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Abstract

We provide a system dynamics implementation of a dynamic ecological economics model. Dynamic economic models are often constrained to use functions, such as the Cobb-Douglas function, chosen "conveniently" to allow for analytic solutions. The C-D function, however, suffers from its fixed elasticity that does not allow for the substitutability between man-made capital and natural capital to change, which is vital for economic sustainability.

Using system dynamics removes this constraint and enables more realistic ecological economics models containing functions not amenable to analytic solution. The base model is the natural resource and population growth model developed by Brander and Taylor (1998) that employs a Lotka-Volterra type structure and strictly follows economic theory in all aspects of its formulation. To make the model more realistic and to enable the consideration of critical environmental issues, we discuss and employ model extensions inspired by modern economics theory. One extension is to use a CES production function with a dynamic substitutability parameter that enables the *study of long-term sustainability of the modeled economy. The model does not have an analytic solution, necessitating a simulation approach. Importantly, under certain conditions the system dynamics implementation robustly returns to equilibrium after disturbances.*

Keywords: ecological economics, economic theory, limits to growth, non-equilibrium

1. Introduction

An important topic in economics is the dynamic processes through which an economy could outgrow its supporting ecosystems and lead to a collapse. One school of thought focuses on the interaction between economic growth and the dynamics of resource depletion, replenishment, substitution, etc. A model of these dynamics in the closed economy of Easter Island, developed by Brander and Taylor (1998), has attracted considerable attention (hereafter, the BT model). Since its initial appearance, the BT model has generated many descendants (Dalton and Coats, 2000; Erickson and Gowdy, 2000; Maxwell & Reuveny, 2000; Reuveny and Decker, 2000; Pezzey & Anderies, 2003; Prskawetz, Gragnani, and Feichtinger, 2003; Basener and Ross, 2005; Dalton et al., 2005; Nagase and Mirza, 2006; D'Alessandro, 2007; Basener et al., 2008; Croix and Dottori, 2008). These models critically examine and extend model specifications for population growth, substitutability, innovation, capital accumulation, and property rights. BT-type models can be classified as a combination of a static general equilibrium model, plus a differential equation-based simulation model that creates time dynamics.

In environmental and ecological economics, the necessity of systems thinking, for example, the importance of positive and negative feedback loops are widely acknowledged. Meanwhile, there is ample scope for further application of system dynamics (henceforth SD) in the field of environmental and ecological economics. One reason for the limited use of SD in economics so far is its often limited use of economic theory. Nordhaus et al. (1992), for example, harshly criticized the "Limits to Growth" model by Meadows et al. (1972), in part because its logic seemed to largely disregard economic theory.

In the present research, we develop a BT-type system dynamics model that is fully compatible with economic theory to improve is potential acceptance as a tool for environmental and ecological economics. Our model implements several of the published BT model extensions, and in each case the model was calibrated to achieve steady state. The calibration process did not require any of the key equations (for production, demand, and other aspects) to be modified from their published forms. Our model allows us to study both the effect of exogenous shocks to the stability of the system and the effect of the evolution of endogenous variables on the system's dynamic responses. Since the substitutability of man-made capital for natural capital changes over time (Beltratti, 1997), we pay our particular attention to the modeling of innovation in our models to help address the impact of technological progress on substitutability.

A major contribution of this paper is to illustrate a practical way to enhance ecological economics models by employing more realistic nonlinear functions rather than linear or some limited types of nonlinear functions that are often "conveniently" chosen because they allow for analytic derivation of equilibrium values. Our models do not require the existence of equilibrium values or that equilibrium states be maintained. The usual mathematical assumptions required to derive equilibrium values are not as critical for BT-type models, because the users of these models focus less on steady state and more on the dynamic evolution of the key indicator of the economic agents' well-being and sustainability.

Section 2 introduces the original BT model, including an SD implementation. Section 3 provides a substantial literature review summarizing and criticizing the original BT model and its various descendants, based on the synthesis paper to be presented in August 2010 at the International Society for Ecological Economics 2010 Conference in Germany (Nagase and Uehara, 2010). The literature review provides the detailed sources for the structure, equations, and key parameter values for our extended BT-type model in section 4. Section 4 presents our system dynamics implementation of an extended BT-type model, along with preliminary testing results and future directions. Section 5 provides discussion and conclusions, including the contributions that system dynamics can make to the field of ecological economics.

2. Background: The BT model

The BT model is a model to describe the collapse of the population of Easter Island. Easter Island was a small, closed economy in the sense that there was little interaction with other economies. It is said that economy in Easter Island collapsed due to the depletion of natural resources (mainly forests) as shown in Figure 1.

The economy of the BT model has the following characteristics. The economy has a renewable resource that is used to produce a harvested good.

The resource dynamics are determined by resource growth and harvesting activities. An additional input is labor, or population, and population growth is endogenously driven by a fertility function. The economy is decentralized in a sense that the relative prices of the goods and wages are determined by market forces. Meanwhile, although people as consumers individually maximize utility, the original BT model has one sector-level aggregate production function for each sector that is linear in labor, given the sizes of the existing resource stock and population. Another important characteristic of the model is that, as with many economic models, the economy is in equilibrium in each period. Hence there is no observable adjustment process towards equilibrium within each period.

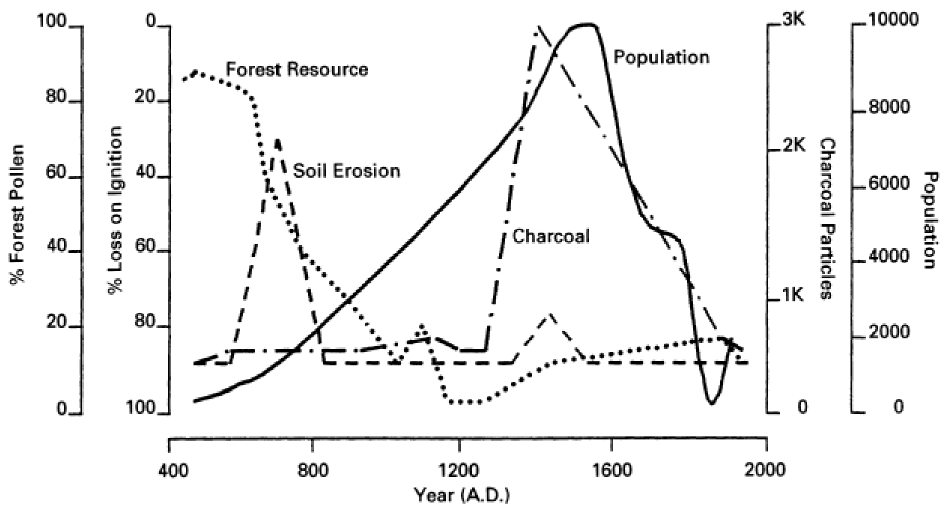

Figure 1. Behavior over Time for key Easter Island metrics
Source: Bahn, P., and J. Flenley, 1992

The aforementioned characteristics are consistent with a fully-decentralized economy where each individual as a consumer makes consumption choices independently and as a worker allocate his/her labor hours as an independent producer. Individuals in this economy behave in a myopic manner in a sense that they do not maximize their utility across multiple time periods and instead focus only on the current period, in contrast to the group of dynamic models in which economic agents are assumed to carry out infinite-time-horizon optimization of their choices. The model is interesting in part because its simplicity allows researchers to easily incorporate additional variables to address more fully various issues surrounding economic growth and sustainability, and because despite the simplicity its behavior is potentially quite volatile.

Mathematically the model described is represented as follows.

a) Consumer's choice

The BT model is a representative agent model, i.e., all individuals are assumed to be identical. An individual's optimal consumption choice is given by

$$\max_{\{h,m\}} \quad u = h^{\beta} m^{1-\beta} \qquad s.t. \quad ph + m = w$$

where u, h, m, p, w, and β are consumer's utility, his/her consumption of the harvested good, that of the manufactured good, relative price of the harvested good, his/her income(wage), and a parameter that indicates his/her preference for the harvested good, respectively. The solution of this problem yields each consumer's demand for the two goods as:

$h^D = \dfrac{\beta w}{p}$ and

$m^D = (1-\beta)w$.

Therefore, the aggregate demand for the two goods are given by $H^D = Lh^D$ and $M^D = Lm^D$ where L is the given population size in the given period.

b) Production Function

The production functions for the two sectors are given by:

$H^p = \alpha S L_H$ and

$M^p = L_M$,

where endogenous variables are H^p and M^p, the production levels of the harvested good and manufactured good, and L_H and L_M, the sizes of labor force assigned to the two sectors (note that $L_H + L_M = L$). Exogenous variables are α, a parameter representing the productivity in the harvested good sector, and S, the natural resource stock size that is fixed in any given period (as L).

The model assumes no explicit rental cost for using S, possibly representing a communal property ownership of the open-access resource stock. Assuming perfect labor mobility across the two sectors, the marginal revenue product of labor in both sectors must equal the wage, i.e., $p\alpha S = w = 1$, or equivalently, the price of the resource good must equal its unit cost of production, $p = w/\alpha S$.

c) Equations of Motion

The two stock variables S and L evolves from one period to another. Their evolution is given by the following two equations:

$$\frac{dS}{dt} = rS\left(1 - \frac{S}{S_{\max}}\right) - H \text{, and}$$

$$\frac{dL}{dt} = L\left(b - d + \phi\frac{H}{L}\right),$$

where parameter *r*, is the regeneration rate of the natural resource, S_{max} is the carrying capacity of the resource stock, *H* is the equilibrium harvested good level ($=H^D = H^p$), ϕ *is the parameter representing* the impact of per-capita consumption of the harvested good on population growth, and finally $b–d$ is the net birth rate.

This simple model has an analytic solution for the static equilibrium. Therefore, to run a model we can use the reduced-form equations:

$$\frac{dS}{dt} = rS\left(1 - \frac{S}{S_{\max}}\right) - \alpha\beta LS \text{, and}$$

$$\frac{dL}{dt} = L(b - d + \phi\alpha\beta S).$$

Using the reduced form, a system dynamics version can be drawn as shown in Figure 2.

Applying the set of parameter values provided by Brander and Taylor (1998), the simulation results shown in Figure 3 resemble qualitatively the reference behavior shown in Figure 1.

3. Background: Literature Review

Beltratti (1997) summarizes prior research regarding economic growth and the relationship with natural resources. Two of his main propositions for the direction of future research are: the interaction between endogenous population dynamics and resource use, and the relationship between endogenous innovation and substitutability among factors of input. Unlike other economic growth models that consider resources but not population dynamics (*e.g.*, Dasgupta and Heal, 1974; Eliasson and Turnovosky, 2004; Economides and Philippopoulos, 2008), BT-type models specify population as functions of endogenous variables such as the consumption level of the harvested good, changes in the resource stock size, and economic benefits provided by children. Although the original BT model lacks any specification for innovation processes, its descendents incorporate exogenous or endogenous innovations that enhance efficiency in harvesting or manufacturing activities, productivity of land resources, and the growth rate and carrying capacity of the natural resource stock.

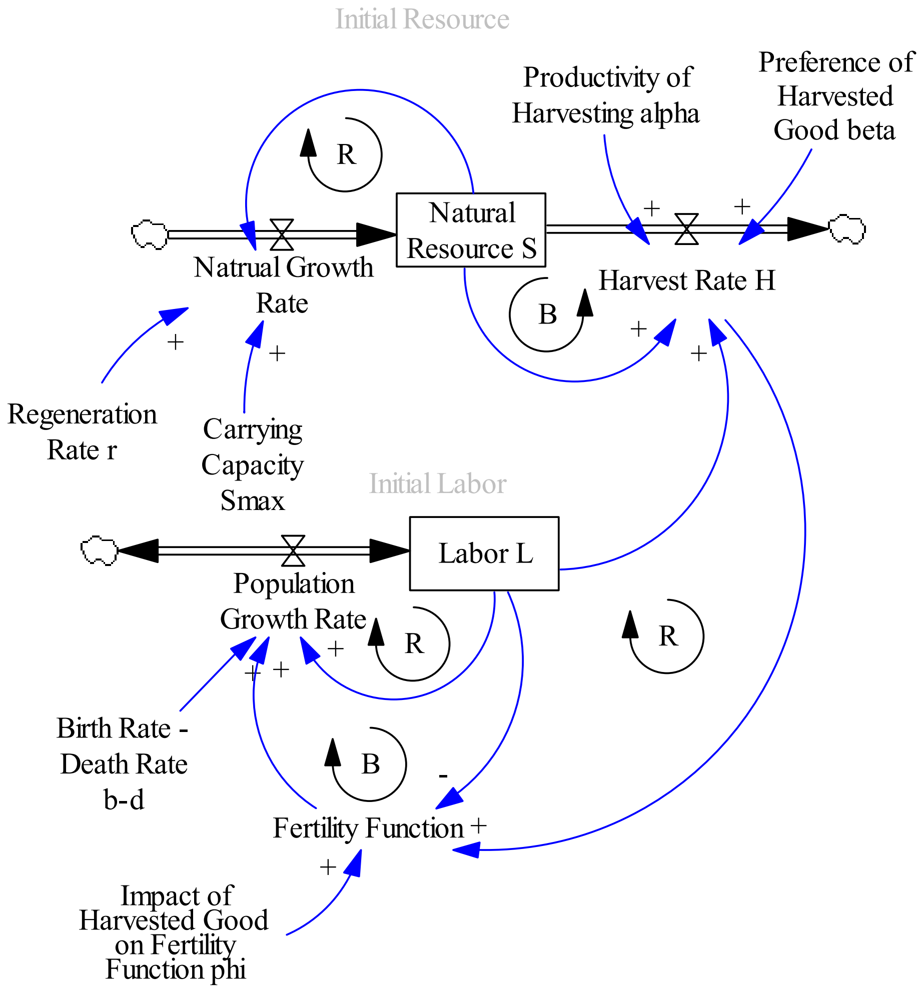

Figure 2: System dynamics flow diagram for BT model

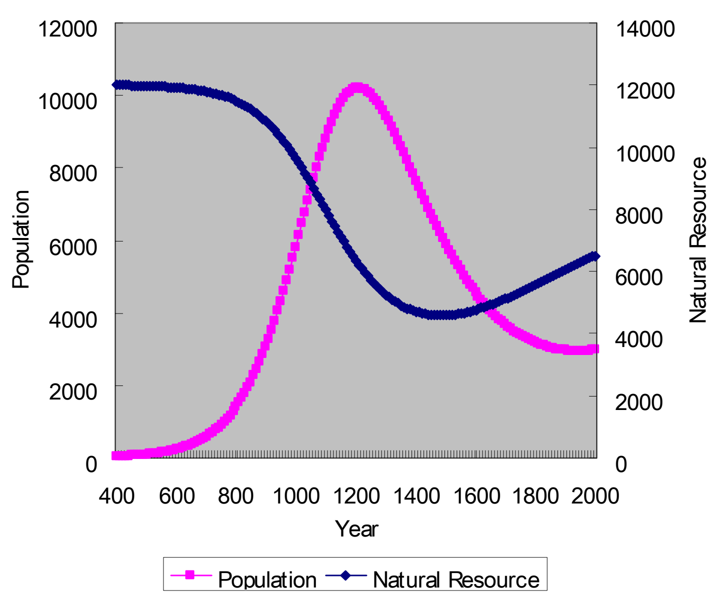

Figure 3: Time behavior plot for system dynamics version of BT model

Both casual observation and data suggest that at the start of an industrializing economy, resource inputs and output levels are closely correlated with each other. But as the economy develops, resource inputs tend to be replaced by capital (Ayres, 1998). To be consistent with this stylized fact, an enhanced BT-type model should allow for the substitutability between natural and man-made inputs to change over time.

Another attribute that is essential for addressing substitutability is the accumulation of man-made capital, a principal feature of Ramsey growth models (Ramsey, 1928). A few of the BT-type models in the literature (c.f., Erickson and Gowdy, 2000; Anderies, 2003) do in fact incorporate manufactured capital accumulation.

3.1. Population Growth Submodel

Although population growth has been treated as exogenous in many studies on economic growth and natural resources, endogenous population dynamics is indispensable for models whose purpose is to address resource scarcity problems. Empirical case studies support that there is a feedback mechanism between population growth and natural resources (*e.g.*, Diamond, 2004). In general, population dynamics models use ordinary differential equations:

$$\frac{dL/L}{dt} \equiv f(\text{weather, food, predators, etc.})$$

where L denotes the population size.

As shown below, population change per time period is typically defined as the sum of fertility at the individual level per time interval. Since a feedback mechanism between population and natural resource is essential, it is better to discuss population dynamics along with resource dynamics. The most popular framework for modeling this type of predator-prey interactions has the following structure called the Lotka-Volterra framework (Turchin, 2003):

dS/dt = "prey growth in the absence of predators" – "total killing rate by predators"

where S denotes the natural resource stock and

dL/dt = "predator growth (or decline) in the absence of prey" + "conversion of eaten prey into new predators."

The basic idea is that the right-hand side of each equation consists of two parts. The first part of each equation indicates the independence of one stock variable from the other, while the second part shows the interdependence between the two stock variables.

The original Lotka-Volterra model is a pure resource-consumer system, meaning that the resource stock (prey) grows exponentially (excluding the predation) and that the predator population also grows exponentially (the net growth rate is subject to the consumption of prey). Meanwhile, as shown in Section 2, the original BT model uses the Volterra (1931) Model in which the growth rate of a natural resource is logistic (excluding the harvest), constrained by its own density, and the population growth rate depends on per-capita consumption of the prey (as cited in Turchin, 2003).

The original BT model expresses Malthusian population dynamics in which population growth consists of two parts, the exogenously given net birth rate ($b\text{–}d$) and the fertility rate ϕ that affects the population growth only with nonzero level of H/L. Since $b\text{–}d$ is assumed to be negative, in the absence of harvest from the nature the population will be extinct.

This population growth function has two notable traits. First, population growth rate is linear in H/L, which implies that the more they eat the more they produce offspring. This may contradict situations in some developed countries where there is negative relationship between income level and population growth. Second, it assumes that consumption of the manufactured goods such as medicine, fishing equipment, boats, agricultural equipment, etc. does not affect the population growth. However, the effects of the consumption of manufactured goods on population growth does matter when one takes into account substitutability issues and the effects of capital accumulation.

Descendants of the BT-type model fall into two groups regarding population dynamics. One group uses the Lotka-Volterra framework, with slight modification. The second group employs population growth functions that are very different from those used in the original BT model.

Compared with the archaeological evidence from Easter Island, population in the original BT model peaks far too early. To explain this gap and to improve the fitness of the model, the authors introduce several changes, none of which satisfactorily resolve the issue. Other researchers provide further modifications to the specifications for fertility, resource scarcity, death rates, conflict and other factors (D'Alessandro, 2007; Reuveny and Decker, 2000;, Maxwell and Reuveny, 2000; and Prskawetz et al, 2003).

In contrast, some researchers elect to depart entirely from the Lotka-Volterra framework and adopt the logistic predation model, originally proposed by Leslie (1948) and expressed by Basener and Ross (2005) and Basener et al. (2008), as follows:

$$\frac{dS}{dt} = r\left(1 - \frac{S}{K}\right)S - \bar{h}L$$

$$\frac{dL}{dt} = \left[a\left(1 - \frac{L}{S}\right)\right]L$$

where a and r are the intrinsic growth rate of population and natural resource, respectively, and $\bar{h}$ is a fixed level of per-capita consumption of the harvested good. These models do not have the fertility component that represents the conversion of eaten prey into new predators, and they tend to show better fitness to the archeological data. An additional advantage of this population function is that it avoids the BT model's tendency to create arbitrarily large population growth, because the logistic function caps population growth based on a carrying capacity. Meanwhile, because the per-capita consumption level of the harvested good remains constant in these models, scarcity does not affect individuals' economic activities, in direct contradiction with neoclassical economic theory.

Thus, current thinking suggests that the Lotka-Volterra framework should be retained, since it is more consistent with neoclassical economics regarding population growth. Our model introduces a manufactured good with the population growth function in order to capture the effect of broader economic activities on the population dynamics, and also to constrain population growth by natural resource availability.

3.2. Substitutability

The results on empirical studies on the elasticity of substitution between man-made input and natural resources (σ) is mixed. Nordhaus and Tobin (1972) calibrate a value of 2.0 for σ at the macro level for the US economy, implying that the conditions for strong sustainability were not met at the time. Markandya and Pedroso-Galinato (2007) use a nested CES production function and a multinational database and provide updated estimates for σ. Their use of more recent data yields values of 1.00 between most factors and .37 between capital and energy. As economies develop, the relationship between energy and capital evolves from complementary to substitution (Ayres, 1998), therefore the changes in estimated values are not particularly surprising. If one stands by the premise of strong sustainability, then for a theoretical model to depict such and economy the value of σ must be less than one between man-made and natural inputs (Lawn, 2003).

Although several BT-type models incorporate innovation, none of them address substitutability, which is peculiar, since the primary purpose of the BT-type models is to study sustainability. Particularly problematic in the original model is that neither natural resources nor man-made capital enters the production function of the manufactured good.

Other BT-type models employ variations of Cobb-Douglas (C-D) functions. Among them, Anderies' (2003) model is the most general in a sense that both H and M are functions of labor and manufactured capital (A):

$$H = E_H S^{\alpha_S} L_H{}^{\alpha_H} A_H{}^{1-\alpha_H} \text{ and}$$

$$M = E_M L_M{}^{\alpha_M} A_M{}^{1-\alpha_M} ,$$

where E_H and E_M are efficiency factors, α_S, α_M and α_H are between 0 and 1, and $A_H + A_M = A$.

While introducing man-made capital is critical to address substitutability, the equations above do not allow the model to address the changes in substitutability between inputs for M, because with C-D functions, σ is one. Nagase and Mirza (2006) employ a CES function $M = [\theta H_M{}^{\rho} + (1-\theta) L_M{}^{\rho}]^{1/\rho}$ where H_M denotes the amount of harvested good used as input. Their study provides sensitivity analyses with respect to different values of σ and finds that reduced substitutability negatively affects the population size, individual's well-being, and the volatility of the system.

A different approach by Prskawetz et al. (2003) adopts a production function $H(S, L) = eSL_H(fL_H + S)^{-1}$, where e and f are positive parameters. As with C-D or CES, this function exhibits diminishing returns and has a constant σ (= 0.5). A unique feature of this function is that, for a given level of an input, the output is asymptotically bounded from above as the other input level goes to infinity.

An ideal model structure would make σ endogenous. Also, introducing Prskawetz et al.'s (2003) production function for the manufactured good has an advantage, because it let the system's resource stock size caps the output level, consistent with the notion of strong sustainability. Combined with the introduction of man-made capital as an input, this structure would allow us to examine the trade-off between man-made and natural capital under the strong sustainability criterion.

3.3. Innovation

The economics literature of endogenous technological change (ETC) or induced technological change (ITC) strongly supports these phenomena, theoretically and empirically. In theory, economic agents respond to changes in relative prices that signal the state of relative resource scarcity, and findings of empirical studies on the historical evolution of technologies are consistent with the theory (*e.g.*, Khatri et al 1998; Thirtle et al. 1998; Popp, 2002).

Because the original BT model does not depict innovation processes, the authors' analysis on the effect of innovation is limited to the comparative statics analysis for exogenously given technological changes. In their model, a better harvesting technology (larger α) reduces the steady-state resource stock size S^*, meanwhile, the effect of an increase in α on L^* depends on the size of the steady-state resource growth. An innovation in biotechnology (increases in r or K) boosts the steady-state population size L^*. The negative effect of innovation in the harvesting sector is consistent with other studies. As shown above, Anderies (2003) adopts C-D production functions for H and M, with efficiency factors E_H and E_M. The author also introduces into his model η, a parameter that represents negative impacts of harvesting activities on the resource base. According to this model, higher productivity (larger E_H or E_M) increases the likelihood of population overshooting and collapse, and reduced externalities (smaller η) fails to prevent this scenario without systemic changes in the feedback loop between resource use and population. D'Alessandro (2007) also finds that larger α reduces the resilience of the internal steady state and increases the risk of a collapse of S.

Two of the BT-type models introduce clear specifications for innovation processes. Reuveny and Decker (2000) employ time-dependent logarithmic and exponential growth functions for K, r, and α. Their simulation results show two intuitively sound results: innovation in harvesting technology, ceteris paribus, can cause a population crash due to resource depletion, and higher resource growth rates, ceteris paribus, can sustain larger population sizes. Dalton et al. (2005) introduce ETC through two differential equations: $\alpha_t = \alpha_{t-1}[1 + \xi_\alpha \lambda(dL/L)]$ and $r_t = r_{t-1}[1 + \xi_r \lambda(dL/L)]$ for $dL > 0$ and $\alpha_t = \alpha_{t-1}$ and $r_t = r_{t-1}$ otherwise, showing that changes in L (embodiment of the existing knowledge and experience with technologies) affect the sizes of α and r. Compared with the original BT model, making α and r endogenous following these rules ,ceteris paribus, worsens the feast-famine cycle.

These studies generally indicate that stimulating harvesting technology could be bad for the system, whereas bio-technologies may be good. However, these models do not show the effect of continuous innovation driven by scarcity and market prices on the stability of the system and its agents' well-being, due to the lack of variables that would allow the technology parameters to evolve in response to changing relative scarcity of productive resources, including natural or man-made capital. By introducing scarcity- and policy-driven ETC, this type of model can help us understand the interactions between population, resource use, and the stability of the economy.

3.4. Capital Accumulation

Two aspects of the existing BT-type models that hinder the introduction of man-made capital accumulation are property rights and time preferences. Motivating agents to preserve resource stocks requires proper assignment of property rights and their secure ownership. A weak property right regime provides little assurance to individuals that their property ownership will be secure, discouraging them to invest for the future. Accumulation and maintenance of any form of capital (man-made or natural) takes place only when agents in the economy care about the future; in the BT model, agents are myopic. Some of the descendent models attempt to address these issues. Anderies' (2003) production functions of H and M include man-made capital (A), and capital accumulation follows the standard definition of the difference between the endogenously determined investment level and the exogenously given capital depreciation. However, the model does not address the effect of an ETC on input substitutability. Good and Reuveny (2006) change the consumer's choice into a dynamic, multi-period optimization, potentially allowing the authors to introduce savings activities and man-made capital accumulation as typically done in Ramsey growth models; however, their model does not include man-made capital.

3.5. Modeling Approach

Dynamic modeling often faces trade-offs between the complexity of real economic activity and the need to make assumptions to simplify the mathematics. Certain functions are popular because one can obtain analytic solutions, but they do not necessarily represent the intended relationships between the relevant variables.

Historically most of the BT-type models employ linear or C-D production and utility functions. These functions are easy to solve for equilibrium outcomes but restrict the models' ability to address substitutability issues. Nagase and Mirza (2006) employ CES functions and show that reducing substitutability between man-made and natural inputs/goods increases instability in the population and resource dynamics and reduces individual welfare. However, in their analysis, values of σ are given exogenously and they do not evolve over time. Given that the critical aspect of substitutability is not its static value but its rate of change over time (Beltratti, 1997), introducing endogenous innovation processes into a model can help address the impact of technological progress on substitutability.

Neoclassical optimal growth models tend to employ linearly homogeneous functions so that steady-state growth rates can be expressed in per-capita terms. However, whether short-run and long-run steady states must exist in a model of population and resource dynamics is a point of debate. While this is the generally accepted approach, in reality an economy may never reach a steady state due to a continuous change processes and disruptive forces that cause instability, such as sudden, non-marginal environmental changes (Scrieciu, 2007; Barker, 2008). With few exceptions, BT-type models represent the blending of a static general equilibrium model and a differential equation-based simulation model. Such models typically require period-by-period static equilibrium. Resorting to numeric solutions can at least relax the above-mentioned constraints on the choice of functional forms and expand the model's scope of analyses.

3.6. Summary

A powerful BT-inspired dynamic simulation model of a small economy would have the following features:

3.6.1. Population growth: a Lotka-Volterra framework that explicitly incorporates a neoclassical economics view of population growth, with growth rates being a function of both harvested and manufactured goods, can account for the broader effects of economic activities on population.

3.6.2. Substitutability: Sustainability is a major issue in population and resource dynamics, and a model must address substitutability issues through endogenous evolution processes for the degrees of substitutability.

3.6.3. Innovation: Scarcity-driven endogenous innovation processes are essential to understand the inherent forces that direct resource allocation and also to reveal the system's transitional adjustment processes.

3.6.4. Capital Accumulation: Man-made capital as an input is essential to address substitutability issues. An equation of motion for capital accumulation driven by relative scarcity of inputs would be consistent with both economic theory and the motivation behind the development of this type of model.

3.6.5. Modeling Approach: Decentralized optimization by economic agents through market transactions is a more realistic modeling approach than a central planner's maximizing the infinite sum of the representative agent's welfare. Even within the standard framework of the BT-type model one can employ less "convenient" functions and rely on numerical methods to calculate solutions over time.

4. Implementing Extended Model in System Dynamics

We begin with the extensions to provide a base model with features described in 3.6. Although the features described in 3.6.2 and 3.6.3 are not incorporated into the base model, the foundation provided by the base model allows for these features to be easily added by making certain parameters dynamic rather than static.

4.1. Model Equations

With the selected extensions added, the model no longer has an analytic solution for the steady state values. Optimizations must be carried out to determine the steady state.

<u>1. Consumer optimization</u>

Consumers are save certain portion of money (sY) for investment that is used for formulation of man-made capital.

$$\max u = h^{\beta} m^{1-\beta} \quad s.t.\ p_H h + p_M m = (1-s)Y$$

<u>2. Harvesting Sector</u>

Harvesting sector solves the same profit maximization problem as the original BT model.

$$\max \pi(L_H) = p_H \alpha S L_H - w L_H$$

<u>3. Manufacturing Sector</u>

Manufacturing sector uses a CES production function, which enables us to study substitutability between natural capital H_M and man-made capital K. Although a CES function gives constant elasticity of substitution, it is possible to make it change dynamically by making substitution parameter ρ endogenous. For example, we can change it as a function of scarcity of natural resource such as the price of harvested good, p_H.

$$\max \pi(L_M, H_M) = p_M a L_M [\gamma H_M^{\rho} + (1-\gamma) K^{\rho}]^{\frac{1}{\rho}} - wL_M - p_H H_M - rK$$

4. Equations of Motion

We have three equations of motion, including capital formation. In addition, we assume that fertility rate is affected not only by harvested good but also by manufactured good (e.g., medical equipment.).

$$\frac{dS}{dt} = rS\left(1 - \frac{S}{S_{\max}}\right) - \left[H_M + H_H\right]$$

$$\frac{dL}{dt} = L\left(b - d + \phi_1 \beta \frac{H_D}{L} + \phi_2 \frac{M}{L}\right)$$

$$\frac{dK}{dt} = sY - \delta K$$

4.2. System Dynamics Implementation of Extended BT-type Model

In general, economics does not use models without analytic solutions. System dynamics (SD) can contribute in two ways. First, SD models can let the system solve the optimization problem, using the more realistic assumption that it does in fact take time to reach equilibrium in the real system, in contrast to instantaneous optimization used in the original BT model. Second, in SD the optimization processes are meaningful and are "white box" (the logic is transparent), and therefore the Baker criterion is satisfied. Among the descendants of the BT model, Good and Reuveny (2006) use numerical approaches (the Pontryagin formulation and the Davidon-Fletcher-Powell algorithm) to solve the dynamic optimization problem. However, these searching methods are black box and their searching processes may have little realistic meaning. In contrast, optimization in system dynamics strives to represent the actual processes taking place in the real system.

The basic idea is to let the system to find L_H, L_M and H_M with associated prices and wages to satisfy the following first order conditions and market conditions obtained from the above-mentioned model.

4.2.1. First order conditions

$$\frac{\partial \pi}{\partial L_H}: \; w = p_H \alpha S$$

$$\frac{\partial \pi}{\partial L_M}: \quad w = p_M a[\gamma H_M^{\rho} + (1-\gamma)K^{\rho}]^{\frac{1}{\rho}}$$

$$\frac{\partial \pi}{\partial H_M}: \; p_H = p_M a L_M \gamma H_M^{\rho-1}[\gamma H_M^{\rho} + (1-\gamma)K^{\rho}]^{\frac{1}{\rho}-1}$$

4.2.2. Market Equilibrium Conditions

$$\text{Harvested Goods Market}: \alpha S L_H = \frac{L(1-s)\beta M}{P_H} + H_M$$

$$\text{Manufactured Goods Market}: \; a L_M [\gamma H_M^{\rho} + (1-\gamma)K^{\rho}]^{\frac{1}{\rho}} = \frac{(1-s)(1-\beta)YM}{P_M} + \frac{sYM}{P_M}$$

4.2.3. Optimization Approach

The basic idea for solving the optimization problems is sometimes described as "hill-climbing" and is formally referred to as a gradient method. In the model, both an "indicated" value and an actual value are calculated, along with their difference (indicated value minus the actual value). A flow *into* the actual value is also represented (this is the rate of change or derivative of the actual value). This flow equals the difference divided by an adjustment time parameter. This is a simple first order "goal-seeking" or "balancing" feedback loop. This sort of logic is repeated for each variable that is allowed to seek a dynamic equilibrium value.

4.2.4. Causal Loop Diagram

Vensim reports that the model has 371 loops, mostly balancing. Clearly, a simple and straightforward causal loop diagram is not possible for this model. Instead, we have colored a few of the loops on the flow diagram shown in the next section. We recognize that having this many loops indicates that the yet-to-be-completed verification and validation process will be essential to establish model credibility.

4.2.5. Model Flow Diagram

The system dynamics flow diagram, shown in Figure 4, is complex and would benefit from additional attention to minimizing line crossings, judicious use of shadow variables, colors, line weights, etc. This process is well underway and will be completed shortly, and prior to the arduous test process that will commence shortly.

With the exception of the variables subject to dynamic optimization, the equations are entirely drawn from the economics literature discussed in Sections 2 and 3. The logic for the optimization variables was described in Section 4.2.3. The parameters were also taken from the literature whenever possible, and initial estimates for the adjustment time constants and the initial values for the state variables were entered and modified experimentally (within plausible ranges) such that steady state is attained in a few time periods.

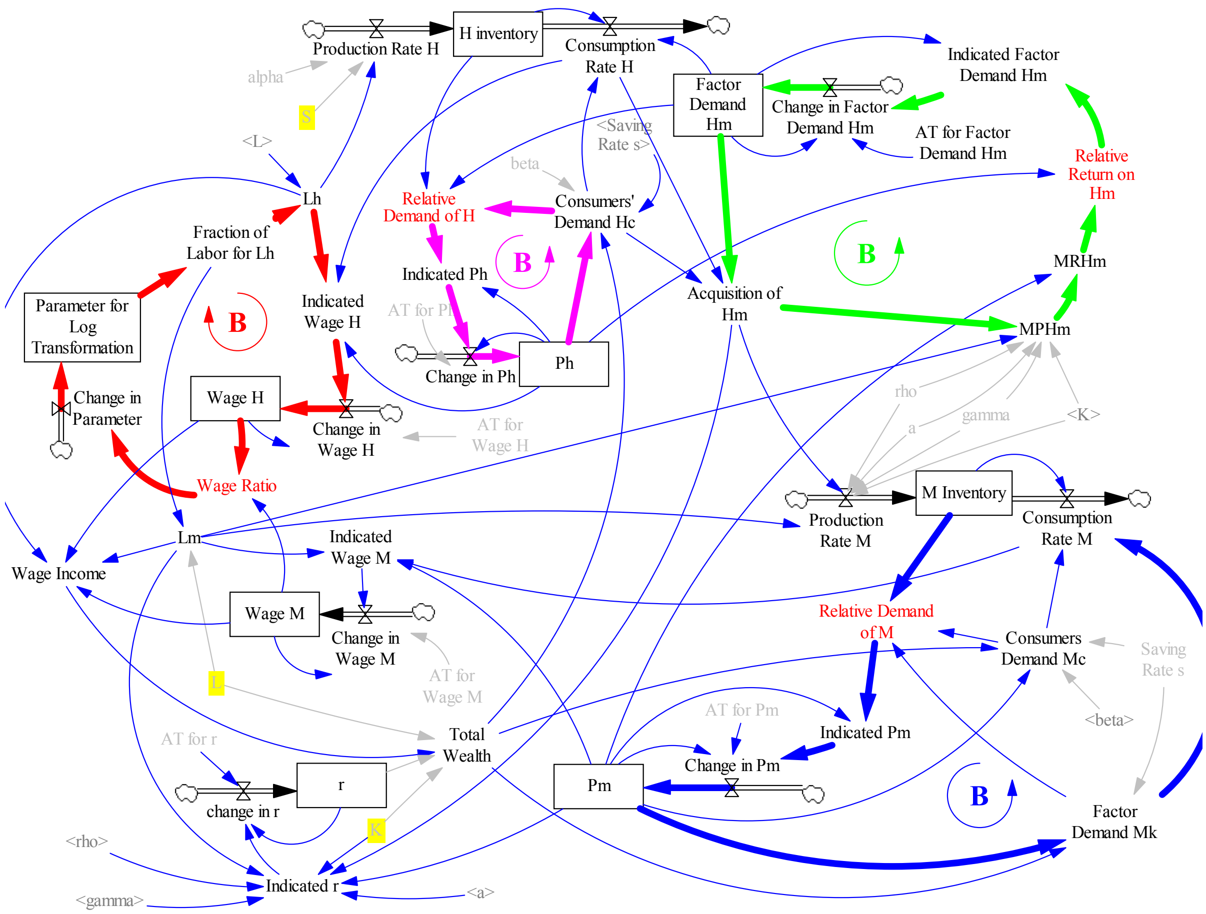

Figure 4: Extended BT Model System Dynamics Flow Diagram. 100's of loops.

4.3. Preliminary Results

Figure 5 shows a preliminary result that helps to establish the potential value of the model as a research tool because key ratios come into equilibrium at a value equal to one.

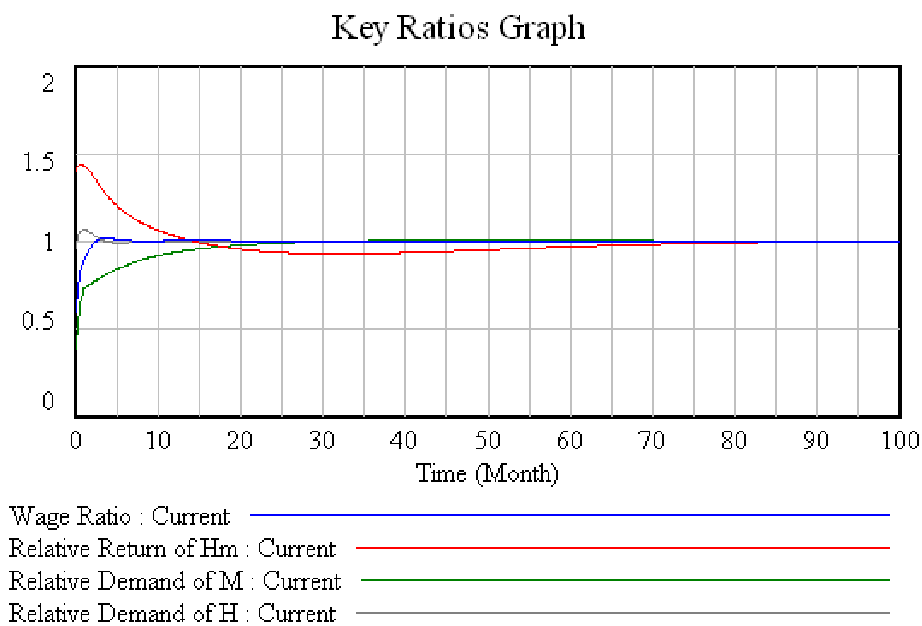

Figure 5: Baseline model run showing four key ratios achieving and remaining at a value of one, as required by economic theory.

Figure 6 shows a second run in which the model has been disturbed from equilibrium at time 50 using a STEP function to change the value of k from 1 to 2.

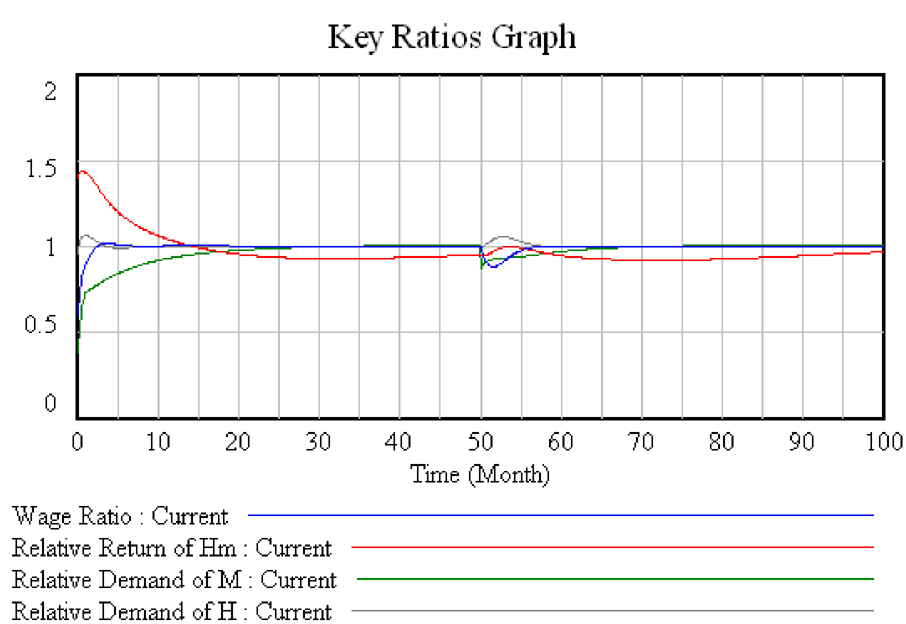

Figure 6: Model results when k is doubled at time 50

As shown in Figure 6, the doubling of k (manmade capital), has only a modest impact, and the model quickly compensates, except for the Relative Return of Hm, which responds much more slowly, indicating a potential parametric problem.

A bigger concern, however, as shown in Figure 7 is that prices and wages spiral upwards without limit, led by the price of the harvested good.

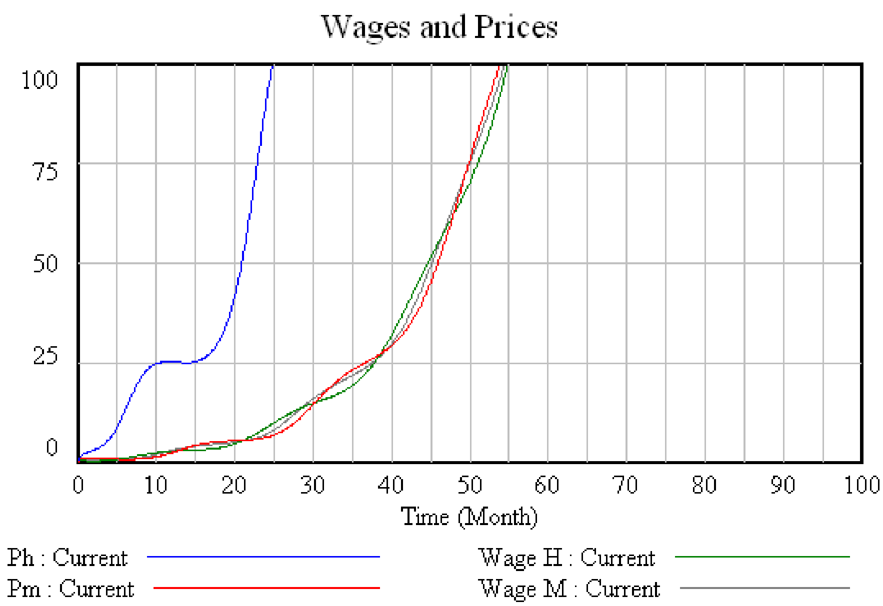

Figure 7: Prices and wages spiral upward without limit

Many approaches to stabilizing prices were tested, including introducing the accumulation of savings, which introduces a lag, but the only approach found thus far to stabilize prices and wages is to add a fraction of the current production amount to the inventory amount when calculating current price. Experimentally, when the fraction is much less than .5, prices and wages increase; whereas when the fraction is much greater than .5, prices and wages decline. As shown in Figure 8, when the fraction is set to .55, prices and wages are stable. We do not have theoretical support for this approach, nor for the particular value that results in stability. However, research regarding the behavior of beer game players has found that people seem to "account for" perhaps 1/3 of the product in the supply chain when making decisions (citation needed). It seems plausible that when more of the supply chain is considered, product will seem less scarce, and prices will be less likely to increase.

4.4. Preliminary Model Robustness Testing

The model is currently being tested following the guidelines outlined in Sterman (2000). First, several key parameters were varied to determine model robustness. Although the model is highly sensitive to the "fraction of production

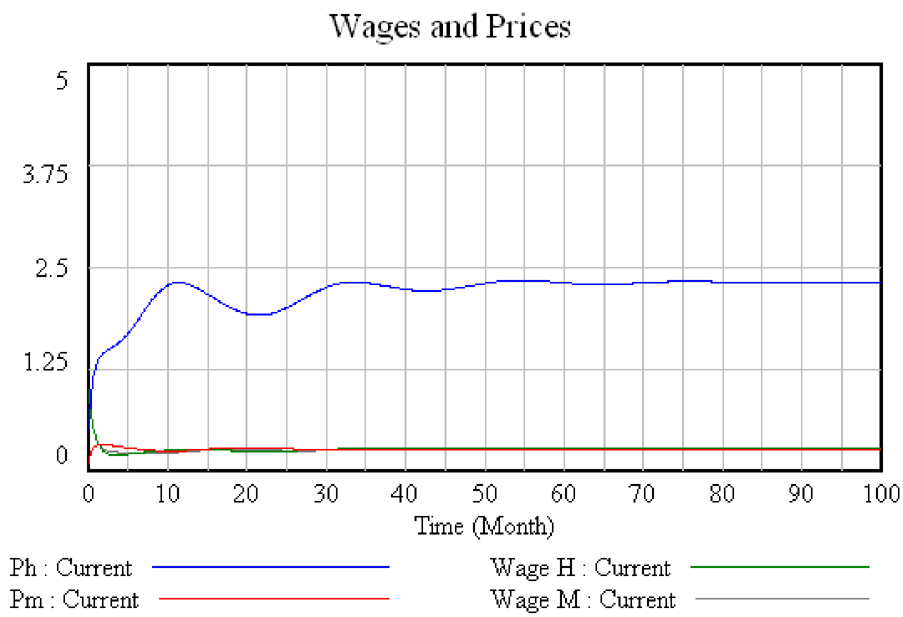

Figure 8: Prices and wages when the expectation of future production is considered

considered" parameter described in Section 4.3, model sensitivity to several other parameters is moderate, including the productivity of harvested good (alpha), the savings rate (S), gamma, the elasticity of substitution (rho), the preference for harvested good (beta), and the various adjustment time parameters.

4.5. Future Work

Once the inflation issues have been addressed to our satisfaction, and the model has been properly tested, we plan to then enhance the model in order to make K, L, and S endogenously dynamic so that the model can serve as a useful new tool for ecological economics research.

5. Discussion

Despite the problems with inflation, the incomplete model testing, and the scarcity of data regarding adjustment times, we are optimistic that the model will prove to be robust and will eventually be able to serve as the foundation for conducting informative experiments regarding the interplay between economic and environmental systems operating under constrained resources.

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