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Dialectics and Catastrophe[1]

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1. Introduction

The Catastrophe Theory of René Thom and E. C. Zeeman' suggests a mathematical interpretation of certain aspects of Hegelian and Marxist dialectics.[2] Specifically, the three 'classical' dialectical principles[2], (1) the transformation of quantity into quality, (2) the unity and struggle of opposites, and (3) the negation of negation, can be modeled with the seven 'elementary catastrophes'[3] given by Thom, especially the catastrophes known as the 'cusp' and the 'butterfly'. Far from being empty metaphysics or scholasticism, as critics have argued, the dialectical principles embody genuine insights into a class of phenomena, insights which can now be expressed within a precise mathematical formalism. This fact does not, however, support the claim that these principles, possibly modified or supplemented, constitute the laws of motion for human thought and for natural and social processes - or even just the last of these.

There is, of course, an enormous and diverse literature on dialectics. The three Hegelian 'laws' will be focused upon, somewhat arbitrarily, because they offer a clear context for discussion. These laws are certainly only one particular reification of a more general framework for analysis and synthesis, which has been extensively developed by writers and philosophers associated with the Communist movement[4], the independent left, and the academic research community. This paper will give only a preliminary demonstration of the close relationship which exists between dialectics and catastrophe theory. A more systematic examination of the dialectical literature from a catastrophe theoretic perspective will be undertaken at a later date.

Some remarks are in order on the nature of catastrophe theory. This theory can be used in a variety of ways ranging from (a) rigorous applications, where the underlying assumptions of the theory can be validated, and where quantitative explanation and prediction is sought, to (b) cases where catastrophe models are invoked a *priori*, but are empirically and quantitatively assessed, to (c) more qualitative modeling of phenomena using the catastrophe archetypes (with perhaps an aspiration to future quantitative treatment), to (d) purely symbolic or metaphoric use of the visual imagery of the theory. Most of the discussion in this paper will be at the qualitative end of this spectrum, but more mathematical aspects of Thom's theory are also relevant. For example, phase transitions, such as the boiling or freezing of liquids, are frequently cited in the Marxist literature as examples of dialectical phenomena, and these phenomena can be described by catastrophe theory (Dodson, 1976; Schulman and Revzen, 1972) at quite a rigorous and quantitative level of analysis.

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[1] *Sociocybernetics*, F. Geyer and J, van der Zouwen, eds., Martinus Nijhoff, The Hague, The Netherlands, 1978, pp. 129-154.

[2] *On posting this paper to the web in 2009*: This paper was written in the 1970s and was influenced by its zeitgeist. I would write a different paper today, but much in it remains useful, especially its demonstration that catastrophe theory can formalize some aspects of two types of dialectical processes.

In the next section, a brief account of catastrophe theory is given by introducing the cusp; the most widely used of the catastrophe types. An interpretation of the three classical dialectical laws is developed in terms of the cusp (Section 3), and applied to the writings of Marx and Engels on political economy and history (Section 4). Here it is necessary to acknowledge in advance - indeed to stress - that the catastrophe-theoretic interpretation does not add new content to the Marxian analyses, but merely highlights their underlying coherence, via a rich system of visual metaphor. That is, like dialectics, catastrophe theory provides a language for modeling and a method of exposition. In Section 5, the butterfly catastrophe is introduced and used to describe a dialectic different from that of the cusp, one in which the struggle of opposites can lead to the creation or dissolution of an independent synthesis. Finally, Section 6 gives a summary and indicates directions for future work.

2. The cusp catastrophe

Thom's theory is about transitions from continuous cause-effect relationships to discontinuous ones. Specifically, the theory describes seven ways (‘elementary catastrophes’) in which either one or two ‘behavior’ variables (effects) can change discontinuously as a result of continuous variation of up to four ‘control’ parameters (causes)[5]. The seven catastrophe types, in order of increasing complexity, are: fold, cusp, swallowtail, butterfly, hyperbolic umbilic, elliptic umbilic, and parabolic umbilic. This paper will consider the cusp and butterfly (Figures 1 and 4), which offer fundamentally different interpretations of dialectics. The other elementary catastrophes may also model dialectical phenomena, but will not be discussed.

In the cusp catastrophe, there are one behavior variable and two control parameters whose relationship can be modeled with the ‘catastrophe machine’ (Zeeman, 1976, 1977) shown in Figure la.

In this model, two rubber bands are attached to a disk which can rotate about its center. The other end of the first rubber band is fixed while the second has a free end (the ‘control point’) which can move about in the plane of the figure. The rotation angle of the disk (x) is the behavior variable, and the control parameters are the coordinates (a, b) of the control point. If this point is moved across the cusp-shaped ‘bifurcation set’ from point 1 to point 7, the disk angle will change gradually until point 6, where it will undergo a discontinuous rotational jump (‘catastrophe’).

Figure 1b gives one interpretation of what is happening: The system is assumed always to be at equilibrium in a local minimum (initially, α) of some energy function. As the control point moves, the topography of the energy function changes. At point 2, i.e., when the control point first enters the bifurcation set[6], an inflection point appears which, at 3, deepens into a second minimum, β. This minimum corresponds to a different equilibrium value for the angular variable, x. At point 4, the depth of the two minima are equal, but at 5, the second state is actually preferred, i.e. if one agitated the disk sufficiently, it would jump from α to β. But if the control point moves gradually, the system does not switch to the new state until its own local minimum disappears (point 6).

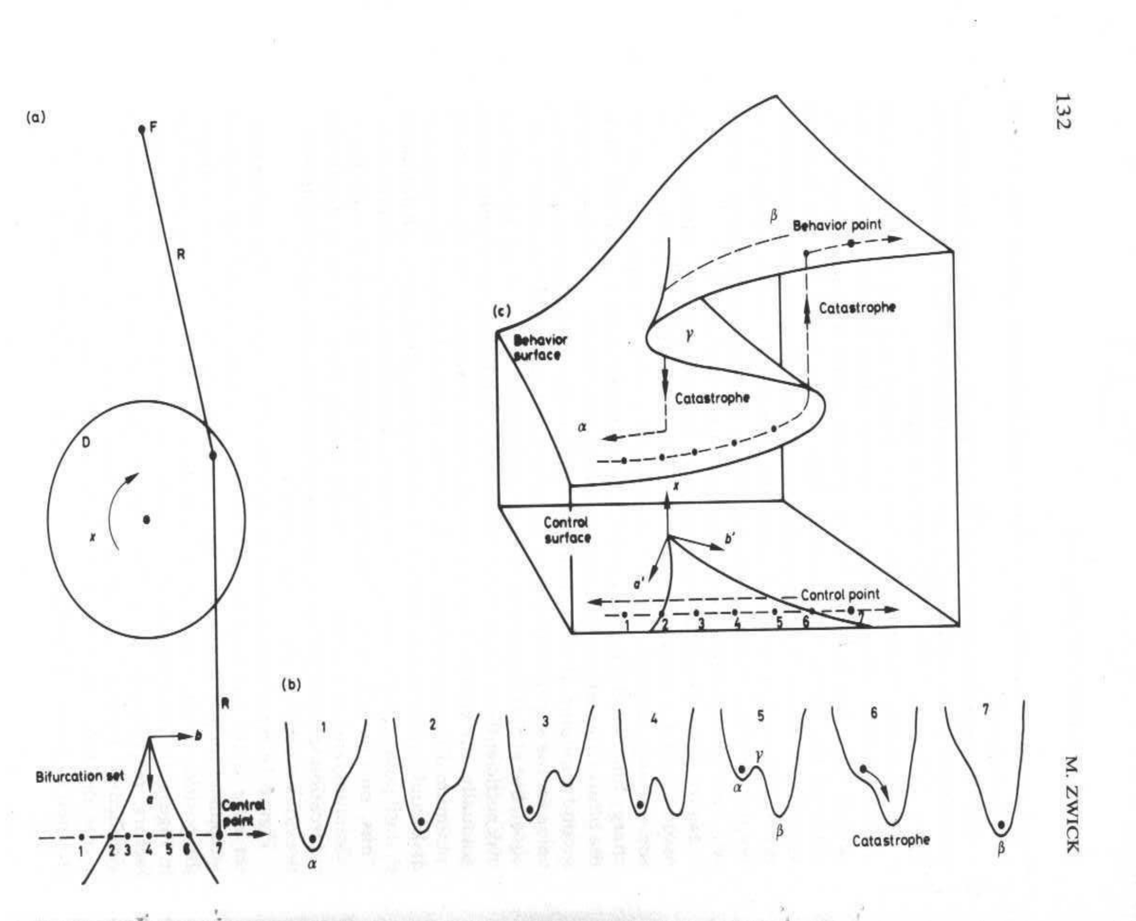

Figure 1. A model of the cusp catastrophe (Zeeman, 1976, 1977).*

Figure la. A ‘catastrophe machine.’ F is a fixed point; D, a disk; R, a rubber band; x, the behavior variable; a and b are splitting and normal factor coordinate axes which specify the location of the control point. As the control point crosses the bifurcation set, the rotation angle of the disk changes discontinuously (a ‘catastrophe’).

Figure 1b. Changes in the energy function for the motion of the control point shown in la. α and β are energy minima, γ an energy maximum; inflection points occur at 2 and 6.

Figure 1c. The control and behavior surfaces. As the control point moves across the bifurcation set (same as in la) on the control surface, the behavior point follows above it on the behavior surface. The upper and lower sheets of the behavior surface correspond to the α and β minima; the central sheet is inaccessible to the behavior point and corresponds to γ. The height of the behavior point specifies the rotation angle, x. See Section 2 for additional discussion.

*For Figures 1-4, dashed arrows are control point trajectories. Double arrows indicate rapid motion. Solid arrows specify coordinate axes.

If one reversed the direction of motion, the disk would rotate slowly, but remain in the range of values we call minimum β, until point 2, where it would undergo a discontinuous flip to minimum α. Thus, on one side of the bifurcation set (point 1) there is only the α minimum, and on the other side (point 7) only the β minimum. Inside the bifurcation set, the system can occupy either minimum, depending upon the previous motion of the control point.

Figure 1c represents additional aspects of this process. The bifurcation set here is the same as the one shown in Figure la, but the diagram introduces an alternative coordinate system which may be used to specify the position of the control point. (For the coordinate system shown earlier in Figure la, the parameters a' and b' are called 'splitting' and 'normal' factors, respectively. In the present arrangement, a' and b' are called 'conflicting' factors.) As the control point moves on a 'control surface' across the bifurcation set, a 'behavior point' moves along with it, constrained to lie on the 'behavior surface' directly above the control point.[7]

The vertical height of the behavior point gives the value of the behavior variable, x, and the catastrophe which occurs at point 6 corresponds to this point jumping from the lower sheet onto the upper. (For motion in the reverse direction, from 7 to 1, it jumps downward, as shown.) Note that the intermediate region (γ) in the S-shaped fold is inaccessible; this corresponds to the local maxima shown in Figure1b. Also, a path which does not actually cross the bifurcation set, or which enters and leaves this region at the same boundary, will not result in any discontinuity.

The occurrence of the jump at different points for forward and reverse paths is an illustration of the property of '*hysteresis*'. The existence inside the bifurcation set of two possible equilibrium states (or, more generally, the split of the behavior surface into upper and lower sheets) is referred to as '*bimodality*'. The fact that the system does not change its state immediately after point 4, even though minimum β is favored, and does so only when it crosses the other boundary of the bifurcation set, is called '*delay*'. The vertex of the cusp in the control surface, and the corresponding point at which the behavior surface bifurcates, are known 'singularities'. Very small changes in the motion of the behavior point near the singularity can lead to passage onto either the lower or upper surface; this is known as '*divergence*'.

It should be stressed that the path of the control point is in no way dictated by the theory, but must be provided anew for each phenomenon being modeled. Also a coordinate system for the control point must be chosen. The reader might glance ahead to Figures 3a and 3b to see some other possible control point trajectories. The first of these uses conflicting control factors, each favoring one particular equilibrium state. (The bifurcation set is then an arena of conflict.) The second uses normal and splitting factors. The first parameter effectively determines which state the system is in, while the second specifies the separation between these states i.e. the gap between the upper and lower behavior surfaces.

Actually, the properties of 'delay', and the discontinuity of the transition between the two possible minima, are not strictly required in the cusp catastrophe, even when the bifurcation set is completely traversed. Some systems may undergo a transition at or close to the moment when a deeper minimum appears (e.g. immediately beyond point 4 in Figure 1). In such cases the transition occurs inside the bifurcation set, and this is known as the 'Maxwell' convention. Also if the system consists of many units, each of which can undergo an independent transition between the two states, then the population of units, taken collectively, can exhibit a smooth transition curve. This would resemble a hypothetical path parallel to the one shown in Figure 1, but ‘behind’ the singularity, i.e. before the behavior surface has bifurcated into two sheets.

These are the essentials of the cusp catastrophe. It might be instructive to cite a few of the applications which have been made of this model. The phenomenon of phase transitions has already been mentioned as a standard example of dialectics, given by Hegel, Engels, and most, if not all, subsequent Marxist writers on this subject. Here temperature and pressure are conflicting factors. The phenomenon exhibits delay in transitions from supersaturated states, but more typically follows the Maxwellian mode. Other uses or illustrations of the cusp, with varying degrees of mathematical and empirical elaboration, have included the analysis of the nerve impulse, the heartbeat, stock market cycles, embryological differentiation, Euler buckling, military conflict and peacemaking, neurological and physiological rhythms, light caustics, and so on (Zeeman, 1976, 1977).

3. The dialectical laws

3.1. Quantity and quality

The relationship of the cusp catastrophe to the classic laws of dialectics summarized in Figure 2.

The first law, ‘the transformation of quantity to quality,’ is close to, though not exactly synonymous with, the generation of discontinuous effects from continuous causes. The two behavioral modes of the cusp are usually qualitatively distinct, as in the phase transition example, where the two regions of density correspond to the gas and liquid states. From a dialectical perspective, change of the quantitative nature, involving ‘mere’ increase or decrease, which does not alter the basis character of the system cannot go on indefinitely, but at a certain point (Hegel's ‘nodal line’), always leads to a qualitative transformation (or ‘leap’). Water, when heated, does not go on getting hotter and hotter indefinitely, but at a certain critical temperature, begins to turn into steam, and undergoes a qualitative change from liquid to gas. The dialectical description is identical to the catastrophe theoretic one. Hegel's ‘nodal line’ is the bifurcation set, and his ‘leap’ is the catastrophe.

The transformation of quantity into quality can also be seen in the cusp property of divergence, wherein small quantitative differences in the path of the control point are amplified and yield qualitatively different results. Evolutionary speciation, a phenomenon which exhibits this property, has recently been the subject of catastrophe theoretic study (Dodson, 1976; Waddington, 1974). And, as Graham notes (1971):

> ‘To Marx and Engels, Darwin’s theory of evolution was an important illustration of the principle of the transition from quantity to quality. This tenet as a part of the Hegelian dialectic preceded Darwin, of course, but Marx and Engels considered Darwinism a vindication of the dialectical process. In the course of natural selection, different species developed from common ancestors; this transition could be considered an example of accumulated quantitative changes resulting in a qualitative change, the latter change being marked by the moment when the diverging groups could no longer interbreed.’

3.2. Interpenetration of opposites

In dialectics, the concept of ‘development’ is distinguished from that of growth to indicate that real change is not a smooth process, but one in which phases of gradual evolution are interrupted by breaks in continuity. This development is said to take place through the unity and struggle - or to use Engel's original formulation, through the mutual interpenetration - of opposites. Linked opposites appear in the cusp model in several ways (Figure 2b):

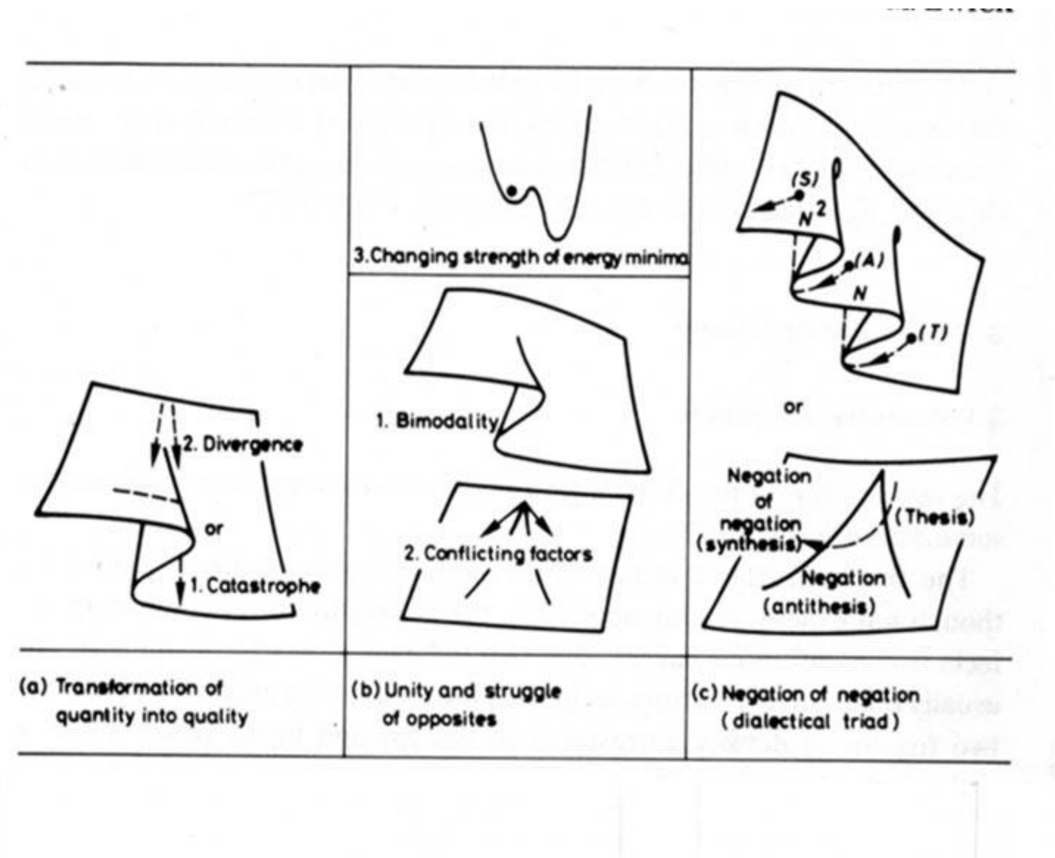

Figure 2. The three dialectical laws interpreted in terms of the cusp.

Figure 2a. The transformation of quantity into quality is illustrated in the property of catastrophe or that of divergence, though some phenomena may exhibit both of these.

Figure 2b. The unity and struggle of opposites is manifest in the three linked properties which are shown, except that control factors may sometimes be given in terms of normal and splitting factors.

Figure 2c. The negation of negation may be given the first or the second interpretation above. (The lower diagram is not the control surface for the upper.) See Section 3 for additional discussion.

(1) The behavior surface is bimodal beyond the point of singularity, and the two behavioral possibilities overlap inside the region of the bifurcation set. Also, the inaccessibility of the central surface (γ in Figure 1c) indicates the impossibility of compromise (intermediate values of the behavior variable) between the two modes.

(2) The control parameters, when given as conflicting factors (e.g. temperature and pressure in the phase transition case) oppose one another but act jointly on the system. If one factor is dominant or exclusively present, it will lead the system into one or the other of the alternate states. Since the regions of dominance overlap, if both factors are approximately in balance, there is a struggle of opposites.

(3) Inside the bifurcation set, the potential function has two minima, which arise from the interpenetration of the domains of influence of the conflicting factors. One minimum corresponds to the actual state of the system, the other to a potential alternative state.[8] In the changing relative strength of the two minima, there is the struggle between the old which is dying away or disappearing and the new which is being born or developing. In the reversal of the relations of dominance between the opposites, one quality comes to replace another.[9]

This developmental process is actually the 'internal content' of the transformation of quantity into quality, i.e. after a long series of gradual changes, the victory of the new over the old occurs suddenly and discontinuously. (Actually, both dialectics and catastrophe theory allow also for qualitative change which is continuous but at least more rapid than the events which lead up to it.) After the qualitative change or catastrophic ‘leap’, the process continues to unfold and to complete itself, according to the detailed circumstances of the situation.

3.3. Negation of negation

The third dialectical law can be given at least two different interpretations in terms of the cusp (Figure 2c). (1) The process continues beyond the first transformation, i.e. each negation represents a qualitative leap. This can be represented by concatenating two or more cusps. (If they are connected in a spiral pattern, this will also suggest the idea that the second negation restores some earlier condition, but at a higher level.)

(2) The negation of negation can also be seen within a single cusp in the following sequence. Initially, there is a single potential minimum, and corresponding behavioral mode, and the uncontested dominance of one of the conflicting factors. Upon entry into the bifurcation set, this condition is negated, and replaced with contradiction, bimodality, and strife. Finally, the struggle of opposites culminates in a qualitative jump: the system experiences the second negation, unimodality is restored - but in the new state.

Here also is an interpretation of the familiar triad of thesis, antithesis, and synthesis, which is closely related to the principle of negation of negation. The triad is not a prominent feature of the dialectics of Engels and Marx, who mocked its 'wooden' uses, but it is found, at least implicitly, in their writings. It can be given two possible interpretations. In the first, thesis, antithesis, and synthesis are the three surfaces of two concatenated cusps. In the second, the region outside the bifurcation set (prior to entry into it) is the thesis, the region inside it is the antithesis, i.e. the domain of contradiction, and the region on the other side of it (into which the control point emerges, causing the ‘leap’)is the synthesis.

4. The Cusp of contradiction

4.1. Dialectics of capitalism

The cusp interpretation of the dialectical principles can be illustrated with some examples selected from (or related to) the writings of Marx and Engels on political economy (Engels, 1941, 1968; Marx, 1904, 1947, 1967, 1968).

Generally any system which undergoes a discontinuous change between one stable state and another can be modeled with the cusp, so we can apply it to the transition between systems of production, and specifically to the transition from capitalism to socialism which Marx predicted and worked towards. There are two ways in which this transition is described. One might be called the 'deep structure' of the process, and is cast in terms of an underlying dialectic between forces and relations of production. The second provides the 'surface structure,' i.e. what is plainly visible. Here the focus is on the actual process by which the relations of production are altered, and on the agents of historical change, the social classes.

The struggle between the principal opposing classes can obviously be represented on the cusp, as shown in Figure 3a, where the strength of the bourgeoisie and the proletariat are the conflicting control factors. The path of the control point in this figure approximately characterizes the dialectical conception of how systems develop and become transformed. First, one control variable increases and establishes its dominance. This process then gives rise inevitably to a growth in strength of an opposing factor, and simultaneously to a gradual slowing in the increase of the first. That is, the success of the system leads invariably, first to the emergence and then to the intensification of inner contradictions. Finally, the second factor, via a catastrophic jump, achieves dominance, and the system is transformed.

The analysis given by Turner (1974) of ‘the dialectical causal imagery’ gives one possible account of the sequence of events through which this process passes. The stages are approximately as follows:

(1) An initial form of social organization; in the present case, the emergence of the capitalist property relations;

(2) Domination of the propertied social classes over other classes;

(3) Objective opposition of interests between classes over distribution of property and power;

(4) Consciousness of this opposition of interests by the dominated class;

(5) Politicization of the subjugated population and increased tension;

(6) Revolutionary conflict;

(7) Social reorganization and the redistribution of property and power.

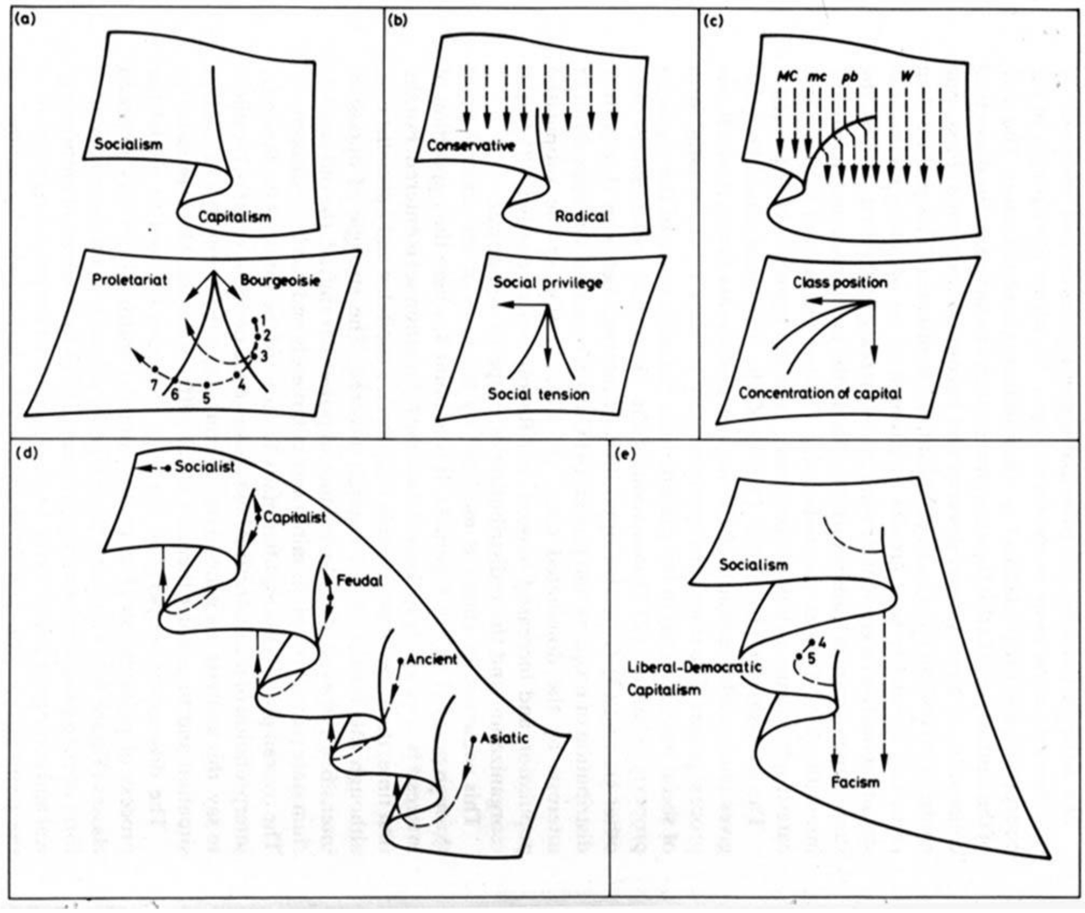

Figure 3. Some Marxian applications of the cusp.

Figure 3a. Class struggle leading to socialism represented as a trajectory on the control surface. Conflicting factors are the strengths of the bourgeois and working classes. The behavior variable is some of the socialization of production. The control point path with the smaller are shows an unlikely yet conceivable alternative trajectory.

Figure 3b. Polarization. Political orientation, the behavior variable, becomes increasingly polarized along lines of power and privilege as social tension grows (e.g. point 5 in the trajectory shown in 3a.)

Figure 3c. Proletarianization of the petty bourgeois (pb) and minor capitalists (mc) with the concentration of capital and expansion of monopoly. MC represents major capitalists; W, workers. Similar to the polarization shown in 3b, but altered cusp geometry gives a shifting boundary between the dominant and subordinate classes.

Figure 3d. A modified and highly idealized Marxian view of historical stages, showing mainly discontinuous changes between systems of production, but the possibility also of alternative routes.

Figure 3e. The secondary cusp of fascism. Entry into the bifurcation set of the capitalism-socialism transition, e.g. point 4 and 5 in the trajectory of 3a, can trigger a strong reaction. This can generate a new topological singularity, with a 'secondary' cusp which splits the capitalism surface into liberal democratic and fascist alternatives. Reversal of the control point motion, either before or after socialism is reached, may cause a sudden transition not to democratic capitalism but to fascism. See discussion in Section 4.

This sequence of stages is modeled by the path of the control point through points 1 to 7 in Figure 3a. It is at point 4, where the opposition of interests is consciously perceived, that the bifurcation set is entered. For the first time, there exist two possible equilibria, a capitalist and a socialist one, although the former is still strongly favored. The struggle of opposites intensifies, but even after the balance of power has shifted, the old equilibrium state persists, until ‘its minimum' completely and suddenly disappears. The overall process exemplifies many features of the catastrophe theoretic interpretation of the dialectical laws, given in the previous section. Needless to say, this analysis is a general one; it can be applied to any dyadic conflict situation and to conflict theories which differ from the Marxian model.

The divergence property of the cusp may also be used to model the process of polarization (Figure 3b), and the shifting boundary between classes (Figure 3c). This latter figure makes use of a slightly more complex form of cusp geometry, used by Zeeman (1977) for analysis of embryological and ecological phenomena. Marx's account of the proletarianization of the petty-bourgeoisie and of the inevitable decrease in the number of capitalists is simply pictured. The figure almost suggests the surprise of those, who, conceiving of themselves among the privileged, suddenly find the reality of their economic position to be otherwise.

In the deep-structure account of the transition between systems of production, the development of the forces of production of society at a given stage leads invariably to the emergence of contradictions between these forces and the social relations through which production is organized. These relations, which initially promote the growth of the productive forces, become outmoded, and block their further development; the contradiction is finally resolved in a ‘synthesis’ in which the system is restructured according to a new and more advanced pattern of social relations. The use of the cusp to model this level of analysis is not obvious, but the forces of production and the capitalist relations which organize them might also be taken as conflicting factors.

The development of the former, past a certain point, tends, according to Marx, to favor a socialist system, and is linked to the growing strength of the working class, while the latter acts as a conservative factor on behalf of the capitalist class and system of production. A trajectory similar to the one shown in Figure 3a would symbolize how, after the demise of the feudal order, the emergence of capitalist productive relations promotes the growth of the productive forces, which in turn later outstrips and/or weakens these relations. Again, the region inside the bifurcation set represents the inner contradiction which is generated between the productive forces and relations, and the simultaneous existence of both an actual equilibrium state of capitalism and a potential one of socialism. This use of the cusp is less compelling than its earlier application to the struggle of opposing classes, but perhaps these two conceptions can be merged by speaking of the conflicting factors as ‘private appropriation’ versus ‘social production.’

4.2. Progress and retrogression

As the cusp may be used to represent the transition between systems of production, the historical stages of Marx, from the Asiatic to the Socialist (or Communist), can be shown by concatenating cusps, as in Figure 3d, thereby also illustrating successive instances of the principle of the negation of negation. The figure also suggests the possibility of moments of choice and alternative routes. The motion of the control point is not fixed by the theory, (Thom's or, for that matter, Marx's), and though the general path given in Figure 3a may be the ‘ideal type’ for dialectical development, deviations are to be expected. The result of the struggle of opposites is said to be, at best, only generally predictable; in any specific case, one cannot foresee the course of events.

A variety of such deviations is possible. Indeed in Turner's analysis mentioned earlier, transitions from stages 1 to 2, and 2 to 3 are, for Marx, relatively unproblematic, but transitions from stages 3 to 4, 4 to 5, and 5 to 6 are subject to 'intervening empirical conditions.' Entry to the cusp region is not actually guaranteed, since those factors which are necessary for the opposition of interests to become consciously perceived may be absent. The working class may simply organize to secure better working conditions but not develop class consciousness, in which case the system will remain outside the bifurcation set, and the possibility of an alternative equilibrium state will not come into existence. It is also conceivable that after entering the bifurcation set, the control point might retrace its steps as the result of internal and/or external influences. A feedback relation may exist in the region of the bifurcation set between the energy function, which represents the relative strength or probability of the actual and potential systems states, and the motion of the control point. That is, recognition of the existence of the new but unrealized equilibrium possibility may strengthen one or the other of the contending parties (or both). Thus, motion towards the catastrophe-generating boundary may be accelerated, or, to the contrary, retarded - or even reversed.

But a system which went far in the direction of change, and was forced backwards towards its initial state, would be unlikely to look as it had in the beginning. Entry into the bifurcation set of the capitalism-socialism transition may induce a 'secondary' cusp to form (Figure 3e) in the 'capitalist surface' to produce a new pair of alternatives: a capitalist system with liberal democratic politics or one coupled to a fascist regime. The sensed direction of motion of the system towards the new potential minimum provokes a reaction, which, if sufficiently powerful, generates the second topological singularity, and a new potential equilibrium state in the opposite direction. Now, reversal of the direction of motion of the control point and/or the erosion - or possibly the disappearance - of the equilibrium surface of democratic capitalism can lead, via a discontinuous change, to fascism. So, too, can the reversal of a transition to socialism which has already been accomplished.

Other variations are possible. There may be random influences, either internal or external to the system, which affect the motion of the control point. For example, where the forces and relations of production are taken as the conflicting factors, a sudden drop in the strength of the capitalist relations, e.g. due to war, may trigger an early catastrophe, even when neither the productive relations nor the productive forces are well developed (alternative trajectory shown dotted in Figure 3a). The catastrophe theoretic model is only a skeletal framework for analysis and exposition; one may add to it additional features such as feedback, random fluctuations, and so on.

5. The butterfly of reconciliation

5.1. The butterfly catastrophe

The butterfly catastrophe is more complex than the cusp, having four control parameters (a, b, c, d) which affect a single behavioral variable. The new control variables, c and d, called the '*bias*' and '*butterfly'* factors, are discussed below. As before, the first two parameters may be chosen either as conflicting or as splitting and normal factors. It is impossible to represent this form completely in two dimensions, so Figure 4a shows selected two-dimensional sections.

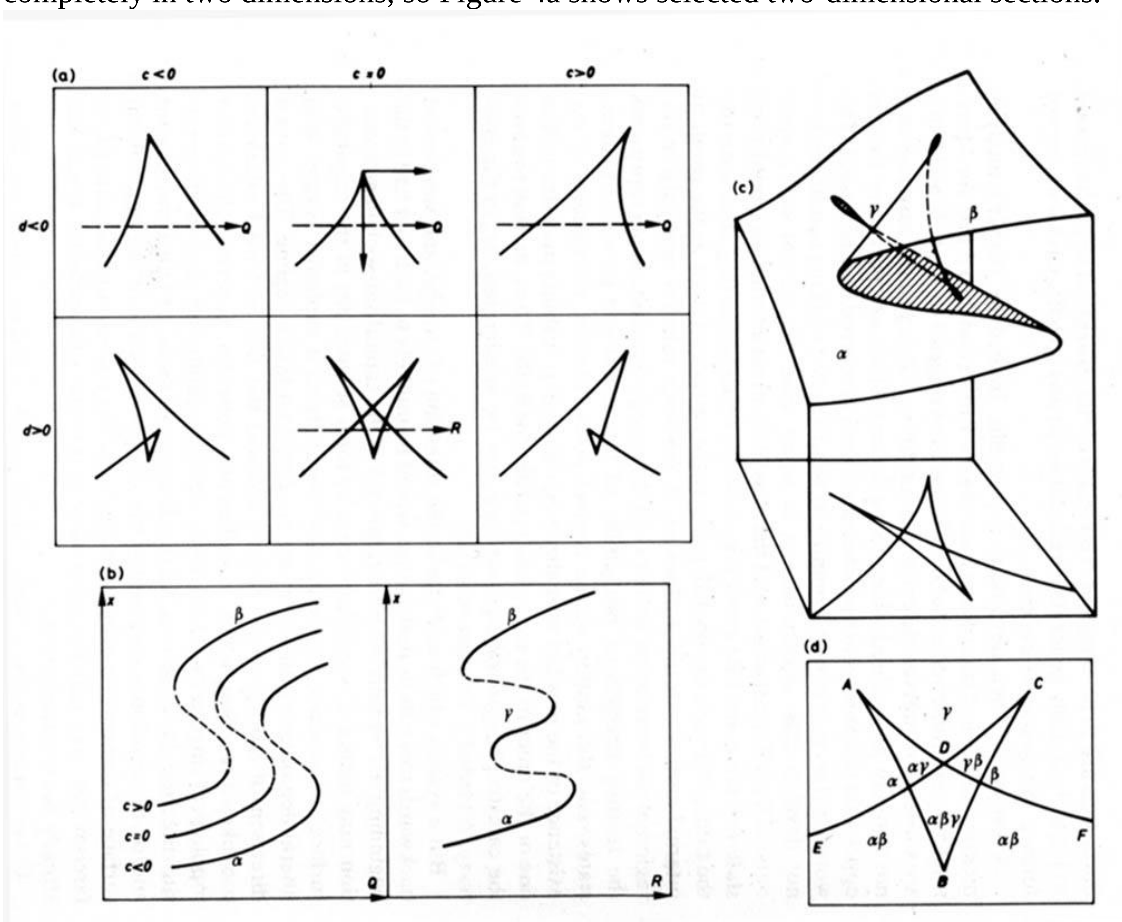

Figure 4. The butterfly catastrophe.

Figure 4a. Sections of the bifurcation set in the (a, b) plane for different values of c, the bias factor, and, d the butterfly factor.

Figure 4b. Sections of the behavior surface for the dashed paths Q and R. For negative values of the butterfly factor, cusp-type curves with two minima are shifted by the bias factor. For positive values, a third minimum emerges.

Figure 4c. Control and behavior surfaces for c =0, d >0.

Figure 4d. Behavioral minima of all locations on the control surface. See Section 5.1.

For negative values of the butterfly factor, the butterfly catastrophe reduces to a cusp which is swung in one direction or another by the bias parameter. For positive values of the butterfly factor, the complexity of this catastrophe becomes manifest: the bifurcation set looks roughly like two cusp bifurcation sets, linked together at a third vertex. As shown in Figure 4b, trajectories across the bifurcation set give either two equilibrium minima, which may be regarded as ordinary cusp alternatives, or three such minima, when an intermediate 'compromise' state is added between the other two. Figure 4e shows the equilibrium and behavior surfaces for zero bias and a positive butterfly factor, and Figure 4d shows the combinations of equilibrium minima which exist at all locations on the control surface. Note that the compromise minimum which exists everywhere in the ‘outer pocket,’ ABC, becomes the only possible state in the 'inner pocket,' ADC.

As already suggested, the dialectic does not always progress towards more ‘advanced’ states of social development. In the critical dialectic of Sartre (Desan, 1965; Sartre, 1977), a different kind of process occurs, which might be modeled as a circular trajectory on the cusp. Sartre portrays how individuals, initially in a condition of isolation and alienation (‘*serialité’*),are impelled to organize, driven by their own need and catalyzed by the existence of opposing groups around them. From a primitive union (a *‘groupe en fusion”*) there evolves a true 'group' held together by 'oath (the pledge and commitment of individual members) and *‘terreur’* (the threat of the collectivity and the fear of leaving it). The group cannot withstand the need for further consolidation, and develops in the direction of institutionalization and bureaucratization, which eventually completes the circle and returns the individual to alienation and powerlessness. With the formation of the group, the system enters, as it were, the bifurcation set. Somewhere in this process, whether with delay and sudden transformation, or more continuously, the behavioral state shifts from the helplessness and autonomy of individual action, to the power and rigidity of collective action. Charisma becomes routinized, ideas turn into their opposites, and the solution becomes part of the problem.

5.2. Two kinds of dialectic

From the perspective of catastrophe theory, the overwhelming emphasis in Marxian dialectics on contradiction and opposition reflects an unwarranted fixation on the cusp. The butterfly catastrophe also exhibits a dialectic, but one in which the struggle of opposites may be reconciled. The previous interpretations of the dialectical principles in terms of the cusp hold as well for the butterfly, since the latter includes the cusp as a special case. However, the additional features of the butterfly catastrophe offer an alternative conception of the second and third dialectical principles. (The interpretation of the transition from quantity to quality is just augmented by the possibility of discontinuous jumps to the compromise state). Within the bifurcation set of the butterfly catastrophe, there is no longer a struggle of opposites in which either one or the other side must be victorious, but the possibility also of a stable intermediate state. This possibility is absent in the cusp where the intermediate sheet in the behavior surface corresponds to an unstable maximum. The possibility of compromise appears by virtue of the two additional control variables, first the bias parameter which can balance the strength of the conflicting factors, then the butterfly parameter which can induce the creation of the new stable minimum. A volatile dyad is changed into a precarious triad and then into a stable tetrad. The negation represented by the struggle of opposites within the cusp bifurcation set is itself negated, not necessarily by the victory of one conflicting factor over the other (though this remains a possibility), but rather by the appearance of a third stable minimum, which in some cases will preempt the domain of the conflicting minima. Similarly, the triad of thesis, antithesis, and synthesis can be interpreted in a new and simple way: The two outer surfaces of the butterfly are the thesis and antithesis, and the pocket of compromise is the synthesis.

One may object that a true 'synthesis' or 'reconciliation' is not simply a compromise between conflicting alternatives. This is true. Dialectical discourse, drawing upon the richness of natural language, and reflecting the concreteness and context-dependence of human experience, cannot be totally represented within an abstract mathematical formalism. Still, Thom's topological theory does recognize the necessity for an independent impulse which forges the synthesis, in addition to the bias parameter which merely adjusts the relative balance between the conflicting forces and cannot by itself give rise to a synthesis. Compromise may also be reached not gradually, but by a qualitative 'leap,' one no less dramatic than that which characterizes the victory of one of the opposites. Also, in many cases, a genuine synthesis will involve some aspect of compromise, i.e. there will often be some variable whose middle range reflects the reconciliation of opposites. In the transition of quantity into quality, quantity does not disappear but acquires qualitative significance. For example, in the phase transition example, the qualitatively different states of liquid and gas are still differentiated one from another through the quantitative behavioral variable, density.

There are two distinguishable types of dialectic, one which results in victory of one of the opposing forces, and a second which gives rise to a compromise or synthesis. Both of these conceptions have been advocated by interpreters of dialectics. Although the differences between these conceptions had serious political repercussions in Communist party history, [10] they do not seem to have been distinctly articulated, or at least have not gained general acceptance among writers on dialectics. Here catastrophe theory offers the possibility of some clarification. Some dialectical phenomena are best modeled with the cusp; others are more appropriately grasped with the butterfly. (Undoubtedly, some may call for other catastrophe types.) Considering that the cusp is appropriate to conflict in which either one side or the other must be dominant, while the butterfly allows for the possibility of compromise or reconciliation, the issue of which conceptualization is the appropriate one for some concrete social phenomenon is not entirely academic. Unfortunately Thom's theory does not offer any account, or at least one comprehensible to a general reader, of how and under what conditions it is possible to transform a cusp into a butterfly.

5.3. Synthesis and fragmentation

A 'butterfly of reconciliation' may be illustrated using the account of Curle (1972) of stages of conflict resolution, as follows:

> 'The prototypical unpeaceful relationship is that of a master and slave, where the slave is ignorant of the enormity of his position and of the fact that it could ever be changed.... This situation can be altered only by what I broadly term education, implying some growth of awareness of his position in the slave.
>
> Once the slave (or dominated group) is aware, he (or it) struggles to reach a position of equality with his master (or the ruling group) so that the relationship can be reordered in accordance with principles of justice.. This is the stage

of confrontation. These two methods, education and confrontation, constitute... the revolutionary stages of peacemaking, whose primary aim is to reduce the imbalance of power... They are followed by three processes that are more appropriate to equal rather than unequal parties in conflict. By techniques of *conciliation*, hostile individuals are brought to the point where they perceive each other with less unreasonable fear and hostility so they can, with some hope of success, begin the process of *bargaining* which leads to a settlement of the dispute and a resolution of the conflict. Finally there is a stage of *development* in which the negative absence of hostilities is transformed into a positive collaboration... [and] cooperation.'

This sequence might be compared with Turner's description of dialectics, mentioned earlier. It is represented in Figure 5, with some alteration, as a particular trajectory on the butterfly.[11]

The first stages are purely cuspian, and bring the system to state of conflict within the bifurcation set. Point 4 marks the appearance of the possibility of compromise, with the emergence of small positive values for the butterfly factor, but the compromise is not actually accessible, since the control point is in a region having only the two cusp minima. In the next stage (point 5), all three states are available, and with transition to the inner pocket region (point 6), the synthesis is actually achieved. In the development stage, the risk of catastrophic loss of agreement is lessened, and a continuous range of cooperative behavior becomes possible. However, as long as the topological singularity, and its associated butterfly morphology, do not actually disappear, there is always the possibility of retrogression - either to a state of uneasy compromise or to the actual resumption of conflict.

Indeed, it is possible for the process to run in reverse. A creative synthesis (of the interests of contending parties, of ideas, or of values) which suffers a distortion of the balancing factor or a weakening of the integrating factor may fragment into unreconciled opposites, one or the other of which eventually gains dominance.

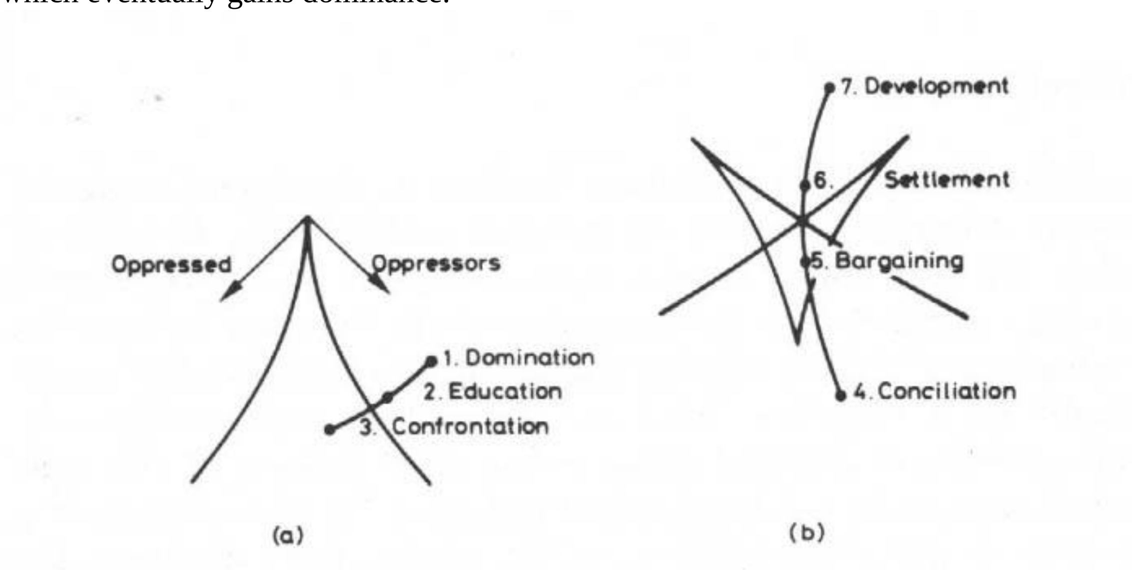

Figure 5. Conflict resolution interpreted on the butterfly (a) Revolutionary stages. (b) Relations among equals. The revolutionary stages are shown in terms of a changing relative balance of conflicting factors, representing the strength and/or advantage of the oppressed and oppressor groups. This cuspian situation becomes transformed at point 4 to a true butterfly as the butterfly factor takes on positive values (not actually shown in the figure.) The trajectory continues on to stage 7 with further changes in the conflicting and butterfly factors. See Section 5.3 and Note 11 for additional discussion.

6. Conclusions

To recapitulate, the classical dialectic laws can be interpreted using the elementary catastrophes known as the cusp and butterfly. In terms of the cusp, the first law is illustrated in divergence and discontinuous jumps; the second law, in the bimodality of the behavior surface, the conflicting control factors, and the changes in the energy function as the bifurcation set is traversed. The third law, and the triad of thesis / antithesis / synthesis, is exhibited either in the three surfaces of two concatenated cusps, or in the three distinct regions in the control plane of a single cusp. A particular trajectory of the control point illustrates the dialectical conception that development gives rise to contradictions within a system, which lead to its transformation. In the butterfly, divergence and discontinuous jumps occur between three, rather than two, minima. The struggle of opposites is no longer absolute. The negation of negation and the dialectical triad may refer to either the effect variables or the causal parameters, i.e. either the three equilibrium regions of the behavior surface or the two conflicting variables plus the combined action of bias and butterfly factors. A particular motion of the control point illustrates the dialectical process by which conflicts may be resolved. Both the cusp and the butterfly can display not only the evolution of systems but their involution; not only progressive development and liberation, but also degeneration and rigidification. It should be clear that the qualitative use of catastrophe theory, like dialectics, depends extensively upon the opinions and values of the investigator.

From the perspective of catastrophe theory, both the complete rejection of dialectics as vague or metaphysical, and the claims of universality made on its behalf by official Communist philosophy, are unfounded. A more accurate view would be that dialectics and catastrophe theory are modes of inquiry and exposition. They provide general models about process and are ‘systems’ frameworks.[12] They are applicable to a variety of phenomena, but are hardly complete or universal. Also, they must be supplemented by concrete knowledge of the phenomena being described, else they indeed dissipate into flights of fancy or rigidify into dogma.

The present paper ranges over a broad array of subjects and is necessarily brief. Many of the topics touched upon, need (and can) be given fuller treatment, and this will be undertaken in subsequent papers. It would be of interest, in terms of the present perspective, to survey systematically - and attempt a taxonomic classification of - the varying interpretations and examples given of dialectic in its voluminous literature. The philosophical affinity between catastrophe theory and dialectics could be explored more fully. Thom's aversion toward the Newtonian world-view which emphasizes the continuity of differential equations and the reducibility of phenomena is close in spirit to the dialectical rejection of ‘mechanistic’ materialism.

The use of catastrophe theory models in the social sciences is just beginning, and here interaction with the rich Marxist literature might be stimulating to both. One possible quantitative application might be the extension of Zeeman's discussion of stock market cycles (1977) to business cycles in general. Most uses of Thom's theory will, however, remain qualitative or even just symbolic or evocative, but as long as a *priori* validity is not claimed for such analyses on the basis of Thom's topological proofs, catastrophe theory will be more illuminating than misleading. Whether truly quantitative and rigorous use of the theory is possible in the social sciences, only time will tell.

In closing, it must be acknowledged that dialectics and catastrophe theory overlap only partially. Catastrophe theory hardly stands in need of dialectical legitimation or interpretation, and a great deal, probably the most socially relevant, and also the most subtle, aspects of dialectics cannot be encompassed within (or even illuminated by) any formal mathematical theory. Still, there is a deep connection between these two modes of thought. In an age inundated by ever-increasing knowledge about ever more minute aspects of reality, one should be grateful for, and tolerate the limitations of, those thought forms which are very general, and which, having the capacity to illuminate individual and social experience, can become also personally meaningful.

Notes

1. Thom (1975) is the author of the theory, but his writings are less accessible for the non-mathematician than the work of his principal 'disciple', Zeeman (1976, 1977).The latter's book contains a host of applications, at various levels of mathematical difficulty, to the physical, biological, and social sciences, and also a current bibliography.

2. This paper draws upon a variety of primary and secondary references on the dialectical principles (Cornforth, 1975; Engels, 1970, 1973; Graham, 1971; Hegel, 1951; Kursanov, 1967; Marcuse, 1960; McGill and Parry, 1948; Stalin, 1940; Venable, 1966; Wetter, 1958), but it is unfortunately utterly beyond its scope to take up the differences of interpretation among these sources. The author has made extensive use (especially in Section 3) of the essay of Cornforth, which is a short, clear, and 'orthodox' Marxist presentation, and one in which the similarities of dialects to catastrophe theory are strikingly manifest.

3. The word 'catastrophe' in this paper is nearly always used as a technical term. It is the name that Thom has given to his topological theory, and to the seven archetypal forms which it encompasses. The ordinary meaning of the word, signifying something disastrous, should not generally be inferred, although occasionally it may be appropriate. Nor should the term be associated with any particular ideological position which has been held in the intellectual/political history of dialectics.

4. Within official Soviet and eastern European philosophy, dialectics and materialism are, of course, wedded together, but only dialectics will be considered in this paper.

5. Strictly speaking, the 'control' and 'behavior' variables can have a reciprocal relationship which departs from that of simple cause and effect.

6. Technically, the bifurcation set refers to the cusp-shaped boundary, which is the locus of points on the control plane at which sudden changes can occur in the behavior variable. However, this term is also used more loosely to denote the region inside this boundary.

7. Some mathematical details: The energy function has the form:

$$V=x^4/4 - ax^2/2 + bx,$$

and Figure 1b plots V versus x for the sequence of values of the parameter, b, corresponding to the path indicated for the control point (the other parameter is held constant). The rate of change of the behavioral variable is assumed to vary with the gradient of V, and so the behavior surface is given by the equilibrium equation:

$$- \varepsilon\, dx/dt = dV/dx= 0 = x^3-ax+b$$

where ε is small. In the limit of $\varepsilon \rightarrow 0$, return to the behavior surface after some displacement from it, such as occurs in catastrophe jumps, is instantaneous. The motion of the control point will typically be specified by equations of the form:

da/dt = f(x, a, b)
db/dt =g(x, a, b)

These are not furnished by the theory, but must be determined for each phenomenon being modeled.

8. This illustrates the dialectical ‘categories’ of ‘possibility’ and ‘actuality.’ Cusp interpretations can be given to other category pairs: ‘Essence and appearance’ might be illustrated by the distinction between control and behavior variables. ‘Necessity and contingency’ are reflected in the underlying topological structure of the cusp surface, which is fixed, contrasted with certain geometrical distortions which are allowed; or in random effects on an otherwise deterministic motion of the control point, which make the moment of catastrophe unpredictable, or cause divergence.

9. This is the classic codification of Stalin (1940). The dialectic can be a powerful a subtle tool for social criticism, but this does not preclude its crystallization into dogma or its ornamental use to legitimate a totalitarian order.

10. Deborin, for example, was attacked in part for advocating the reconciliation of opposites; Bukharin also had such a view (Wetter, 1958).

11. The first two control parameters are chosen as conflicting factors, which represent the strengths of the dominating and dominated groups. It is more difficult to suggest the nature of the bias and butterfly factors, but they relate perhaps to whatever forces exist which serve the cause of justice, and to those commonalities, deeper than the overt struggle, which bind together the contenders.

12. The more general affinity which exists between Marxism and systems theory is coming to be increasingly recognized, (e.g. Amburgey and McQuarie, 1977; Kirschenmann 1970; Merrill, 1977; Wallerstein, 1974).

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References

Amburgey, T. and D. McQuarie, ‘System Change in Karl Marx's Model of Socio-economic Formation.’ *General Systems* 22, 99-1031 1977

Cornforth, M., *Materialism and the Dialectical Method*, New York: International, 1975.

Curle, A., *Mystics and Militants*, London: Tavistock, 1972.

Desan, W., *The Marxism of Jean-Paul Sartre*, Garden City: Doubleday, 1965.

Dodson, M. M., ‘Darwin's Law of Natural Selection and Thom's Theory of Catastrophes.’ *Mathematical Biosciences* 28, 243-274, 1976.

Engels, F., *Anti-Duhring*, New York: International, 1970.

Engels, F., *Dialectics of Nature*, New York: International, 1973.

Engels, F., *Ludwig Feuerbach*, New York: International, 1941.

Engels, F., *Socialism*: Utopian and Scientific. Marx and Engels, Selected Works, New York: International, 1968.

Fowler, D. H., ‘The Reimann-Hugoniot Catastrophe and the Van der Waals Equation’ in C. H. Waddington (ed.), *Towards a Theoretical Biology*, Vol. 4, Chicago: Aldine-Atherton, 1-7, 1972.

Graham, L. R., *Science and Philosophy in the Soviet Union*, New York: Vintage, 1971.

Hegel, G. W. F., *Science of Logic*, W. H. Johnston, and L. G. Struthers (trans.). New York: Macmillan, 1951.

Kirschenmann, P. P., *Information and Reflection*, New York: Humanities, 1970.

Kursanov, G., *Fundamentals of Dialectical Materialism*, Moscow: Progress, 1967.

Marcuse, H., *Reason and Revolution*, Boston: Beacon, 1960.

Marx, K., A *Contribution to the Critique of Political Economy*, Chicago: Kerr, 1904.

Marx, K., *Capital*, Vol. 1-3, New York: International, 1967.

Marx, K. and F. Engels, *The German Ideology*, New York: International, 1947.

Marx, K. and F. Engels, *Manifesto of the Communist Party in Marx* and Engels, *Selected Works*, New York: International, 1968.

McGill, V. U. and W. T. Parry, ‘The Unity of Opposites: A Dialectical Principle.’ S*cience and Society* 12, 418-444, 1948.

Merrill, J., Marxism and Systems Theory: A New Intellectual Convergence?' unpublished manuscript, March 25, 1977.

Sartre, J. P., *Critique of Dialectical Reason*, A. Sheridan-Smith (trans.). Honolulu: University of Hawaii, 1977.

Schulman, L. S. and M. Revzen, Phase Transitions as Catastrophes.' *Collective Phenomena* 1, 43-47, 1972.

Stalin, J., *Dialectical and Historical Materialism*, New York: International, 1940.

Thom, R., *Structural Stability and Morphogenesis*, Reading: Benjamin, 1975.

Turner, J. H., *The Structure of Sociological Theory*, Homewood: Dorsey, 1974.

Venable, V., *Human Nature*, The Marxian Mew, Cleveland: Meridian, 1966.

Waddington, C. H., 'A Catastrophe Theory of Evolution.' *Annals of the New York Academy of Sciences* 231, 32-42, 1974.

Wallerstein, I., *The Modern World System*, New York: Academic, 1974.

Wetter, G. A., *Dialectical Materialism*, Westport: Greenwood, 1958.

Zeeman, E. C., ‘Catastrophe Theory', *Scientific American* (April): 65-83, 1976.

Zeeman, E. C., *Catastrophe Theory: Selected Papers* 1972-1977, Reading: Addison-Wesley, 1977.