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Recoil Distributions in Particle Transfer

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Classical Thomas peaks in various fast second-order particle transfer processes are quantum mechanically broadened by energy nonconservation in the intermediate states of collision. This quantum broadening is considered in observable velocity distributions of recoil particles.

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Energy nonconservation in the intermediate states of a system of colliding particles may be large if the collision time is small. Thus energy-nonconserving effects could be significant for reactions in which intermediate states play an important role at high velocities. Fast particle transfer is such a reaction. In fast particle transfer reactions it is now well known¹⁻⁸ that second Born terms, which include a sum over the intermediate states of the system, dominate over other Born terms including the first Born term, which excludes contributions from intermediate states. The theoretical prediction that second Born terms can dominate even at moderately high velocities in a certain range of projectile scattering angles was confirmed by the observation of the Thomas peak in cross sections, differential in the scattering angle of the projectile, for particle capture.⁹⁻¹² In this paper we detail the quantum broadening of classical Thomas peaks in various second-order transfer processes and give a physical interpretation. For three-body transfer, the broadening of the Thomas peak occurs along a locus determined by overall energy and momentum conservation. If a fourth particle is present this locus broadens into a ridge, and, in addition, there arises a secondary ridge whose crest corresponds to energy conservation in intermediate states of the collision. The classical Thomas peak is obtained by enforcing both overall energy and momentum conservation and conservation of energy of the intermediate states. We concentrate on velocity distributions of the recoil particle since in atomic collisions the quantum broadening for the recoil particle tends to be easier to observe than corresponding broadening for the projectile.

Particle transfer in a one-step process is classically forbidden by conservation of energy and momentum at high collision velocities. Consequently, the simplest classical mechanism for particle transfer is the two-step process proposed¹³ by Thomas in 1927. The kinematic diagram corresponding both to the classical model of Thomas and to second Born terms is shown in Fig. 1, where particle 1,

2, or 3 scatters twice. Here, since particles 1 and 2 go off together with the same velocity, the entire collision is coplanar. If all the masses and the incident velocity $\mathbf{v}$ are known, then there are six unknowns, $\mathbf{v}'$, $\mathbf{v}_f$, and $\mathbf{v}_3$ as defined by Fig. 1. Conservation of momentum gives two equations of constraint for each collision. Conservation of overall energy gives a fifth constraint. And conservation of energy in the intermediate state gives a sixth constraint. With six equations of constraint, all six unknowns may be completely determined. For example, for $p^+ + H \rightarrow H + p^+$ it is easily verified that $\alpha = (m_e/M_p)\sin(60^\circ)$, $\beta = 60^\circ$, and $\gamma = 120^\circ$, where $M' = \tilde{M}_f = M_2 = m_e$, an electron mass, and $M_1 = M_3 = M_p$, a proton mass. The standard Thomas peak at $\alpha = 0.027^\circ$ has been observed.¹²

In general, conservation of overall momentum and energy gives three equations of constraint in the four unknowns $\mathbf{v}_f$ and $\mathbf{v}_3$, namely,

$$M_1 \mathbf{v} = (M_f + \tilde{M}_f) \mathbf{v}_f + M_3 \mathbf{v}_3, \quad (1)$$

$$\frac{1}{2} M_1 v^2 = \frac{1}{2} (M_f + \tilde{M}_f) v_f^2 + \frac{1}{2} M_3 v_3^2, \quad (2)$$

FIG. 1. Particle transfer diagram. The intermediate mass M' may equal M_1 , M_2 , or M_3 . The mass of the upper (lower) particle on the diagram is taken as M_f ($\tilde{M}_f$) in text and M_f ($\tilde{M}_f$) may equal either M_1 (M_2) or M_2 (M_1).

where M_f ($\tilde{M}_f$) is the mass of the upper (lower) particle in the final bound-state system shown in Fig. 1, in which M' is the mass of the intermediate particle (M_1 , M_2 , or M_3). From Eqs. (1) and (2) it is easily shown that the velocity of the recoil particle is constrained by the condition

$$\begin{aligned} 2\hat{\mathbf{v}}_3 \cdot \hat{\mathbf{v}} &= 2\cos\gamma = \frac{M_1 + M_2 + M_3}{M_1} \frac{v_3}{v} - \frac{M_2}{M_3} \frac{v}{v_3} \\ &= \frac{M_1 + M_2 + M_3}{M_1 M_3} M_2 \frac{M'}{M_2} K - \frac{M_2}{M'} \frac{1}{K} \\ &= r \frac{M'}{M_2} K - \frac{M_2}{M'} \frac{1}{K}, \end{aligned} \quad (3)$$

where $r = (M_1 + M_2 + M_3)M_2/M_1 M_3$. Here K is a dimensionless ratio of momenta defined¹⁴ by

$$K = \frac{M_3 v_3}{M' v}. \quad (4)$$

Equation (3) specifies the values of v_3 allowed by overall energy and momentum conservation independent of the intermediate states of the system.

In the simplest two-step processes, illustrated in Fig. 1, we impose conservation of energy in the intermediate states,¹⁵ namely,

$$E_{\text{total}} = \frac{1}{2} M_1 v^2 = \frac{1}{2} M' v'^2 + \frac{1}{2} M_f v_f^2. \quad (5)$$

After some algebra, we obtain for $M' = M_1$, M_2 , or M_3 the general condition for intermediate energy conservation, namely,

$$K = 1. \quad (6)$$

This corresponds to $v_3 = M'v/M_3$. The classical Thomas peak occurs when both Eqs. (3) and (6) are satisfied.

If the collision time Δt is small, then the energy uncertainty ΔE will be large and the constraint of intermediate energy conservation may be violated within $\pm \Delta E$. For particles of charge Z_1 , $-Z_2$, and Z_3 corresponding to M_1 , M_2 , and M_3 in Fig. 1, we estimate $\Delta E \cong \hbar/\Delta t = \hbar v'/\Delta r = \Lambda v' \hbar/a_0$. Here a_0/Λ is a size characteristic of the system, where $\Lambda = Z_i Z_j$ is an interaction strength in units of the square of the electron charge with Z_i as the charge of the intermediate particle and Z_j as the larger of the projectile or target nuclear charges. Within the constraint of overall energy and momentum conservation, particle transfer is permitted within intermediate energies of order $\pm \Delta E$ away from the classical Thomas peak.

In two-step processes for transfer, the intermediate particle can in general be particle 1, 2, or 3. For the case where the intermediate particle is particle 2, i.e., the transferred particle, we show in Fig. 2 the loci for the overall energy and momentum conservation at various values of r . We also show the intermediate-energy-conserving Thomas peak at $K=1$. Even when the classical Thomas peak is kinematically forbidden, the tail of

FIG. 2. Overall energy- and momentum-conserving loci at $2\cos\gamma = rK - 1/K$ for various values of r , and intermediate-energy-conservation locus at $K=1$ when the intermediate particle is the captured particle, i.e., $M' = M_2$. These loci are invariant under the interchange of M_1 and M_3 . In the equivalent figures for $M' \neq M_2$, the only change is a shift by M'/M_2 of the locus of intermediate energy conservation. The uncertainty principle permits particle transfer within intermediate energies of $\pm \Delta E$ of the Thomas peak along the locus of overall momentum and energy conservation.

the broadening may extend into the physical region¹⁶ if the energy uncertainty in the intermediate states ΔE is large enough.

To illustrate the nature of second-order ridges, the distribution of recoil particles, $d\sigma/dv_3$, given by a second Born quantum calculation¹ for the four-body reaction $p^+ + \text{He} \rightarrow \text{H} + \text{He}^{++} + e^-$ is shown in Fig. 3. Here $M_1 = M_p$ and $M' = M_2 = M_3 = m_e$ so that $r \cong 1$. In this case the He nuclear charge Z_T localizes the two electrons but plays no direct role in the transfer process. The presence of this fourth role in the transfer process broadens the constraint of overall three-body energy and momentum conservation. A sharp ridge corresponding to overall energy and momentum conservation, and a broad ridge cresting at $v_3 = v$ (or $K=1$) are both evident for 200-MeV proton impact shown in Fig. 3. The broad ridge corresponds to a pole in the Green's function.² The crest of this ridge occurs when the intermediate energy equals the total energy. The Thomas peak is found at the intersection of these two ridges. The widths of both ridges are consistent with the uncertainty principle. The broad energy-conserving ridge has a width $\Delta E \cong Z_T v'$

FIG. 3. Differential cross section $d\sigma/dv_3$ in the velocity of the recoil electron for $p^+ + \text{He} \rightarrow \text{H} + \text{He}^{++} + e^-$ for 200-MeV p^+ . Here v_0 is the Bohr velocity. The sharp ridge, which permits energy nonconservation in intermediate states, intersects the broad energy-conserving ridge at $v_3 = v$ and $\gamma = 90^\circ$. At lower projectile energies both ridges are broader and weaker: The broad peak is indistinct at energies near or below a few MeV where experiments are feasible.

$\times \hbar/a_0$, corresponding to the above discussion with $v' \cong v \cong v_3$. The sharp ridge has a width given by the momentum distribution of the target electron which is proportional to Z_T . This width goes to zero¹⁷ as $Z_T \rightarrow 0$.

There is also broadening about the Thomas peak in α , easily found by repeating the development above. However, for common atomic processes the fractional energy width about the Thomas peak at $K=1$ in α , i.e.,

$$\Delta E/E_f \cong [\Lambda v' \hbar / \frac{1}{2} m' v^2 a_0] [m' M_3 / (M_1 M_3 - m'^2)],$$

is usually small, while the fractional energy width in γ ,

$$\Delta E/E_3 \cong [\Lambda v' \hbar / \frac{1}{2} m_f v^2 a_0] (M_3/m'),$$

is usually large. Thus, a ridge in α is usually less prominent than a ridge in γ .

Energy-nonconserving second Born amplitudes have been previously examined in the case of the Thomas peak for electron capture from atoms by protons.¹⁸⁻²² It was shown¹⁸ that the energy-nonconserving and energy-conserving amplitudes obey a dispersion relation characteristic of resonant processes.¹⁹ It was also found²⁰ that at very large v , half of the total cross section for particle transfer is due to energy-nonconserving terms. This implies that half of the total cross section associated with recoil distribution also comes from intermediate energy nonconservation.

Although the effects that we have considered have not

yet been observed experimentally, double-differential measurements for 1-MeV protons on helium are considered to be technically feasible.²³ As observation of the classical Thomas peak advanced our understanding of second-order effects in particle transfer processes, observation of the effects discussed in this paper could serve to probe energy-nonconserving aspects²⁴ of the intermediate states through which a system passes during a collision.

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¹⁴Alternatively, one could use $K' = M_3 v_3 / M_2 v$, independent of the intermediate mass M' . Then Eq. (3) is more simply $2 \cos \gamma = K' - 1/K'$. However, we choose $K = M_3 v_3 / M' v$ since $K=1$ corresponds to intermediate energy conservation for all M' .

¹⁵Of course momentum is conserved in each intermediate step for central potentials, e.g., $|\mathbf{R} - \mathbf{R}'|^{-1}$, which are translationally invariant.

¹⁶Certain symmetries may, however, impose restrictions: Some cancellation can occur, for example, when particles 1 and 2 have equal masses much smaller than M_3 and opposite charges. Compare R. Shakeshaft and J. M. Wadehra, Phys. Rev. A **22**, 968 (1980); N. C. Deb, J. H. McGuire, and N. C. Sil, Phys. Rev. A **36**, 1082 (1987).

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²⁴We note that the energy fluctuation may be computed classically, namely,

$$\Delta\mathcal{E} = E' = E_{\text{total}} = (K^2 - 1)E_{\text{total}} \left[\frac{M_f M'}{M_1 M_3} \right] \left[\frac{\tilde{M}_f + M_3}{M_1 + M_2} \right].$$

If a cross section were to remain large (e.g., due to higher Born terms) for some $\Delta\mathcal{E}$ greater than the quantum uncertainty ΔE , then it would be possible to study cross sections at specific non-conserving intermediate energies to a relative accuracy of $\Delta E/\Delta\mathcal{E}$.