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Puddle jumping: Spontaneous ejection of large liquid droplets from hydrophobic surfaces during drop tower tests

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Large droplets and puddles jump spontaneously from sufficiently hydrophobic surfaces during routine drop tower tests. The simple low-cost passive mechanism can in turn be used as an experimental device to investigate dynamic droplet phenomena for drops up to 10^4 times larger than their normal terrestrial counterparts. We provide and/or confirm quick and qualitative design guides for such “drop shooters” as employed in drop tower tests including relationships to predict droplet ejection durations and velocities as functions of drop volume, surface texture, surface contour, wettability pattern, and fluid properties including contact angle. The latter is determined via profile image comparisons with numerical equilibrium interface computations. Water drop volumes of 0.04–400 ml at ejection speeds of -0.007 – 0.12 m/s are demonstrated herein. A sample application of the drop jump method is made to the classic problem of low-gravity phase change heat transfer for large impinging drops. Many other candidate problems might be identified by the reader. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4963686>]

NOMENCLATURE

Bo	= Bond number $\equiv \rho g_o V_d^{2/3} / \sigma$, where ρ is the density difference across the fluid interface
g	= acceleration field strength
g_o	= terrestrial gravitational acceleration, 9.8 m/s^2
H	= characteristic capillary height, $2(\sigma / \rho g_o)^{1/2}$
L_d	= puddle length scale
R_c	= Cassie (outer) circle radius
R_d	= droplet radius, $(3V_d / 4\pi)^{1/3}$
R_p	= puddle radius
R_s	= spherical solid surface (dish) radius of curvature
R_w	= Wenzel (inner) circle radius
$\mathcal{R}$	= solid substrate curvature parameter, $1 - R_d / R_s$
t	= time
t_{dt}	= low-g freefall time duration of drop tower
t_j	= drop jump time scale, $(\rho H R_p^2 / \sigma)^{1/2}$
T_p	= hot plate surface temperature
u	= droplet impact velocity
U	= drop/puddle jump velocity
$\tilde{U}$	= numerical or simple analytic prediction of puddle jump velocity
$\tilde{U}_{max}$	= maximum simple analytic model prediction of puddle jump velocity
$\tilde{U}_{w/c}$	= Wenzel-Cassie puddle jump velocity
V_w	= remnant of volume of original puddle volume on Wenzel surface after jump
V_d	= drop/puddle volume

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- V_{dmin} = minimum drop volume to jump, $(\sigma/\rho g_o)^{3/2}$
 V_{dmax} = maximum drop jump volume for drop tower, $\leq \sigma t_{dt}^2/\rho$
 We = impact Weber number, $\rho u^2 R_d/\sigma$
 We_j = measured drop jump Weber number, $\rho U^2 R_d/\sigma$
 $\tilde{We}_j$ = predicted drop jump Weber number, $\rho \tilde{U}^2 R_d/\sigma$
 α = roll-off angle
 ϕ = ratio of Wenzel-to-Cassie wetted areas, $(R_w/R_p)^2$
 μ = dynamic viscosity
 ρ = liquid density
 σ = surface tension
 θ = contact angle
 θ_{Cassie} = Cassie state contact angle
 θ_{Wenzel} = Wenzel state contact angle
 τ = terrestrial rebound contact time, [Richard *et al.* \(2002\)](#)
 ψ = Wenzel to capillary height ratio, $(R_w/H)^2$

I. INTRODUCTION

As recently introduced and reviewed by [Wollman *et al.* \(2016\)](#), the drop jump or puddle jump mechanism provides a no-moving-parts method for investigations of large length scale drop physics. Such investigations are of fundamental interest in their own right, but they are particularly applicable to the low-g fluid systems aboard spacecraft where such large unearthy droplets are commonplace in fuel tanks, coolant systems, and water processing equipment.

A simple drop tower experiment rig is shown in Fig. 1 where a 2 ml blue-dyed water droplet is placed on a hydrophobic surface achieving a Cassie state. To various degrees, releasing the experiment into freefall results in a “rolling-up” of the “puddle” across the surface. The vertical asymmetry of the geometry allows the drop to push off and away from the surface as shown incrementally in Fig. 2. [Kirko *et al.* \(1970\)](#) were the first to report such observations in early drop tower tests employing immiscible liquids (i.e., mercury in hydrochloric acid). A wide variety of drop-in-air demonstrations are highlighted by [Weislogel \(2012\)](#) and [Wollman *et al.* \(2016\)](#). The drop ejection process is related to the second half of droplet rebound and bounce phenomena from super-hydrophobic surfaces (see [Richard *et al.* \(2002\)](#) and [de Ruiter *et al.* \(2015\)](#), and references contained therein) and the drop jump phenomena observed when drops coalesce on hydrophobic surfaces ([Farhangi *et al.*, \(2012\)](#)). The inverse problem of bubbles jumping “downward” from flat surfaces in drop tower experiments is also discussed by [Wollman *et al.* \(2016\)](#) with related work pursued by [Suñol and González-Cinca \(2010\)](#). Numerical investigations of the drop jump phenomena are conducted by [Zhang *et al.* \(2014\)](#).

FIG. 1. (a) Simple drop tower experiment rig with (b) magnified view of drop jump surface.

FIG. 2. Drop jump of 2 ml water puddle from a hydrophobic surface with $\theta \approx 148^\circ$. (a) At $t = 0$ s the puddle is largely flat due to gravity. Following release of the experiment into freefall (b) the drop rolls up, (c) jumps from the surface at fixed velocity U , and (d) and (e) exhibits damped oscillations dependent on droplet volume and fluid properties.

II. APPLICATIONS OF THE DROP JUMP MECHANISM

Because the drop jump process is so simple, we are interested in applying it to numerous low-gravity drop dynamics investigations. For example, the experiment rig in Fig. 1 includes a polished aluminum plate positioned directly above the hydrophobic drop jump surface. This plate is heated prior to the test to any desired temperature to study large drop impacts on heated surfaces in a microgravity environment. Fig. 3 provides test results performed at interesting plate temperatures: Fig. 3(a) shows an ambient impact and damped oscillation ($T_p = 25^\circ\text{C}$), 3(b) nucleate boiling with a majority of mass loss by bubble rupture and satellite droplet ejection ($T_p = 179^\circ\text{C}$), 3(c) transition boiling ($T_p = 239^\circ\text{C}$), and 3(d) the Leidenfrost condition with miniscule heat transfer ($T_p = 306^\circ\text{C}$). Adhesions, splashing, bouncing, and boiling in numerous combinations are observed. A broad literature exists for this important problem (Inadat and Yang, 1993), but the drop jump mechanism via drop tower facility permits the study of drops with up to 10^4 times the volume of typical terrestrial drops with accompanying large, slow, easily filmed, macro-capillary dynamics. The freefall environment also allows the study of low impact Weber numbers approaching zero impact velocity and with certifiably negligible buoyancy effects.

Without using the drop jump method, similar freefall drop impact experiments are performed, for instance, by Qiao and Chandra (1995) who employ still flash photography during 0.055 s “drop tests” (15 mm free fall distance) with $V_d \approx 0.0017$ ml. The longer duration of a designated drop tower facility (22.2 m and 2.1 s in our case) permits at least a 38-fold longer viewing time for drops up to 200 000 times larger and achieves impact Weber numbers $We = \rho u^2 R_d / \sigma$ an order of magnitude smaller, where We is a measure of the inertia present at impact relative to the restoring force of surface tension.

The breadth of experimental parameters in such investigations is large when considering surface materials and properties, fluid types, wall super-heat, liquid sub-cooling, air saturation, impact angle, oscillation mode at impact, and more. Rein (1993) discussed the importance of incident angle and droplet shape (prolate or oblate) on impact. In another study, Lee *et al.* (2013) examined dynamic behavior of an impinging droplet on a hot surface by varying the Weber number. Inada *et al.* (1989) studied mechanisms of the “miniaturization” of 6.3 and 33.5 μl sessile droplets following impact with a heated plate.

In a natural way, drop volume, impact velocity, and impact oscillation mode are controlled output parameters of the drop jump method. Following this line of inquiry, the present investigation seeks to report some of the performance limits and design guides restricted to the drop jump method when employed as a drop deployment device for any number of drop dynamics investigations. Distilled water with surface tension $\sigma = 71.9$ mN/m and density $\rho = 1000.6$ kg/m³ is used in all tests performed herein. Nonwetting Cassie-state contact angles between $120^\circ \leq \theta \leq 150^\circ$ are achieved. The NASA SpotLight-16 image analysis software of Klimek and Wright (2006) is used to digitize all droplet dynamics including center of mass displacements to determine drop jump velocity U , usually by threshold tracking. Most experiments are recorded via HD Panasonic 900M video camera at 60 fps. When needed, a Nikon J1 with 30 mm VR lens at 400 fps is employed.

FIG. 3. A 2 ml water drop with initial temperature 23 °C launched via puddle jump impinges against a heated mirror polished aluminum plate: (a) $T_p = 25$ °C, (b) 179 °C (nucleate boiling with significant droplet ejection), (c) 239 °C, and (d) 306 °C (the Leidenfrost temperature). Times after impacts are provided in seconds. Images recorded at 400 fps.

Diffuse background lighting is provided by Green LED Lighting Solutions' 24 VDC LED backlight panels.

Up to four methods are used to measure the steady jump velocities reported herein: automated area centroid tracking, automated threshold tracking of the average of the top and bottom location of the drop, and sometimes manual tracking of the same. True mass center measurements could be made only for rare cases of axial symmetry. Between 126 and 840 images per drop jump experiment could be analyzed. For sufficient distances from the hydrophobic surfaces, the methods achieved nearly identical results with ± 2 pixels accuracy leading to velocity errors significantly less than $\pm 5\%$ depending on magnification. Precision mass $\pm 0.1\%$ and precision graduated syringe $\pm 1\%$ methods were used to determine drop volumes. The drop deployment methods were calibrated for each experimenter and found repeatable to $\pm 1\%$. We note that slight evaporation of the deployed drops could be observed when drop mass was measured adding typically $\leq 1\%$ to an over-prediction in drop volume. Over 550 drop tower tests were conducted in support of this work with individual experiments repeated as many as 10 times with nominal reproducibility of $\pm 3\%$, which is approximately equivalent to the total individual experiment error of approximately $\pm 3\%$. (Note that by 10 repeat experiments we mean 10 "identically" prepared surfaces, 10 "identically" deployed drops of

“identical” volume, and 10 “identical” image processing procedures to find the nominal experiment repeatability of $U \pm 3\%$.) We note that errors in surface preparation such as poor hydrophobic coating coverage, incomplete coating curing, contamination, local Wenzel states, and asymmetric initial conditions produce obvious jump velocity outliers that are not presented.

III. DROP JUMP CHARACTERISTICS AND MEASUREMENTS

Focusing strictly on the passive drop jump mechanism arising from the step reduction in effective gravity level common to drop tower facilities, we identify the primary parameters of the system as (1) drop volume, (2) surface hydrophobicity, and (3) surface topology. The impact of gravity on the process is measured by the initial 1- g_o Bond number of the system $Bo = \rho g_o V_d^{2/3} / \sigma$. The drop tower release method and drop rig apparatus can also play a significant role in the drop jump process by introducing and promulgating acoustic ringing in the system during freefall. Such effects are unavoidable, complex, minimized by design, and presently neglected. Thus, drop volume, surface hydrophobicity, and surface topology are treated herein as control parameters to design drop jump systems for desired drop velocities, drop volumes, drop mode shapes, and more. We first present the impact of drop volume on the experimentally measured drop jump velocity. Simple estimates are forwarded for jump time, minimum and maximum jump volume, and maximum expected jump velocity. The methods used to produce large hydrophobic surfaces are then discussed followed by a selection of observations of potentially useful drop jump characteristics arising from the unique control of hydrophobic surface topology at such large capillary length scales.

IV. DROP VOLUME, V_d

For comparison with the 2 ml drop jump of Fig. 2, Figs. 4 and 5 provide image sequences of 30 and 95 ml drop jump tests, respectively. The specific wetting conditions of the surfaces will be discussed shortly. From Figs. 2, 4, and 5 it is clear that the presence of inertia increases with drop volume leading eventually to bubble ingestion, geyser formation, and highly perturbed undulating puddle jumps.

For $V_d \leq 20$ ml, the impact of drop volume on jump velocity is shown in Fig. 6 with a selection of representative scaled images of the drops at the point of ejection provided for reference. We note that typical terrestrial droplet volumes are $\lesssim 0.1$ ml. The drop volume is an easily controlled variable affecting drop jump in such drop tower tests. The drop jump velocity first increases with

FIG. 4. A 30 ml puddle jump.

FIG. 5. Formation of a geyser at $t = 0.35$ s for a slightly asymmetric 95 ml puddle jump.

FIG. 6. Drop jump velocity U as a function of drop volume V_d (laser etched surface #4). Select images at point of separation shown approximately to scale.

drop volume. A single frame is presented in Fig. 7 for four small drops of incremental volume tested simultaneously in this regime. As drop volume increases above 2 ml, a sharp decline in U is observed in Fig. 6. A wider range of drop jump data is presented in Fig. 8 for U and Fig. 9 for We . Images of the drops at the point of separation are added for comparisons in Fig. 9. The smallest image shown is for a 0.6 ml drop which is already nearly 10 times larger than typical terrestrial drops. These results will be discussed further in connection with simple model predictions.

A. Minimum drop jump volume, V_{dmin}

For sufficiently hydrophobic Cassie-state conditions, the impact of drop volume is manifest through the magnitude of Bond number, where $Bo \gg 1$ implies initially gravity-dominated, flat, disc-like puddle states that are certain to jump. Drop jump is less likely as Bo decreases in magnitude below $Bo \sim 1$ because in such instances the drop initial condition is nearly as dominated by surface tension as the low-g state of the drop tower test. Thus, we expect

$$V_{dmin} \sim (\sigma / \rho g_o)^{3/2}, \quad (1)$$

and for water, a lower volume limit of the drop jump phenomena ≈ 0.02 ml is predicted. Such a limit for $V_d \approx 0.03$ ml is identified in Fig. 6, where $Bo \approx 1.32$. Lower drop volume limits are expected for higher contact angles. Nonetheless, such a threshold is expected for all wetting conditions since it is more energetically favorable for the drops to adhere to surfaces regardless of the contact angle (Satterlee and Reynolds, 1964).

FIG. 7. Drop jump velocity U resulting from drop tower test as a function of drop volume V_d (3D printed hydrophobic surface). Image taken at $t = 0.39$ s into drop test.

FIG. 8. Comparison of experimental drop jump velocity U from a dished 3D printed hydrophobic surface ($R_s = 0.352$ m) with numerical and simple theoretical predictions. Relevancy for the simple theory leading to Eqs. (5), (B1) and (B2) is identified by the onset of the large puddle limit $6^{1/3}Bo^{-1/2} < 1$. U_{max} from Eq. (7) is the asymptote for Eq. (5). Drop dimensions (< 5 mm) begin to rival hydrophobic surface post spacing (≈ 0.5 mm) for $V_d \lesssim 0.1$ ml, which is also the upper limit of terrestrial drop volumes tested by Richard *et al.* (2002). The conditions where geysers are observed are identified by $6 \lesssim V_d \lesssim 110$ ml. Three solid square data points for large drops on flat laser-etched surfaces (#4) are provided for reference.

B. Drop jump transients

Because the drop jump process requires time to develop, an upper droplet volume limit arises depending on the low- g time afforded by the particular drop tower facility. For example, for the 30 ml puddle jump shown in Fig. 4 the large droplet manages to leave the surface during the first 0.66 s of the 2.1 s drop tower test. Knowledge of this time is helpful for drop tower experiments wishing to exploit the puddle jump method.

In a similar manner to terrestrial drop rebound experiments, during the jump period the large puddles roll up as capillary waves travel radially inward eventually colliding and forming a jet or geyser, which in turn may break up forming large and/or small satellites. For the 95 ml puddle jump shown in Fig. 5, the initial planform shape of the puddle is not perfectly circular, which results in an asymmetric two-pronged 1.8 m/s geyser observed at time $t = 0.35$ s and $t = 0.41$ s, as observed by Pearson *et al.* (2013). At 60 fps we find that for droplets up to approximately 10 ml no geysers form,

FIG. 9. Comparison of experimental jump We_j with numerical and simple theoretical predictions. Data re-plotted from Fig. 8. Select drop jump images at separation are shown below drop tower data to approximate scale for comparisons. Satellite drops of miniscule volume in the geyser regime are not shown.

while for drops $6 \text{ ml} \lesssim V_d \lesssim 100 \text{ ml}$ they do. No geysers are observed for large puddles of 200, 300, 400, and even 500 ml. To be reported in a separate publication, subsequent experiments with higher contact angle hydrophobic surfaces suggest that geyser formation persists at higher volumes.

C. Drop jump time scale, t_j

The related terrestrial problem of contact time τ for a bouncing droplet impacting a hydrophobic surface is studied by Richard *et al.* (2002) and de Ruiter *et al.* (2015), and numerous others. The drop jump time of the present study is quite similar to the drop recoil and rebound phase in such investigations. In the current drop tower tests the initial condition of the large droplet is affected by gravity, but gravity does not impact the drop jump process itself. The large drops jump quickly and by ignoring viscosity ($\mu UR_p/\sigma H \ll 1$) in addition to low-gravity ($\rho g R_p H/\sigma \ll 1$), a scale momentum balance between inertia and surface tension gives $\rho U^2/R_p \sim \sigma/R_p H$, where R_p is the 1- g_o large puddle radius defined by $\pi R_p^2 H \equiv V_d$, $H \equiv 2(\sigma/\rho g_o)^{1/2}$ is the characteristic puddle capillary height, and g is the acceleration level during freefall ($g \leq 10^{-4} g_o$). Thus, $U \sim (\sigma/\rho H)^{1/2}$ and we identify the drop jump time scale as $t_j \sim R_p/U \sim (\rho H R_p^2/\sigma)^{1/2}$. For large disc-shaped puddles, $V_d \sim H R_p^2$ such that for large drops we expect the jump time scale to be

$$t_j \sim \left(\frac{\rho V_d}{\sigma} \right)^{1/2}, \quad (2)$$

which is similar to the contact time of Richard *et al.* (2002) for terrestrial drop rebound experiments

$$\tau \approx 2.6 \left(\frac{\rho R_d^3}{\sigma} \right)^{1/2} \approx 1.27 t_j. \quad (3)$$

Our experimentally measured drop jump values for t_j are presented in Fig. 10 for water drops on identical laser-etched hydrophobic surfaces varying only the drop volume V_d . The data of Richard *et al.* (2002) with similar contact angle are also provided in the figure. Fair agreement with both Eqs. (2) and (3) is observed over the full range of drop volumes tested, which is extended by approximately 4 orders of magnitude by the present data. The contact time τ of Eq. (3) and the half-contact time $\tau/2$ are overlaid in Fig. 10 for comparisons. Observations of the figure show that t_j is not simply half the rebound contact time. Systematic over-predictions in jump time t_j may be present for the largest puddles. The form of Eq. (2) is easy to remember.

D. Maximum drop jump volume, V_{dmax}

Eq. (2) may be inverted to determine the largest characteristic puddle volume that can be expected to jump from a hydrophobic surface in the drop tower freefall time available t_{dt} ,

FIG. 10. Drop tower tests results (open circles) for drop jump time t_j from laser etched surface (#4) as function of V_d from image data collected at 400 fps. The terrestrial measurements of drop impact and rebound by Richard *et al.* (2002) (solid circles) are divided by two and overlaid for comparison. A 4-order of magnitude extrapolation of the simple Eq. (2) model does well in general as seen in panel (b), with possible over-predictions beginning for $V_d > 200 \text{ ml}$ as observed in panel (a).

$$V_{dmax} \leq \sigma t_{dt}^2 / \rho. \quad (4)$$

For example, for $t_{dt} = 2.1$ s, for water, we find $V_{dmax} \approx 300$ ml, which is approximately 10 000 times larger than a typical raindrop (~ 0.03 ml). This result corroborates with experiments for our maximum drop jump volume of 400 ml presented in Fig. 10(a).

V. SIMPLE EQUILIBRIUM MODELS FOR PREDICTED DROP JUMP VELOCITY, $\tilde{U}$

Simple analytic and numerical equilibrium methods may be quickly deployed to predict puddle jump velocity $\tilde{U}$ by equating the total surface energies SE_1 of the flattened drop immediately following release into freefall with the long duration spherical configuration surface energy SE_2 plus kinetic energy KE_2 , the latter which is used to compute $\tilde{U}$ since $KE = \rho V_d \tilde{U}^2 / 2$ and $SE_1 = SE_2 + KE_2$. All dynamic effects are ignored with expectations of over-predictions. Equilibrium contact angles, fluid properties, and initial gravity vector are imposed to compute the mechanical surface energy of the initial drop shape. The calculations may be performed for ideally flat surfaces as well as for the slightly dished surfaces employed in the drop tower experiments to keep the large puddles centered. The steady state spherical surface energy is then subtracted and drop velocity $\tilde{U}$ calculated. Only the simplest of the theoretical models is outlined below with further details for both theoretical and numerical results provided in [Appendices A–C](#).

A. Simple analytic model for drop jump velocity, $\tilde{U}$

The drop jump velocity is estimated analytically in the large puddle volume limit by assuming a sufficiently non-wetting puddle of cylindrical disk shape with radius R_1 and height H . We ignore viscous losses due to shear, roll-up, receding contact line motion, and air drag. We assume the drop ultimately achieves a spherical configuration of radius $R_2 = R_d$ and that no satellite drops are formed. Non-spherical distortions of the drop during the jump are eventually damped by viscous forces which are also ignored. These idealized initial and final states are depicted schematically in Figs. 11(b) and 11(c).

Using the notation of Figs. 11(b) and 11(c), the surface energies for liquid-solid (ls), gas-liquid (gl), and solid-gas (sg) for cylindrical disk and spherical shapes are $SE_1 = [(\sigma A)_{ls} + (\sigma A)_{gl} + (\sigma A)_{sg}]_1$ and $SE_2 = [(\sigma A)_{gl} + (\sigma A)_{sg}]_2$, respectively. From Young's equation, we have $\sigma_{sl} - \sigma_{sg} = -\sigma_{gl} \cos \theta$, where $R_1 = R_p = (V_d / \pi H)^{1/2}$, $R_2 = R_d = (3V_d / 4\pi)^{1/3}$, and $H \approx 2(\sigma / \rho g)^{1/2}$. From $KE = SE_1 - SE_2$ we find

$$\tilde{U} = \left(\frac{\sigma g}{\rho} \right)^{1/4} \left[1 - \cos \theta + 2 \left(\frac{\pi H^3}{V_d} \right)^{1/2} - 6^{2/3} \left(\frac{\pi H^3}{V_d} \right)^{1/3} \right]^{1/2} \equiv \left(\frac{\sigma \tilde{W} e_j}{\rho R_d} \right)^{1/2}. \quad (5)$$

In the large puddle limit $(\pi H^3 / V_d)^{1/3} \ll 1$, and for $\pi/2 \leq \theta \leq \pi$, Eq. (5) reduces to

$$\tilde{U} \approx \left(\frac{\sigma g}{\rho} \right)^{1/4} (1 - \cos \theta)^{1/2}. \quad (6)$$

FIG. 11. Simplified large drop jump model: (a) Actual initial condition with capillary height $\sim H$, (b) state 1 idealized cylindrical disc initial condition with contact angle θ along the solid-liquid surface where $R_1 = R_p$, and (c) state 2 final zero-gravity rigid body spherical configuration traveling at predicted velocity $\tilde{U}$, where $R_2 = R_d$.

When $\theta \approx \pi$ the maximum puddle jump limit is found to be

$$\tilde{U}_{\max} \approx \left(\frac{4\sigma g}{\rho} \right)^{1/4}. \quad (7)$$

The dependence of $\tilde{U}$ on θ for super-hydrophobic surfaces is weak and when $\theta \approx 148^\circ \pm 10^\circ$, we observe only a 4% reduction in $\tilde{U}$ when compared to a perfect non-wetting surface of $\theta = 180^\circ$. Substituting for H we note that the large puddle limit may be recast as $(\pi H^3/V_d)^{1/3} \equiv 6^{1/3} \text{Bo}^{-1/2} \ll 1$, where $\text{Bo} \equiv \rho g R_d^2/\sigma$ is the Bond number based on spherical drop radius R_d . In this limit we also note that the drop jump Weber number $\tilde{\text{We}}_j \equiv \rho \tilde{U}^2 R_d/\sigma \rightarrow 2 \text{Bo}^{1/2}$. These expressions are used to write Eq. (5) as

$$\tilde{\text{We}}_j = \text{Bo}^{1/2} [1 - \cos \theta + 2 \cdot 6^{1/2} \text{Bo}^{-3/4} - 6 \text{Bo}^{-1/2}]. \quad (8)$$

Eqs. (5) and (7) are added to Fig. 8 providing fair agreement with experimental results, but only in the restricted regime of the analysis assumptions—above the initiation of the large puddle limit and below the limit of geyser formation (significant non-spherical distortions). Fortunately, this is a range of drop volumes that make optimal use of the brief time afforded by typical drop towers. Fig. 9 includes $\tilde{\text{We}}_j$ based on predicted velocity $\tilde{U}$ from Eq. (5). In comparing terrestrial drop bounce and drop tower drop jump phenomena, since $\tilde{\text{We}}_j$ is determined by idealized equilibrium state assumptions, $\tilde{\text{We}}_j$ predicts that the rebound velocity would be equal to the impact velocity. Thus, drop jump dynamics for a given value of $\tilde{\text{We}}_j$ might be expected to be similar to drop rebound dynamics when impact Weber number $\text{We} = \rho u^2 R_d/\sigma \approx \tilde{\text{We}}_j$.

In reference to schematics of Figs. 11(b) and 11(c), the terms in brackets in Eqs. (5) and (8) in the order left to right represent the energies of the top surface area of the disk drop, the bottom surface of the disk drop, the disk side walls, and the spherical surface, respectively. The large puddle jump velocity limit of Eq. (7) is a constant, independent of drop volume. For water we find $\tilde{U}_{\max} \approx 0.23$ m/s. However, $\tilde{\text{We}}$ continues to rise with puddle volume through R_d , or Bo , as observed in Eq. (8) and Fig. 9. The dependence of $\tilde{U}$ on $\theta \geq 130^\circ$ is weak. This equilibrium analytical approach may also be pursued numerically using exact numerical surface energies for the initial surface configuration without the cylindrical disc assumption as discussed in Appendix A. Eq. (5) agrees well with the flat surface numerical solution for large drop volume.

B. Other puddle jump velocity predictions, $\tilde{U}_{\text{dished}}$ and $\tilde{U}_{\text{w/c}}$

The analytic model may be extended to address both flat and dished hydrophobic surfaces as well as drop jump phenomena for mixed Wenzel and Cassie wetting conditions. Sample calculations are provided in Appendices B and C yielding closed form expressions apt for comparisons to the experimental results in Figs. 8 and 9.

VI. SURFACE HYDROPHOBICITY

A. Hydrophobic surfaces

Sufficiently hydrophobic Cassie state surfaces are desirable, if not essential, for the drop jump process. For the experiments of this work, we seek to quickly create large low-cost hydrophobic surfaces for a variety of drop jump investigations. Though an ever-increasing number of methods to create hydrophobic surfaces abound, such as DRIE (Sun *et al.*, 2010), hydrophobic metals (Vorobyev and Guo, 2015), hydrophobic aerosols (Ogihara *et al.*, 2012), and numerous commercially available coatings, in this work we construct hydrophobic surfaces by either laser etching acrylic sheets using a 60W CO₂ Universal Laser Systems VLS4.60, or by 3D printing with a 30 μm resolution 3D Systems Projet 3500. The requirement of surface feature control at the micro-scale is relaxed due to the large dimensions of the liquid drops under investigation here. The laser cut surfaces are created by choosing power setting q and cutter speed c , pillar width w , and pillar spacing d identified in Table I. For the surfaces of this work, we choose $q = 36$ W and $c = 0.1$ m/s. The combination of these parameters determines the channel depth h . The polymer surfaces are then cleaned using a warm soap solution,

TABLE I. Properties and dimensions of hydrophobic surfaces shown in Fig. 12. Uncertainties on α are $\pm 10\%$. Representative values for drop jump velocity U listed for $V_d = 2.0$ ml drop.

Surface #	Manufacture technique	d (mm)	h (mm)	w (mm)	θ (°)	α (°)	U (m/s)
1	3D print	0.50	0.5	0.5	144	1.7	0.086
2	3D print	0.25	0.5	0.5	138	2.2	0.063
3	3D print	0.25	2.0	0.5	132	2.0	0.070
4	Laser etch	0.50	0.6	0.4	148	2.8	0.120

rinsed with aqueous methanol, and air dried. They are lightly spray-coated at a 25 cm working distance with PTFE aerosol (King Controls, 2014) and dried with compressed air. A selection of magnified images of such surfaces is provided in Fig. 12 and characteristic properties of the surfaces are listed in Table I. Because the laser cut process is far less controlled than other methods such as photochemical etching or 3D printing, a wide variety of surface types is empirically tested using static contact angle θ and dynamic roll-off angle α measurements.

The example surface of Fig. 12(d) (#4) is employed in the majority of tests reported here as it is found to establish an adequate, easily reproduced surface with contact angle $\theta \approx 148 \pm 1^\circ$ and roll-off angle $\alpha \approx 2.8 \pm 0.4^\circ$ for a 2 ml drop. Such surfaces are sufficiently hydrophobic such that it is difficult to balance the drop on a handheld horizontal surface. As a recourse, we frequently post-process the surfaces in an oven over shallow molds (dishes) of spherical radius R_s to assure the desired contour to accurately position the puddles before the drop tower tests.

B. Measurement of contact angle θ and roll-off angle α

Average static non-wetting contact angles are measured by comparing profile images of drops of known volume to numerically computed droplet shapes of the same volume. The *Surface Evolver* algorithm of Brakke (1992) is employed via the SE-FIT graphical user interface (Chen *et al.*, 2011). The contact angle θ is varied in the computations until the interface profile error with the experiment images is minimized. Example simulations are shown in Fig. 13 for 2 ml and 0.03 ml water droplets on a laser etched surface with computed contact angles $\theta \approx 148^\circ$ and $\theta \approx 135^\circ$, respectively. The numerical predictions are essentially coincident when overlain.

As often reported, droplet roll-off angles provide a convenient means of comparing the hydrophobicity of various surfaces (Smyth, 2010). For our large drops/puddles and large featured surfaces, the roll-off angle α is a strong function of the drop volume as shown in Fig. 14.

VII. ADDITIONAL DROP JUMP OBSERVATIONS: EFFECTS OF HYDROPHOBIC SURFACE TOPOLOGY

A. Hydrophobic surface wettability patterns

In Fig. 15(a) is sketched a profile view of a laser etched surface where an inner circular region establishes an effective contact angle that is lower than the non-wetting area outside the circle.

FIG. 12. 3D printed ((#1)–(#3)) and laser etched (#4) surfaces employed in majority of experiments conducted. Each surface is identified by reference number with properties listed in Table I.

FIG. 13. (Left to right) SE-FIT simulations, overlays, and experimental images for (a) a 2 ml puddle on a hydrophobic laser cut surface (ref. Surface #4) with $\theta \approx 148^\circ$ and (b) a 0.03 ml droplet on the same surface where $\theta \approx 135^\circ$.

FIG. 14. Roll-off angle α as function of drop volume V_d and surface type (ref. Fig. 12, Table I).

FIG. 15. Laser etched acrylic sheet with hydrophobic surface (Cassie state) of diameter $2R_c$ and partial hydrophilic surface (Wenzel state) of diameter $2R_w$: (a) schematic and (b) perspective image of a 2 ml puddle. By varying $\phi \equiv (R_w/R_p)^2$ we control the drop jump velocity.

Fig. 15(b) shows a perspective image of a 2 ml blue-dyed water drop on such a surface. By placing a drop centered on this circle with a volume that spans the wetting discontinuity, it is possible to passively tune both the volume and velocity of the jumping drop. Here we present results only when Wenzel states are established within the inner circle of radius R_w and Cassie states within the outer circle of radius R_c . Fig. 16 presents a time sequence of the drop jump process under these conditions for 2 ml drops with $R_w = 3.15$ mm and $R_w = 4.5$ mm, respectively. We define R_p by the large puddle limit, where $\pi R_p^2 H = V_d$ such that, for large puddles, R_p is always slightly greater than R_c . Expectedly, the larger the area ratio $\phi \equiv (R_w/R_p)^2$ of Wenzel-to-Cassie wetted areas the lower the jump velocity. Fig. 17(a) presents drop jump velocity as a nearly linearly decreasing function of ϕ . A remnant of the original puddle of volume V_w remains on the Wenzel surface, the approximate volumes of which are plotted in Fig. 17(b). It is the degree of connection of the jumping drop with this adhered volume that controls the jump velocity. For values $\phi > 0.36$, no jump occurs. For values $\phi \approx 0.306$ and 0.322 , the filament connecting the jumping drop to the adhered remnant can

FIG. 16. A 2 ml drop jump from a hydrophobic surface with a Wenzel state area at the center: (a) $R_p = 11.2$, $R_w = 3.1$ mm ($\phi = 0.077$) and (b) $R_p = 11.0$, $R_w = 4.5$ mm ($\phi = 0.17$). The larger the Wenzel state radius R_w the slower the jump velocity. Remnant volumes V_w are 0.22 and 0.26 ml, respectively. For the remnant we observe $\theta_w \approx 115 \pm 5^\circ$.

FIG. 17. (a) Drop jump velocity for 2 ml drops as a function of Wenzel state coverage ratio ϕ , approximately fit by $U = 0.078 - 0.226\phi$ (m/s). Larger values for ϕ produce drops that do not jump. Open symbols represent ejected drop velocities; solid symbols represent rebounded drop velocities following collision of the ejected drop with the remnant drop. A drop of $\phi = 0.36$ jumps with $U \approx 0$ m/s. Larger values of ϕ do not jump. Eq. (C1) demonstrates qualitative agreement. (b) Wall-bound remnant drop volume as a function of ϕ , with approximate fit $V_w = 2.094\phi$ (ml) in the linear range $\phi < 0.32$.

FIG. 18. A 2 ml drop jumps from a Cassie-Wenzel surface with $\phi = 0.306$. At this critical condition the retarding capillary filament force is greater than the developing ejected drop inertia ($t = 0.33$ s). As the drop reverses direction towards the solid surface the rivulet ruptures (0.41 s), the separated drop impacts the remnant drop (0.76 s), rebounds, and proceeds away from the surface at a low velocity of 0.004 m/s (1.03 s). The remnant volume $V_w = 0.62$ ml for this test.

exert a force large enough to drive the eventually detaching drop back towards the solid surface. As shown in Fig. 18, and as observed in other forced droplet ejection applications (Tanguy *et al.*, 2012), in such cases the ejected drop bounces off the adhered remnant surface and continues its slow motion away from the surface. Negative velocities are recorded in such cases as shown in Fig. 17(a), but these vectors are reversed following rebound off the remnant drop surface. The rebound drop velocity is also reported in Fig. 17(a), which fits the overall linear trend. For these tests $\theta_{Cassie} = 148^\circ$ and $\theta_{Wenzel} \approx 115 \pm 5^\circ$. Extremely low, even zero, velocity drop jumps may be

FIG. 19. Simultaneous drop jump for four 0.5 ml water puddles from 3D printed surfaces of varying curvature at $t = 0.57$ s. The curvature function $\mathcal{R} \equiv R_d/R_s$ from left to right is 0.84, 0.68, 0.52, and 0.37, where $R_s = 31.3, 15.6, 10.4$, and 7.8 mm, respectively.

obtained in this manner, which may be of interest in subsequent investigations that hope to control the Weber number of impinging droplets to, say, heated surfaces.

B. Hydrophobic surface curvature, $\mathcal{R}$

Variations in surface topology at lengths on the order of the droplet length scale L_d (for $Bo \gg 1, L_d \sim (\rho V_d^2 g / \sigma)^{1/4}$; for $Bo \sim 1, L_d \sim V_d^{1/3}$; for $Bo \ll 1$, drops do not jump for our range on contact angles) can also be exploited to control drop jump velocity, drop oscillation mode, and vector. Fig. 19 provides a single frame of 0.5 ml drop jumps from 4 surfaces tested simultaneously in a drop tower experiment. These 3D printed hydrophobic surfaces increment macro-curvature as sketched below each drop in the figure, where $\mathcal{R} \equiv 1 - R_d/R_s$ (Moláček and Bush, 2012). Drop jump data for $0.5 \leq V_d \leq 2$ ml are shown for $0 \lesssim \mathcal{R} \lesssim 0.84$ in Fig. 20. The degree of reproducibility in such tests is high, typically within $\pm 3\%$ for a 2 ml drop with $\mathcal{R} = 0.5$. We identify a nearly linear dependence of drop jump velocity on $\mathcal{R}$ as shown in Fig. 20, where $U \approx 0.103\mathcal{R} + 0.012$ (in m/s) provides an adequate estimate for design purposes.

FIG. 20. Drop jump velocity U as a function of 3-D printed hydrophobic surface curvature function $\mathcal{R} \equiv 1 - R_d/R_s$ for selected drop volumes with $\theta \approx 145 \pm 10^\circ$ and hydrophobic surface spherical radius of curvature R_s . A linear relationship for $U(\mathcal{R})$ provides fair agreement for drop volumes $0.5 \leq V_d \leq 2.0$ ml for $\mathcal{R} \leq 0.85$. Predictions of U from Eq. (B2) are provided for each test point with an “x” symbol. The linearly fit slopes are in favorable agreement.

FIG. 21. Depression patterns in hydrophobic surfaces tested providing a variety of jump characteristics: oblique jumps, spinning jumps, fissions, re-combinations, impacts with rebounds, and multi-mode oscillations.

FIG. 22. Image sequence for a 10 ml puddle jump from a hydrophobic circular trough (ref. Fig. 21(c)). In this case, the toroidal rivulet jumps, splits, and sends two approximately equal volume drops away from the surface at $\pm 8^\circ$ oblique angles.

FIG. 23. Image sequence for a 9 ml puddle jump from a hydrophobic sawtooth trough (ref. Fig. 21(f)). The sawtooth segment jumps with significant inward off-mass-center velocity components that produce a spinning drop jump with $U = 0.05$ m/s.

C. Hydrophobic surface topology: Initial puddle shape

By subsequently heating and deforming the laser etched sheets over molds, it is possible to create a variety of initial puddle shapes providing further control of drop jump characteristics such as oblique jump vectors, spin, and oscillation modes. A sub-set of patterns tested is shown in Fig. 21, the results of which only three are briefly presented herein. For example, oblique jumps are displayed in Fig. 22 where a 10 ml circular loop puddle/rivulet (ref. Fig. 21(c)) jumps sending two drops at $\pm 8^\circ$ from normal angles in opposing directions. In Fig. 23, a 9 ml rivulet in a sawtooth segment trough (ref. Fig. 21(f)) spins as it jumps. The ends of the rivulet are drawn inwards colliding off-center. The drop jump velocity for this spinning drop is $U = 0.05$ m/s compared to a 9 ml drop jump from a flat hydrophobic surface which achieves $U \approx 0.090$ m/s. In Fig. 24, a 4 ml straight line segment rivulet (ref. Fig. 21(a)) jumps at $U = 0.04$ m/s with a distinctly different modal oscillation of ≈ 4.0 Hz when compared to normal 4 ml circular jump with $U = 0.10$ m/s and ≈ 2.5 Hz oscillation. Thus, by controlling the initial configuration of the drop/puddle/rivulet via hydrophobic surface topology, myriad complex interactions, oblique drops, spinning drops, and drops of varied oscillation modes may be ejected using the passive drop jump approach.

FIG. 24. Image sequence of a 4 ml rivulet puddle jump out of a hydrophobic straight segment trough (ref. Fig. 21(a)). The cylindrical drop shapes at $t = 0.33, 0.47$, and 0.60 s are essentially identical only rotated 90° . The ≈ 4 Hz non-axisymmetric mode oscillation compares to ≈ 2.5 Hz axisymmetric mode for a 4 ml planar circular puddle jump as expected.

VIII. DISCUSSION

Discrepancies of all drop jump velocity predictions with the experimental data in Figs. 8 and 9 are due primarily to the neglect of free surface distortions and viscous dissipation during and following drop jump. Best agreement is observed only for $0.5 \lesssim V_d \lesssim 2$ ml with significant discrepancies outside this range. Viscous dissipation due to the receding contact line, roll-up, and shear has been investigated for the drop impact and recoil problem by Attané *et al.* (2007) and others. For the drop jump problem, when viscous shear dominates dissipation, it may be argued that to neglect viscous dissipation the quantity $(\mu^2/\rho^2 g_o V_d)^{1/2}$ must be exceedingly small. This condition is readily achieved for the large water drops of this investigation. For large puddle volumes $2 \text{ ml} \lesssim V_d$, the missing energy in the system must therefore reside in the large distortions of the free surface at the point of separation, which are subsequently damped by viscosity during flight. As an example, in Fig. 9, the distorted surface of the 2 ml drop shown at separation increases the surface area of the jumped drop, which in turn reduces the energy available for bulk kinetic motion. By image analysis we estimate an approximately 7% over-prediction in kinetic energy (i.e., We_j) for this case, which is comparable to the 12% over-prediction observed in the figure. Higher harmonics may account for the remainder, but significant further work along these lines is straightforward and may be pursued. As observed in Fig. 8, at lower drop volumes $V_d \lesssim 0.15$ ml, discrepancies of the experimental data with the numerical models are currently attributed to stick slip dissipation at the receding contact line where the characteristic surface roughness length scale ≈ 0.5 mm (pillar spacing) is significant compared to the characteristic drop radius $R_d \lesssim 3.3$ mm.

Figures 8 and 9 display the drop tower data for drop jump velocities from primarily a single dished 3D-printed hydrophobic surface with $R_s = 0.352$ m. Flat laser-etched #4 surface data (solid square symbols) are provided only for the largest drop volumes to indicate empirical differences expected due to surface curvature as well as to illustrate visually how large these low-gravity “droplets” can be relative to terrestrially tested droplets. Flat surface tests are increasingly difficult as drop volumes increase due to the degree of level required prior to each drop tower test. Large drops require slightly dished surfaces to prevent the drops from rolling away. However, alternative methods to establish symmetric circular puddles on flat hydrophobic surfaces may be pursued enabling comparable plots to Figs. 8 and 9 for flat surfaces and volumes up to approximately 400 ml.

IX. SUMMARY

The drop jump method exploiting a drop tower provides a simple no-moving-parts deployment method for large droplet dynamics investigations. For sufficiently hydrophobic Cassie states ($\theta \geq 130^\circ$), we show drop volume limits $0.04 \lesssim V_d \lesssim 400$ ml and drop velocity limits $-0.007 \lesssim U \lesssim 0.12$ m/s for water in a 2.1 s drop tower. Zero velocity ejections may be achieved. We show adequate predictions of minimum drop jump volume $V_{dmin} \sim (\sigma/\rho g_o)^{3/2} \approx 0.02$ ml, Eq. (1), maximum drop volume $V_{dmax} \sim \sigma t_{dt}^2/\rho \approx 300$ ml, Eq. (4), and certain large drop jump velocity limits $U \leq (4\sigma g_o/\rho)^{1/4} \approx 0.23$ m/s, Eq. (7). Experimental drop jump velocities are lower due to increasing dissipation during the roll-up, geysers, drop break-up, and free drop fundamental and higher harmonic capillary wave regimes. Geysers are observed for a/symmetric puddles of volumes $6 \lesssim V_d \lesssim 110$ ml. For system design purposes, for drop volumes $0.04 \leq V_d \leq 400$ ml we find

the time for drop jump t_j adequately obeys the scale relationship $t_j \approx (\rho V_d / \sigma)^{1/2}$, Eq. (2). For $0.5 \lesssim V_d \lesssim 2$ ml the simple equilibrium model over-predicts U by $\lesssim 15\%$, Eq. (3), while a numerical equilibrium model that takes greater account of local initial interface curvature over-predicts U by $\lesssim 10\%$, Eq. (B2).

In general, the jumped droplets/puddles are enormous when compared to normal terrestrial droplet investigations; over 50-fold larger when comparing drop diameter, and over 100 000-fold larger comparing drop volume. For such uniquely large capillary fluidic bodies, the drop tower approach identifies a path for further research investigations. The drop jump velocity is not only controlled by drop volume. In this work we show that contact angle, wettability pattern, and hydrophobic surface curvature can be varied providing additional passive control of the drop jump velocity and dynamics. We find that drop jump is common for Cassie states when $\theta > 130^\circ$ with little impact of increased contact angles in the range $150^\circ \lesssim \theta \leq 180^\circ$. By control over initial puddle shape, we demonstrate a variety of drop oscillation modes, oblique jumps, and spinning jumps. We provide simple expressions for drop jump as a function of initially combined Wenzel-Cassie states, Eq. (C1), Fig. 17, and curved hydrophobic surface, Eq. (B1), Fig. 20. We also demonstrate a fast static equilibrium numerical tool for accurate non-wetting contact angle measurement as well as a means of approximating maximum drop jump velocities while precisely determining energy losses during the process.

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APPENDIX A: EQUILIBRIUM NUMERICAL METHOD

The SE-FIT software (Chen *et al.*, 2011) which runs the *Surface Evolver* algorithm (Brakke, 1992) is used as an efficient static equilibrium numerical tool to predict puddle jump velocity $\tilde{U}$ in the manner described in Section V. Predictions of $\tilde{U}(V_d)$ and $\tilde{W}_{ej}(V_d)$ are provided in Figs. 8 and 9 for both flat and dished hydrophobic surfaces. The numerical results are more accurate than the simple analytical results because no simplifications to the initial 1- g_o interface state and resulting surface energies are necessary. However, the numerical solution still expectedly over-predicts the experiments, the difference serving as a quick and quantitative measure of viscous dissipation and transient free surface creation during the drop jump process. Typical runtimes are less than 120 s/pt using an Intel Xeon 2.26 GHz 4-core desktop computer.

APPENDIX B: SIMPLE ANALYTIC MODEL FOR DISHED SURFACE PUDDLE JUMP VELOCITY, $\tilde{U}_{dished}$

The SE-FIT numerical method is applied in Figs. 8 and 9 for both flat and dished hydrophobic surfaces, as shown schematically in Figs. 11 and 25, respectively. The simple analytic model of $KE = SE_1 - SE_2$ applied to the latter yields

$$\tilde{U}_{dished} = \left[\frac{2\sigma}{\rho V_d} (SE_1 - SE_2) \right]^{1/2} \equiv \left(\frac{\sigma \tilde{W}_{e_{dished}}}{\rho R_d} \right)^{1/2}, \quad (B1)$$

where $SE_1 = 2\pi h R_s (1 - \cos \theta) - \pi h^2 + 2\pi H (2h R_s - h^2)^{1/2}$ and $SE_2 = (6\pi^{1/2} V_d)^{2/3}$ with

$$h = R_s - H - 2(R_s^2 + H^2)^{1/2} \sin(\pi/6 - \beta)$$

and

$$\beta = \frac{1}{3} \tan^{-1} \left(\frac{(4\pi^2 H^2 R_s^2 (3R_s^2 + 2H R_s + 3H^2) + 12\pi V_d (R_s^3 - H^3) - 9V_d^2)^{1/2}}{2\pi R_s^3 - 2\pi H^3 - 3V_d} \right).$$

FIG. 25. Simplified large drop jump model on a spherically “dished” surface of radius R_s : (a) Actual initial condition with capillary height $\sim H$ and dished surface depth h_s , (b) state 1 idealized cylindrical disc initial condition with contact angle θ along the solid-liquid surface where $R_1 = R_p$, and (c) state 2 final zero-gravity rigid body spherical configuration traveling at predicted velocity $\tilde{U}$, where $R_2 = R_d$.

A less cumbersome form of Eq. (B1) is found for large dish radius limit $h/R_s \ll 1$, which yields

$$\tilde{U}_{dished} \approx \left[\frac{4\pi\sigma H}{\rho V_d J} \left(1 - \cos\theta + 2J^{1/2} - J \left(\frac{9}{2} \right)^{1/3} \left(\frac{V_d}{\pi H^3} \right)^{1/3} \right) \right]^{1/2} \equiv \left(\frac{\sigma \tilde{W} e_{dished}}{\rho R_d} \right)^{1/2}, \quad (\text{B2})$$

where $J \equiv H/[R_s((1 + V_d/\pi H^2 R_s)^{1/2} - 1)]$. In reference to Figs. 25(b) and 25(c), in the order left to right, the terms in the parentheses of Eq. (B2) represent the energies of the top surface area of the disk drop, the weak spherical cap bottom surface of the drop, the disk side walls, and the jumped spherical drop surface, respectively. As observed in Figs. 8 and 9, Eqs. (B1) and (B2) using the experimental value $R_s = 0.352$ m agree well with the equilibrium numerical solutions for large drop volume. We note also that Eqs. (B1) and (B2) recover the flat surface result of Eq. (5) in the large puddle limit $(\pi H^3/V_d)^{1/3} \ll 1$ when hydrophobic surface curvature is small, $V_d/\pi H^2 R_s \ll 1$. Unlike Eq. (B2), Eq. (B1) is not limited to slightly dished hydrophobic surfaces and large puddles, making it appropriate for comparisons to the highly curved surface drop jump tests such as those presented in Fig. 20. Eq. (B1) is overlaid on the plot where it identifies the correct qualitative behavior, the differences again being a quantitative measure of energy loss during the process.

APPENDIX C: SIMPLE ANALYTIC MODEL FOR WENZEL-CASSIE PUDDLE JUMP VELOCITY, $\tilde{U}_{w/c}$

As sketched in Fig. 26, if we apply this same simple puddle jump energy approach to the Wenzel-Cassie wettability patterns of Fig. 15 we find

FIG. 26. Simplified large drop jump model for Cassie-Wenzel drop jump: (a) Actual initial condition with Wenzel surface identified, (b) state 1 idealized cylindrical disc initial condition, and (c) state 2 final zero-gravity rigid body spherical drop jump configuration traveling at velocity $\tilde{U}_{w/c}$, where $R_2 = R_d$. The remnant drop volume V_w resides on the Wenzel region with θ_w .

$$\tilde{U}_{w/c} = \left(\frac{\sigma g}{\rho} \right)^{1/4} \left[1 - (1 - \phi) \cos \theta_c + 2 \left(\frac{\phi}{\psi} \right)^{1/2} - 6^{2/3} \left(\frac{\phi}{\psi} \right)^{1/3} \left(1 - \frac{f_2 \phi \psi^{1/2}}{3} \right)^{2/3} - 2 f_1 \phi \right]^{1/2}, \quad (C1)$$

where $\phi \equiv (R_w/R_p)^2$, $\psi \equiv (R_w/H)^2$, θ_c is the Cassie state contact angle, θ_w is the effective Wenzel state contact angle, and f_1 and f_2 are dimensionless geometric functions characterizing the remnant drop free surface area and volume, respectively,

$$f_1 \equiv \frac{1 - \cos \theta_w}{\sin^2 \theta_w} \quad \text{and} \quad f_2 \equiv \frac{(1 - \cos \theta_w)^3 (2 + \cos \theta_w)}{\sin^3 \theta_w}. \quad (C2)$$

In reference to Figs. 15, 26(b), and 26(c), in the order left to right, the terms in brackets in Eq. (C1) represent the energies of the top surface area of the disk drop, the effective Cassie-Wenzel state bottom surface of the disk drop, the disk side walls, the reduced spherical drop surface, and the remnant drop surfaces, respectively. In the large puddle limit $(\pi H^3/V_d)^{1/3} = (\phi/\psi)^{1/3} \ll 1$, and assuming $\theta_c \geq 150^\circ$ and $\theta_w \approx 90^\circ$, Eq. (B2) reduces to

$$\tilde{U}_{w/c} \approx \left(\frac{4\sigma g}{\rho} \right)^{1/4} \left(1 - \frac{\phi}{2} - \phi \cos \theta_w \right)^{1/2} \approx \left(\frac{4\sigma g}{\rho} \right)^{1/4} \left(1 - \frac{\phi}{4} + \dots \right), \quad (C3)$$

the right hand side of which anticipates a negative linear slope form in Fig. 17(a) for small ϕ . Eq. (C1) is added to Fig. 17(a), where it is again only helpful in a qualitative sense to predict U , while in a quantitative sense to predict dissipation.

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