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Transform method of processing for speckle strain-rate measurements

Donald D. Duncan, Sean J. Kirkpatrick, F. Fausten Mark, and Lawrence W. Hunter

We have developed a highly sensitive method for measuring thermal expansion, mechanical strain, and creep rates. We use the well-known technique of observing laser speckle with a pair of linear array cameras, but we employ a data-processing approach based on a two-dimensional transform of the speckle histories from each camera. This technique can effect large gauge sizes, which are important in the assessment of the spatial statistics of creep. Further, the algorithm provides simultaneous global estimates of the strain rates at both small- and large-scale sizes. This feature may be of value in the investigation of materials with different short- and long-range orders. General advantages of our technique are compact design, modest resolution requirements, insensitivity to slow surface microstructure changes (as seen with oxidation), and insensitivity to zero-mean-noise processes such as turbulence and vibration. Herein we detail the theory of our technique and the results of a number of experiments. These tests are intended to demonstrate the performance advantages and limitations of the transform method of processing speckle strain-rate data.

Key words: Creep, thermal expansion, strain rates, speckle metrology.

Introduction

Here we present a unique objective speckle strain-gauge technique for the extraction of strain-rate estimates. We provide a discussion of the theory behind what we term the transform technique¹ and show its performance in a number of experiments.

Our measurement concept is illustrated in Fig. 1. The dog-bone figure represents the specimen under tensile stress. It is illuminated normally with a laser beam, and the illuminated portion of the specimen constitutes the gauge size. A pair of linear array detectors is symmetrically opposed at an angle θ with respect to the specimen normal, and both are in the plane of the incident laser beam. Yamaguchi²⁻⁴ showed that the speckle motion as sensed by a single

camera is given by

$$\begin{aligned} \delta x(\theta) = & a_x \left(\frac{L \cos^2 \theta_s}{L_s \cos \theta} + \cos \theta \right) \\ & - a_z \left(\frac{L \cos \theta_s \sin \theta_s}{L_s \cos \theta} + \sin \theta \right) \\ & - L \left[\epsilon_{xx} \left(\frac{\sin \theta_s}{\cos \theta} + \tan \theta \right) - \Omega_y \left(\frac{\cos \theta_s}{\cos \theta} + 1 \right) \right], \end{aligned} \quad (1)$$

where L is the distance between the specimen and the detector, L_s is the distance between the specimen and the source (i.e., the radius of curvature of the incident wave front), θ_s is the angle of the source with respect to the specimen normal, ϵ_{xx} is the linear strain in the plane of the laser beam and the detector, a_z is the out-of-plane movement, and Ω_y is the specimen rotation about the y axis. By using the illustrated configuration and subtracting the speckle movements as viewed by the two cameras, we arrive at the formula for differential speckle motion (before and after stress):

$$\delta A_x \equiv \delta x(\theta) - \delta x(-\theta) = -2L\epsilon_{xx} \tan \theta - 2a_z \sin \theta. \quad (2)$$

A proper choice of the observation distance and angle can make the out-of-plane motion described by the

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Fig. 1. Illustration of the measurement configuration.

second term negligible with respect to the first. We assume in the following discussion that this condition is met. Alternatively, the complement of the configuration shown in Fig. 1 can be used (one detector, two laser beams), in which case the out-of-plane term in Eq. (2) can be made to disappear altogether.

With this procedure and using correlations to determine speckle motion, Yamaguchi²⁻⁴ demonstrated a sensitivity of 10 microstrain (10 $\mu\epsilon$). A variation of Yamaguchi's technique is used by Barranger⁵ for the measurement of creep of furnace-heated specimens. One may achieve some strain-resolution improvement by interpolating the cross-correlation function between the reference and sample exposures.⁶ Another resolution-improvement technique is the interpolation of the speckle pattern itself.⁷ Improved resolutions obtained by these means, however, require that the speckle patterns be oversampled, i.e., that the speckles be large with respect to the camera pixel size. Large speckles can be obtained with either large observation distances or with small gauge sizes. In the interest of keeping the measurement configuration compact, we prefer a small observation distance. Further, in many instances small gauge sizes are acceptable or even desirable. However, to assess the spatial statistics of creep adequately, one must use large gauge sizes.⁸

Transform Method of Processing

As we mentioned above, the conventional approach to calculating the differential speckle pattern motion at the two detectors has relied on the calculation of the cross correlation between the reference speckle pattern (before stress) and the signal speckle pattern (after stress). However, because the objective is simply to estimate a lateral shift in a noisy signal, several alternatives to the cross-correlation technique are possible. Analogously to speckle pattern photography,⁹ one may, for example, add (or subtract) the two speckle patterns, Fourier transform, and inspect for the sinusoidal modulation. This approach also suggests homomorphic processing algorithms such as cepstral analysis.¹⁰ Our approach differs from these previous approaches that consider only pairs of signals. Our processing concept uses what we call a stacked speckle history. One such speckle history is illustrated in Fig. 2, which shows a

Fig. 2. Example of a stacked speckle history.

sequence of 256 one-dimensional speckle patterns stacked one atop another. These data are for an object strained at a uniform rate from approximately 15 $\mu\epsilon$ at the top of the image to approximately -15 $\mu\epsilon$ at the bottom, where each row represents 256 pixels. From Fig. 2 one can see a lateral shift in the speckle sequence and a gradual decorrelation in the speckle patterns as one proceeds from top to bottom. This display is much like that obtained by Oulamara *et al.*¹¹ within another context.

The visual similarity between the data displayed in Fig. 2 and the appearance of unprocessed synthetic aperture radar (SAR) data¹² is striking. The analogy is particularly strong between the fast (range) axis and the slow (Doppler) axis for the SAR data, and the fast (speckle) direction and the slow (temporal) direction of the speckle history. Naturally, the question arises as to how the desired information manifests itself in this display. Of course, the answer is simply that the time rate of the speckle pattern shift is given by the tilt of the corrugated structure. The standard method of processing SAR data therefore suggests a simple method of processing: a two-dimensional Fourier transform implemented with a fast-Fourier-transform (FFT) algorithm. Figure 3 is an example of such an operation. In Fig. 3, spatial frequency is along the horizontal axis and temporal frequency is along the vertical; dc is in the center. The information that we desire is given simply by the slope of the bright line running through dc, i.e., the time rate of the speckle pattern shift. Under the assumption of negligible out-of-plane motion, the time rate of strain [see Eq. (2)] is then given by

$$\dot{\epsilon}_{xx} = \frac{m_2 - m_1}{2L \tan \theta}, \quad (3)$$

trated in Fig. 1, we have for the differential Young fringe shift,

$$\delta A_x = \delta x(\theta) - \delta x(-\theta) = -2L\epsilon_{xx} \tan \theta. \quad (13)$$

This is recognized as the same result as Eq. (2) under the assumption of negligible out-of-plane motion. In this derivation we have ignored the effects of curvature of the incident wave front and of rigid-body motions. A more detailed derivation, which accounts for the various rigid-body motions,² leads to Eq. (1) and shows that this differential measurement configuration actually eliminates these effects.

For the more general case in which there may be many point scatterers (see Fig. 5 for definitions of the geometrical variables), the observed intensity pattern consists of the interference patterns for all pairwise separations. By defining the separation and center-of-gravity variables,

$$s_{ij} = a_i - a_j, \quad (14a)$$

$$\sigma_{ij} = \frac{a_i + a_j}{2}, \quad (14b)$$

we find that the resulting intensity term for the ij th pair is given by

$$2 \cos[k(R_i - R_j)] \\ = 2 \cos\left[\frac{2\pi s_{ij}}{\lambda L} (L \sin \theta + x \cos \theta - \sigma_{ij})\right]. \quad (15)$$

At this point the purpose of this generalization should be apparent. Because of the linearity of the Fourier transform and the fact that a fully developed speckle pattern can be viewed as the interaction between all possible point scatterer pairs, the transform method of processing for strain rate is seen to exploit fully the spatial as well as the temporal information in the recorded signals.

To proceed further, we assume a constant strain rate so that the separation and center-of-gravity variables are given by

$$s_{ij} = s_{ij0}(1 + \dot{\epsilon}t), \quad (16)$$

$$\sigma_{ij} = \sigma_{ij0}(1 + \dot{\epsilon}t), \quad (17)$$

Fig. 5. Geometry for a single point scatterer.

where $\dot{\epsilon}$ is the time rate of strain. Substituting these definitions into the argument of the cosine function of Eq. (15), one obtains the following:

$$2 \cos[k(R_i - R_j)] \\ = 2 \cos[2\pi(f_0 + tf_t + xf_s + xtf_{st} - t^2f_{tt})], \quad (18)$$

where

$$f_0 = \frac{s_{ij0} \sin \theta}{\lambda} \left(1 - \frac{\sigma_{ij0}}{L \sin \theta}\right), \quad (19a)$$

$$f_s = \frac{s_{ij0} \cos \theta}{\lambda L}, \quad (19b)$$

$$f_t = \frac{\dot{\epsilon}s_{ij0} \sin \theta}{\lambda} \left(1 - \frac{2\sigma_{ij0}}{L \sin \theta}\right), \quad (19c)$$

$$f_{st} = \frac{\dot{\epsilon}s_{ij0} \cos \theta}{\lambda L}, \quad (19d)$$

$$f_{tt} = \frac{\dot{\epsilon}^2 s_{ij0} \sigma_{ij0}}{\lambda L}. \quad (19e)$$

The sizes of the spatiotemporal and quadratic time terms dictate the ultimate (large-strain-rate) applicability of this speckle technique in general. If one ensures that these terms are small with respect to the remaining terms, then the problem is separable. In this case, the time rate of the Young fringe movement is given by the quotient of the ordinate and the abscissa, f_t/f_s .

The bound on the term involving the center of gravity is more restrictive and apparently introduces a bias into the estimate of temporal frequency f_t . The differential measurement geometry, however, removes this bias.

Figure 6 shows an application of the transform method for a simulation¹⁶ that uses five discrete scatterers with six unique separations. Another feature of the simulation was the addition of vibration noise and thermal detector noise. With this simulation we were able to quantify the superiority of the transform algorithm over the conventional correlation approach.

Details of the Transform Algorithm

We have described the concept of the transform processing technique; now we elaborate on the actual algorithm. We have experimented with several processing approaches. The first relies on the use of a two-dimensional FFT.¹ In this case we transform each row in the desired segment of the speckle history by using a FFT (Hanning window, $N = 1024$ with zero filling), and the columns are transformed in a similar manner. Each of the columns in the transform domain represents a cut (at constant spatial frequency) through the focused line, as illustrated in Fig. 3. We determine the location of this line within each column by calculating its center of gravity. In addition, a measure of the signal-to-noise ratio (SNR)

Fig. 6. Result of the application of the transform algorithm for several discrete scatterers.

in each column is calculated. This figure of merit is simply the ratio of the peak value to the width of the line.

Another processing scheme that we have experimented with is a hybrid. It uses the nonparametric FFT spectral estimator in the spatial direction (the rows), and a parametric spectral estimator in the temporal direction (the columns). For each of the rows in the desired segment of the speckle history, the same transform technique as described above is used. In the temporal direction, however, an autoregressive (AR) spectral estimator¹⁷ is used (modified covariance; number of records, 10–20; number of poles, 3–5). For each column in the resulting focused image (for a fixed spatial frequency), the peak value of the image is located.

The advantage of parametric spectral estimation techniques is that they make use of *a priori* knowledge of the process. Specifically, we know in this case that the focused line image will be highly localized. After the nonparametric transformation in the spatial direction (using the FFT), we know that transformation in the temporal direction should produce a narrow line spectrum at each spatial frequency. Furthermore, parametric techniques are particularly appropriate for processes displaying a lack of stationarity. By the very nature of the autoregressive estimators, they produce a spiky spectrum. Thus, good resolution in the temporal direction can be obtained without the use of extended observation periods.

The next step in the algorithm is the determination of the position of the focused line. The general approach is a weighted-least-squares¹⁸ (WLS) fit to the selected points in each column. In the case of the FFT algorithm, the weight is simply the SNR for the selected point. For the nonparametric algo-

rithm, the weight is the value of the image at the peak.

To determine the scale-size dependence of strain rate, we perform a least-squares fit for temporal frequency parameterized on spatial frequency, $f_t(f_s)$. Making use of the fact that the focused image is symmetric about the dc point, we can use only the data in the right half-plane. We incorporate *a priori* knowledge of symmetry by forcing the regression line through the origin. For example, under the assumption of no scale-size dependence of strain rate, we find that the regression model to be fit is

$$f_t(f_s) = mf_s. \quad (20)$$

An example of a model incorporating the possibility of scale-size dependence of strain is

$$f_t(f_s) = mf_s + \beta f_s^2. \quad (21)$$

In this case, the resulting strain-rate estimate is given by

$$\dot{\epsilon}(f_s) = \frac{m_2 - m_1}{2L \tan \theta} + \left(\frac{\beta_2 - \beta_1}{2L \tan \theta} \right) f_s, \quad (22)$$

where the subscripts are associated with cameras 1 and 2.

A variation on the approach that uses an individual regression fit to the processed data from each camera is for one to difference the ordinates of the focused lines (at each spatial frequency) to arrive at a display of $\Delta f_t(f_s)$, where $\Delta f_t = f_{t, \text{camera2}}(s) - f_{t, \text{camera1}}(s)$. From our earlier discussions we see that the estimate of strain rate is given by

$$\dot{\epsilon}(f_s) = \frac{\Delta f_t(f_s)/f_s}{2L \tan \theta} \quad (23)$$

One advantage of such an approach is that it is much easier for one to test for significance of a nonlinear parameter in the regression. For example, by making use of the relationship between separation and spatial frequency,

$$f_s = \frac{s \cos \theta}{\lambda L}, \quad (24)$$

we have

$$\dot{\epsilon}(s) = \left(\frac{\lambda}{2 \sin \theta} \right) \frac{\Delta f_t(f_s)}{s}. \quad (25)$$

A power-law model,

$$\dot{\epsilon}(s) = \alpha s^\beta, \quad (26)$$

for example, permits an easy test of the significance of the strain-rate variation with scale size. For homogeneous strain rate, one would find that β is not significant. The disadvantage of this prior differencing approach is that the specification of the weight on

the temporal frequency difference, Δf_t (used in the WLS regression fit), is problematic.

Experiment and Results

We used a pair of CCD cameras I2S, Model iDC161, with 3456 square pixels at a 10.7- μm pitch. The cameras were controlled through individual cards plugged into an IBM PC AT. Integration time was 30 ms. We triggered the exposures by using a Keithley Model DAS-8PGA analog-to-digital (A/D) card that also was mounted on the PC bus. We programmed this A/D card to output a square wave that we subsequently used to trigger the cameras. In addition, we used the A/D card to accrue auxiliary data from the test-set load cell and an extensometer. The sample interval was 0.5 s.

The laser (Spectra Diode Model SDL-5422-H1) is a GaAlAs device that emits at 819 nm and is temperature tunable through the use of a thermoelectric cooler. This diode runs monomode for drive currents less than approximately 120 mA and has a spectral bandwidth of $\leq 0.67 \text{ \AA}$ and a resulting coherence length in excess of 1 cm. We used a drive current of 100 mA, which yielded approximately 70 mW of output power. The output beam is a fan of $10^\circ \times 25^\circ$ full width at half-maximum. We controlled the size of the illuminated stripe by using a lens made specifically for collimating diode laser beams (Melles Griot Model 06GLC002, 8-mm focal length). The shape of this beam is advantageous because a beam elongated in the plane of measurement produces speckles that are elongated perpendicular to the camera array. Thus the speckle patterns as seen by the cameras tend to retain good correlation in the presence of out-of-plane speckle motion.

We conducted several experiments designed to demonstrate the transform technique's ability to resolve strains at various scale sizes. For each of the tests we used a Materials Testing System (MTS) fatigue-testing system (MTS Model 810) to apply a linear load rate to a 1 in. $\times$ 1 in. (2.54 cm $\times$ 2.54 cm) cross-section specimen. A typical load waveform is illustrated in Fig. 7. Two different test articles were used: two 1 in. $\times$ 1 in. cross-section stainless-steel billets, one

Fig. 7. Load waveform for MTS experiments.

Fig. 8. Illustration of test specimens.

with a 4.24-mm-diameter hole drilled in the middle. The specimens are illustrated in Fig. 8. Also shown for each of the specimens is the laser illumination. For each of the test specimens we determined the actual overall strain rate with an extensometer (MTS Model 632.11B-20, 1-in. gauge length). For the uniform sample the strain rate was $3.38 \pm 0.23 \mu\epsilon/\text{s}$ [100 lb/s (45.35 kg/s) load rate], and for the drilled sample it was $3.48 \pm 0.20 \mu\epsilon/\text{s}$ (100 lb/s load rate). These estimates were based on the same time intervals over which the speckle strain-rate estimates were made.

In our experiments the laser impinged the sample normally and was effectively located along a line connecting the cameras, i.e.,

$$L_s = L \cos \theta. \quad (27)$$

For this geometry Eq. (1) becomes

$$\delta x(\theta) = a_x \left(\frac{1}{\cos^2 \theta} + \cos \theta \right) - a_z \sin \theta - L \left[\epsilon_{xx} \tan \theta - \Omega_y \left(\frac{1}{\cos \theta} + 1 \right) \right]. \quad (28)$$

The observation distance, L , was 0.82 m; the nominal viewing angle, θ , was 25° for the tests that used the uniform billet and 23° for those that used the drilled billet.

Because our cameras have a large number of pixels and because for each speckle history we used only 512 pixels, we were led to consider the possibility of using a double differential measurement technique. That is, the 512 pixels at one end of the camera would constitute a measurement at one angle, θ_1 , and the 512 pixels at the other end would constitute a measurement at another angle, θ_2 . Under the assumption that angle $\theta' = \theta_1 - \theta_2$ is small, one can show that the time rate of strain is given by the expression

$$\dot{\epsilon}_{xx} = \left[\frac{\delta A x(\theta_2) + \delta A x(\theta_1)}{4L \tan^3 \theta} \right] + \left[\frac{\delta A x(\theta_2) - \delta A x(\theta_1)}{4L \theta' \tan^2 \theta} \right]. \quad (29)$$

Note that this result makes no assumption about out-of-plane motion; the double differencing eliminates these contributions. Our efforts along these lines were largely unsuccessful because of the small angular difference between the pixels at the two ends of each camera ($\theta' = 2.2^\circ$). The use of (effectively) four cameras, however, does yield information about the consistency of the strain-rate estimates, and it can yield good estimates of the confounding factors in the measurement, i.e., the rigid-body terms in Eq. (1) that are being subtracted. Specifically, by adding the speckle motions at symmetrically opposed angles, we obtain an estimate of the rigid-body terms:

$$\delta x(\theta) + \delta x(-\theta) = 2a_x \left(\frac{1}{\cos^2 \theta} + \cos \theta \right) + 2L\Omega_y \left(\frac{1}{\cos \theta} + 1 \right). \quad (30)$$

Information of this nature is useful in establishing the uncertainties in the estimates of the specimen strain. Specifically, one would want these rigid-body terms to be small with respect to the strain term so that precision is not lost in the subtraction operation.

Uniform Billet

Initial calculations used 20 records of data at 0.5 Hz for a total observation time of 10 s. The resulting total strain based on the extensometer data was 33.8 $\mu\epsilon$. Results of the various processing approaches are summarized in Table 1. We also performed strain-rate estimates based on ten data records and thus inferred from the extensometer data a total strain over the observation period of 16.9 $\mu\epsilon$. In Table 1 all confidence intervals are at the 80% level. For each processing approach the results are quoted

for the plus cameras (pixels at $\theta + \theta'$) and the minus cameras (pixels at $\theta - \theta'$).

The use of the FFT-FFT algorithm on 20 samples provided consistent strain-rate estimates that agreed closely with the extensometer data. The use of 20 data records, the FFT-AR algorithm (three-pole autoregressive estimator in the temporal direction), and a power-law fit with prior differencing produced exponents with P values of 0.66 and 0.11. We were thus led to conclude that the strain rate as measured over the illuminated stripe was homogeneous. The application of the FFT-AR algorithm and subsequent WLS fits yielded estimates that were within the uncertainty interval of the extensometer estimates. Increasing the number of parameters in the AR algorithm seemed to decrease the consistency of the estimates as calculated at the two angles. Finally, for the estimates based on ten records, the FFT-FFT algorithm began to yield inconsistent results. For ten data records, the FFT-AR algorithm produced estimates that displayed a 15% difference for the two angles but an average that was within the uncertainty in the extensometer-based estimates. For these short data records the power-law fits provided erratic results regardless of which transform algorithm was used. We therefore restricted all subsequent efforts to the use of a first- or second-order WLS polynomial fit algorithm.

Drilled Billet

Here again we began our strain-rate estimates with 20 data records. Based on the extensometer estimate of 34.8 $\mu\epsilon$, the inferred strain rate over this observation interval was 3.48 $\mu\epsilon/s$. Table 2 contains a summary of the various processing approaches for this test subject. Under the assumption of a homogeneous strain rate at all scale sizes, we found that the FFT-FFT algorithm with a WLS fit using 20 data records yielded results with a 12%

Table 1. Summary of Results for a Uniform Billet

Algorithm	No. of Samples	Model	Estimated Strain Rates ($\mu\epsilon/s$)
Extensometer	20	First-order WLS	3.38 ± 0.23
FFT-FFT	20	First-order WLS	3.44 ± 0.15
			3.44 ± 0.11
FFT-AR, three poles	20	Power law	$3.06 s^{-0.0156 a}$
			p value for slope = 0.66
			3.63 ± 0.13
			$4.38 s^{-0.0469 a}$
			p value for slope = 0.11
			3.70 ± 0.11
FFT-AR, three poles	20	First-order WLS	3.64 ± 0.06
FFT-AR, five poles	20	First-order WLS	3.58 ± 0.05
FFT-FFT	10	First-order WLS	4.09 ± 0.04
			3.44 ± 0.12
			1.56 ± 0.15
			-0.21 ± 0.14
FFT-AR, three poles	10	First-order WLS	3.69 ± 0.08
			3.16 ± 0.10

^aScale size s , expressed in meters, is $0 \leq s \leq 17$ mm.

Table 2. Summary of Results for a Drilled Billet

Algorithm	No. of Samples	Model	Estimated Strain Rates ($\mu\epsilon/s$)
Extensometer	20	First-order WLS	3.48 ± 0.20
FFT-FFT	20	First-order WLS	3.30 ± 0.07
			3.72 ± 0.07
FFT-AR, three poles	20	First-order WLS	3.36 ± 0.04
FFT-FFT	20	Second-order WLS	3.71 ± 0.05
			$4.07-53.16 s^a$
			$3.17 \leq \dot{\epsilon} \leq 4.07$
			$4.28-40.0 s$
			$3.60 \leq \dot{\epsilon} \leq 4.28$
FFT-AR, five poles	20	Second-order WLS	$4.04-45.5 s^a$
			$3.27 \leq \dot{\epsilon} \leq 4.04$
			$4.12-25.8 s^a$
			$3.68 \leq \dot{\epsilon} \leq 4.12$
FFT-FFT	10	First-order WLS	2.26 ± 0.12
			1.95 ± 0.15
FFT-AR, three poles	10	First-order WLS	3.43 ± 0.08
			3.40 ± 0.07

^aScale size s , expressed in meters, is $0 \leq s \leq 17$ mm.

variation for the two angles but with an average that agreed closely with the extensometer data. The FFT-AR transform that used three poles agreed closely also.

The use of the FFT-FFT algorithm and a second-order polynomial fit produced strain-rate estimates that varied from an average (for the two angles) of 3.39 to 4.18 $\mu\epsilon/s$. We discuss the interpretation of these results in the next section. Experiments with the FFT-AR transform showed the best results with five poles. These results agreed closely with those from the FFT-FFT algorithm. Again, for the shorter data records the FFT-FFT algorithm began to yield poor results, whereas the FFT-AR transform yielded consistent results that agreed closely with the extensometer estimates.

In experimenting with the various approaches listed above, we came to the conclusion that for the longer data records the FFT-FFT algorithm provided the most robust performance. When the data records became short, however, the FFT-AR transform yielded superior performance. The one unfortunate aspect of parametric estimators, however, is the degree of subjectivity in the choice of the number of parameters in the model. Various rules of thumb suggest that the number of poles for short data records be of the order of one third to one half the number of data records.¹⁷ For this application, in which we are certain that all the spectral energy is located in a narrow band (the focused bright line), we found that the fewer number of poles tended to suffice and that results became erratic for the number of poles exceeding one third the number of data records. One factor that was of use in the determination of the number of poles to use was the consistency of the estimates provided by the two angles.

These results are remarkable when one considers the magnitude of the speckle motion for the strains used herein. For the uniform test specimen, the total strain over the observation period was 16.9 $\mu\epsilon$. For an object in pure strain, the resulting speckle motion is 6.66 μm [see Eq. (13)]. This is only approximately 60% of a single pixel. Despite this small motion, the autoregressive estimator was able to provide a stable estimate of the strain rate. Note that this figure of 16.9 $\mu\epsilon$ does not constitute the absolute sensitivity of this technique. Rather, the strain-rate resolution can be augmented substantially through the appropriate choice of observation distance and angle.

Interpretation

We now develop a model for the scale-size dependence of strain. This model is consistent with our view of the speckle pattern as being due to the superposition of all possible scatter pairs on the rough object and the fact that the transform method provides global estimates of strain rate as a function of spatial frequency, i.e., scale size.

Consider first a conventional strain gauge of linear

dimension s , which we place on an object undergoing stress. Assume further that the object has spatially varying mechanical properties and thus strain (rate). The effective strain rate as registered by this strain gauge is given by

$$\dot{\epsilon}_{\text{eff}}(\sigma, s) = \frac{1}{s} \int_{\sigma-s/2}^{\sigma+s/2} dx \dot{\epsilon}(x), \quad (31)$$

where the mean location of the strain gauge is σ , and the strain gauge effectively measures the average strain rate over the dimension s , centered at σ . This is, in effect, what the strain measurement of Sharpe¹⁵ produces. Because the speckle pattern can be considered to be caused by all possible scatter pairs over the illuminated region, the strain rate as inferred by the transform method is

$$\dot{\epsilon}_{\text{meas}}(s) = \frac{\int_{-\infty}^{\infty} d\sigma \dot{\epsilon}_{\text{eff}}(\sigma, s) I(\sigma + s/2) I(\sigma - s/2)}{\int_{-\infty}^{\infty} d\sigma I(\sigma + s/2) I(\sigma - s/2)}, \quad (32)$$

where I is the specimen illumination. This expression is consistent with the fact that the focused line is simply the autocorrelation function of the illumination intensity.¹⁹ Note that the transform algorithm directly provides an estimate of such an autocorrelation. The denominator ensures proper normalization. For example, when the effective strain rate is independent of position and gauge size (homogeneous test specimen), this expression reduces to

$$\dot{\epsilon}_{\text{meas}}(s) = \dot{\epsilon}. \quad (33)$$

Finally, this expression can be shown to reduce to the proper result for a discrete number of scatterers. (The autocorrelation function yields the set of unique separations for all the scatterer pairs.)

Now, given the measured strain rate, what can we say about the actual strain rate on the test specimen? First, we note that the effective strain rate will vary more quickly in the sum coordinate, σ , than the separation coordinate, s . This situation suggests that we may make the approximation

$$\dot{\epsilon}_{\text{eff}}(\sigma, s) \approx \dot{\epsilon}_{\text{slow}}(s) \dot{\epsilon}_{\text{fast}}(\sigma). \quad (34)$$

A reasonable choice for such a factorization is the product of the effective strain-rate behaviors along the two axes, $\sigma = 0$ and $s = 0$:

$$\dot{\epsilon}_{\text{eff}}(\sigma, s) \approx \frac{\dot{\epsilon}_{\text{eff}}(\sigma, 0) \dot{\epsilon}_{\text{eff}}(0, s)}{\dot{\epsilon}_{\text{eff}}(0, 0)}. \quad (35)$$

However, from Eq. (33),

$$\dot{\epsilon}_{\text{eff}}(\sigma, 0) = \dot{\epsilon}(\sigma), \quad (36)$$

so that our quasi-homogeneous effective strain rate is given by

$$\dot{\epsilon}_{\text{eff}}(\sigma, s) \approx \frac{\dot{\epsilon}(\sigma)\dot{\epsilon}_{\text{eff}}(0, s)}{\dot{\epsilon}(0)}. \quad (37)$$

In this case the measured strain rate is given by

$$\begin{aligned} \dot{\epsilon}_{\text{meas}}(s) &\approx \dot{\epsilon}_{\text{eff}}(0, s) \\ &\times \frac{\frac{1}{\dot{\epsilon}(0)} \int_{-\infty}^{\infty} d\sigma \dot{\epsilon}(\sigma) I(\sigma + s/2) I(\sigma - s/2)}{\int_{-\infty}^{\infty} d\sigma I(\sigma + s/2) I(\sigma - s/2)}. \end{aligned} \quad (38)$$

The term involving the integration over σ will be slowly varying in the separation variable s , with the result that the behavior of the measured strain rate will be dominated by the behavior of the effective strain rate,

$$\dot{\epsilon}_{\text{eff}}(0, s) = \frac{1}{s} \int_{-s/2}^{+s/2} dx \dot{\epsilon}(x). \quad (39)$$

When the intensity pattern has a Gaussian profile (as it did here), it is easily shown that the term in approximation (38), which involves the quotient of integrals, is independent of scale size s . In this case, the measured strain rate is

$$\dot{\epsilon}_{\text{meas}}(s) \approx C \dot{\epsilon}_{\text{eff}}(0, s), \quad (40)$$

where C is a constant.

To get a feeling for the validity of our results, we considered a similar problem of a circular hole in a thin sheet. This is a problem for which an analytical solution exists.⁸ Predictions of the longitudinal strain rate for this problem are displayed in Fig. 9. These results are for a lateral distance from the

center of the hole of 1.5 hole radii. Note that the longitudinal strain rate is highest opposite the hole but is actually less than that for the homogeneous sheet. This is merely a reflection of the fact that the stress flow lines are deflected by the presence of the hole.

Figure 10 is a display of the effective strain rate [Eq. (39)] for this problem. These results are plotted for scale size normalized to hole radius. Our previous estimates of the strain-rate variation assumed scale sizes of 0–17 mm, which in normalized terms is 0–8 hole radii. The thin-sheet problem therefore suggests a strain-rate variation of roughly a factor of 1.8:1 from small- to large-scale size. Although our predictions yielded somewhat less variation than this, the correspondence between the actual strain rates for a thin sheet and our block is only qualitative.

Discussion

We have developed a highly sensitive method for measuring thermal expansion, mechanical strain, and creep rates. We use the well-known technique of observing laser speckle with a pair of linear array cameras, but we employ a data-processing approach based on a two-dimensional transform of the speckle histories from each camera. This technique can effect large gauge sizes, which are important in the assessment of the spatial statistics of creep. Further, the algorithm provides simultaneous global estimates of the strain at both small- and large-scale sizes. This feature may be of value in the investigation of materials with different short- and long-range orders. General advantages of our technique are compact design, modest resolution requirements, insensitivity to surface microstructure changes (as seen with oxidation), and insensitivity to zero-mean-noise processes such as turbulence and vibration.

Further, we have shown that the technique of Yamaguchi² can be considered a generalization of Sharpe's technique.¹⁵ Specifically, we showed that the speckle pattern can be viewed as arising from all possible scatter pairs over the illuminated region of the rough surface. This permits generalization of

Fig. 9. Longitudinal strain rate along a line 1.5 hole radii from the center of a hole in a thin sheet.

Fig. 10. Effective strain rate as a function of scale size.

our measurement technique to geometries not specifically considered by Yamaguchi, for example, scatter from small fibers that may be used to reinforce composite materials. This is, in fact, the thrust of our current measurement efforts. Traditionally, these fiber scatter measurements have been attempted in the backscatter direction. On the basis of the material presented herein, however, it is clear that these measurements can also be considered in the forward-scatter direction. Measurements of this kind can produce large signal levels with modest laser powers.

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