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Exploratory Reconstructability Analysis of Accident TBI Data

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Abstract

This paper describes the use of reconstructability analysis to perform a secondary study of traumatic brain injury data from automobile accidents. Neutral searches were done and their results displayed with a hypergraph. Directed searches, using both variable-based and state-based models, were applied to predict performance on two cognitive tests and one neurological test. Very simple state-based models gave large uncertainty reductions for all three DVs and sizeable improvements in percent correct for the two cognitive test DVs which were equally sampled. Conditional probability distributions for these models are easily visualized with simple decision trees. Confounding variables and counter-intuitive findings are also reported.

Keywords: reconstructability analysis; machine learning; OCCAM; information theory; traumatic brain injury; health care analytics

1 Introduction

This paper reports the application of reconstructability analysis (RA) to the secondary analysis of TBI data. Secondary analysis of health care data can be useful when the clinical population exhibits unexplained variability in outcomes that are not resolved by the primary analysis. Also, the long time and considerable expense needed to complete a study make additional examination of the data desirable. Both of these conditions are highly relevant to traumatic brain injury: TBI is a serious and prevalent clinical condition for which unexplained variation in outcome unfortunately persists despite decades of research; moreover the volume of existing TBI data provides a unique opportunity for secondary analyses (National Center for Injury Prevention and Control, 2015; Samadani & Daly, 2016). The substantive aim of this study is to discover unexpected relationships in the data and to contribute to ongoing efforts of the Brain Trauma Evidence Based Consortium (BTEC) to develop a dynamic model of brain trauma and a new clinically useful TBI classification system. The methodological aim of this study is to further develop RA methodology and demonstrate its capabilities.

RA (Ashby, 1964; Klir, 1985, 1986; Krippendorff, 1986; Zwick, 2004) is a probabilistic graphical modeling technique, a fusion of information theory and graph theory. Graphs define the models that are considered, and information measures quantify the models' predictive efficacy. In these graphs, a node is a variable and a link is a relation (an association) between two or more variables. If relations link only two nodes, this is an ordinary graph; if relations can link more than two nodes, it is a hypergraph. One is interested in models that are hypergraphs because one is interested in associations between more than two variables.

RA is explicitly designed for exploratory modeling, having the capacity to detect non-linear and multivariate interactions that are not hypothesized in advance. Two types of exploration are available: (a) directed searches which aim to discover models that are predictive of some dependent variable (DV) given a set of independent variables (IVs), and (b) neutral searches in which no IV/DV distinction is made and the aim is to discover associations that exist between any of the variables. The principal focus of this paper is on directed studies, but some results from neutral explorations are also reported.

RA models are also conceptually transparent: a directed RA model is simply a conditional probability distribution of a dependent variable (DV), given the composite state of a set of independent variables (IVs); a neutral RA model is simply a joint probability distribution. As a probabilistic graphical modeling method, RA overlaps with log-linear modeling, logistic regression, and Bayesian networks. Where it overlaps with these similar methods, it is equivalent to them (Zwick, 2012), although RA has unique features not present in these other methods, and these other methods have unique features not available in RA. All of these probabilistic graphical modeling methods differ from other machine learning methods, such as support vector machines and neural networks, which are designed for continuous variables. The reason RA is attractive for secondary data analysis is that other data analysis methods are often not well designed for exploration, have more limited model types, have difficulty with nominal variables or with stochasticity, or are not conceptually transparent.

2 Data

The data analyzed here, obtained from Megan Preece (2012), is on patients with traumatic brain injury resulting from automobile accidents (Preece et al, 2010, 2011, 2013). There are 52 variables, divided into five types, labeled as P, Y, G, C, and N variables, where P = patient characteristics (17 variables), Y = symptoms, i.e., subjective reports (25 variables), G = signs, i.e., objective indicators (4 variables), C = cognitive deficits (5 variables), N = neurologic deficits (1 variable). The sample size is 337, reduced to 175 or fewer when missing data are excluded.

The aim of the study is primarily to predict specific deficit (C or N) variables from P, Y, and G variables and from the other deficit variables, and secondarily to look for associations among any of the variables. In this paper, we report the prediction of two C variables – the neuropsychological Digit Symbol Substitution Test (DSST), abbreviated as Cdg (N = 255), and the Spatial Reaction-Time Test (RT) normalized for age and sex, abbreviated as Cnr (N = 210) – and one N variable – the Visual Acuity Test, abbreviated as Nlr (N = 154). The DSST is a paper and pencil or online task requiring the patient to match symbols with their corresponding digits under timed conditions. It is considered to be sensitive to brain injury and to concussion in particular. The RT test, less complex than DSST, assesses how quickly the patient responds to visual stimuli. The variables involved in the predictive models discussed in this paper, as IVs or DVs or both, are listed in Table 1

(Table 1)

The first letters of the variables indicate their variable types. The table lists, after the variable abbreviations, their original cardinalities, although many variables were rebinned to lower cardinalities in the analysis. For some records, values of some variables were missing. Being missing is included as an additional possible state; so, for example, binary variables with some values missing are listed as having cardinality three.

3 Methodology

This section provides a brief summary of the main features of reconstructability analysis. RA calculations in this study were performed using the Occam software package developed at Portland State University (PSU) (Willett & Zwick, 2004). This package takes standard text input and provides easily interpretable output. It is web-accessible and can be run either in real time, where it provides html output, or in batch (off-line) mode, in which it emails results to the user as a csv file. This software package runs on PSU servers and is openly available for non-commercial research and educational uses.

Being based in information theory, RA is inherently a nominal data method, but can be applied also to continuous variables if their values are discretized (binned). Binning procedures are available in many commercial and public domain software packages; a utility program is also available at the RA web site (Zwick, 2017), which outputs a data file in Occam input format. Occam also allows easy rebinning (aggregating existing bins) in the input file. The RA web site includes an Occam user manual and access to many publications that make use of RA methodology.

An RA model is simpler – has fewer degrees of freedom (df) – than the data, but captures much of the information in the data. RA searches for good models are of two types: directed and neutral. Directed searches consider models that predict a dependent variable (DV) from a set of independent variables; neutral searches consider models that do not make any IV-DV distinction. This paper reports results from both types of searches, but the principal results are those from the directed searches.

In directed searches, a candidate model is compared to a reference model, which is either the independence model, for which no IV predicts the DV, or the data, for which all the IVs predict the DV in a single interaction effect. For example, consider three IVs, A, B, and C, and one DV, Z. The independence model, at the bottom of the lattice of structures, is ABC:Z, where the colon means 'and.' This model says that there is a relation between A, B, and C, but no relation between any of these IVs and Z. The data, at the top of the lattice of structures, is ABCZ, in which there is a four-way interaction effect where A, B, and C collectively predict Z. In the present study, the independence model is the chosen reference.

In neutral searches, replace Z by variable D and assume that no IV-DV distinction is being made for the four variables A, B, C, and D. The independence model for the neutral search, i.e., the bottom of the lattice of structures, is A:B:C:D. Note that when ABC:Z is chosen as the independence model for the directed search in which Z is a DV, there is no concern for relations that may exist between the IVs A, B, and C, so the 3-way ABC relation is built in to every directed model to allow for such relations.

A relation includes all its projections (embedded relations). ABC thus includes AB, AC, and BC, and the univariate margins, A, B, and C. The order of the relations in a structure is arbitrary, and the order of the variables in a relation is also arbitrary. For example, Z:BAC is identical to ABC:Z.

An example of a directed search model intermediate between the independence model and the data is ABC:BZ, which says that there may be a relation between A, B, and C, which is non-predictive since it doesn't involve the DV, and there may also be a predictive relation between B and Z. The ABC in ABC:BZ is called the 'IV component' since it includes all the IVs, and in Occam output, the model is referred to as IV:BZ. In directed search models, an IV component is always included to allow for relations among the IVs. When a predictive relation – here BZ – is included in a model, this does not mean that the relation is strong; it just means that this relation is being modeled. In the tables below showing model search results, the "IV" component is often omitted from the model names for simplicity.

Models with one predicting relation, e.g., ABC:BZ, do not have loops, while models with multiple predicting relations, e.g., ABC:AZ:BZ, have loops. (The loop here consists of AZ, ZB, and BA; the last of these is embedded in ABC). In this latter model, AZ and BZ are separate, but they are not simply additive contributions to the prediction of Z. A conventional three-way interaction effect between A, B, and Z would be represented by an ABZ relation, as in model ABC:ABZ, but the AZ and BZ relations in ABC:AZ:BZ also constitute a (lesser) type of interaction effect (Zwick, 2011). Models without loops are computationally simple, since they can be fit algebraically. Models with loops can present challenging computational space and time demands, since they must be fit iteratively. For many variables, nearly all models have loops. One drawback of Bayesian networks (BN) is that they cannot have loops; RA, by contrast, encompasses such models, though RA in turn doesn't consider all BN models (Zwick 2011).

Models are subsets of variables, each subset indicating a projection of the data that is preserved in the model. The above models are all 'variable-based.' Another type of model includes components that specify specific states of variables (Jones, 1985; Johnson & Zwick, 2000; Zwick & Johnson 2004; Johnson 2005). An example is ABC:

Z: A_1B_2Z. The first two components of this model, namely ABC and Z together define the variable-based independence model. Addition of the A_1B_2Z component, however, makes this a state-based model. This third component means that the probability that A = 1, B = 2, and any value of Z is either unusually high or unusually low. State-based models pick out informationally salient states. In results reported below, the independence part of the state-based model is often (in the above, ABC:Z) omitted for simplicity.

The predictive success of (equivalently, the information captured in) a model is quantified by %ΔH, the reduction of uncertainty (Shannon entropy) of the DV if one knows the values of the predicting IVs. Like variance, H is a measure of spread, here the spread of a probability distribution, but unlike variance, because Shannon entropy contains a logarithm term, low values of uncertainty-reduction, even as low as 8%, can indicate big effect sizes. This is illustrated in Table 2 for the DV, Z, and the IV, A. The marginal distribution of Z is {.5, .5}, but knowing A the conditional distribution of Z is either {.33, .67} or {.67, .33}. This change of Odds from 1:1 to 1:2 or 2:1 is a big effect, but it is only an uncertainty reduction of 8%.

(Table 2)

Uncertainty reduction is the central information theoretic measure of predictive efficacy, but since it is useful to compare RA results to other methods that don't generate this measure. Occam reports also the more general accuracy measure of %correct (%c) and the related measures of true and false positives and negatives, sensitivity, and specificity. Uncertainty reduction roughly tracks with %correct – the more the uncertainty of the DV is reduced, the higher the accuracy of prediction tends to be – but these measures do not track perfectly. Moreover, they track best when the marginal probability distribution of the DV is approximately uniform. For skewed distributions, models can reduce uncertainty but still not improve accuracy. In such cases, the real predictive strength of the model is its uncertainty reduction, not its %correct. Uncertainty reduction, for example, registers the difference, for a binary variable, between predicting a state because it has a probability of .55 or because it has a probability of .95, despite the fact that both probability values give the same prediction and thus contribution to %correct. This point is illustrated in the analysis below of the DV, Nlr.

A good model has high uncertainty reduction or %correct; it also has low complexity, defined as degrees of freedom, or low Δdf, the difference between df(model) and df(reference), where the reference here is independence.. These two aspects of goodness oppose one another, so a good model is really one that optimally trades off accuracy (uncertainty reduction, information captured) and simplicity. This tradeoff is either explicit, as in the Bayesian Information Criterion (BIC) and the Akaike Information Criterion (AIC), which compute weighted sums of error and complexity (the opposites of accuracy and simplicity), or the tradeoff is implicit, as in a Chi-square p-value calculation, also a standard way of selecting a model.

BIC penalizes more for complexity than AIC, and is thus more conservative than AIC. A third model selection criterion in Occam is 'Incremental p-value,' which uses Chi-square p-values to pick models. The IncrP model is the model with the highest uncertainty reduction whose difference from (the bottom reference of) independence is statistically significant, and for which a path exists from independence to the model in which every incremental increase in complexity is statistically significant. BIC and AIC are given in Occam output as differences between these measures for the reference minus their values for the model,as follows

$$\Delta \text{AIC} = \Delta \text{LR} + 2\,\Delta \text{df} \qquad (1)$$

$$\Delta BIC = \Delta LR + \ln(N)\, \Delta df \qquad (2)$$

where ΔLR is the change in likelihood ratio Chi-square between the bottom reference and the model, i.e.,

$$\Delta LR = 2\, N\, \Sigma\, p \ln [\, q(\text{reference}) / q(\text{model})\,] \qquad (3),$$

where p is the observed probability distribution of the data, q(model) is the calculated distribution of the model, and q(reference) is the calculated distribution of the reference model of independence, Δdf is the increase in degrees of freedom from the bottom reference, and N is the sample size. Large positive differences of ΔAIC and ΔBIC indicate good models. In this study the highest ΔBIC model was always selected as the best model.

Occam offers three types of searches that differ in refinement and thus predictive power: (1) a coarse search, using variable-based models without loops, which have only one predicting relation, e.g., IV:BZ; (2) a fine search, using variable-based models with loops, which have multiple predicting relations, e.g., IV:AZ:BZ; and (3) an ultra-fine search, which uses state-based models, e.g., IV:Z:A1B2Z. Coarse searches are fast and can handle many variables; fine searches are slow and can handle at most 100s of variables; ultra-fine searches are very slow, and can handle only fewer than 10 variables. Differences between these three searches are illustrated in Figure 1.

(Figure 1)

In this figure, a dashed level represents the model selected by the search. Fine searches consider more models, at smaller increments of Δdf, than coarse searches, and ultra-fine searches more models than fine searches. More refined searches are advantageous because they can yield more complex and thus more predictive models that are still statistically justified, or they might yield models that are equally predictive but simpler (smaller Δdf) than those obtained from less refined searches. Also, a more refined search can discover interaction effects between variables that are not seen in less refined searches. The above figure illustrates the first of these possible benefits: the fine search selects a more complex, and thus more predictive, model that is not considered by the coarse search; and the ultra-fine search selects a still more complex model that is not considered by the fine search.

The search procedure for both neutral and directed searches begins at some initial model, either the independence model at the bottom of the Lattice of Structures or the data itself which is at the top of the Lattice of Structures. In the research reported in this paper, the reference model was always the bottom model. Starting from this model, Occam identifies all the 'parents' of this model at the next level up – all models with the smallest increase in complexity from the starting model. Of these parents, some number – set by the parameter 'width' – are selected by some search criterion and retained. The parents of these width models are then generated, and the best width models are retained at this next level. The process proceeds from 'level 0' of the starting model up to a number of levels specified by the parameter 'levels'. This 'beam search' is schematically represented in Figure 2.

(Figure 2)

4 Results

4.1 Neutral searches

A neutral coarse search was done, and its BIC model was:

PijGpc: PijGgc: PijGxc: Pag: PsxYcv: PyePed: PyePri: YpnYem: YemYds: YddYds: YdaYds: YdsPph: GhlPri: PulPri: PriPph: **PriCdg**: **PriNlr**: PmdPpkGpc: PpkPph: PphGpl: PphPqe: PphPqv: PphPlg: **PphCsr**: PphYcv: **PphPiq**: PphGpt: GpcPnp: **GpcChp**: **GpcCsc**: GpcYhs: GpcYdz: GpcYna: GpcYns: GpcYsd: GpcYfa: GpcYir:

GpcYdp: GpcYax: GpcYfr: GpcYfg: GpcYcn: GpcYtk: GpcYbr: GpcYls: GpcYdv: GpcYrs: GpcYaz: GpcYrm: PlgPac: **CnrCsr**

This model consists of 50 associations (plus 1 independent variable, Pag), where all but one association (Pmd Ppk Gpc) is pairwise. Fifteen associations in red have p-values ≤ 0.05; eight more in purple have p-values ≤ 0.10. Six associations that involve C and N variables are in bold, but only two of these, namely Pph Csr and Cnr Csr are statistically significant at the 0.05 level. The graph of this model is shown in Figure 3.

(Figure 3)

Table 3 shows, for each of the 15 associations, how predictive each member of the pair of variables is of the other member. The first two columns list the abbreviations of the variables, whose identities are given in the last two columns. The third and fourth columns indicate the reduction of uncertainty of variable 2 given variable 1 and the reduction of uncertainty of variable 1 given variable 2. This is followed by the p-value of the association and its sample size. The following two columns give the *increase* in %correct in predicting variable 2 given variable 1, and in predicting variable 1 given variable 2, where this increase is measured from the reference of the %correct obtained from the independence models.

(Table 3)

The strengths of the associations are strong but most associations are fairly obvious. Only the two associations, shown in the table in blue, namely {previous concussion, dizzy} and {previous concussion, frustrated} are somewhat novel and thus potentially interesting. One association, namely {reaction time, previous head injury} appears obvious, but will turn out upon further examination to be counter-intuitive in the directionality of the association.

4.2 Directed searches

Directed coarse, fine, and ultra-fine searches were done for three DVs: the Digit Symbol Substitution Test (Cdg), the Normalized Reaction Time Test (Cnr),and the Visual Acuity Test (Nlr). For these DVs, a final best model was selected from the ultra-fine search, and for this model, the conditional probability distribution of the DV, given the predicting IVs, is shown. For Cdg and Cnr, this best model is also summarized in a decision tree.

Predicting performance on Digit Symbol Substitution test

Table 4 presents the results of coarse, fine, and ultra-fine searches that attempt to predict Cdg after this DV has been rebinned to two states, roughly equal in probability. In listing the models, the table omits the non-predicting IV component.

For the coarse search, the six top single predicting IVs are listed with their complexities (Δdf), the p-values that assess the significance of their difference from independence, their %reduction of DV uncertainty (%ΔH), their %correct (%c), and their ΔBIC from independence. The single predictors are ordered by their uncertainty reductions, which is different from the order of their ΔBIC values, since ΔBIC considers not only uncertainty reduction but also complexity.

(Table 4)

The table shows that Pij (patient injury type) is the best single predictor in terms both of uncertainty reduction and ΔBIC, but these two measures differ in their ranking of Pye (years of education). Pye is the fifth best predictor in terms of uncertainty reduction, but the second best in ΔBIC, because it adds only 1 degree of freedom to the independence model.

In the fine search, BIC picks a model with Pij and Pye as predictors, not surprisingly since these are, by ΔBIC, the first and second best single predicting IVs in the coarse search. The fine search results illustrate the fact that BIC selects simpler models (Δdf = 4) than AIC (Δdf = 9) and IncrP (Δdf = 10). The additional degree of freedom in the IncrP model beyond the AIC model is due to adding Psx (sex) as an additional predictor.

The ultra-fine (state-based) search gives BIC model

IV: Cdg : $\mathbf{Pij_2}$ $\mathbf{Cnr_1}$ Cdg : $\mathbf{Pye_0}$ Cdg.

This very simple (Δdf = 2) model includes all three predictors from the more complex (Δdf = 9) AIC fine search model, but it selects only one state of each of these predictors as salient. It also shows Pij and Cnr interacting in their prediction of Cdg, which is not seen in the AIC fine search model. (This illustrates the point made earlier that a more refined search may discover interaction effects not observed in less refined searches.)

This ultra-fine BIC model is only about half as predictive (%ΔH = 13.5) as the fine BIC model (%ΔH = 25.5), but it is also half as complex. (Δdf = 2 as opposed to 4). Using the most conservative criterion to select models, either of these two BIC models could be chosen as the 'best model,' but because the state-based model has an additional predictor (Cnr), and is thus potentially more interesting, it has been selected as the Cdg best model.

Table 5 shows the conditional probability distribution, p(Cdg | Pij Pye Cnr), for the data and for this best model. The DV states, Cdg_0 and Cdg_1, mean low and high Digit Symbol scores, respectively, so a high probability of Cdg_0 indicates a cognitive deficit. Alongside the conditional probability values, the table lists for each composite IV state the probability of a high score divided by the probability of a low score, namely

$$\text{Odds} = p(Cdg_1 \mid Pij\ Pye\ Cnr) / p(Cdg_0 \mid Pij\ Pye\ Cnr) \quad (4)$$

High Odds values are good outcomes, low Odds are poor outcomes, while Odds near 1 have IV conditional probabilities that are close to the marginal probabilities for the whole sample. To the right of the Odds column is the p-value that assesses the significance of the difference between conditional and marginal probabilities.

(Table 5)

Comparing the (shaded) 3rd and 4th rows of Table 5 shows that for orthopedic (control) injuries and high education, difference in performance (in bold) on the Reaction-time Test (Cnr) does not predict any difference in the Odds. Comparing the (shaded) 3rd and 7th rows shows that for high education and fast reaction time, difference in injury type (Pij) – either head injury or merely orthopedic (in italics) –also does not predict an Odds difference. All three of these rows (IV states) have the same Odds, namely 2.7.

The conditional probability distribution for this state-based model can be understood to result from integrating the distributions of the separate components of this model, namely IV: Cdg: **Pij_2 Cnr_1 Cdg** and IV: Cdg: **Pye_0 Cdg.** The component distributions are shown in Table 6. The distribution for the first component shows that Odds are low (0.33) for patients with head injuries and slow reaction times. The distribution for the second component shows that Odds are low (0.5) for patients with low education. Above Table 5, for the full model, integrates these two effects.

(Table 6)

The table for the full model can be summarized in the decision tree shown in Figure 4. The leaves of the tree are the Odds values followed by the p-value. Odds with significant p-values (at or near a 0.05 cutoff level) are shown in larger font. The decision tree can be summarized verbally as follows. For all patients, education predicts performance on the Digit Symbol Substitution Test: more education predicts better performance. Education is thus a confounding variable for the Digit Test in discriminating concussion, and must be controlled for. This is not surprising, given the complexity of the DSST. For orthopedic injury patients, reaction time does not predict digit symbol score. For patients with mild head injury, fast reaction time predicts better digit symbol performance beyond the influence of education.

(Figure 4)

Predicting performance on the Normalized Reaction Test

Table 7 shows results of coarse, fine, ultra-fine searches for the Normalized Reaction-time Test (Cnr) after this DV has been rebinned to two equally sampled bins.

(Table 7)

For the coarse search, the table lists models selected by the three criteria, rather than tabulating the best single predictors. Three IVs show up in these models: Cdg, performance on the Digit Symbol Substitution Test (since Cnr predicts Cdg, it's not surprising that Cdg also predicts Cnr); Gpt, amnesia; and, for the IncrP model, also Pph, previous head injury. These IVs show up as 3- and 4-way joint interaction effects.

The fine search BIC model, Cdg Cnr : Gpt Cnr, includes Cdg and Gpt as separate rather than as joint predictors, but, the more aggressive AIC and IncrP criteria highlight a Cdg Gpt Cnr interaction effect, and also add Pph plus two additional IVs not found in the best coarse models: Pri, recent illness, in the AIC model, and Pye, years of education, in the IncrP model.

The ultra-fine search retains several of the IVs found in the coarse search, but indicates specific states of these variables: Pph1 is previous head injury, Cdg1 is high Digit Test score; Gpt1 is the absence of amnesia. Note that this $\Delta df = 2$ ultra-fine BIC model has a higher uncertainty reduction ($\%\Delta H = 12.4$) than the more complex ($\Delta df =$

3) coarse BIC model (%ΔH = 10.6) and the equally complex (Δdf = 2) fine BIC model (%ΔH = 8.8). Adding back IV: Cnr, the independence part of the ultra-fine model, the full state-based best Cnr model is

$$\mathbf{IV : Cnr : Pph_1\ Cdg_1\ Cnr : Cdg_0\ Gpt_1\ Cnr.}$$

Table 8 shows the conditional probability distribution for this model. The Odds value is the probability of fast (normal) reaction time divided by the probability of slow reaction time, given a particular IV state, i.e.,

$$\text{Odds} = p(\text{Cnr}_0 \mid \text{Pph Cdg Gpt}) \,/\, p(\text{Cnr}_1 \mid \text{Pph Cdg Gpt}) \qquad (5)$$

Again, high values of Odds are good, low values point to a deficit, and values near 1 indicate similarity to the marginal probability distribution of the overall sample.

(Table 8)

Comparing the (shaded) 2nd and 4th rows of 0shows that for those patients who score low on the Digit Symbol Substitution Test and have amnesia, the presence or absence of a previous head injury does not matter: both have Odds = 0.2. Comparing the shaded 7th and 8th rows shows that if the patient has had a previous head injury and scores high (normal) on the Digit Symbol Test, the absence or presence of amnesia also does not matter: both have Odds = 2.7.

The table can be summarized in the decision tree shown in Figure 5 which shows Odds (on the left) and p-values (on the right). To summarize this decision tree: for low performance on Digit Symbol Test, amnesia predicts slow reaction time. For normal performance on Digit Symbol Test, previous head injury increases the probability of fast (normal) reaction time; this latter result is anomalous.

(Figure 5)

Predicting performance on the Visual Acuity Test

Table 9 shows results of coarse, fine, ultra-fine searches for the Visual Acuity Test (Nlr, the Logarithm of minimum angle of Resolution) after this DV has been rebinned to two equally sampled bins.

(Table 9)

For the coarse search, the table lists models selected by the three criteria. Two IVs show up in these models: Ycv, corrected vision, and Pye, years of education. The fine search adds Gpt, amnesia, as a predicting IV. The BIC models of the coarse and fine searches are the same: only Ycv predicts Nlr. In the ultra-fine BIC model, selected as the best model,

$\mathbf{IV : Nlr : Pye_0\ Ycv_1\ Nlr : Pye_1\ Gpt_1\ Nlr}$

Ycv interacts separately with both Pye and Gpt, illustrating the fact that state-based models can often detect interaction effects not detected by variable-based models. Also, the uncertainty reduction, 32.4%, for this ultra-fine model, which has $\Delta df = 2$, is almost as large as the uncertainty reduction, 36.1%, for the fine AIC model, which has $\Delta df = 5$, and much larger than the uncertainty reduction, 11.7%, of the coarse and fine BIC models. This illustrates the enhanced power of state-based modelling.

Note that no model does better than the independence reference model %correct of 95.5%. This illustrates the point made above that when probability distributions are highly skewed predictive models often make the same predictions as the reference model, so their %correct measures show no improvement. However, predictive models can differ substantially from the reference model in their conditional probability distributions, and thus provide valuable predictive information about risk. This predictive information is registered by the %uncertainty measure of model goodness.

Table 10 shows the conditional probability distribution for this best model. The Odds value here is defined slightly differently than before; it is

$$\text{Odds} = p(\text{Nlr}_1 \mid \text{Pye Ycv Gpt}) / p(\text{Nlr}_0 \mid \text{Pye Ycv Gpt}) \qquad (6)$$

Now, *low* values of Odds are good, *high* values point to a deficit, and values near 1 indicate similarity to the marginal probability distribution of the overall sample.

(Table 10)

Comparing the (shaded) 3rd and 4th rows of Table 10 shows that low education and correct vision predicts low visual acuity. For these patients, the presence or absence of amnesia does not matter in that both have Odds = 0.39. Small but not statistically significant effects of the presence or absence of amnesia are shown elsewhere in the table.

5 SUMMARY

This analysis illustrates the type of results that can be obtained from exploratory modeling with RA and demonstrates the possibility of using RA to better understand – and potentially ultimately to improve – clinical outcomes. Analyses can be done at three different levels of refinement. Models are conceptually transparent, being simply conditional probability distributions of a DV given the states of IV predictors. The distributions can be readily summarized with easily interpretable decision trees.

This analysis of Preece data is a test bed for future analyses of other TBI data, which hopefully will include other types of IVs, such as imaging, genomic, and proteomic measures. Specific findings reported here are tentative and should be subjected to confirmatory tests with new data. This is particularly true of the anomalous finding in the Cnr model in which previous head injury predicted better reaction-time scores than the absence of previous injury. One possible explanation of this anomaly is that prior exposure to the Reaction Time test introduces a practice effect. But if reaction time is so vulnerable to a practice effect that it no longer discriminates concussed from non-concussed, then it's probably not an appropriate measure for this purpose. Another finding of potential interest is the indication by the Cdg model that level of education may be a confounding factor in assessing TBI patients with the Digit Symbol Test.

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Table 1 Variables in directed models discussed in this paper

Ped	8	highest level of education
Pij	5	Injury group (patient or control)
Pph	3	Previous head injury
Pri	3	Recent illness
Psx	2	Sex
Pye	6	Years of education
Ycr	3	Corrected vision
Ggc	4	Glasgow coma scale
Gpt	3	Post traumatic amnesia
Cdg	7	Digit Symbol Substitution neuropsychological test
Csr	6	Spatial Reaction Time test (reaction time to visual stimuli)
Cnr	6	Spatial Reaction Time test normalized for age and sex
Nlr	4	Visual Accuity Test (Logmar: logarithm of minimum angle of resolution)

Table 2 Illustration of small uncertainty reduction but big effect size

	Z_0	Z_1	
A_0	.67*.5	.33*.5	0.5
A_1	.33*.5	.67*.5	0.5
	0.5	0.5	

Table 3 Predictive success for associations found in neutral search

v1	v2	%ΔH(2|1)	%ΔH(1|2)	p-value	N	Δ%c(2|1)	Δ%c(1|2)	v1	v2
Ggc	**Pij**	**34.5**	**86.5**	**0.000**	**196**	**9.7**	**7.7**	**glasgow coma scale**	**Injury patient/control**
Gxc	**Pij**	**32.9**	**12.6**	**0.000**	**280**	**20.4**	**14.3**	**external cause**	**Injury patient/control**
Ped	**Pye**	**41.3**	**34.8**	**0.000**	**248**	**32.3**	**27.4**	**highest educ level**	**years of education**
Yem	Ypn	6.4	6.1	0.000	218	5.0	2.3	emotional problems	painscale
Yds	Yem	6.0	27.8	0.000	210	3.8	0.0	stress	emotional problems
Ydd	**Yds**	**43.6**	**26.0**	**0.000**	**210**	**1.4**	**1.9**	**depression**	**stress**
Yda	**Yds**	**54.7**	**32.6**	**0.000**	**210**	**0.0**	**2.9**	**anxiety**	**stress**
Pmd	**Ppk**	**50.7**	**57.6**	**0.000**	**230**	**28.3**	**15.7**	**current medications**	**painkillers**
Gpc	**Pnp**	**57.0**	**100.0**	**0.000**	**52**	**11.5**	**30.8**	**previous concussion**	**# previous concussion**
Pac	**Plg**	**26.5**	**12.3**	**0.000**	**201**	**0.0**	**12.4**	**caused accident**	**case litigated**
Cnr	**Csr**	**48.6**	**48.3**	**0.000**	**210**	**34.3**	**31.0**	**reaction time norm**	**reaction time**
Psx	Ycv	6.5	8.8	0.000	197	2.0	0.0	sex	corrected vision
Gpc	Ydz	13.7	21.9	0.003	52	0	9.6	previous concussion	dizzy
Csr	Pph	5.3	2.3	0.010	187	5.3	4.8	reaction time	previous head injury
Gpc	Yfr	9.1	17.3	0.011	52	1.9	9.6	previous concussion	frustrated

Table 4 Digit Symbol Test (Cdg) model searches

Model	Δdf	p	%ΔH	%c	ΔBIC
REFERENCE (independence)					
Cdg	0	1.00	0.0	50.9	0.0
COARSE[&] (single predictors)					
Pij Cdg	3	0.00	11.9	68.3	47.6
Ped Cdg	7	0.00	11.7	65.0	5.9
Ggc Cdg	3	0.00	5.6	65.0	18.3
Cnr Cdg	5	0.00	3.5	60.8	6.1
Pye Cdg	1	0.00	3.0	68.3	27.9
Csr Cdg	5	0.00	2.5	63.3	0.4
FINE[*]					
Pij Cdg : Pye Cdg	4	0.00	25.5	72.9	BIC
Pij Cdg : Pye Cdg : Cnr Cdg	9	0.00	32.8	76.7	AIC
Pij Cdg : Pye Cdg : Cnr Cdg : Psx Cdg	10	0.00	32.9	76.3	IncrP
ULTRA-FINE[#]					
$Pij_2\ Cnr_1\ Cdg : Pye_0\ Cdg$	2	0.00	13.5	68.6	BIC

Pij = patient injury type	Pye = years of education
Ped = education level	Csr = Spatial Reaction Test
Ggc = Glasgow coma scale	Psx = sex
Cnr = Norm. Spatial Reaction Test	

[&]N = 240

[*]N = 240, |Cnr| = 6, including missing

[#]N = 275, |Cnr| = 2, no missing

Table 5 Best Cdg model

Full SB model = IV: Cdg: **Pij_2 Cnr_1 Cdg : Pye_0 Cdg**

				Conditional probabilities of DV					
IV states				**Data**		**Model**			
Pij	*Pye*	*Cnr*	*N*	*Cdg_0*	*Cdg_1*	*Cdg_0*	*Cdg_1*	*Odds*	*p*
orthop	low	fast	18	0.5	0.5	0.59	0.41	0.7	0.41
orthop	low	slow	22	0.68	0.32	0.59	0.41	0.7	0.36
orthop	high	**fast**	38	0.21	0.79	0.27	0.73	***2.7***	0.01
orthop	high	**slow**	20	0.35	0.65	0.27	0.73	**2.7**	0.05
head	low	fast	15	0.53	0.47	0.59	0.41	0.7	0.45
head	low	slow	24	0.88	0.13	0.86	0.14	0.2	0.00
head	high	fast	18	0.33	0.67	0.27	0.73	*2.7*	0.06
head	high	slow	20	0.6	0.4	0.62	0.38	0.6	0.26
			175	0.49	0.51	0.49	0.51	1.00	

IVs Pij (patient injury type): 1 orthopedic (control) vs 2 head injury

Pye (years of education): 0 low vs 1 high

Cnr (Normalized Reaction-time Test): 0 fast (normal) vs 1 slow (deficit)

DV Cdg (Digit Symbol Test): 0 low (deficit) vs 1 (high, normal)

Table 6 Components of best Cdg model

Full SB model = IV: Cdg: **Pij_2 Cnr_1 Cdg : Pye_0 Cdg**

1st component = IV: Cdg: **Pij_2 Cnr_1 Cdg**

Conditional probabilities of DV

IV states			Data		Model			
Pij	*Cnr*	*N*	*Cdg_0*	*Cdg_1*	*Cdg_0*	*Cdg_1*	*Odds*	*p*
orthop	fast	56	0.3	0.7	0.4	0.6	1.5	0.19
orthop	slow	42	0.52	0.48	0.4	0.6	1.5	0.26
head	fast	33	0.42	0.58	0.4	0.6	1.5	0.32
head	slow	44	0.75	0.25	0.75	0.25	**0.33**	0
		175	0.49	0.51	0.49	0.51	1.00	

Aggregated 1st component

Pij	*Cnr*	*N*	*Cdg_0*	*Cdg_1*	*Cdg_0*	*Cdg_1*	*Odds*	*p*
not head-slow		131	0.4	0.6	0.4	0.6	1.5	0.01
head	slow	44	0.75	0.25	0.75	0.25	**0.33**	0
		175	0.49	0.51	0.49	0.51	1.00	

2nd model component = IV: Cdg: **Pye_0 Cdg**

IV states		Data		Model			
Pye	*N*	*Cdg_0*	*Cdg_1*	*Cdg_0*	*Cdg_1*	*Odds*	*p*
low	79	0.5	0.5	0.67	0.33	0.5	0
high	96	0.21	0.79	0.34	0.66	**1.9**	0
	175	0.49	0.51	0.49	0.51	1.00	

IVs Pij (patient injury type): 1 orthopedic (control) vs 2 head injury
Pye (years of education): 0 low vs 1 high
Cnr (Normalized Reaction-time Test): 0 fast (normal) vs 1 slow (deficit)

DV Cdg (Digit Symbol Test): 0 low (deficit) vs 1 (high, normal)

Table 7 Normalized Reaction Test (Cnr) model searches

Model	Δdf	p	%ΔH	%c	N=175
REFERENCE					
Cnr	0	1.00	0.0	50.9	
COARSE					
Cdg Gpt Cnr	3	0.00	10.6	64.6	BIC, AIC
Pph Cdg Gpt Cnr	7	0.00	13.1	66.9	IncrP
FINE					
Cdg Cnr : Gpt Cnr	2	0.00	8.8	64.6	BIC
Pri Cnr : Pph Cnr : Cdg Gpt Cnr	6	0.00	14.7	70.3	AIC
Pye Cnr : Pph Cnr : Cdg Gpt Cnr	5	0.00	12.9	67.4	IncrP
ULTRA-FINE					
Pph_1 Cdg_1 Cnr : Cdg_0 Gpt_1 Cnr	2	0.00	12.4	64.8	BIC

Cdg = Digit Symbol Substitution Test   Pri = recent illness

Gpt = amnesia;   Pye = years education

Pph = previous head injury

Table 8 Best Cnr model

				Conditional probabilities of DV					
IV states				Data		Model			
Pph	*Cdg*	*Gpt*	*N*	*Cnr_0*	*Cnr_1*	*Cnr_0*	*Cnr_1*	*Odds*	*p*
no	low	no	20	0.4	0.6	0.52	0.48	1.1	0.92
no	low	yes	19	0.16	0.84	0.16	0.84	**0.2**	0.00
yes	low	no	30	0.57	0.43	0.52	0.48	1.1	0.90
yes	low	yes	18	0.17	0.83	0.16	0.84	**0.2**	0.00
no	high	no	24	0.50	0.50	0.52	0.48	1.1	0.91
no	high	yes	13	0.61	0.39	0.52	0.48	1.1	0.93
yes	high	*no*	38	0.76	0.23	0.73	0.27	*2.7*	0.01
yes	high	*yes*	14	0.64	0.36	0.73	0.27	*2.7*	0.09
			176	0.51	0.49	0.51	0.49	1.0	

IVs Pph (previous head injury): no vs yes

Cdg (Digit Symbol Substitution Test): low(deficit) vs high (normal)

Gpt (amnesia): no vs yes

DV Cnr (Reaction-time Test): Cnr_0 fast (normal) vs Cnr_1 slow (deficit)

Table 9 Nlr model searches

Model	Δdf	p	%ΔH	%c	N=154
REFERENCE					
Nlr	0	1.00	0.0	95.5	
COARSE					
Ycv Nlr	1	0.00	11.7	95.5	BIC
Pye Ycv Nlr	3	0.00	25.0	95.5	AIC, IncrP
FINE					
Ycv Nlr	1	0.00	11.7	95.5	BIC
Pye Ycv Nlr : Pye Gpt Nlr	5	0.00	36.1	95.5	AIC
Ycv Nlr	1	0.00	11.7	95.5	IncrP
ULTRA-FINE					
Pye_0 Ycv_1 Nlr : Pye_1 Gpt_1 Nlr : Nlr	2	0.00	32.4	95.5	BIC

Nlr = visual acuity (Logarithm of minimum angle of resolution)

Ycv = corrected vision

Pye = years of education

Gpt = amnesia

Table 10 Best Nlr model

				Conditional probabilities of DV					
	IV states			Data		Model			
Pye	*Ycv*	*Gpt*	*N*	*Nlr_0*	*Nlr_1*	*Nlr_0*	*Nlr_1*	*Odds*	*p*
low	no	no	33	1	0	1	0	0	0.23
low	no	yes	22	1	0	1	0	0	0.32
low	yes	no	9	0.67	0.33	0.72	0.28	0.39	0
low	yes	yes	5	0.8	0.2	0.72	0.28	0.39	0.01
high	no	no	38	1	0	1	0	0	0.2
high	no	yes	21	0.9	0.1	0.91	0.09	0.10	0.31
high	yes	no	15	1	0	1	0	0	0.42
high	yes	yes	11	0.91	0.09	0.91	0.09	0.10	0.47
			154	0.95	0.05	0.95	0.05	0.05	

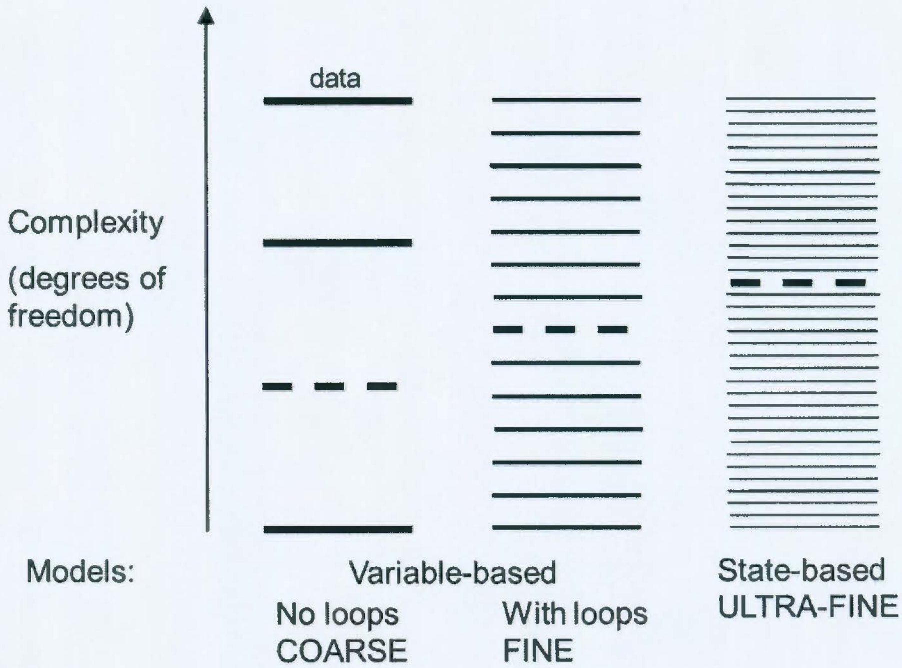
IVs Pye (years of education): 0 low vs 1 high
Ycv (corrected vision): 0 no vs 1 yes
Gpt (amnesia): no vs yes

DV Nlr (Visual acuity: logarithm of minimum angle of resolution): 0 normal vs 1 deficit

Odds Unlike previous tables, here **low** values of Odd are favorable

List of Figures

Figure 1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Complexity
(degrees of freedom)
Models:
Variable-based
No loops
COARSE
With loops
FINE
State-based
ULTRA-FINE

Figure 2

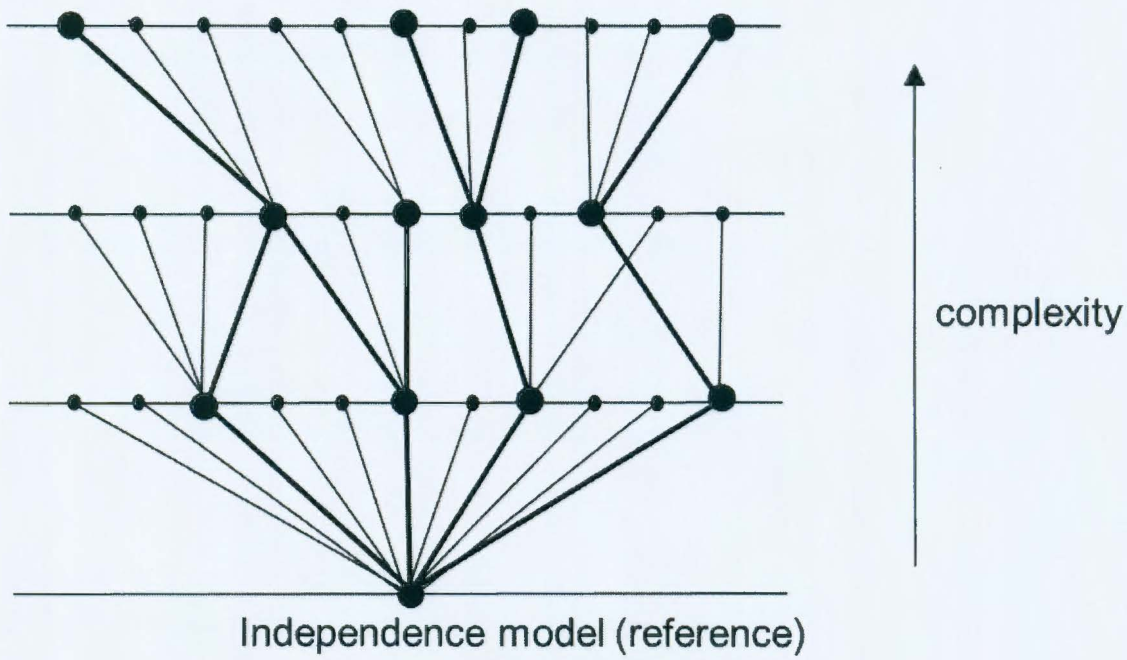
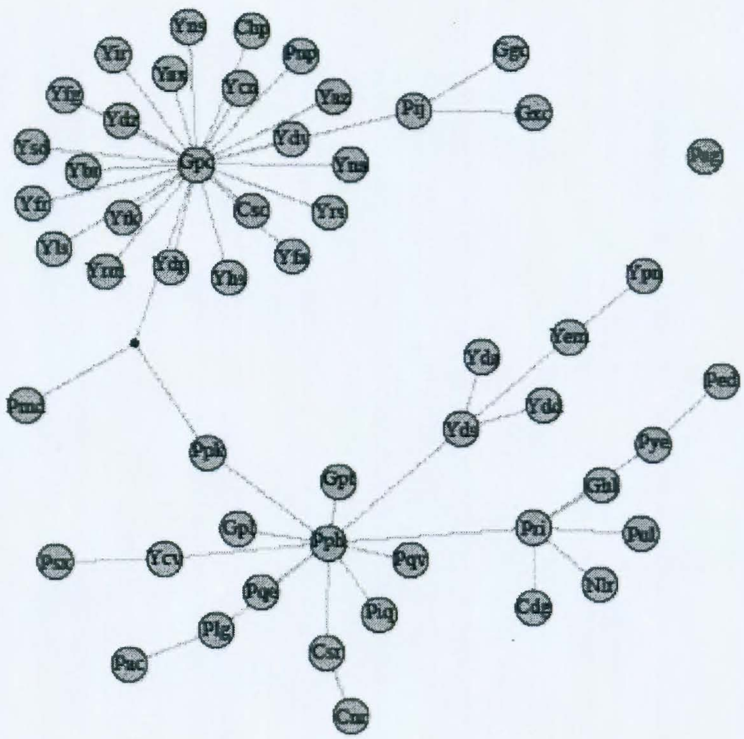


Figure 3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Ydu
Ycx
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Yrs
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Ppb
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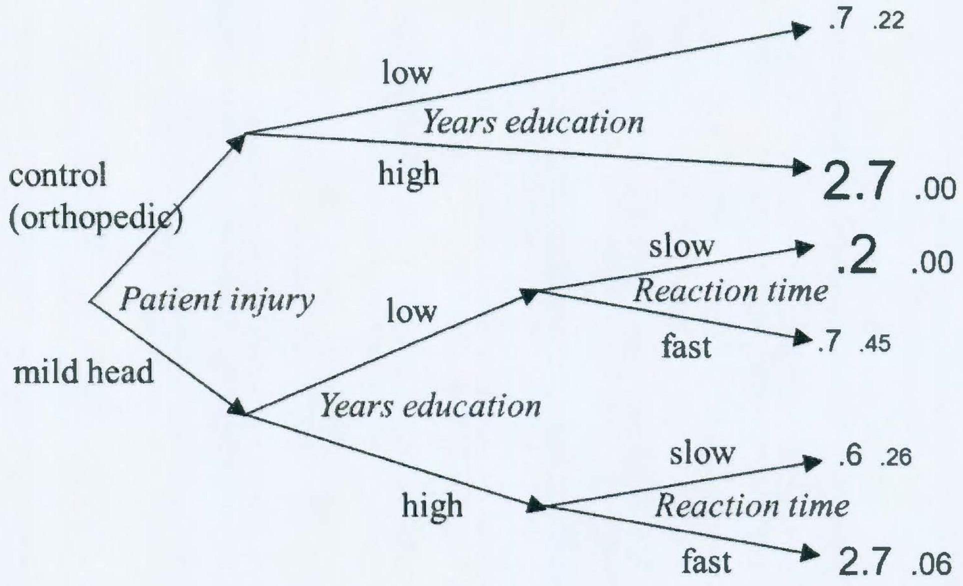
Ped
Pul
Cde

Figure 4


.7 .22
low
Years education
control
(orthopedic)
high
2.7 .00
slow
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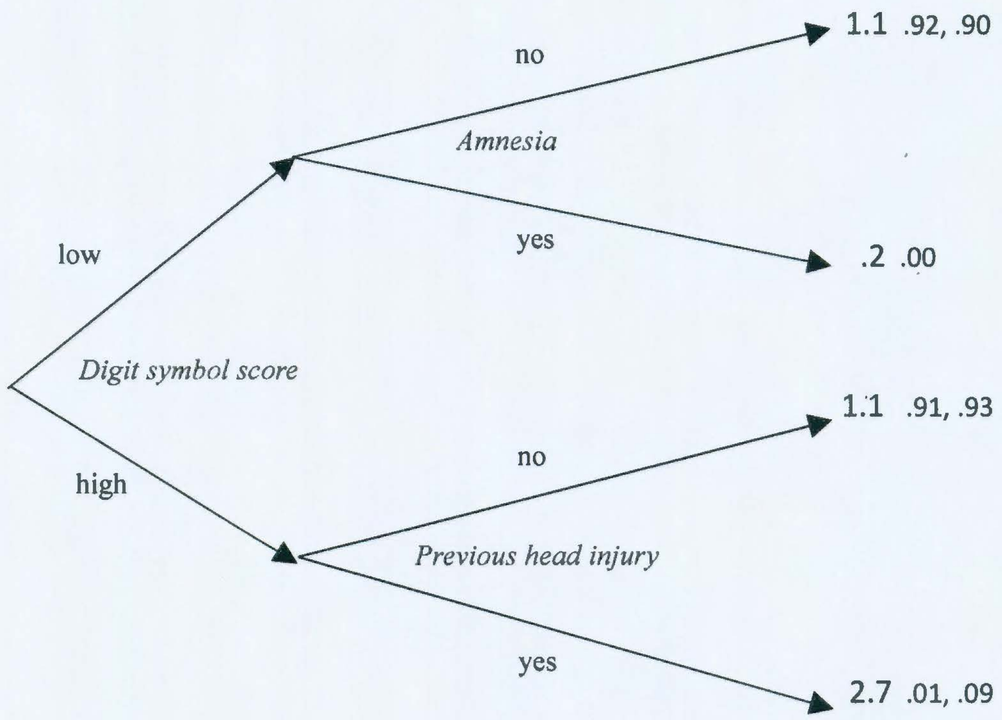
Patient injury
low
Reaction time
fast
.7 .45
mild head
Years education
slow
.6 .26
high
Reaction time
fast
2.7 .06

Figure 5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1.1 .92, .90
Amnesia
yes
.2 .00
low
Digit symbol score
high
no
1.1 .91, .93
Previous head injury
yes
2.7 .01, .09

Martin Zwick is a Professor of Systems Science at Portland State University. Prior to taking his current position at PSU, he was a faculty member in the Department of Biophysics and Theoretical Biology at the University of Chicago, where he worked in macromolecular structure and mathematical crystallography. In the 1970's his interests shifted to systems theory and methodology. Since 1976 he has been on the faculty of the PSU Systems Science Ph.D. Program and during the years 1984-1989 he was the program head. His current research interests are in discrete multivariate modeling (reconstructability analysis), theoretical biology, and systems theory and philosophy.

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Rosemary Tracie Nettleton is a Data Systems Administrator in Tillamook Oregon. In 2013 Tracie completed a Master of Bioinformatics and Computational Biology degree at Oregon Health and Science University where her focus was bio-image processing. Tracie's previous research experience is in the effects of drug and alcohol addiction. Her published papers discuss the respiratory effects of methadone, morphine and buprenorphine in the neonate. Tracie also holds a degree in Chemical Engineering from Washington State University and prior to her research career, worked as a process engineer in the silicon industry. In her current position, Tracie manages the client database and electronic health record for a behavioral health agency on the Oregon coast.