Latent Space Models for Temporal Networks

Jasper Alt
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Latent Space Models for Temporal Networks

Jasper Alt

Department of Computer Science
Portland State University

References

- This talk based on work (with Rajesh Venkatachalapathy) submitted to NetSci 2019.

- A fuller manuscript (with Rajesh) is in progress.

- Peres et. al. “Mobile Geometric Graphs” (2010)

- Holme, P. “Modern Temporal Network Theory: A Colloquium”
Outline

- Homophily and latent space
- Random geometric graphs and Brownian motion
- Mobile geometric graphs
- Implementation
- Observations
- Current and future work
0. Social Networks

- Maps of relationships between actors

- What do the actors tell us about the networks?

- What do the networks tell us about the actors?
1. Homophily
“Birds of a feather flock together”

- Actors with common attributes are more likely to form ties

- Ties between actors may indicate common attributes

- Typically expressed in an informal way

- Latin prefix, Greek Suffix

- Idiophily?

2. Latent Space
- Can we state something more formal?

- Combinations & intensities of attributes assign the actors to coordinates in a *latent space*

- More attributes in common, or attributes with similar intensities, mean the actors are closer together in this space.

- Tobler's Law: Everything is related to everything else, but near things are more related than distant things.

- (Q. Are there *any* networks that do *not* correspond to a latent space?)
Wikipedia: The political compass
3. Homophily in Latent Space

The graph at right is described by an $n \times n$ adjacency matrix $A$, where $n = 10$. The matrix entries are defined as:

$$A_{ij} = \begin{cases} f(d_{ij}) & \text{if } f(d_{ij}) > m \\ 0 & \text{otherwise} \end{cases}$$

where $f(d_{ij}) =$

- $\frac{1}{1 + e^{0.5(5 - d_{ij})}}$
- $1 - \frac{d_{ij}}{2}$

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4. Latent Space Dynamics

- We know attributes can change.

- This means the actors' positions in latent space change.

- There are many possible ways this could happen.

- In social contexts, we expect nontrivial dynamics.
People influence each other and are influenced by social forces.

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<thead>
<tr>
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<tbody>
<tr>
<td>Ancient, advanced civilizations, such as Atlantis, once existed</td>
<td>39.6</td>
<td>55.0</td>
<td>56.9%</td>
<td>17.3%</td>
</tr>
<tr>
<td>Aliens have visited Earth in our ancient past</td>
<td>27.0</td>
<td>35.0</td>
<td>41.4%</td>
<td>14.4%</td>
</tr>
<tr>
<td>Places can be haunted by spirits</td>
<td>46.6</td>
<td>52.3</td>
<td>57.7%</td>
<td>11.1%</td>
</tr>
<tr>
<td>Aliens have come to Earth in modern times</td>
<td>24.7</td>
<td>26.2</td>
<td>35.1%</td>
<td>10.4%</td>
</tr>
<tr>
<td>Bigfoot is a real creature</td>
<td>13.5</td>
<td>16.2</td>
<td>20.7%</td>
<td>7.2%</td>
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<tr>
<td>Some people can move objects with their minds</td>
<td>19.1</td>
<td>25.0</td>
<td>26.2%</td>
<td>7.1%</td>
</tr>
<tr>
<td>Fortune tellers and psychics can foresee the future</td>
<td>14.1</td>
<td>19.4</td>
<td>17.2%</td>
<td>3.1%</td>
</tr>
</tbody>
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*Chapman University Survey of American Fears*
5. Null Models

- Network science has some big null models.

- Erdos-Renyi — random graphs

- Configuration models — random with fixed degree sequence

- Stochastic block models — random with some community structure
Wikipedia: Erdos-Renyi model
$k_1 = 3 \quad k_2 = 2 \quad k_3 = 2 \quad k_4 = 1$

Wikipedia: Configuration model
- Good null models show what can be explained by chance.

- Random dynamics are useful for testing non-random alternatives.

- A good null model for latent space network dynamics will have random dynamics.
• This will make it easier to spot networks that don't.

6. Temporal Networks

• We typically understand static networks to be ‘snapshots’ of dynamic networks.

• Real networks are temporal.

• Static networks can be instantaneous snapshots or aggregate over a window of time.

• What is a null model for temporal networks?

• Random, but time dependent
Building Blocks

6. Random Geometric Graphs

- Graphs
- Geometric
- Random
• Nodes are placed using some *point process*

• (say, uniformly at random)

• Nodes within some radius of one another are connected.

• Studied in the context of wireless networks.

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7. Brownian Motion

• Move a normally distributed distance in either direction.

• Like a particle in a fluid.

• Shows up in physics, statistics, quantitative finance, etc.

• Preserves the intensity of point processes.
8. Mobile Geometric Graphs

- Random geometric graphs with Brownian motion

- Brownian motion adds time dependence while preserving randomness

- Very little studied

- Existing work focuses on the mathematical statistics of their communication properties.

Contd