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Scott A. Wells
Portland State University

Gregory K. Savage
Portland State University

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Scott A. Wells and Gregory K. Savage
Department of Civil Engineering
Portland State University
Portland, Oregon 97207-0751
Voice (503) 725-4276 Fax (503) 725-4298
http://www.ce.pdx.edu/~wellss
e-mail: scott@eas.pdx.edu

Abstract

During the initial stages of a Buchner funnel or specific resistance test, gravity drainage occurs prior to application of the pressure differential. Some allow time for a small cake to form by gravity drainage. Filtrate data from the gravity drainage period can be used to determine constitutive properties of the cake under a hydrostatic pressure gradient. The constitutive properties that define the structure of the cake include the permeability and porosity as functions of the applied stress. Equations governing the drainage rate during a gravity filtration experiment assuming a constant and a non-constant average cake permeability and cake porosity were developed. Numerical solutions were shown predicting the gravity drainage rate given known constitutive relationships. Also, a procedure was shown illustrating how constitutive relationships could be determined using gravity drainage data.

Introduction

During the initial stages of the Buchner Funnel test, gravity drainage occurs prior to application of the pressure differential. Some researchers recommend allowing time for a thin cake to form by gravity prior to application of the pressure differential (Vesilind, 1979). Christensen and Dick (1985a, 1985b) and Wells and Dick (1988) evaluated the impact of allowing a gravity drainage period (or cake formation period) on computed values of specific resistance.

This research note explores the theory and application of gravity filtration data prior to application of the pressure differential in a Buchner funnel test.

Governing Equations: Constant Average Cake Permeability and Porosity

For flow through a series of cake layers of varying permeability, Greenkorn (1982) showed that Darcy's Law could be written as

$$Q = \frac{A\Delta p}{m\Sigma r_i^* L_i} = \frac{A\Delta p}{m\Sigma \frac{L_i}{k_i}} = \frac{\bar{k} A\Delta p}{mL} \tag{1}$$

where
Q : flow rate through cake
A : cross-sectional area of filtration cell
Δp : applied pressure differential across entire cake
r_i^* : inverse permeability of layer i
L_i : length of layer i
m : dynamic viscosity
L : total cake length $= \Sigma L_i$
$\bar{k}$: average permeability of cake

For gravity filtration, the applied pressure differential and the cake length is a function of time, such that Darcy's Law can be written as

$$Q = \frac{\bar{k}A}{m}\frac{\Delta p(t)}{L(t)} = \frac{\bar{k}A}{m}\frac{\rho g h(t)}{L(t)} \tag{2}$$

where h : distance from top of cake to free water surface

Using mass conservation, the relationship between L(t) and h(t) can be determined as follows:

For gravity drainage, the rate of filtrate production is equal to the change of volume of the initial slurry (assuming no solids are lost through the filter medium), i.e.,

$$Q = -\frac{\Delta h}{\Delta t}A \tag{3}$$

where Δh : change in height of slurry
Δt : change in time

The initial mass of solids in the slurry is

$$\text{Mass}_{\text{initial}} = \forall C_i = h_i \text{A} C_i \tag{4}$$

where h_i : initial height of slurry
C_i : initial concentration of slurry
$\forall$: volume of slurry added to filtration cell

If the cake is formed at an average concentration of C_c , then the initial mass is equal to the mass of solids in the slurry (at concentration C_i) and the mass of solids in the cake (at concentration C_c), such that

$$h_i \text{A} C_i = h C_i \text{A} + L C_c \text{A} \tag{5}$$

Simplifying and solving for L,

$$L = \frac{C_i}{C_c}\left(h_i - h\right) \tag{6}$$

Also, note that C_c can be described by the porosity of the cake, ε_c, as

$$C_c = r_s \left(1 - \in_c\right) \tag{7}$$

where r_s : solid density

Also, the initial concentration of suspended solids can be determined using the initial porosity, ε_i, as

$$C_i = r_s \left(1 - \in_i\right) \tag{8}$$

Then substituting Eqs. 3 and 6 into Eq. 2 and simplifying,

$$\frac{dh}{dt} = \frac{-\bar{k} rgh}{m \frac{C_i}{C_c}\left(h_i - h\right)} \tag{9}$$

Using an initial condition that $h = h_i$ at $t = 0$, the solution to Eq. 9 assuming $\bar{k}$ and C_c are not a $f(t)$ is an implicit equation of the form

$$h \exp\left(\frac{-h}{h_i}\right) = h_i \exp\left(\frac{at}{h_i} - 1\right) \tag{10}$$

where $$a = \frac{-\bar{k}}{m}\frac{rg}{C_i / C_c} = \frac{-\bar{k}\,rg}{m}\frac{(1-\in_c)}{(1-\in_i)} \tag{11}$$

Hence, if data of *h(t)* are determined from experimental results, the value of $\bar{k}$ and $\in_c$ can be determined assuming they are constant as a *f(t)*. Note that Eq. 10 can be rearranged as

$$t = \frac{\ell n\left(\frac{h}{h_i}\right) - \frac{h}{h_i} + 1}{\frac{a}{h_i}} \tag{12}$$

There is also a minimum value of $h_{\min}$ or conversely a maximum value of L. These can be determined from a mass balance as in Eq. 6 such as

$$L_{\max} = \frac{C_i}{C_c}\left[h_i - h_{\min}\right] \tag{13}$$

or

$$h_{\min} = \left(\frac{\frac{C_i}{C_c} h_i}{1 + \frac{C_i}{C_c}}\right) \tag{14}$$

The relationship between L and h is shown in Figure 1.

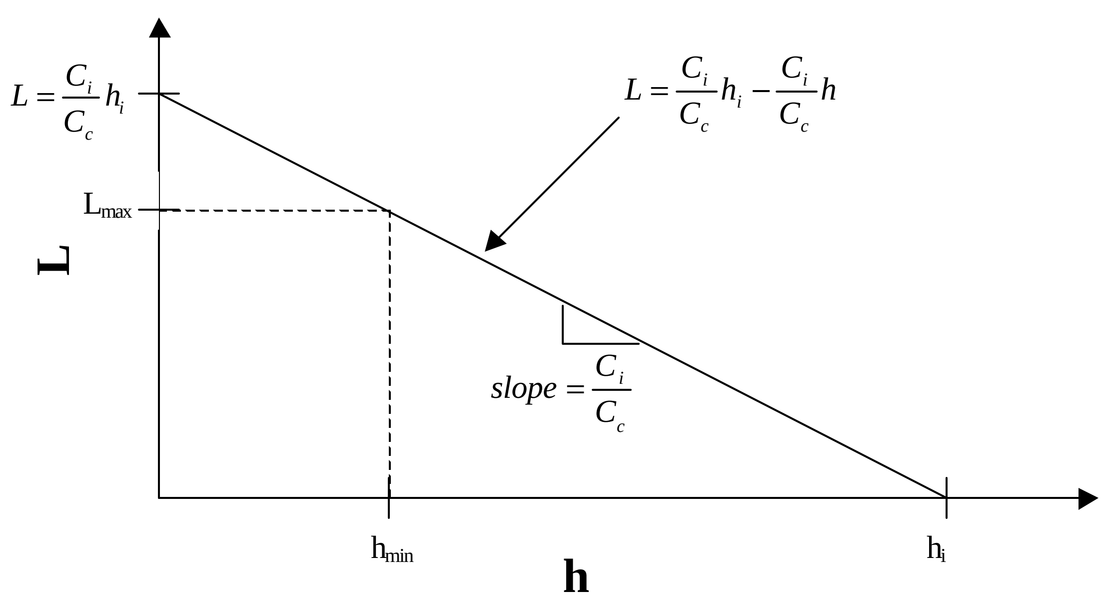

Figure 1. Relationship between cake length L and height of slurry in filtration vessel.

Governing Equations: Non-Constant Average Cake Permeability and Porosity

The average cake permeability and porosity ($\bar{k}$ and $\in_c$) can be assumed to be functions of time by assuming appropriate constitutive relationships. For example, the permeability can be described as a function of porosity, i.e., *k = f(Î)*, and the relationship between pressure differential and cake porosity can be defined using the definition of the average cake coefficient of volume compressibility, m_v, such that

$$m_v \;=\; -\frac{\P \in}{\P r} \;=\; \frac{\P \in}{\P s'} \tag{15}$$

where p : pure water pressure

s': effective stress $= \Delta p_{\text{total}} - p$

Any functional relationships for k and m_v can be chosen, but for mathematical convenience and according to data from Wells (1990a), the following forms were chosen:

$$\bar{k} = a \exp\left(b \in_c\right) \tag{16}$$

$$m_v \;=\; c \exp\left(d \in_c\right) \tag{17}$$

where $a\ [L^2],\ b[-],\ c[ML^{-1}T^{-2}],\ d[-]$: empirical constants.

Substituting Eq. 17 into Eq. 15, integrating, using $\Delta p = rg\left(h - h_i\right)$, and solving for $\in_c$,

$$\in_c = -\frac{1}{d}\,\ell n\left[-cdrg\left(h - h_i\right) + \exp\left(-\mathrm{d} \in_{\mathrm{i}}\right)\right] \tag{18}$$

Then, rewriting Equation 8 by substituting Eq. 7 and Eq. 16,

$$\frac{dh}{dt} = \frac{-\bar{k} rgh}{m \, {}^{C_i}\!/_{C_c} \left(h_i - h\right)} = \frac{-a \exp\left(b \in_c\right) rgh\left(1 - \in_c\right) r_s}{mC_i\left(h_i - h\right)} \tag{19}$$

Then substituting Eqs. 18 and 8 into Eq. 19, we have the final differential equation for *h*,

$$\frac{dh}{dt} = \frac{-arhg\left[\exp\left(-\frac{b}{d}\ell n\left\{-cdrg\left(h - h_i\right) + \exp\left(-d \in_i\right)\right\}\right)\right]\left[1 + \frac{1}{d}\ell n\left\{-cdrg\left(h - h_i\right) + \exp\left(-d \in_i\right)\right\}\right]}{mr_s\left(1 - \in_i\right)\left(h_i - h\right)} \tag{20}$$

This non-linear ordinary differential equation can be solved numerically.

Numerical Results

Eqs. 12 and 20 were solved for a given set of constitutive relationships for kaolin clay shown in Table 1. Eq. 12 was solved implicitly for h as a f(t). Eq. 20 was solved using a Runge-Kutta numerical technique. Figure 2 shows the resulting height of fluid in a column or Buchner funnel and Figure 3 shows the expected filtrate volume as a function of time for both Eq. 12 and Eq. 20.

Table 1 . Constitutive parameters for kaolin clay (Wells, 1990a).

Parameter values for Eqs. 16 and 17	Value	Units
a	2.04E-20	m^2
b	28.9	-
c	2.0E-16	$kg/m/s^2$
d	28.9	-

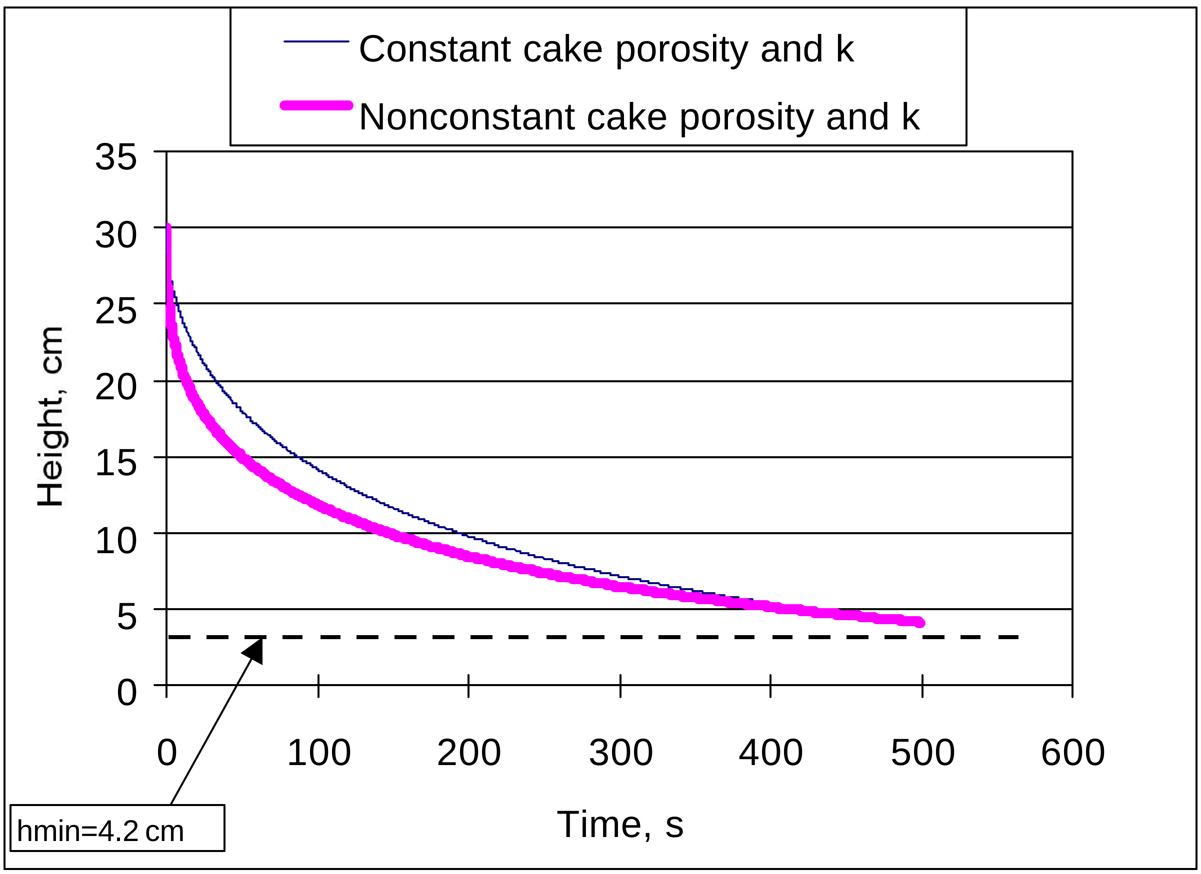

Figure 2. Predictions of slurry height from Eq. 12 and 20 for constitutive parameters from Table 1, ε_c=0.7, A=150 cm^2, ρ_s=1.5 g/cm^3, h_i=30 cm, ε_i=0.95, C_i=75 kg/m^3, μ=0.01 $g/cm^2/s$, ρ=0.9982 g/cm^3.

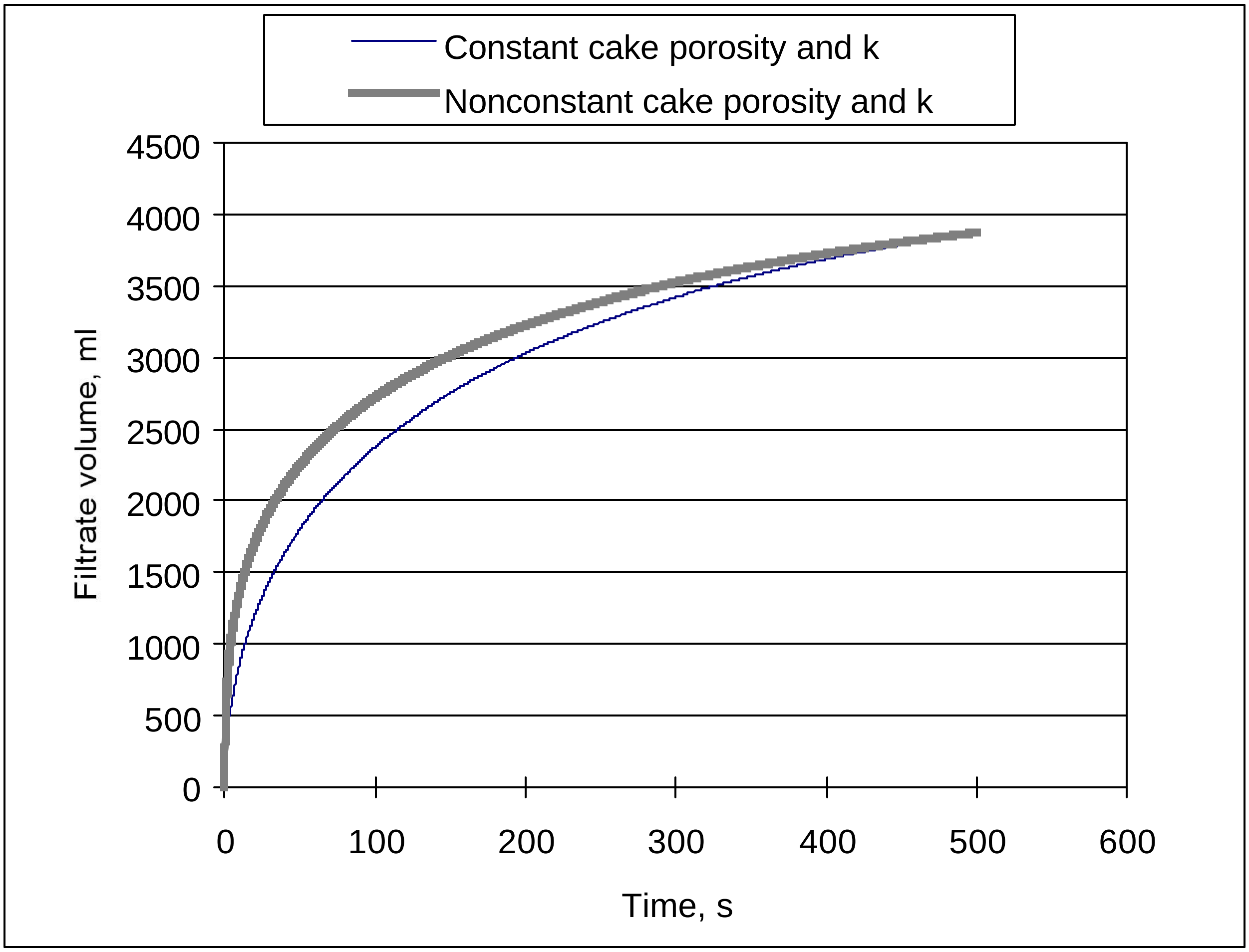

Figure 3. Predictions of filtrate volume from Eq. 12 and 20 for constitutive parameters from Table 1, ε_c=0.7, A=150 cm^2, ρ_s=1.5 g/cm^3, h_i=30 cm, ε_i=0.95, C_i=75 kg/m^3, μ=0.01 g/cm^2/s, ρ=0.9982 g/cm^3.

Determination of Constitutive Properties from Gravity Drainage Data

The inverse problem is to determine the permeability and porosity of the cake given the drainage rate. Eqs. 16 and 17 introduced 4 empirical coefficients necessary to specify the constitutive properties. Other constitutive relationships also usually require a minimum of 4 parameters (Vorobjov et al., 1993). The technique would be similar to one developed by Wells (1990b) where a non-linear, least-squares curve fitting procedure was used to determine those coefficient values with the minimum error.

Summary

The rate of filtrate production data from a gravity filtration experiment can be used to determine constitutive properties of the slurry. These properties include cake porosity and cake permeability as a function of pressure differential. Equations determining the drainage rate over time were presented for the both constant and non-constant average cake porosity and permeability. The constant average cake permeability and porosity model predicted a slower rate of filtrate production than the non-constant porosity and permeability model. These models can be used to determine slurry properties by using laboratory filtrate data to calculate the relationship between permeability and porosity and porosity and effective stress. Slurry properties allow researchers to compare slurry characteristics and to develop complex models of the dewatering behavior.

References

Christensen, G. L. and Dick, R. I. (1985a) "Specific Resistance Measurements: Methods and Procedures," Journal of Envir. Engr., ASCE, 111(3), 258-271.

Christensen, G. L. and Dick, R. I. (1985b) "Specific Resistance Measurements: Nonparabolic Data," Journal of Envir. Engr., ASCE, 111(3), 243-257.

Greenkorn, R.A. (1982) Flow Phenomena in Porous Media, Marcel Dekher, New York.

Vesilind, P. A. (1979) Treatment and Disposal of Wastewater Sludges, Ann Arbor Science, Ann Arbor, MI.

Vorobjov, E. I.; Anikeev, J. V.; and Samolyotov, V. M. (1993) "Dynamics of filtration and expression: new methods for combined analysis and calculation of the process with due account of the cake consolidation dynamics and the filter medium compressibility," Chemical Engineering and Processing, Vol 32, 45-51.

Wells, S. A. and Dick, R. I. (1988) "Synchrotron Radiation Evaluation of Gravity Sedimentation Effects Prior to Dewatering," Proceedings ASCE-CSCE National Conference on Environmental Engineering, Vancouver, British Columbia, 845-852.

Wells, S. A. (1990a) "Modeling and Analysis of Compressible Cake Filtration," Ph.D. Dissertation, Dept. of Civil and Environmental Engineering, Cornell University, Ithaca, N.Y.

Wells, S. A. (1990b) "Determination of Sludge Properties for Modeling Compressible Cake Filtration from Specific Resistance Tests," Proceedings A.S.C.E. National Environmental Engineering Conference, Washington, D.C., 125-131.