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Vibrational Modes in Acoustic Gallery Scanning Probe Microscopy

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Abstract:

A distinct characteristic in Acoustic Gallery Scanning Probe Microscopy (AG-SPM) constitutes the use of its supporting structural frame as an acoustic resonant cavity for monitoring the nanometer-sized amplitude of its stylus-probe. Although very straightforward in its implementation, its amplitude detection sensitivity could be improved by a more thorough understanding of its working principle mechanism, as well as by a more systematic procedure to attain a closer matching between one of the cavity's acoustic resonances and the probe's natural frequency. Herein, a description of the working principle of the AG-SPM is attempted from a vibrational-mode analysis perspective, and a successful specific procedure is presented to maximize the AG acoustic response. Such an improvement in sensitivity will make AG-SPM a better metrology instrumentation to study probe-sample shear-force interactions at nanometer scale separation distances, as well as its role in the fabrication of nanostructures via SPM.

Keywords:

Acoustic Feedback; SPM; Acoustic Gallery Sensing; Vibrational Modes; Tuning Fork

1. INTRODUCTION

There exists an interest for developing opto/acoustic imaging devices and sensors, which can be applied in diverse areas including medicine and electronics [1, 2]. As the current trend of shrinking devices continues, detecting the wave-amplitude of the probing radiation with high sensitivity becomes crucial. Among the diverse strategies to achieve low signal levels detection, it is worth mentioning the effort to harness the confinement of waves into reduced spaces to, thus, more efficiently exploit their constructive interference. One way of achieving such a confinement is to use “perfect mirrors” whose working principle is based on the total internal reflection principle (different than the one that uses Bragg reflection as occurs in dielectric-coated parallel mirrors). This method works over a broader range of frequencies, and can be used to make resonators confined in three-dimensions, provided one can force the wave inside the cavity to efficiently interfere with themselves [3, 4]. This approach is being pursued to build opto-mechanical resonators [5], directional optical lasers[6], including nanodevices used in frontier research areas for cooling opto-mechanical systems at quantum levels [7, 8]. The work presented herein addresses the

wave confinement strategy from an acoustic perspective. More specifically, we focus on attaining a further development of Acoustic Gallery Scanning Probe Microscopy (AG-SPM) [9], a technique recently introduced by our group, where the oscillatory motion of its tuning fork probe is tracked with an acoustic transducer located away from the probe. In AG-SPM the acoustic waves generated (at the probe-cavity junction) by its dithering probe end up confined on the walls of its structural supporting frame (the latter resembling closer a cavity of cylindrical shape). By judiciously placing an acoustic transducer on the perimeter of the cavity for maximum signal detection, AG-SPM is able to monitor the nanometer-size oscillations of its sturdy piezoelectric tuning fork probe. In this article we report efforts aimed at obtaining a better analytical understanding of the AG-SPM working principle, as well as investigating new strategies for improving its detection sensitivity. The goal is making AG-SPM to become not only a more effective SPM imaging instrumentation, but also an analytical apparatus to characterize the properties of mesoscopic fluids confined between two solid boundaries, which appear to play a relevant role in interfacial friction phenomena [10], as well as in the fabrication of nanostructures[11]. In order to gain a better perspective of the technique, we provide first a brief background about how the AG-SPM idea germinated.

1.1 Acoustic Gallery Scanning Probe Microscopy

Scanning probe microscopy (SPM) comprises a collection of proximal-probe imaging techniques widely used to characterize the surface of materials with nanometer scale resolution. The working principle of SPM-imaging is based on its ability to raster scan a surface with a sharp tip probe. To perform this detailed excursion, the tip is mounted to a voltage-controlled XYZ piezoelectric-positioner going around the contours of the sample's topographic features. Oscillatory-voltages V_x and V_y are applied to execute a pre-determined lateral motion, while an automated feedback voltage-control (that respond to a specific device and mechanism that senses the probe-sample distance, as will be described below) dictates the proper V_z actuation. The latter allows to move the tip up and down, but maintaining its close proximity to the sample's surface landscape. The recorded V_x , V_y and feedback V_z voltages are used to reconstruct, via software, the sample's topographic image $z = z(x, y)$ after a proper voltage-displacement calibration of the voltages and XYZ-positioner displacements. A sharper probe leads to a finer image resolution.

To sense the probe sample-distance (required to implement the feedback control mentioned above) one particular type of SPM uses a quartz tuning fork (QTF) of 32.768 kHz nominal resonance frequency, which has a very stiff spring constant ($\sim 25,000$ N/m) and has very low damping characteristics (mechanical quality factor $Q \sim 2000$ or higher at ambient conditions). In this SPM, a sharp stylus is attached to one of the TF tines (typically oriented vertically); upon applying a driving voltage, the probe oscillates laterally (~ 1 nm amplitude) and the tuning fork, due to its piezoelectric characteristics, gives rise to an ac-current. The latter is synchronously detected with a lock-in amplifier [10]. It turns out that the amplitude of the probe's lateral oscillations decreases monotonically as the tip travels vertically towards the horizontally oriented surface. The physics principles governing probe-sample shear-force interactions is not yet well understood, however, the method is widely exploited to implement a V_z feedback voltage to control the probe-sample distance by simply monitoring the probe's oscillations. Because of the mechanical rigidity of the setup along the vertical direction, the probe does not experience sudden jumps towards the sample, as occurs in typical AFMs, thus providing well-controlled probe-sample distance. The latter is a very important metrology feature for studying the distance dependence of shear-force interactions. In QTF-SPM the vertical position of the probe is monitored by detecting the electrical admittance while driving the TF at its expected mechanical resonance. However, the TF's intrinsic capacitance convolutes

the electrical admittance detection; as a result, it is not possible to determine directly the exact state of TF's mechanical motion. The latter compromises the metrology capabilities of the technique. While corrections to suppress this shortcoming have been introduced,[12] it is highly desirable to design much simpler TF-based detection methods.

AG-SPM [9] offers an alternative to circumvent the shortcomings associated with the TF's inherent capacitance by relying instead in an acoustic transducer to monitor the oscillatory motion of a SPM probe. As schematically illustrated in **Figure 1**, the simplicity of AG consists in using its own supporting frame as an acoustic cavity where the mechanical waves, generated at the TF-cavity junction and subsequently travelling towards the cavity, are detected by an acoustic transducer judiciously located at the periphery of the cavity. AG is able to detect the TF tines' lateral motion with nanometer-sized amplitude precision. Notice that this detection method resembles the ability to hear whisper conversations at the galleries of a cathedral even when the source is located at extraordinary distances [13].

Although very straightforward in its implementation, AG-SPM would benefit from a better analytical description of its working principle, which could eventually lead to better implementation strategies and find out the condition of maximum detection sensitivity. Aware of the multiple bulk and surface waves that can be excited in a given non-homogeneous and non-symmetrical cavity, AG-SPM will be tested, as a first step, with cavities of different shape but maintaining some degree of cylindrical symmetry. The dimension of cavities will be kept very similar; and an attempt will be made to describe the acoustic spectral response from the existent analytical models. On the other hand, since AG-SPM relies on an ideally perfect matching between the cavity's resonance frequency and the probe's mechanical resonance frequency, which seldom occurs, we describe a method to overcome this shortcoming and, hence, improve AG-SPM detection sensitivity.

2. EXPERIMENTAL SETUP

Figure 1 shows the experimental setup for acoustically monitoring the nanometer-sized oscillations of a probe attached to an electrically driven tuning fork. The setting has been described previously with very much detail [9]; herein we provide just a succinct description, accompanied with additional information of some modifications introduced to independently test the frequency response of the cavity alone, as well as improvements in the data acquisition process via LabVIEW, as shown in the bottom-left diagram of **Figure 2**.

2.1 Characterization of the Cavity's Mechanical Response

The left side of the top diagram in **Figure 2** shows an electrically driven piezoelectric plate (12 mm×12 mm×2 mm) that is used to mechanically excite the cavity in the 20 kHz to 40 kHz range (as to include the 32.768 kHz TF's operating frequency) while simultaneously monitoring its acoustic spectral response. The right side of the diagram shows an alternative way to excite the cavity by using an electrically driven TF instead. In this last setting, the TF is held by a piezoelectric tube ¹ and, altogether, attached to a

¹ Piezo tube from EBL Products, Inc. EBL 3, 0.5 mm thick, 40 mm long, 20 mm outside diameter.

Figure 1. **Left:** Schematic views of the acoustic gallery (AG) experimental setup for sensing the nanometer-sized lateral vibrations amplitude of a sharp stylus that is attached to an electrically driven piezoelectric tuning fork (1). As the tip vibrates, the tube that holds the TF picks up mechanical waves, which travel towards the cavity where they interfere. An acoustic transducer (2) is judiciously placed on the periphery of the cavity, at a site of maximum constructive interference. **Right:** Synchronous detection of acoustic waves via a lock-in amplifier (4) referenced to the TF driving frequency. The simultaneous detection of the TF's electrical admittance is optional.

Figure 2. **Top:** Typical size of the cavities analyzed herein. The excitation is either with a piezo-plate (PP) (left side diagram) or with a tuning fork (TF) (right side diagram). **Bottom right:** Schematic of the experimental setup for testing the frequency response, where the cavity is excited not by a TF (as in **Figure 1**) but instead by an electrically driven piezoelectric plate. **Bottom-left:** Typical acoustic spectral response in magnitude (trace with circles) and phase (solid trace).

stainless steel cylindrical cavity shell but electrically isolated by a disk made of macor ceramic material². The piezoelectric tube plays here no other role than as a holder of the TF and as a medium through which the TF vibrations are transmitted to the steel cavity. Alternatively, in this second experimental setting (top right side of **Figure 2**) the TF is interchanged, back and forth, with a piezoelectric plate as the excitation source. This is done intentionally to test the cavity's spectral response each time the natural frequency of the tuning fork is purposely varied by a few Hertz (as described in more detail in the "Matching Frequency" section below). The bottom-right side shows the lock-in synchronous detection of

² MACOR Machinable Glass Ceramic is a registered trademark of Corning Incorporated, Corning, NY. MACOR is composed of approximately 55% fluorophlogopite mica and 45% borosilicate glass. Young's Modulus (at 25 °C) equal to 66.9 GPa, DC volume resistivity (at 25 °C) greater than 1016 ohm-cm.

Figure 3. Stainless-steel cylindrical shells used in the present work as acoustic cavities. The scale bar is 2.5 cm.

the acoustic signal, where the driving signal generator is interfaced via GPIB and controlled by LabView graphic program interface. The bottom-left side of the figure displays a typical acoustic spectral response; an ideal cavity would have a significant acoustic response at the particular TF operating frequency.

We can identify four main components in **Figure 1** and **Figure 2**: (i) A driven element, which in **Figure 1** is constituted by a piezoelectric TF of 32.768 kHz nominal resonance frequency and in **Figure 2** by a piezoelectric plate. A signal generator (SR DS-345) supplies a programmable frequency and programmable ac voltage amplitude to the piezoelectric element. (ii) An acoustic cavity, which is in mechanical contact with the TF or the piezo-plate. (iii) An acoustic transducer (DECI SE 25-P 426 that has a 3 mm diameter sensitive area) placed at the periphery of the cavity. (iv) A lock-in amplifier (SR-844 or SR-850) for synchronously detecting the acoustic signal; unless noted otherwise, we typically used a 30 ms time constant. In a typical procedure, an ac-voltage of 10 mV rms amplitude is applied across the terminals of the TF, which makes the tines to laterally oscillate with a few nanometers amplitude (~ 4 nm when driven at its resonance frequency). The vibrations of the TF or the piezo-plate, respectively, are transmitted as acoustic waves to the circular frame, where, we expect, they establish sites of constructive interference around the cavity's periphery; the maxima are detected by the acoustic transducer via lock-in synchronous detection. In the case of **Figure 2**, the process is automated via LabVIEW and using a General Purpose Interface Bus (to control of the signal generator settings) and a data acquisition card (NI 6008 DAQ). Additionally, for control and comparison purposes, the system also monitors the TF's electrical admittance.

2.2 The Acoustic Cavity

Figure 3 displays circular (A, D) and regular hexagonal (C, D) stainless-steel cylindrical-shell cavities used in these experiments; **Table 1** shows their corresponding dimensions. Cavities C and D have tapered external walls. The planar geometry of the external walls in B and C aimed at optimizing their contact area with the flat sensitive area of the acoustic sensor. A plastic spring clamp is used to keep a tight contact between the transducer and the cylindrical cavity³, and a minute amount of vacuum-grease layer is sandwiched between the sensor and the cavity-wall in order to improve the acoustic coupling. The notches on the cavities A and B respond to the fact that they are already part of a SPM system, and have

³ We tried a variety of clamps of different spring constant, since pressing the acoustic sensor against the cavity too tight or too loose leads to a very poor coupling of sound from the cavity to the acoustic transducer.

just been taken apart for analysis purposes. Notice, the test cavities were designed with slightly different shapes and very similar sizes so any of them could relatively easy be integrated into an existent SPM.

Table 1.

	Cavity A Circular	Cavity A' Circular	Cavity B Hexagonal external walls	Cavity C Hexagonal and tapered external walls	Cavity D Conical external walls
External diameter	48.8 mm	50.8 mm			31 mm (Top edge) 51 mm (Bottom edge)
External perimeter	153 mm	159.6 mm	145 mm	148 mm (Top side) 168 mm (Bottom side)	97.5 mm (Top side) 160 mm (Bottom side)
Internal diameter	31.5 mm	31.5 mm	31.5 mm	31.5 mm	25.4 mm
Internal perimeter	99 mm	99 mm	99 mm	99 mm	79.8 mm
Height	69 mm		69 mm	69 mm	69 mm

2.3 Calibration of Piezoelectric TF's Resonant Frequency

Once the spectral response of a given cavity is determined, it would be ideal that the resonance frequency of the TF (with a probe attached to it) matches one of the acoustic spectral peaks; however this seldom happens. This is aggravated by the fact that when a probe is attached to one of the tine of the TF, the resulting resonance frequency of the (TF + probe) system depends on many parameters such as attachment procedure, length of the stylus probe, *etc.* To address this situation, we have established a very simple method to tune the TF's resonance frequency (within a 2 kHz range), which consists in modifying slightly its mass by adding a tiny amount of ethyl cyanoacrylate adhesive ⁴ to the side of one of the tines. This procedure causes a “red shift” of the resonance frequency, and, since a cavity spectrum displays so many peaks, it is usually possible to find one of lower frequency than the initial TF's natural frequency. The procedure is also accompanied by a decrease of the probe's oscillation amplitude due to damping effect from the viscous glue; but we demonstrate herein that such loss is more than compensated by the increase of acoustic signal when the modified TF's natural frequency matches the cavity maximum response.

3. EXPERIMENTAL RESULTS

3.1 Acoustic Spectral Response From Cylindrical Cavity Shells

Figure 4 shows three superimposed spectra obtained by placing the acoustic sensor at different locations around the perimeter of the cavity, respectively. Notice, when the sensor is located at a particular position on the periphery of the cavity we may obtain a high acoustic response at a given frequency, but the

⁴ Ethyl cyanoacrylate adhesive, better known as super glue. This adhesive was chosen because it requires only 30 minutes waiting period between trials in order to obtain a reproducible result.

response at the same frequency with the sensor located at a different place may be much lower. For example, the peak at 22 kHz is higher for the “open triangles” trace, but smaller for the “circles” trace. Notice also that this role is reversed at, for example, 36.8 kHz.

Figure 4. Three superimposed acoustic spectra of the cylindrical circular cavity-A (of radius $R=24.3$ mm) obtained with the acoustic sensor located at three different positions on the cavity, respectively. The arrows on the left side are to compare the acoustic response amplitude at 22 kHz from the triangle spectrum (high value) and the circles spectrum (low value); notice the reversal in the amplitude response at 36.8 kHz (arrows on the right side). Notice also the absence of peaks in the 28.5 – 33.5 kHz range.

To verify the sensitivity of the modes to the dimensions of the cavity, we also tested the frequency response of a cavity similar to cavity-A except that its external diameter is 2 mm larger (hence its thickness also larger); all the other dimensions being the same. We will refer to this cavity as cavity-A'. Notice that the location of peaks (dashed traces) for A' in general appear to change relative to the peaks for A (solid traces); but some peaks appear to stay immune to the size change (see for example the peaks around 28 kHz and 34.5 kHz). This observation may be helpful in identifying modes that depend only on the radial dimension; *i.e.* differentiating them from the ones that depend only on the height of the cavity.

Figure 5. Comparison between three acoustic spectra (randomly superimposed solid traces) obtained from the circular cylindrical cavity A and three superimposed spectra (broken-line traces) from cavity A'. Their outside diameter (OD) differ by 2 mm; $OD_{A'} > OD_A$.

On the other hand, the 28.5 kHz – 34.5 kHz range appears to be immune to receiving new peaks upon

this change in the radial dimension change; in **Figure 5** there are no peaks in that range for cavities A and A'. Although this particular characteristic may be instructive and helpful for identifying modes present in the cavities, both cylinders would not be useful for an AG-SPM system that uses a TF of 32.768 kHz natural frequency. It would be ideal to design a cavity with an acoustic response peak close to the in turn TF resonance frequency. For that reason, although guided initially by trials and errors, we built several hexagonal cylindrical cavities C and D. (The hexagonal design responded in part also to the convenience of having a better coupling between the flat cavity walls and the flat sensing area of the acoustic transducer). Their acoustic response is displayed in **Figure 6**; notice both spectra display one particular acoustic response peak (marked with an arrow in the figure) that is very close to the TF's 32.768 kHz nominal natural frequency.

Figure 6. Superimposed acoustic spectra for cavities of external hexagonal shape. **Left:** Cavity B of external walls vertically straight. **Right:** Cavity C of tapered external walls. The former shows a peak at 32.5 kHz, the latter shows a peak at 31.5 kHz; both peaks (marked with an arrow) are close to the TF's 32.768 kHz nominal natural frequency.

The somewhat simpler spectral response of the hexagonal conical cavity around the TF's 32 kHz resonance frequency prompted us to also test another conical cavity but of circular shape. The result is presented in **Figure 7**.

Figure 7. Three superimposed acoustic spectral response from the circular conical shell cavity, displaying a well isolated peak at 31.7 kHz, conveniently close to the 32.768 nominal natural frequency of the tuning fork.

Figure 8. Superimposed spectral response of cavity-D (solid traces) obtained using a piezo-plate as excitation source (magnitude indicated on the left vertical scale). The diamond trace indicates the peak values of the cavity response acquired using a TF as the excitation source; each discrete data point corresponds to a TF tuned to a different natural frequency (magnitude of the signal indicated on the right vertical scale).

3.2 Matching Frequency

Next we use a piezoelectric tuning fork to excite the acoustic cavity (see top diagram in **Figure 2**). The TF is firmly attached to a piezoelectric tube, and both together assembled to the cavity, constituting a setting that is very similar to the original SPM system, which inspired the AGS idea [9]. Since, in general, a cavity's peak frequency response does not match the TF's natural frequency, we proceeded to fine tune the natural frequency of the TF in order to maximize the cavity's acoustic response. We selected cavity-D for this principle demonstration given its isolated peak frequency response at 31.7 kHz, which facilitates tracking its acoustic response as the tuning fork's natural frequency is purposely changed.

The procedure consists in applying a small drop of ethyl cyanoacrylate adhesive⁵ to one of the TF-tines, which typically produces a "red shift" change between 50 Hz to 200 Hz (a decrease in the magnitude of the TF oscillation is also observed). A 10-minute drying time ensures a reproducible measurement of the frequency change. The "diamond" trace in **Figure 8** summarizes the result of this procedure after implementing it several times. For further control and comparison purposes, after each addition of mass we also recorded the cavity's spectral response but using a piezo plate as the driving source. For this purpose, the piezo-plate interchanged position with the TF. However, changing a given source back and forth creates some degree of variations in the acoustic response. We therefore graph multiple spectral responses, which were taken after each interchange of excitation source. Put all together we observe in **Figure 8** (left scale) that the different spectra display a similar trend. This whole procedure allows us to tune the TF natural frequency beyond a 2 kHz range.

⁵ Ethyl cyanoacrylate adhesive, better known as super glue. This adhesive was chosen because it requires only 30 minutes waiting period between trials in order to obtain a reproducible result.

4. ANALYSIS

An analytical description of the acoustic response from an original AG-SPM, a system of inhomogeneous material composition and non-symmetric components, would be a very difficult task. That is the reason why we arbitrarily selected to analyze only the top side of such a system, which has a geometrical shape close to a cylindrical shell (**Figure 2**) and, hence, more amenable for analytical modeling. Still, as it turns out, the acoustic frequency response of an apparently simple structure, such as cavity-A, is already somewhat sophisticated, displaying multiple peaks even in the reduced 20–40 kHz range (**Figure 5**). Nonetheless, taking into account the dimensions of the cavities displayed in **Figure 3**, we still attempt to justify analytically that indeed acoustic modes should be detected in that reduced frequency range. As a background, let's first evaluate an order of magnitude of the acoustic wavelength involved in our experiments. In bulk stainless steel (Young modulus $E = 20.5 \times 10^{10} \text{ N/m}^2$, density $\rho = 7850 \text{ Kg/m}^3$, and Poisson ratio $\nu = 0.28$) the longitudinal and transverse acoustic wave velocities are $c_l = 5.77 \times 10^3 \text{ m/s}$ and $c_t = 3.19 \times 10^3 \text{ m/s}$ respectively⁶; at excitation frequencies around 32 kHz the corresponding wavelengths are $\lambda_l = 18 \text{ cm}$ and $\lambda_t = 10 \text{ cm}$, respectively. Notice, these two values are close to the perimeter ($\sim L_{ext} = 15 \text{ cm}$) of the tested acoustic cavities (**Figure 2**).

4.1 Plate Model Approach

On a first approximation, the waves in the tested cavities resemble more like wave propagation inside a plate; hence, the velocities involved will be different than the ones in the bulk. Consider for example cavity-A. If we mentally cut this cylindrical shell along a vertical line, and then unfold it to a flat surface, we would obtain a plate of length $L_{ext} = 2\pi R_{ext} = 15.3 \text{ cm}$ (or $L_{int} = 9.9 \text{ cm}$ if we take the inner radius instead), width $H = 6.9 \text{ cm}$, and thickness $\Delta R = 8.6 \text{ mm}$. Accordingly, we have to consider velocities associated to stretch motion (longitudinal and transverse oscillations parallel to the plate) and bending motion (oscillations perpendicular to the plate). The velocities for the waves in this first group are $c_{l,stretch} = 5.32 \times 10^3 \text{ m/s}$ and $c_{t,stretch} = 3.19 \times 10^3 \text{ m/s}$, which can be used to estimate the frequencies of standing waves⁷.

Along the side of length $L_{ext} = 15.3 \text{ cm}$ the frequency required to establish a full wavelength would be $f_1 = c_{l,stretch}/L_{ext} = 34.8 \text{ kHz}$ and $f_2 = c_{t,stretch}/L_{ext} = 20.8 \text{ kHz}$. Notice these two frequencies fit within the range of analysis in **Figure 5**. If we had used $L_{int} = 9.9 \text{ cm}$ instead, the frequencies would be $f_3 = 53.7 \text{ kHz}$ and $f_4 = 32.2 \text{ kHz}$ respectively. The former is outside of the analyzed frequency range, but if the requirement were to establish one-half wavelength oscillations instead then the frequency would be $f_5 = 26.5 \text{ kHz}$. Similarly, the frequencies required to establish one half wavelength along the side of length $H = 6.9 \text{ cm}$ are $f_6 = 38.5 \text{ kHz}$ and $f_7 = 23.1 \text{ kHz}$.

On the other hand, the velocity of *bending* waves in a plate displays dispersion, $f =$

⁶ In stainless steel the velocity of longitudinal elastic waves in a bulk medium is given by $c_l = (E/\rho)^{1/2}[(1 - \nu)/((1 + \nu)(1 - 2\nu))]^{1/2} = 5.77 \times 10^3 \text{ m/s}$, while the transverse elastic waves propagate at speed by $c_t = (E/\rho)^{1/2}[1/(2(1 + \nu))]^{1/2} = 3.19 \times 10^3 \text{ m/s}$.

⁷ Stretching longitudinal oscillations propagate with speed $c_l = (E/\rho)^{1/2}[1/(1 - \nu^2)]^{1/2} = 5.32 \times 10^3 \text{ m/s}$. Stretching waves with oscillations perpendicular to the direction of propagation (but still in the plane of the plate) travel with speed $c_t = (E/\rho)^{1/2}[1/(2(1 + \nu))]^{1/2} = 3.19 \times 10^3 \text{ m/s}$.

Figure 9. Cross sections and coordinates to describe axisymmetric vibration modes in a cylindrical shell.

$(k^2/2\pi)\sqrt{E/\rho}\sqrt{h^2/[12(1-\nu^2)]}$, the frequency being proportional to k^2 instead of k as occur in the case of stretching waves. A mode with one-half wavelength fitting along L_{ext} would have $f = 0.887 \times 10^3$ Hz. Notice the frequencies of the lowest bending waves frequencies are much smaller than the lowest frequencies associated to the stretching waves; this is expected since, by comparison, the plate offers less resistance to bending than to stretching elongations. Among the higher modes that can be calculated with the previous expression, only $f_8 = 31.9$ kHz fits inside our range of analysis⁸. If we use L_{in} instead, we obtain $f_9 = 33.9$ kHz⁹; and along the side of length $H = 6.9$ cm we obtain $f_9 = 39$ kHz¹⁰.

Overall, we have found up to nine potential standing wave modes whose frequencies fit within the 20 kHz - 40 kHz bandwidth in which cavity-A was tested. But some of the individual frequencies are not reproduced experimentally, which is an indication that the flat plate model needs some refinement.

4.2 Axisymmetric Free Vibrations

A better approximation can be obtained from analytical solutions associated to waves established in thin cylindrical shells (of radius R , height L , and thickness h). An important feature of shells, that distinguish them from plates, is that bending is accompanied with stretching; that is deformation of plates which are curved in the undeformed state have properties that are fundamentally different from those of flat plates [14]. It is worth then to explore analytical solutions describing vibrations in cylindrical shells and to compare them with our experimental results. For simplicity, let's consider first axisymmetric free vibrations (Figure 9), *i.e.* free flexural vibrations modes of the cylinder where the radial w displacements do not depend on the polar coordinate θ , and have zero circumferential displacement p . Any cross section of the cylinder will then be a circular ring, with several (n) half-waves established along the shell axis (only one half-wavelength is schematically shown on the left side of Figure 9). The equation governing these particular type of waves is given by [15] $\partial^4 w / \partial z^4 + 4\beta^4 w = -(\rho h / D)(\partial^2 w / \partial t^2)$, where $D \equiv Eh^3/12(1-\nu^2)$ is the flexural rigidity (or stiffness) of the shell, and $\beta^4 \equiv Eh/4R^2D = 3(1 -$

⁸ The other bending mode frequencies along a side of length $L_{ext} = 15.3$ cm are 3.55 kHz, 7.9 kHz, 14.2 kHz, 22.2 kHz, and 31.9 kHz, 43.5 kHz.

⁹ The other bending mode frequencies along a side of length $L_{in} = 9.9$ cm are 33.9 kHz, 53 kHz.

¹⁰ The other bending mode frequencies along a side of length $H = 6.9$ cm are 4.36 kHz, 17 kHz, 39 kHz, 69 kHz.

$v^2)/R^2h^2$. This equation admits solutions of the form $w(x,t) = \sin(n\pi z/L) \sin(2\pi f_n t)$ provided that, $f_n = (1/2\pi R) \sqrt{(E/\rho)} \sqrt{1 + \mu \lambda_n^4}$, where $\mu = (h/R)^2/12(1 - v^2)$ and $\lambda_n = n(\pi R/L)$.

For cavity-A (of radius $R = R_{ext} = 24.4$ mm, thickness $h = 8.6$ mm, and height $L = 68.8$ mm) we obtain $f_1 = 33.6$ Hz and $f_2 = 37.6$ Hz for $n = 1$ and $n = 2$ respectively. (For a 0.1 mm change in the radius, the calculated frequency of a mode changes by about 150 Hz). Although these solutions correspond to a particular boundary condition where the radial elongation w is zero at $z = 0$ and $z = L$, it turns out that for cylinders not too short (*i.e.* discarding those satisfying $R/L > 1$) the first mode at least is expected not to significantly depend on the boundary conditions [15]. Although the two frequencies calculated above fall within our 20 kHz - 40 kHz range of analysis, still it is difficult (with such a partial information) to match them univocally with some particular experimental peaks displayed in Figure 5. For this reason we expand our analysis to check more sophisticated modes; those that depend on the polar coordinate.

4.3 Asymmetric Flexural Vibrations

For a circular cylindrical shell, let's consider modes of the form,

$$\begin{aligned} w_{mn}(x,t) &= (\cos m\theta) \left(\sin \frac{n\pi z}{L} \right) \sin \omega t \\ p_{mn}(x,t) &= (\sin m\theta) \left(\sin \frac{n\pi z}{L} \right) \sin \omega t \\ u_{mn}(x,t) &= (\cos m\theta) \left(\cos \frac{n\pi z}{L} \right) \sin \omega t \end{aligned} \quad (1)$$

where m refers to the number of half waves along the polar direction, and n refers to the number of half-wavelength along the z -axis. These modes satisfy the boundary conditions of a cylinder simply supported at its extremes at $z = 0$ and $z = L$. (We will assume again that the modes depend weakly on the specific boundary conditions). The natural frequencies $f_{mn} = \omega_{mn}/2\pi$ can approximately be obtained from the following expression,

$$\omega_{mn}^2 = \frac{E}{\rho R^2(1 - v^2)} \frac{(1 - v^2)\lambda_n^4 + a^2(\lambda_n^2 + m^2)^4}{m^2 + (\lambda_n^2 + m^2)^2} \quad (2)$$

where $a^2 = (h/R)^2/12$ and $\lambda_n = n(\pi R/L)$.

Table 2 displays the values of the lowest asymmetric flexural nm -modes obtained from expression (2), using the dimensions of the cavities A and A' (they are identical, except for their 1 mm difference in their external radius and corresponding change in thickness.) The value for the radius R has been taken, arbitrarily, equal to the external radius of the cavity. The modes that fall within our 20 kHz to 40 kHz experimental range of analysis are marked with an asterisk. Some interesting features can be obtained from these results:

i) For a given value of n (more specifically $n=1$ and $n=2$), the frequency first decreases and then increases as m increases. That is, the lowest frequency does not occur at the lowest value of m . This “deep” in the frequency behavior as a function of m just illustrates the complicated task one can be involved if trying to interpret the acoustic spectral response without a proper analytical model.

ii) For $n=1$, as m increases the resonant frequency jump drastically from ~ 33 kHz down to 18 kHz and then back up to 35 kHz, thus leaving a large spectral range with a flat response. This could possibly

Table 2. Fuzzy if-then rules

n	m	Cavity A Freq. (Hz)	Cavity A' Freq. (Hz)
1	0	33621	32393
1	1	18345	18630
1	2	18807	19720
1	3	34938*	36379*
1	4	59381	61556
1	5	91082	94208
2	0	37668*	37534*
2	1	34363*	35184*
2	2	35782*	37914*
2	3	49610	52557
2	4	73168	76892
2	5	104562	109238
3	0	51662	54466
3	1	52599	55962
3	2	58449	62882
3	3	72817	78192
3	4	95935	102183
3	5	126998	134231

Cavity A: R_{ext} = 24.4 mm; thickness = 8.6 mm. Cavity A': R_{ext} = 25.4 mm; thickness = 9.6 mm.

be one of the reasons why the spectrum in **Figure 4** displays a flat response in the 28.5 kHz - 33.5 kHz range.

iii) The modes for cavity-A marked with an asterisk in **Table 2** have frequencies around 35 kHz, a region where the spectrum in **Figure 4** also shows several peaks. These peaks are located just at the right side of the 28.5 kHz - 33.5 kHz range where there is a flat zero-response. With the exception of the mode $m = 0$, the model predicts that the frequency should increase when comparing cavity-A and cavity-A' ¹¹. This is not in contradiction with the result in **Figure 5** (where the peaks from cavity-A and cavity-A' are superimposed), since the 28.5 kHz - 33.5 kHz range still has a flat zero-response (*i.e.* no peak moved to a lower frequency, thus in agreement with what was predicted by the model).

Despite the consistency of the asymmetric flexural vibrations model with the results outlined above, expression (3) is still unable to predict the peak responses in the 20 kHz to 28.5 kHz range. One possible reason may lie in the arbitrariness of using the external radius of the cavity as the value for R in expression (2) to calculate the values presented in **Table 2**. Maybe we should identify an effective radius (with a value in between R_{max} and R_{min}). This is explored in **Table 3**, which lists the frequencies of the different modes corresponding to different radii, but keeping the thickness of the cavity constant. Notice that for radii values around $R = 0.020$ m (the average of R_{max} and R_{min}), the $n = 1, m = 2$ mode has frequencies in the 21 kHz to 28 kHz range. But adopting any of these radii would prevent us from explaining the peaks around and above 35 kHz. We are facing then the limitations of the model that is applicable only to thin

¹¹ As the radius increases, the frequency of the mode in principle should rather decrease. But in the case of cavity A' not only its radius increased but also its thickness. An increase in thickness makes the cavity more robust and with a higher resonance frequency.

Table 3.

n	m	$Freq\ (Hz)$ ($R_{in}=0.0157\ m$)	$Freq.\ (Hz)$ ($R=0.018\ m$)	$Freq.\ (Hz)$ ($R=0.020\ m$)	$Freq.\ (Hz)$ ($R=0.022\ m$)	$Freq.\ (Hz)$ ($R_{ext}=0.0244\ m$)
1	0	51826	45397	40902	37229	33621
1	1	18205	18220	18312	18370	18345
1	2	35437	28545*	24366*	21367*	18807
1	3	77010	60069	49570	41818	34938
1	4	136124	105282	86143	71986	59381
1	5	212325	163609	133374	111005	91082
2	0	54538	48471	44289	40921	37668
2	1	41325	39397	37724	36132	34363
2	2	51654	45200	41273	38384	35782
2	3	91063	74279	63929	56326	49610
2	4	149696	118896	99804	85700	73168
2	5	225732	177029	146809	124458	104562
3	0	64997	59997	56672	54080	51662
3	1	63100	59560	56953	54767	52599
3	2	76452	69531	65088	61662	58449
3	3	114384	97663	87313	79657	72817
3	4	172319	141572	122522	108450	95935
3	5	248074	199393	169195	146868	126998

shells with low thickness-to-radius ration, but in our case that factor is 0.35.

We have then reached a point in which we encounter very much limitations in the model used to describe our experimental results, even though we have limited ourselves to analyze just a section of the AG-SPM hardware with the highest symmetry. We come to the conclusion that it may be more appropriate to use computer simulation, in which there will be no approximations in the model, and the calculation can be compared with existent SPM stages.

5. CONCLUSIONS

The working principle mechanism of AG-SPM has been analyzed using as a model asymmetric flexural vibrations in cylindrical shells. These modes would be excited by an electrically driven quartz tuning fork thus establishing localized maxima and minima acoustic vibrations around the SPM's structural frame. The model is chosen to fit the geometrical shape of the frame, which resembles closer a cylindrical shell cavity. Different cavities of gradually different sizes and shapes were tested. Despite their similar dimensions (including two circular cavities whose radii differed by only 1 mm) and slightly different shape (cylinders of straight or tapered external walls), we found that their experimental spectral acoustic response displayed significant disparity among each other. The observed sensitivity of the spectral response to fine variations in the cavity dimensions is corroborated by the model, which for cavities of circular shape predicts changes in the resonance peaks of about 150 Hz for a 0.1 mm change in radius. For a cavity of 25 mm radius, this represents a change in 0.4 %. This fine sensitivity of the spectral response made difficult to identify and track analytically the multiple isolated peak responses due to the relatively coarse geometrical variations among the tested cavities. Still, the model predicted the existence of resonant peaks within the experimentally analyzed 20 – 40 kHz range. The model also revealed that

the lowest resonant frequency does not occur at the lowest values of the mode indices n, m ; instead as the value of m increases monotonically (for a fixed n) the corresponding discrete resonance frequencies first decrease drastically and then increase back again, thus leaving behind an empty frequency range gap. As intricate as this latter feature may appear to be, it could be the reason why the spectral response from the cylindrical cavities A and A' display a flat zero-response in the 28.5 kHz - 33.5 kHz range. Despite all the merits, as rationalized above, of the thin cylindrical cavity model used here, the observed high sensitivity of the spectral response to the cavity's dimensions and shape suggests that more realistic models (of non-symmetric shape component and inhomogeneous material composition) are needed. It would be then highly recommended to explore the use of numerical analysis as the next step in the investigation of particular modes existent in a given AG-SPM structural frame.

On the other hand, we have successfully been able to track the cavity response using different TFs of corresponding different natural frequencies as the driving sources. The response to the TF's single frequency faithfully tracked the cavity's spectral response. Retrospectively, based on these new results, it appears that the ASG-SPM has not been used very efficiently in the past. The method described here offers the possibility to gain a much higher sensitivity. The significance of these outcomes resides in the corresponding improvement in detection sensitivity of the AG-SPM to analyze more accurately probe-sample shear-force interactions at nanometer separation distances.

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