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Scalar-wave approach for single-mode inhomogeneous fiber problems^{a)}

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It has generally been accepted that accurate results may be obtained using the scalar-wave approach to solve problems dealing with inhomogeneous multimode guided-wave structures. The problem of the applicability of the scalar-wave approach to obtain wave propagation characteristics in single-mode fiber or integrated optical circuit guides with inhomogeneous index profiles is examined in this paper. It is shown that if certain limiting conditions are satisfied, the scalar-wave approach will yield valid results for single-mode structures. These limiting conditions are usually satisfied by many practical single-mode inhomogeneous fibers or IOC structures.

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It has been universally accepted that the full set of Maxwell equations resulting in the vector-wave equation must be used to treat waveguides supporting single mode. This requirement confines the analytical treatment to only a few simple structures. The advent of optical-fiber guides as viable communication links as well as the dawn of small high-bandwidth integrated optical circuits demand that the analytical horizon be expanded. It is recognized that many more problems can be solved if only the scalar-wave equation needs be considered. The purpose of this paper is to investigate in depth the conditions under which the scalar wave equation can be used instead of the vector wave equation and to demonstrate, with concrete examples, the validity of the scalar-wave approach¹ in providing accurate results for the graded index fiber guide.

Starting with the vector-wave equation for the electric field vector $\mathbf{E}$ in the fiber structure,

$$\nabla \times \nabla \times \mathbf{E} - \omega^2 \mu_0 \epsilon \mathbf{E} = 0, \quad (1)$$

where ω is the frequency of the wave, μ_0 is the permeability, and $\epsilon(\mathbf{r})$ is the inhomogeneous permittivity of the structure; and making use of the vector identity

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \quad (2)$$

and the relation

$$\nabla \cdot \mathbf{E} = -\epsilon^{-1} \nabla \epsilon \cdot \mathbf{E}, \quad (3)$$

one has

$$\nabla^2 \mathbf{E} + \omega^2 \mu_0 \epsilon \mathbf{E} - \nabla(\epsilon^{-1} \nabla \epsilon \cdot \mathbf{E}) = 0. \quad (4)$$

Rewriting Eq. (4) gives

$$\nabla^2 \mathbf{E} + \omega^2 \mu_0 \epsilon_0 \left\{ \frac{\epsilon}{\epsilon_0} \mathbf{E} - \left[\frac{1}{\omega^2 \mu_0 \epsilon_0} \nabla \left(\frac{1}{\epsilon} \nabla \epsilon \cdot \mathbf{E} \right) \right] \right\} = 0. \quad (5)$$

The relative importance of the terms within the curly brackets can be determined from the following:

$$\frac{\epsilon}{\epsilon_0} \mathbf{E} = O\left(\frac{\epsilon}{\epsilon_0}\right) \mathbf{E}, \quad (6)$$

$$\frac{1}{\omega^2 \mu_0 \epsilon_0} \nabla \left(\frac{1}{\epsilon} \nabla \epsilon \cdot \mathbf{E} \right) = \frac{1}{k_0^2} O\left(\frac{\nabla \epsilon}{\epsilon} \cdot \nabla \mathbf{E}\right) = O\left(\frac{\epsilon/\epsilon_0}{k_0 l} \mathbf{E}\right), \quad (7)$$

where O means the "order of magnitude" and l is the smaller of the distances over which ϵ/ϵ_0 and E change appreciably. For single-mode fiber structures, the values of ϵ/ϵ_0 and $k_0 l$ are typically in the range

$$\epsilon/\epsilon_0 = O(2), \quad (8)$$

$$k_0 l \simeq (2\pi/\lambda)l = O(10^2 \text{ or } 10^3),$$

$$l \simeq O(10-100 \mu), \quad \lambda \simeq O(1 \mu) \quad (9)$$

It follows that the second term within the curly brackets in Eq. (5) is several orders of magnitude smaller than the first term, $(\epsilon/\epsilon_0)\mathbf{E}$. It is therefore justifiable to neglect the second term and write Eq. (5) in the form

$$\nabla^2 \mathbf{E} + k_0^2 (\epsilon/\epsilon_0) \mathbf{E} = 0 \quad (10)$$

The physical significance of replacing Eq. (5) by Eq. (10) is this. By discarding the term $\nabla(\epsilon^{-1} \nabla \epsilon \cdot \mathbf{E})$, we are neglecting any depolarization effects that may occur. This means that the wave retains the linear polarization it has at the source, which is evidenced by the fact that Eq. (10) can be reduced to a scalar equation by writing $\mathbf{E}(\mathbf{x})$ in the form

$$\mathbf{E}(\mathbf{x}) = \hat{e}_p u(\mathbf{x}), \quad (11)$$

where $\hat{e}_p$ is a unit vector in the direction of the initial polarization of the wave. Substituting Eq. (11) in Eq. (10), we find that $u(\mathbf{x})$ satisfies the scalar-wave equation

$$\nabla^2 u + k_0^2 (\epsilon/\epsilon_0) u = 0. \quad (12)$$

This equation with the boundary condition on the initial sur-

^{a)}Partially supported by Hughes IR & D.

FIG. 1. Comparison of the scalar-wave results with vector-wave results for the dispersion characteristics of HE_{11} and HE_{21} modes. The data points are scalar-wave results and the solid curves are exact vector-wave results. The core index variations are given by $n_1 = n_2' \epsilon^{1/2} = n_2' [b_0 - b_2 \rho^2]^{1/2}$, where $\rho = r/a$, n_2' is approximately the index of the cladding and a is the core radius.

face, and the radiation condition at infinity, completely specifies $u(x)$, from which we can then obtain the electromagnetic field vectors $\mathbf{E}$ and $\mathbf{H}$.

To verify the fact that the scalar-wave approach may be used to obtain accurate results for the case of wave propagation along single-mode inhomogeneous fiber structures, we shall now consider the special case of the dominant mode propagating in a fiber with a parabolic radial index profile. This case was chosen because exact vector-wave solutions exist for this problem. By comparing our results based on the scalar-wave approach with the exact results, one may verify the applicability of this scalar-wave approach to single-mode problems.

Vector-wave solutions exist for radially inhomogeneous fibers.^{2,3} Two methods can usually be used to obtain the propagation characteristics of dominant modes in these types of fibers: (1) the radially inhomogeneous cylinder is subdivided into a number of concentric layers and the problem is solved by matching the solution for each homogeneous layer at the subdivided boundaries;² (2) the problem for the radially inhomogeneous cylinder is formulated in terms of a set of four coupled first-order differential equations for the transverse field components, and direct numerical integration is then performed to obtain the propagation constants of the lower-order modes.³ Both methods have been used to obtain the dispersion characteristics of the dominant HE_{11} mode for a parabolic index profile fiber. Results are shown in Fig. 1. The normalized propagation constant $\beta c / n_2' \omega$ is plotted against the normalized frequency $n_2' \omega a / c$ in Fig. 1 for the following index profile:

$$n = n_2' b_0^{1/2} \left(1 - \frac{1}{2} \frac{b_2}{b_0} \rho^2 \right), \quad \text{for } \rho \leq 1, \quad (13)$$

$$n = n_2' b_0^{1/2} \left(1 - \frac{1}{2} \frac{b_2}{b_0} \right), \quad \text{for } \rho \geq 1, \quad (14)$$

with $\rho = r/a$, a is the radius of the inhomogeneous core, ω is the frequency of the wave, c is the speed of light in vacuum, and n_2' , b_0 , and b_2 are given constants. Data given in Fig. 1 will be used to check the accuracy of the scalar-wave results.

The scalar-wave solution for the propagating wave in a parabolic index profile medium can be written in the form⁴

$$u = u_0(x, y) \exp \left[-i \frac{2\pi n_0}{\lambda} - (m + n + 1)(n_2/n_0)^{1/2} \right] z, \quad (15)$$

where the propagating field is assumed to be linearly polarized, $u_0(x, y)$ is a real Hermite-Gaussian function, $\lambda = 2\pi c / \omega$, m and n are integers (0, 1, 2, ...), and the index profile is given in the standard Gaussian beam notation

$$n = n_0 \left[1 - \frac{1}{2} (n_2/n_0) a^2 \rho^2 \right]. \quad (16)$$

The values of n_0 and n_2 in this expression are related to n_2' , b_0 , and b_2 given previously by the relationships

$$n_0 = n_2' b_0^{1/2}, \quad (17)$$

$$(n_2/n_0) a^2 = b_2/b_0. \quad (18)$$

The propagation constant β for the field can be obtained readily from Eq. (15),

$$\beta = 2\pi n_0 / \lambda - (m + n + 1)(n_2/n_0)^{1/2} \quad (19)$$

or

$$\frac{\beta c}{n_2' \omega} = \sqrt{b_0} - (m + n + 1) \left(1 - \frac{1}{b_0} \right)^{1/2} \left(\frac{c}{n_2' \omega a} \right). \quad (20)$$

This is the analytic result for the normalized propagation constant as a function of the normalized frequency for various modes ($m, n = 0, 1, \dots$) in a parabolic index guide based on the scalar-wave equation. Numerical results calculated according to this equation are also displayed in Fig. 1 by the dashed curves. It is clearly seen that excellent agreement

FIG. 2. Comparison of the analytic scalar-wave results with the numerical FFT scalar-wave results for the adjusted phase shift of a Gaussian beam propagation along a parabolic index profile fiber with index $n(r) = n_0 [1 - \delta(r/a)^2]$. The data points are numerical FFT scalar-wave results and the solid lines are analytic scalar-wave results. $a = 50 \mu\text{m}$ and $n_0 = 1.5$. Computations were carried out for various free-space wavelengths $\lambda = 0.6, 0.8, 1.0$, and $1.2 \mu\text{m}$ and the corresponding beam radii $b = 10.03, 11.58, 12.95$, and $14.20 \mu\text{m}$ for $\delta = 5 \times 10^{-4}$ case and $b = 5.64, 6.51, 7.28$, and $7.97 \mu\text{m}$ for the $\delta = 5 \times 10^{-3}$ case [$\delta = (n_2/n_0) a^2 (1/2)$].

between the scalar-wave results and the vector-wave results is obtained. Only when the operating frequencies are near the cutoff frequency of the mode does any difference exist. In fact, the difference is *not* caused by the inadequacy or inaccuracy of the scalar-wave approach, but rather by the difference in our choice of index profiles for the vector and the scalar cases. In the vector case the index profile is no longer parabolic for $\rho \gg 1$, while in the scalar case it is always parabolic. By extending the parabolic index profile beyond $\rho = 1$ for the vector case, we can show (according to our computations) that the difference becomes very small indeed. It appears, therefore, that if the conditions described by Eqs. (8) and (9) are satisfied, excellent results may be obtained using the scalar-wave equation.

Let us now consider the case of the numerical solution of the scalar-wave equation by the FFT (fast Fourier transform) technique.¹ Of course, one must recognize that the important feature of this approach is in its ability in solving problems with nonparabolic index profiles; but to determine whether this approach may be used to treat single-mode problems, we shall compare the numerical results obtained according to this FFT approach for the parabolic index profile case with those obtained according to the analytic scalar-wave solution. Since we have already shown that the analytic scalar-wave solution is as good as the vector-wave solution for the situation under consideration, we can safely conclude that if the numerical FFT scalar wave solution checks out with the analytic scalar-wave solution, then this numerical FFT scalar-wave approach may be used with confidence for single-mode fibers with arbitrary index profiles as long as the limiting conditions specified earlier are satisfied.

Figure 2 shows the "adjusted phase shift" of the beam as it propagates down a parabolic index profile fiber. Adjusted phase shift is defined as the difference between the phase shift of a plane wave in an n_0 medium, $(2\pi n_0/\lambda)z$, and the actual phase shift of the beam; i.e., $\Phi_{\text{adjusted}} = (2\pi n_0/\lambda)z - \beta z$. The expression for the index profile is given by Eq. (16) with $n_0 = 1.5$ and $(n_2/n_0)a^2 = 0.005$. The data points refer to the numerical results obtained by the numerical FFT scalar-wave approach while the solid lines were calculated according to the formula

$$\Phi_{\text{adjusted}} = (2\pi n_0/\lambda)z - \beta z = (m + n + 1)(n_2/n_0)^{1/2}z, \quad (21)$$

which is the analytic expression [Eq. (15)] obtained according to the scalar-wave solution discussed earlier. For the dominant HE_{11} mode, $m = 0$ and $n = 0$. So, $\Phi_{\text{adjusted}} = (n_2/n_0)^{1/2}z$. It can be seen that excellent agreement was achieved.

In conclusion we can safely state that our numerical FFT scalar-wave approach can be used with confidence to solve single-mode problems dealing with fiber or IOC structures with general inhomogeneous index profiles provided that the following conditions are satisfied:

(a) The depolarization effects are negligible, or

$$\left| \frac{1}{\omega^2 \mu \epsilon_0} \nabla \cdot \frac{1}{\epsilon} \nabla \epsilon \cdot \mathbf{E} \right| \ll \left| \frac{\epsilon}{\epsilon_0} \mathbf{E} \right|, \quad (22)$$

where ϵ is the dielectric permittivity, $\mathbf{E}$ is the electric field vector, μ and ϵ_0 are, respectively, the free-space permeability and permittivity, and ω is the frequency of the wave. Physically, Eq. (22) implies that the index profile of the fiber varies little over distances of the order of the wavelength.

(b) The paraxial ray approximation may be used, or the factor $(\partial^2 A / \partial z^2)(\mathbf{x}, z)$ is negligible compared to the factor

$$\left(2ikn_0 \frac{\partial}{\partial z} + \nabla_t^2 + k^2(n^2(\mathbf{x}, z) - n_0^2) \right) A(\mathbf{x}, z),$$

where

$$u = \exp(ikn_0 z) A(\mathbf{x}, z).$$

This condition means that the complex amplitude $A(\mathbf{x}, z)$ varies much more rapidly transverse to the direction of propagation than it does along the direction of propagation, which is satisfied for fields propagating at small angles to the z axis.

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