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# Prospective Elementary Mathematics Teacher Content Knowledge: An Introduction

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> *The first and fundamental requisite for every teacher is that he have thorough command of the subject matter which he teaches; that he have mastered it so well that he speaks with his own authority; only so can he hope to lead the pupil to the corresponding feeling of independent mastery.*
>
> *– J. W. A. Young, 1920*

This Special Issue on the mathematical content knowledge of prospective elementary teachers (PTs) provides summaries of the extant peer-reviewed research literature from 1978 to 2012 on PTs' content knowledge across several mathematical topics, specifically whole number and operations, fractions, decimals, geometry and measurement, and algebra. Each topic-specific summary of the literature is presented in a self-contained paper, written by a subgroup of a larger Working Group that has

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collaborated across several years, resulting in this Special Issue sharing the final work. The authors hope this summative look at prospective teacher content knowledge will be of interest to the mathematics education community and will be a useful resource when considering future research as well as designing mathematics content courses for prospective elementary teachers.

The following sections in this issue provide background information on our overarching framework for the mathematical content knowledge of prospective elementary teachers as well as our rationale for conducting the summary of research. We briefly describe the intent and history of the Working Group that conducted the summaries, followed by the methods utilized in the summary process. Finally, we provide a description of what follows in each subsequent paper and close with our intentions of how this Special Issue might be used by our readers.

## Background

The mathematical preparation of K–8 students is a challenge both in the United States and internationally. Studies from many countries report students coming away from their elementary education having memorized facts and procedures with varying degrees of success but not developing robust mathematical conceptions or flexibility in their reasoning (e.g., Reys et al., 1999; Stigler & Hiebert, 1999). Students who struggle in school mathematics have limited career options. Even those who perform well in mathematics courses are unlikely to enjoy mathematics or take an interest in science, technology, engineering, and mathematics careers if they experience the subject as dry and procedurally focused. Mathematics instruction can emphasize conceptual understanding and the engagement in mathematical practices. In order to positively influence the

direction of mathematics teaching and learning, our elementary teachers must be adequately prepared.

Research over the last few decades has shown that the work of teaching mathematics requires a different knowledge base than the mathematical knowledge required for other professions (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Conference Board of the Mathematical Sciences [CBMS], 2012). Ball and her colleagues identify this knowledge as *Mathematical Knowledge for Teaching* (MKT), which they define as the "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (Ball et al., 2008, p. 399). They developed a framework for MKT consisting of two domains, subject matter knowledge and pedagogical content knowledge (see Figure 1).

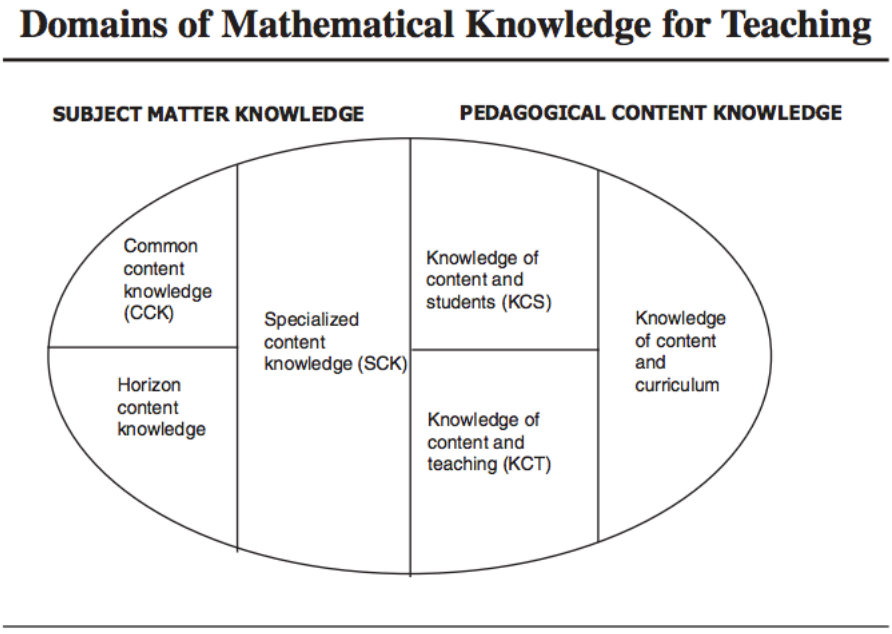


*Figure 1*. Mathematical knowledge for teaching framework (Ball et al., 2008, p. 403).

For these summary papers, we chose to focus primarily on what could be considered prospective teachers' subject matter knowledge, or *content knowledge*. Included in mathematics content knowledge are *Common Content Knowledge* (CCK), which is described as the mathematical knowledge that everyone should know; *Specialized Content Knowledge* (SCK), described as the mathematical knowledge that is special to the work of teaching; and *Horizon Content Knowledge*, which is an understanding of how mathematical topics fit together and make up a curriculum. In addition to these three types of knowledge, we also include *Knowledge of Content and Students* (KCS), which involves understanding students' thinking and difficulties with mathematics. While Ball and her colleagues include KCS in pedagogical content knowledge, we argue that understanding children's thinking can help in the development of PTs' specialized content knowledge, and thus should be included in a summary of PTs' content knowledge.

Unfortunately, PTs often do not come to teacher education with adequate subject matter knowledge (e.g., Ball, 1990). Even after having taken their college mathematics courses, many PTs do not reason mathematically in flexible or sophisticated ways (e.g., Yang, Reys, & Reys, 2009). These deficiencies in mathematics content knowledge are also seen in practicing elementary teachers in the United States and many other countries (e.g., Ma, 1999; Reys et al., 1999). Thus, there is evidence that PTs' mathematics content knowledge does not necessarily improve on its own once they become teachers. Rather, the responsibility falls on mathematics teacher educators to help PTs develop a strong base in their content knowledge during their college years.

Researchers, teacher educators, and organizations have noted the need to improve PTs' mathematics content knowledge and have called for efforts to that effect (Ball, 1990;

CBMS, 2001, 2012; Mathematics Teacher Preparation Content Workshop Program Steering Committee, 2001). Research is necessary to support these efforts. In particular, research concerning PTs' mathematical thinking in specific content areas would inform instruction. As the authors of *The Mathematical Education of Teachers* observe, "The key to turning even poorly prepared prospective elementary teachers into mathematical thinkers is to work from what they do know" (CMBS, 2001, p. 17). Mathematics teacher educators need to understand the conceptions with which PTs enter their classrooms in order to design instruction that builds on those conceptions (Brown, Bransford, & Cocking, 1999).

By reviewing mathematics education research concerning PTs' content knowledge, we seek to establish what is known up to this point in time. Summarizing what is known also enables us to identify areas for further research. We envision these summary papers to be a starting point for future directions in research on PTs' content knowledge and the development of that knowledge, as well as providing information useful in the design and development of mathematics courses for PTs.

## Brief History and Intent of the Working Group

The current set of authors has participated in one or more Working Groups at the Psychology of Mathematics Education–North American (PME-NA) Chapter, the National Council of Teachers of Mathematics (NCTM), or the Association of Mathematics Teacher Educators (AMTE) meetings on a regular basis since 2007 and has presented at several of those meetings (e.g., NCTM 2007, PME-NA 2009, AMTE 2009, AMTE 2010), as well as at the International Congress on Mathematical Education (ICME 2012) and PME 2013. At PME-NA 2007, the Working Group agreed on the need for the construction of a research base for the study of prospective teacher content knowledge. This included a need to summarize

existing (completed and current) peer-reviewed research and to develop a research agenda. At PME-NA 2009, the Working Group grew in membership and allowed for the work of summarizing the existing literature to be divided into five content areas: whole number and operations, fractions, decimals, geometry and measurement, and algebra. The group was divided into five subgroups with each focusing on one of the five content areas. The ultimate goal was to have each group produce a summary paper of the existing literature on prospective teacher content knowledge. A secondary goal was to establish continued collaborations and to grow professionally through developing our pedagogy, especially as it relates to the teaching of content courses for prospective elementary teachers. To achieve the first goal we designed the following research questions:

1. What research has been conducted on elementary prospective teachers' content knowledge?
2. What have we learned about prospective teachers' content knowledge?

At AMTE 2010, the group met to refine the guidelines for creating these individual paper summaries, and at PME-NA 2010, a rough draft of the combined summaries was refined. Over the years, we continued this collaboration and worked together to refine the methodology and the parameters of the overarching study.

## Methods

As the Working Group met over the years, the scope of the research review was extended. Initially in 2007, the Working Group focused on a "current perspective" to provide an in-depth description of what is known about prospective elementary teachers' content knowledge from a review of peer-reviewed research articles of the last decade. Initially this time period was 1998–2008, which was eventually extended to include 2010,

as the work of the group continued over several years. At a subsequent PME-NA meeting, the Working Group decided to include a "historical look" to describe what was known in the specific content area prior to 1998. Lastly, the Working Group made the decision to update the current perspective to include recent peer-reviewed articles in 2011 and provide a "view of the horizon" to present future directions that built upon prior sections by examining peer-reviewed articles in 2012 and conference proceedings published in 2011–2012. While the actual review process did not follow a chronological timeline, in this Special Issue, each content group presents its summaries of the literature on mathematics content knowledge of prospective teachers in the following three time periods: *A Historical Look, A Current Perspective*, and *A View of the Horizon*. Thus, the description of the common methods will follow this chronological timeline as well.

**A Historical Look**

For the time period prior to 1998, each content group conducted an ERIC search using general search terms such as *preservice, prospective, elementary, teacher, education,* and *content knowledge*, as well as specific content search terms such as *number, whole number, addition, subtraction, geometry,* and *algebra*. Combinations of search terms were entered into the ERIC database. Since all countries do not use the same grade-level classification system as the U.S., we decided to look at findings from studies of PTs preparing to teach children aged 3–14 to account for cases with combined middle and elementary certifications. The title and abstract of each research article resulting from the two searches were read to determine whether the article focused on elementary PTs' mathematics content knowledge. If the title and abstract did not suffice to make a determination of fit, reviewers read the whole article.

**A Current Perspective**

For the time period between 1998 and 2011, we conducted an ERIC search using the same keywords described in the historical perspective above. Likewise, a determination of fit for each article was made using the same process as described above. A list of 23 journals from which articles were found for summarizing was compiled.

Subsequently, it was brought to our attention that one article published between 1998 and 2011 in one of the 23 journals was not included in the results from our ERIC search. We then found that ERIC does not contain all years for all journals included in the database. Thus, each journal was then carefully reviewed for additional articles focusing on PTs' content knowledge between 1998 and 2011 to ensure all articles focusing on PTs' content knowledge in those identified journals were found. This review produced more articles that were published in a year not included in ERIC database or were not indexed with any of the previously listed search terms.

**A View of the Horizon**

For the final time period, 2011–2012, we conducted an ERIC search for the year 2012 using the same process described earlier. In addition, we reviewed the conference proceedings from both the International Group and the North American Chapter of the Psychology of Mathematics Education (PME and PME-NA) for the years 2011 and 2012. For this review, each content group carefully searched for keywords in the titles and abstracts of all papers found in the proceedings. If the title and abstract did not suffice to make a determination of fit, reviewers read the whole paper.

**Inclusion/Exclusion Criteria**

The Working Group established exclusion criteria across all content groups and excluded articles that had (a) a general description of content knowledge that lacked specific attention to three primary content areas (thus, our claims are restricted to these three content areas): numbers and operations (including whole numbers, fractions, decimals, and operations), geometry and measurement, and algebra; (b) a sole focus on perceptions about mathematics not connected to content knowledge needed for teaching (we make no claims about PTs' beliefs in this Special Issue); (c) a focus on describing classroom practice or activities with a lack of attention to research design methods; and (d) a primary focus on high school PTs, mathematics majors, or inservice elementary teachers (our claims are restricted to prospective elementary teachers). For each content group, at least two researchers met to discuss the inclusion/exclusion of articles in their related content area. All disagreements about inclusion/exclusion into the database were resolved through discussion.

**Database**

The database included peer-reviewed research articles focusing on the mathematical content knowledge of elementary PTs in any of the content areas described earlier. The studies in our database listed the reference, content area, research questions, study type, research design, lens or approach used, selection criteria, description of participants, conditions of and procedures for data collection, data analysis, findings, and conclusions and implications.

## Description of What Follows

The results of the Working Group are summarized in each of the subsequent papers in this Special Issue. Each paper focuses on a different content topic (whole numbers and operations, fractions, decimals, geometry and measurement, and algebra) and is organized into the following categories: historical, current, and horizon. The papers are presented as literature reviews in terms of what is known regarding PTs' content knowledge.

### A Focus on Whole-Number Concepts and Operations

The research illustrates that prospective teachers' knowledge of this topic is largely dependent upon standard algorithms for solving a given type of task. In addition, they struggle with justifying why the algorithms work. The authors note that more research is needed regarding the types of conceptions PTs have when entering teacher education programs, as well as a need to document how their understanding develops.

### A Focus on Fractions

Research shows that PTs are often able to solve fraction problems algorithmically but not justify the algorithm or represent the situation with a correct model, such as a word problem or diagram. In addition, PTs' understanding of fractions, in general, tends to be limited, in that most think of fractions only in terms of a part-whole interpretation. More research is needed to better understand PTs' conceptions of fractions and ways to improve their understanding, as well as to document how their understanding develops.

### A Focus on Decimals

Though PTs may be able to successfully solve computational problems with decimals, they tend to lack a conceptual understanding of the structure of decimals. Historical and current research illustrates that PTs' difficulties stem from their

understanding of place value, incorrectly transferring whole number algorithms to decimals, and with their understanding of the density of decimals. Future research is needed to systematically examine how PTs' understanding of decimals develops.

**A Focus on Geometry and Measurement**

Prior research has documented that PTs' understanding of geometry and measurement is limited largely to memorized procedures. Though the research literature does not address every topic within geometry and measurement, the research that has been done suggests that (a) the van Hiele levels, (b) dynamic geometry software, and (c) methods fostering PTs' understanding of concept images and definitions can be useful in improving PTs' conceptual understanding of these topics. Future research is needed to address ways to develop PTs' understanding of geometry and measurement as well as to address the topic gaps that still exist in the literature.

**A Focus on Algebra**

The summary of the algebra research shows that PTs can readily use symbolic representations with variables, expressions, and equations; however, they have difficulties with interpreting and connecting various representations to each other or to a problem situation. In addition, PTs' computational methods are often inflexible, inefficient, or incorrect. Recent research suggests developing PTs' understanding by focusing instruction around justifying multiple representations and solution methods. However, more research is warranted regarding how this understanding develops, as most has focused on either PTs' incoming conceptions or analysis of pre/posttests.

A common theme throughout the papers is that PTs' understanding of each topic is limited to using algorithms, and difficulties lie in their ability to justify why the algorithms

work. In addition, previous research has focused chiefly on describing PTs' conceptions of these content topics, whereas recent research is moving toward documenting how PTs' mathematical understandings develop. More research is needed as each summary paper in this Special Issue illustrates that the research base regarding PTs' understanding is limited.

## Final Thoughts

The intent of this Special Issue is to summarize and share what research suggests regarding the mathematical content knowledge of prospective elementary teachers. Have they met the expectations described in Young's 1920 statement of mastering the subject so well that prospective teachers can lead their students "to the corresponding feeling of independent mastery"?

Given the summary information provided, we believe this Special Issue could be a resource in:

- graduate course work and seminars, prompting ideas for future research directions and topics;
- the design and development of current and future research on the mathematical content knowledge of PTs, providing a summary of the extant research literature through 2012 in the selected mathematical topics;
- the design of mathematics content courses for PTs, providing information about PTs' common misconceptions, as well as strategies and tools that may help PTs work through the misconceptions and develop a better understanding of the mathematics;

- the design of content-specific professional development for elementary teachers, where information on misconceptions and strategies and tools for learning may still provide useful information for improving the teachers' content knowledge.

Thus, we hope this Special Issue will be a useful reference for future research as well as strategies for practice related to the development of the mathematical content knowledge of prospective elementary teachers.

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