2018

Ideas and Graphs: the Tetrad of Activity

Martin Zwick
Portland State University, zwick@pdx.edu

Citation Details
Zwick, Martin (2018). “Ideas and Graphs [Post-print], “International Journal of General Systems, 47 (7): 731-750. https://doi.org/10.1080/03081079.2018.1510921

Ideas and Graphs: the Tetrad of Activity

Martin Zwick

A graph can specify the skeletal structure of an idea, onto which meaning can be added by interpreting the structure. This paper considers several directed and undirected graphs consisting of four nodes, and suggests different meanings that can be associated with these different structures. Drawing on John G. Bennett's "systematics," specifically on the Tetrad that systematics offers as a model of "activity," the analysis formalizes and augments the systematics account and shows that the Tetrad is a versatile model of problem-solving, regulation and control, and other processes. Discussion is extended to include hypergraphs, in which links can relate more than two nodes, and the possibility of a "reconstructability analysis of ideas" is suggested.

Keywords: graphs, ideas, tetrad, activity, John G Bennett, systematics, number symbolism, Charles Sanders Peirce, Talcott Parsons, hypergraphs, reconstructability analysis, Lattice of Structures

1. Introduction

"God made the integers; all else is the work of man - Kronecker (Bell 1986)

Graphs can be associated with ideas, different graphs with different ideas. In this paper I discuss ideas from a number of researchers in the natural and social sciences and engineering. I include graphs in which links between nodes are directed or undirected. I focus in this paper on graphs involving four nodes, and show that four-node graphs can represent the skeletal structures of different complex ideas. For four nodes considered pairwise there are 6 possible links, and each link can be present or absent in the graph, so there are $2^6 = 64$ possible undirected graphs. If one can assign a direction to each of these 6 links, then an AB link could be directed from A to B or from B to A or the direction might be left unspecified, so there are $3^6 = 729$ possible directed graphs (if bidirectional links are precluded). Only a small number of undirected or directed graphs are discussed here, but these should be sufficient to show that different ideas can be associated with different graphs.

It is useful to expand the notion of graphs to include hypergraphs, in which links can connect more than two nodes. There are 114 undirected hypergraphs for four nodes (Zwick 2004), and many more directed hypergraphs. Hypergraphs are considered in the data modeling methodology of reconstructability analysis, and the possibility of developing a "reconstructability analysis of ideas" is discussed below.

This study is based on the "systematics" of John G. Bennett (1956, 1961, 1966, 1993) which was further developed by Anthony Blake (1997, 1998, 1999) and others (Systematics.org 2018); more particularly on the concept in systematics of the Tetrad. Bennett (1897-1974) is a little-known British scientist, philosopher, and religious teacher, whose system-building efforts bear comparison with that of the more widely known philosopher, Alfred North Whitehead, who also attempted to synthesize ideas from religion, philosophy, and science. As a modern version of number symbolism, a

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

traditional mode of thought occurring widely in many cultures of both West and East, systematics has strong and explicit affinities with systems theory (Bennett 1963, 1970). Bennett refers to the categories of systematics, namely the Monad, the Dyad, etc., as "systems," and the structures of these systems to which meanings are assigned are graphs, namely nodes connected by links. In Bennett's terminology, nodes that are linked are called "terms" that are "mutually relevant"; in systems terminology, these are "elements" connected by "relations." In this paper, "term" and "element" are used interchangeably, as are "link" and "relation." A set of relations or a relation involving all of the terms is referred to as a "system."

Central to Bennett's undertaking is the hypothesis that a "natural" meaning might be associated with a multi-term relation that depends only on its ordinality, i.e., the number of its constituent terms. A corollary to this hypothesis is that embedded lower ordinality relations and sets of these sub-relations can be associated with meanings that derive from the meaning of the overall multi-term system.

Bennett's hypothesis might be validated deductively or inductively. A deductive approach would require that one derive the natural meaning of the system directly from the terms and their relation. This would entail deducing a particular *model* of a formal system from the formal system itself, i.e., deriving semantics from syntax. One cannot imagine doing this if one considers the formal system alone, but if one considers that this formal system is used by human beings, and if one believes that evolution has conferred upon our cognitive apparatus some built-in patterns of interpreting experience (as proposed, for example, by Kant or, more recently, by Chomsky), then an inherent connection between syntactic structure and meaning is not inconceivable. After all, we experience meaning in music, and this is no doubt connected to the mathematical structure of the music (and to the human mind and body). An inductive approach, by contrast, would require a survey, in the cultural expressions of different societies, of occurrences of relations among a particular number of terms. If this approach found meanings that were universal or at least ubiquitous, then such meanings would be candidates for the "natural" meanings of relations with particular ordinalities.

Bennett's systematics was not deductive. It was partially inductive, as he mined the literatures of different cultures for ideas useful to his project, but the degree to which he was able to give empirical support to the meanings that he assigned to particular integers varies considerably over the different integers. Mainly, his approach was intuitive, although the extensive use that he made of systematics in his magnum opus, *The Dramatic Universe,* attests to the flexibility and generativity of this philosophical framework.

While systematics was developed outside the context of contemporary philosophy of science, its Monad, Dyad, and Triad resemble Charles Sanders Peirce's (1868) notions of Firstness, Secondness, and Thirdness. The precise similarities and differences between these three categories of Bennett and Peirce, the progression for Bennett from the Monad to the Dyad, Triad, and Tetrad (Bennett goes up to twelve, the Duodecad), the progression for Peirce from Firstness to Secondness to Thirdness (Peirce stops at three), and the question of whether a category of Fourthness could consistently and productively be added to Peirce's framework will be the subject of future explorations. The focus of the paper is only on one category of Bennett's: the Tetrad. This category by itself, however, can take on multiple graph structures. These different structures can be associated with different meanings, and these associations illuminate the elusive relationship between syntax and semantics. The graph structures presented in this paper are summarized in Table 1; all the examples come from the (natural and social) sciences.

Table 1 Examples of the Tetrad discussed in this paper

SOURCE		SUBJECT
Arrow	Figure 3	decision theory: aggregation of ordinal preferences
Parsons	Table 4, Figure 5	action: in general & in society and social systems
Ozbekhan	Table 5	planning: policy, decision making, administration
Kauffman, Zwick	Figure 6	scientific categories: matter/energy/information/utility
Browder	--	kinds of mathematics
Zwick	Figure 7	societal fundamentalisms
Wiener	Figure 8	feedback control: illustrated by thermostat
Miller	Figure 9(a)	adaptive decision/control
Ashby, Zwick	Table 6, Figure 10(a)	genesis of control
MacLean	Figure 10(c)	triune brain model
Lendaris	Figure 10(d)	neural networks: approximate dynamic programming
Diamond	Figure 11	diachronic adaptive failure
Jenkins	Table 7	phases of systems engineering

2. The Tetrad of systematics; applications

2.1 System and its terms

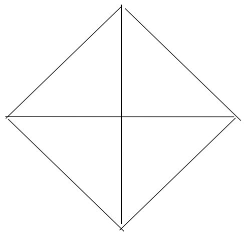
The literature of Systematics offers varying formulations of the Tetrad which differ in details. Figure 1 is a close approximation to Bennett's representation of the Tetrad. The four elements ("terms") in this system are ground (actual), instrument (practical), direction (theoretical), and goal (ideal), labelled A, B, C, and D, respectively.

The six undirected links ("interplays") between the elements are here labelled AB, AC, AD, BC, BD, and CD. When links are directed, labels are underlined, e.g., $\underline{AB}$ means $A \rightarrow B$ and $\underline{BA}$ means $A \leftarrow B$. AB and BA, when *not* underlined, however, are equivalent, and do not imply any directionality. A graph consisting of multiple dyadic links has these links separated by colons (":"). Thus, $\underline{AB}$:$\underline{BC}$:$\underline{CD}$ means the specific directed graph $A \rightarrow B \rightarrow C \rightarrow D$, while AB:BC:CD means a path either from A to D or from D to A or one of six other possible meanings. (There are three links and two possible directions for each link, so there are eight possibilities if one disallows bidirectional links.)

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

Figure 1 Tetrad (Bennett 1966) [1]

Four-term System:	TETRAD
Systemic Attribute:	ACTIVITY
Term Designation:	SOURCE
Term Characters:	
MOTIVATIONAL:	*ground* (actual)
	goal (ideal)
OPERATIONAL	*direction* (theoretical)
	instrument (practical)



1st Order Connectivities: INTERPLAYS (The six interplays are lines in the diagram.)

"Activity," the system attribute for the Tetrad, means activity that is purposive. Although such activity might refer to the behaviour of any organism, and might even be applied to processes that do not involve living systems, Bennett presents the Tetrad, as well as the other categories of systematics, primarily in the context of human action. This is suggested by the basic distinction he makes between motivational (ground, goal) and operational (instrument, direction) terms and by his correlation of the terms of the Tetrad with Aristotle's Four Causes (Bennett 1966). This correlation is shown in Table 2 with an alternative possible correlation.

Table 2 Two correlations of the Bennett's Tetrad with Aristotelian causes

		Bennett	(alternative)
goal	Ideal	Formal	Final
direction	Theoretical	Final	Formal
instrument	Practical	Efficient	Efficient
ground	Actual	Material	Material

2.2 Interplays and partitions

Between the limits of the overall system (tetradic relation) and the four individual terms, there are many structures that involve undirected or directed relations between pairs or – if one allows hypergraphs – triplets of terms. About these structures Bennett speaks only of undirected dyadic relations ("interplays"). The six interplays are listed in Table 3; the interplays most salient for Bennett are the vertical and horizontal axes of motivation (ground-goal) and operation (instrument-direction), respectively.

---

[1] Bennett puts instrument on the left and direction on the right. I prefer to reverse these locations because goal→ground more closely parallels direction→instrument than instrument→ direction since English is read from top to bottom and from left to right.

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

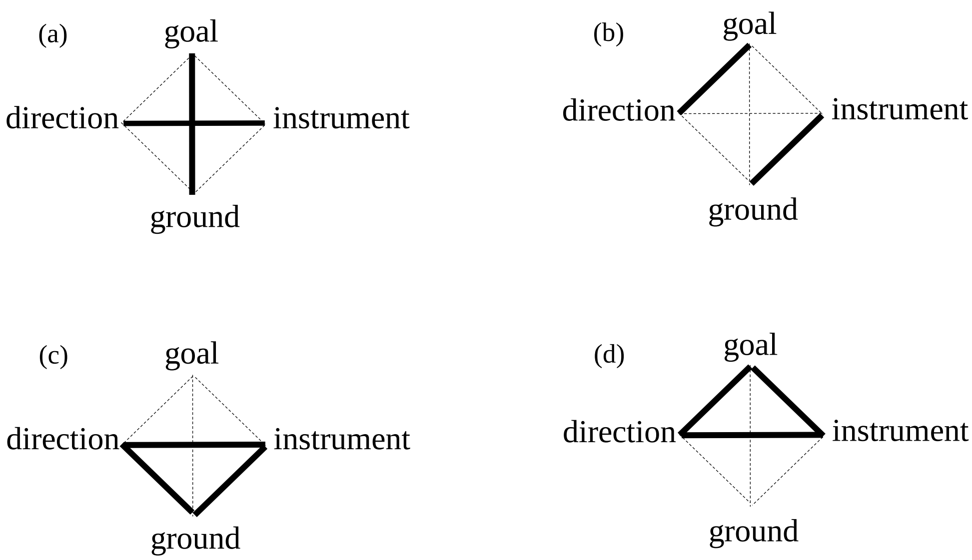
Table 3 Interplays (Bennett)

ground-goal	AD	Motivation
ground-direction	AC	Governance
ground-instrument	AB	Skill
goal-direction	DC	(not given by Bennett, but suggested here: Understanding)
goal-instrument	DB	Integrity
direction-instrument	CB	Operation

As a conjunction of these two axes, the Tetrad has the graph structure AD:BC, i.e., ground-goal : direction-instrument, shown in Figure 2(a). There are other 2:2 partitions possible. For example, structure AB:CD, i.e., ground-instrument : direction-goal, shown in Figure 2(b), is mentioned below in the section that discusses Talcott Parsons' theory of action.

Figure 2 Partitions

(a) & (b) are 2:2 partitions; (c) & (d) are 3:1 partitions



One can consider also 3:1 partitions. Bennett displays the partition, ground-instrument-direction: goal, as a tetrahedron with a triangular ground-instrument-direction as its base and goal as its apex; this is shown in Figure 2(c). In this figure the apex is connected to the base by dotted lines to suggest the pyramidal structure, but since these structures are partitions, the apex is not actually linked to the base. Apex and base might represent a distinction between an ideal which may not yet exist, and a triadic ground-instrument-direction relation which does exist. This triad might then be considered to expand the first term of the ground-goal motivation interplay: more exists than the actual state of affairs (ground) and this more (direction and instrument) provides the possibility for the ground to be transformed into the goal. In this first partition, ground-instrument-direction is a subgraph labelled ABC.

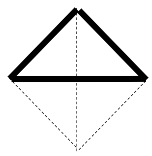
Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

The second partition, shown in Figure 2(d) and written as BCD:A, is useful for describing the Tetrad as a model of control. This partition distinguishes between the system that is controlled (ground) and a controlling triad (goal-direction-instrument). Instrument is the term in this triad that is in direct contact with ground. The purpose of control is to bring ground in conformity to goal. The strategy (theory) that governs the application of instrument to ground is provided by direction.

This same 3:1 partition can also be useful in applying the Tetrad to decision theory, more specifically to the aggregation of ordinal preferences that is the subject of the Arrow Impossibility Theorem. Arrow (1950) showed that for ordinal, as opposed to cardinal, utilities, when there are more than two alternatives, one cannot aggregate individual preferences among alternatives in a way that always yields a collective preference that is simultaneously decisive, egalitarian, and rational. An aggregation procedure that is decisive is one that results in a definite choice, i.e., that is not subject to ties or deadlocks; this excludes, for example, allowing individual voters veto power. An aggregation process that is egalitarian is one that accords equal weight to the preferences of individual voters (or to groups of voters of the same size). An aggregation process that is rational is one that is transitive and whose outcome is insensitive to the voting agenda and to "excluded alternatives."[2]

A graph depicting this three-way conflict pointed out in Arrow's theorem is shown in Figure 3. Here, ground is the set of preferences that need to be aggregated. The upper triad reflects the three conflicting requirements for a successful aggregation. The requirement for decisiveness is practical; the requirement for being egalitarian is ideal; the requirement for being rational is theoretical.

Figure 3 Arrow impossibility theorem



[2] Transitivity means that if there is collective preference for A over B and for B over C, there must be collective preference for A over C. Insensitivity to agenda means that if voting occurs in steps, e.g., first between two of these alternatives and then between the winner and the third alternative, the result should be independent of which pair of alternatives is considered first. Insensitivity to excluded alternatives means that the outcomes should not depend on how voters rank alternatives with no realistic possibility of being collectively chosen.

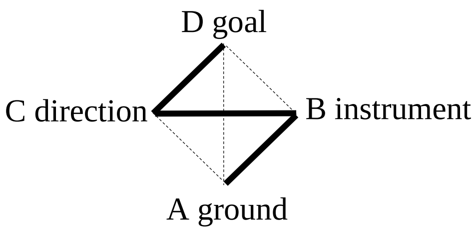
Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

2.3 Hierarchy

Interplays are only pairwise relations, and partitions divide the system into subgraphs. Graphs that link all four terms are obviously also of interest, and the simplest of these are sequences that order the four terms lineally. There are 4*3*2*1 = 24 such directed sequences. Bennett does not explore these tetradic sequences, but he does explore all six sequences for the Triad. He represents the terms of the Triad by numbers 1 (active), 2 (passive), and 3 (neutralizing), and gives specific interpretations to the triadic sequences 123, 132, 213, 231, 312, 321. This paper does a similar analysis of several directed four term sequences and by doing so supplements the systematics literature on the Tetrad.

One important sequence that *is*, however, implicitly discussed in this literature arrays the terms of the Tetrad in a hierarchical dimension with ground and goal at its limits and instrument and direction at intermediate points. This is shown in Figure 4 as an undirected graph which can be read going up from ground to goal (AB:BC:CD) or going down from goal to ground (DC:CB:BA). The undirected graph AB:BC:CD (where the individual relations are undirected and the order of relations is also arbitrary) can represent either direction or both. The zig-zag path in Figure 4 conveys an additional non-hierarchical idea: although direction is closer to goal and thus higher than instrument which is closer to ground, there is a secondary sense (in the idea of a motivational axis) in which direction and instrument are on the same level. The hierarchical sequence of Figure 4 is actually not explicitly given by Bennett, but is implicit in his discussion of the Tetrad, and features prominently in Blake's work.

Figure 4 Hierarchy



2.3.1 Action (Parsons)

A clear example of this AB:BC:CD hierarchy is Talcott Parsons' theory of action (1966, 1971). Parsons writes: "Action consists of the structures and processes by which human beings form meaningful intentions and, more or less successfully, implement them in concrete situations." Although Bennett's idea of "activity" is broader than Parsons' notion of "action," since the former might apply to behaviour of other organisms and even to some non-living phenomena, most of Bennett's examples are in fact also drawn from the human sphere.

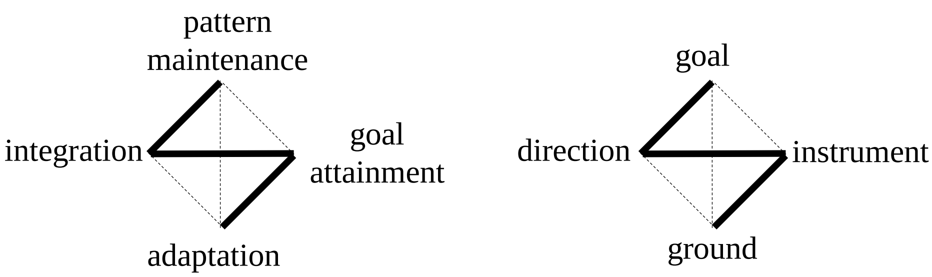
The first column of Table 4 lists the elements of action in general. The second column gives Parsons' interpretation of these elements as they occur in what he called Societies. One element of Society is the Social System, relative to which Culture, Personality, and Organism are environments. The third column applies Parsons's scheme recursively to elements of the Social System. The parallelism between Parsons' action and Bennett's activity is shown in Figure 5.

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

Table 4 Parsons' systems of action
The columns are hierarchies: the first row is the top; the fourth row is the bottom. At high levels information is salient; at low levels matter/energy is salient.

Action	Society	Social System[3]
Pattern Maintenance	Cultural System	Institutionalized Cultural Patterns
Integration	Social System	Community
Goal Attainment	Personality System	Polity
Adaptation	Behavioral Organism	Economy

Figure 5 Parsons' and Bennett's Tetrads



(a) Parsons (b) Bennett

The hierarchical order in Parsons' "action" is the same as the hierarchical order of the systematics Tetrad. Descending the hierarchy produces the following sequence:

- D. Pattern Maintenance is goal; in Society it is accomplished by the Cultural System, the societal component "concerned with the … controlling patterns of the system"; in the Social System, it is accomplished by culturally determined institutions.
- C. Integration is direction, provided to the Society by the Social System and provided to the Social System by the Community.
- B. Goal Attainment is instrument, implemented for the Society by the Personality of individuals – Parsons notes that "all action is the action of individuals" – and implemented for the Social System by the Polity. (The salient word that characterizes this component is "attainment," not "goal," which here has the narrow sense of specific objectives.)
- A. Adaptation is ground, performed for Society by the Behavioral Organism "which adapts to the broad conditions of the … physical environment," and performed for the Social System by the Economy. Adaptation partakes of the character of goal, but constitutes a lower end in contrast to the higher end of Pattern Maintenance.

[3] Parsons views his scheme as fractal, i.e., his action tetrad can manifest at different scales.

Parsons assigns high information to the top of his hierarchy and high energy to the bottom, these two interacting via cybernetic relations. In cybernetic control, he writes, “systems high in information but low in energy regulate other systems higher in energy but lower in information.” The sequence going down thus specifies a “hierarchy of controlling (informational) factors”; the sequence going up specifies the “hierarchy of conditioning (matter-energy) factors.” Parsons’ writings also make significant use of the pairwise interplays (to use Bennett’s terminology) between his four action terms. An exploration, joining the ideas of Bennett and Parsons, of distortions of the societal Tetrad is offered in (Zwick 2014).

Applying the 2:2 partition shown in Figure 2(b) to Parsons’ Tetrad for the Social System, gives the partition economy-polity: community-culture. The lower pair of terms corresponds to what Habermas (1987) called “the system”; the upper pair corresponds to what he called the “lifeworld.” In the idea of “socio-technical systems,” pioneered by Emery and Trist (1965), Habermas’ “system” is the technical part, while Habermas’ “lifeworld” is the socio part.

2.3.2 Planning (Ozbekhan)

A framework proposed by Ozbekhan (1971) for “planning as a hierarchical system” exhibits the AB:BC:CD hierarchical order shown in Figure 4, the downwards cybernetic control spoken of by Parsons, and the 1:3 partitioning of a system controlled at three levels of Figure 2(d). Ozbekhan’s framework is summarized in Table 5.

Table 5 Planning hierarchy (Ozbekhan)

Bennett	Ozbekhan	
goal	Self-Organizing Level (Normative)	policy making
direction	Optimizing & Learning Level (Strategic)	executive decision making
instrument	Control Level (Operational)	administrative functions
ground	Process	

The normative level (goal) is concerned with determining ends: what “ought” to be done. The strategic level (direction) concerns the relationship between “known options and their possible alternative consequences,” namely what “can” be done; this clearly requires some model of the entire control process. The operational level (instrument) is concerned with implementation: the “how” of what is to be done. Ozbekhan describes the structure of the plan and the working of controls at the three structural levels as follows [levels are labeled as they are elsewhere in this paper]:

> The structure of the plan has three hierarchically related levels:
> B. an operational level at which the plan is mainly mechanistic in character
> C. a higher strategic level at which the plan is anticipatory in character
> D. a still higher normative level at which the plan is telic in character…

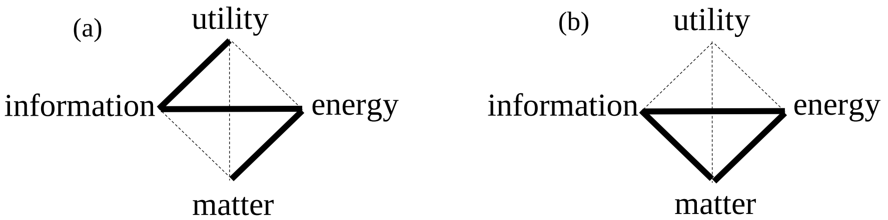
These controls work in the following way:
D. the normative plan, to fulfill its specific function, depends on the operations of all (the levels below it…;
C. the strategic plan, to fulfill its specific function, relies on the mechanisms of the operational plan and on the environmental inputs below it…;
B. the operational plan, to fulfill its specific function, relies on inputs from the environment…

2.3.3 Matter-Energy-Information-Utility

Parsons' cybernetic hierarchy calls to mind a more abstract application of the Tetrad that answers the question that the systems theorist Stuart Kauffman once posed (1998): "Matter, energy, information ... [and then] what?!" Historically, these three scientific categories emerged sequentially. Interest in the underlying nature of materiality can be traced back to the Greeks; thermodynamics, the science of energy, was developed in the 19th century; information as a scientific category was not recognized until the middle of the 20th century. Kauffman wondered what new categories might supplement this triad. An answer offered here – a 4th term – is "utility," whose conceptualization occurred at the same time as the conceptualization of information as a basic category. In the crystallization of the systems movement after World War II, Information (Communication) Theory of Shannon and Weaver (1949) formalized notions of information and Game and Decision Theory of von Neumann (1944) formalized notions of utility.

Bennett's Tetrad can be used to organize these four categories into a whole, shown in Figure 6(a): matter is ground (material cause), energy is instrument (efficient cause), information provides direction (formal cause), and utility is goal (final cause). Energy governs transformation of matter, information governs energetic interactions, and, in the domain of living systems the pursuit of utility – evolutionary fitness – governs the generation and utilization of information. This upwards hierarchy, <u>AB</u>:<u>BC</u>:<u>CD</u>, which reflects the historical sequence of scientific acquisition of these basic categories, captures basic relations among them, and amplifies Parsons' cybernetic ideas.

Figure 6 Scientific categories



The notion of "utility" as a fourth fundamental category in scientific explanation also bears on another question posed by Kauffman (1998): "What is required to be able to say that a system 'acts on its own behalf'?" The answer, again, is utility, whose evolutionary variant, fitness, expresses the idea of action by a system on its own behalf or on behalf of similar systems. Utility is quintessentially biological, but matter, energy,

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

and information apply also to non-living systems. In evolution, utility is the end, relative to which information-energy-matter are means. Figure 6(b), the utility:matter-energy-information partition, which exemplifies the generic goal : ground-instrument-direction partition of Figure 2(c), captures the structure of this idea.

2.3.4 Kinds of Mathematics (Browder)

Felix Browder (1975), writing on the nature of mathematics, proposed four kinds of mathematics which also manifest the hierarchy of Figure 4. Mathematics-I

> includes all the counting, measuring, and calculation which is part of the life process for almost all human beings in our society, as well as the systems of calculation and measurement which underlie the organization of every economic system beyond the most primitive stage where money is introduced.

This is mathematics as ground (A). Mathematics-II is "the use of known mathematical techniques and concepts to formulate and solve problems in other intellectual disciplines. In terms of day-to-day practice, this is the primary function of mathematics in the physical sciences and more recently in the biological and social sciences." This is mathematics as instrument (B). Mathematics-III is "mathematical research…the investigation of the concepts, methods, and problems of the diverse mathematical disciplines." This is mathematics as direction (C). Mathematics-IV is "the vision of mathematics as the ultimate and transparent form of all human knowledge and practice." This is the Pythagorean and Platonic vision of the mathematical forms as an intellectual goal (D).

3. Other directed tetradic graphs

There are many other tetradic graphs besides the hierarchical AB:BC:CD. Among these are graphs that have "leading parts" and graphs that represent lineal paths.

3.1 Leading parts

3.1.1 Societal fundamentalisms

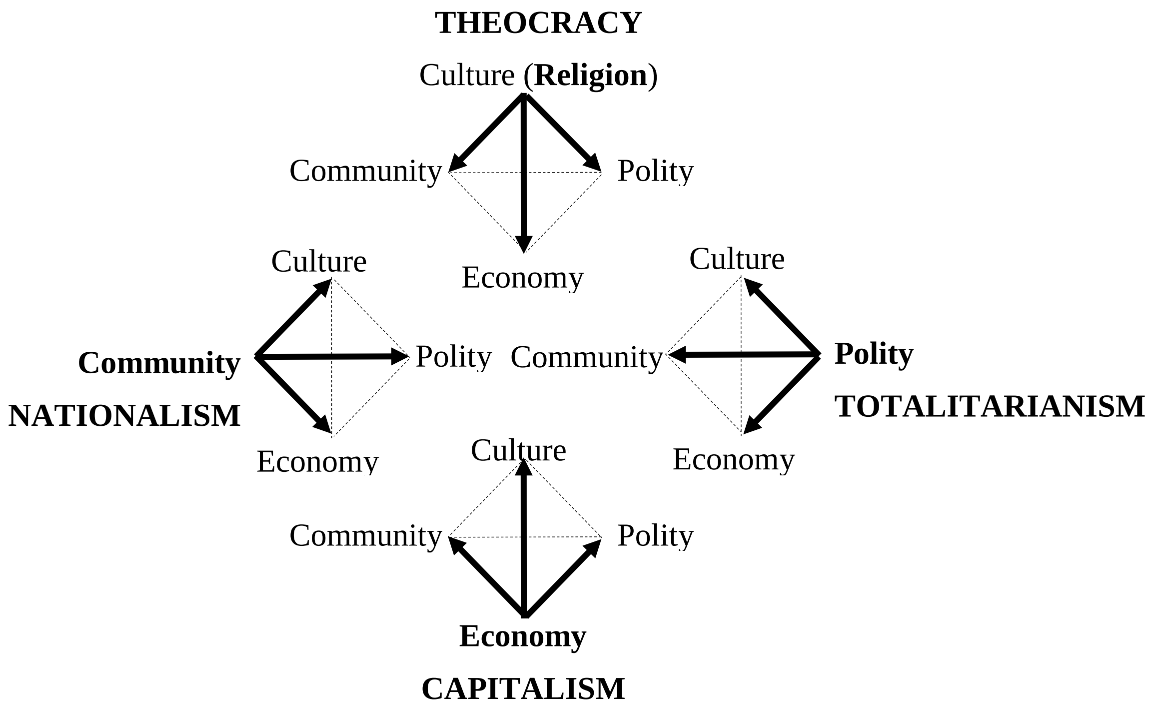
Von Bertalanffy (1979) noted that some systems show "leading parts," elements that are more important than other elements. For Parsons, an ideal Society would reflect some optimal balance of differentiation and integration of the Tetradic elements. Each would have some autonomy, but each would also be constrained by the others.[4] The presence of a leading part, however, would represent the dominance of one element over the others, and reflect a "fundamentalism." The fundamentalisms produced by the hegemony of each element of Parsons' Tetrad are shown in Figure 7.

---

[4] Deviations from an ideal balance of autonomy and mutual constraint are the subject of the study of Parsons' tetrad from the perspective of Bennett in (Zwick 2014).

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

Figure 7 Leading parts & fundamentalisms



In Totalitarianism, Polity (instrument) is the leading part: government controls society, economy, and culture; this graph would be written as BA:BC:BD. The Soviet Union under Stalin was an example. In "pure" Capitalism, Economy (ground) would be the leading part, represented by the graph AB:AC:AD (democratic Capitalism, however, represents a fusion of two organizing principles, one centered in the economy, the other defined by the Community-Polity axis). In Theocracy, Culture (goal), more specifically religion, is hegemonic, as depicted by graph DA:DB:DC. A society whose leading part is Community (direction), shown as CA:CB:CD, might represent a fundamentalism of Nationalism. These are sociological *ideal types*. Societies that perfectly illustrate any of these individual types are rare; actual societies are mixtures. Still, one can sometimes identify a leading part in societal structures, one element of the Tetrad that is more salient than the other three.

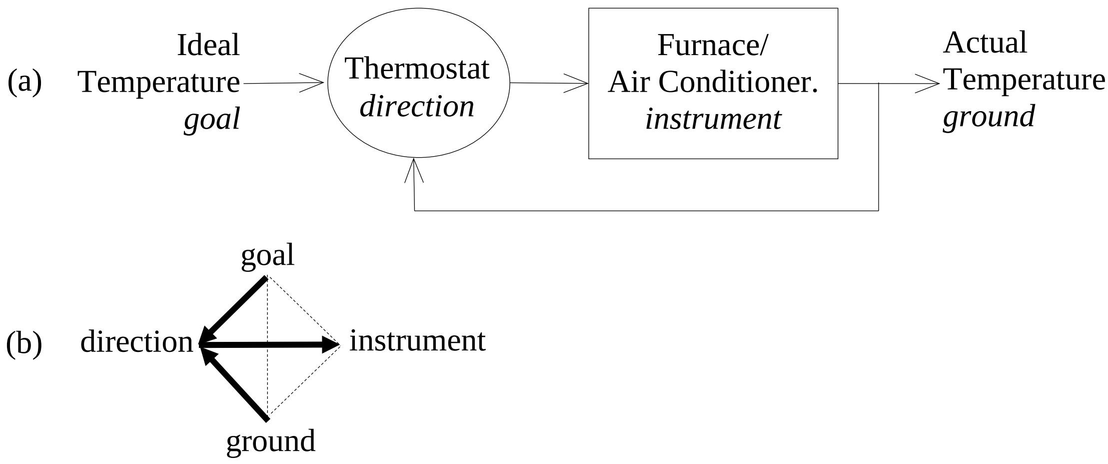
3.1.2 Feedback Control

A leading part does not have to represent a distortion due to hypertrophy; it may alternatively represent a benign centralization. For example, direction is the leading part of error-controlled feedback systems, exemplified by the thermostat, shown in Figure 8(a). In the thermostat, the goal is the ideal temperature, which is an input to the thermostat control unit. The ground is the actual temperature, also an input to this unit. The unit itself provides the direction, i.e., the governance of the process, and it does so by sending instructions to an instrument, such as a furnace or air conditioner. Note that the lineal sequence (ignoring feedback) of goal-direction-instrument-ground is the hierarchical order for the Tetrad, and that the centrality of direction in this feedback control system accords with this term – "thermostat" – being used as emblematic of the whole system. The graph, DC:AC:CB, for the thermostat is shown in Figure 8(b), and one could add a BA relation to this path to indicate the action of instrument on ground.

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750. https://doi.org/10.1080/03081079.2018.1510921

Figure 8 Thermostat system

(a) as a negative feedback system; (b) as a graph

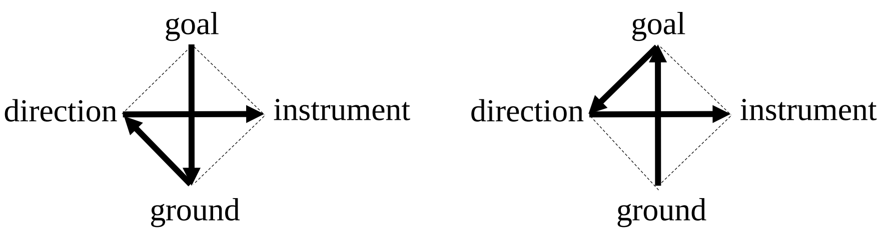


3. 2 Lineal paths

3.2.1 Control (Miller), problem solving

Another interesting and simple type of graph is a lineal order of elements. For example, "control" and "problem-solving" have similar lineal graphs, <u>DA</u>:<u>AC</u>:<u>CB</u> and <u>AD</u>:<u>DC</u>:<u>CB</u>, respectively, shown in Figure 9.

Figure 9 Control, problem solving



(a) Control (b) Problem solving

James G. Miller (1965) writes about the first of these:

> Every adaptive decision is made in four stages: (i) Establishing the purpose or goal whose achievement is to be advanced by the decision; (ii) analysing the information relevant to the decision [ground]; (iii) synthesizing a solution selecting the alternative action or actions most likely to lead to the purpose or goal [direction]; and (iv) issuing a command signal to carry out the action or actions [instrument].

"Adaptive decision" is here called "control." In control, goal is compared to ground, and the difference between them is fed to direction, which provides instructions to instrument. If one augments <u>DA</u>:<u>AC</u>:<u>CB</u> by adding the fact that instrument alters ground, after which the process repeats, this changes the graph to <u>DA</u>:<u>AC</u>:<u>CB</u>:<u>BA</u>.

Problem-solving is similar to control, except that it begins with a ground that is unsatisfactory. This ground is compared to a goal which is preferred. While control is initiated by the intention to achieve an explicit ideal that is potential, problem-solving is initiated by the desire to correct something that is actual. The transition to direction and then to instrument are identical in both control and problem-solving, but beginning with – that is, emphasizing – the actual in problem-solving is different from beginning with the ideal in control. In the language of political change, the problem-solving orientation generally motivates ameliorative reform; the orientation towards control represents the aspirations of utopian or revolutionary action. Problem solving in Figure 9(b) is shown as $\underline{AD}$:$\underline{DC}$:$\underline{CB}$; it too can be augmented by adding the action of instrument on ground, to give $\underline{AD}$:$\underline{DC}$:$\underline{CB}$:$\underline{BA}$, which is a cycle.

Control and problem solving both include a theory $\rightarrow$ practice (direction $\rightarrow$ instrument) relation. The *reverse* relation, namely practice $\rightarrow$ theory, is central to learning, modeling, and theory building, processes whose goal is realization of values (normative modeling) or improvement of understanding (descriptive modeling). In such processes, one compares ground and goal, then moves to instrument (e.g., practical investigations), and from there to direction (theory).

3.2.2 Genesis of control (Ashby, MacLean, Lendaris)

In Figure 2(d), control is represented as an instrument-direction-goal triad that governs some ground that is being controlled, and in Figure 8(a), the sequence of steps through which control is implemented is specified by a lineal graph. Both of these figures are synchronic, describing a control system that is already present. How this system comes about – its diachronics – is also of interest, and can also be modelled by the Tetrad.

Control comes into being historically. There is first some underlying process or ground, either internal or external, and then the possibility of control through some instrument. One might posit that instrument initially is blind, with external (natural) selection causing the survival of instrumental responses which are fortuitously effective. At this stage, there is no internal representation of the effectiveness of instrumental action. Natural selection preserves those responses which are adaptive and thus performs the role of goal, but this performance is external to the system. If there is time for several possible responses by the instrument to be tried, and if responses that don't achieve the goal are not lethal, it can be valuable for the system to have some internal representation of states that are preferred, since if such a representation exists, the instrument could try different actions randomly, sticking with an action that achieves the goal but randomly trying another action otherwise. This "trial and error" learning, in Ashby's terminology (1952, 1956), is called "Hunt and Stick Regulation." If the system has a capacity for memory, it can store successful responses to different environmental challenges, and the use of such memory to guide action is a primitive direction component. This evolutionary story is summarized in Table 6, and the graph showing the sequential addition of terms is shown in Figure **10**(a).

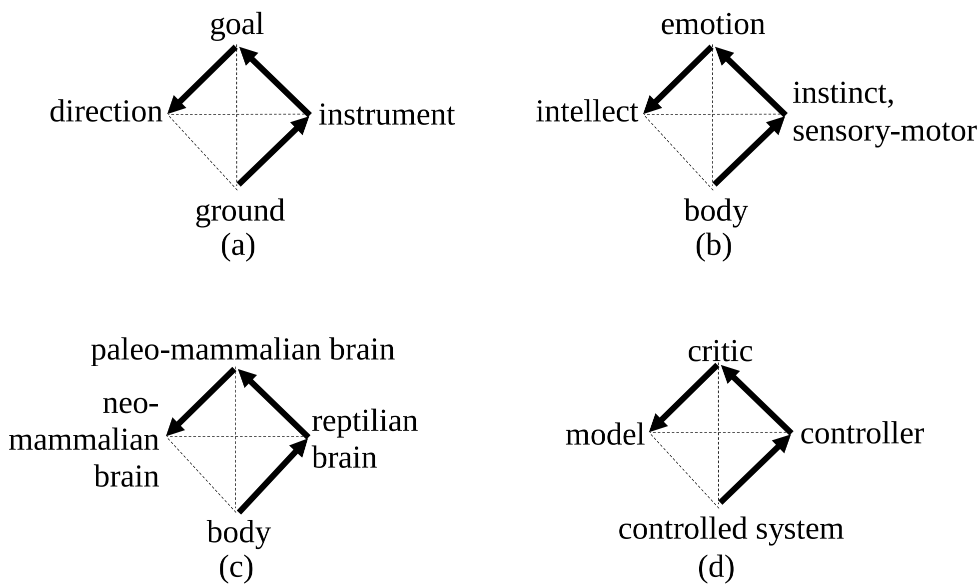
Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

Table 6 Evolutionary genesis of control

Stages	
4. ground-instrument-goal-direction	Hunt & stick regulation with memory
3. ground-instrument-goal	Hunt & stick regulation
2. ground-instrument	Adaptation through natural selection
1. ground	--

Figure 10 Genesis of control

(a) General scheme; (b) Cognitive system; (c) MacLean brain model; (d) Approximate dynamic programming (Lendaris)



In higher organisms, one might see the sequence shown in Figure **10**(a) in the relation to body of instinct and sensory-motor function, emotion, and intellect. Body is ground. Instinct and sensory-motor function is instrument, which is the repertoire of early evolutionary adaptations. Adding emotion internalizes goal. Adding intellect provides direction, the possibility of modeling control and its outcomes. This is depicted in Figure **10**(b). In MacLean's (1990) triune-brain model, the evolution of the brain proceeded in precisely this sequence: first instinct and motor functions (reptilian brain) emerged, then emotional functions (paleo-mammalian brain), then intellectual functions (neo-mammalian brain). This is depicted in Figure 10(c).

A similar sequence is implicit in the neural net (NN) scheme of Approximate Dynamic Programming (ADL) (Lendaris & Neidhoefer 2004), shown in Figure 10(d). In a neural net with an ADL architecture, there are components that play the roles of instrument, goal, and direction, namely the "controller," the NN component (instrument) that interfaces directly with the controlled system (ground), the "critic," the NN component (goal) that assesses expected utilities, and the "model," the NN component (direction) that models the effects of the controller on the controlled system.

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

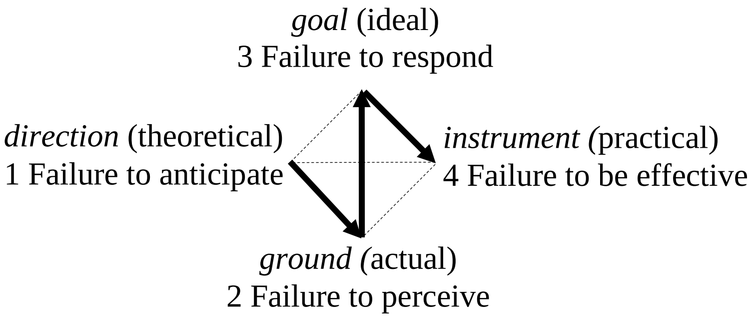
A minimal control system would involve just a controller (instrument). Adding a critic (goal) would augment the power of this minimal control system. The further addition of a model component (direction) would augment the power of this system still further. The diachronic sequence, AB, then AB:BD, then AB:BD:DC, models the evolutionary genesis of control.

3.2.3 Diachronic adaptive failure (Diamond)

Systems do not always successfully adapt. They may fail to control what needs to be controlled; they may fail to solve problems that need to be solved. Synchronic adaptive failure might be attributed to errors in specification of what is (ground), or what should be (goal), or how what should be might be accomplished (direction), or the means by which this can be achieved (instrument). Synchronic adaptive failure can result from errors in any of these.

Diachronic adaptive failure, however, begins with failure to anticipate the future. This is an inadequacy of theoretical understanding, a failure in direction. Beyond this failure, there may be the failure to perceive what is actually occurring; this is a failure in ground. Or to respond to what is perceived; this is a failure of goal. Or if there is a response to the perceived gap between actual and ideal, the response may not be effective; this is a failure of instrument. This CA:AD:DB scheme, from Jared Diamond (2005), is shown in Figure 11

Figure 11 Diachronic adaptive failure (Diamond)



4. Hypergraphs

4.1 Representation

In the discussion above, examples of the Tetrad are shown as graphs, in which nodes have only dyadic links. However, in four term systems there is the possibility of triadic links which can be considered if the analysis is generalized to hypergraphs, which allow such links. (Aside from the Tetrad itself, which is the holistic integration of the four terms, there are no other tetradic relations.) The 3:1 partitions of the Tetrad were shown in Figure 2(c) & (d) as three dyadic links plus a monad, but these partitions have a more general representation as a triadic link plus a monad: the partition goal : direction-instrument-ground is D:CBA, and the partition ground : instrument-direction-goal is A:BCD. Similarly, the thermostat graph was shown in Figure 8(b) as the graph AC:DC:CB, but it is better modelled as the hypergraph ADC:CB, since the thermostat unit does not actually have separate dyadic relations with the two inputs, but rather integrates the two via a triadic relation. One might also add to this BA, the action of instrument on ground, to give the hypergraph ADC:CB:BA.

Zwick, Martin (2018). "Ideas and Graphs, "International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

A different representation will be used here for hypergraphs: boxes will represent links and lines will represent nodes; this is illustrated in Figure 12 for the 3:1 partitions and for the thermostat. The box-line representation in this figure depicts the hypergraph topology without specifying the identities of the elements. Such topologies are here called "general" structures. If one in addition specifies the identities for all the elements, i.e., labels the lines, one obtains the "specific structures" encompassed by the general structure. One general structure thus typically encompasses multiple specific structures, where these specific structures involve permutations of the terms. (However, the specific structure ABC:ABD:ACD:BCD has no different permutation, so its general structure has only this one instantiation.) Also, each general or specific structure, if undirected, encompasses multiple directed structures.

Figure 12 Box (relation) and line (element) system representations

(a) A simple neutral hypergraph with a triadic relation; (b) a directed hypergraph

(a) (b)

4.2 Systems Engineering (Jenkins)

Another illustration of hypergraph structures is afforded by Jenkins' (1976) description of the "broad stages in the development of a systems engineering project." Jenkins speaks of four phases, summarized in Table 7. The first phase "starts with a common-sense analysis of what is going on, and why, and whether it might have been done better. Then the system and its objectives have to be defined and data gathered about its likely performance." This is ground, but since Jenkins also includes in this phase the specifications of objectives, this phase also involves goal. In the second, theoretical, phase, "a quantitative model has to be built and used to simulate or explore a number of different ways of operating the system, finally ... optimizing the system." This is direction. The third phase concerns the practical, i.e., implementation, realizing the system to be built and checking it for performance, reliability, etc. This is instrument. In the final phase, the system is in routine operation. "The effectiveness of the operational system will need to be assessed, and if unsatisfactory, the system 'tuned', or reoptimized, to operate in an environment which may turn out to be different from that for which it was designed." If the targeted goal is not reached, ground (the environment in which the system operates) is reconsidered. Because the first and fourth stages include both ground and goal, the process is summarized in the hypergraph ADC:CB:BDA, which has two triadic relations and one dyadic relation.

Table 7 Phases of Systems Engineering (Jenkins)

4. Operation	*goal (ground)*
3. Implementation	*instrument*
2. Systems Design (Synthesis)	*direction*
1. Systems Analysis	*ground (goal)*

5. The reconstructability analysis of ideas

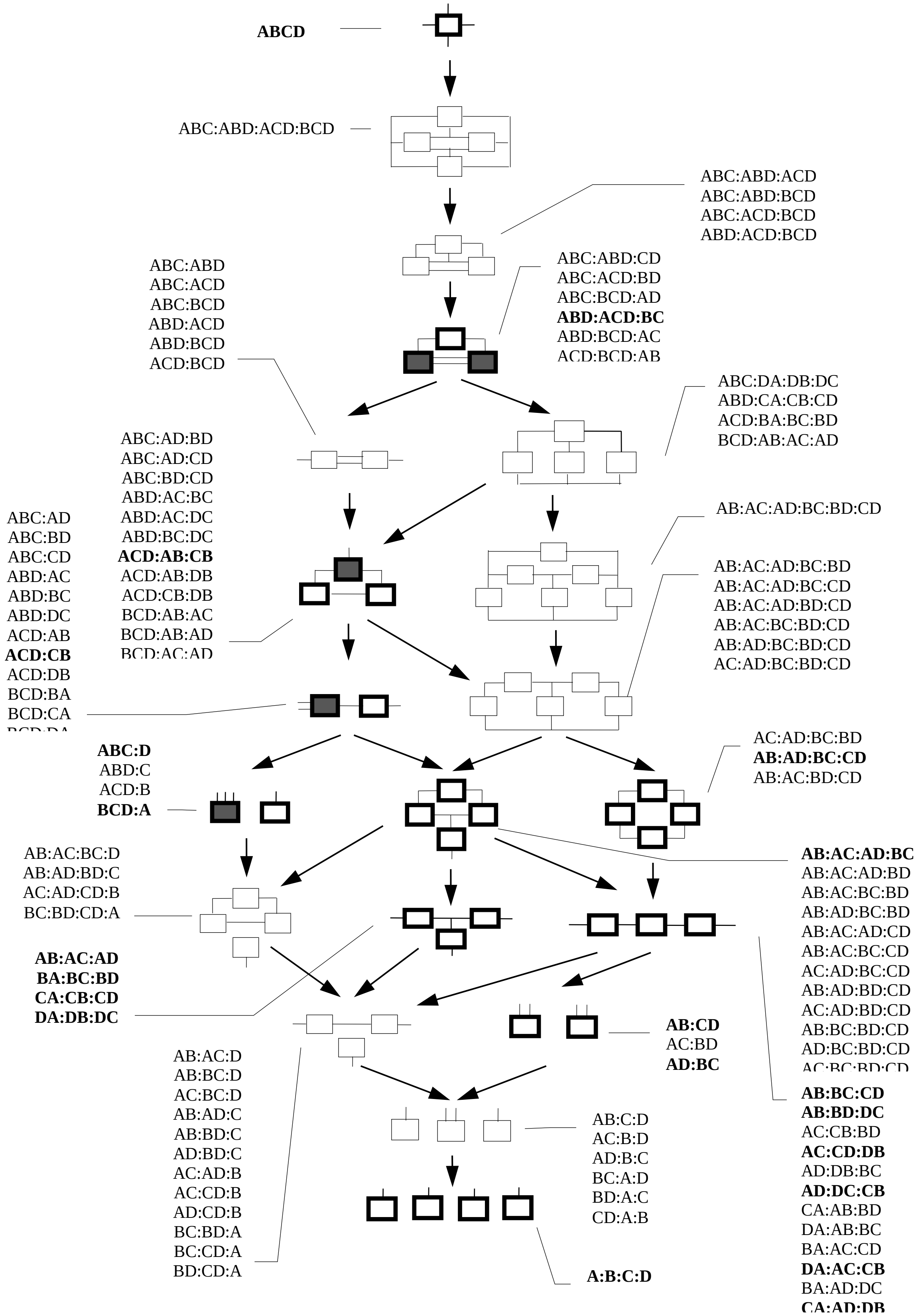
The undirected general and specific structures for four elements (terms) are shown in the Lattice of Structures of Figure 13. This Lattice can be thought of as either the set of downward decompositions of the top structure, which is the tetradic system as a whole, or the set of upwards compositions from the bottom structure of the lattice, in which the four elements are independent of one another. In Figure 13, boxes and lines depict general structures. Lines are elements (terms); boxes are relations between elements. A line may branch to connect to multiple boxes, but as long as it is not interrupted by a box, all parts of this branching are the same element. To each general structure in Figure 13 there is attached a list of the specific structures included in the general structure. For four elements, there are 20 general structures and 114 specific structures. All the structures in the figure are undirected; for each undirected structure there are many directed structures, but these are not shown. The general and specific structures in bold are those discussed in this paper, in their undirected or directed forms.

The data modeling method known as reconstructability analysis (Ashby 1964; Klir 1985, 1986; Krippendorff 1986; Zwick 2004) allows one to choose one or more hypergraphs from the Lattice of Structures to model a four-variable *quantitative* multivariate data set. Here, however, the Lattice of Structures is used in a *qualitative* way. Elements are not variables and links (boxes) are not quantitative relations; rather, elements here are *concepts* and links are *qualitative* relations between concepts. The top structure in the Lattice, which plays the role of data, is here the qualitative and holistic relation between ground, goal, direction, and instrument that Bennett conceptualized as "activity." This paper associates particular hypergraphs in the Lattice with particular *types* of activity, presented in the work of various researchers.

This application of the Lattice of Structures to systematics enables the "reconstructability analysis *of ideas*." Bennett's interpretation of the six dyadic links of Table 3 only touches upon the structural possibilities of the Tetrad. The Lattice of Structures augments his approach by offering many syntactic structures that can be associated with the Tetrad. Even without assigning directions to links, there are 114 different (specific) structural possibilities; if one assigns directions to links, there are very many more. Such a "reconstructability analysis of ideas" is not necessarily tied to Bennett's interpretation of the Tetrad as "activity." It could readily be applied to any tetradic system property based on four terms, and a Lattice of Structures can also be defined for systems with other ordinalities. Systematics, which aims at the study of the composition and decomposition of ideas, is enhanced with this Lattice. Normally such study is inhibited by the fact that one cannot get semantics from syntax, but if one hypothesizes meanings for the top and bottom of the Lattice of Structures, one can recognize meanings which are plausible to associate with intermediate structures, since these intermediate structures are bracketed by structures whose meanings are available.

Figure 13 Lattice of 4-element undirected structures

Box (relation) and line (element) figures are general structures. Specific structures are listed for each general structure. Structures with boxes (relations) in bold are cited in this paper; in these structures, triadic relations are filled in with grey.



Zwick, Martin (2018). “Ideas and Graphs, “International Journal of General Systems, 47 (7): 731-750.
https://doi.org/10.1080/03081079.2018.1510921

6. Discussion

This paper samples a number of directed and undirected graphs and hypergraphs that are different structural forms of Bennett's Tetrad, and comments on the meanings that can be associated with these structures. Additional examples can be found in the systematics literature. *Elementary Systematics* (1993) offers an accessible introduction that discusses and gives examples of each number system up to the Pentad. For the source of this systematics enterprise, and to see the Tetrad embedded in a complex philosophical framework that also deploys systems with other ordinalities, see Bennett's magnum opus, *The Dramatic Universe* (1956, 1961, 1966).

This paper has three aims: (a) to provide empirical support to Bennett's intuition that the Tetrad of "activity," whose terms are ground, goal, direction, and instrument, has wide application, (b) to show that the different Tetradic graphs and hypergraphs offer syntactic structures for different manifestations of "activity," (c) to expand Bennett's framework by offering a methodology that can be applied not only to other conceptions of the Tetrad but also to conceptual systems of different ordinality.

While the Tetrad of "activity" does seem to be ubiquitous, this does not imply that tetradic structures are restricted to this particular meaning. Many other meanings of four-term systems can be found in the cultural expressions of human societies. To give just one example from a source that is close to systems thinking: the "semiotic square" of Greimas[5] has no apparent relation to Bennett's Tetrad. A question yet to be explored is whether or not such other tetradic meanings are as ubiquitous – as "archetypal" – as Bennett's "activity" [6]; also whether or not there might be some overarching framework that integrates multiple different tetrads.

Bennett's Tetrad was based mainly on intuition. While this paper offers empirical support for this intuition and expands Bennett's framework, it does not provide any deductive justification for his particular interpretation of the Tetrad and its component terms. Nonetheless, Bennett's four-term system reflects a deep insight. This is demonstrated by its generativity, which is illustrated by the variety of applications discussed in this paper. This sample of applications is not large but it shows that this particular structure is relevant to the work of such diverse thinkers as Ashby, Arrow, von Bertalanffy, Diamond, Habermas, Kauffman, Lendaris, MacLean, Miller, and Parsons.

Acknowledgements

This paper is a result of many stimulating discussions with Anthony Blake; the author's debt to him is more extensive than citations can reflect.

---

[5] The semiotic square is illustrated by the tetrad of white, black, non-white, non-black. The square distinguishes between contrariety (white vs black, non-black vs non-white), complementarity (white and non-black, black and non-white), and contradiction (white vs non-white, black vs non-black) (Chandler 2002).

[6] For example, one candidate for another archetype, the Tetrad of higher-lower (or some instantiation of this dyad) and male-female, is ubiquitous in religious symbolism.

References

Arrow, Kenneth J. (1950), "A Difficulty in the Concept of Social Welfare," Journal of Political Economy, 58(4), 328–346.

Ashby, W. Ross (1952). *Design for a Brain*. London: Chapman & Hall LTD.

Ashby, W. Ross (1956). *An Introduction to Cybernetics*. London: Chapman & Hall LTD.

Ashby, W. R. (1964) "Constraint Analysis of Many-Dimensional Relations." *General Systems Yearbook*, 9, pp. 99-105.

Bell, Eric Temple (1986). *Men of Mathematics*. New York: Simon and Schuster.

Bennett, John G. (1993). *Elementary Systematics: A Tool For Understanding Wholes*. David Seamon, ed. Bennett Books, Santa Fe.

Bennett, John G. (1956). *The Dramatic Universe: Vol. One: The Foundations of Natural Philosophy*. London: Hodder & Stoughton.

Bennett, John G. (1961). *The Dramatic Universe: Vol. Two: The Foundations of Moral Philosophy*. London: Hodder & Stoughton.

Bennett, John G. (1963). "Systematics and General Systems Theory," Systematics, 1(2). http://www.systematics.org/journal/vol1-2/GeneralSystems.htm

Bennett, John G. (1966). *The Dramatic Universe: Vol. Three: Man and His Nature*. London: Hodder & Stoughton.

Bennett, John G. (1970). "Systematics and Systems Theories," Systematics, 7(4). http://www.systematics.org/journal/vol7-4/GST.htm

von Bertalanffy Ludwig. (1979). *General System Theory*, New York: George Braziller.

Blake, Anthony (1997). "Systematics: A Method of Understanding Complex Wholes." Charles Town WV: DuVersity.

Blake, Anthony (1998). "Systematics: A Method of the Intelligible." Charles Town WV: DuVersity Publications.

Blake, Anthony (1999). "Three Essays on Systematics." Charles Town WV: DuVersity Publications.

Browder, Felix E. (1975). "Is mathematics relevant? And if so, to what?" University of Chicago Magazine, Spring 1975, pp. 11-16.

Chandler, Daniel (2002). *Semiotics: the basics*. New York: Routledge.

Diamond, Jared (2005). *Collapse: How Societies Choose to Fail or Succeed*. New York: Viking.

Emery, F.E. and Trist, E.L. (1965). "The Causal Texture of Organizational Environments." Reprinted in Emery, F., ed., *Systems Thinking*. Baltimore: Penguin, pp. 241-257.

Habermas, Jürgen (1987). The Theory of Communicative Action. Vol. II: Lifeworld and System, T. McCarthy (trans.). Boston: Beacon.

Jenkins, Gwilym M. (1976). “The Systems Approach.” In *Systems Behavior* (2nd Edition), ed. by John Beishon and Geoff Peters, New York: Harper and Row, pp.78-104.

Kauffman, Stuart (1998). Talk at International Conference on Complex Systems, Nashua, Massachusetts, Oct 25-30. Also, https://www.edge.org/conversation/stuart_a_kauffman-the-adjacent-possible

Klir, George (1985). *The Architecture of Systems Problem Solving*. New York: Plenum Press.

Klir, George (1986) “Reconstructability Analysis: An Offspring of Ashby’s Constraint Theory”, *Systems Research*, 3 (4), pp. 267-271.

Krippendorff, K. (1986). *Information Theory. Structural Models for Qualitative Data (Quantitative Applications in the Social Sciences Monograph #62*. (Beverly Hills: Sage).

Lendaris, George G. & Neidhoefer, J.C. (2004). “Guidance in the Use of Adaptive Critics for Control.” Chapter 4 in Si, Barto, Powel, & Wunsch, *Handbook of Learning and Approximate Dynamic Programming*, 97-124, New York: John Wiley IEEE Press.

MacLean, Paul D. (1990). *The Triune Brain in Evolution: Role of Paleocerebral Functions*, New York: Plenum.

Miller, James G. (1965). “Living Systems: Cross-Level Hypotheses.” Behavioral Science, vol. 10, no. 3, p. 394.

von Neumann, John & Morgenstern, Oskar (1944). *Theory of Games and Economic Behavior*. New York: Wiley.

Ozbekhan, Hasan (1971). “Planning and Human Action.” In Paul Weiss, ed., *Hierarchically Organized Systems in Theory and Practice*, Hafner, New York, pp. 123-230.

Parsons, Talcott (1966). *Societies: Evolutionary and Comparative Perspectives*. Englewood Cliffs New Jersey: Prentice-Hall, 5-29.

Parsons, Talcott (1971). *The Systems of Modern Societies*. Englewood Cliffs New Jersey: Prentice-Hall.

Peirce, Charles S. (1868). “On a New List of Categories.” Proceedings of the American Academy of Arts and Sciences, 7.

Shannon, Claude & Weaver, Warren (1949). *Mathematical Theory of Communication*. University of Illinois Press, Urbana.

https://www.systematics.org/ (2018)

Wiener, Norbert (1961). *Cybernetics*. Cambridge: MIT Press.

Zwick, Martin (2004). “An overview of reconstructability analysis.” Kybernetes, vol. 33, pp. 877-905.

Zwick, Martin (2014). "Complexity Theory and Political Change: Talcott Parsons Occupies Wall Street." In *Complexity and the Human Experience, Modeling Complexity in the Human and Social Sciences*, Paul A. Youngman & Mirsad Hadzikadic, editors, CRC Press, pp. 141-160.

Zwick, Martin (2018). “Ideas and Graphs, “International Journal of General Systems, 47 (7): 731-750. https://doi.org/10.1080/03081079.2018.1510921