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Final Report

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16. Abstract

The objective of this project is to develop data-driven, decision-making models for shared-mobility system design and operation. Specifically, we will use shared e-scooters as a representative system, with the ultimate goal of facilitating an electric shared-mobility revolution that promises a more sustainable future.

In the past few years, shared e-scooter systems have gained increased popularity around the world because of their benefits to health, traffic congestion, the environment, and accessibility. As of 2018, approximately 100 U.S. cities have launched shared e-scooter programs, accounting for 38.5 million trips. However, the business model to manage e-scooter sharing remains nascent, with many challenges still poorly addressed and outstanding. As a result, we propose to solve several urgent questions that arise at the company and policymaker levels for e-scooter sharing (e.g., planning, operations), by developing a data-driven optimization model to provide decision makers with a robust solution that enables low cost and high service quality. In line with the NITC themes, these research results have the potential to provide e-scooter companies with new decision-making tools and methodologies to effectively design and operate shared e-scooter systems, and thus help to ensure system reliability and cost effectiveness.

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EXECUTIVE SUMMARY

In the past few years, shared e-scooter systems have gained increased popularity around the world because of their benefits to health, traffic congestion, the environment, and accessibility. As of 2018, approximately 100 U.S. cities have launched shared e-scooter programs, accounting for 38.5 million trips. However, the business model to manage e-scooter sharing remains nascent, with many challenges still poorly addressed and outstanding. In this project, we propose to solve several urgent questions that arise at the company and policymaker levels for e-scooter sharing (e.g., planning, operations), by developing a data-driven optimization model to provide decision makers with a robust solution that enables low cost and high service quality. Specifically, we develop a two-stage stochastic programming (SP) model for the planning of the e-scooters in the presence of demand uncertainty. In the first stage, we address the major planning decisions for an e-scooter network, including "how many e-scooters are needed in the network and in which locations"? The second-stage is evaluated daily based on the planning decisions from the first stage and the uncertain demand realizations, while its objective is to minimize the costs of an extended spacial-temporal-SoC network (SoC stands for state-of-charge). We then apply the sample average approximation method to solve the two-stage SP problem, and compare the proposed model with some benchmark planning approaches in a numerical study.

In line with the NITC themes, these research results have the potential to provide e-scooter companies with new decision-making tools and methodologies to effectively design and operate shared e-scooter systems, and thus help to ensure system reliability and cost effectiveness.

1. Introduction

Shared mobility is the future of the customer transportation industry. The main benefits of the shared mobility agenda are threefold. First, it is an economic solution for many customers who cannot afford the high costs of vehicle ownership. Second, it is environmentally friendly, and it allows less resources to be used more efficiently. And third, it offers a novel solution to the problem of traffic congestion, which is a major issue in many large cities. Micromobility solutions, in particular e-scooter and e-bicycle share programs, are popular due to their effectiveness, especially for the last mile trips (e.g., trips between homes and metro stations). From one side, the technological advances in recent years, especially in the fields of communication (e.g., 5G networks), electronics (e.g., smartphones and on board GPS devices), and energy storage (e.g., lithium-ion batteries) have allowed for the implementation and manufacturing of very economical yet efficient e-scooters and e-bicycles. Moreover, from customers' perspective, e-scooters and e-bikes are much easier to access and utilize compared to the traditional bike-station setup. In addition, the governments in larger cities have provided financial incentives to such solutions, with an effort to solving the high traffic congestion problem and promoting clean means of transport. As a result, many cities across the globe are adapting the e-scooter and e-bike resolution to their transportation portfolio. For instance, the City of Tucson launched a pilot program in 2019 with 1,000 e-scooters which average about 600 trips a day [1].

Successful operation of the e-scooters from a managerial point of view requires both careful planning and operational strategies for the e-scooter renting company. Such strategies for instance aim at decreasing the amount of unused (idle) e-scooters by carefully placing them in popular locations and re-balancing them on a tentative schedule. In particular and from a day-to-day operational perspective, careful strategies minimize the operational costs for the company while also increasing the costumer satisfaction rate (e.g., by having enough e-scooters available at potential demand locations). From a longer-term planning perspective, it is necessary to consider e-scooter demand patterns (e.g., on a seasonal basis) and devise planning strategies accordingly that lead to long-term economic benefits while achieving the efficient operation of the system.

Hereafter, we first lay out the literature review for designing a set of planning decisions for micromobility sharing systems. The idea of bike sharing has been scrutinized as early as 1965 when a group of bicycles were painted white and left in the city of Amsterdam for people to use for free. However, within a month these bikes were either stolen or lost. Indeed, the idea of shared mobility was not successful until smart technologies such as electronic card readers were used to unlock bikes from a storage rack to prevent theft. Moreover, many of the early bike-share projects failed due to the need for the presence of dock stations, which required manpower and had limited capacity. Recently, and due to the advancement of personal smartphones, the idea of shared bike systems has become more prevalent. In particular, the development of GPS technology and fast internet (4G, 5G) has made it possible to establish dockless bicycle or scooter sharing facilities that decreases operational costs by eliminating the need to establish and operate dock stations. Moreover, e-scooter sharing is a fairly recent technology that is currently booming due to the lowering of the costs of lithium-ion batteries and the recent technological advancements of smartphone and communication technologies [2, 3]. Consequently, the literature on the e-scooter planning problem is relatively sparse. However, bike sharing and e-bike sharing (docked or dockless) have been going on for a longer time, which are similar to e-scooters from the perspective of planning and operation [4].

In particular, these groups of small, lightweight vehicles operated at speeds typically below 25 km/h and personally driven by users are commonly referred to as micromobility. The planning problem for the micromobility technologies has a long history. In summary, a chronological list of the shared micromobility technologies and research is classified as follows:

- Docked sharing: Bike-sharing and electric bike sharing [5]
- Dockless sharing: Free-floating bike sharing [6] and e-scooter sharing [7]
- Customer oriented sharing: Dockless sharing that is customer operated (i.e., it considers customer charging incentive, relocation incentive) [8].

Next, we present a modeling and methodology overview for the e-scooter sharing and micromobility planning problem.

Modeling and Methodology Literature: Two-stage stochastic programming (SP) is a popular technique often used for modeling planning decisions under uncertainty. This problem is particularly popular for formulating long-term planning decisions, such as renewable planning and transportation planning. The reason is that its particular two-stage form allows for modeling two sets of decisions simultaneously. In particular, a *here-and-now* decision in the first stage and a *wait-and-see* decision in the second stage. Consequently, two-stage SP modeling tool has been widely adopted to address micromobility planning problems. For instance, [9] proposes a two-stage SP model where the first stage is the re-balancing planning and the second stage is expected costs in terms of fulfilled and unfulfilled demands. In addition, [10] proposes two-stage and multistage SP models for determining the optimal number of bikes to assign at each station at the beginning of each operational period.

Other methodologies besides the two-stage SP have also been used in the literature to address the micromobility planning problem. For instance, [11] applies agent-based simulation to find an optimal rebalancing strategy for bike sharing systems. And [12] develops a mixed-integer linear programming (MILP) formulation for bike relocation problem, which is solved via a branch-and-cut algorithm. Moreover, [7] proposes an MILP model for assigning e-scooters to chargers and solves the problem using a college admission heuristic algorithm, which is compared with a black hole optimizer heuristic algorithm. Furthermore, others, such as [8], use a Markov chain paired with deep reinforcement learning techniques for rebalancing of dockless bike sharing systems, where the user is incentivized to do the rebalancing.

In this study, we develop a two-stage SP model for the planning of the e-scooters in the presence of demand uncertainty. In particular, we address the major planning decisions for an e-scooter network (i.e., “how many e-scooters are needed in the network and in which locations?”, “how many charging facilities are needed in the network and at which locations and at what capacities?”, and, lastly, “how do we rebalance the network periodically so the e-scooters are available when and where they are needed?”). These planning decisions are formulated in the first stage of the problem. The second stage is evaluated daily based on the planning decisions from the first stage and the uncertain demand realizations. These operational decisions for an e-scooter network involve renting out the e-scooters to the customers, charging the e-scooters, and rebalancing or relocating e-scooters in the course of a day. The second-stage problem is modeled to minimize the costs of an extended spacial-temporal-SoC network (SoC stands for state-of-charge). This model also is designed to provide economic incentive to customers to walk to a neighboring location to rent out the e-scooters when there is no availability at their current location. The demands in the second-stage problem are considered uncertain to more accurately represent the nature of the problem. We apply the sample average approximation method [13] to solve the two-stage SP problem, and compare the proposed model with some benchmark planning approaches in a numerical study at the end.

2. Mathematical Modeling

In this section, the e-scooter planning problem is modeled as a two-stage stochastic optimization problem. The general form for a two-stage stochastic program is given as follows:

$$\min_{x \in \mathscr{X}} [\mathscr{C}_1(x) + \mathbb{E}[Q(x,\xi)]]. \tag{1}$$

where x is the first-stage decision, ξ is the vector of uncertainty, $\mathscr{C}_1(x)$ is the first-stage objective function, $\mathscr{X}$ is the feasibility set of the first-stage problem, and $Q(x,\xi)$ is the optimal value of the second-stage problem. For the e-scooter planning problem, the first-stage decision variables x consist of the long-term (monthly or yearly) planning decisions, while the second-stage optimization problem involved in $Q(x, \xi)$ represents the operational decisions of the network, given a first-stage decision x and uncertain parameter ξ. The detailed mathematical models for the first- and second-stage problems are introduced in subsections 2.1 and 2.2, respectively. Lastly, the overall two-stage program for the e-scooter planning is summarized in subsection 2.2.

2.1 First-stage planning model

The objective of the first-stage problem is to minimize the investment costs which, in particular, includes investments costs for installing charging facilities, for adapting relocation schedules into the network, and increasing the e-scooter fleet size. Let $\mathscr{L}$ denote the set of all the locations in the network. Some of these locations can be selected to host charging facilities. Let φ represent the set of potential enterprise charging facility locations. Then the binary decision u_i, $\forall i \in \varphi$ is introduced such that $u_i = 1$ if a charging facility is allocated at location i, otherwise $u_i = 0$. Moreover, an integer variable q_i indicates the capacity of the charging facility at location i. Next, we assume that the relocations follow some *schedules* which indicate the time-wise relocation path (i.e., the locations that the relocation vehicle visits at specific times to collect or drop off e-scooters). And let $\mathscr{K}$ to be a set of potential schedules that one or some of them are selected by the service company to do the relocating task. The binary variable $p_k, \forall k \in \mathscr{K}$ is introduced such that $p_k = 1$ indicates the relocation schedule k is selected and is active in our model. Each relocation schedule $k \in K$ has a certain capacity indicated by an integer variable o_k. Finally, consider z_i, $\forall i \in \varphi$ to denote the number of e-scooters allocated to location i at the beginning of the day. Furthermore, we summarize the associated parameters in the first-stage in Table 1.

With the introduction of the variables and parameters above, the first-stage model for the e-scooter planning problem is represented as follows:

Table 1. Parameters and decision variables in the first stage.

Para. & Vari.	**Description**
$\hat{c}^p_t / \hat{c}^q_t$	Electricity price of purchasing active / reactive power from the transmission system at period t.
c_i^{CF}	The fixed cost of installing a charging facility at location $i \in \varphi$.
c_i^{CV}	The variable cost of installing a charging facility at location $i \in \varphi$.
c_i^{RF}	The fixed cost of employing a relocation schedule $k \in K$.
c_i^{RV}	The variable cost of employing a relocation schedule $k \in K$.
c_i^{EA}	The cost associated to deploying one e-scooter at location $i \in \varphi$.
B_i	The maximum e-scooter fleet size at location $i \in \mathscr{L}$.
M_i	The maximum capacity for charging facility at location $i \in \varphi$.
M	The total capacity for charging facilities.
G_k	The maximum capacity for the relocation schedule $k \in K$.
G	The total capacity for the relocation schedules.

$$
\begin{aligned}
\min_{x:=\{\mathbf{z},\mathbf{p},\mathbf{o},\mathbf{u},\mathbf{q}\}} \quad & \sum_{i\in\varphi}(c_i^{CF}u_i + c_i^{CV}q_i + c_i^{EA}z_i) + \\
& \sum_{k\in K}(c_k^{RF}p_k + c_k^{RV}o_k) + \mathbb{E}Q(x,\xi) && && \text{(2a)} \\
s.t. \quad & q_i \le M_i u_i, && \forall\, i \in \varphi, && \text{(2b)} \\
& q_i \ge u_i, && \forall\, i \in \varphi, && \text{(2c)} \\
& \sum_{i\in\varphi} q_i \ge M, && \forall\, i \in \varphi, && \text{(2d)} \\
& z_i \le B_i, && \forall\, i \in \varphi, && \text{(2e)} \\
& o_k \le G_k p_k, && \forall\, k \in K, && \text{(2f)} \\
& o_k \ge p_k, && \forall\, k \in K, && \text{(2g)} \\
& \sum_{k\in K} o_k \le G, && && \text{(2h)} \\
& u_i, p_k \in \{0,1\}, z_i, q_i, o_i \in \mathbb{Z}_+ \cup \{0\}, && \forall i \in \varphi, \forall k \in K && \text{(2i)}
\end{aligned}
$$

where the objective is to minimize the total investment costs and the expected second-stage cost, $Q(x,\xi)$, which is described in detail in section 2.2. Constraints (2b) impose an upper bound on the size of the charging facility at location $i \in \varphi$. Constraints (2c) described the relationship between the binary and integer variables u_i and q_i, indicating that if a location is selected for installing a charging facility, it must have non-zero charging capacity. Also, an upper bound on the total capacity of all charging facilities via constraints (2d). Moreover, the number of e-scooters allocated to location $i \in \varphi$ at the beginning of the day is restricted via constraint (2e). Furthermore, constraints (2f) and constraints (2g) describe the relationship between p_k and o_k (similar to constraints (2c)).

2.2 Second-stage operational model

After making the planning decision for the e-scooter network in the first stage (the long-term planning decisions), the e-scooter operations need be adjusted to meet demands after realization of uncertainty. In the second stage we aim to minimize the operational costs, including the cost of vehicle movement in the spacial-temporal-SoC network (e.g., relocation and charging costs) and a penalty cost for unserved demands. In this subsection, we first introduce the spacial-temporal-SoC network and its characterizing arcs, and then we present the mathematical reformulation for the second-stage problem.

2.2.1 Spatial-Temporal-SoC Network

A spatial-temporal-SoC network is developed to represent the operations of the system. Let $\mathscr{L}$ denote the set of all the locations in the network, and $\mathscr{T} = \{1,2,..,T\}$ and $\mathscr{S}_p = \{0,1,\dots,s_p\}$ denote the sets of time periods and state-of-charge levels, respectively. Here, T denotes the number of operational time-periods in a day, and s_p is the full capacity of the battery (type p) when charged. Let $d_{ijtt'}$ be the number of trips demanded from i at time t to j at time t', where obviously $t' \geq t + t_{ij}^m$, and t_{ij}^m is the minimum travelling time required from i to j. We assume SoC changes linearly in time for charging and discharging. Also, SoC consumption is assumed to be linear with the traveling time, and we denote the number of SoC used (discharged) per time period by s^d. Moreover, s_c^c and s_e^c are the number of SoC increased per time period when charging by a customer or enterprise, respectively. We further assume an e-Scooter cannot be rented when the SoC falls below a certain threshold so it must be charged, relocated or idle in the current location. There is also a degradation cost for batteries, which we consider to be relative to the number of SoC that an e-Scooter uses/charges, and it has a unit cost of c_p^{deg}.

Let r_{ij} be the revenue generated by a trip from i to j, and ω_{ik} be the discount rate for customers to walk from i to k (we assume it is proportional to the physical distance). We denote by $c_{ij}^{rlcn,k}$ the relocation cost from i to j through schedule k, c_i^{idle} the idle cost at location i, and c_{it}^{chrg} the charging cost at location i and time t.

The network is then modeled as a graph $G = (V,E)$. The nodes $v_{its} \in V$ denote the state of the e-scooters (i.e., location $i \in \varphi$ time $t \in T$ SoC $s \in S$), and the arcs $e \in \mathscr{E}$ represent the movement of the e-scooters in the network. We consider the following type of arcs in $\mathscr{E}$:

1. **Rental arcs** $e = (v_{its}, v_{jt's'})$ for $d_{ijtt'} > 0$, $s' \geq 0$, and $t' - t \geq t^m_{ij}$, with demand amount $d_{ijtt'}$ and cost $-r_{ij}(t'-t)$. To be a demand arc, an arc should satisfy $s \geq s' + (t'-t)s^d$, and also s be above SoC threshold. Flows on these arcs represent the rental movement of e-scooters with SoC s being rented from location i starting at time t, and being returned to location j at time t' and SoC s'.

2. **Routed Rental arcs** $e = (v_{k\tilde{t}s}, v_{jt's'})$ for $d_{ijtt'} > 0$ and in case when there are no available e-scooters in location i, the customer is encouraged to walk to a neighboring location $\tilde{i} \in \mathscr{L}_i$, where $\mathscr{L}_i$ denoted the neighboring locations to zone i, and rent from there. However, the starting time for this trip after walking from i to $\tilde{i}$ changes to $\tilde{t} = t + t^w_{i\tilde{i}}$, where $t^w_{i\tilde{i}}$ denotes the walking time from zone i to $\tilde{i}$, and the actual e-scooter travel from location $\tilde{i}$ to j is performed at a discounted rate to enable an economic incentive for the rider. Cost is $-(1-\omega_{i\tilde{i}})r_{\tilde{i}j}(t'-t-t^w_{i\tilde{i}})$. For example if $\omega_{i\tilde{i}} = 5\%$, then the revenue for the trip from $\tilde{i}$ to j is calculated at a 95% rate.

3. **Relocation arcs** $e = (v_{its}, v_{jt's})$ for $t' \geq t + t^m_{ij}$. The relocation schedules are selected in the first-stage. Each schedule denotes "a utility vehicle that goes a certain path during the day and can do relocation along that path." For instance, Table 2 shows two demonstrative schedules that could be implemented for a network of $T = 5$ time periods and 3 locations $L = \{A, B, C\}$. The cost for relocating an e-scooter from i to j through schedule $k \in K$ is noted by $c^{rlcn,k}_{ij} t^m_{ij}$. Moreover, each schedule has a capacity that is also selected in the first-stage model.

Table 2. Example of schedule.

Time Period	1	2	3	4	5
Schedule 1	A	B	C	A	B
Schedule 2	A		B		A

4. **Idle arcs** $e = (v_{its}, v_{i,t+1,s})$ for $i \in \mathscr{L}$, and $1 \leq t \leq T$ with cost c^{idle}_i, representing e-scooters staying in location i from period t to $t+1$ and with the SoC of s.

5. **Charging arcs** To model the charging arcs in our network, we introduce charging locations (or shadow locations) corresponding to the charging locations from the first-stage (i.e., φ). Total locations are denoted by $\mathbb{L} = \mathscr{L} \bigcup \varphi$. To perform the act of charging, an e-scooter must move to and back from the shadow locations. Therefore, there will be (1) transportation arcs (movement of e-scooters between the physical locations and the charging locations) and

(2) the actual charging arcs, which increase the SoC of batteries. Therefore, the following arcs are added to consider an enterprise-wide charging capability into the network.

- **Enterprise Charger Transportation arcs** $e = (v_{its}, v_{\hat{j},t+t^E_{i\hat{i}},s})$ for collecting the e-scooters from the physical locations and taking them to the charging facilities (racks) only if available at that location, and $e = (v_{\hat{j},ts}, v_{i,t+t^E_{i\hat{i}},s})$ for taking the e-scooters back to the physical locations after they are charged. Here, $t^E_{i\hat{i}}$ is the average time that it takes to collect the e-scooters from a physical location i and take them to the charging facility $\hat{i}$ by the service company.
- **Enterprise Charging arcs** $e = (v_{\hat{i},ts}, v_{\hat{i},t+1,s+s^c_e})$ for charging facility $\hat{i} \in \hat{\varphi}$, and $0 \leq s + s^c_e \leq s_p$, and flows on these arcs mean that e-scooters are being charged from s to $s + s^c_e$ at charging zone $\hat{i}$ in one time period between t to $t+1$. Note that the enterprise charging can only happen at the locations in the network (regions in the city) that are allocated charging capabilities by the first-stage decision. Therefore, a collaboration between the planning of the relocation schedules and charging facilities is needed to charge e-scooters in an efficient manner.

Finally, customer-wide charging capability could also be included into the network. That is, instead of the enterprise collecting the e-scooters from a region and charging them at the charging facilities in that region (only at the locations allowed by the first-stage decision), now the customers could take the e-scooters to their home and charge them. The following arcs are introduced in the network to reflect customer-wide charging:

- **Customer Charger Transportation arcs** $e = (v_{its}, v_{\hat{i},t+1,s})$ for going from physical locations to charging locations and $e = (v_{\hat{i},tp}, v_{i,t+1,p})$ for coming back to physical locations. Here, we assume that the customer takes the e-scooter home to charge so charging node $\hat{i}$ should be the same as the physical location where the e-scooter is located. Moreover, an e-scooter can be moved by the customers for charging if the SoC level of the e-scooter is less than full (i.e., $s < p$), and the e-scooters are fully charged when returned to the network by the customers.
- **Customer Charging arcs** $e = (v_{\hat{i},t,s}, v_{\hat{i},t+1,s+s^c_c})$ for shadow zone $\hat{i}$, and $0 \leq s + s^c_c \leq s_p$, and flows on these arcs mean that the e-scooter is being charged from SoC s to $s + s^c_c$ in zone i in one time period between t to $t+1$. Here, c^{chrg}_{it} is the charging cost incurred by a customer at location i and time t. This cost to the company can be interpreted as an incentive or discount to the customers. There is also a degradation cost here for every SoC level changed.

2.2.2 Mathematical Modeling of Second Stage

The unit flow cost and capacity of each arc are summarized in Table 3. For each realization of the uncertain demand $d_{ijtt'}$, the recourse decisions $y_e \geq 0 \; \forall e \in \mathscr{E}$, represents e-scooter movement on the network indicating the flow on the Rental, Routed Rental, Idle, Relocation, and Charging arcs. Let the slack variable $w_{ijtt'}$ represent the unsatisfied demands corresponding to $d_{ijtt'} \geq w_{ijtt'}$. In vector notation $\mathbf{y} = (y_e \geq 0 \; \forall e \in \mathscr{E})^\top$, and $\mathbf{w} = (w_{ijtt'} \geq 0 \; \forall d_{ijtt'} > 0, i, j \in \mathscr{L}, t, t' \in T)^\top$. The respective costs are 1) c_1 in Table 3 cost of vehicle movement in the spacial-temporal-SoC network (i.e., Rental, Routed Rental, Relocation (transsshipment), Idle, Charging), and 2) c_2 expected cost of unserved demands. For unserved demands in particular, $c^{\text{lost}}_{ijtt'}$ denotes the penalty for losing demand from i at time t to location j at time t'. Furthermore, p denotes the maximum state-of-charge of battery. We also let $\delta^+(v_{its})$ and $\delta^-(v_{its})$ denote the set of arcs leaving and entering node v_{its}, respectively. Moreover, let the set $\mathscr{D}^{Rent}_{ijtt'}$ denote the set of Rental or Routed Rental arcs corresponding to demands from location i at period t to location j at period t', and $\mathscr{D}^{Char}_{\hat{i}t}$ denote the set of charging arcs at charging location $\hat{i}$ at period t, and $\mathscr{D}^{Sche}_{kt}$ denote the set of Relocation Schedule arcs corresponding to schedule k that are active at period t. Furthermore, each relocation schedule and charging capacity has their corresponding capacities which are decided in the first stage (o_k, $k \in K$, $q_i \; i \in \varphi$). Then, the full mathematical model is as follows:

Table 3. Unit flow costs.

Type of Arc	Cost per unit flow
Rental arc	$-r_{ij}(t'-t)$
Routed Rental arc	$-(1-\omega_{i\tilde{i}})r_{kj}(t'-t-t^w_{i\tilde{i}})$
Relocation arc	$c^{rlcn,k}_{ij} t^m_{ij}$
Idle arc	c^{idle}_i
Cstmr Charging Transport. arc	
Cstmr Charging arc	$(s'-s)c^{chrg}_{it} + c^{deg}_p$
Srvc Charging Transport. arc	$c^{trans}_{serv} + c^{deg}_p$
Srvc Charging arc	$c^{chrg}_{serv} + c^{deg}_p$

$$
\begin{aligned}
Q(x,\xi) = \min_{\mathbf{y},\mathbf{w}} \; & c_1^\top \mathbf{y} + c_2^\top \mathbf{w} && && \text{(3a)}\\
s.t. \quad & \sum_{e\in\delta^+(v_{i,1,p})} y_e = z_i, && \forall\, i \in \mathbb{L}, && \text{(3b)}\\
& \sum_{e\in\delta^-(v_{its})} y_e - \sum_{e\in\delta^+(v_{its})} y_e = 0, && i \in \mathbb{L}, t \in \mathscr{T}/1,T, s \in \mathscr{S}_p, && \text{(3c)}\\
& \sum_{e\in\delta^-(v_{i,T,p})} y_e = z_i, && \forall\, i \in \mathbb{L}, && \text{(3d)}\\
& \sum_{e\in\mathscr{D}^{\mathsf{Rent}}_{ijtt'}} y_e + w_{ijtt'} = d_{ijtt'}, && \forall\, i,j \in \mathscr{L}, t \in \mathscr{T}, t' = t + t^m_{ij} && \text{(3e)}\\
& \sum_{e\in\mathscr{D}^{\mathsf{Sche}}_{kt}} y_e \le o_k, && \forall t \in \mathscr{T}, k \in \mathscr{K} && \text{(3f)}\\
& \sum_{e\in\mathscr{D}^{\mathsf{Char}}_{\hat{i}t}} y_e \le q_i, && \forall t \in \{2,3,\ldots,T\}, i \in \varphi && \text{(3g)}\\
& y_e = 0 && \forall e \in \delta^+(v_{i,1,p}), s \neq s_p, s \in \mathscr{S}_p, && \text{(3h)}\\
& y_e = 0 && \forall e \in \delta^-(v_{i,T,p}), s \neq s_p, s \in \mathscr{S}_p. && \text{(3i)}
\end{aligned}
$$

The objective is to minimize the total cost. Constraints (3b), (3c) (3d) balance the flow of e-scooters in the network. Constraint (3e) is added for meeting the demands. Constraint (3f) is to limit the amount of relocation per each schedule to its capacity. The number of e-scooters being charged simultaneously at a particular charging location is bounded from above by constraint (3g). Lastly, constraints (3h) and (3i) are introduced based on the assumption of the problem that the battery SoC should be full at the beginning and end of the planning horizon.

2.3 Two-stage stochastic program of e-scooter planning problem

The overall two-stage stochastic program of the e-scooter planning problem is then formulated as follows:

$$\min_{x\in\mathscr{X}} [\mathscr{C}_1(x) + \mathbb{E}[Q(x,\xi)]]. \tag{4}$$

This problem is, in particular, a two-stage stochastic mixed-integer program where both the first- and second-stage problems are mixed-integer linear programs and the demand parameters in the second stage are random. In general, two-stage stochastic mixed-integer programs are computationally challenging to solve exactly for large-scale instances. Next, a sample-based approximation of the problem is discussed.

When historical data are available for the random variables, one popular approximation approach for solving the stochastic programming problem (4) is the sample average approximation method [13], which is to replace the underlying probability distribution of the random variables with their empirical distribution obtained from the historical samples. However, the resulting approximation problem is a large-scale mixed-integer problem, and therefore, it is computationally challenging to be solved in practice. To alleviate this issue, we adopt a scenario reduction technique to further approximate the stochastic programming problem (4), where a subset of the data set is employed instead of the entire data set. In this report, we refer the solution approach for solving the two-stage e-scooter planning problem (4) as sample average approximation (SAA) method.

3. Numerical Results

In this section, a set of computational results for the two-stage stochastic e-scooter planning problem are reported. In particular, the SAA results for solving the planning problem are reported as compared with a deterministic version of the model where the underlying uncertainty is overlooked. First the characteristics and data for a hypothetical network regarding the e-scooter planning problem are introduced. The purpose for the development of this network is to perform tangible analysis on the two-stage model performance. In the entire section, the planning horizon is considered to be one year long. The raw demand data for the test were from the City of Tucson. All numerical tests were implemented on a computer with an Intel Core i7-7700 CPU and 16 GB memory. All optimization problems were solved by Gurobi solver in Python.

3.1 Problem setup and parameters

We consider an extended spatial-temporal-SoC network with seven locations, 24 time intervals and 10 SoC levels. The maximum number of e-scooters that can be allocated to each location is as follows; $\{B_1 = 40, B_2 = 20, B_3 = 40, B_4 = 28, B_5 = 20, B_6 = 32, B_7 = 20\}$. The cost of deploying one e-scooter is assumed to be $\$400$. A set of 12 relocation schedules for the problem are considered, as presented in Table 13 in Appendix A. The total relocation capacity is assumed to be $G = 25$, which is to be divided among multiple schedules. Each of these schedules can have a minimum of five and a maximum of 10 e-scooters on board at one time. That is to reflect the size of the vehicle that carries the e-scooters through the schedules. Moreover, a total charging capacity of $M = 25$ is available which is to be assigned to different charging facilities. The minimum and maximum sizes of each charging facility are set to be five and 10 e-scooters at a time, respectively. Moreover, the fixed costs for installing charging facilities at each of the locations and for deploying each of the relocation schedules are given in Tables 4 and 5, respectively. Note that the costs here are on a yearly basis and they need to be converted to daily costs to be comparable with the second-stage costs in the problem (4). In the numerical tests, an interest rate of zero is assumed for this conversion (i.e., simply diving the yearly costs by number of days in a year). Table 4 indicates that no charging facilities are to be installed at locations L4 and L7. The variable costs for installing a charging facility and for deploying relocation schedules are set at $5\$$ per e-scooter and $2\$$ per e-scooter, respectively.

Table 4. Fixed costs of installing charging facilities (yearly).

Location	L1	L2	L3	L4	L5	L6	L7
Cost ($)	9,125	7,300	8,030	-	11,680	12,775	-

Table 5. Fixed costs of deploying relocation schedules (yearly).

Location	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
Cost ($)	5110	4015	3650	2190	5110	4015	3650	2190	5475	5840	6205	7300

For the second-stage problem, the problem setting is as follows. The minimum travelling time between the locations t_{ij}^m is assumed to follow a discrete uniform distribution as *Unif*$(1,2)$. The unit costs for each arc (i.e., relocation, idle, charging, and rental arcs) are given in Table 6.

Table 6. Unit flow costs.

Arc Parameter	Cost ($)
$c_{ij}^{rlcn,k}$	*Unif*$(1.7,2.9)$
c_i^{idle}	*Unif*$(0.05,0.15)$
r_{ij}	$-Unif(2,6)*t_{ij}^m$
c_{serv}^{chrg}	*Unif*$(0.3,0.5)$
c_{serv}^{trans}	0.4
c_{it}^{chrg}	*Unif*$(1.5,2.1)$

The demands are uncertain with an empirical distribution (based on historical data). However, this data set (from the City of Tucson) did not report starting and finishing time for the e-scooter trips. As a result, the data were adjusted to follow typical e-scooter demand trends, similar to other data sets that are available online such as at [14–16]. Figure 1 represents the modified demands in one region at different time periods of a day. The demands at each location are first satisfied by

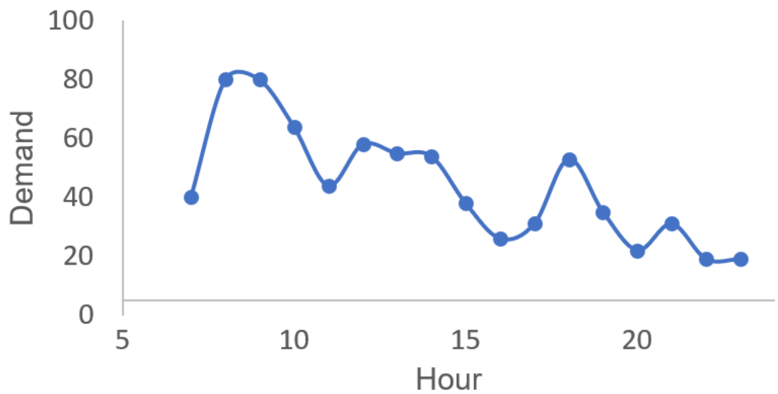

Fig. 1. E-scooter demand over a day.

the e-scooters at that location. If no e-scooters are available at the current location, the customer is encouraged to walk to a neighboring location to rent e-scooters. The walking time to a neighboring region is considered to be one time interval for all cases. The discount rate for customers who walk from i to a neighboring location $\tilde{i}$ to rent out e-scooters is set to 20% at all locations, that is $\omega_{i\tilde{i}} = 0.20\ \forall i \in \mathbb{L}$, and the cost of lost demand c_{ij}^{lost} is considered to be 50% more than the income for that demand, that is $c_{ij}^{lost} = 1.5 r_{ij}$. That is to consider an extra cost for customer dissatisfaction.

3.2 Computational study and optimal costs

In this section, the computational performance of the two-stage e-scooter planning problem (4) is reported.

Optimality and customer rejection costs. First, the sample average approximation of the model (4) is compared with a deterministic version of the model, that is model (4) with a deterministic demand parameter. The "optimal value" and "CPU times" for each method are reported in Table 7. Additionally, out-of-sample simulation results, particularly "customer rejection costs" and "average second-stage (operational) costs", are presented to evaluate the performance of the planning decisions for each method. For the in-sample test, a set of 10 data samples are selected randomly from the data set for the SAA method. These samples are then averaged to obtain one average demand for the deterministic model. For the out-of-sample, a set of additional 100 samples from the data set are then selected (excluding the initial selected samples in the in-sample test) to analyze the quality of the solutions obtained from both methods. The optimal first-stage solutions obtained from both methods are then evaluated by fixing the first-stage variables on the second-stage operation problem.

In terms of optimal values, the results in Table 7 demonstrate that the SAA

Table 7. Optimality and performance for the stochastic vs deterministic cases.

	Two-stage model (4)		Simulation performance	
	Optimal value ($)	CPU time (s)	Average cost ($)	Customer rejection cost
SAA	-758.032	1000	-1199.11	314.19
Deterministic	-891.394	1	-1024.98	437.76

method achieves higher costs than the deterministic version of the model. Note that the negative costs indicate an income. Also note that these two models are not directly comparable, as their second stage varies significantly. Therefore, next and to evaluate the quality of the solutions obtained from each method (on equal grounds), out-of-sample simulation tests are performed. The out-of-sample simulation results show that the SAA planning decision leads to better operational performance compared to the deterministic planning decision. More precisely, the SAA planning decision results in higher profit (lower operational cost and higher income). It is observed that the solution obtained from the SAA method has significantly less customer rejection costs. Thus, the customer satisfaction is significantly higher under the planning strategies from the SAA method. At last, the results in Table 7 indicate that the benefits from the SAA method come at a cost of higher CPU times than the deterministic case, which is due to the larger size of the optimization problem using the SAA approach (larger set of constraints and integer variables).

Optimality and relaxed second stage. Relaxing the second-stage integer variables (y, w) to continuous variables could be a potential way of dealing with the high computational times for the SAA formulation of the problem. Next, a relaxed version of the SAA problem is solved and compared with the original SAA problem, and the results are shown in Table 8.

Table 8. SAA results for the relaxed vs integer second-stage variables.

	Model (2)	Relaxed second-stage
CPU time (s)	1000	62
First-stage investment costs	397.9	396.81
Second-stage (average) costs	-1483.42	-1483.14

From Table 8, we can observe that relaxing the second-stage integer variables in this planning problem does not affect the optimal values significantly. On the other hand, relaxing the variables leads to much faster computational results. These results are essential in implementing further efficient methodologies (such as benders decomposition) to speed up the solution procedure for the SAA formulation of the two-stage planning problem.

3.3 Sensitivity analysis

In this section, sensitivity analysis results (i.e., the effects of varying some of the parameters of the model on the optimal costs) are reported. In particular, sensitivity analysis with respect to the walking distance parameter (ω), maximum capacity of the relocation schedules (G), maximum capacity of the charging facilities (M), and the maximum e-scooter fleet size ($\sum_{\forall i} B_i$) are presented in Tables 9, 10, 11, and 12, respectively. For each case, the detailed optimal cost components (i.e., the first-stage planning and the second-stage operational costs) are reported.

First, and regarding the walking distance parameter ω, let all of the locations incur the same discount rate (i.e., $\omega_{i\tilde{i}} = \omega \; \forall i \in \mathbb{L}$ and ω is either 0.1 or 0.4) to represent a low and aggressive discount strategy respectively. The detailed costs are reported in Table 9 for the two strategies regarding the economic incentive to customers who walk to a neighboring location when there are no e-scooters readily available at their current location. It is observed that increasing the discount rate to 40% leads to an increase in the total costs for the two-stage planning problem. This increase occurs in both the first-stage and second-stage costs. Moreover, the cost of lost demand in the system is also increased as a result of the aggressive discount rate to the customers. Lastly, and regarding the first-stage costs, the results from Table 9 indicate that that the aggressive strategy leads to investing more into the system infrastructure, that is charging facilities, relocation schedules, and e-scooter fleet size, in order to avoid a shortage of e-scooters. Consequently, the first-stage costs are high. Secondly, and as a result of such investments, the system is more likely to provide e-scooters at the location of the customers. Thus, the system is more likely to satisfy the demands directly through profitable Rental arcs and not rely much on the less profitable Routed Rental arcs.

Table 10 reports sensitivity results for the case of varying the upper bound on the relocation schedules capacity parameter (G) in constraints (2h). Increasing this bound from 15 to 25 does not changes the total costs of the system significantly. In particular, we observe that the fixed cost associated with relocation schedules remains constant, and there is only a small increase in the variable costs of the relocation schedules. That means, for instance, a larger vehicle is assigned to some existing relocation schedules.

Next the sensitivity analysis is performed with regards to the maximum charging capacity parameter (M) from constraints (2d). Table 11 summarizes the detailed optimal costs of problem (4) with three levels of maximum charging capacities. As the maximum charging capacity parameter increases from 20 to 30, the total costs of the system decrease. In particular, the first-stage investment costs increase, but that leads to a significant increase of the income in the second stage. The customer rejection costs of the planning horizon decrease when increased levels of charging capacities are available for planning. It is also important to note that the relocation capacities of the system, as well as the e-scooter fleet size, have

Table 9. Sensitivity analysis of economic incentive.

		Economic incentive (ω)	
		0.1	0.4
Total costs		-1106.78	-1066.45
First-stage investment costs		398.81	428.54
Charging facilites	Fixed	87.0	112.0
	Variable	125.0	125.0
Relocation Schedule	Fixed	28.0	34.0
	Variable	40.0	42.0
E-scooter flee		118.81	115.54
Second-stage (average) costs		-1505.59	-1494.99
Rental		-2040.46	-2234.57
Routed rental		-234.3	-5.07
Relocation		211.4	150.07
Idle		98.02	96.83
Charging	Electricity	221.92	223.16
	Transport	96.0	104.4
Customer rejection cost		141.84	170.18

increased, and that all has lead to increased profit in the system. Therefore, this strategy (i.e., increasing the maximum charging capacity parameter) is strongly supported by our planning model (4).

From the second-stage operational perspective, it is important to note that the increased investment in charging facilities has translated to a decrease in the relocation costs. That is because the increased fixed cost of charging facilities means opening new charging facilities in new locations, and that indicates that the e-scooters are more likely to be charged in their own location, so the need for transportation in the second stage is decreased.

Last but not least, Table 12 displays the effect of changing the maximum e-scooter fleet size (or maximum allowed e-scooters), that is $\sum_i B_i$, on the total costs. It is observed that decreasing the maximum allowed e-scooter fleet size leads to less efficient plans for the system. In particular, by limiting the maximum allowed e-scooter flee size from 200 to 100 a 10% increase in the total cost is observed. Decreasing this limit to 50 leads to a 70% increase in the costs. This increase in costs mainly occurs at the second stage when the cost of customer rejection has increased while the rental income has decreased. And that is straightforward as a result of a shortage of e-scooters in the system.

Table 10. Sensitivity analysis of relocation capacity.

		Relocation Capacity (G)	
		15	25
Total costs		-1082.63	-1085.52
First-stage investment costs		390.99	397.9
Charging facilities	Fixed	87.0	87.0
	Variable	125.0	125.0
Relocation Schedule	Fixed	28.0	28.0
	Variable	30.0	38.0
E-scooter flee		120.99	119.9
Second-stage (average) costs		-1473.62	-1483.42
Rental		-2131.26	-2117.37
Routed rental		-121.1	-131.43
Relocation		195.25	194.87
Idle		103.82	100.26
Charging	Electricity	221.6	222.21
	Transport	97.2	96.8
Customer rejection cost		160.86	151.23

The results in the last column of Table 12 correspond to the event where the total number of e-scooters in the system is enforced to be equal to 200 (that is by setting the inequality constraints in (2e) to equality constraints). Furthermore, it is observed that the second-stage costs, in particular idle costs, have increased significantly (about $\$100$ per day) by this strategy, while the total renting income remains almost the same. This indicates that the last column is reporting a suboptimal solution. It can be concluded that suboptimally distributing many e-scooters into the system not only increases the investment costs, but also adds to the operational costs of the system.

4. Conclusions and Future Research Directions

In this study, we developed a two-stage stochastic programming model for the planning of e-scooters systems in the presence of demand uncertainty. In particular, we addressed the major planning decisions for an e-scooter network (i.e., "how many e-scooters are needed in the network and in which locations?" "how many charging facilities are needed in the network and at which locations and

Table 11. Sensitivity analysis of charging capacity.

		Charging Capacity		
		20	25	30
Total costs		-769.7	-1085.52	-1378.96
First-stage investment costs		325.64	397.9	470.44
Charging facilites	Fixed	55.0	87.0	112.0
	Variable	100.0	125.0	150.0
Relocation Schedule	Fixed	28.0	28.0	34.0
	Variable	38.0	38.0	48.0
E-scooter flee		104.64	119.9	126.44
Second-stage (average) costs		-1095.34	-1483.42	-1849.4
Rental		-1738.58	-2117.37	-2524.1
Routed rental		-131.43	-131.43	-103.47
Relocation		217.57	194.87	196.51
Idle		91.44	100.26	94.8
Charging	Electricity	179.63	222.21	265.34
	Transport	76.8	96.8	119.6
Customer rejection cost		209.23	151.23	101.91

at what capacities?", and, lastly, "how do we rebalance the network periodically so the e-scooters are available when and where they are needed?"). Through numerical results, we showed that our optimal planning of the e-scooter system will lead to minimum planning and operational cost and high demand satisfaction rates. Overall, we provided an optimal planning tool for decision makers when e-scooter planning decisions are not trivial due to uncertainties in data and the size of the problem. In the future, this research can be extended to the following directions. First, to increase the robustness in uncertainty modeling, we may model uncertainty through a distributionally robust optimization framework, in which the distribution of the uncertain variables is assumed to be unknown but belongs to an ambiguity set (i.e., probability distribution family). In this case a minimum of first-stage plus the expected second-stage costs under the worst-case distribution is pursued. An event-wised ambiguity set (e.g., different seasons have different ambiguity sets) with a Wasserstein distance metric can be used to model the e-scooter demand uncertainty. Another direction is to develop decomposition approaches to solve the e-scooter planning problem (4) when a large network is considered, such as Benders decomposition approach.

Table 12. Sensitivity analysis of fleet size.

		E-scooter flee size				Unforced
		50	75	100	200	200
Total costs		-333.85	-743.58	-973.63	-1085.52	-853.64
First-stage investment costs		348.78	336.58	361.65	397.9	521.55
Charging facilities	Fixed	112.0	80.0	80.0	87.0	112.0
	Variable	125.0	125.0	125.0	125.0	125.0
Relocation Schedule	Fixed	28.0	28.0	28.0	28.0	34.0
	Variable	38.0	36.0	36.0	38.0	38.0
E-scooter flee		45.78	67.58	92.65	119.9	212.55
Second-stage (average) costs		-682.63	-1080.16	-1335.28	-1483.42	-1375.19
Rental		-1324.38	-1764.21	-2011.17	-2117.37	-2273.79
Routed rental		-63.58	-65.49	-108.95	-131.43	-30.13
Relocation		118.17	149.92	181.69	194.87	182.3
Idle		17.4	34.08	62.69	100.26	256.24
Charging	Electricity	140.93	188.4	214.83	222.21	224.46
	Transport	42.4	66.8	93.2	96.8	110.0
Customer rejection cost		386.42	310.34	232.42	151.23	155.73

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Appendix

A. Data for relocation schedules

The following relocation schedules are assumed for the problem. In particular, the stops for each schedule in particular the time and location of each schedule are given in the Table 13. If a schedule is selected then it repeats during the whole operational day after a rest of 1 periods. For example, when the operational periods $T = 24$ then “S1” starts at time period 1 at location L1 and finished at time period 23 at L3 after repeating six times.

Table 13. Relocation schedules.

Schedules	Stop 1	Stop 2	Stop 3	Stop 4
S1	(1,L1)	(2, L2)	(3, L3)	
S2	(1,L1)	(2, L2)	(4, L3)	
S3	(1,L1)	(3, L2)	(6, L3)	
S4	(1,L1)	(4, L2)		
S5	(1, L5)	(2, L6)	(3, L7)	
S6	(1, L5)	(2, L6)	(4, L7)	
S7	(1, L5)	(3, L6)	(6, L7)	
S8	(1, L5)	(4, L6)		
S9	(1,L1)	(2, L2)	(3, L3)	(6, L6)
S10	(1,L1)	(2, L2)	(3, L3)	(6, L7)
S11	(1,L1)	(2, L3)	(4, L5)	(6, L6)
S12	(1,L1)	(2, L3)	(4, L5)	(6, L7)

B. Online data sets

Below we present a list of data sets available online on shared mobility. For each data set some brief description is mentioned.

- Louisville data open access [14].
  It includes several types of data. Main data set includes following: Starting time, end time, trip duration, trip distance, starting GPS and end GPS (rounded 3 decimals).

- Bike-share (Docked and Dockless) and E-scooter Systems [17].
  Large scale metadata on planning level across country (by year and city served). Sharing mobility in general, both dockless and docked.

- Escooter data, City of Chicago [15, 16].
  E-scooter data includes GPS as well.

- Texas, Austin scooter and bicycle data [18–20]. More than 9 million rows of data. Includes the following information on each trip: Starting time, end time, starting location and end location.