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Full State Revivals in Linearly Coupled Chains with Commensurate Eigenspectra

J. J. P. Veerman
Portland State University, veerman@pdx.edu

Jovan Petrovic
University of Belgrade

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Full State Revivals in Linearly Coupled Chains with Commensurate Eigenspectra

J. J. P. Veerman[a,b], J. Petrovic[c,*]

[a]*Fariborz Maseeh Dept. of Math. and Stat., Portland State Univ., Portland, OR, USA*
[b]*CCQCN, Dept of Physics, University of Crete, 71003 Heraklion, Greece*
[c]*Vinca Institute of Nuclear Sciences, University of Belgrade, 12-14 Mike Alasa, 11000 Belgrade, Serbia*

Abstract

Coherent state transfer is an important requirement in the construction of quantum computer hardware. The state transfer can be realized by linear next-neighbour-coupled finite chains. Starting from the commensurability of chain eigenvalues as the general condition of periodic dynamics, we find chains that support full periodic state revivals. For short chains, exact solutions are found analytically by solving the inverse eigenvalue problem to obtain the coupling coefficients between chain elements. We apply the solutions to design optical waveguide arrays and perform numerical simulations of light propagation thorough realistic waveguide structures. Applications of the presented method to the realization of a parallel bus for quantum states, perfect transfer and edge oscillations are proposed and considered in detail.

Keywords:
linear chains, periodicity, state transfer

1. Introduction

Coherent transfer of quantum states is one of basic requirements for the construction of quantum computer [1]. Coherence preserves relative phases between quantum states allowing for their interference. The transfer of quantum states can be realised either via moving particles that act as qubit carriers, e.g. ions [2], or via stationary solid-state media with qubits that

*Corresponding author
Email address: jovanap@vin.bg.ac.rs (J. Petrovic)

propagate through them, e.g. integrated optical circuits [3]. Scalability and portability of future computers have steered investigations towards the latter, in which qubits are transferred by chains of linearly coupled waveguide arrays (WGAs)[4] or quantum dots [5]. Propagation through linearly coupled chains is, in general, quasi-periodic without full revivals of the initial state and hence without a possibility of faithful transfer of a qubit across the chain. In isolated noiseless stationary chains, such dephasing is a consequence of incommensurability of the eigenfrequencies of the chain. Although it is a reversible unitary process and not irreversible decoherence, it prevents state reconstruction and perfect transfer.

It has been suggested [6] and demonstrated in cold atoms [7] that the dephasing can be mitigated by a dynamic control of the propagation along the chain by interventions to the system Hamiltonian. However, implementation of this method in miniature solid state systems that hold a great promise as the future quantum computer hardware (e.g. integrated photonic circuits, quantum dots) requires fine interventions along the chain and represents a major technological challenge. Alternatively, couplings between the elements of a translationally invariant chain can be tailored while leaving the system Hamiltonian constant during the state evolution. For example, coherent dynamics of quantum states through a Heisenberg chain of linearly coupled atomic Zeeman states [8] and along an invariant optical WGA [4] have been engineered by controlling the strength and ratios of their coupling coefficients, respectively. Unlike in atomic and other spin systems, in which only the next-neighbour coupling is allowed by selection rules, in chains without selection rules (WGAs), higher order couplings are present. Since the higher-order coupling does not cause decoherence, the periodic dynamics are possible but require intricate engineering of coupling coefficients [9].

Mathematically, conditions of periodic transport and perfect revivals are satisfied when the coupling matrix eigenvalues are commensurate [10, 11]. Hereafter we refer to these chains as commensurate chains. The chains with less than four elements are always commensurate. The eigenvalues of longer finite chains are commensurate only for certain ratios of their coupling coefficients. The key challenge in chain engineering is finding these ratios and that requires a solution to the non-trivial inverse eigenvalue problem. Inverse eigenvalue problems are analytically solvable for a small number of cases and are, in general, of polynomial complexity [12]. Inverse solutions are not unique and a concrete choice of solution depends on the targeted transfer dynamics.

Realizations of coherent transport in linear chains mostly aim at the perfect transfer of the state amplitude (in literature referred to as perfect transfer) without restrictions on phase. As a result of extensive theoretical studies of the perfect transfer, a general solution [13] and several particular solutions have been proposed [11, 14]. Curiously, reported practical realizations mostly rely on solutions found in nature in the form of Bloch oscillations [15], atomic spin chains [8], or their emulation by optical lattices [16]. However, in order to realise a faithful transfer of a many-qubit state, the relative phase of all qubits must be preserved. Therefore, the restrictive conditions of the perfect transfer do not suffice and it is necessary to find solutions that reconstruct both the amplitude and phase of a quantum state. To the best of our knowledge, general solutions have been derived only for 4- and 5-element chains (WGAs) with mirror symmetry [17].

Here, we address the problem of the complete state reconstruction in fixed chains and find solutions by tailoring the coupling coefficients. We take the general approach to the inverse eigenvalue problem and analytically solve it for 4- and 5-element chains without symmetry constrains and for 7- and 9-element chains symmetric around the centre. Unlike earlier approaches to the chain design that propose modification of the uniform chain [18] or solutions that asymptotically approach maximum transfer fidelity [19], the solutions proposed are exact and enable new design completely independent of uniform chains. We use the analytic solutions to numerically obtain the full state reconstruction in realistic WGAs realizable by direct laser writing [20, 21]. Examples include the perfect transfer of a many-qubit state, an asymmetric array that supports the full state revivals and edge state Rabi coupling. Since the discretization inherently present in the analytical model approximates a WGA by a set of waveguides of infinitesimally small diameter, we discuss limitations encountered in design of realistic WGAs.

The paper is structured as follows. The mathematical model of a chain with next-neighbour linear coupling and condition of its commensurability are given in Section 2. In the same section, analytic solutions of the inverse eigenvalue problem for general and symmetric chains are derived. In Section 3, the analytic solutions are used to design realistic symmetric and asymmteric WGAs that support coherent transport with full state revivals. Further, an application of commensurable WGAs in construction of parallel data buses is suggested. Conclusions are given and other chain configurations of interest are outlined in Section 4.

2. Commensurable next-neighbour coupled chains

Evolution of a state through a linear chain of n elements with the next-neighbour coupling is modelled by the Schrödinger equation of the form:

$$i\frac{\partial\psi(t)}{\partial t} = A_n\psi(t) \tag{1}$$

where ψ is the vector state in $\mathbb{R}^n$ and A_n is a real nxn tridiagonal symmetric matrix that accounts for coupling between neighbouring elements. We assume that there are no loss or gain along the chain, hence that A is Hermitian. In what follows, we will consider various n-dimensional cases of this eq.(1)."

Periodicity of solutions requires all eigenvalues ω_j, $j = 1, n$ of A to be commensurate, i.e. that there are integers n_j such that

$$\forall\, j \;:\; \frac{T}{2\pi}\omega_j = n_j. \tag{2}$$

Period of oscillation is defined as the smallest T for which the above equation holds for all eigenvalues of the chain. It is the smallest period with which the state can oscillate. However, if only some of the eigenvalues are excited, state revivals can occur with a longer period.

Two and three element chains have unconditional periodicity. Their solutions are well-known and heavily applied in atomic and laser physics, see e.g. the theory of a three-level atom coupled by resonant laser light [22].

2.1. 4-element chain

We use the shortest commensurate chain that permits incommensurate eigenvalues to illustrate the procedure used for solving the inverse eigenvalue problem. We start from a coupling matrix with real nearest-neighbour interaction acting on $\mathbb{R}^4$ of the general form:

$$A_4(a_1, a_2, a_3) \equiv \begin{bmatrix} 0 & a_1 & 0 & 0 \\ a_1 & 0 & a_2 & 0 \\ 0 & a_2 & 0 & a_3 \\ 0 & 0 & a_3 & 0 \end{bmatrix}. \tag{3}$$

It is easy to verify that the eigenvalues $\omega_{1,2,3,4}$ of A_4 are given by:

$$\left[\begin{array}{c} \frac{1}{2}\sqrt{2{a_3}^2+2{a_2}^2+2{a_1}^2+2\sqrt{{a_3}^4+2{a_3}^2{a_2}^2-2{a_3}^2{a_1}^2+{a_2}^4+2{a_2}^2{a_1}^2+{a_1}^4}} \\ -\frac{1}{2}\sqrt{2{a_3}^2+2{a_2}^2+2{a_1}^2+2\sqrt{{a_3}^4+2{a_3}^2{a_2}^2-2{a_3}^2{a_1}^2+{a_2}^4+2{a_2}^2{a_1}^2+{a_1}^4}} \\ \frac{1}{2}\sqrt{2{a_3}^2+2{a_2}^2+2{a_1}^2-2\sqrt{{a_3}^4+2{a_3}^2{a_2}^2-2{a_3}^2{a_1}^2+{a_2}^4+2{a_2}^2{a_1}^2+{a_1}^4}} \\ -\frac{1}{2}\sqrt{2{a_3}^2+2{a_2}^2+2{a_1}^2-2\sqrt{{a_3}^4+2{a_3}^2{a_2}^2-2{a_3}^2{a_1}^2+{a_2}^4+2{a_2}^2{a_1}^2+{a_1}^4}} \end{array}\right]. \tag{4}$$

From (2) we see that the chain is commensurate if and only if there are 2 non-negative integers n_1 and n_2 with the greatest common divisor (GCD) equal to 1 for which

$$\begin{aligned} \frac{T}{2}\sqrt{2x^2+2\sqrt{x^4-4u^4}} &= 2\pi n_1 \\ \frac{T}{2}\sqrt{2x^2-2\sqrt{x^4-4u^4}} &= 2\pi n_2. \end{aligned} \tag{5}$$

In this case the eigenvalues are $(\pm n_1, \pm n_2)$ scaled by an arbitrary scalar. Writing the frequency as $q = \frac{2\pi}{T}$ and solving the above equation we obtain that the coupling coefficients are

$$\begin{aligned} a_1 &= \frac{q}{2}\left(\epsilon_1\sqrt{(n_1+n_2)^2-s^2}+\epsilon_2\sqrt{(n_1-n_2)^2-s^2}\right) \\ a_2 &= qs \\ a_3 &= \frac{q}{2}\left(\epsilon_1\sqrt{(n_1+n_2)^2-s^2}-\epsilon_2\sqrt{(n_1-n_2)^2-s^2}\right) \end{aligned} \tag{6}$$

where ϵ_1 and ϵ_2 in $\{-1, +1\}$, s an arbitrary real, and the frequency $q = 2\pi/T$ an arbitrary non-zero real.

Note that for any eigenspectrum given by n_1 and n_2 an infinite number of chains can be constructed by choosing different s and $\epsilon_{1,2}$. Not all values of s will give real values for the a_i. However, all will give eigenvalues with the ratios $\{\pm n_1, \pm n_2\}$.

An interesting example occurs when we choose degenerate eigenvalues, $n_1 = n_2$. Then we are forced to choose $s = 0$ and hence $a_2 = 0$, meaning that the chain decomposes into two 2-element chains with the same coupling coefficients $a_1 = a_3$.

2.2. 5-element chain

We turn to the commensurability of $A_5(a_1, a_2, a_3, a_4)$ given by

$$A_5(a_1, a_2, a_3, a_4) \equiv \begin{bmatrix} 0 & a_1 & 0 & 0 & 0 \\ a_1 & 0 & a_2 & 0 & 0 \\ 0 & a_2 & 0 & a_3 & 0 \\ 0 & 0 & a_3 & 0 & a_4 \\ 0 & 0 & 0 & a_4 & 0 \end{bmatrix}. \qquad (7)$$

The central eigenvalue of this chain in always 0. We assume that the other eigenvalues relate to one another as $(\pm n_1,\ \pm n_2)$ and repeat the procedure from the previous section. If $a_1^2 - a_4^2 \neq 0$, then we obtain the commensurate array with the coupling coefficients in the form:

$$\begin{array}{rcl} a_1^2 & = & q^2 s^2 \\ a_2^2 & = & q^2 \left(\frac{(n_1^2 n_2^2 - s^2 t^2) - (n_1^2 + n_2^2 - (s^2 + t^2)) s^2}{t^2 - s^2} \right) \\ a_3^2 & = & q^2 \left(\frac{-(n_1^2 n_2^2 - s^2 t^2) + (n_1^2 + n_2^2 - (s^2 + t^2)) t^2}{t^2 - s^2} \right) \\ a_4^2 & = & q^2 t^2 \end{array} \qquad (8)$$

where s and t are arbitrary reals, and q is an arbitrary non-zero real.
If $a_1^2 - a_4^2 = 0$ the solution assumed the form:

$$\begin{array}{rcl} a_1^2 & = & q^2 n_1^2 \\ \begin{pmatrix} a_2 \\ a_3 \end{pmatrix} & = & \sqrt{q^2(n_2^2 - n_1^2)}\, R_\phi \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{array} \qquad (9)$$

where q is an arbitrary non-zero real and R_ϕ is a rotation by an arbitrary angle ϕ.

In the case of a chain that is mirror symmetric around the centre ($a_j = a_{n-j}$, $j = 1, 2$), the eigenvalues are easily calculated to be: $\{0, \pm a_1, \pm\sqrt{2a_2^2 + a_1^2}\}$. If $a_1 \cdot a_2 \neq 0$, the commensurability condition reduces to:

$$\begin{array}{c} a_1^2 = q^2 n_1^2 \\ a_2^2 = q^2 \frac{n_2^2 - n_1^2}{2}. \end{array} \qquad (10)$$

If $a_1 = 0$, the system becomes a trivially periodic 3-element chain with two outer elements remaining uncoupled to the others. If $a_2 = 0$ the system

decomposes into two pairs of coupled elements that oscillate independently at both ends od the initially considered 5-element chain.
For a chain with an equidistant energy spectrum, reverse engineering renders a solution from the family of Clebsch-Gordan coupling coefficients, e.g. for a symmetric 5-element array $n_2 = 2n_1$, $a_1 = qn_1$, and $a_2 = qn_1\sqrt{\frac{3}{2}}$. Such chains describe coupling of atomic spins [8] and have been used to construct optical couplers [17].

2.3. Symmetric 7-element chain

Now we turn our attention to the real, symmetric, nearest neighbour interaction with left-right symmetry acting on $\mathbb{R}^7$.

$$A_7(a_1, a_2, a_3) \equiv \begin{bmatrix} 0 & a_1 & 0 & 0 & 0 & 0 & 0 \\ a_1 & 0 & a_2 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & a_3 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & a_3 & 0 & 0 \\ 0 & 0 & 0 & a_3 & 0 & a_2 & 0 \\ 0 & 0 & 0 & 0 & a_2 & 0 & a_1 \\ 0 & 0 & 0 & 0 & 0 & a_1 & 0 \end{bmatrix} \tag{11}$$

and calculate the coupling coefficients that render the spectrum $(0, \pm n_1, \pm n_2, \pm n_3)$:
If $a_3 \neq 0$:

$$\begin{aligned} a_1^2 &= q^2 \left(\tfrac{n_2^2 n_3^2}{n_2^2 + n_3^2 - n_1^2} \right) \\ a_2^2 &= q^2 \left(n_1^2 - \tfrac{n_2^2 n_3^2}{n_2^2 + n_3^2 - n_1^2} \right) \\ a_3^2 &= q^2 \left(\tfrac{n_2^2 + n_3^2 - n_1^2}{2} \right) \end{aligned} \tag{12}$$

where q is an arbitrary positive real.
If $a_3 = 0$, the chain is decomposed into two 3-element chains with period $T = \frac{2\pi}{\sqrt{a_1^2 + a_2^2}}$.

2.4. Symmetric 9-element chain

In the spirit of the previous sections, we solve the inverse problem for a mirror symmetric 9-element chain and obtain that the system with $A_9(a_1, a_2, a_3, a_4)$

is commensurate if the a_i satisfy:
$\underline{\text{If } a_3 \neq 0 \text{ and } a_4 \neq 0}$:

$$\begin{aligned} a_1^2 &= q^2 \left(\frac{n_1^2 n_2^2 (n_3^2+n_4^2-n_1^2-n_2^2)}{(n_1^2+n_2^2)(n_3^2+n_4^2-n_1^2-n_2^2)-(n_3^2 n_4^2-n_1^2 n_2^2)} \right) \\ a_2^2 &= q^2 \left(\frac{n_3^2 n_4^2-n_1^2 n_2^2}{n_3^2+n_4^2-n_1^2-n_2^2} - \frac{n_1^2 n_2^2 (n_3^2+n_4^2-n_1^2-n_2^2)}{(n_1^2+n_2^2)(n_3^2+n_4^2-n_1^2-n_2^2)-(n_3^2 n_4^2-n_1^2 n_2^2)} \right) \\ a_3^2 &= q^2 \left(\frac{(n_1^2+n_2^2)(n_3^2+n_4^2-n_1^2-n_2^2)-(n_3^2 n_4^2-n_1^2 n_2^2)}{(n_3^2+n_4^2-n_1^2-n_2^2)} \right) \\ a_4^2 &= q^2 \left(\frac{n_3^2+n_4^2-n_1^2-n_2^2}{2} \right). \end{aligned} \tag{13}$$

$\underline{\text{If } a_4 = 0}$, the matrix consists of three diagonal blocks two of which are 4-element chains and one block is the number 0.
$\underline{\text{If } a_3 = 0}$, there are also three diagonal blocks, that represent 3-element chains.

3. Coherent state transfer through optical waveguide arrays

We apply the above theory to achieve coherent transport through optical WGAs. These arrays are accessible by modern direct laser writing [21] and lithography [23] fabrication techniques. Indeed, perfect transfer through the WGAs with equidistant eigenvalues has been demonstrated experimentally [16, 4]. The absence of loss due to the straightness of waveguides is one of important advantages of the WGA-based couplers over the circuits composed of nested optical couplers [24]. Here, we first describe the physical system in more detail and then apply exact general solutions to construct WGAs capable of full state revivals, a parallel data bus for optical quantum or classical computer, the first non-trivially commensurate asymmetric WGA and edge Rabi coupling.

A typical dielectric optical waveguide consists of a region with refractive index higher than the index of the substrate that enables light confinement in transversal direction and propagation along the guide. Waveguides couple to one another via evanescent fields with the strength that decays exponentially with the distance between them, [25]. Once the coupled coefficients of a chain are determined by the method described in Section 2, a WGA can be constructed by engineering the inter-waveguide separations as described in [16, 17].

Since the coupling coefficients are derived for a discrete model in Section 2 that assumes a perfect transversal confinement of a state to a chain element, we paid spatial attention to work close to this condition. This can be achieved in two ways: by working with high index-contrast waveguides and distant enough waveguides. Since the effective coupling in the former case requires extremely long propagation lengths, we chose to work with WGAs with large spacing between the guides. Fundamental modes of distant waveguides are strongly confined to individual waveguides and the fundamental modes of closely-spaced waveguides form a supermode extending through all of them, Fig. 1b. All simulations with exception of the last example, were performed for the separation-to-diameter ratio around 2.2, which also secures weak waveguide coupling.

The Rabi frequency q that features in expressions for matrix elements a_i as a free scaling parameter is here set to 1. All simulated WGA were composed of identical circular waveguides with diameter 8.2 μm, core refractive index 1.45 and substrate refractive index 1.445. Input modes were eigenmodes of individual waveguides at 1550 nm. To transfer the information through an array, we encode it in the amplitude and phase of a light wavefunction $\psi(z) = (\psi_1(z)e^{i\phi_1(z)}, \psi_2(z)e^{i\phi_2(z)}, ...\psi_n(z)e^{i\phi_n(z)})$. The time in Eq. 1 is replaced by the space coordinate z along which the light propagates. Numerical simulations were performed using the finite-difference beam propagation method and transparent boundary conditions [26].

3.1. Example 1: Phase and amplitude revivals

An example of a symmetric 5-waveguide array inversely engineered to support full periodic state revivals is shown in figure 2. To prove the restoration of both the amplitude and phase of the wavefunction, we launch two input vector states of the same amplitude and different phases 0 and $\pi/2$ into the WGA. The WGA is constructed to have eigenvalues $0, \pm 2, \pm 3$ and to satisfy (10). The relative phase of input modes determines the propagation dynamics (light trajectory through a WGA). Nevertheless, the full revivals of the input state occur at $4\pi m$, $m = 1, 2, 3...$ irrespectively of the input phase, figure 2. The state vector redraws the same closed contour in the complex plane. The 'cleanness' of the contour clearly distinguishes arrays with periodic dynamics from the arrays with quasi-periodic dynamics, whose state vectors never repeat the same path but fill in a subspace of the complex plane instead (see e.g. [27]).

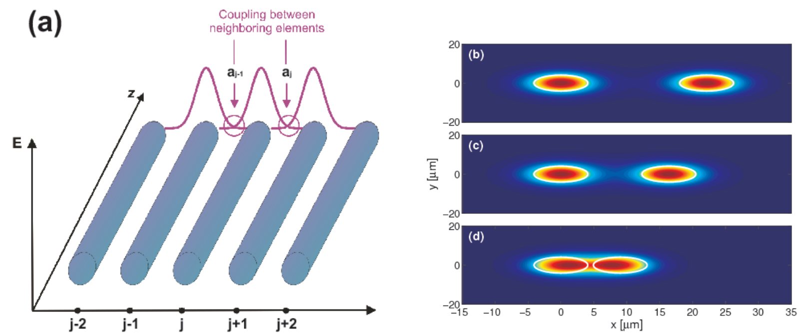

Figure 1: (a) A waveguide array. Profiles of electric field amplitudes $|E|$ of supported modes sketched along z axis display evanescent field overlap responsible for coupling. Coupling coefficients a_i correspond to those in matrices $\mathbf{A}$. (b)-(d) Intensity ($I \propto |E|^2$) profiles of the fundamental modes of pairs of circular waveguides separated by 2.7 D, 2.0 D and 1.1 D, respectively, where D is the waveguide diameter. Red colour corresponds to the maximum and blue colour to the minimum intensity. White contours show waveguide cross sections.

Further, use the transfer fidelity to estimate the accuracy required for revivals. It is defined as $F(z) = |\langle\psi(z)|\psi(0)\rangle$— and is calculated for a range of input conditions. In the worst case,fidelity drops for 5% when the length of the array is 5% of the revival length. Taking into account that the usual distance between the neighbouring waveguides is a few microns, a fidelity within 1% from 1 can be achieved with a resolution of a few tens of nanometers, which is achievable by the direct laser writing technique.

3.2. Example 2: Parallel data bus

The most exploited periodic solutions have been those used to realise the perfect state transfer from the beginning to the end of the chain. It implies the transfer with fidelity 1 of the state amplitude (energy) and is achieved if the ratio of differences of subsequent eigenvalues ω_n is rational, $\omega_i - \omega_j = (2m_n + 1)\pi/L_0$, where m_n is an integer and L_0 the chosen state transfer length [14, 28]. The chain supporting such a transfer is referred to as quantum wire or quantum data bus. The procedure for the general inverse solution of this problem has been given in [13]. However, explicit solutions are still a few. Several systems have been proposed by choosing the eigenvalue scalings $\omega_n \propto n$, n^2, $n(n+1)$ where $n = 1, 2, 3....$ The linear scaling renders a well-known array with spin-coupling coefficients formally equivalent to Clebsch-Gordan coefficients [29, 30]. Analytical solutions have

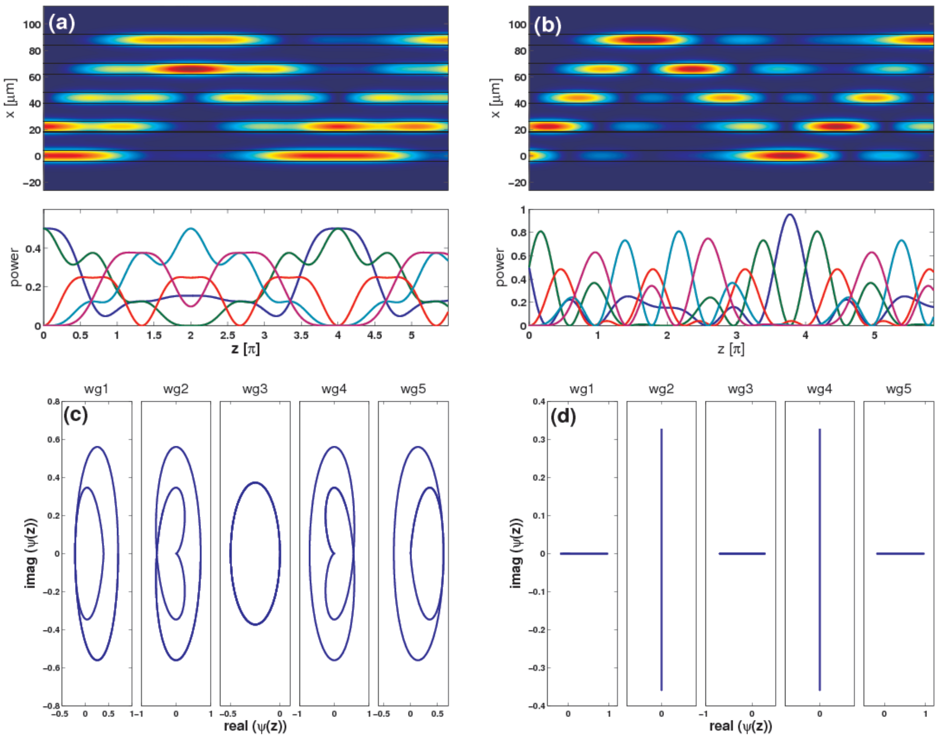

Figure 2: Results of numerical simulations of light propagation through a commensurable 5-waveguide array with $n_1 = 2$, $n_2 = 3$. Input states have different relative phases: (a,c) $\psi(0) = (1, 1, 0, 0, 0)$ and (b,d) $\psi(0) = (1, e^{-i\pi/2}, 0, 0, 0)$. (a,b) Upper graphs: intensity profiles (red colour corresponds to the highest intensity). Lower graph: $|\psi(z)|^2$ obtained by solving (1). (c,d) Rotation of the state vector ψ in complex plane. Colour coding of the lines is blue, green, red, cyan and purple going from the bottom waveguide up, respectively.

been reported for short chains with 6, 7, 8 and 16 elements [31]. In laboratory, the perfect state transfer through chains with more than 3 elements has been demonstrated in optical waveguide arrays (WGAs) [4], and between Zeeman states of atoms constituting a Bose-Einstein condensate [8]. Realisations in other physical systems, such as quantum dots, have been proposed [32].

Here we use the commensurability conditions to expand the set of known analytic solutions for perfect transfer. Since a WGA can support periodic dynamics during which the perfect transfer to the end waveguide never occurs, the condition of perfect transfer is not equivalent to the periodicity condition and the explicit solutions for perfect transfer given below are subsets of solutions derived in Section 2. The perfect transfer can be realized in 5-elements WGAs that satisfy (10) and have even n_2 and $n_1 < n_2$. Similarly, the condition (12) and even n_1, odd n_2, odd n_3, and $n_2 < n_1 < n_3$, guarantee the perfect transfer between the end waveguides of a 7-element array. These solutions include known Heisenberg chains with equidistant eigenvalues. An example of such a chain with ($n_1 = 2$, $n_2 = 1$, $n_3 = 3$) is shown in figure 3a.

Further, we propose a parallel optical data bus that can transfer a number of states simultaneously. It is based on a commensurate WGA acting as a multiport device that supports both the parallel input and read-out from n waveguides. Here, instead of limiting the consideration to the perfect (energy) transfer from one to the other end of a WGA, we exploit coherent transfer of the complex wavefunction ψ for all possible input states. Any WGA with a commensurate eigenspectrum can perform this function. In Fig. 3, we show two examples. The first is the Heisenberg-like chain and the second is an array with Fibonacci sequence of eigenvalues $n_1 = 13$, $n_2 = 5$, $n_3 = 8$. Of course, the eigenvalues do not have to posses any particular regularity apart from that dictated by the formulas in Section 2 and the condition of no loss/gain. Indeed, all WGAs given as examples throughout this Section are potential solutions for construction of a parallel bus.

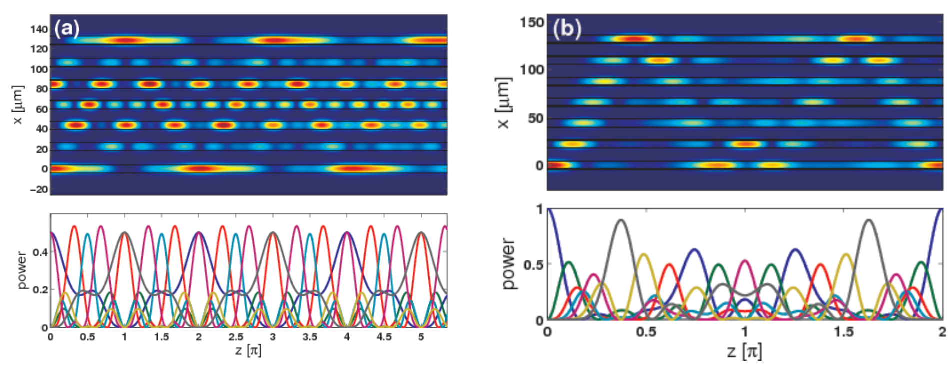

Figure 3: Results of numerical simulations showing the complete transfer through a 7-waveguide array with (a) equidistant eigenvalues $n_1 = 1$, $n_2 = 2$, $n_3 = 3$ and input vector state $\psi(0) = (1, 0, 0, 0, 1, 0, 0)$, (b) Fibonacci eigenvalues $n_1 = 3$, $n_2 = 8$, $n_3 = 5$. The input vector state is $(1, 0, 0, 0, 0, 0, 0)$. $|\psi(z)|^2$ obtained by solving (1). Colour coding of the lines is blue, green, red, cyan, purple, olive green and black going from the bottom waveguide up, respectively.

Another chain that supports a parallel read-out are atoms with different spins that can be imaged simultaneously using the Stern-Gerlach scheme [8]. However, construction of an arbitrary input state comprising different atomic spin levels requires sophisticated coherent control techniques, [7].

3.3. Example 3: Asymmetric arrays

While the chains reported in literature treat only symmetric arrays, we observe that the full state revivals are possible in asymmetric arrays that are constructed without any *a priori* restrictions on coupling coefficients. Solutions in Section 2.1 and 2.2 enable construction of such arrays. An example in Fig. 4 shows a 4-guide array with commensurate eigenvalues $n_1 = \pm 1$, $n_3 = \pm 3$, and arbitrarily chosen parameters $s = \sqrt{3}$, $\epsilon_1 = 1$ and $\epsilon_2 = -1$. It is composed of the WGs separated by $L_{2,3} = 0.977 L_{1,2}$ and $L_{3,4} = 0.954 L_{1,2}$. To produce the required small difference in inter-waveguide separations, a resolution of a few percent is required. For usually used separations of several microns, such resolution is achievable by femtosecond laser fabrication technique.

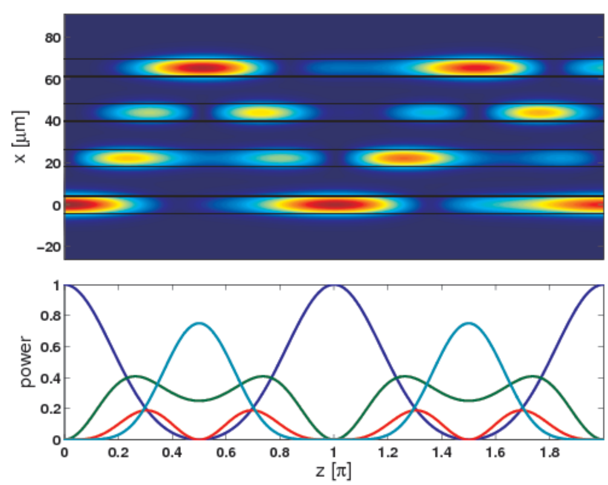

Figure 4: Results of numerical simulations of light propagation through a commensurable asymmetric 4-waveguide array with eigenvalues set by $n_1 = 1$, $n_2 = 3$ and the input vector state $\psi(0) = (1, 0, 0, 0)$. Upper graph shows intensity profiles (red colour corresponds to the highest intensity). Waveguides are shown by black lines. Lower graph: $|\psi(z)|^2$ obtained by solving (1). Colour coding of the lines is blue, green, red and cyan from the bottom waveguide up, respectively. Small deviations from the periodic propagation are due to the limited mode confinement in a realistic waveguide and numerical artifacts.

3.4. Example 4: Reduced arrays

We further explore asymptotic cases of effective reduction of an array in terms of the number of elements. When some inter-waveguide separations are much smaller than the others, propagation dynamics comprises slow and fast oscillating fields, favourizing propagation through the closely spaced waveguides. For instance, in a 7-waveguide array with eigenvalues that satisfy the condition $n_3 >> n_1 >> n_2$, the light power is confined to the 4 outer waveguides at any point of propagation, figure 5. According to the formulas given in 12 the above condition renders an array with a stronger coupling between the inner than the outer elements, $a_3 >> a_2 >> a_1$, with the density of waveguides decreasing from the centre of the array outwards. Note that, if cut at the any length that corresponds to an odd multiple of $\pi/2$, this WGA can be used as an equal 1x4 splitter (splits a signal into 4 with equal amplitudes).

4. Conclusions

We have investigated finite chains with linear coupling as means of coherent state transfer in quantum computers. Coherent propagation with periodic state revivals is achieved by reversely engineering couplings between the neighbouring elements of a chain to yield its eigenspectrum commensurate. The inverse eigenvalue problem is solved analytically for general chains with 4 and 5 elements and for symmetric chains with 7 and 9 elements. These solutions comprise particular cases actively studied to achieve the perfect state transfer from one to the other end of the chain. However, the presented solutions provide a much wider and still under-explored possibilities for coherent propagation and revivals of both the state amplitude and phase through a set of parallel channels. The full inverse solution is demonstrated on experimentally accessible optical WGAs. In such a system, coupling coefficients are controlled by tailoring the inter-waveguide distances. Practical limitations neglected in a discrete analytical model are considered. Presented analytical and numerical solutions offer a plenitude of possibilities for engineering of WGAs that are now realizable by mature laser fabrication technologies. They are characterised by low loss and robustness of architecture. Finally, besides the mathematical challenge to find analytical solutions for linear chains with larger and even number of elements, there is a considerable interest in coherent transfer through two dimensional and closed chains, e.g. multicore fibres.

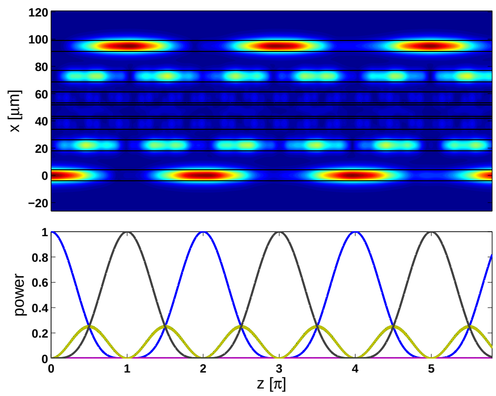

Figure 5: Results of numerical simulations showing light dynamics in 7-waveguide array with $n_1 = 100$, $n_2 = 1$, $n_3 = 10000$ that reduces to a 4-waveguide array. The input vector state is $(1, 0, 0, 0, 0, 0, 0)$. $|\psi(z)|^2$ obtained by solving (1). Colour coding of the lines is blue, green, red, cyan, purple, olive green and black going from the bottom waveguide up, respectively.

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