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Eigenvector pruning method for high resolution beamforming

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This paper introduces an eigenvector pruning algorithm for the estimation of the signal-plus-interference eigenspace, required as a preliminary step to subspace beamforming. The proposed method considers large-aperture passive array configurations operating in environments with multiple maneuvering targets in background noise, in which the available data for estimation of sample covariances and eigenvectors are limited. Based on statistical properties of scalar products between deterministic and complex random vectors, this work defines a statistically justified threshold to identify target-related features embedded in the sample eigenvectors, leading to an estimator for the signal-bearing eigenspace. It is shown that data projection into this signal subspace results in sharpening of beamforming outputs corresponding to closely spaced targets and provides better target separation compared to current subspace beamformers. In addition, the proposed threshold gives the user control over the worst-case scenario for the number of false detections by the beamformer. Simulated data are used to quantify the performance of the subspace estimator according to the distance between estimated and true signal subspaces. Beamforming resolution using the proposed method is analyzed with simulated data corresponding to a horizontal line array, as well as experimental data from the Shallow Water Array Performance experiment. © 2015 Acoustical Society of America. [<http://dx.doi.org/10.1121/1.4930568>]

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I. INTRODUCTION

Given the central role played by hydrophone arrays in modern sonar systems,^{1,2} the development of data processing techniques for the detection and localization of potential targets of interest in the water column constitute an active area of research.³ For applications using passive-array configurations for target azimuth estimation, efforts have been directed to the study of high-resolution adaptive beamforming methods which compute data-driven steering vectors optimally designed according to a variety of criteria.^{1,2,4} In addition, significant work has been conducted for the implementation of robust beamformers, designed to reduce the detrimental impact on performance that comes from (often unavoidable) experimental conditions such as snapshot-deficiency^{2,5} or mismatch between presumed/actual target-generated wavefronts impinging on the array.^{2,6–8} The focus of this paper is the introduction of a beamforming technique that increases resolution in azimuth estimation for loud contacts, in scenarios involving large-aperture arrays operating in dynamic environments which limit the availability of locally stationary data. The technique belongs to the family of subspace beamformers^{9–12} and it is based on an iterative algorithm for the extraction of target-related features buried in sample eigenvectors obtained from few data samples.

A typical sonar application consists of vertical or horizontal line arrays (HLAs) with N hydrophones that perform spatial sampling of the acoustic field in the water column. This field comprises a mixture of propagating wavefronts originated from multiple discrete targets of interest, interferers, and underwater noise.¹ Long arrays with aperture length of many wavelengths are preferred over short arrays due to the potential for increased signal gain and angular resolution that allows discerning between closely spaced targets. Despite this advantage, data processing for large apertures poses unique challenges related to the availability of measured data for the computation of optimal steering weights:^{13–15} on one hand sonar operation in dynamic environments limits the time interval for collection of M data snapshots, since stationarity can be affected by factors such as fast maneuvering targets, random array deformations, and time-dependent sound speed variations in the water column. On the other hand, the accuracy of estimated inter-sensor covariances, data sample eigenvalues, and eigenvectors (required in most adaptive techniques) is compromised by the lack of sufficient statistically independent data snapshots. Therefore, array processing methods that require very few data snapshots are on high demand.

Subspace beamformers are known to exhibit robustness against mismatch between the assumed form of the steering vector and the actual target spatial signature.^{1,12} An important step in their implementation is the estimation of the signal (or equivalently, the noise) eigenspace,^{9–12} which is

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generally constructed using a subset of the unaltered sample eigenvectors. To determine the rank of the estimated signal subspace, eigenvalue-based statistical tests have been proposed for the asymptotic regime $M > N$.¹⁶ More recently, signal subspace estimators applicable to the $M \sim N$ regime have been developed in the context of random matrix theory^{5,17} (in which N/M is constant as $N \rightarrow \infty$), based on asymptotic properties of the sample eigenspectra.

Unlike these eigenvalue-based approaches, the work presented here utilizes theoretical statistical properties of certain functions of noise-only N -dimensional vectors.¹⁸ Deviation from these theoretical results provides a basis for identifying nonrandom target-related features (i.e., wavefronts) buried in the structure of sample eigenvectors computed from $M \ll N$ data snapshots. By iterative subtraction of such wavefronts from the original sample eigenvectors, a matrix with columns that are statistically likely to span the noise subspace is obtained and used to compute a projector for the signal subspace. The approach used in this paper to identify source-related features differs from previous eigenvector-based signal detectors tested for $N \sim M$,¹⁹ which are based on parametric models of the signal eigenstructure for closely spaced targets.

This paper is organized as follows: Section II introduces notation of the snapshot model and the subspace beamformer. Section III explains preliminary theoretical results on the statistics of N -dimensional random vectors, which are utilized in Sec. IV for the design of a signal subspace estimator. Section V illustrates the application of the proposed beamformer with simulated and experimental data, followed by final comments in Sec. VI.

II. SNAPSHOT MODEL AND BEAMFORMING

Data at an N -sensor array can be modeled as the vector^{1,15}

$$\mathbf{y}_m = \sum_{q=1}^Q \mathbf{v}_{\phi_q} \xi_{qm} + \mathbf{w}_m, \quad (1)$$

where the integer subindex m indicates discretized time, $\mathbf{w}_m$ is the vector of background noise with each entry modeled as a zero-mean complex Gaussian random variable with variance σ_w^2 , Q is the number of targets at unique azimuths ϕ_q in the water column, ξ_{qm} is the amplitude of the q th target modeled as a zero-mean complex Gaussian random variable with variance σ_q^2 matching the target strength, and

$$\mathbf{v}_\phi = [1 \quad e^{-jk \sin \phi} \quad \dots \quad e^{-jk(N-1) \sin \phi}]^H / \sqrt{N} \quad (2)$$

is the steering vector at azimuth ϕ for spatial wavenumber k . The population covariance obtained as the expected value $\mathbf{R} = E[\mathbf{y}_m \mathbf{y}_m^H]$ can be split into contributions from the *signal* and *noise* subspaces as¹

$$\mathbf{R} = \sum_{n=1}^Q \lambda_n \mathbf{u}_n \mathbf{u}_n^H + \sum_{n=Q+1}^N \lambda_n \mathbf{u}_n \mathbf{u}_n^H, \quad (3)$$

where $\mathbf{u}_n$ and λ_n are the n th population eigenvector and eigenvalue, respectively, which are unobservable quantities

when working with experimental data. Similar to Eq. (3), the sample covariance matrix obtained from M snapshots is defined as

$$\begin{aligned} \hat{\mathbf{R}} &= \sum_{n=1}^{\bar{Q}} \hat{\lambda}_n \hat{\mathbf{u}}_n \hat{\mathbf{u}}_n^H + \sum_{n=Q+1}^N \hat{\lambda}_n \hat{\mathbf{u}}_n \hat{\mathbf{u}}_n^H \\ &= \frac{1}{M} \sum_{m=1}^M \mathbf{y}_m \mathbf{y}_m^H, \end{aligned} \quad (4)$$

where $\hat{\mathbf{u}}_n$ and $\hat{\lambda}_n$ are the n th sample eigenvector and eigenvalue, respectively, and $\bar{Q}$ is an estimate of the number of sources.

In this work, subspace beamforming is implemented by two main steps: first, data snapshots are projected into a rank- $Q^{(\cdot)}$ eigenspace. Second, minimum variance adaptive beamforming (MVDR) with diagonal loading^{4,8} is applied to the sample covariance matrix estimated from the projected data. The entire process is indicated by the following notation:

$$\begin{aligned} \mathbf{y}_m^{(\cdot)} &= \mathbf{P}^{(\cdot)} \mathbf{y}_m : \text{Projection,} \\ B^{(\cdot)}(\phi) &= \mathbf{w}^{(\cdot)}(\phi)^H \hat{\mathbf{R}}^{(\cdot)} \mathbf{w}^{(\cdot)}(\phi) : \text{Beamforming,} \end{aligned} \quad (5)$$

where the projector

$$\mathbf{P}^{(\cdot)} = \sum_{n=1}^{Q^{(\cdot)}} \mathbf{u}_n^{(\cdot)} (\mathbf{u}_n^{(\cdot)})^H \quad (6)$$

is obtained from a set of $Q^{(\cdot)}$ orthonormal basis $\mathbf{u}_k^{(\cdot)}$ (which could be population eigenvectors, sample eigenvectors, or other forms defined below) and^{1,4}

$$\mathbf{w}^{(\cdot)}(\phi) = \frac{(\hat{\mathbf{R}}^{(\cdot)} + \nu)^{-1} \mathbf{v}_\phi}{\mathbf{v}_\phi^H (\hat{\mathbf{R}}^{(\cdot)} + \nu)^{-1} \mathbf{v}_\phi} \quad (7)$$

is the azimuth-dependent adaptive weight vector with ν indicating the loading level⁸ commonly added to rank-deficient covariance matrices prior to inversion. The sample covariance matrix $\hat{\mathbf{R}}^{(\cdot)}$ in Eq. (5) is obtained from the projected data as

$$\hat{\mathbf{R}}^{(\cdot)} = \frac{1}{M} \sum_{m=1}^M \mathbf{y}_m^{(\cdot)} (\mathbf{y}_m^{(\cdot)})^H. \quad (8)$$

In Sec. V results obtained by implementing four different projectors are compared. Specifically,

- (1) The benchmark beamformer, computed from the true signal eigenspace projector $\mathbf{P}^{\text{true}} = \sum_{n=1}^Q \mathbf{u}_n (\mathbf{u}_n)^H$ using the population eigenvectors.
- (2) The diagonally loaded MVDR beamformer,⁸ $B^{c_1}(\phi)$, obtained from the projector $\mathbf{P}^{c_1} = \sum_{n=1}^{Q^{c_1}} \hat{\mathbf{u}}_n \hat{\mathbf{u}}_n^H$ where the rank $Q^{c_1} = M$.
- (3) The subspace beamformer $B^{c_2}(\phi)$, obtained from the projector $\mathbf{P}^{c_2} = \sum_{n=1}^{Q^{c_2}} \hat{\mathbf{u}}_n \hat{\mathbf{u}}_n^H$ where the rank

$Q^c = \max(1, Q^{\text{NE}})$ and Q^{NE} is the eigenvalue-based rank estimator introduced by Nadakuditi and Edelman.²⁰

- (4) The eigenvector-pruning beamformer $B^{\text{evp}}(\phi)$, obtained from $\mathbf{P}^{\text{evp}} = \sum_{n=1}^{Q^{\text{evp}}} \mathbf{u}_n^{\text{evp}} (\mathbf{u}_n^{\text{evp}})^H$. The procedure to compute the orthonormal basis $\mathbf{u}_n^{\text{evp}}$ and rank Q^{evp} is the subject of this paper, as detailed in Sec. IV.

III. STATISTICS OF NOISE-ONLY EIGENSAPCES

The rank of the signal subspace is often estimated by methods based on the sample eigenvalues $\hat{\lambda}_n$ ($n=1, \dots, N$),^{16,20,21} since the population eigenstructure $\lambda_1 \geq \dots \geq \lambda_Q > \lambda_{Q+1} = \dots = \lambda_N = \sigma_w^2$ (which would allow perfect estimation of Q) is nonobservable. Rather than an eigenvalue-based approach, this paper proposes the estimation of the signal subspace based entirely on the information content of the N -dimensional sample eigenvectors. To this end, consider the null hypothesis of noise-only data corresponding to a source-free environment where $\mathbf{R} = \sigma_w^2 \mathbf{I}_N$. For this case, the unitary matrix $[\hat{\mathbf{u}}_1 \dots \hat{\mathbf{u}}_N]$ is Haar distributed²² for which individual entries exhibit statistics consistent with independent complex Gaussian variables with zero mean and variance $1/N$ for large N .²³ Based on previous results,¹⁸ the probability distribution function (pdf) of $\alpha = \cos^{-1}(|\hat{\mathbf{u}}_1^H \mathbf{v}_\phi|)$ for any $n \in [1, \dots, M]$ and arbitrary ϕ is known, which provides statistical bounds to decide whether $\hat{\mathbf{u}}_n$ is in agreement with the null hypothesis, or rather there is evidence of the presence of target-related information.

To illustrate this concept, Fig. 1(a) shows a histogram of $\cos^{-1}(|\hat{\mathbf{u}}_1^H \mathbf{v}_0|)$ (normalized as a pdf) from 5000 Monte Carlo realizations from a signal-free sample covariance matrix, which is shown to converge to the theoretical pdf¹⁸

$$f_\alpha(\alpha) = C \cos \alpha (\sin \alpha)^{2N-2}; \quad 0 \leq \alpha \leq \pi/2, \quad (9)$$

where $C = 1 / \int_{\alpha=0}^{\pi/2} \cos \alpha (\sin \alpha)^{2N-2} d\alpha$ is a normalizing constant. The result in Fig. 1(a) is relevant in this work since it provides necessary conditions for an eigenvector to be considered “signal free.” For eigenvectors with strong information content related to a target at azimuth ϕ , a left-shifted histogram is expected since $\alpha \rightarrow 0$ as $\hat{\mathbf{u}}_n \sim \mathbf{v}_\phi$. This is shown in Fig. 1(b) for data corresponding to a single source of strength 3 dB at $\phi = 0^\circ$.

From Fig. 1 it is clear that informative (signal-related) eigenvectors are responsible for shifting the realizations of α toward zero. Then, f_α provides a statistical rule to decide if a sample eigenvector contains features resembling $\mathbf{v}_\phi$

$$\alpha : \begin{cases} \leq T_F : \text{informative eigenvector,} \\ > T_F : \text{noise-only eigenvector,} \end{cases} \quad (10)$$

where T_F is a user-defined threshold that must be chosen based on a trade-off between signal detection and false detection rate. For example, in the source-free case in Fig. 1(a), the number of false detections (i.e., instances in which $\alpha \leq T_F$) can be reduced to virtually zero if $T_F = 1.25$. On the other hand, if a signal exists as in Fig. 1(b), the probability of detection would be only 50% since roughly half of

FIG. 1. (a) Comparison between Eq. (9) (solid line) and a histogram built from 5000 Monte Carlo realizations of $\alpha = \cos^{-1}(|\hat{\mathbf{u}}_1^H \mathbf{v}_0|)$ for noise-only data simulated with an array of $N = 80$ hydrophones and $M = 10$ snapshots per realization. (b) Illustration of the left-shifted histogram that results from including a 3 dB target at azimuth angle $\phi = 0^\circ$.

the histogram realizations in this example have $\alpha \leq 1.25$. In Sec. IV, an algorithm for extraction of source-related features which scans over all possible azimuths and all sample eigenvectors is introduced.

IV. AN EIGENVECTOR PRUNING ALGORITHM

Since $\alpha \leq T_F$ provides statistical evidence that the wavefront $\mathbf{v}_\phi$ is a significant component of the n th sample eigenvector, evidence of other azimuth-dependent wavefronts can be obtained by iteratively scanning over all eigenvectors ($n = 1, \dots, M$) and all azimuths. The following algorithm is proposed:

- (1) Given M snapshots, compute the sample eigenvectors $\hat{\mathbf{u}}_1, \dots, \hat{\mathbf{u}}_M$.
- (2) Compute $\mathbf{v}_{\phi_g}$ for $g = 1, \dots, G$ azimuths equally spaced over the range $-\pi/2 \leq \phi \leq \pi/2$.
- (3) Initialize $\mathbf{s}_n$: $\mathbf{s}_n = \hat{\mathbf{u}}_n$ for $n = 1, \dots, \min(M, N)$.
- (4) Find $\phi_{\min}$, the incoming wavefront angle at which $\alpha_{\min} = \cos^{-1}(|\mathbf{s}_n^H \mathbf{v}_{\phi_g}|)$ is minimum for $n = 1, \dots, \min(M, N)$ and $g = 1, \dots, G$.
- (5) If $\alpha_{\min} \leq T_F$, compute the stripped vectors $\mathbf{s}_n = \mathbf{s}_n - (\mathbf{v}_{\phi_{\min}}^H \mathbf{s}_n) \mathbf{v}_{\phi_{\min}}$ for $n = 1, \dots, \min(M, N)$ and go back to step (4). Otherwise go to step (6).
- (6) Compute the resulting non-orthogonal vectors $\mathbf{g}_n = \hat{\mathbf{u}}_n - \mathbf{s}_n$, which are statistically likely to span the same eigenspace corresponding to the dominant population eigenvectors $\mathbf{u}_n, n = 1, \dots, Q$.

- (7) Obtain a set of orthonormal vectors $\mathbf{u}_n^{\text{evp}}$, $n = 1, \dots, Q^{\text{evp}}$ of the dominant eigenspace, constructed from the singular value decomposition of $\mathbf{g}_n$, $n = 1, \dots, \min(M, N)$. Here, Q^{evp} is determined by the column rank of $\mathbf{g}_n$.

The crucial part of the proposed algorithm is the removal of wavefronts in step (5) according to the threshold T_F : once the decision to remove $\mathbf{v}_{\phi_{\min}}$ has been made based on analysis of the n th eigenvector, the wavefront is also removed from all remaining eigenvectors. By doing so the algorithm minimizes the effect of target leakage that takes place due to inaccurate estimation of the sample eigenvectors from limited data, which can be significant for $M \ll N$. By step (7), the orthonormal basis $\mathbf{u}_n^{\text{evp}}$ are likely to span the signal subspace. The quality of this approximation depends on the value of T_F : small values result in estimated eigenspaces that lined up well with the true eigenspace corresponding to the louder sources in the water column, while large T_F promote the inclusion of quiet sources in the estimated eigenspace at the expense of also incorporating noise.

In Sec. V, subspace estimation performance for the case of simulated data is quantified by the distance between the subspaces spanned by $\mathbf{U}^{\text{true}} = [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_Q]$ (i.e., the true signal subspace) and $\mathbf{U}^{(\cdot)} = [\mathbf{u}_1^{(\cdot)} \dots \mathbf{u}_{Q^{(\cdot)}}^{(\cdot)}]$, according to the *projection distance metric*^{24,25}

$$D^{(\cdot)} = \frac{\sqrt{(Q - \text{trace}[\mathbf{P}^{\text{true}} \mathbf{P}^{(\cdot)}])}}{\sqrt{Q}}. \quad (11)$$

Here, Eq. (11) is normalized by $\sqrt{Q}$ so $0 \leq D^{(\cdot)} \leq 1$ with $D^{(\cdot)} = 0$ indicating perfect performance of the subspace estimator while $D^{(\cdot)} = 1$ taking place when the subspaces defined by $\mathbf{U}^{\text{true}}$ and $\mathbf{U}^{(\cdot)}$ are orthogonal to each other. Note that other metrics such as the maximum and minimum correlation²⁵ can be used to quantify performance. However, these metrics are strongly affected by individual principal angles: the maximum (respectively, minimum) correlation metric depends exclusively on the sine of the smallest (respectively, largest) principal angle, which can result in overly optimistic (respectively, pessimistic) performance. Likewise, the overlap metric given by the angle between subspaces (Risteski and Trenčevski²⁶) can be overly pessimistic since the cosine of the largest principal angle appears as a factor. The projection metric used in this work is preferred since it includes contributions from all principal angles and it is not strongly affected by individual ones.²⁵

Prior to demonstrating the application of the proposed algorithm to beamforming in Sec. V, it is worth commenting on the expected performance relative to T_F . In addition to successful detection of true targets, the number of false peaks at target-free azimuths is an important parameter to be considered. The results in Fig. 1(a) show that for a simple detection algorithm that compares a single eigenvector against a fixed $\mathbf{v}_{\phi_g}$, the expected number of “detections” for which $\alpha < T_F$ over N_{MC} Monte Carlo realizations is $N_{\text{peaks}} = \text{round}(N_{\text{MC}} F_{T_F})$, where

$$F_{T_F} = \int_0^{T_F} f_x(\alpha) d\alpha. \quad (12)$$

However, for the algorithm described above, the relation between N_{peaks} and F_{T_F} is more complicated since multiple eigenvectors are simultaneously considered, as well as due to the fact that once a “detection” is found in the n th eigenvector, the corresponding $\mathbf{v}_{\phi_g}$ is simultaneously removed from all eigenvectors. Despite this complexity, the expected value and standard deviation of N_{peaks} for the proposed algorithm can be obtained by Monte Carlo simulations with $\mathbf{R} = \mathbf{I}$ for any chosen T_F , since the algorithm does not require knowledge of σ_w as it is based on normalized sample eigenvectors. Table I shows examples of the expected N_{peaks} for different combinations of N , M , and F_{T_F} .

As a final comment in this section, it was pointed out to the author by an anonymous reviewer that the proposed pruning algorithm is a particular case of the matching pursuits method by Mallat and Zhang.²⁷ In matching pursuits, a function $f(t)$ is decomposed into a sum of weighted waveforms selected from a redundant dictionary of patterns $g_{\gamma_n}(t)$ (e.g., sinusoids, wavelets consisting of dilations and translations of a Gaussian function). The selection proceeds by finding $g_{\gamma_0}(t)$, the pattern with largest projection $\langle g_{\gamma_0}(t), f(t) \rangle = \int_{-\infty}^{\infty} f(t) g_{\gamma_0}(t) dt$. Then, $f(t) = \langle g_{\gamma_0}(t), f(t) \rangle g_{\gamma_0}(t) + R_0(t)$ where $R_0(t)$ is the residual at iteration 0. This procedure is then applied iteratively to subsequent residuals, with a stopping criterion at the l th iteration based on the residual’s energy compared to the energy of the original function [i.e., stop when $\int_{-\infty}^{\infty} |R_l(t)|^2 dt \ll \int_{-\infty}^{\infty} |f(t)|^2 dt$, as in Mallat and Zhang,²⁷ Eq. (35)]. In the subspace estimator proposed here, the redundant dictionary consists of the complex exponentials $\mathbf{v}_{\phi_g}$, which are selected by searching for the largest $\cos^{-1}(|\langle \mathbf{s}_n, \mathbf{v}_{\phi_g} \rangle|)$ [step (4)]. Likewise, the stripped vectors $\mathbf{s}_n$ at step (6) parallel the final residual $R_l(t)$ in Eq. (15) from Mallat and Zhang.²⁷ There are two differences between both approaches: first, the proposed pruning algorithm is applied simultaneously to $\min(M, N)$ signals with the purpose of reducing leakage of target signatures among all eigenvectors. Second, the stopping criterion makes use of the statistics of N -dimensional noise-only vectors described in Sec. III (as opposed to an energy-based criterion), resulting in an

TABLE I. Mean and standard deviation of the expected number of detections (N_{peaks}) for the proposed beamformer B^{evp} as a function of the number of array hydrophones N , the number of data snapshots M , and F_{T_F} [defined in Eq. (12)].

N	M	F_{T_F}	N_{peaks}	
			Mean	Standard deviation
100	10	0.005	11	2
		0.01	17	2
	20	0.005	19	3
		0.01	31	3
80	10	0.005	8	2
		0.01	14	2
	20	0.005	15	2
		0.01	25	3

intuitive connection between the number of false detections and the stopping criterion [step (5) of the pruning algorithm].

V. EIGENVECTOR-PRUNING BEAMFORMING

This section uses simulated data and the metric D^{evp} to show the performance of $\mathbf{P}^{\text{evp}}$ compared to the classic approach in which subspace projectors are constructed from a subset of the sample eigenvectors. Application of $\mathbf{P}^{\text{evp}}$ to beamforming is illustrated with simulated and experimental data.

A. Simulated data

Zero-mean, complex Gaussian data snapshots¹ with covariance $\mathbf{R}$ were simulated for a HLA with $N = 100$ sensors at half-wavelength spacing at an arbitrary frequency. The environment consists of an isovelocity water column with sound speed 1500 m/s and uncorrelated white background noise with $\sigma_w^2 = 1$. A total of $Q = 5$ uncorrelated far-field targets with parameters indicated in Table II are considered. The far field-assumption is adopted here for simplicity, although it is not a requirement since the replica vector $\mathbf{v}_\phi$ can be modified accordingly to account for spherical wavefronts from near-field sources without altering the algorithm described in Sec. IV.

Note that for a HLA in a shallow water environment, multipath propagation does not affect the azimuth of target-generated wavefronts impinging the array (assuming negligible horizontal refraction), but only its amplitude as a result of wavefront interference between multipaths. For this reason, the simulations in this section only consider direct paths which provide a baseline of the performance of B^{evp} for arrivals of expected power σ_q^2 .

Sample covariance matrices in this section are computed with $M = 20$ snapshots generated according to Eq. (1). Figure 2 shows $N_{\text{MC}} = 100$ Monte Carlo realizations of the beamformers B^{c1} , B^{c2} , and B^{evp} (left sub-panels), as well as the metric $D^{(\cdot)}$ for each subspace estimator (right sub-panels). Compared to B^{c1} , B^{c2} shows a reduction on the background noise levels as a result of projecting the data into $\mathbf{P}^{c2}$ which has rank $Q^{c2} < Q^{c1}$. Despite this improvement, separation of closely spaced targets is not achieved due to two factors affecting B^{c2} : first, the low source levels used in this simulation result in subspace rank underestimation with $Q^{c2} = 1$ in agreement to the results in Nadakuditi and Edelman,²⁰ Fig. 5, for similar simulation parameters

TABLE II. Simulation parameters for performance evaluation of the beamformers B^{c1} , B^{c2} , and B^{evp} . This study considers $Q = 5$ far-field stationary targets with azimuth ϕ_q and strength $10 \log(\sigma_q^2/\sigma_w^2)$ in dB with respect to the background noise level.

q	Strength (dB)	ϕ_q (°)
1	4	0
2	4	1
3	3	4
4	5	5
5	3	8

FIG. 2. Beamforming results and eigenspace distance for (a) MVDR B^{c1} ; (b) MVDR B^{c2} ; and (c) proposed B^{evp} with $F_{T_F} = 0.005$, corresponding to data projections into subspaces of rank M , $Q^{c2} < M$, and Q^{evp} , respectively. The simulated environment consists of five sources with azimuth and strength indicated in Table II. Results are normalized to a maximum of 0 dB and displayed with dynamic range $[-17, \dots, 0]$ dB. B^{evp} yields better angular resolution, allowing visualization of the closely spaced targets at most Monte Carlo realizations.

($N = 128$, $M = 20$). Second, significant signal leakage into eigenvectors $[\mathbf{u}_{Q^{c2}+1}, \dots, \mathbf{u}_M]$ is expected due to low sample size, as the sample eigenvectors are highly inconsistent with the population eigenvectors. Figure 2(c) shows that B^{evp} yields better angular resolution, allowing visualization of the closely spaced targets at azimuth pairs $[0^\circ, 1^\circ]$ and $[4^\circ, 5^\circ]$ at most Monte Carlo realizations. Subspace estimation performance also suggests advantage of the proposed approach, yielding D^{evp} always smaller than D^{c1} and D^{c2} and closer to the ideal value of 0.

Details of the beamformer outputs B^{c1} , B^{c2} , B^{evp} , and B^{true} are shown in Fig. 3, corresponding to the 70th Monte Carlo realization from Fig. 2. For this realization, $D^{\text{evp}} = 0.12$ (i.e., small distance between true and estimated eigenspaces) which explains the similarity between B^{evp} and B^{true} . Here, B^{evp} results in 17 peaks above the typical beamformer output level around -45 dB, close to the expected

FIG. 3. (Color online) (a) Detail of the 70th beamforming realization from Fig. 2, comparing B^{c1} , B^{evp} , and B^{c2} to the benchmark beamformer B^{true} . Vertical dotted lines indicate the true azimuth of five sources considered in the simulation. (b) Zoom in of (a) showing the beamformer output around the true azimuths. Results are normalized to a maximum of 0 dB.

$N_{\text{peaks}} = 19$ indicated in Table I. In this case, a target tracking algorithm should consider all 17 peaks as potential target detections until new realizations are made available for further confirmation or removal.

Figure 3(b) shows that for this particular Monte Carlo realization, both B^{evp} and B^{c1} exhibit peaks around each of the five targets. However, the azimuth of some of these peaks is biased with respect to their true values (see, for example, B^{evp} at $\phi = 0^\circ$, 4° , and 5° and B^{c1} at $\phi = 0^\circ$ and 5°). To compare B^{c1} and B^{evp} in terms of this bias, target detection was applied to each Monte Carlo realization from Figs. 2(a) and 2(c). The detection method consists of searching for peaks in the beamformer output within a window of length δ_s ,

$$|\phi_q - \phi| \leq \delta_s, \quad (13)$$

centered around the true target azimuths ($q = 1, \dots, 5$). For each target, a detection is defined as the peak with the azimuth closer to ϕ_q . Note that there are instances in which no candidate peaks are found within the search window, leading to missing detections. The results of target detection by this method with $\delta_s = 0.5^\circ$ are shown in Fig. 4 for B^{c1} (open circles) and B^{evp} ($\times$), in which both beamformers exhibit similar spreading of detections around the true azimuths. The statistics of these results are summarized in Table III in terms of the mean μ_{ϕ_q} of the estimated target azimuths (taken over 100 Monte Carlo realizations), their standard deviation σ_{ϕ_q} , and the detection rate Λ_q (i.e., number of detections/100 Monte Carlo realizations, computed for each target). Both beamformers result in similar mean values (i.e., in agreement up to the first decimal point), while B^{evp} exhibit slightly larger standard deviations and significantly higher detection rates for most targets.

FIG. 4. Target azimuth estimation corresponding to B^{c1} (open circles) and B^{evp} ($\times$), obtained by applying peak detection to the beamforming results in Figs. 2(a) and 2(c), respectively. True azimuths ϕ_q are shown as dashed lines.

The results in Fig. 2 are for sources with comparable power levels. However, experimental data often include more challenging cases in which quiet targets of interest are masked by loud targets located at nearby azimuths. To test this case, Fig. 5 shows beamforming performance after increasing the levels of sources 1 and 3 in Table II from 4 dB and 3 dB to 10 dB and 15 dB, respectively. As in Fig. 2, in most cases B^{evp} properly resolves all targets with reduced levels of background noise. An interesting item is observed for the results in Fig. 5(b), since in this case B^{c2} only detects two targets. The reason for this is that the increased source levels σ_1^2 and σ_3^2 result in $Q^{NE} = 2$ for most of the realizations, as well as better alignment of the top-2 eigenvectors $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$ in the directions $\phi_1 = 0^\circ$ and $\phi_3 = 4^\circ$. Therefore, this is a case in which B^{c2} would highly benefit from augmenting the estimated signal eigenspace rank to include quieter sources, by forcing $Q^{c2} > Q^{NE}$.

B. Experimental data

The B^{evp} beamformer was also applied to data from the Shallow Water Array Performance (SWAP) experiment.²⁸ For this experiment a fixed 500-hydrophone HLA with inter-

TABLE III. Summary of target detection results from Fig. 4: true target azimuth ϕ_q , estimated mean/standard deviations (μ_{ϕ_q} and σ_{ϕ_q} , respectively) in degrees ($^\circ$), and detection rates Λ_q .

ϕ_q	B^{c1}			B^{evp}		
	μ_{ϕ_q}	σ_{ϕ_q}	Λ_q	μ_{ϕ_q}	σ_{ϕ_q}	Λ_q
0	0.03	0.16	80	0.00	0.22	88
1	1.01	0.15	73	1.03	0.23	89
4	4.00	0.18	72	3.94	0.18	83
5	5.01	0.15	86	4.96	0.18	96
8	7.99	0.13	100	8.01	0.14	94

FIG. 5. Results similar to Fig. 2, with simulated data obtained by increasing the power levels of sources 1 and 3 in Table II from 4 dB and 3 dB to 10 dB and 15 dB, respectively. Despite the proximity of loud and quiet targets at 0° and 1° as well as at 4° and 5° , B^{evp} in (c) resolves all five targets.

element spacing of ~ 1.75 m was deployed off the east coast of Florida at a depth of about 250 m in a location with heavy ship traffic. The array design frequency is 424 Hz considering the measured sound speed of 1485 m/s at the water–sediment interface.²⁸ Acoustic data containing arrivals from multiple moving ships were collected at a sampling rate of 1 kHz. In addition, Automatic Identification System (AIS) data available from some of the ships at the time of the recordings are used in this section to provide independent confirmation of beamforming performance.

Figure 6 shows B^{c1} and B^{evp} corresponding to 180 min of data collected on September 7, 2007 between 10:00 a.m. and 13:00 p.m. UTC (Coordinated Universal Time). For this example, data from a sub-aperture of $N=90$ hydrophones were processed by applying short time Fourier transform on non-overlapping data intervals of length 1 s. The results in Fig. 6 correspond to the average of B^{c1} (or B^{evp}) over frequencies $f=355$ Hz and $f=380$ Hz, although similar results were obtained using single frequencies. Time-dependent sample covariance matrices with $M=20$ snapshots/

FIG. 6. (a) B^{c1} applied to 180 min of data from the SWAP experiment with $N=90$ and $M=20$; (b) corresponding B^{evp} processor with $F_{T_F}=0.005$, showing a reduction on the number of suspected false detections compared to (a); (c) B^{evp} processor with $F_{T_F}=0.05$. The double arrow indicates an example in which increasing F_{T_F} allows improved detection of a weak target at the expense of increasing the number of false detections at other azimuths/times. Results are normalized to a maximum of 0 dB and displayed with dynamic range $[-30, \dots, 0]$ dB.

covariance were used to compute the beamformer B^{c1} as well as B^{evp} for $F_{T_F}=0.005$ and $F_{T_F}=0.05$, as shown in Figs. 6(a), 6(b), and 6(c), respectively. Similar to the simulated examples, B^{evp} provides sharper target detections compared to B^{c1} , improving separation of suspected targets. As shown in Table I, increasing F_{T_F} can help to detect weak targets at the expense of also increasing the number of false detections, as observed for the ship track at $(97 \text{ min} < t < 113 \text{ min}, 74^\circ > \phi > 54^\circ)$ and indicated by a double arrow.

Details of the results in Fig. 6 are shown as a zoom-in view in Fig. 7. Here, AIS shipping lanes that seem to be in agreement with the beamformers are shown as superimposed dashed lines, while dotted lines show AIS tracks that could not be identified by either B^{c1} or B^{evp} . Comparison of B^{c1} and B^{evp} in Figs. 7(a) and 7(b), respectively, shows that B^{evp} yields lower side lobe levels that allow to better distinguish individual ship lanes. For example, the short track around $t=80$ min and $\phi=-50^\circ$ (arrow 1) is more clearly identified in B^{evp} . There is also an instance in which an evident shipping lane not confirmed by AIS data at $(77 \text{ min} < t < 87 \text{ min}, -50^\circ < \phi < -35^\circ)$ indicated by arrow 2 is clearly identified by B^{evp} , while appearing masked by side lobes in B^{c1} .

As a comparison of azimuth resolution between B^{evp} and B^{c1} the 3 dB beam width $\eta^{(\cdot)}$ along AIS ship tracks was

FIG. 7. (Color online) Zoom in of Fig. 6, with AIS ship tracks superimposed as dashed lines (tracks confirmed by B^{evp}) and dotted lines (tracks undetected by beamforming): (a) B^{c1} ; (b) B^{evp} ; (c) difference $\eta^{c1} - \eta^{evp}$ of the beam width corresponding to the selected target [solid line in (a) and (b)]: the predominance of positive values indicates that B^{evp} generally results in sharper target detections compared to B^{c1} . (d) through (f) show similar analysis applied to earlier tracks. Numbered arrows indicate targets discussed in the text.

obtained. To this end, target detections are identified using the method described in Sec. V A, with a search window given by Eq. (13). Since θ_q is not available for experimental data, the search window is centered around ϕ_{AIS} , the target azimuth according to AIS data. Once the peak at ϕ_{peak} closer to ϕ_{AIS} has been identified, the 3 dB beam width is given by $\eta^{(c)} = \phi_H - \phi_L$, where ϕ_H and ϕ_L satisfy $|B^{(c)}(\phi_{peak} - \phi_L)|/|B^{(c)}(\phi_{peak})| = 0.5$ and $|B^{(c)}(\phi_{peak} + \phi_H)|/|B^{(c)}(\phi_{peak})| = 0.5$, respectively. Figure 7(c) shows the difference $\eta^{c1} - \eta^{evp}$ for $F_{Tf} = 0.005$ and $\delta_s = 2^\circ$, yielding positive values in the majority of cases as a result of sharper target detections obtained by B^{evp} . As an additional example, Figs. 7(d)–7(f) apply the same analysis to earlier tracks, in which similar improvement in azimuth resolution was also achieved by B^{evp} . Except for a few cases, B^{evp} results in typical beamforming peaks that are between 0.6° and 1.3° thinner than B^{c1} in both examples considered in Fig. 7.

VI. FINAL COMMENTS

This paper introduces a beamforming technique that improves localization performance in snapshot-deficient sonar experimental scenarios. The processor is based on estimation of a set of orthonormal basis that represent the signal subspace more accurately than by simply considering a subset of the sample eigenvectors. The core of the estimator is an eigenvector pruning algorithm that identifies angle-dependent wavefronts for removal from the sample eigenspace. Wavefront tagging for removal is theoretically grounded on the statistical properties of scalar products between the assumed replica vector and random vectors. After pruning, the remaining (presumably signal-free) vectors are used for estimation of the signal-plus-interferer eigenspace projector.

Results using simulated data corresponding to multiple far-field targets show that signal eigenspace representation by the proposed orthonormal basis is superior than the classic approach based on sample eigenvectors. The improved performance (quantified by an eigenspace-distance metric) translated into better azimuth resolution for closely spaced targets in the water column.

Subspace estimation was highly efficient for the simulated examples in this work, likely due to the lack of mismatch between the data and the plane-wave steering vector. A study of the robustness of the method to experimental factors such as array deformation, phase errors due to spherical wavefronts from near-field sources, and hydrophone calibration errors will be an interesting subject for further investigation. However, the improved angular resolution and reduced side lobe levels already observed when applying the proposed method to experimental data suggest robustness of the method to typical experimental uncertainties.

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