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Abstract

Design of sedimentation tanks for solid-liquid separation is often dependent on assuming ideal flow conditions. But the geometry of the tank and density currents as a result of temperature and suspended solids influences the fluid mechanics of the tank and can result in significant deviations from ideal flow.

A two-dimensional radial flow model was proposed which incorporated the effect of density currents resulting from temperature and suspended solids differentials within the tank. The numerical model predicted the steady-state, layer-averaged radial flow and layer depth.

The model solution and field data showed that the momentum and suspended solids of the inflow caused a density current that moved along the clarifier bottom. In the winter, this density current rose as a result of warm inflow water after the initial momentum had decreased and the suspended solids had settled.

Introduction

Design of sedimentation tanks for water and wastewater treatment processes are often based on the assumption of uniform unidirectional flow through the tank as shown in Figure 1. Dick (1982) showed though that many full-scale sedimentation tanks do not follow ideal flow behavior. Because of uncertainties in the hydrodynamics of clarifiers, designers typically use safety factors to account for this non-ideal flow behavior (Abdel-Gawad and McCorquodale, 1984).

Non-ideal flow behavior can be the result of the following (DeVantier and Larock, 1987; Tay and Heinke, 1983; Wells, 1990):

1. inlet and outlet geometry
2. inflow jet turbulence
3. dead zones in the tank
4. resuspension of settled solids
5. density currents caused by suspended solids and temperature differentials within the tank.

Steady-state models of currents in clarifiers have been developed for rectangular clarifiers by Ostendorf (1986) and for circular clarifiers by Abel-Gawad and McCorquodale (1984) and Shamber and Larock (1981). In these cases, the effect of density differences due to temperature and suspended solids stratification were ignored. Later, DeVantier and Larock (1987) used a modified form of Rodi's (1980) k- turbulence model to predict currents in a circular clarifier at steady-state. The effect of buoyancy on tank hydrodynamics was considered of secondary importance and was ignored in their final model formulation. McCorquodale (1997) summarized the state-of-the-art in modeling clarifier hydrodynamics and showed that currently available clarifier models do not account for the impact of temperature differences within the tank caused by atmospheric heating or cooling. Hence, in this study, the relative importance of the impact of temperature differentials within a tank on clarifier hydrodynamics were evaluated.

Mathematical Modeling of the Two-Layer Flow

A hydrodynamic model of the density currents in a 2-layer system can be characterized by the continuity and momentum equations for each layer assuming a radial coordinate system as shown in Figure 2. The turbulent time-averaged Navier-Stokes equations for radial coordinates (Sabersky, et al. 1989) were simplified making the following assumptions:

1. axisymmetric flow, i. e., $\overline{V} = 0$ and / = 0 where $\overline{V}$ is the turbulent time average of the velocity in the -direction [m/s];

2. molecular terms were negligible compared to the turbulent flux terms;

3. flow was incompressible;

4. fluid density changes were neglected with the exception of body force term (the Boussinesq approximation);

5. $\partial P_a / \partial r = 0$; where $\overline{P}_a$ is the time averaged atmospheric pressure [kPa].

The resulting general mass continuity equation in cylindrical coordinates was then: $\frac{\partial \overline{w}}{\partial z} + \frac{1}{r}\frac{\partial (r\overline{u})}{\partial r} = 0$ (1)

where w: vertical velocity [m/s]

u: radial velocity [m/s]

r: radial coordinate [m]

z: vertical coordinate [m]

and overbars represent the turbulent time-average.

The resulting r-momentum equation in cylindrical coordinates was: $\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial r} + \overline{w}\frac{\partial \overline{u}}{\partial z}$

$$= -\frac{1}{\overline{\rho}}\frac{\partial \overline{P}}{\partial r} + \frac{1}{\overline{\rho}}\frac{\partial}{\partial z}\left(\overline{\rho}\,\overline{w'u'}\right) + \frac{1}{\overline{\rho}r}\frac{\partial}{\partial r}\left(r\overline{\rho}\,\overline{w'u'}\right) \quad (2)$$

where t: time [s]

$\overline{P}$: time average fluid pressure [kPa]

w': turbulent fluctuating vertical velocity [m/s]

u': turbulent fluctuating radial velocity [m/s]

: fluid density [kg/m^3]

The z-momentum equation simplified to the hydrostatic equation by assuming negligible vertical accelerations:

$$\overline{P} = \overline{P}_a + g\int_z^h \overline{\rho}\,dz = \overline{P}_a + \overline{\rho}g(h-z) \quad (3)$$

where g: acceleration due to gravity [m/s^2]

h: total depth of the tank [m]

Assuming a flow regime with an inlet baffle as in Figure 3, Equations 1-3 were simplified as follows (following the approach of Schijf and Schonfeld, 1953):

- the z-momentum equation (hydrostatic equation) was substituted into the x-momentum equation
- shear stress changes in the r-direction were neglected.

The resulting continuity and r-momentum equations in cylindrical coordinates for each layer after simplification were:

<u>Layer 1</u> ($h \geq z \geq h_2$) :

Continuity Equation $$\frac{\partial \overline{w}_1}{\partial z} + \frac{1}{r}\frac{\partial (r\overline{u}_1)}{\partial r} = 0 \quad (4)$$

Momentum Equation: $$\frac{\partial \overline{u}_1}{\partial t} + \overline{u}_1\frac{\partial \overline{u}_1}{\partial r} + \overline{w}_1\frac{\partial \overline{u}_1}{\partial z} = -\frac{g}{\overline{\rho}_1}\frac{\partial}{\partial r}[\overline{\rho}_1(h-z)] + \frac{1}{\overline{\rho}_1}\frac{\partial \tau(z)}{\partial z} \quad (5)$$

<u>Layer 2</u> ($h_2 \geq z \geq 0$):

Continuity Equation: $$\frac{\partial \overline{w}_2}{\partial z} + \frac{1}{r}\frac{\partial (r\overline{u}_2)}{\partial r} = 0 \quad (6)$$

Momentum Equation: $$\frac{\partial \overline{u}_2}{\partial t} + \overline{u}_2\frac{\partial \overline{u}_2}{\partial r} + \overline{w}_2\frac{\partial \overline{u}_2}{\partial z} = -\frac{g}{\overline{\rho}_2}\frac{\partial}{\partial r}[\rho_2(h-z)] + \frac{1}{\overline{\rho}_2}\frac{\partial \tau(z)}{\partial z} \quad (7)$$

After layer averaging in the z-direction, assuming steady-state, and simplifying, the layer equations (Equations 4-7) become:

<u>Layer 1:</u> $$\frac{Q_1^2}{4g\pi^2 h_1^2 r^2}\left[\frac{2}{r} + \frac{1}{h_1}\frac{dh_1}{dr} - \frac{1}{r}\right] = \frac{dh_1}{dr} + \frac{dh_2}{dr} + \frac{h_1}{2\rho_1}\frac{d\rho_1}{dr} + \frac{(\tau_i - \tau_s)}{g\rho_1 h_1} \quad (8)$$

<u>Layer 2:</u> $$\frac{Q_2^2}{4g\pi^2 h_2^2 r^2}\left[\frac{1}{r} + \frac{1}{h_2}\frac{dh_2}{dr}\right] = \frac{\rho_1}{\rho_2}\frac{dh_1}{dr} + \frac{h_1}{\rho_2}\frac{d\rho_1}{dr} + \frac{dh_2}{dr} + \frac{h_2}{2\rho_2}\frac{d\rho_2}{dr} + \frac{(\tau_b - \tau_i)}{g\rho_2 h_2} \quad (9)$$

where $_{1,2}$: densities in the upper and lower layers, respectively [kg/m^3]

Q_1,Q_2: flow rates in the upper and lower layers, respectively, where Q_1=(1-D_v)Q_o and Q_2=$D_v Q_o$ [m^3/s]

Q_o: tank inflow rate [m^3/s]

D_v: vertical dilution factor at baffle entrance (if =1, no dilution)=(Q_o+$Q_{entrained}$)/Q_o [-]

h_1,h_2: upper and lower layer heights, respectively [m]

$_{b,i,s}$: shear stress at the bottom, interface, and at the surface, respectively

[kPa]

The surface shear stress was assumed to be zero (implying no wind effects or surface ice formation) and the other shear stresses were calculated using (Grubert, 1989)

$$\tau_b = \frac{f}{8}\frac{\rho_2}{h_2^2}Q_2\left|Q_2\right| \quad (10) \qquad \tau_i = \frac{f_i}{8}\frac{\rho_2+\rho_1}{2r^2}\left(\frac{Q_1}{h_1}-\frac{Q_2}{h_2}\right)\left|\frac{Q_1}{h_1}-\frac{Q_2}{h_2}\right| \quad (11)$$

where f_i: interfacial friction factor (=0.01) [-]

f: bottom Darcy-Weisbach friction factor (=0.01) [-]

Numerical Solution and Results

These equations were solved simultaneously for h_1 and h_2 by using a fourth order Runge-Kutta method. Experimental data used in solving the above equations was taken from a study by LaLiberte (1990). This study was performed at Bend, Oregon at the Bend Wastewater Control Plant on its uncovered secondary clarifier during a period of strong winter cooling. Temperature and suspended solids data were taken as a function of depth and radial position within the tank. These data showed that an inflow entered the tank with a suspended solids about 2000 mg/l and a temperature over 1^{o}C warmer than the tank surface. After the initial momentum of the inflow decreased and solids settled, the inflow rose as a buoyant plume. The resulting temperature distribution is shown in Figure 4. Data from this study in Bend Oregon that were used in the numerical model predictions are summarized in Table 1.

Table 1. Experimental data from LaLiberte (1990) for the Bend, Oregon Wastewater Control Plant secondary clarifer. Data were taken during a period of low flow and high surface cooling from 2-4 am on 3/5/89.

Parameter	Value from field study
Flow rate, m^3/s	0.03
Tank depth at centerwell, m	4.6
Tank depth at periphery, m	3.5
Inlet baffle radius, m	2.1
Inlet baffle depth, m	2.4
Tank radius, m	12.2
Area of tank, m^2	467
Tank volume, m^3	1900

Inflow temperature, ^{o}C	13.7
Outflow temperature, ^{o}C	12.5
Reduction in suspended solids from inflow to outflow, mg/l	2000 mg/l

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Model predictions of the layer height are shown in Figure 5 for the Bend, Oregon secondary clarifier with an inlet baffle. For this simulation the suspended solids were assumed to settle out linearly with distance along the tank. A more comprehensive model of the sedimentation process (like the models summarized by McCorquodale, 1997) would be necessary to predict this variation in suspended solids. These results though are meant to illustrate the temperature impact on the flow field. Figure 5 contrasts the layer height with and without suspended solids for a temperature loss of 1.2^{o}C over the tank. The suspended solids keep the warmer, but still denser inflow, near the bottom. But as the solids settle as the momentum of the radial flow diminishes, temperature then influences the trajectory of the plume. If there were no suspended solids in the inflow, temperature becomes more important in determining the location of the interface. Hence, the shape of the interface is a function of the flow rate, inflow suspended solids, temperature difference across the tank. Temperature data shown in Figure 4 also illustrate the effect of buoyancy on the rising of the flow near the middle of the tank.

Summary and Conclusions

A mathematical model of the turbulent flow in a circular tank was developed assuming a radial coordinate system, steady-state, and averaging over each vertical layer. The effect of longitudinal density differences between the vertical layers as a result of temperature and suspended solids differences was also incorporated into the mathematical model. The mathematical model showed that

- the inlet baffle controlled the hydraulics of the dense underflow by forcing the inflow to follow the bottom of the tank
- temperature differences on the order of 1^{o}C could cause the inflow to be deflected upward as a result of buoyancy effects

The mathematical model agreed qualitatively with field data showing that temperature differentials within the clarifier can cause the inflow to rise after both the initial momentum of the inflow has decreased and the suspended solids have settled.

This work shows that ideal flow in a clarifier (as shown in Figure 1) may not be realistic for conditions where inlet baffles direct the suspended solids laden flow downward. More complex mathematical models of these processes have been developed that model this flow field. But during conditions where temperature differentials exist within clarifiers, such as during periods of winter cooling, the effect of temperature can also be significant and should therefore be included in models of these processes.

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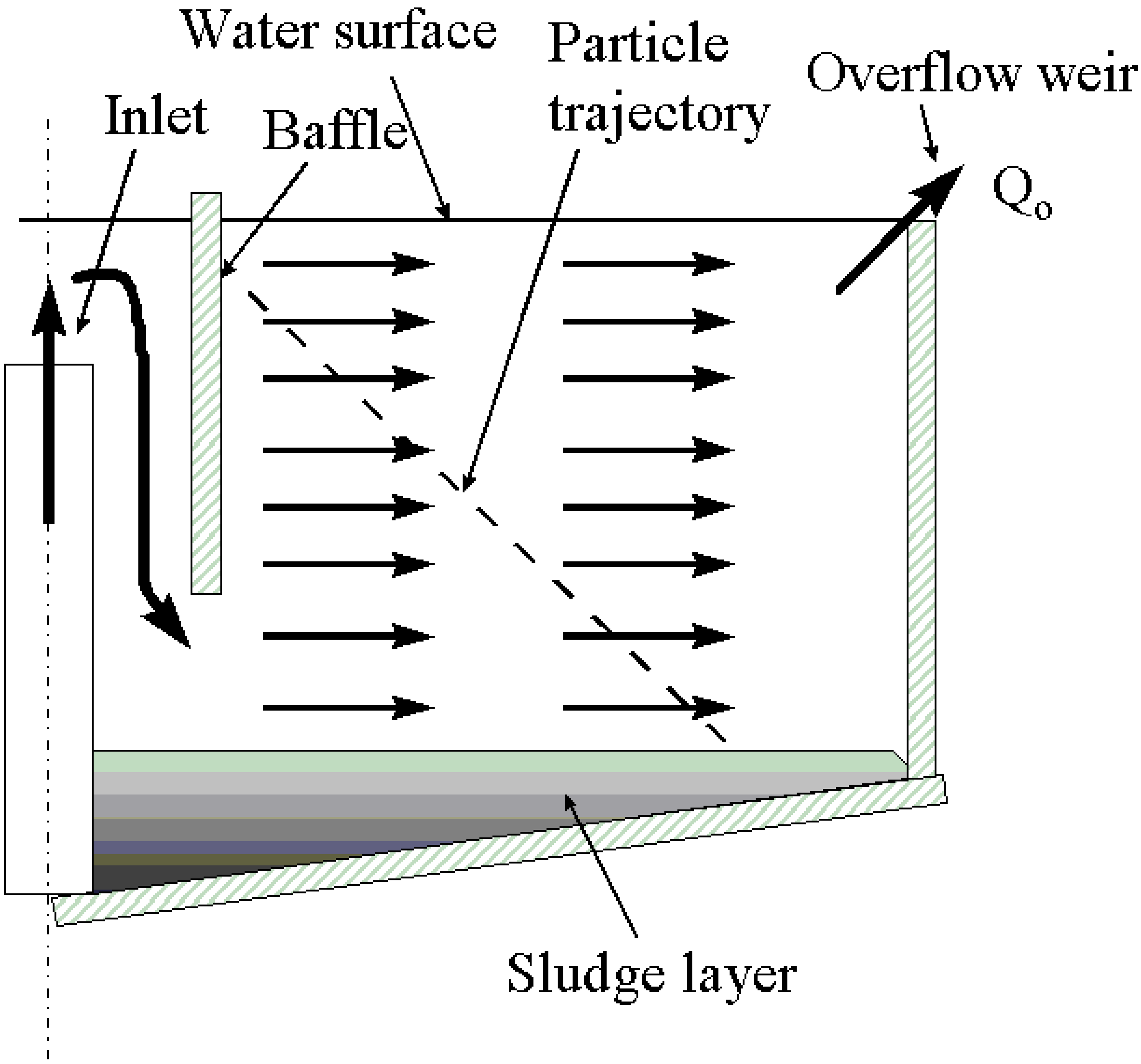

Figure 1. Uniform unidirectional flow in a baffled clarifier.

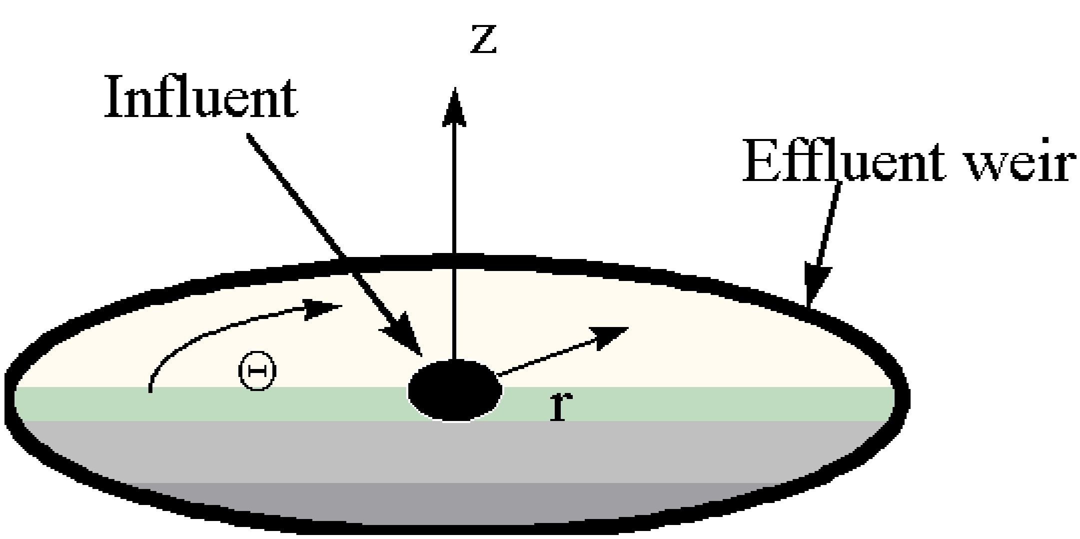

Figure 2. Coordinate system for radial flow in a circular clarifier.

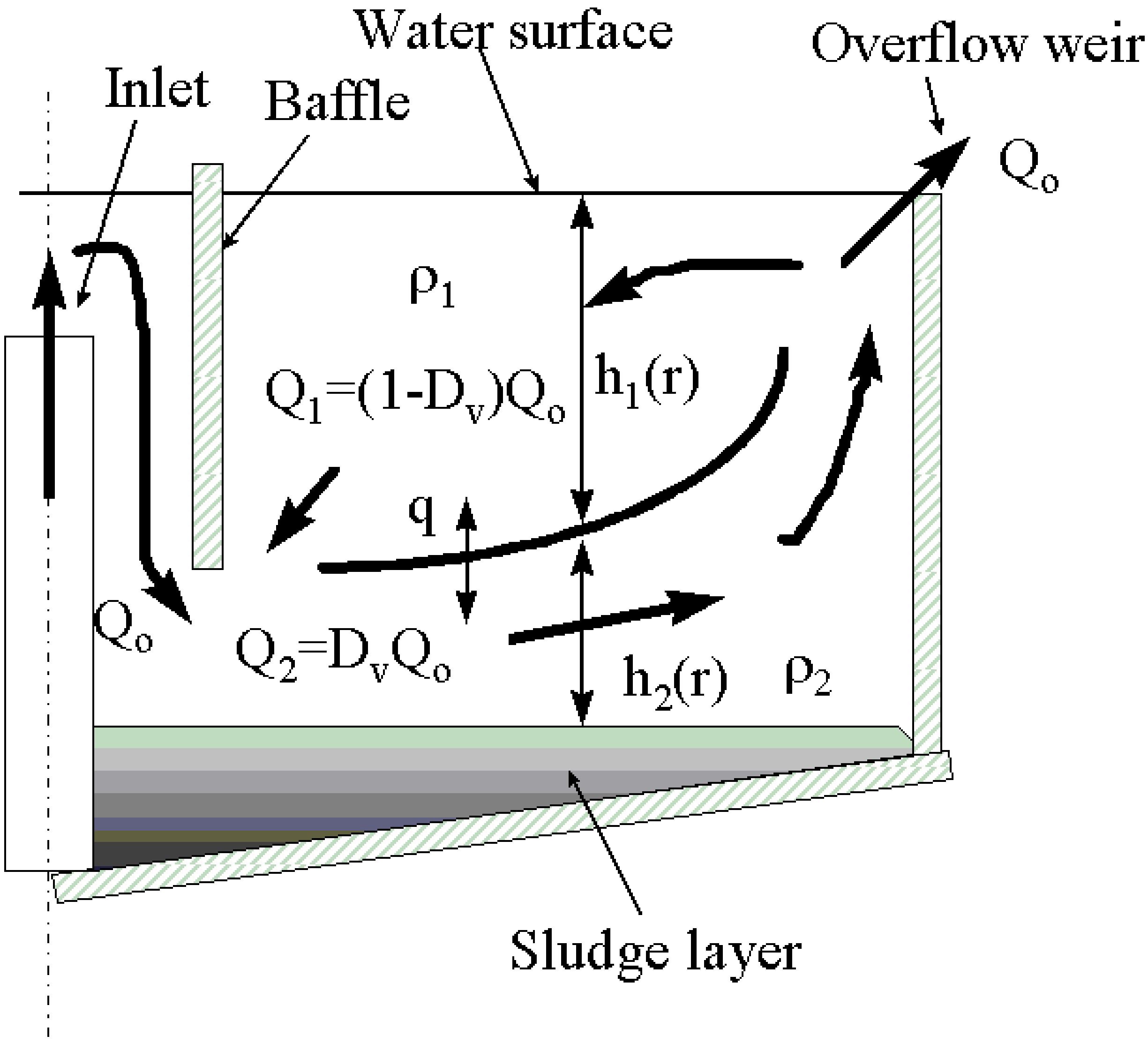

Figure 3. Definition sketch for flow model in a baffled clarifier.

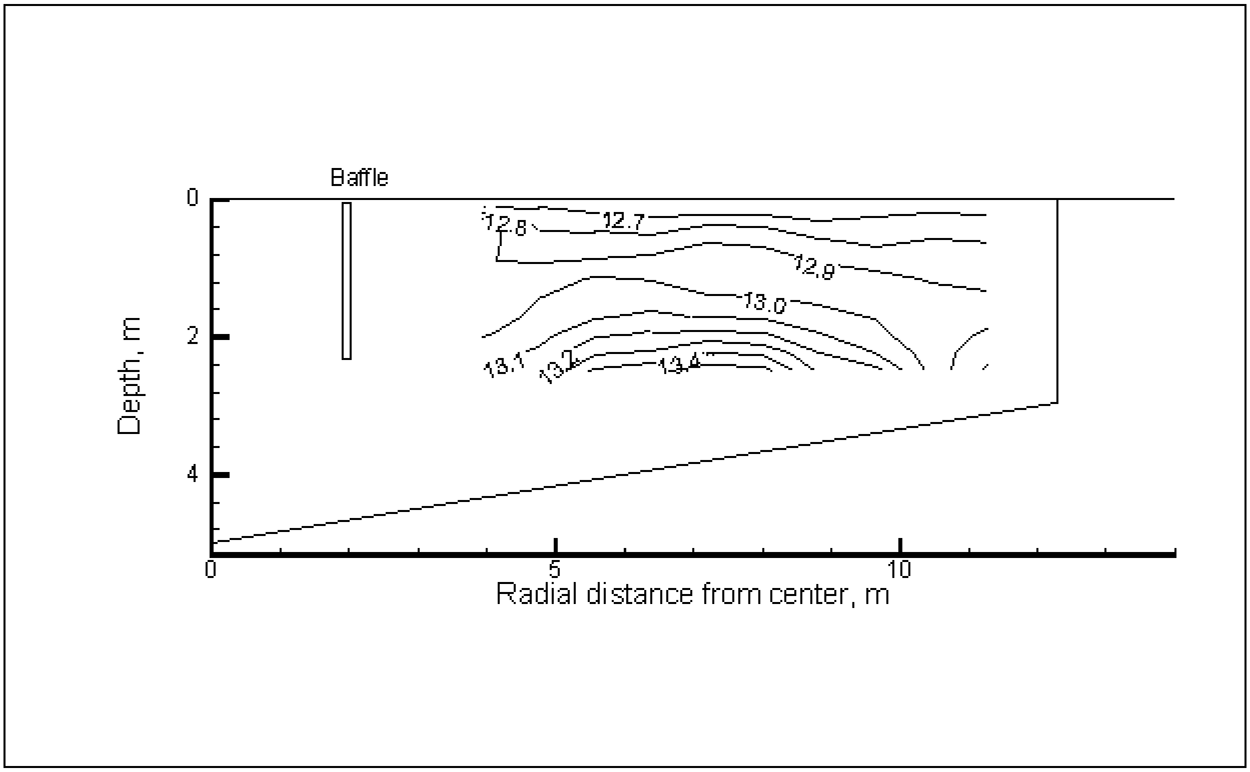

Figure 4. Temperature profile across the secondary sedimentation tank at the Bend treatment plant on 3/5/89 at 2:46 am (Wells and LaLiberte, in-print).

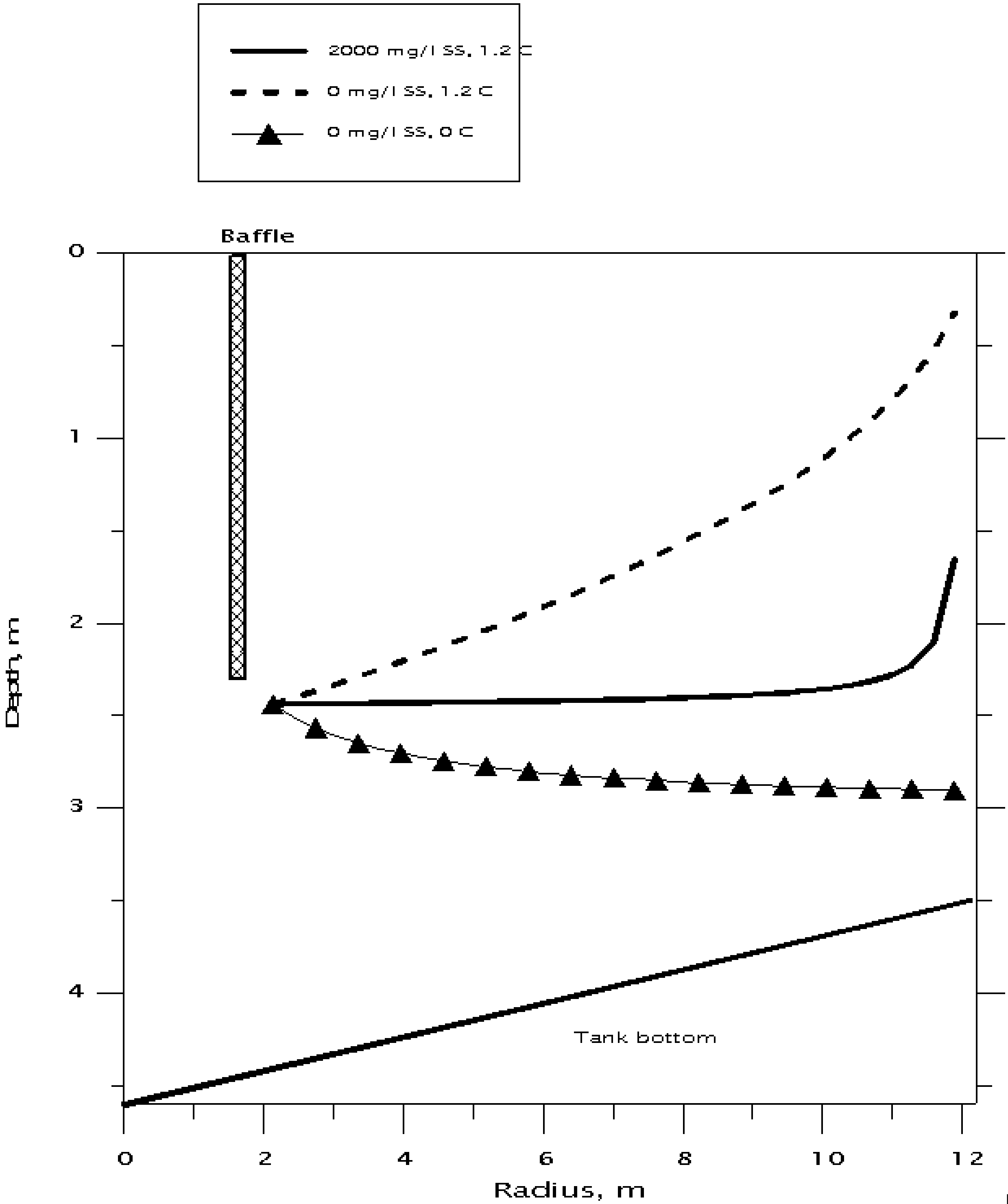

Figure 5. Model predictions of interface layer height as a function of radial position for Q=0.03 m^3/s, D_V=1.5, for the Bend secondary clarifier comparing model predicted interface height with and without suspended solids and temperature effects.