7-2018

Introduction to Reconstructability Analysis

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http://www.pdx.edu/sysc/research_dmm.html

ISSS 2018, Corvallis, July 22-27

WHAT IS RA?

- Reconstructability Analysis (RA) = a probabilistic graphical modeling methodology

- RA = Info theory + Graph theory

- Graphs, applied to data, are models:
- node = variable; link = relationship

- RA uses not only graphs (a link joins 2 nodes), but hypergraphs (a link can join >2 nodes)

WHY RA MIGHT BE OF INTEREST TO YOU 1/2

- Can detect many-variable or non-linear interactions not hypothesized in advance, i.e., it is explicitly designed for exploratory search
- Transparent (not black box), easily interpretable
- Designed for nominal variables
- Can also analyze continuous variables via binning
- Prediction/classification, clustering/network models
- Time series, spatial analyses
- Overlaps common statistical & machine-learning methods (but has unique features)

WHY RA MIGHT BE OF INTEREST TO YOU 2/2

- Web-accessible user-friendly software (OCCAM)
- Analyses at 3 levels of refinement:
  - coarse (very fast, *many* variables)
  - fine (slower, 100s of variables)
  - ultra-fine (slow, < 10 variables)
- Standard application: frequency data $f(A_i, B_j, C_k, Z_l)$
- Variety of non-standard capabilities
  - Data: set-theoretic relations & mappings
  - Predict continuous variables
  - Integrate multiple inconsistent data sets
  - Regression-like Fourier version

PAST/PRESENT RA APPLICATIONS

- *BIOMEDICAL*

  Gene-disease association, disease risk factors, gene expression, health care use & outcomes, dementia, diabetes, heart disease, prostate cancer, brain injury, primate health, surgery

- *FINANCE-ECONOMICS-BUSINESS*

  Stock market, bank loans, credit decisions, apparel analyses, market segmentation

- *SOCIAL-POLITICAL-ENVIRONMENTAL*

  Socio-ecological interactions, wars, urban water use, rainfall, forest attributes

- *MATH-ENGINEERING*

  Logic circuits, automata dynamics, genetic algorithm & neural network preprocessing, chip manufacturing, pattern recognition, decision analysis

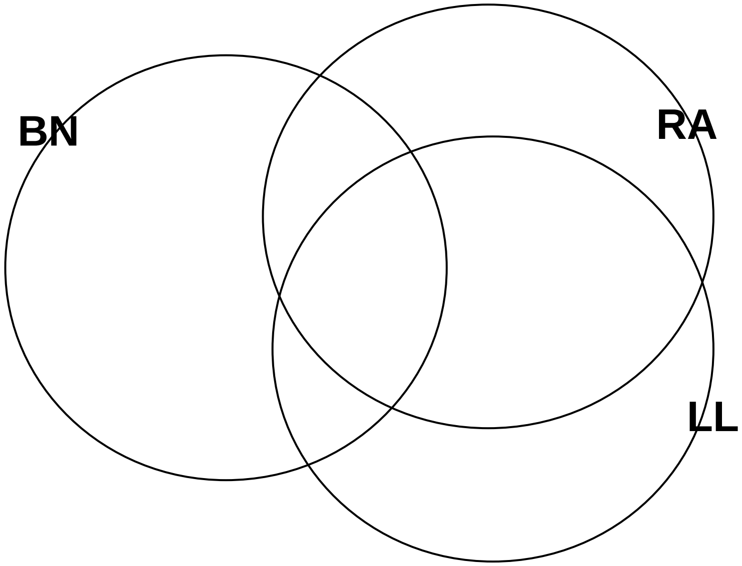
- *OTHER*

  Textual analysis, language analysis

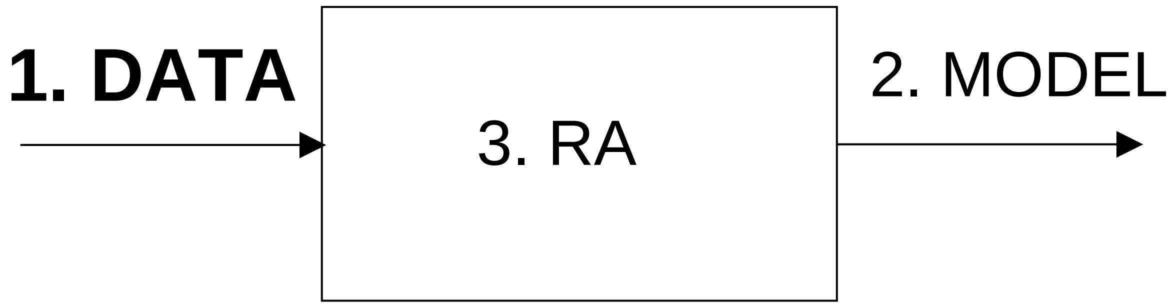
OVERLAP with STATISTICAL, MACHINE LEARNING METHODS

Relation to log linear (LL) (& logistic regression) models & to Bayesian networks (BN)

Where methods overlap, they are equivalent



1. input data to RA
   - form of data (cases X variables)
   - data cases indexed by individual, time, space
2. model output from RA
3. basics of RA
4. for more information



FORM OF DATA

Variables

- Type: nominal; bin if continuous (continuous DV needn't be binned)

- Number: few variables to 100s (in principle, to 1000s or more)

- Distinctions:

  directed system
  - *Predict/classify* a DV (output) from IVs (inputs)

  neutral system
  - No IV-DV distinction: association, clustering / network

FORM OF DATA

- frequency(A_i,B_j,C_k,Z_l) or individual cases

				frequency
A_0	B_0	C_0	Z_0	13
A_0	B_0	C_0	Z_1	2
A_0	B_0	C_1	Z_0	9
A_0	B_0	C_1	Z_1	11
…	…	…	…	__
				N

N = sample size

	A	B	C	Z
$case_1$	A_0	B_0	C_0	Z_0
$case_2$	A_1	B_2	C_3	Z_1
…				
$case_N$	A_0	B_0	C_0	Z_0

Cases are indexed by
individual (in a population),
time, or
space

frequency(ABCZ) / N = p_{data}(ABCZ)

DATA ***CASES INDEXED BY INDIVIDUAL*** **(#ID)**

ID	,0,0,ID
APOE	,2,1,Ap
Gender	,2,1,Sx
Education	,3,1,Ed
AgeLastExam	,3,1,Ag
rs1801133	,3,1,A
rs3818361	,4,1,B
rs7561528	,3,1,C
rs744373	,3,1,D
rs6943822	,3,1,E
rs4298437	,3,1,F
rs7012010	,3,1,G
rs11136000	,3,1,H
rs10786998	,4,1,J
rs11193130	,4,1,K
rs610932	,3,1,L
rs3851179	,3,1,M
rs3764650	,4,1,N
rs3865444	,4,1,P
Dementia	,2,2,Z

DEMENTIA EXAMPLE

Z = 0 no disease; Z = 1 disease

#ID	Ap	Sx	Ed	Ag	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Z
101	0	0	2	2	1	1	0	1	2	2	1	1	2	0	1	1	2	2	1
103	0	0	2	1	0	2	2	0	1	1	1	2	2	0	1	1	0	1	0
111	0	1	2	1	2	2	1	1	0	1	1	2	1	1	2	2	0	1	0
112	0	0	2	2	2	2	1	1	1	2	1	1	0	2	2	0	0	2	0
118	0	1	0	2	2	2	2	0	0	1	1	1	.	.	1	1	0	2	0
120	0	1	2	2	1	2	1	1	0	1	1	2	1	1	1	2	0	.	1
121	0	0	2	2	2	2	1	1	2	0	0	0	2	0	1	1	1	.	1
122	0	0	1	2	1	2	1	1	2	0	0	2	2	0	1	1	1	1	0
123	0	0	2	2	2	2	2	0	1	1	0	0	2	0	2	1	0	1	1

. . .

DATA CASES INDEXED BY TIME

	X	Y	Z
t-4	--	--	--
t-3	0	1	2
t-2	3	4	5
t-1	6	7	8
t	9	10	11

original data

A	B	C	X	Y	Z
--	--	--	--	--	--
--	--	--	--	--	--
0	1	2	3	4	5
3	4	5	6	7	8
6	7	8	9	10	11

transformed data

Values are labels for variable states at particular times

XYZ = generating variables

Apply mask (here # lags = 2) to data

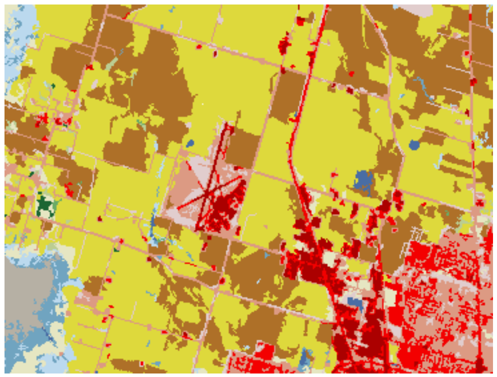
Mask adds lagged variables, ABC(t) = XYZ(t-1)

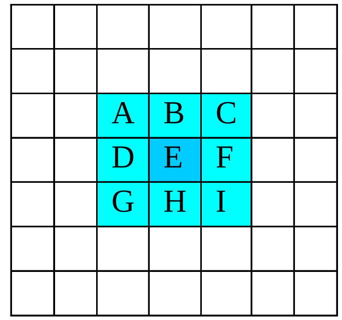
E.g., A(t-1) = X(t-2), labeled 3

Masking: time series → atemporal sample

DATA ***CASES INDEXED BY SPACE*** *: 1 generating variable*

A,14,1,A
B,14,1,B
C,14,1,C
D,14,1,D
E,14,2,E
F,14,1,F
G,14,1,G
H,14,1,H
I,14,1,I





Moore neighborhood

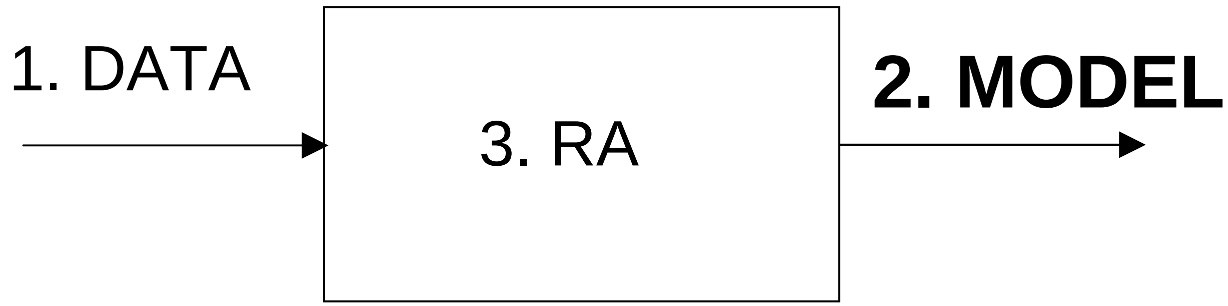
E = DV
A,B,C,D,F,G,H,I = IVs

IVs & DV have 14 possible states

#A	B	C	D	**E**	F	G	H	I
71	71	71	71	71	71	71	71	71
71	71	71	71	71	71	71	71	71
71	71	71	71	71	71	71	71	71
71	71	71	71	71	71	71	71	71
71	71	71	71	71	71	71	71	71
71	71	71	71	71	71	71	71	71
71	71	71	71	71	71	71	71	71
71	71	71	71	71	71	71	71	71
71	71	71	95	71	95	71	71	71
95	71	95	95	71	95	71	71	71
95	95	95	95	95	71	71	71	95
71	95	95	90	95	95	71	95	95
95	95	90	90	71	95	95	95	95
95	90	90	90	95	90	95	95	90

...

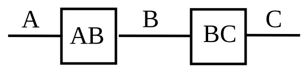
1. input data to RA
2. *model output from RA*
    - *model = structure (hypergraph) applied to data (GT)*
    - *types of structures (GT)*
    - *selecting a model (IT)*
    - *model = (conditional) probability distribution (IT)*
3. basics of RA
4. for more information

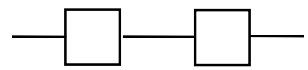


MODEL = STRUCTURE* *APPLIED TO DATA

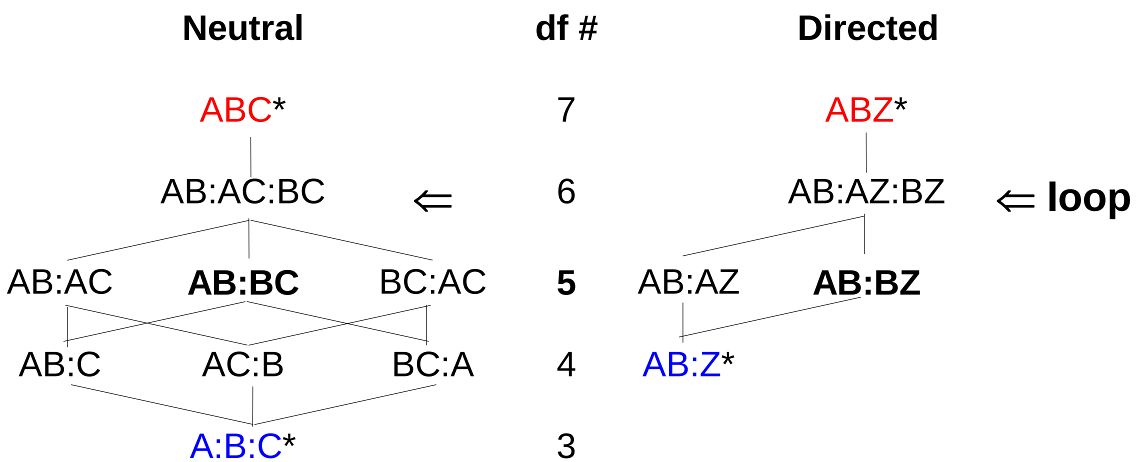
A structure (graph or hypergraph) is a set of relationships (GT)

Specific structure **AB:BC** *General* structure





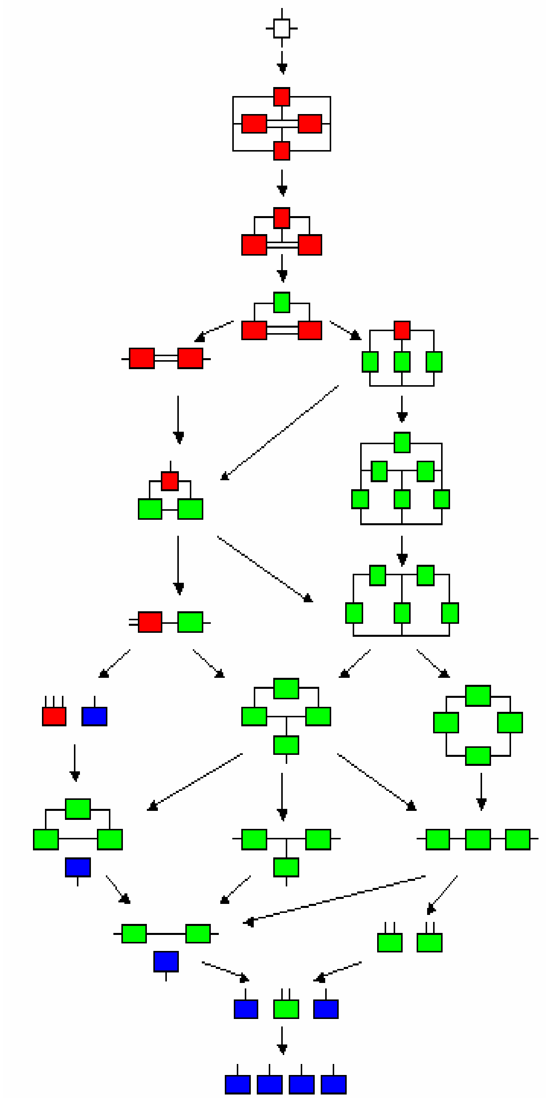
LATTICE OF *SPECIFIC* STRUCTURES (3 variables)

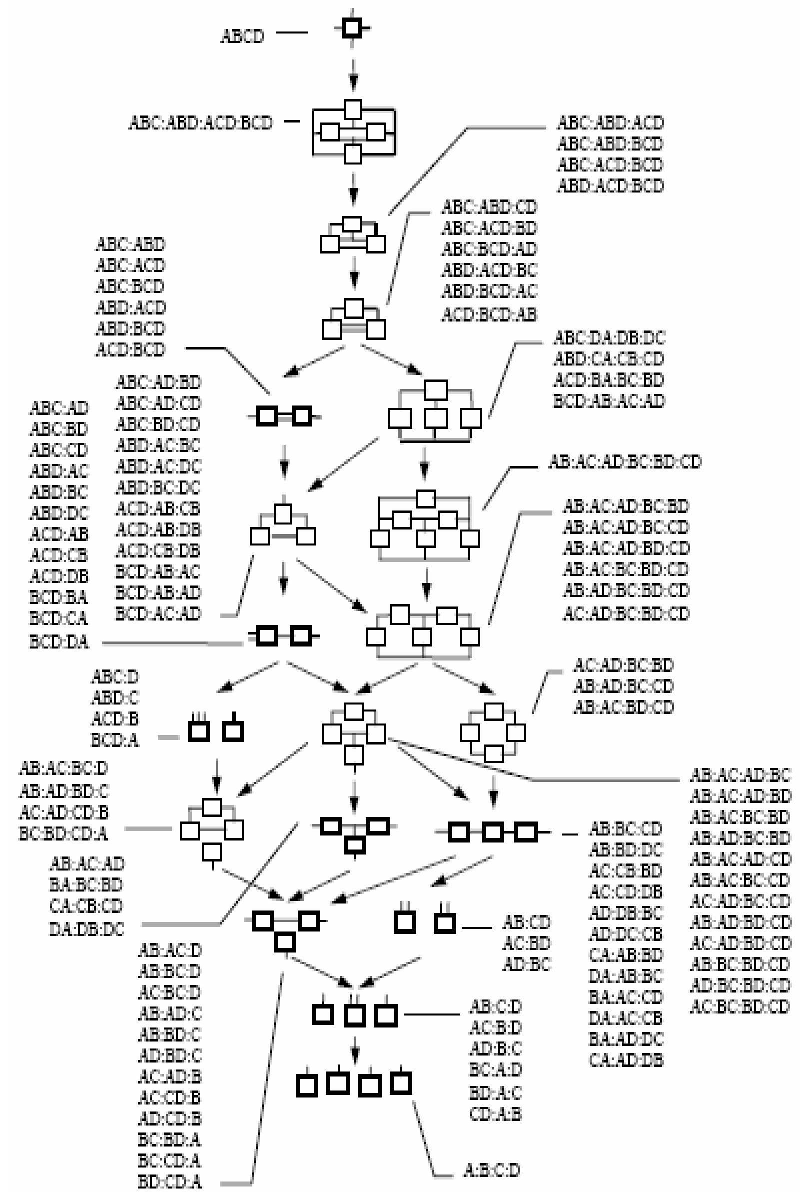


* **Reference model is data or independence**

\# df (degrees of freedom) values are for binary variables

***STRUCTURES** 4 variables (GT)*





***STRUCTURES** (GT)*

Combinatorial explosion

# variables	3	4	5	6
# general structures	5	20	180	16,143
# specific structures	9	114	6,894	7,785,062
(where 1 variable is DV)	5	19	167	7,580
(1 DV, no loops)	4	8	16	32

NEED INTELLIGENT HEURISTICS TO **SEARCH LATTICE**

Can analyze 100s of variables, & for simple models, many more.

TYPES OF STRUCTURES (GT)

FOR PREDICTION / CLASSIFICATION (directed system)

- Variable-based
  - no loops — *many* variables (fast) *[coarse]*

    IV:ACZ — simple prediction, feature selection

  - with loops — up to 100s of variables (slow) *[fine]*

    IV:ABZ:BCZ — better prediction

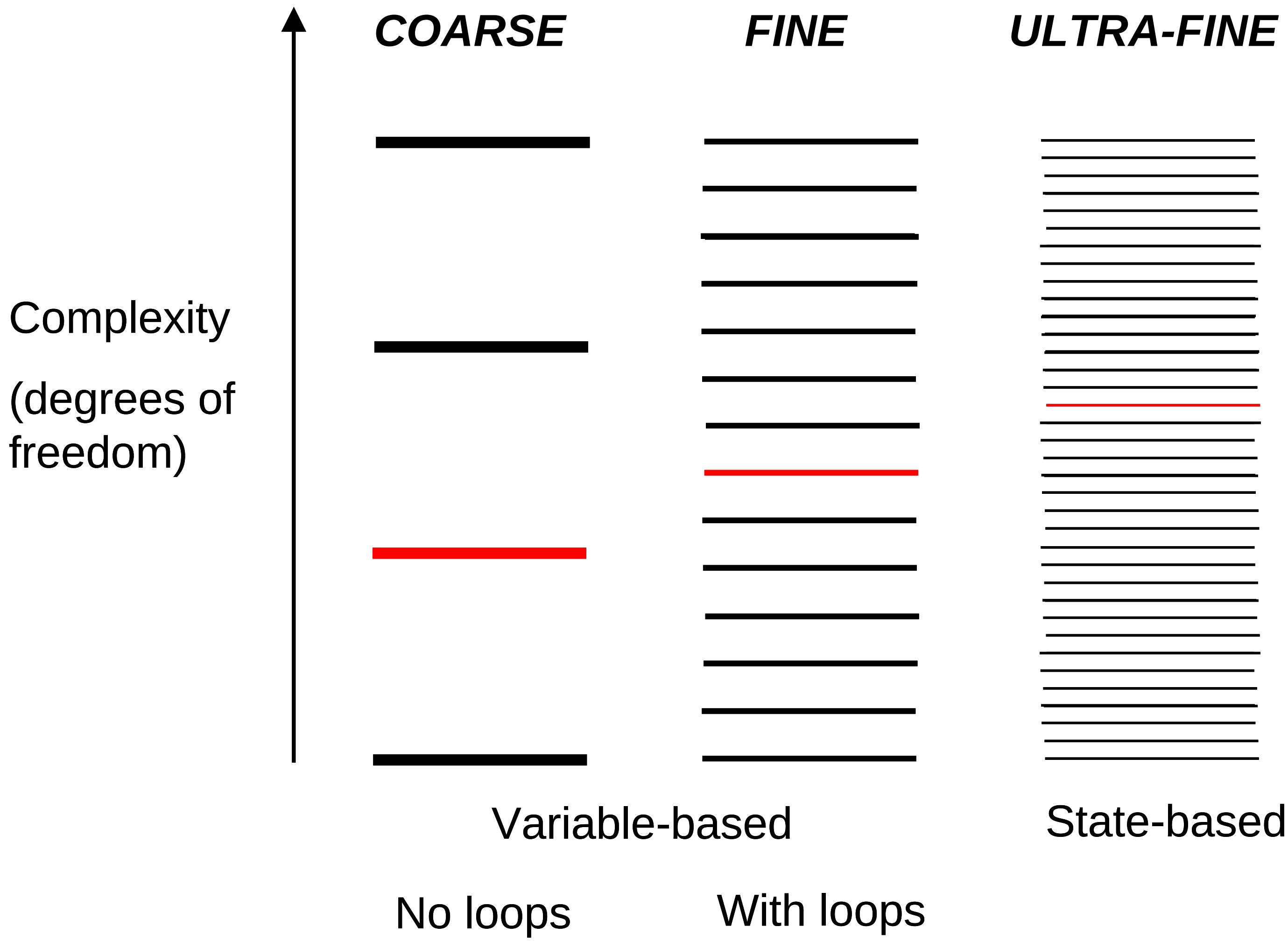
- State-based — < 10 variables (v. slow); *[ultra-fine]*

  IV:Z: A_1B_1Z : $B_2C_3Z_1$ — best prediction; detailed models

"IV" = ABC (all IVs); Z = DV

All directed system models include an IV component

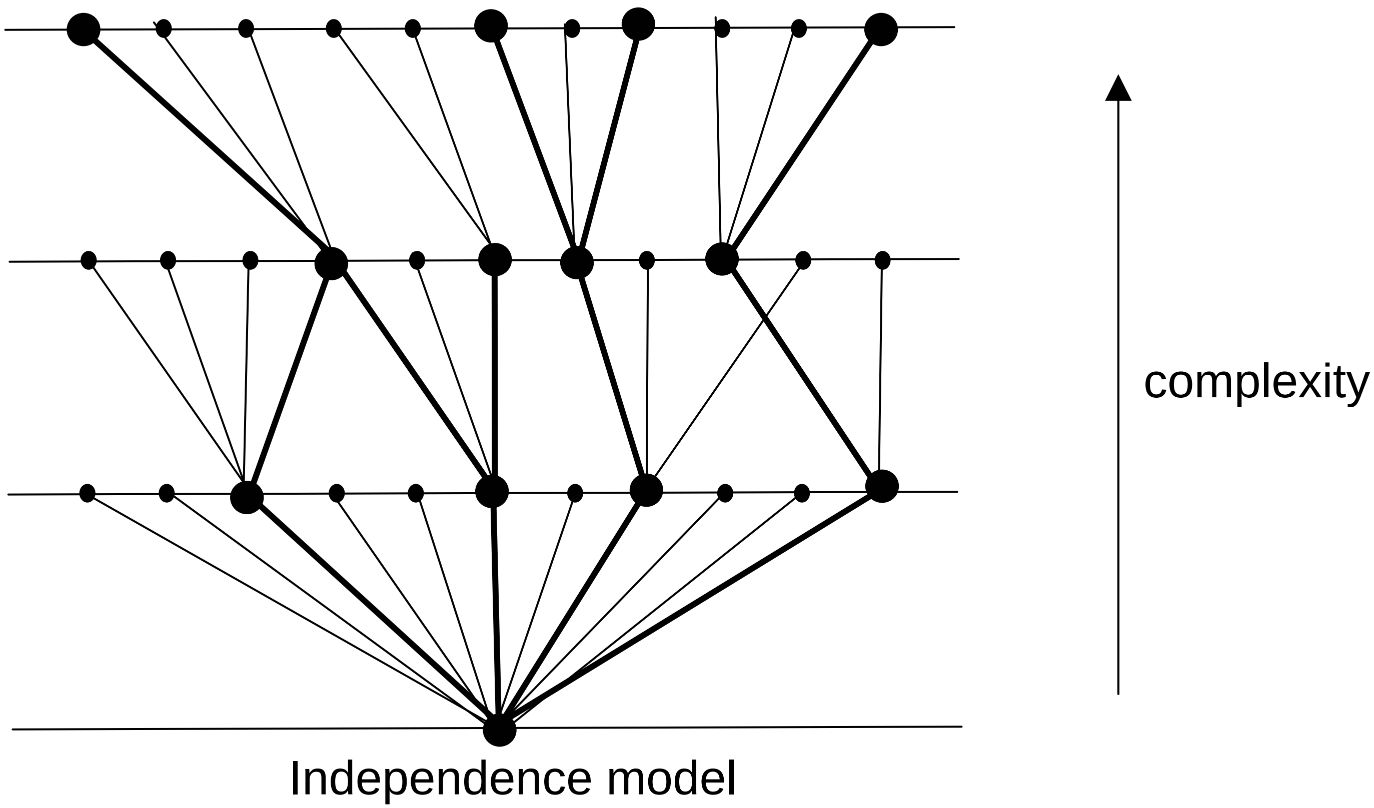
TYPES OF STRUCTURES (GT)



SEARCHING LATTICE OF STRUCTURES

beam search, levels = 3, width = 4 (node = model)

(there are many other search algorithms)



MODEL = PROBABILITY DISTRIBUTION ***(IT)***

for directed system: *conditional* distribution
for neutral system: joint distribution
gotten by applying data to a structure

Directed system:

- Model = calculated *conditional* probability distribution, e.g., $p_{IV:AZ:BZ}(Z_l \mid A_i\ B_j\ C_k)$
- Distribution gives rule to predict DV (Z) from IVs (A,B,C) (e.g., rule = 0 means predict Z_0)

SELECTING A MODEL (IT)

1. High information (or low error) in model

   <u>For directed system</u>

   – *Info-theory measure: high ΔH, <u>reduction of uncertainty of DV</u>*
   – *Generic measure: high %correct, accuracy of prediction*

2. Low complexity: df, degrees of freedom

3. Information $\leftrightarrow$ complexity tradeoff

   – Statistical significance (Chi-square p-values)
   – Integrated measures: AIC, BIC

     (Akaike & Bayesian Information Criteria)
   – BIC a conservative selection criterion

UNCERTAINTY REDUCTION: ***SIMPLE EXAMPLE***

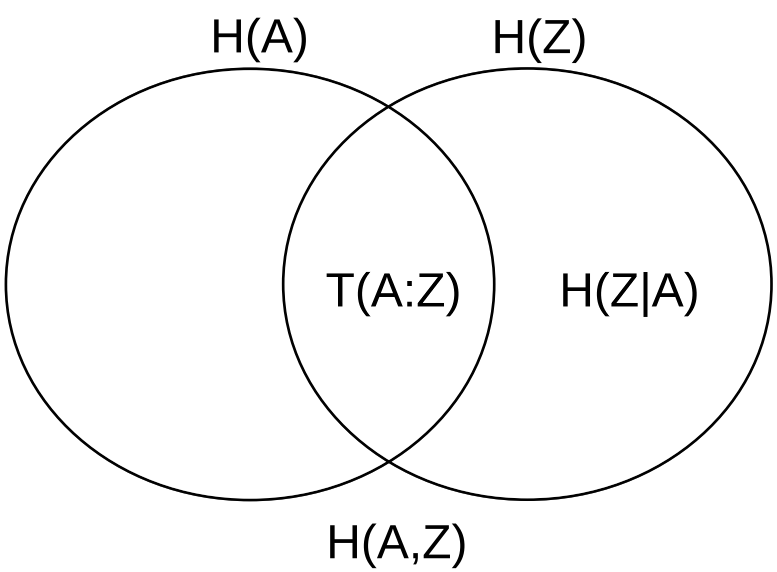
2 variables: IV=A; DV = Z; T(A:Z)=mutual information (association)

- *Uncertainty reduction* is *like* variance explained

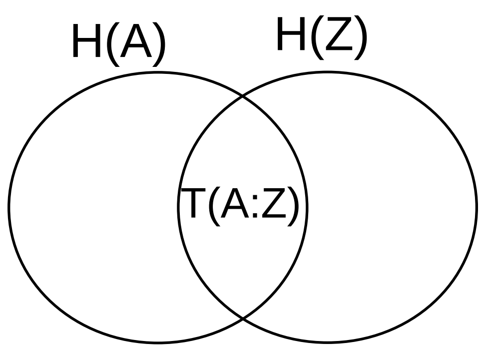
  Model AZ = predict Z, i.e., reduce H(Z), by knowing A

- Uncertainty *reduced* = T(A:Z); uncertainty *remaining* = H(Z|A)

  ΔH = T(A:Z) / H(Z) *fractional uncertainty reduction* (will express in %)



UNCERTAINTY REDUCTION: SIMPLE EXAMPLE



	Z_0	Z_1	
A_0	**.67*.5**	**.33*.5**	.5
A_1	**.33*.5**	.67*.5	.5
df=3	.5	.5	

- $p(Z_1)/p(Z_0)$= 1:1, not knowing A $\rightarrow$ 2:1 or 1:2, knowing A
- $\Delta H(Z) = T(A:Z) / H(Z) = 8\%$
- 8% reduction in uncertainty is *large* (unlike variance!)

SELECTING A MODEL *DEMENTIA EXAMPLE*

Criterion	model	ΔH(%)	Δdf	%c	ΔBIC
Variable-based (with loops)					
BIC	IV: Ap Z : Ed Z : K Z	16	5	70	59
p-value	IV: Ap Z : Ed Z : K Z : C Z : L Z	18	9	71	
AIC	IV: B Ap Z : Ed Z : K Z : C Z	20	11	72	
State-based					
BIC	(model below; each interaction = 1 df)	20	6	72	81

IV:Z: Ap_1Z : Ed_0Z : K_2Z : $Ap_0Ed_2C_2Z$: $Ap_0Ed_1C_2K_1Z$: $Ap_0Ed_1C_0K_1Z$

Models integrate multiple predicting interactions

IV = ApEdCKL… (all the independent variables); %c(IV:Z) = 52

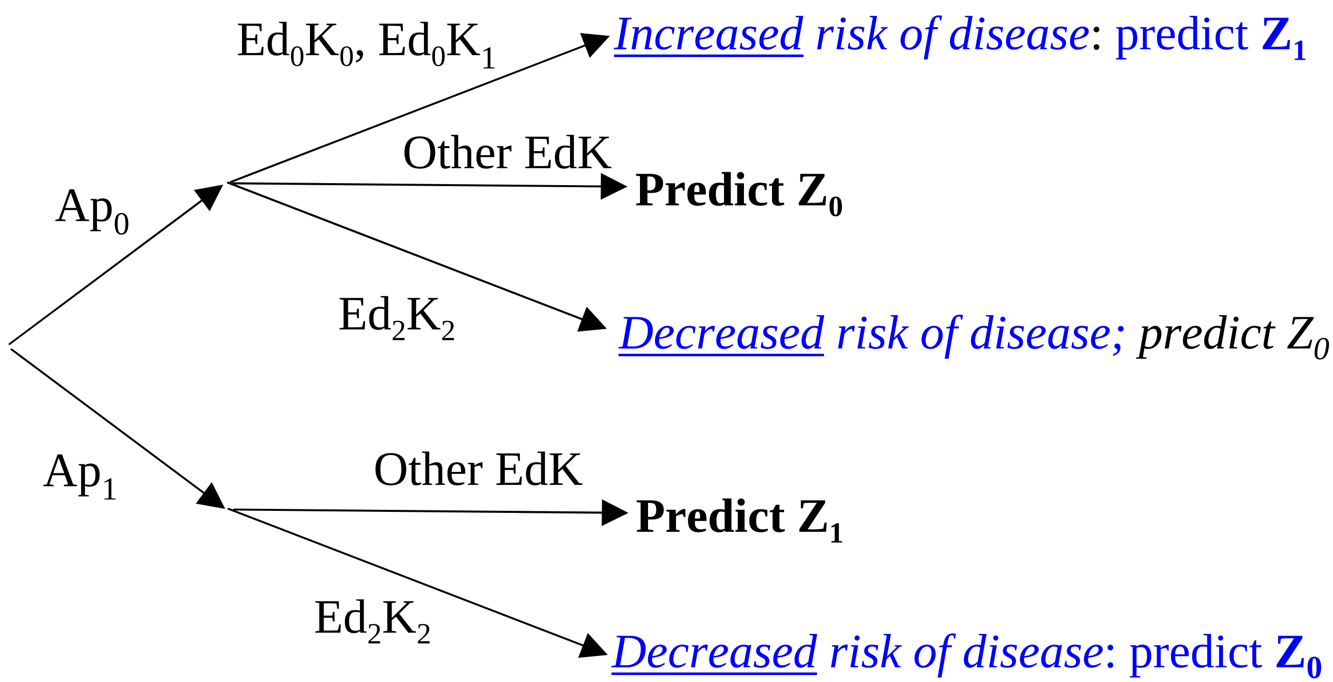
PROBABILITY DISTRIBUTION DEMENTIA EXAMPLE

				DATA		MODEL	IV:ApZ:EdZ:KZ					
IV				obs p(Z | IV)		calc p(Z | IV)			(p-value)	correct		(p-value)
Ap	Ed	K	freq	Z_0	Z_1	Z_0	Z_1	rule	p_{rule}	#	%	p_{Ap}
0	0	0	4	0.0	1.000	.122	.878	1	0.131	4	100.0	**0.028**
0	0	1	8	.125	.875	.124	.876	**1**	**0.033**	7	87.5	**0.002**
0	0	2	4	.250	.750	.294	.706	1	0.409	3	75.0	0.138
0	1	0	31	.645	.355	.616	.384	0	0.198	20	64.5	0.707
0	1	1	37	.622	.378	.619	.381	0	0.147	23	62.2	0.714
0	1	2	23	.783	.217	.827	.173	**0**	**0.002**	18	78.3	0.072
0	2	0	66	.636	.364	.640	.360	**0**	**0.023**	42	63.6	0.894
0	2	1	61	.656	.344	.644	.357	**0**	**0.025**	40	65.6	0.942
0	2	2	33	.848	.152	.842	.158	**0**	**0.000**	28	84.8	**0.020**
0	--	--	267	.648	.352	.648	.352	**0**				
1	0	0	1	.000	1.000	.026	.974	1	0.343	1	100.0	0.571
1	0	1	7	.143	.857	.026	.974	1	**0.012**	6	85.7	0.134
1	0	2	2	.000	1.000	.074	.926	1	0.228	2	100.0	0.514
1	1	0	13	.308	.692	.234	.766	1	0.055	9	69.2	0.709
1	1	1	24	.167	.833	.237	.763	1	**0.010**	20	83.3	0.633
1	1	2	11	.545	.455	.478	.522	1	0.884	5	45.5	0.146
1	2	0	32	.219	.781	.254	.746	1	**0.005**	25	78.1	0.732
1	2	1	39	.256	.744	.256	.744	1	**0.002**	29	74.4	0.735
1	2	2	17	.529	.471	.504	.496	**0**	0.973	9	52.9	**0.040**
1	--	--	146	.281	.719	.281	.719	**1**				
			413	.518	.482	.518	.482	0		291	70.5	

PROBABILITY DISTRIBUTION ***DEMENTIA EXAMPLE***

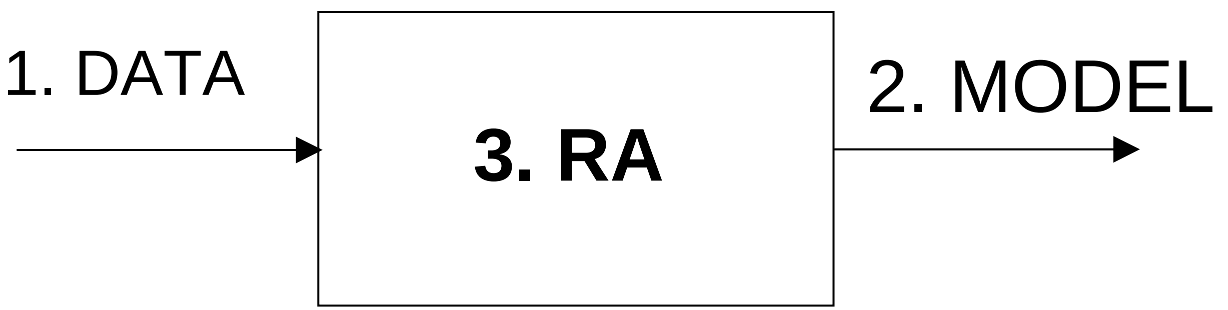
Decision tree from conditional probability distribution

(Increase or decrease of risk given by odds ratios.)



1. input data to RA
2. model output from RA
3. basic RA algorithms *(IT, inside the black box)*
   - generate model
   - evaluate model

4. for more information



GENERATE MODEL

frequencies shown, not probabilities

data: observed ABC (df=7)

	C_0		C_1	
	B_0	B_1	B_0	B_1
A_0	143	77	253	182
A_1	227	46	411	139

1478

model: calculated $ABC_{AB:BC}$

	C_0		C_1	
	B_0	B_1	B_0	B_1
A_0	142	72	254	188
A_1	227	52	409	134

3.*Evaluation*

1.*Projection*

2.*Composition*

	B_0	B_1	
A_0	396	259	655
A_1	638	185	823
	1034	444	

	B_0	B_1	
C_0	370	123	493
C_1	664	321	985
	1034	444	

model: AB:BC (df=5)

GENERATE MODEL

- *Projection* = sum frequencies or probabilities

- *Composition*

Maximize model entropy *subject to* model constraints

Model entropy: $H(p_{model}) = - \Sigma\, p_{model} \log_2 p_{model}$

E.g., for model AB:BC, *maximize* $H(p_{AB:BC})$ *subject to*

$$p_{AB:BC}(AB) = p_{data}(AB)$$

$$p_{AB:BC}(BC) = p_{data}(BC)$$

Composition is critical computational step; done

(a) Algebraically (very fast) loopless models

(b) Iteratively (Iterative Proportional Fitting) models with loops

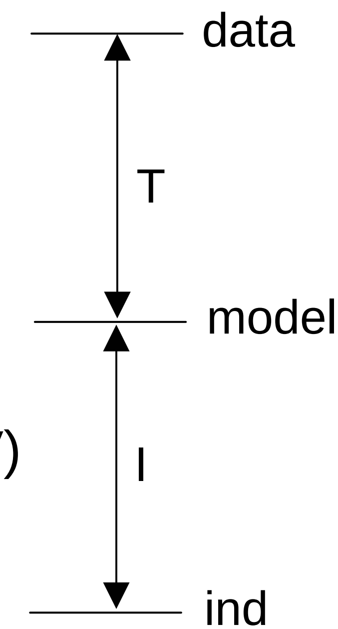
***EVALUATE MODEL** (1/2)*

- *Evaluation* (1 = data dependent; 2 = data independent)

1. *[ref=data]*

error, T_{model} $= H_{model} – H_{data}$

$= \Sigma\, p_{data} \log_2(p_{data}/p_{model})$



[ref=<u>ind</u>ependence]

information, I_{model} $= H_{ind} – H_{model}$

$= \Sigma\, p_{data} \log_2(p_{model}/p_{ind})$

uncertainty reduction $= H(DV) - H_{model}(DV \mid IV)$

2. *[ref=<u>ind</u>ependence]*

complexity $= \Delta df = df_{model} – df_{ind}$

EVALUATE MODEL (2/2)

Trade off information (or error) & complexity, define best model criterion, via:

Use likelihood ratio Chi-square, LR = k N T

- p-values from ΔLR, Δdf, Chi-square table

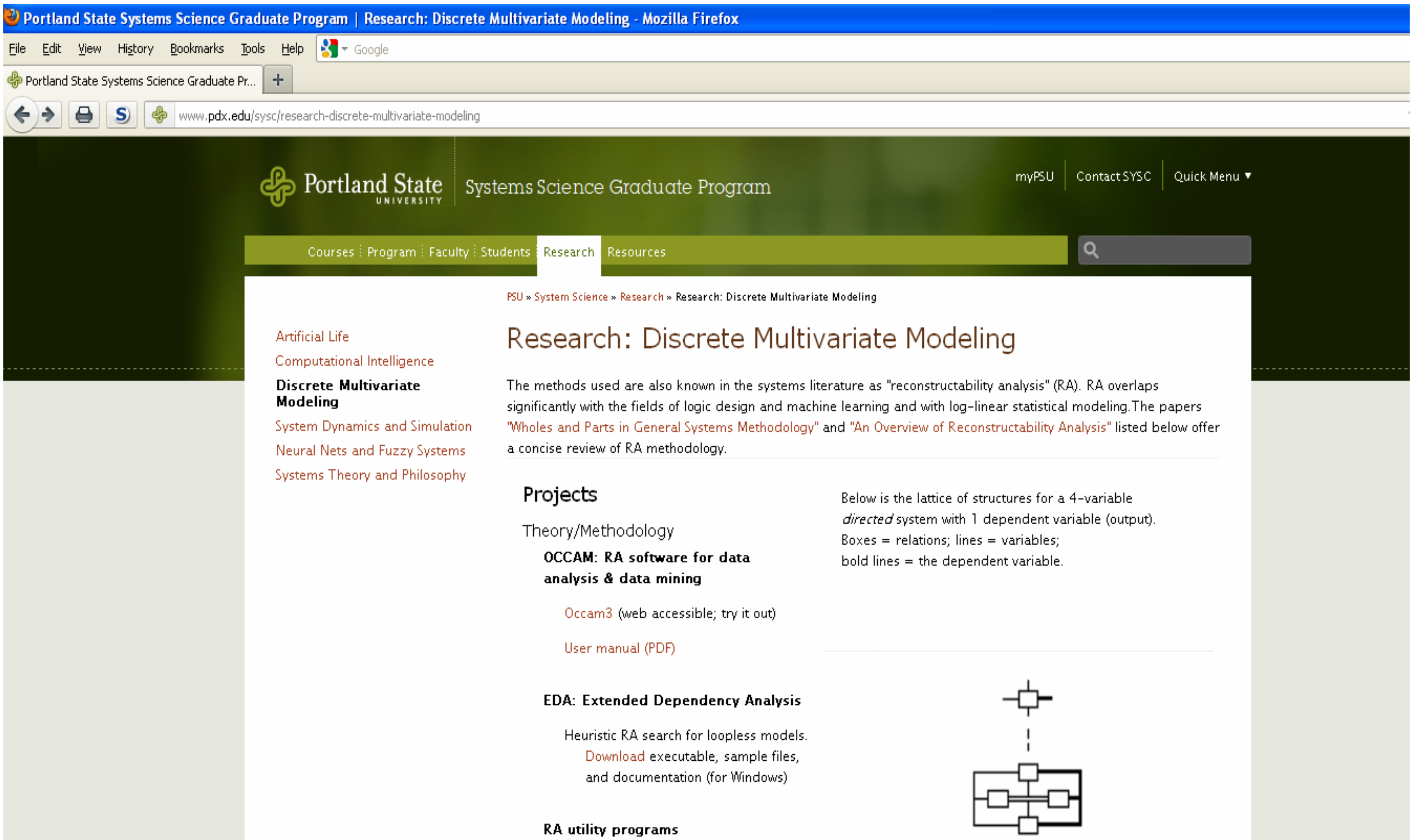
Or linear combinations of information & complexity

- ΔAIC = ΔLR + 2 Δdf
- ΔBIC = ΔLR + ln(N) Δdf

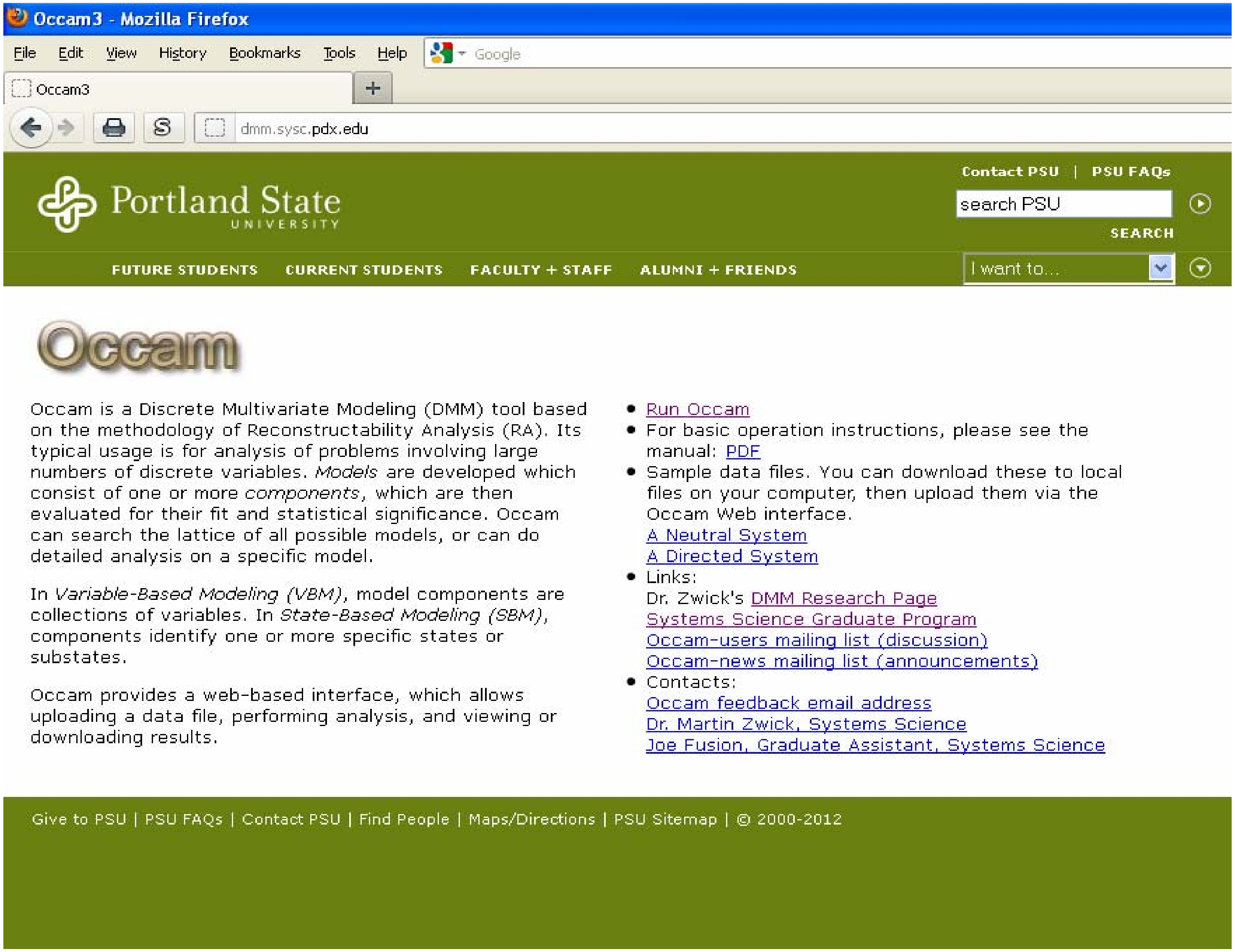
1. input data to RA
2. model output from RA
3. basic RA algorithms
4. for more information
   - DMM (RA) web page
   - Software: OCCAM
   - MORE INFORMATION ON RA

DMM (RA) WEB PAGE

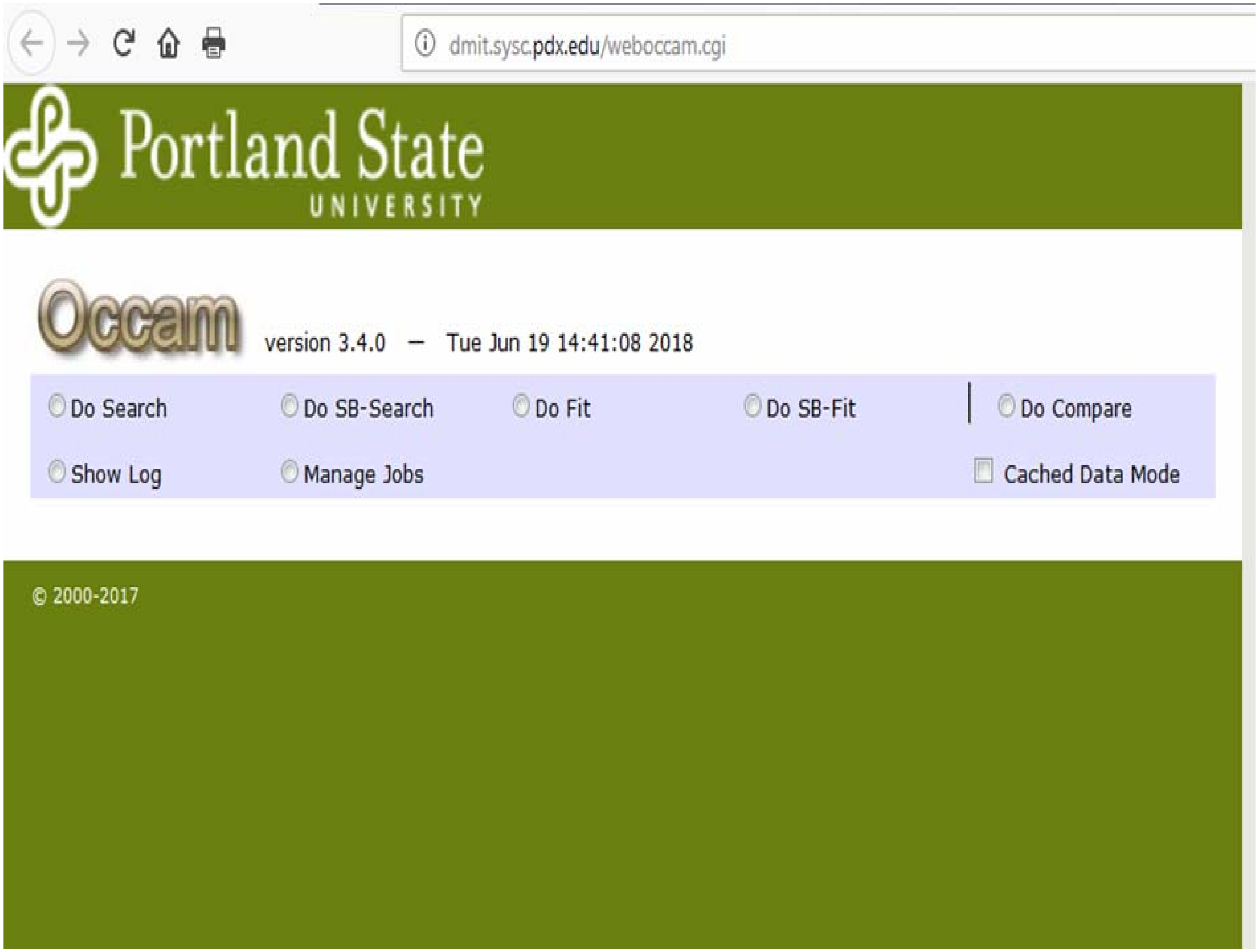
http://pdx.edu/sysc/research-discrete-multivariate-modeling



SOFTWARE: OCCAM *(access on DMM page)*



OCCAM Initial Screen



BASIC OCCAM ACTIONS

- Search = exploratory modeling, examine many models, find best or good ones

  (OCCAM actions: Search, SB-Search)

- Fit = confirmatory modeling, look at one model in detail (see probability distribution) & use for prediction

  (OCCAM actions: Fit, SB-Fit)

(OCCAM actions: Show Log, Manage Jobs = managerial functions)

INFORMATION ON RA

- Review articles on DMM page
  - "Wholes & Parts in General Systems Methodology" (accessible)
  - "An Overview of Reconstructability Analysis" (encompassing)
- Krippendorff, Klaus (1986). Information Theory. *Structural Models for Qualitative Data* (Quantitative Applications in the Social Sciences Monograph #62). New York: Sage Publications.
- *International Journal of General Systems*
- *Kybernetes*, Vol. 33, No. 5/6 2004: special RA issue

- OCCAM is available for use

  (but consult with me before doing anything other than variable-based models without loops)

- Plan to make OCCAM open-source; contact me if you would like to be involved

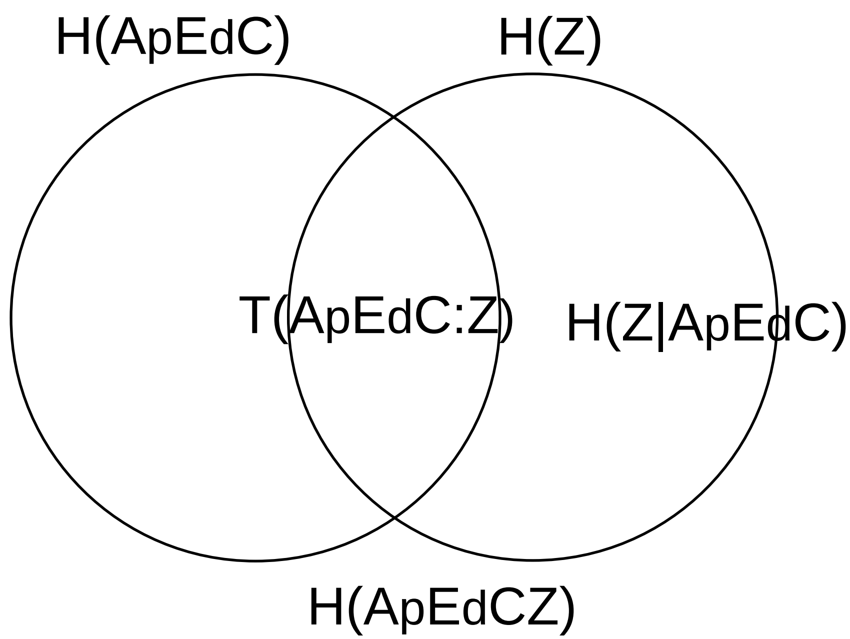
- zwick@pdx.edu

- *Thank you.*

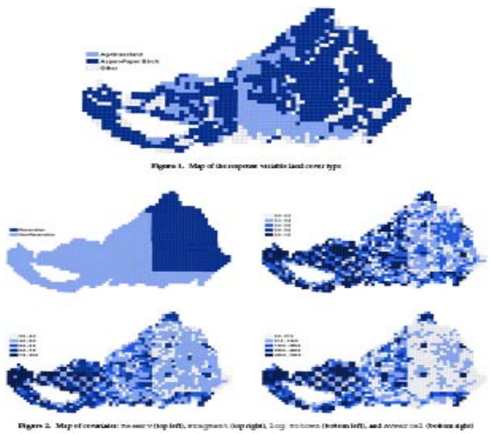
UNCERTAINTY REDUCTION: DEMENTIA EXAMPLE

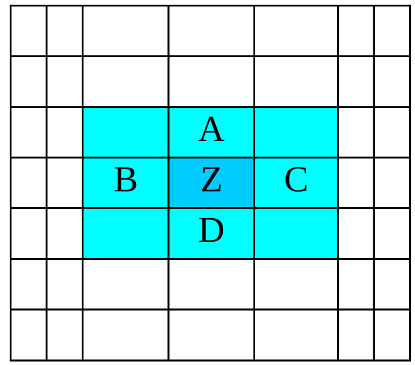
Criterion	model	ΔH(%)	Δdf	%c
BIC	IV:ApZ:EdZ:CZ	16	5	70

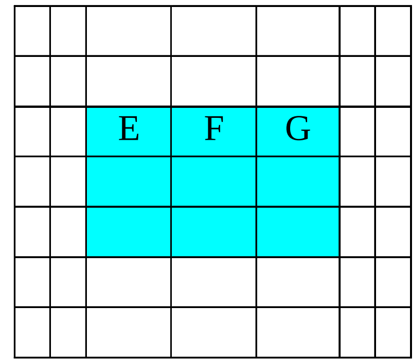
$\Delta H = T_{IV:ApZ:EZ:CZ}(ApEdC:Z) / H(Z) = 14\%$

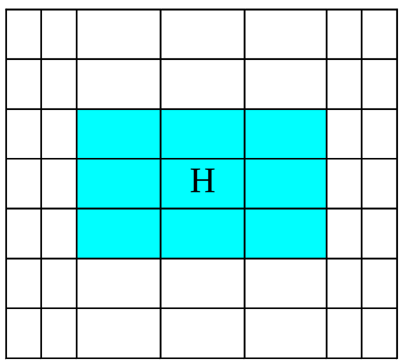


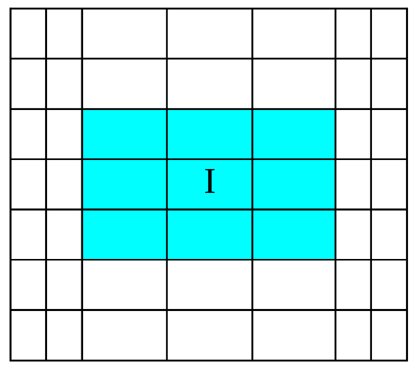
HYPOTHETICAL MODEL ***SPATIAL EXAMPLE***

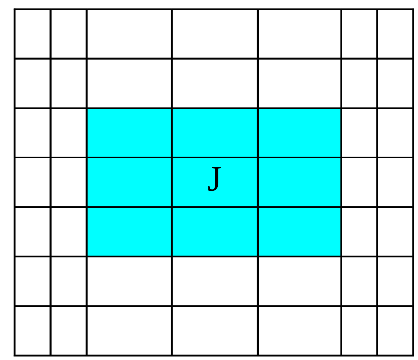












- In 5-generating-variables spatial example, model *could* be:
- IV: ABCD Z:EFG Z:HIJ Z