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Reorganizing Algebraic Thinking: An Introduction to Dynamic System Modeling

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Abstract

System Dynamics (SD) modeling is a powerful analytical method used by professional scientists, academics, and governmental officials to study the behavior patterns of complex systems. Specifically through use of the Stella software, it is a method that I and others have used for over two decades with high school, and even middle school, math and science students. In this paper I describe an introduction to SD modeling intended for an algebra class (in either middle or high school). In the body of the paper, a nested sequence of simple bank account examples, increasing in complexity, is used to demonstrate a comparison between using a closed form approach and using Stella to mathematize each situation. The comparison, showing equivalent recursive equations, closed form equations, and Stella modeling diagrams, is designed to give the reader (algebra teacher, mathematics education decision-maker, researcher, or whomever) an accessible introduction to understanding Stella model diagrams and the mathematical engine operating under the “hood” of the software. In particular, I highlight the limitations of closed form equations to capture the needed problem elements beyond a certain level of complexity, even when the problem is still simple enough for analysis by quite young students using Stella. In the final section, I discuss how, once students become comfortable with the software, the level of sophistication of problems they can analyze (including complex problems) by designing and building Stella models is extensive, significantly beyond what they can analyze with equations. Then I point to limitations in the traditional math curriculum, manifest in the Common Core State Standards, in terms of failure to prepare students for modeling of complex dynamic systems, and the related failure to exploit the potential of new representational resources.

Key words: algebraic thinking; system dynamics modeling; mathematical modeling; mathematization; complexity in models; software; Common Core State Standards; traditional math curriculum; limitations of traditional curricula; young students’ mathematical reasoning; technology in mathematics education

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Introduction

In this paper I show, using a nested sequence of simple examples increasing in complexity, why math teachers need not rely solely on the current symbolic representation (closed form equation) for studying dynamic problems in algebra.

Mathematizing story problems into closed form has, historically, been difficult for students (Schwartz & Yerushalmy, 1995). However, new technologies can provide alternate and more visual representations of many functions studied in algebra, making algebraic concepts accessible to more students. While the closed form representation of problems has served us well in the past, many of the problems our students will face, as adults, will require the ability to understand and make decisions about complex systems (Lesh, 2006). The goal of this paper is to support an evolution in teaching strategy and the content employed so as to make the study of complex systems accessible to algebra students.

“Change is accelerating, and as the complexity of the systems in which we live grows, so do the unanticipated side effects of human actions, further increasing complexity” (Stermann, 1994). Our nation and the global community face serious problems such as global warming, soaring national debt, unsustainable consumption of natural resources, daunting health costs for families, rising numbers of children in poverty, environmental impacts on health, and more. How will our children be able to successfully address these problems if teachers do not have strategies that are designed to help students build understanding of dynamic systemic problems?

Using technology, specifically Stella, it is possible to have students represent and analyze problems that would typically have been out of their mathematical reach using traditional closed form equation approaches (Blume & Heid 2008;

Fisher, 2011a; Pea, 1987). In what follows, I first describe Stella's visual interface and present a series of models using that software. Based on this presentation, I argue how the use of the software extends the range of possibilities for modeling, and in so doing reorganizes algebraic thinking. Finally, I consider some extensions beyond the scope of this paper, reflecting on the unchanging nature of the mathematics curriculum in the light of the need to understand dynamic systems, and the new possibilities offered by technology.

A Graduated Series of Stella Models

Stella Icons

The Stella SD software uses four main icons. One icon operates as an accumulator of “stuff” over time. This “stuff” can be physical, like the number of cars in a city, or abstract, like “concern about child homelessness.” The accumulator is identified as a “stock” shown as a rectangular icon. The stock depicts an important variable of interest and represents an aspect of the *state* of the system. See Figure 1.

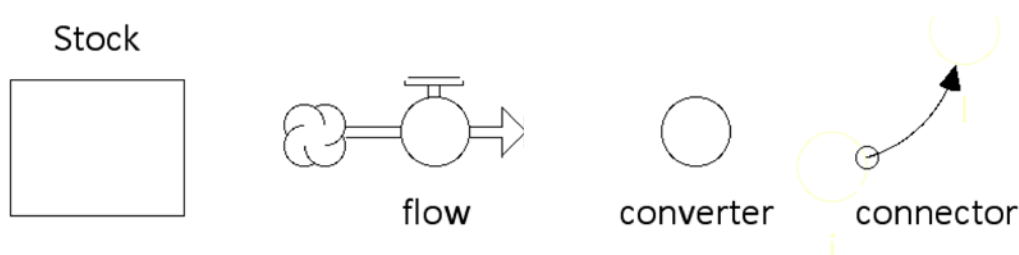


Figure 1: The modeling icons used in the Stella modeling software.

As will become clear in the examples that follow, Stella models situations dynamically through updating repeatedly how the state of the system repeats over time (similar to recursive equations). The time step is changeable, with precision

increasing as it is made smaller; calculus represents the limiting case as the time step approaches zero).

A “flow” icon represents a rate of change in a stock. If the flow arrowhead is pointing toward/away from the stock, a positive value (within the icon) represents the *rate* of increase/decrease of the stock value.

Another icon called a “converter” could represent either a parameter value or a non-stock variable whose value is computed using a formula.

Finally, there are icons called “connectors” that link converters to flows, converters to other converters, stocks to flows, or stocks to converters. They act like telephone lines, communicating numeric information between components so formulas can be updated each time step (calculation interval).

A series of simple finance scenarios will be used to show how it is possible to think about a problem using different symbolic representations, and will show some of the advantages of each representation. The intention of the lesson sequence is not to have a teacher use all three methods with students. Rather, the teacher would just have students build the Stella diagram, modify it as the lessons progress, and analyze the output of each model to answer the questions.

First Lesson, Linear Growth: Depositing Money in a Shoebox

Twelve year old Demitre wants to save money to buy a bicycle and helmet that cost \$198. His grandmother gave him \$50 on his last birthday. He has a regular allowance of \$5 per week for doing some chores around his house. He wants to know how long he will have to save his money in order to buy the bike and helmet. Demitre has not yet studied algebra so he might determine how long he has to wait using a recursive sequence of calculations, as shown below in the second column of Table 1.

Table 1: Calculations for the savings needed to purchase the bicycle and helmet.

Week	Money in shoebox (recursive calculation)	Money in shoebox (calculation leading to algebraic equation)
Starting amount of money:	\$50	\$50
Money after 1 week	$\$50 + \$5 = \$55$	$\$50 + \$5 = \$55$
Money after 2 weeks	$\$55 + \$5 = \$60$;	$(\$50 + \$5) + \$5 = \$50 + 2*\$5 = \60
Money after 3 weeks	$\$60 + \$5 = \$65$;	$(\$50 + 2*\$5) + \$5 = \$50 + 3*\$5 =$ $\$65$
...
Money after 30 weeks	$\$195 + \$5 = \$200$;	$(\$50 + 29*\$5) + \$5 = \$50 + 30*\$5$ $= \$200$

Both calculations would lead Demitre to the same conclusion, thirty weeks of saving are required. The calculation shown in the middle column is intuitive, if cumbersome. The calculations shown in the third column are even more cumbersome but lead to a pattern that can be written as a closed form linear formula, $M_t = 50 + 5t$, the type we want students to learn in algebra, because it is useful for mathematical thinking for future courses.

We could represent the calculations in the middle column using the recursive formula, $M_t = M_{t-1} + 5$. Pictorially, students could draw a box (or stock) where the money is being stored and draw an inflow to the box showing that a constant amount of money is being deposited into the box each week. (See Figure 2, left diagram.)

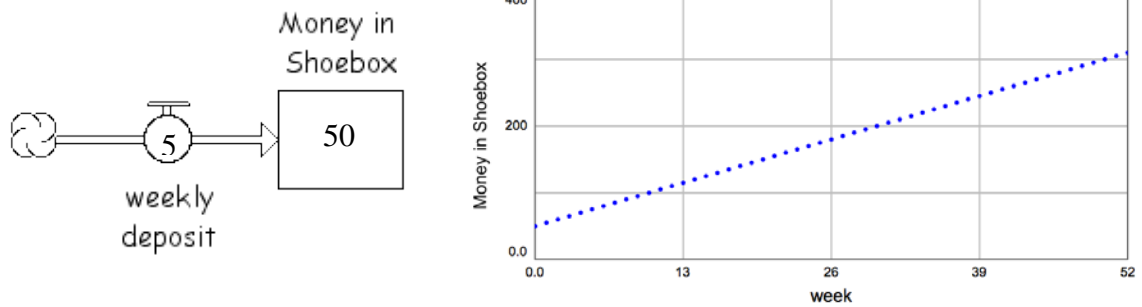


Figure 2: A pictorial representation of depositing money in a shoebox, and the graph of the amount of money in the shoebox over time. Note: the user can drag the cursor over the graph to read the values of the dependent and independent variables at each point.

Demitre thinks he may want to spend a little of his allowance each week, perhaps \$2 on treats. So now, how long will it take him to save for his bicycle and helmet? See Table 2.

Table 2: Calculations for savings needed to purchase the bicycle and helmet if \$2 are spent each week.

Week	Money in shoebox (recursive calculation)	Money in shoebox (calculation leading to algebraic equation)
Starting amount of money	\$50	\$50
Money after 1 week	$\$50 + \$5 - \$2 =$ \$53	$\$50 + \$5 - \$2 = \53
Money after 2 weeks	$\$53 + \$5 - \$2 =$ \$56	$(\$50 + \$5 - \$2) + \$5 - \$2 = \$50 +$ $2*\$5 - 2*\$2 = \$56$
...

Money after 10 weeks	$\$77 + \$5 - \$2 =$ \$80	$(\$50 + 9*\$5 - 9*\$2) + \$5 - \$2 =$ $\$50 + 10*\$5 - 10*\$2 = \80
...
Money after 50 weeks	$\$197 + \$5 - \$2 =$ \$200	$(\$50 + 49*\$5 - 49*\$2) + \$5 - \$2 =$ $\$50 + 50*\$5 - 50*\$2 = \200

We see that Demitre will now need to save for fifty weeks to meet his goal.

Again, the middle column is the most intuitive method of hand calculation for someone who does not know algebra. The third column shows how one might recognize the pattern necessary to write the algebraic formula to summarize this savings plan, $M_t = 50 + 5t - 2t = 50 + 3t$.

The recursive formula for the middle column calculation is $M_t = M_{t-1} + 5 - 2$. (I will purposefully not simplify the arithmetic. The diagram in Figure 3 shows the increase and decrease separately.) If we want to draw a picture of what is happening in this situation we could draw a figure similar to Figure 2, but add an outflow to represent the constant spending that is occurring. See Figure 3.

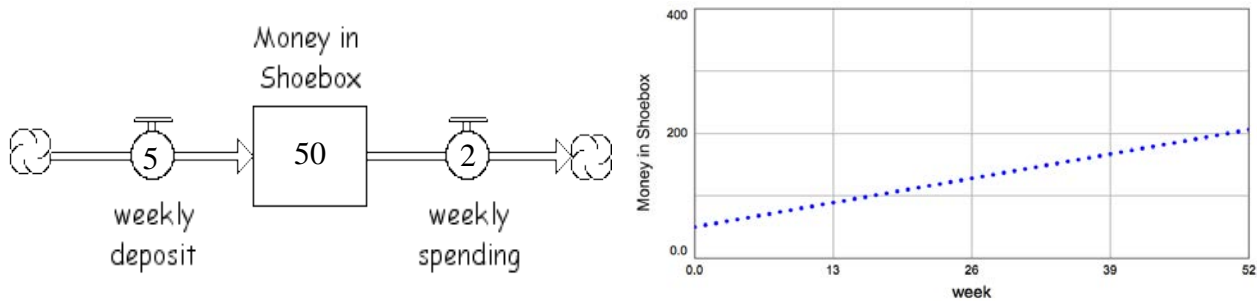


Figure 3: A pictorial representation of depositing money to and spending money from a shoebox, and the graph of the money in the shoebox over time.

Second Lesson, Exponential Growth: Putting the Money in the Bank

Demitre's older sister, Helena, says Demitre should put his money in the bank because he will collect interest, which will shorten the amount of time he will need to save to purchase his bike and helmet. She explains how interest works. To make the problem easier for him to understand she assumes the yearly interest will be 10%. (10% is an unrealistic yearly interest amount but is used here for convenience.) She also says that, since he will want to know how much will be saved each week, she will assume that the interest is calculated weekly by the bank. That means the weekly interest rate will be about $\frac{10}{52}\%$ or 0.0019. She tells him they will only consider interest and no deposits or withdrawals for this initial interest example. See Table 3.

Table 3: Calculations for savings needed to purchase the bicycle and helmet placing money in the bank at 10% annual interest, compounded weekly.

Week	Money in bank (recursive calculation) decimals rounded for convenience	Money in bank (calculation leading to algebraic formula)
Starting amount of money	\$50	\$50
Money after 1 week	$\$50 + 0.0019(\$50) =$ \$50.095	$\$50 + 0.0019(\$50) = \$50(1.0019) =$ \$50.095
Money after 2 weeks	$\$50.095 +$ $0.0019(\$50.095) =$ \$50.19	$(\$50 + 0.0019(\$50)) + 0.0019(\$50$ $+ 0.0019(\$50)) = \$50(1.0019)^2 =$ \$50.19
...
Money after 10	$\$50.86 +$	$\$50(1.0019)^9 +$

weeks	$0.0019(\$50.86) =$ \$50.96	$0.0019(\$50(1.0019)^9) =$ $\$50(1.0019)^{10} = \50.96
...		
Money after 50 weeks	\$54.87 + $0.0019(\$54.87) =$ \$54.97	$\$50(1.0019)^{49} +$ $0.0019(\$50(1.0019)^{49}) =$ $\$50(1.0019)^{50} = \54.98
...
Money after 1040 weeks (about 20 years)	\$359.33 + $0.0019(\$359.33) =$ \$360.02	$\$50(1.0019)^{1039} +$ $0.0019(\$50(1.0019)^{1039}) =$ $\$50(1.0019)^{1040} = \360.02

Again, the middle column appears more intuitive. The third column shows the pattern for the algebraic formula if one were considering compounding interest weekly for 1040 weeks, i.e., $M = 50(1.0019)^t$, where $t = \text{weeks}$.

The recursive formula for the middle column in table 3 is $M_t = M_{t-1} + 0.0019 * M_{t-1}$. If we wanted to look at a picture that would follow the recursive thinking in column two of Table 3 for this interest bearing account, we might draw the diagram shown in Figure 4, on the left.

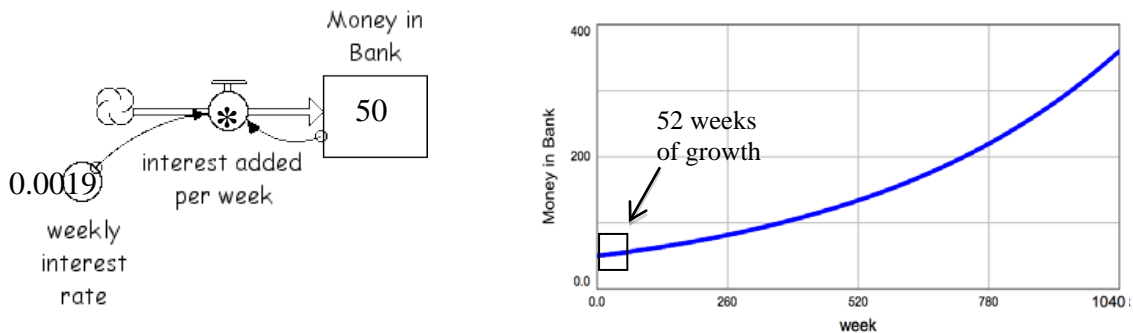


Figure 4: A picture showing how the interest on an interest bearing bank account might be calculated and added to the account. The graph shows the amount of money in the bank over 1040 weeks.

What the diagram in Figure 4 shows is that Demitre is starting with \$50 in the bank and is adding interest each week. The interest is calculated by taking the current amount of money that is in the bank and multiplying it by the weekly interest rate of 0.0019. The asterisk in the valve part of the inflow indicates that the software is multiplying the two factors that point to it. When the interest is added, in any given week, the Money in Bank is increased by the interest amount and is therefore larger when the subsequent interest calculation is made, just as is shown in the second column (recursive calculation) of Table 3.

Up to this point the equation and Stella representations were both useful. Now, in the next lesson, we start to see how we can easily continue to expand the problem using SD but not when using a closed form equation approach.

Third Lesson, Constant and Exponential Change: Adding Interest to Demitre's Original Savings Plan

Demitre has been convinced by his sister that placing his money in the bank is a good idea. But he still wants to make his weekly deposits of \$5 and wants to be able to take out \$2 each week for incidentals. Demitre will probably not find a bank that will give him 10% interest, but again we will keep this interest rate, for convenience. Let's see how we can calculate the weekly status of his money now.

Table 4: Calculating the money in the bank with 10% annual interest, compounded weekly, \$5 deposited per week and \$2 withdrawn per week.

Week	Money in bank (recursive calculation) decimals rounded for	Money in bank (calculation leading to algebraic equation)

	convenience	
Starting amount of money	\$50	\$50
Money after 1 week	$\$50 + 0.0019(\$50) + \$5 - \$2 = \$53.095$	$\$50(1.0019) + \$5 - \$2 = \53.095
Money after 2 weeks	$\$53.095 + 0.0019(\$53.095) + \$5 - \$2 = \$56.196$	$\$53.095(1.0019) + \$5 - \$2 = \56.196 there is no longer a <i>convenient</i> pattern to follow when we combine exponential and constant change
...		
Money after 10 weeks	$\$78.07 + 0.0019(\$78.07) + \$5 - \$2 = \$81.22$	
...		
Money after 46 weeks	$\$195.26 + 0.0019(\$195.26) + \$5 - \$2 = \$198.63$	<i>No simple closed form equation</i>

The recursive formula used for column 2 of Table 4 is $M_t = M_{t-1} + M_{t-1} * 0.0019 + 5 - 2$. It shows he could make the purchase four weeks sooner, by placing his money in the bank. Twelve year old Demitre may not be able to follow all the calculations shown in column 2 of Table 4, but if we showed him the diagram in Figure 5 he would probably be able to understand the logic of the flow of money that is shown.

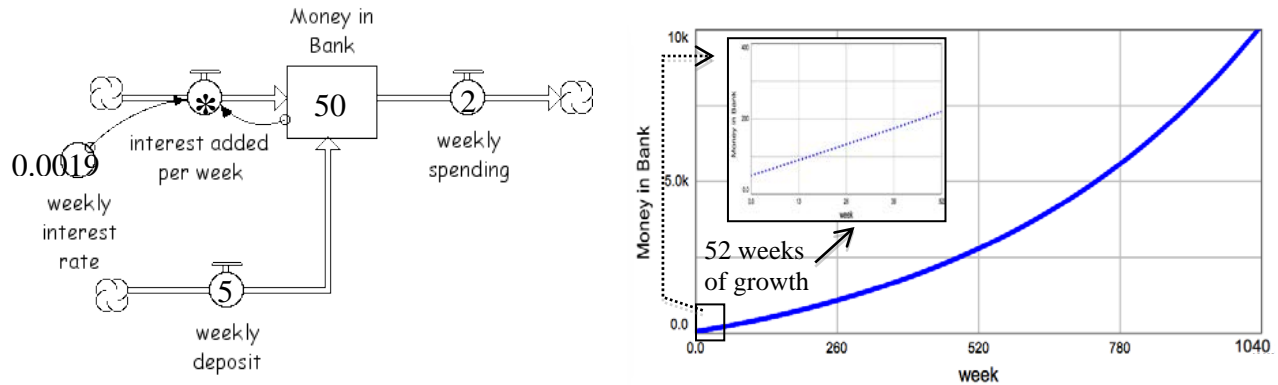


Figure 5: A picture of Demitre's savings plan, with interest being added to his bank account each week, and a constant amount in weekly deposits, and (constant) weekly spending. The larger graph shows the exponential behavior of Money in the Bank (over 20 years), as one would expect. The inset graph shows the amount of Money in the Bank over 52 weeks, which appears linear but is also exponential.

Extending Lesson 3

Demitre's parents want him to develop good financial habits. They are willing to increase his allowance to \$6 per week if he will put \$1 per week in a savings account in the bank that will earn 12% annual interest (compounded weekly). If he were just to put this extra dollar in a savings shoebox he knows he would have saved \$46 in his shoebox at the end of 46 weeks (when he bought his bike and helmet). Placing the extra dollar in his checking account and having the bank automatically transfer \$1 to his savings account each week should work out better for him, since he is earning interest on his accounts. How much more money will he have in savings after 46 weeks, if he follows this plan? (answer: \$2.47) A possible model diagram and graphical output are shown in Figure 6. There are many scenarios that could be tested with the model in Figure 6 – different interest on checking and/or savings account, different savings amounts, different spending amounts, etc.

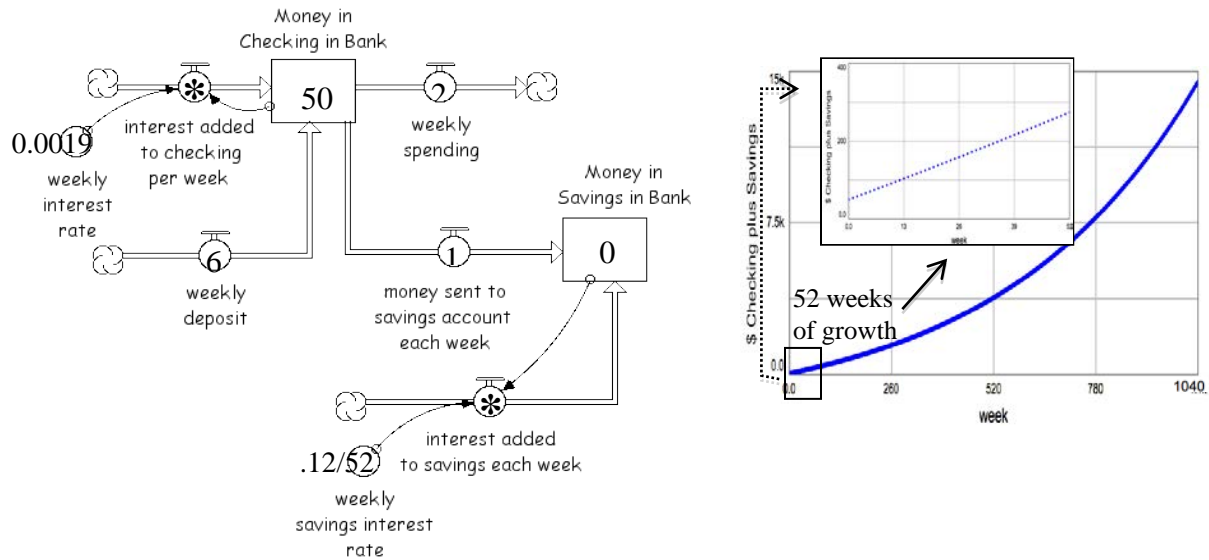


Figure 6: A picture (diagram) of Demitre’s checking and savings account system.

The graph shows the sum of both the checking and savings accounts.

To summarize, the value of the new approach and software (Stella) for mathematizing and analysis include:

- a visual, icon-based method of defining variables and differentiating their purposes in the problem,
- specifying variables using a naming procedure that includes full words or phrases,
- a structural design that lays out, in schematic form, the relationship and dependencies between the variables and parameters in the problem, allowing a more pictorial view of the overall problem structure ,
- a dynamic approach to defining functions (i.e., focus on rates of change),
- a quick approach to testing “what-if” scenarios, altering model structure or parameters, and re-running the model to view the change in behavior graphically or numerically,

In short, Stella provides a representation that has been used by adult professionals to perform complex systems analysis, but that is still accessible to a broad section

of high school students who can, therefore, also perform complex systems analysis (Fisher, 2011b).

Reorganizing Algebraic Thinking

How can a claim be made that creating SD models reorganizes algebraic thinking?

The claim is made from 20 years of action research in multiple algebra classes.

Four statements will be listed to suggest why the claim is made. Consider the first two (of four) statements:

- SD modeling provides an alternate, process oriented symbolic representation for mathematizing dynamic problems.
- SD modeling representation is more schematic in appearance and so more pictorial. Dependencies of one variable on another are visually identified. Full words/phrases are used to label the icons.

Research has shown that multiple representations can improve understanding of a mathematical concept provided the representations are linked, one representation reinforcing or complementing the information learned from other representations (Heid & Blume, 2008). Introducing the new SD diagram representation is no exception. When linear functions are studied in class and the SD linear model diagram is introduced its structure and component definitions are described in relation to its graphical and numerical output and then compared to the closed form equation representation. A similar process occurs when introducing the exponential Stella model.

Moreover, the SD model diagram is more visual, and the identification of component parts much less cryptic than used in closed form equations. Bishop (1989, in Dreyfus 1991) said, "... there is value in emphasizing visual representations in all aspects of the mathematics classroom." Dreyfus (1991) also adds that the value of visualization in mathematics classrooms is underrated and consequently underutilized.

Statement three hints that SD modeling can provide a different lens for viewing functions that can have an important influence on their mathematical thinking, especially as it applies both to interpreting dynamics in the world around them and in preparation for future mathematics classes.

- Creating SD models requires algebra students to consider functions from a dynamic perspective (from their rate of change and as accumulations).

“The move from a static model in an inert medium, like a drawing, to dynamic models in inter-active media that provide visualization and analytic tools is profoundly changing the nature of inquiry in mathematics and science” (Bransford, Brown, Cocking, Donovan, & Pellegrino, 2000). Moreover, a focus on functions in terms of their rates of change, and on those dynamic variables that act as accumulators, provides an experience that one could consider a conceptual calculus approach. This approach is not typical in algebra, currently.

Finally, the fourth statement is the most powerful. If students are able to study different and more sophisticated problems, it cannot help but change how they think about the world and how they see the value of mathematics in viewing the world.

- Using SD modeling tools like Stella, it is possible for students to study problems that would be beyond their mathematical level using equations alone.

Having students build models that give them access to problems that are beyond those typically presented in algebra class exposes students to new applications of the mathematics they are studying. Bransford et al. (2000) recommend that schools look for technology programs that can be used as a “tool to support knowledge building.” The final example in this paper is just one very small example of the knowledge building that can occur with SD modeling that I and other teachers have used with secondary school algebra students.

There are more statements that can be made suggesting ways SD modeling reorganizes algebraic thinking, but those are best identified and supported in conjunction with modeling examples that make the statements clear, so lie beyond the scope of this paper.

Discussion

In this paper I have presented the most elementary beginnings of a modeling path to help students think differently about situations that surface algebraic concepts. The first three examples (shown in lessons 1 and 2) in this article are exact models, simple enough to be expressed both by closed form equations and Stella diagrams. These three examples are intended to give teachers an idea about how they might start introducing a modeling approach for studying dynamic systems. That path takes teachers through the familiar territory of applications of functions that are part of their current curriculum. The first three examples show the connection of the new representation (the Stella model diagram) to the recursive equation representation as well as the closed form equation representation. The difference that the diagram approach brings at this point (besides the visual nature of the diagram) is that the functions presented are described in terms of their dynamic behavior, linear function behavior evolving from constant rates of change and exponential function behavior evolving from proportional rates of change.

The two examples shown in lesson 3, however, indicate how easy it is to mathematize problems/applications that require a combination of functions when using the diagramming representation of Stella. The complexity of the diagram has grown very slightly in these last two examples, but the complexity of the situation modeled has grown such that it is no longer possible to mathematize the problems using closed form representation. Mathematizing with closed form equations typically requires simplifying the problem, sometimes extensively, to be

able to bring the problem to an approachable level for students. The SD approach allows more complexity to be retained and is ideal to use to model natural phenomena that are not necessarily simple.

It is possible to design Stella diagrams that will produce not only linear and exponential function behavior, as shown in the previous examples, but also, for example, quadratic, convergent (goal-seeking/asymptotic), logistic, and sinusoidal function behavior. It is then possible to combine these Stella diagram segments to study larger problems in the same way that small LEGO® structures are combined to develop larger LEGO systems. Some scenarios I have used as model-building lessons in algebra classes include the study of drug pharmacokinetic dynamics, population and resource depletion dynamics, and predator/prey dynamics, to name a few.

Another benefit of the SD approach is that it more directly assesses student understanding of the problem under study, as the stock/flow diagrams bring to the surface, for analysis and discussion, the student's conception of how the problem is structured, surfacing their "mental models." This visual mapping of the problem provides a vehicle for problem analysis that is inclusive of modeler and others, including students and teachers. The modeling acts both as a mathematization process and a communication process. As such, it allows students access to more sophisticated problems and gives them a vehicle for explaining their work that is more intuitive than using the closed form equation. Access to such dynamic problems, starting with closed form algebraic equations comes later, in calculus. Yet learning the closed form equation in calculus helps only modestly, as most complex real world systems have no closed form solution of any type.

I have used this software in my high school algebra classes for 20 years, in both an inner-city high school and a suburban high school. I have also taught, in both schools (also for 20 years), a year-long course in SD modeling that included

students from grades 9-12 in the same class. The students in these classes built increasingly complex models to study the spread of epidemics, supply and demand dynamics, the growth of cities and jobs, and more. In the last 10 weeks of the course students chose a topic (containing dynamic behavior) of their own from the news or from another class they were taking (the value of this work for interdisciplinary learning is obvious). Students (in teams of two) designed models of nonlinear complex systems, with an emphasis on feedback analysis. They were able to implement potential policies to try to mitigate the undesirable behavior of their system and determine which policies might be most effective. It was very empowering for them. They built a working model, wrote a technical paper explaining their model, and did a presentation. It was the work that students were able to produce that made me an advocate for this modeling approach. Students who were of many differing abilities were able to study problems that were beyond what was generally available in their math and science classes. They saw applications of mathematics and science that were usually relegated to post-secondary study. Samples of student models, technical papers, and presentations can be found at www.ccmodelingsystems.com (select the Students tab).

Stella has also been used as a student model-building tool in some mathematics and science classes at the middle school level for over two decades. Curriculum is available for use in middle school and for use in high school mathematics and science classes (Creative Learning Exchange 2015; Fisher 2005, 2011a). Online professional development courses are available for math and science teachers who want to learn this method of modeling. A detailed alignment of SD modeling and the CCSS-M Practices and the Functions and Modeling Standards can be found at www.ccmodelingsystems.com/res-stds-skills-math.html.

Our mathematics curriculum and practices have changed little in the last hundred years (Schank, 2004). Two national studies of the instructional practices

of middle school mathematics and science teachers were conducted by Stigler and Hiebert (2009). The first study (in 1995) indicated that not much had changed in teacher instructional practice in the past century. A follow-up study (in 1999) indicated teachers were more aware of reform efforts in mathematics but still found no evidence that teacher practices had changed. Stigler and Hiebert stress that schools must become places where teachers as well as students are expected to learn.

The traditional mathematics curriculum starts with arithmetic in the early grades with efforts to generalize arithmetic processes and thinking as students progress into the middle elementary grades. This process becomes more formalized, using symbols, as algebra is introduced in the upper elementary grades. Algebra also introduces the study of functions that are a necessary precursor for the study of calculus (Kaput, 1995). More emphasis on inclusion of statistics in CCSS-M is a welcome and needed enhancement of the curriculum. The CCSS-M standards also place more emphasis on modeling, yet with somewhat vague reference to the value of technology in expanding the breath and depth of problems students could study.

The new representations afforded by icon-based software that can be used to mathematize real-world problems are not mentioned in the modeling section of the CCSS-M standards. Kaput and Roschelle (2013) argue that, within mathematics classrooms and many other contexts, representational access to mathematical concepts has not changed significantly in (at least) the last 100 years. They go on to say “Computational media are reshaping mathematics, both in the hands of mathematicians and in the hands of students as they explore new, more intimate connections to everyday life.”

In some ways the inertia in the use of computing technology (excluding graphing calculators) to enhance the study of real-world problems in the

mathematics curriculum is understandable. Only recently has more widespread access to non-calculator computing devices in schools increased to more acceptable percentages, although much progress still needs to be made. In the last 4 years access to school issued computing devices (not counting graphing calculators) for US students and teachers has increased in percentage from the low 20s to the mid 50s (Molinar, 2015). Moreover, there are other significant barriers to teacher use of technology in the classroom (Philipp, 2007; Wolf & Le Vasan 2008), not the least of which is the fear some teachers have about the potential for a change in the role of the teacher from the disseminator of information to facilitator of the use of information. The research welcomes this change in role but it is one that requires teacher training/retraining, providing teachers experience with the software, helping them determine how best to understand and alter their curriculum for optimal results, and for many helping them develop strategies for the changing management schemes needed in the classroom (Zbiek & Hollebrands, 2008).

Our children deserve access to current analytical methods made approachable by creative technologies, technologies that provide them an alternative approach to mathematize real-world systemic problems. They deserve tools that allow them to “evaluate critically, and act upon, issues of importance in their personal lives and the lives of their communities” (Greer & Mukhopadhyay, 2003).

SD was developed in the mid 1950s. In 1985 Stella was developed via a National Science Foundation grant. Once Stella became available, SD became accessible to pre-college students. SD has been and is currently used to address serious global systemic problems, like global climate change, deforestation, loss of natural resources, health care protocols, and more. Never once, in the 20 years I taught SD in my high school math classes, was I asked why we were learning this

new modeling approach – by either the students or their parents. In fact, more than a few parents lamented they did not have the opportunity to learn this approach when they were in school.

We have proof that high school students can create relatively sophisticated SD models to study complex systemic problems. This proof is supported by the fact that they can also write technical papers explaining how their model was constructed and why it produces its behavior. What is presented in this paper is just the beginning of a very stimulating mathematical pathway. The incorporation of System Dynamics model building activities into my algebra and modeling classes was the most stimulating and impactful experimental process I had every tried in my 30+ years of teaching secondary mathematics. I regularly had to modify my SD modeling requirements as my students continually exceeded my expectations. They taught me so much!

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