, L., Svensen, H., 2011. Controls on
1059 the geomorphic expression and evolution of gryphons, pools, and caldera
1060 features at hydrothermal seeps in the Salton Sea Geothermal Field, south-
1061 ern California. *Geomorphology* 130, 327–342. doi:10.1016/j.geomorph.
1062 2011.04.014.
- 1063 Orvis, K.H., Grissino-Mayer, H.D., 2002. Standardizing the reporting of
1064 abrasive papers used to surface tree-ring samples. *Tree-Ring Research* 58,
1065 47–50.
- 1066 Oseen, C.W., 1910. Über die Stokes'sche formel, und über eine verwandte
1067 Aufgabe in der Hydrodynamik. *Arkiv för matematik, astronomi och fysik*
1068 6.
- 1069 Parfitt, E.A., 2004. A discussion of the mechanisms of explosive basaltic
1070 eruptions. *J. Volcanol. Geotherm. Res.* 134, 77–107. doi:10.1016/j.
1071 jvolgeores.2004.01.002.
- 1072 Parfitt, E.A., Wilson, L., 1995. Explosive volcanic eruptions—IX. The tran-
1073 sition between Hawaiian-style lava fountaining and Strombolian explosive
1074 activity. *Geophys. J. Int.* 121, 226–232. doi:10.1111/j.1365-246X.1995.
1075 tb03523.x.
- 1076 Philpotts, A.R., Shi, J., Brustman, C., 1998. Role of plagioclase crystal
1077 chains in the differentiation of partly crystallized basaltic magma. *Nature*
1078 395, 343–346. doi:10.1038/26404.
- 1079 Piau, J.M., 2007. Carbopol gels: Elastoviscoplastic and slippery glasses
1080 made of individual swollen sponges. Meso- and macroscopic properties,

- 1081 constitutive equations and scaling laws. *J. Non-Newt. Fluid Mech.* 144,
1082 1–29. doi:10.1016/j.jnnfm.2007.02.011.
- 1083 Pignon, F., Magnin, A., Piau, J.M., 1996. Thixotropic colloidal suspensions
1084 and flow curves with minimum: identification of flow regimes and rheo-
1085 metric consequences. *J. Rheol.* 40, 573–587. doi:10.1122/1.550759.
- 1086 Pinkerton, H., Norton, G., 1995. Rheological properties of basaltic lavas
1087 at sub-liquidus temperatures: laboratory and field measurements on lavas
1088 from Mount Etna. *J. Volcanol. Geotherm. Res.* 68, 307–323. doi:dx.doi.
1089 org/10.1016/0377-0273(95)00018-7.
- 1090 Pinkerton, H., Sparks, R.S.J., 1978. Field measurements of the rheology of
1091 lava. *Nature* 276, 383–385. doi:10.1038/276383a0.
- 1092 Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., 1992. Nu-
1093 merical Recipes in C. Second ed., Cambridge University Press, New York.
- 1094 Proussevitch, A.A., Sahagian, D.L., 1996. Dynamics of coupled diffusive and
1095 decompressive bubble growth in magmatic systems. *J. Geophys. Res.* 101,
1096 17447–17455. doi:10.1029/96JB01342.
- 1097 Rifai, R., 2008. Spatial modelling and risk assessment of Sidoarjo mud vol-
1098 canic flow. M.Sc. thesis. Univ. Gadjah Mada. Yogyakarta, Indonesia.
- 1099 Ripepe, M., Harris, A.J., Carniel, R., 2002. Thermal, seismic and infrasonic
1100 evidences of variable degassing rates at Stromboli volcano. *J. Volcanol.*
1101 *Geotherm. Res.* 118, 285–297. doi:10.1016/S0377-0273(02)00298-6.

- 1102 Roscoe, R., 1952. The viscosity of suspensions of rigid spheres. *Br. J. Appl.*
1103 *Phys.* 3, 267–269. doi:10.1088/0508-3443/3/8/306.
- 1104 Rudolph, M.L., Manga, M., 2010. Mud volcano response to the 4 April
1105 2010 El Mayor-Cucapah earthquake. *J. Geophys. Res.* 115. doi:10.1029/
1106 2010JB007737.
- 1107 Rudolph, M.L., Manga, M., 2012. Frequency dependence of mud vol-
1108 cano response to earthquakes. *Geophys. Res. Lett.* 39. doi:10.1029/
1109 2012GL052383.
- 1110 Ryerson, F.J., Weed, H.C., Piwinskii, A.J., 1988. Rheology of subliquidus
1111 magmas: 1. picritic compositions. *J. Geophys. Res.* 93, 3421–3436. doi:10.
1112 1029/JB093iB04p03421.
- 1113 Saar, M.O., Manga, M., Cashman, K.V., Fremouw, S., 2001. Numerical
1114 models of the onset of yield strength in crystal-melt suspensions. *Earth*
1115 *Planet. Sci. Lett.* 187, 367–379. doi:10.1016/S0012-821X(01)00289-8.
- 1116 Sahagian, D., 1985. Bubble migration and coalescence during the solidifica-
1117 tion of basaltic lava flows. *J. Geol.* 93, 205–211. doi:10.1086/628942.
- 1118 Shaw, H.R., Wright, T.L., Peck, D.L., Okamura, R., 1968. The viscosity of
1119 basaltic magma; an analysis of field measurements in Makaopuhi lava lake,
1120 Hawaii. *Am. J. Sci.* 266, 225–264. doi:10.2475/ajs.266.4.225.
- 1121 Sherwood, D.J., Sáez, A.E., 2014. The start of ebullition in quiescent, yield-
1122 stress fluids. *Nuc. Eng. Des.* 270, 101–108. doi:10.1016/j.nucengdes.
1123 2013.12.050.

- 1124 Shields, J.K., Mader, H.M., Pistone, M., Caricchi, L., Floess, D., Putlitz,
1125 B., 2014. Strain-induced outgassing of three-phase magmas during sim-
1126 ple shear. *J. Geophys. Res. Solid Earth* 119, 6936–6957. doi:10.1002/
1127 2014JB011111.
- 1128 Sikorski, D., Tabuteau, H., de Bruyn, J.R., 2009. Motion and shape of
1129 bubbles rising through a yield-stress fluid. *J. Non-Newt. Fluid Mech.* 159,
1130 10–16. doi:10.1016/j.jnnfm.2008.11.011.
- 1131 Sturz, A.A., Kamps, R.L., Earley, P.J., 1992. Temporal changes in mud
1132 volcanoes, Salton Sea geothermal area, in: Kharaka, Y.K., Maest, A.S.
1133 (Eds.), *Water-Rock Interaction*. Balkema, Rotterdam, Netherlands, pp.
1134 1363–1366.
- 1135 Suckale, J., Hager, B.H., Elkins-Tanton, L.T., Nave, J.C., 2010. It takes three
1136 to tango: 2. Bubble dynamics in basaltic volcanoes and ramifications for
1137 modeling normal Strombolian activity. *J. Geophys. Res. Solid Earth* 115.
1138 doi:10.1029/2009JB006917.
- 1139 Suckale, J., Hager, B.H., Elkins-Tanton, L.T., Nave, J.C., 2011. Reply
1140 to the comment by Mike R. James et al. on “It takes three to tango:
1141 2. Bubble dynamics in basaltic volcanoes and ramifications for model-
1142 ing normal Strombolian activity”. *J. Geophys. Res. Solid Earth* 116.
1143 doi:10.1029/2011JB008351.
- 1144 Svensen, H., Hammer, Ø., Mazzini, A., Onderdonk, N., Polteau, S., Planke,
1145 S., Podladchikov, Y.Y., 2009. Dynamics of hydrothermal seeps from the
1146 Salton Sea Geothermal System (California, USA) constrained by tem-

- 1147 perature monitoring and time series analysis. *J. Geophys. Res.* 114.
1148 doi:10.1029/2008JB006247.
- 1149 Svensen, H., Karlsen, D.A., Sturz, A., Backer-Owe, K., Banks, D.A., Planke,
1150 S., 2007. Processes controlling water and hydrocarbon composition in seeps
1151 from the Salton Sea Geothermal System. *Geology* 35, 85–88. doi:10.1130/
1152 G23101A.1.
- 1153 Tabuteau, H., Coussot, P., de Bruyn, J.R., 2007a. Drag force on a sphere in
1154 steady motion through a yield-stress fluid. *J. Rheol.* 51, 125–137. doi:10.
1155 1122/1.2401614.
- 1156 Tabuteau, H., Oppong, F.K., de Bruyn, J.R., Coussot, P., 2007b. Drag on
1157 a sphere moving through an aging system. *Europhys. Lett.* 78. doi:10.
1158 1209/0295-5075/78/68007.
- 1159 Tsamopoulos, J., Dimakopoulos, Y., Chatzidai, N., Karapetsas, G., Pavlidis,
1160 M., 2008. Steady bubble rise and deformation in Newtonian and viscoplas-
1161 tic fluids and conditions for bubble entrapment. *J. Fluid Mech.* 601, 123–
1162 164. doi:10.1017/S0022112008000517.
- 1163 Vanderkluyzen, L., Burton, M.R., Clarke, A.B., Hartnett, H.E., Smekens,
1164 J.F., 2014. Composition and flux of explosive gas release at LUSI mud
1165 volcano (East Java, Indonesia). *Geochem. Geophys. Geosyst.* 15, 2932–
1166 2946. doi:10.1002/2014GC005275.
- 1167 Vona, A., Romano, C., Dingwell, D.B., Giordano, D., 2011. The rheology of
1168 crystal-bearing basaltic magmas from Stromboli and Etna. *Geochimica et*
1169 *Cosmochimica Acta* 75, 3214–3236. doi:10.1016/j.gca.2011.03.031.

- 1170 Walker, G.P.L., 1987. Pipe vesicles in Hawaiian basaltic lavas: Their origin
1171 and potential as paleoslope indicators. *Geology* 15, 84–87. doi:10.1130/
1172 0091-7613(1987)15<84:PVIHBL>2.0.CO;2.
- 1173 Weinberg, R.F., Podladchikov, Y., 1994. Diapiric ascent of magmas through
1174 power law crust and mantle. *J. Geophys. Res.* 99, 9543–9559. doi:10.
1175 1029/93JB03461.
- 1176 Wildemuth, C.R., Williams, M.C., 1984. Viscosity of suspensions modeled
1177 with a shear-dependent maximum packing fraction. *Rheologica Acta* 23,
1178 627–635. doi:10.1007/BF01438803.
- 1179 Wilson, L., Head, J.W., 1981. Ascent and eruption of basaltic magma on
1180 the Earth and Moon. *J. Geophys. Res. Solid Earth* 86, 2971–3001. doi:10.
1181 1029/JB086iB04p02971.
- 1182 Zhou, J.Z.Q., Fang, T., Luo, G., Uhlherr, P.H.T., 1995. Yield stress and
1183 maximum packing fraction of concentrated suspensions. *Rheologica Acta*
1184 34, 544–561. doi:10.1007/BF00712315.

Tables

Table 1. Mud column heights and rheological parameters for all experiments. Fitted rheological parameters for ascending flow curves of mud samples from each experimental run. Reported uncertainties are parameter errors from nonlinear least squares fit. In Experiment 1, mud height was not used to determine sphere terminal velocities. Experiment 3 used the rheological parameters reported for Experiment 2; in Experiment 4, water content of mud away from the zone of bubble bursting (side) was not measured.

Experiment	Mud height h (m)	Consistency K (Pa s ^{n})	Power-law index n (-)	Yield stress τ_c (Pa)	Water wt% (-)
1 (spheres)	–	2.0 ± 0.2	0.71 ± 0.03	93.2 ± 0.3	40.1 ± 0.3
2	0.867	3.6 ± 0.5	0.61 ± 0.03	127.4 ± 0.7	38.5 ± 0.3
3	0.660	–	–	–	–
4	0.880	3.4 ± 0.7	0.61 ± 0.05	102.3 ± 1.1	39.9 ± 0.3
4 (side)	0.880	5.0 ± 0.9	0.51 ± 0.04	115.1 ± 1.3	–

Table 2. Rigid sphere parameters with measured and corrected terminal velocities. Here d is diameter, $\Delta\rho_{\text{eff}}$ is effective density difference, and u_t is terminal velocity. Note that the density difference of steel in mud, without friction or counterweight, is 6350 kg/m^3 .

d (cm)	$\Delta\rho_{\text{eff}}$ (kg/m^3)	u_t (measured) (m/s)	u_t (corrected) (m/s)
4.13	4790	0.23 ± 0.03	0.27 ± 0.04
4.13	5300	0.37 ± 0.07	0.44 ± 0.08
4.13	5890	0.52 ± 0.09	0.64 ± 0.11
4.76	4330	0.29 ± 0.03	0.38 ± 0.04
4.76	5150	0.51 ± 0.06	0.69 ± 0.08
4.76	6060	0.75 ± 0.12	1.07 ± 0.17
5.08	3960	0.27 ± 0.03	0.36 ± 0.04
5.08	5220	0.63 ± 0.08	0.93 ± 0.12
5.08	6110	0.86 ± 0.14	1.32 ± 0.22

Table 3. Fluid rheology controls minimum mobile gas bubble sizes according to the yield criterion given by equation (9). We compute minimum diameters and mobile bubble velocities (at twice the minimum volume) for fluids in this study, natural mud eruptions, and various basaltic magmas.

	Material properties				Mobile bubble properties	
	τ_c (Pa)	K (Pa s ^{<i>n</i>})	n (-)	ρ_L (kg/m ³)	d_{\min} (m)	$u_t (V = 2V_{\min})$ (m/s)
Carbopol, 0.09 wt% (Lavrenteva et al., 2009)	4	5.2	0.35	1000	0.002	10 ⁻⁴
Carbopol, 1.8 wt% (Sikorski et al., 2009)	33.5	10.5	0.55	1000	0.015	10 ⁻²
Salton Sea mud, Experiment 3	~100	4	0.5	1600	0.03	10 ⁻¹
Italian mud volcanoes (Manga and Bonini, 2012)	4–8	2–5	0.22	1400	0.001–0.003	10 ⁻⁴
Lusi early eruption estimate, 70 wt% water	0.1–10	0.1–1	1	1300	10 ⁻⁵ –10 ⁻³	10 ⁻⁵ –10 ⁻²
Lusi late eruption estimate, 30 wt% water	10 ² –10 ³	1–100	0.5–1	1800	10 ⁻² –10 ⁻¹	10 ⁻¹ –10 ⁰
Kilauea lava lake in situ (Shaw et al., 1968)	~10 ²	~700	1	2700	0.01	10 ⁻³
Kilauea basalt in lab (Ryerson et al., 1988)	10 ¹ –10 ³	10 ¹ –10 ²	0.7–0.9	2700	0.001–0.1	10 ⁻³ –10 ⁰
Etna lava flow in situ (Pinkerton and Sparks, 1978) ^a	181	8000	0.76	2700	0.03	10 ⁻⁴
Etna basalt in lab (Pinkerton and Norton, 1995)	78	~3000	~0.5	2700	0.01	10 ⁻⁶
Strombolian bombs in lab (Gurioli et al., 2014)	≤ 10 ⁴	10 ⁹ –10 ¹¹	1	2700	≤ 1.2	10 ⁻⁷ –10 ⁻⁵

^aRheological parameters from Castruccio et al. (2014).

Figures

Figure 1. (a) Location map showing Salton Sea mud volcanoes (magenta volcano glyph). (b) Geomorphic features at the Salton Sea geothermal system. ~ 1 m gryphon, notebook in foreground for scale. (c) Bubbling mud pot, ~ 0.5 m across.

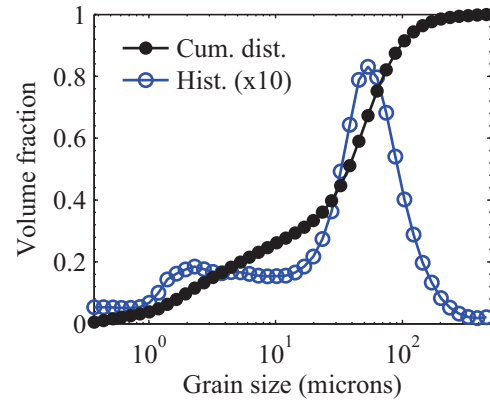


Figure 2. (top) X-ray computed tomography image of Salton Sea mud, image resolution is 1.3 microns/voxel. (bottom) Particle size distribution. Open blue circles show histogram (scaled x10 for clarity) and black filled circles show cumulative size distribution.

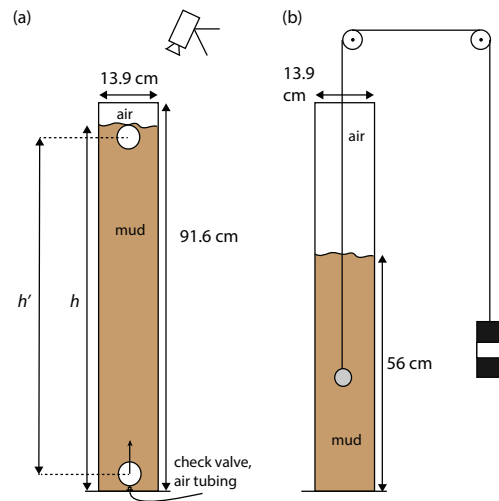


Figure 3. Schematic illustrations and photos of experimental apparatuses for bubble (top) and sphere (bottom) velocity measurement.

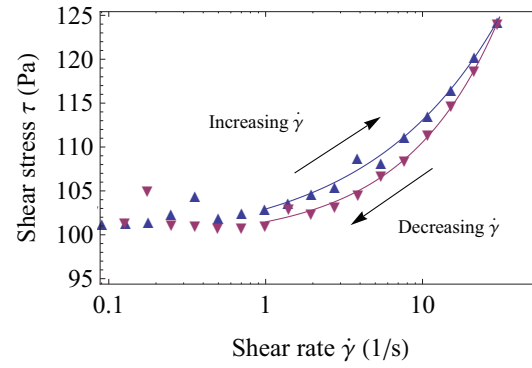


Figure 4. Flow curves for mud samples collected at the end of Experiment 3. Blue and purple curves fit ascending and descending data respectively to the Herschel-Bulkley model, equation (2).

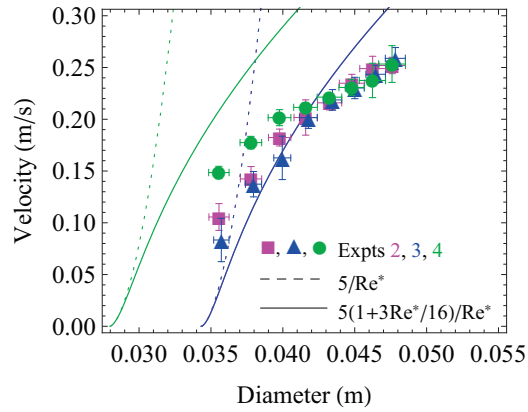


Figure 5. Bubble velocities as a function of volume for mud experiments; rheological properties of each experiments given in Table 1. Solid lines plot the drag empiricism of equation (6) with $A = 5$; dotted lines plot the same empiricism without the Oseen-like inertial correction (i.e., $C_D = 5/Re^*$). Blue curves plot predictions for Experiments 2 and 3 both; green curves plot predictions for Experiment 4.

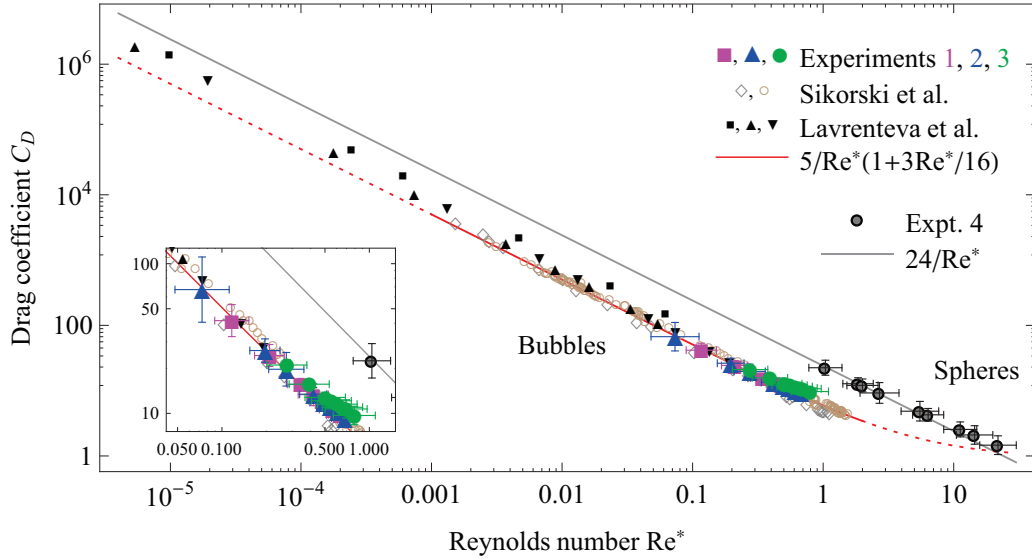


Figure 6. Re^* as a function of C_D for all data considered. Open diamonds and circles plot data for Carbopol with yield stresses 24.1 Pa and 33.5 Pa (concentrations 1.2, 1.8 wt%) respectively (Sikorski et al., 2009). Filled squares, upward pointing triangles, and downward pointing triangles plot data for Carbopol with yield stresses 1.5 Pa, 2.4 Pa, and 4.0 Pa respectively (concentrations 0.07–0.09 wt%) (Lavrenteva et al., 2009). Omitted error bars for data of Sikorski et al. (2009) and Lavrenteva et al. (2009) are comparable to marker sizes. Inset is zoomed into data for air bubbles in Salton Sea mud.

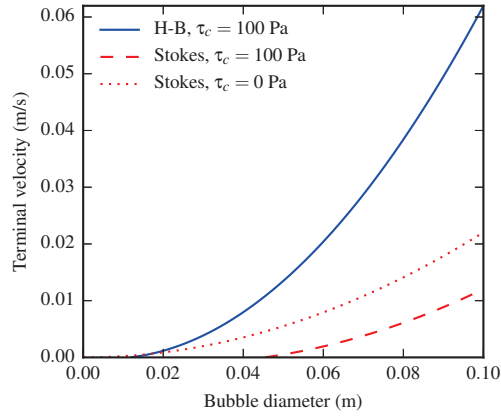


Figure 7. Bubble velocity predictions for fluid with $n = 1$, consistency $K = 1000$ Pa s, yield stress $\tau_c = 100$ Pa, and density $\rho_L = 2700$ kg/m³. H-B model prediction computed from equation (6); the maximum value of Re^* is 0.015. Stokes predictions approximate fluid as Newtonian using $\mu_{\text{eff}}(d)$ (equation (7)). The Newtonian approximation, with or without yield stress, underpredicts bubble velocity by a factor of 5–10 here.

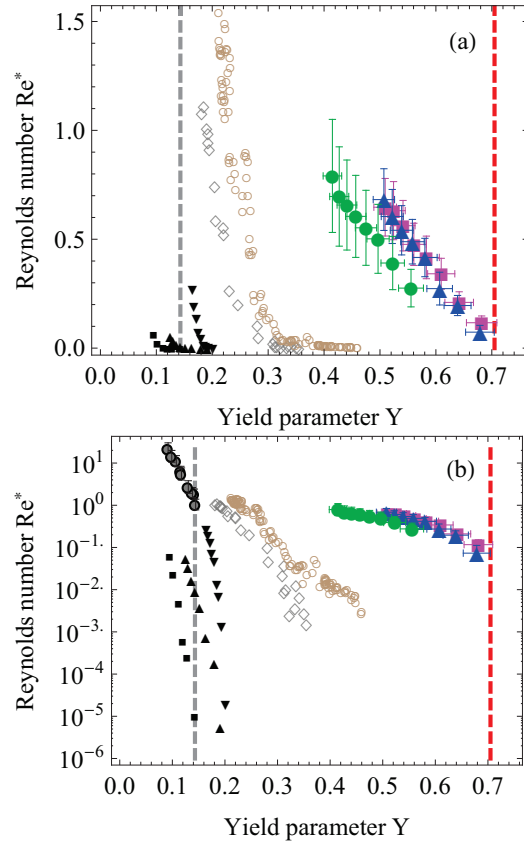


Figure 8. (a) Re^* as a function of Y for bubbles. (b) Re^* as a function of Y in semi-log space for rigid spherical particles and bubbles. Symbols are as in Figure 6; gray dashed line plots $Y_c = 0.143$ for spherical particles; red dashed line plots $Y_c = 0.705$ for bubbles from equation (9) with $A = 5$ and $X(n = 0.6) = 1.38$. The plotted data from Sikorski et al. (2009) disagree slightly with the corresponding plot in their study, as we have applied their shape empiricism in lieu of measured shape data.

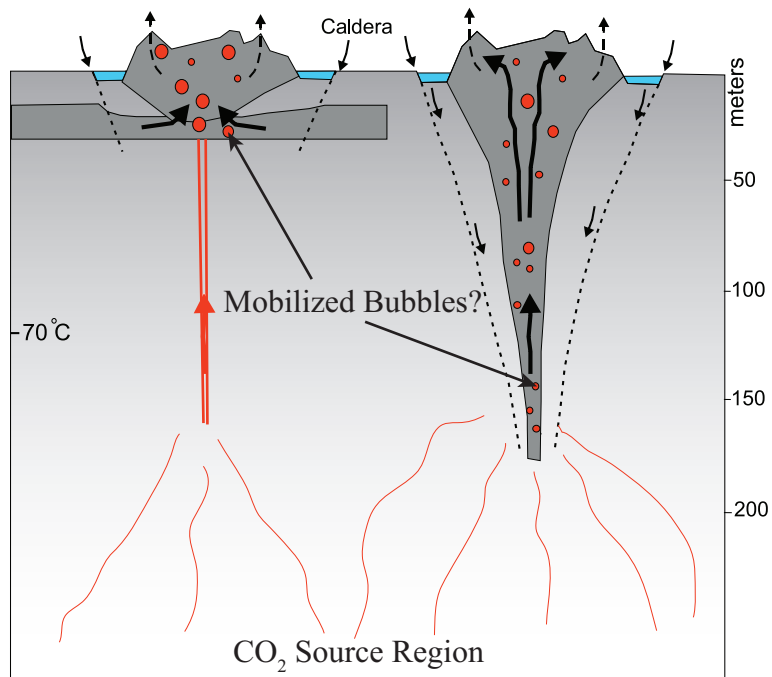


Figure 9. Schematic illustration of two possible mud volcano plumbing systems for the Salton Sea features shown in Figure 1. Modified from Svensen et al. (2009) and Rudolph and Manga (2010). CO₂, shown in red, travels upward through a porous matrix, creating bubbles at more shallow depths. The uppermost 200 m consists of deltaic and lacustrine sediments.

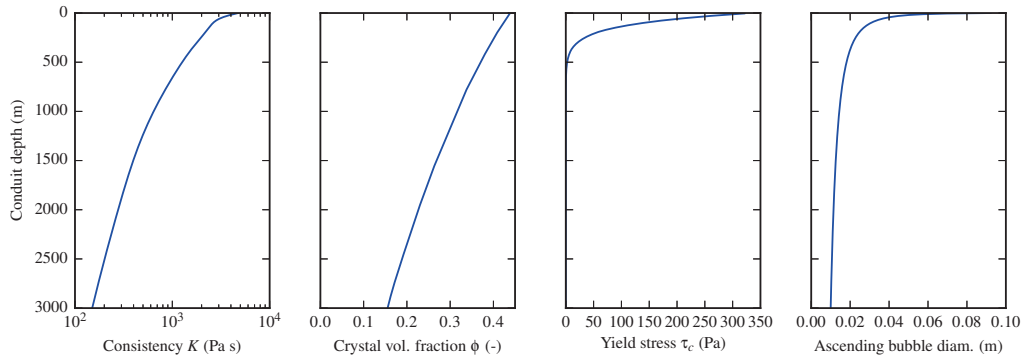


Figure 10. Material parameters for Stromboli conduit model. Consistency K and crystal fraction ϕ from Figures 3, 10 of Beckett et al. (2014); τ_c follows equation (10). Ascending bubble diameter plotted for 1 cm bubble nucleating at depth of 3 km.

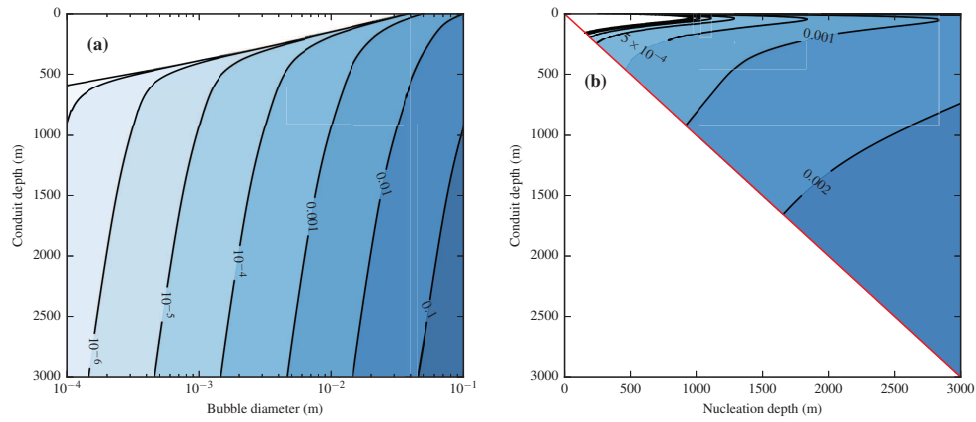


Figure 11. (a) Bubble velocity (m/s) at various sizes and depths in the model conduit. (b) Bubble velocities (m/s) for 1 cm bubbles ascending and expanding from depth; i.e., vertical slices are velocity trajectories of individual bubbles. Bubbles smaller than a few cm are easily immobilized by the onset of a yield stress and increasing consistency K . Bubbles ascending from depth will decelerate and may accumulate and coalesce beneath more viscous shallow melt.

Highlights

- Bubbles are more mobile than rigid particles in muds for the same buoyancy
- A criterion for mobilizing bubbles is bounded for fluids with a yield stress
- Basaltic lavas and natural muds can immobilize centimeter-sized bubbles
- A vertical yield stress gradient in Stromboli's conduit can hinder coalescence