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Statistical Inference for Multimodal Travel Time Reliability

Avinash Unnikrishnan
Portland State University, uavinash@pdx.edu

Miguel Figliozzi
Portland State University, figliozzi@pdx.edu

Subhash C. Kochar
Portland State University, kochar@pdx.edu

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Avinash Unnikrishnan, Ph.D.

Subhash Kochar, Ph.D.

Miguel Figliozzi, Ph.D.

Portland State University

STATISTICAL INFERENCE FOR MULTIMODAL TRAVEL TIME RELIABILITY

Final Report

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by

Avinash Unnikrishnan
Subhash Kochar
Miguel Figliozzi
Portland State University

for

National Institute for Transportation and Communities (NITC)
P.O. Box 751
Portland, OR 97207

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16. Abstract

Travel time reliability is a key metric of interest to practitioners and researchers because it affects travel choice and the economic competitiveness of urban areas. This research focuses on three travel time reliability metrics – buffer index, modified buffer index, and the relative width of travel time distributions. The key novel contributions of this research include using the multivariate delta method to prove that the sampling distributions of the three travel time reliability metrics are asymptotically normal. The asymptotic standard error for the three reliability metrics is derived. The asymptotic normality and the standard error result are used to arrive at a formula for the confidence interval. The derivations are non-parametric since they do not impose any shape requirement on travel time distributions. A case study based on a highway corridor in Portland, OR, is utilized to estimate confidence intervals for the three travel time reliability metrics for several travel time distributions with different shapes and levels of skewness. The performance of the proposed method is compared against several bootstrapping-based confidence intervals with favorable performance. Finally, upper-tailed, lower-tailed, and two-tailed one-sample hypothesis testing procedures are developed, and numerical tests show a positive performance and high statistical power for sample sizes that can be readily obtained.

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EXECUTIVE SUMMARY

Travel time reliability, or “the consistency or dependability in travel times, as measured from day-to-day and/or across different times of the day” (FHWA, 2021), significantly impacts travel behavior. Several metrics have been defined to measure travel time reliability. This research focuses on two families of metrics. The first is the buffer index, which captures a measure of the extra time that the average traveler needs to add to have an on-time arrival probability of 95% (FHWA, 2021). We consider two forms of buffer index – (i) the ratio of 95th percentile travel time to sample average travel time minus one and, (ii) the ratio of 95th percentile travel time to median travel time minus one. We refer to the latter definition of buffer index as modified buffer index in this research. Note that we consider the modified buffer index as in the presence of outliers or when the distribution is skewed as is often the case for travel times, sample median is a better measure of central tendency than sample mean. Therefore, the modified buffer index might provide a more accurate representation of travel time reliability compared to regular buffer index. The second family of metrics considered in this research is called relative width of travel time distributions. The relative width is defined as the ratio of the range of travel times in which 80% of the observations around the median fall into and the median travel time (van Lint and van Zuylen, 2005). Glick & Figliozzi (2017) adopt a similar metric for understanding transit reliability using speed data. Both buffer index and relative widths are sample statistics and, therefore, will vary depending on the travel time samples. This research presents methods to conduct statistical inferences – confidence intervals and one-sample hypothesis tests on the three travel time reliability metrics mentioned above. The methods presented in this project will help account for the variability in the estimated buffer index, modified buffer index, and relative width and attach statistical guarantees.

The first part of this research focuses on methodology to derive confidence intervals for the three travel time reliability metrics. The multivariate delta method, along with select results from the statistical literature on the joint distribution of sample quantiles and sample means, is used to show that the asymptotic distribution of the buffer index, modified buffer index, and relative width is normal. In addition to the distribution, we also derive a formula for the standard error. Given the asymptotic normal distribution and the standard error, this result is used to determine the reliability metrics' confidence interval formula. It is well known that the shape of the travel time distribution can vary based on the time of day, location, day of the week, etc. The asymptotic normal distribution and standard error result are travel time distribution agnostic and do not impose any shape requirement and apply to various travel time distributions.

The travel time data from Portland, OR metropolitan region is used to calibrate a probability distribution. Four other travel time population distributions are generated based on this data. The shapes considered vary from the common right-skewed, symmetric, and less common left-skewed, and bimodal distribution. We generate travel times of different sample sizes using simulation from the five distributions. The performance of the Standard Normal with Asymptotic Standard Error confidence interval is compared against six other bootstrapping confidence intervals. For the buffer index, the Standard Normal with Asymptotic Standard Error confidence interval provides consistent coverage of over 95% for common right-skewed, symmetric, and bimodal travel time distribution shapes. Standard Normal with Asymptotic Standard Errors consistently delivers higher than 95% coverage for all sample sizes for right-skewed and symmetric cases tested for the modified buffer index. Standard Normal with Asymptotic Standard Errors consistently achieves 95% for all cases tested for the travel time relative widths.

The asymptotic normality results and the standard error formula are also used to derive upper-tailed, lower-tailed, and two-tailed one-sample hypothesis tests for the three reliability parameters. We compare the performance of the hypothesis testing procedures with travel time samples from the population travel time distribution of real-world data obtained from the Portland, OR, metropolitan region. Formulas are derived for the p-values and rejection region. Simulation results show that the power of the hypothesis test increases with sample size.

1.0 INTRODUCTION

Travel time reliability has worsened in almost all urban areas in the United States over the last thirty years. According to the Urban Mobility Report, which has been produced since 1987, the number of congested hours and other reliability metrics such as the planning and buffer indices briefly dipped during the COVID-19 pandemic but has already returned to pre-pandemic levels (TTI, 2021).

Travel behavioral studies demonstrate that travelers consider travel time reliability in addition to average travel times in their travel choice decision making (Boyles et al., 2010; Pinjari and Bhat, 2006). This has led to a significant amount of research on factors affecting travel time reliability, trends in travel time reliability (Martchouk et al., 2011; Van Der Loop et al., 2014), and incorporating reliability objectives into transportation planning models (Anderson et al., 2019; Boyles et al., 2010; Khani and Boyles, 2015). In addition to automobiles, reliability metrics are critical for other modes such as transit and bicycles (Glick and Figliozzi, 2017) and freight (Shams et al., 2017), where reliability has a direct impact on costs (Figliozzi et al., 2011).

The Federal Highway Administration (FHWA) formally defines travel time reliability as "the consistency or dependability in travel times, as measured from day-to-day and/or across different times of the day" (FHWA, 2021). A wide variety of metrics has been used for characterizing travel time reliability, such as percent variation, variability index (Lomax et al., 2003), standard deviation (Day et al., 2015), skew and width (van Lint et al., 2005), reliability ratio (Fosgerau and Engelson, 2011), misery index, on-time arrival probability, etc. (Pu, 2011). This research focuses on two popular travel time reliability metrics – buffer index (FHWA, 2021; Lyman and Bertini, 2008) and relative width (van Lint and van Zuylen, 2005). The buffer index is popular and conceptually simple because it tries to capture a measure of the extra time that the average traveler needs to add to have an on-time arrival probability of 95% (FHWA, 2021). The second travel time reliability metric considered in this study considers the relative width of travel time distribution which is defined as the ratio of the range of travel times in which 80% of the observations around the median fall into and the median travel time (van Lint and van Zuylen, 2005). Glick & Figliozzi (2017) use a similar metric for analyzing transit reliability using high-resolution speed data. But like any statistic, the buffer index and relative width have variability, which begs the question, how confident should the traveler be about the estimated buffer index and relative width?

Traditionally, transportation engineers and planners have used point estimates for the buffer index and relative width to compare the reliability of various modes or corridors or the same corridors for different times of the day. For example, a roadway segment with a lower value of the buffer index and relative width of travel time distribution is considered more reliable. However, any sample statistic like the buffer index and relative width will be associated with variability. First, the natural variability associated with random samples will lead to different buffer indices and relative widths. Second, the index estimated variability might be a function of factors such as road and mode type and assumptions regarding the travel time distribution or population from where travel time samples are obtained.

Researchers have proposed different methods to determine confidence intervals (Glick and Figliozzi, 2017; Hou et al., 2012) and hypothesis tests (Spiegelman and Gates, 2005) of speed reliability metrics such as percentiles. However, point speeds are often normally distributed, unlike travel times, whose distribution need not be symmetric (Anderson et al., 2019). Moreover, the buffer index is a function of percentiles and the sample mean and the relative width is a function of different sample quantiles. Thus, the methodologies developed in Glick and Figliozzi (2017) and Hou et al. (2012) utilizing only percentiles are not directly applicable to estimate confidence intervals for the buffer index and the relative width. To date, there are no ready-made procedures to attach statistical guarantees or perform statistical inferences on the travel time buffer index and relative widths. This research is timely because the COVID-19 pandemic and consequent changes in traffic levels have highlighted the need to quickly compare and better understand the behavior of most commonly used traffic reliability measures (Rilett et al., 2021).

We overcome three main challenges in this research. First, the lack of knowledge about the sampling distribution of buffer index and relative width. Second, the absence of a ready-made formula that can be used to estimate the standard error of the buffer index and relative width. Prendergast and Staudte (2017, 2016) have obtained first-order approximation-based estimates of the standard error of the ratio of quantiles. Note that in this study, we consider buffer index involving the ratio of quantile to sample mean and the ratio of quantiles. In this research, using the multivariate delta method, we show that the sampling distribution of the buffer index and the relative width is asymptotically normal and derive a formula for the standard error. This asymptotic normality result is used to arrive at a confidence interval formula for the buffer index and the relative width. The third challenge is the lack of consensus on the population distribution of travel times. Depending on the study and the context, a wide variety of distributions such as lognormal, Weibull, or Burr have been found to be appropriate (Emam and Al-Deek, 2006; Susilawati et al., 2013; Taylor, 2017; Uno et al., 2009). Moreover, the shape of the distribution can also vary – left-skewed, right-skewed, symmetric, bimodal, etc. (Chen et al., 2014; Feng et al., 2012; Guo et al., 2010; Kazagli and Koutsopoulos, 2013). The asymptotic normality-based confidence interval does not impose any shape requirement on travel time distributions. Hence, we develop confidence interval procedures that are general because they are independent of the type of travel time distributions and work for a wide range of distribution shapes. This research compares the performance of the proposed method to estimate confidence intervals against several bootstrapping-based confidence intervals, which also do not require any specific shape or distribution assumptions for travel times. The asymptotic normality result is then used to formulate an upper-tailed, lower-tailed, and two-tailed one-sample hypothesis testing procedure for the buffer index and relative width.

2.0 ASYMPTOTIC DISTRIBUTION OF TRAVEL TIME RELIABILITY METRIC

2.1 INTRODUCTION

In this chapter, we derive the distribution and standard error of the buffer index and relative width using asymptotic theory. A confidence interval formula is presented based on asymptotic distributions. Next, we describe several bootstrapping-based confidence interval procedures.

Let $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ be a random sample from a continuous probability distribution with cumulative distribution function (CDF) $F(.)$, probability density function $f(.)$, mean μ, and standard deviation σ. Note that we do not know the $F(.), \mu$, and σ, they can vary depending on many factors such as signal settings, traffic compositions, time-of-day, day-of-the-week, and weather conditions. The shape can be symmetric or asymmetric, left-skewed or right-skewed. Let ζ_p and β represent the true $100p^{th}$ percentile and the reliability parameter of interest, respectively. For example, if β is the buffer index, then:

$$\beta = \frac{\zeta_p}{\mu} - 1 \qquad (2.1)$$

Let $\hat{\mu}$ and $\hat{\zeta}_p$ denote the sample mean and the sample $100p^{th}$ percentile, respectively. Note that the buffer index utilizes the 95th percentile, but the treatment in this section is more general since it applies to any percentile. The sample estimate of the buffer index $\hat{\beta}$ is given as

$$\hat{\beta} = \frac{\hat{\zeta}_p}{\hat{\mu}} - 1 \qquad (2.2)$$

This research also considers another form of the buffer index which is the ratio of the percentiles. We call this the modified buffer index. Let ζ_{p_1} and ζ_{p_2} represent the true $100p_1^{th}$ and $100p_2^{th}$ percentile, respectively. Let $\hat{\zeta}_{p_1}$ and $\hat{\zeta}_{p_2}$ denote the sample estimate of the $100p_1^{th}$ and $100p_2^{th}$ percentile. The true and sample estimate of the modified buffer index is shown below:

$$\beta = \frac{\zeta_{p_1}}{\zeta_{p_2}} - 1 \qquad (2.3)$$

$$\hat{\beta} = \frac{\hat{\zeta}_{p_1}}{\hat{\zeta}_{p_2}} - 1 \qquad (2.4)$$

For travel time reliability applications, we use the ratio of the 95th percentile to the median which is the 50th percentile.

The third reliability metric considered in this work is called the relative width of travel time distributions (van Lint and van Zuylen, 2005). Let ζ_{p_1}, ζ_{p_2}, and ζ_{p_3} represent the true $100p_1^{th}$, $100p_2^{th}$, and $100p_3^{th}$ percentiles with $p_1 < p_2 < p_3$. Let $\hat{\zeta}_{p_1}$, $\hat{\zeta}_{p_2}$, and $\hat{\zeta}_{p_3}$ denote the sample estimate. The true and sample estimate of the relative width is shown below:

$$\beta = \frac{\zeta_{p_3} - \zeta_{p_1}}{\zeta_{p_2}} \tag{2.5}$$

$$\hat{\beta} = \frac{\hat{\zeta}_{p_3} - \hat{\zeta}_{p_1}}{\hat{\zeta}_{p_2}} \tag{2.6}$$

In this research, we use the 10th, 50th, and 90th percentile for p_1, p_2, and p_3 respectively (van Lint and van Zuylen, 2005).

To find the $100(1-2\alpha)$ confidence interval, we need to determine an upper bound $\overline{r(\boldsymbol{X})}$ and lower bound $\underline{r(\boldsymbol{X})}$ such that the probability of β lying in $\left[\underline{r(\boldsymbol{X})}, \overline{r(\boldsymbol{X})}\right]$ is $100(1-2\alpha)$. If we know the sampling distribution of $\hat{\beta}, F_{\hat{\beta}}(.)$ and its standard error, $\sigma_{\hat{\beta}}$, we can determine the confidence interval.

2.2 MULTIVARIATE DELTA METHOD AND JOINT DISTRIBUTIONS OF SAMPLE QUANTILES AND SAMPLE MEAN

In this section, we first present the multivariate delta method. We then present two theorems – the first one on the joint distribution of sample quantiles and sample mean and the second result on joint distributions of sample quantiles.

2.2.1 Multivariate Delta Method

Suppose that a multivariate vector of statistics $\mathbf{T} = (T_1, T_2, T_3)$ converges asymptotically to a multivariate normal distribution, that is,

$$\sqrt{n}\,(\mathbf{T} - \boldsymbol{\theta}) \xrightarrow{D} \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma}) \tag{2.7}$$

where n is the number of observations, $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$, $\mathbf{0}$ is a three-dimensional null vector and

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix}.$$

Let $h(\mathbf{T})$ be a scalar function of $\mathbf{T}$ with continuous partial derivatives. According to the multivariate delta method (cf. Theorem 5.5.28 in Casella and Berger (2001)),

$$\sqrt{n}[h(\mathbf{T}) - h(\boldsymbol{\theta})] \xrightarrow{D} N(0, \sigma_h^2) \tag{2.8}$$

where

$$\sigma_h^2 = \begin{pmatrix} \frac{\partial h(\boldsymbol{\theta})}{\partial \theta_1} & \frac{\partial h(\boldsymbol{\theta})}{\partial \theta_2} & \frac{\partial h(\boldsymbol{\theta})}{\partial \theta_3} \end{pmatrix} \boldsymbol{\Sigma} \begin{pmatrix} \frac{\partial h(\boldsymbol{\theta})}{\partial \theta_1} \\ \frac{\partial h(\boldsymbol{\theta})}{\partial \theta_2} \\ \frac{\partial h(\boldsymbol{\theta})}{\partial \theta_3} \end{pmatrix} \tag{2.9}$$

provided $\sigma_h^2 > 0$. In particular, if

$$h(\mathbf{T}) = \frac{T_1}{T_2}$$

then it follows from the equations (2.8) and (2.9) that

$$\sqrt{n}\left[\frac{T_1}{T_2} - \frac{\theta_1}{\theta_2}\right] \xrightarrow{D} N\left(0, \frac{\sigma_1^2}{\theta_2^2} - 2\,\frac{\theta_1}{\theta_2^3}\sigma_{12} + \frac{\theta_1^2}{\theta_2^4}\sigma_2^2\right) \tag{2.10}$$

2.2.2 Joint Distribution of Sample Quantiles and Sample Mean

In this subsection, we present two results to derive the asymptotic distribution of the travel time reliability indices considered in this research.

Theorem 1: (Ferguson (1999)) Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution with cumulative distribution function (CDF) $F(.)$, probability density function $f(.)$, mean μ, and finite variance σ^2. Let $0 < p < 1$ and let ζ_p denote the $100p^{th}$ percentile of $F(.)$. Assuming that $f(.)$ is continuous and positive at p and letting $\hat{\mu}$ and $\hat{\zeta}_p$ denote the sample mean and the sample $100p^{th}$ percentile, then

$$\sqrt{n}\begin{pmatrix} \hat{\zeta}_p - \zeta_p \\ \hat{\mu} - \mu \end{pmatrix} \xrightarrow{D} \mathbf{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \frac{p(1-p)}{\mu^2[f(\zeta_p)]^2} & \frac{\tau_p}{f(\zeta_p)} \\ \frac{\tau_p}{f(\zeta_p)} & \sigma^2 \end{bmatrix}\right) \tag{2.11}$$

where

$$\begin{aligned} \tau_p &= \int_{-\infty}^{\zeta_p} (1-p)(\zeta_p - x) f(x)dx + \int_{\zeta_p}^{\infty} p(x - \zeta_p) f(x)dx \\ &= \int_{-\infty}^{\zeta_p} [\zeta_p - p\zeta_p - x + px] f(x)dx + \int_{\zeta_p}^{\infty} [px - p\zeta_p] f(x)dx \end{aligned}$$

$$= p\int_{-\infty}^{\infty} xf(x)dx - p\zeta_p \int_{-\infty}^{\infty} f(x)dx + \zeta_p \int_{-\infty}^{\zeta_p} f(x)dx - \int_{-\infty}^{\zeta_p} xf(x)dx$$
$$= p\mu - p\zeta_p + \zeta_p F(\zeta_p) - \int_{-\infty}^{\zeta_p} xf(x)dx$$
$$= p\mu - \int_{-\infty}^{\zeta_p} xf(x)dx$$

Theorem 2: (Ekström and Jammalamadaka, 2012) Let $X_1, \ldots, X_n$ be independent and identically distributed continuous random variables with cumulative distribution function (CDF) $F(.)$ and probability density function $f(.)$. Let $0 < p < 1$ and let ζ_p denote the $100p^{th}$ percentile of $F(.)$. Let $0 < p_1 < p_2 < p_3 < 1$. Assuming that $f(.)$ is continuous and positive at p_1, p_2, and p_3. Let $\hat{\zeta}_{p_1}, \hat{\zeta}_{p_2}$, and $\hat{\zeta}_{p_3}$ denote the sample percentiles, then

$$\sqrt{n}\begin{pmatrix} \hat{\zeta}_{p_1} & \zeta_{p_1} \\ \hat{\zeta}_{p_2} & -\zeta_{p_2} \\ \hat{\zeta}_{p_3} & \zeta_{p_2} \end{pmatrix} \xrightarrow{D} \mathbf{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}\right) \qquad (2.12)$$

Where $\sigma_{11} = \frac{p_1(1-p_1)}{[f(\zeta_{p_1})]^2}$, $\sigma_{22} = \frac{p_2(1-p_2)}{[f(\zeta_{p_2})]^2}$, $\sigma_{33} = \frac{p_3(1-p_3)}{[f(\zeta_{p_3})]^2}$, $\sigma_{12} = \frac{p_1(1-p_2)}{f(\zeta_{p_1})f(\zeta_{p_2})}$, $\sigma_{13} = \frac{p_1(1-p_3)}{f(\zeta_{p_1})f(\zeta_{p_3})}$, and $\sigma_{23} = \frac{p_2(1-p_3)}{f(\zeta_{p_2})f(\zeta_{p_3})}$.

2.3 ASYMPTOTIC DISTRIBUTION OF TRAVEL TIME RELIABILITY INDEXES

2.3.1 Asymptotic Distribution of Buffer Index

The following result follows from equations (2.10) and (2.11).

Theorem 3: The asymptotic distribution of the buffer index is given as

$$\sqrt{n}\left(\hat{\beta} - \beta\right) \xrightarrow{\boldsymbol{D}} N(0, \sigma_B^2) \qquad (2.13)$$

where

$$\sigma_B^2 = \frac{p(1-p)}{\mu^2\left[f(\zeta_p)\right]^2} - \frac{2\zeta_p\tau_p}{\mu^3 f(\zeta_p)} + \frac{\zeta_p^2\sigma^2}{\mu^4}$$

The sample estimate of σ_B^2, i.e. $\hat{\sigma}_B^2$, can be obtained by using sample estimates of $\mu, \zeta_p, \tau_p, \sigma^2$ which are $\hat{\mu}, \hat{\zeta}_p, \hat{\tau}_p$, and $\hat{\sigma}^2$ respectively. Since we do not know the form of the PDF, we can replace $f(\zeta_p)$ with the kernel density estimate at $\hat{\zeta}_p$. In this work, for the kernel density estimates, we use the Epanechnikov kernel with optimal bandwidth determined based on the quantile optimality ratio assuming an underlying lognormal distribution proposed by Prendergast and Staudte (2017, 2016). Prendergast and Staudte (2017, 2016) show that such a density approximation works very well for unimodal distributions supported in the interval $[0, \infty)$ which is the case for travel times. Also, in $\hat{\tau}_p$, we can replace $\int_{-\infty}^{\hat{\zeta}_p} xf(x)dx$ with $\frac{\sum_{i=1}^{n} x_i I(x_i \leq \hat{\zeta}_p)}{n}$ where $I(x_i \leq \hat{\zeta}_p)$ is an indicator function that takes value 1 when $x_i \leq \hat{\zeta}_p$ and 0 otherwise. The limiting standard error of $\hat{\beta}, \hat{\sigma}_{\hat{\beta}}$ can be estimated by:

$$\hat{\sigma}_{\hat{\beta}} = \sqrt{\frac{\hat{\sigma}_B^2}{n}} \tag{2.14}$$

2.3.2 Asymptotic Distribution of Modified Buffer Index

The following result follows from equations (2.10) and (2.12).

Theorem 4: The asymptotic distribution of the modified buffer index is given as

$$\sqrt{n}\left(\hat{\beta} - \beta\right) \xrightarrow{\boldsymbol{D}} N(0, \sigma_B^2) \tag{2.15}$$

where

$$\sigma_B^2 = \frac{p_2(1-p_2)}{\zeta_{p_1}^2 \left[f\left(\zeta_{p_2}\right)\right]^2} - 2\frac{p_1(1-p_2)\zeta_{p_2}}{f\left(\zeta_{p_1}\right) f\left(\zeta_{p_2}\right) \zeta_{p_1}^3} + \frac{p_1(1-p_1)\zeta_{p_2}^2}{\left[f\left(\zeta_{p_1}\right)\right]^2 \zeta_{p_1}^4}$$

The sample estimate of σ_B^2, i.e. $\hat{\sigma}_B^2$, can be obtained by using sample estimates of ζ_{p_1} and ζ_{p_2} which are $\hat{\zeta}_{p_1}$ and $\hat{\zeta}_{p_2}$ respectively. Since we do not know the form of the PDF, we can replace $f\left(\zeta_{p_1}\right)$ and $f\left(\zeta_{p_2}\right)$ with the kernel density estimate at $\hat{\zeta}_{p_1}$ and $\hat{\zeta}_{p_2}$ respectively. Similar to the buffer index, for the kernel density estimates, we use the Epanechnikov kernel with optimal bandwidth determined based on the quantile optimality ratio assuming an underlying lognormal distribution proposed by Prendergast and Staudte (2017, 2016). The limiting standard error of $\hat{\beta}, \hat{\sigma}_{\hat{\beta}}$ can be estimated as

$$\hat{\sigma}_{\hat{\beta}} = \sqrt{\frac{\hat{\sigma}_B^2}{n}} \tag{2.16}$$

2.3.3 Asymptotic Distribution of Relative Width of Travel Time Distribution

The following result follows from equations (2.9) and (2.12).

Theorem 5: The asymptotic distribution of the relative width is given as

$$\sqrt{n}\left(\hat{\beta} - \beta\right) \xrightarrow{\boldsymbol{D}} N(0, \sigma_B^2) \tag{2.17}$$

where

$$\sigma_B^2 = \frac{p_1(1-p_1)}{\left[f\left(\zeta_{p_1}\right)\right]^2 \zeta_{p_2}^2} + \frac{p_2(1-p_2)}{\left[f\left(\zeta_{p_2}\right)\right]^2}\left[\frac{\zeta_{p_3}-\zeta_{p_1}}{\zeta_{p_2}^2}\right]^2 + \frac{p_3(1-p_3)}{\left[f\left(\zeta_{p_3}\right)\right]^2 \zeta_{p_2}^2} + \frac{2p_1(1-p_2)}{f\left(\zeta_{p_1}\right)f\left(\zeta_{p_2}\right)\zeta_{p_2}}\left[\frac{\zeta_{p_3}-\zeta_{p_1}}{\zeta_{p_2}^2}\right] - \frac{2p_1(1-p_3)}{f\left(\zeta_{p_1}\right)f\left(\zeta_{p_3}\right)\zeta_{p_2}^2} - \frac{2p_2(1-p_3)}{f\left(\zeta_{p_2}\right)f\left(\zeta_{p_3}\right)}\left[\frac{\zeta_{p_3}-\zeta_{p_1}}{\zeta_{p_2}^3}\right]$$

The sample estimate of σ_B^2, i.e. $\hat{\sigma}_B^2$, can be obtained by using sample estimates of ζ_{p_1}, ζ_{p_2}, and ζ_3 which are $\hat{\zeta}_{p_1}, \hat{\zeta}_{p_2}$, and $\hat{\zeta}_{p_3}$ respectively. Since we do not know the form of the PDF, we can replace $f\left(\zeta_{p_1}\right)$, $f\left(\zeta_{p_2}\right)$, and $f\left(\zeta_{p_3}\right)$ with the kernel density estimate at $\hat{\zeta}_{p_1}$, $\hat{\zeta}_{p_2}$, and $\hat{\zeta}_{p_3}$ respectively. Similar to the buffer index, for the kernel density estimates, we use the Epanechnikov kernel with optimal bandwidth determined based on the quantile optimality ratio assuming an underlying lognormal distribution proposed by Prendergast and Staudte (2017, 2016). The limiting standard error of $\hat{\beta}, \hat{\sigma}_{\hat{\beta}}$ can be estimated as

$$\hat{\sigma}_{\hat{\beta}} = \sqrt{\frac{\hat{\sigma}_B^2}{n}} \tag{2.18}$$

2.4 STANDARD NORMAL CONFIDENCE INTERVAL

Given that the distribution of $\hat{\beta}$ is proved to be normal asymptotically, we can determine the standard normal confidence interval using the following steps:

- Evaluate $\hat{\beta}$ for the original sample. Use equations (2.2), (2.4), and (2.6) for the buffer index, modified buffer index, and relative widths respectively.
- Compute $\hat{\sigma}_{\hat{\beta}} = \sqrt{\frac{\hat{\sigma}_B^2}{n}}$. Use equations (2.13), (2.15), and (2.17) for the buffer index, modified buffer index, and relative widths respectively.
- The $100(1-2\alpha)$ standard normal confidence interval is given by $[\hat{\beta} - z_\alpha \hat{\sigma}_{\hat{\beta}}, \hat{\beta} + z_\alpha \hat{\sigma}_{\hat{\beta}}]$ where z_α is the $100(1-\alpha)^{th}$ percentile of the standard normal distribution.

2.5 BOOTSTRAP CONFIDENCE INTERVALS

Bootstrapping is a statistical technique to arrive at point estimates or variance of point estimates using random samples drawn with replacements from the sample data. Bootstrapping can also be thought of as making inferences about population parameters based on random samples from the empirical cumulative distribution function. Since the sample in effect becomes the population, there are no restrictions or requirements on population distributions for bootstrapped confidence interval procedures (Davison and Hinkley, 1997; Efron and Tibshirani, 1993). As travel time distributions have been found to have different distributions and shapes, bootstrap confidence intervals are applicable for travel time reliability metrics. Note that the bootstrap confidence interval methods are applicable for buffer index, modified buffer index, and relative width by choosing the appropriate formulas for β and $\hat{\beta}$ respectively.

Let $\boldsymbol{X}^* = (X_1^*, X_2^*, \ldots, X_n^*)$ be a bootstrap sample from $\boldsymbol{X}$. In other words, $\boldsymbol{X}^*$ is a random sample drawn with replacement from the original observed sample $\boldsymbol{X}$. Let B denote the total number of independent bootstrap samples. While the sample size is finite, B can be arbitrarily large but bounded.

The B independent bootstrap samples are $\boldsymbol{X}^{*\mathbf{1}}, \boldsymbol{X}^{*\mathbf{2}}, \ldots, \boldsymbol{X}^{*\boldsymbol{B}}$. Let $\hat{\zeta}_p^{*b}, \hat{\mu}^{*b}$, and $\hat{\beta}^{*b}$ represent the sample $100p^{th}$ percentile, the sample mean, and the sample buffer index respectively. These parameters are evaluated for the b^{th} bootstrap sample. Let $\hat{\tau}_\alpha$ and $\hat{\tau}_{1-\alpha}$ denote the $100\alpha^{th}$ and $100(1-\alpha)^{th}$ sample percentile of $\hat{\beta}^{*b}$ $\forall b = 1, .., B$.

In this research, we consider the following bootstrapped procedures: (i) Simple bootstrap, (ii) Percentile bootstrap, (iii) Standard Normal with Bootstrapped Standard Errors, (iv) Standard Normal with Log-transformed Buffer Index, (v) Bias-corrected and accelerated (BCa), (vi) Studentized Bootstrap with Asymptotic Standard Errors. While the details and associated proofs can be found in classic statistics textbooks (Davison and Hinkley, 1997; Efron and Tibshirani, 1993), the procedures for the bootstrap confidence intervals are briefly described below for the sake of completeness.

2.5.1 Simple Bootstrap

- Evaluate $\hat{\beta}$ for the original sample.
- Evaluate $\hat{\beta}^{*b}$ for each of the B independent bootstrap samples.
- Determine $\hat{\tau}_\alpha, \hat{\tau}_{1-\alpha}$ which correspond to the $100\alpha^{th}$ and $100(1-\alpha)^{th}$ sample percentile of $\hat{\beta}^{*b}$.
- The $100(1-2\alpha)$ simple bootstrap confidence intervals is given by $[2\widehat{\beta} - \hat{\tau}_{1-\alpha}, 2\hat{\beta} - \hat{\tau}_\alpha]$.

2.5.2 Percentile Bootstrap

- Evaluate $\hat{\beta}$ for the original sample.
- Evaluate $\hat{\beta}^{*b}$ for each of the B independent bootstrap samples.
- Determine $\hat{\tau}_\alpha, \hat{\tau}_{1-\alpha}$ which correspond to the $100\alpha^{th}$ and $100(1-\alpha)^{th}$ sample percentile of $\hat{\beta}^{*b}$.
- The $100(1-2\alpha)$ percentile bootstrap confidence intervals is given by $[\hat{\tau}_\alpha, \hat{\tau}_{1-\alpha}]$.

2.5.3 Standard Normal with Bootstrapped Standard Errors

- Evaluate $\hat{\beta}$ for the original sample.
- Evaluate $\hat{\beta}^{*b}$ for each of the B independent bootstrap samples.
- Compute $\hat{\sigma}_\beta^* = \sqrt{\frac{\sum_{b=1}^{B}\left(\widehat{\beta}^{*b} - \bar{\beta}^*\right)^2}{B-1}}$ where $\bar{\beta}^* = \frac{\sum_{b=1}^{B}\widehat{\beta}^{*b}}{B}$

- The $100(1-2\alpha)$ standard normal confidence interval is given by $[\hat{\beta} - z_\alpha \hat{\sigma}^*_\beta, \hat{\beta} + z_\alpha \hat{\sigma}^*_\beta]$.

2.5.4 Standard Normal with Log-transformed Buffer Index

- Evaluate $\hat{\beta}$ for the original sample.
- Evaluate $\hat{\beta}^{*b}$ for each of the B independent bootstrap samples.
- Log transform the buffer index $\hat{\beta}^{*b}$.
- Compute $\hat{\sigma}^*_\beta = \sqrt{\frac{\sum_{b=1}^{B}\left(\hat{\beta}^{*b} - \bar{\beta}^*\right)^2}{B-1}}$ where $\bar{\beta}^* = \frac{\sum_{b=1}^{B}\hat{\beta}^{*b}}{B}$.
- The $100(1-2\alpha)$ standard normal confidence interval is given by $[\exp(\hat{\beta} - z_\alpha \hat{\sigma}^*_\beta), \exp(\hat{\beta} + z_\alpha \hat{\sigma}^*_\beta)]$.

2.5.5 Bias-corrected and accelerated (BCa)

- Evaluate $\hat{\beta}$ for the original sample.
- Evaluate $\hat{\beta}^{*b}$ for each of the B independent bootstrap samples.
- Determine $\hat{\gamma}_0 = \Phi^{-1}\left(\frac{\sum_{b=1}^{B} I(\hat{\beta}^{*b} \leq \hat{\beta})}{B}\right)$ where $I\left(\hat{\beta}^{*b} \leq \hat{\beta}\right)$ is an indicator function taking value 1 when $\hat{\beta}^{*b} \leq \hat{\beta}$ and 0, otherwise and Φ is the standard normal CDF.
- Let $\boldsymbol{X_j^i}$ represent i^{th} jack-knifed sample or the original sample after removing the i^{th} variable, i.e., $\boldsymbol{X_j^i} = (X_1, X_2, \dots X_{i-1}, X_{i+1}, \dots, X)$.
- Calculate $\hat{\beta}_j^i$ the reliability parameter of the sample $\boldsymbol{X_j^i}$ and $\bar{\beta}_j = \frac{\sum_{i=1}^{n}\hat{\beta}_j^i}{n}$.
- Determine $\hat{\gamma}_1 = \frac{\sum_{i=1}^{n}\left(\bar{\beta}_j - \hat{\beta}_j^i\right)^3}{6\left[\sum_{i=1}^{n}\left(\bar{\beta}_j - \hat{\beta}_j^i\right)^2\right]^{\frac{3}{2}}}$.
- Compute $\alpha_1 = \Phi\left(\hat{\gamma}_0 + \frac{\hat{\gamma}_0 + z_\alpha}{1 - \hat{\gamma}_1(\hat{\gamma}_0 + z_\alpha)}\right)$ and $\alpha_2 = \Phi\left(\hat{\gamma}_0 + \frac{\hat{\gamma}_0 + z_{1-\alpha}}{1 - \hat{\gamma}_1(\hat{\gamma}_0 + z_{1-\alpha})}\right)$ where z_α denotes the $100\alpha^{th}$ percentile of the standard normal distribution.
- Determine $\hat{\tau}_{\alpha_1}, \hat{\tau}_{\alpha_2}$ which correspond to the $100\alpha_1^{th}$ and $100\alpha_2^{th}$ percentile of $\hat{\beta}^{*b}$.
- The $100(1-2\alpha)$ BCa bootstrap confidence interval is given by $[\hat{\tau}_{\alpha_1}, \hat{\tau}_{\alpha_2}]$.

2.5.6 Studentized Bootstrap with Asymptotic Standard Errors

- Evaluate $\hat{\beta}$ for the original sample.
- Evaluate $\hat{\beta}^{*b}$ for each of the B independent bootstrap samples.
- Compute $\hat{\sigma}^*_\beta = \sqrt{\frac{\sum_{b=1}^{B}\left(\hat{\beta}^{*b} - \bar{\beta}^*\right)^2}{B-1}}$ where $\bar{\beta}^* = \frac{\sum_{b=1}^{B}\hat{\beta}^{*b}}{B}$.
- For each bootstrap sample $\boldsymbol{X^{*b}}, b = 1,2, \dots B$.
  - Compute $\hat{\sigma}^{*b}_\beta = \sqrt{\frac{\hat{\sigma}_B^2}{n}}$.

- Determine $t^{*b} = \frac{\widehat{\beta}^{*b} - \widehat{\beta}}{\widehat{\sigma}_{\beta}^{*b}}$.

- Determine $\widehat{T}_{\alpha}, \widehat{T}_{1-\alpha}$ which correspond to the $100\alpha^{th}$ and $100(1-\alpha)^{th}$ percentile of t^{*b}.
- The $100(1-2\alpha)$ studentized double bootstrap confidence intervals is given as $[\widehat{\beta} - \widehat{T}_{1-\alpha}\widehat{\sigma}_{\beta}^{*}, \widehat{\beta} - \widehat{T}_{\alpha}\widehat{\sigma}_{\beta}^{*}]$.

3.0 CASE STUDY

The approaches developed in the previous section are applied to a real-world case study. The data for the case study is from the Portland, OR metropolitan region and was originally collected and analyzed by Anderson et al. (2019). The data belongs to the eastbound direction of the Tualatin Sherwood corridor, which begins at SW Tualatin-Sherwood Road and OR 99W and ends at SW Nyberg Street and I-5 as shown in Figure 3-1. Depending on the section of the corridor, the AADT generally lies in the 20000-30000 range. The final section near I5 has an AADT of slightly more than 40000. The traffic mix is dominated by automobiles, with 68-75% cars (depending on sections) and nearly 20% light trucks.

The travel time data was downloaded from the BlueMAC Transportation Data Systems website (BlueMAC, 2017). In 2016, about 120 Bluetooth detector devices, called BlueMAC devices, were installed in Washington County at intersections on various arterials. The vehicle capture rate of these BlueMAC devices is higher than 10% of the traffic for target corridors (Anderson et al., 2019). The BlueMAC Transportation Data Systems website allowed us to select any two BlueMAC devices, one is the origin and the other the destination, and download all travel time data recorded by these detectors. The data used in this study was not temporally or spatially aggregated. We used travel time data of vehicles detected at the beginning and end of the eastbound direction of the Tualatin Sherwood corridor. For more details on the data and descriptive analysis, see Anderson et al. (2019).

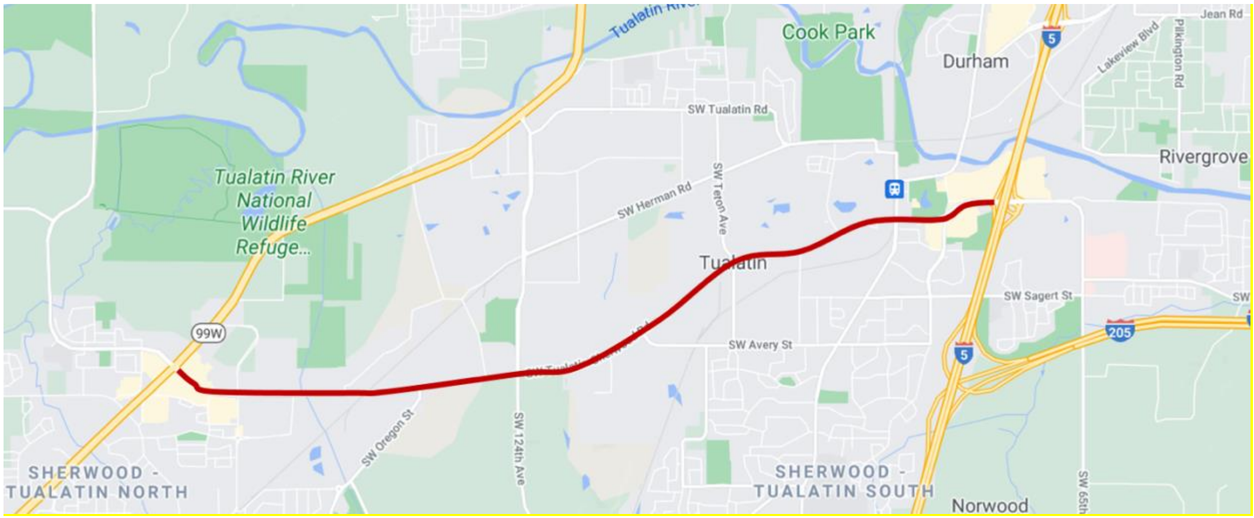

Figure 3-1: Tualatin-Sherwood Corridor

This study uses corridor-level travel time information collected from August 2017 to November 2017 from 6 AM to 7 PM. All the observations lying outside the bounds defined by equation (3.1) were classified as outliers.

$$M \pm 3\,\frac{\sum_{i=1}^{m}|tt_i - M|}{m} \tag{3.1}$$

where tt_i is the travel time observed in the i^{th} trip, M is the median in each 15-minute block of travel times, and m is the number of trips within each block of travel times (Clark et al., 2002; Zhang et al., 2018). After removing the outliers, there are 17491 observations that cover both weekdays and weekends. The density of the travel time is shown below in Figure 3-2. Note the log-lognormal shape of the data but with a small bump on the right tail. This type of shape is not uncommon when analyzing travel time data since congestion skews the data and even create bumps. The distributions analyzed later in the paper consider these two factors: skewness and the potential presence of bumps (technically bimodal distributions) on the right tail.

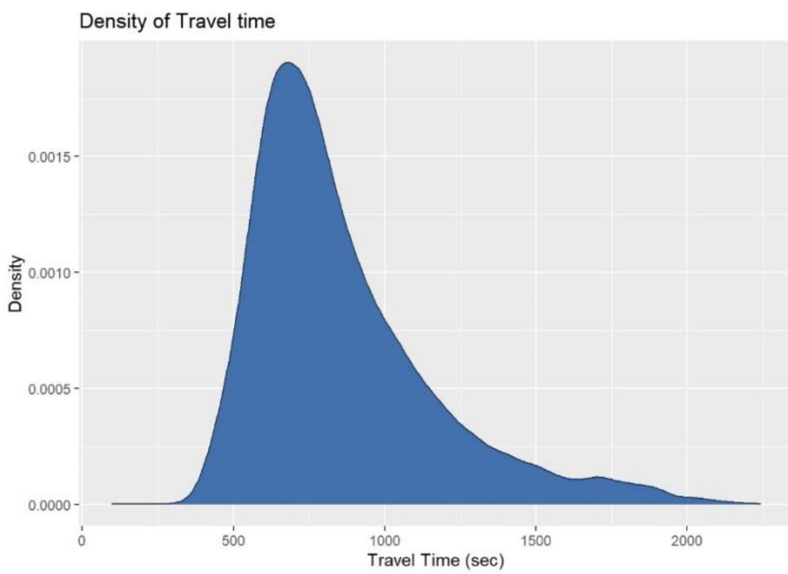

Figure 3-2: Travel Time Density

The Lognormal distribution ($\mu = 6.7034, \sigma = 0.3245$) was a good fit for the data and consistent with the predominant shape. Note that for the lognormal distribution, μ and σ represent the mean and standard deviation of the natural log of travel times. Since Lognormal distribution has been widely used for travel times, we picked Lognormal ($\mu = 6.7034, \sigma = 0.3245$) as the “Base” case denoted as B since travel times in many instances tend to look lognormal and right-skewed. In general, travel time distributions tend to be right skewed. However, in rare occasions, such as the onset of congestion, they can be left skewed(van Lint and van Zuylen, 2005) which we model in Case D. If the time period of analysis is longer than the peak period, you will have the bimodal distribution which we see in Case E. For the sake of conciseness, we generated only four additional cases to cover different shapes and distributions:

- Case A: Lognormal distribution with a higher right skew.
- Case C: Symmetric Normal distribution.
- Case D: Skewed Normal distribution with location parameter $\xi = 1250$, scale parameter $\omega = 400$, and shape parameter $\alpha = -2.5$ for a left-skewed distribution.
- Case E: Bi-modal Normal distribution with the first and second normal centered at 700 and 1200, respectively.

A summary of key parameters and statistics for the five cases A to E are presented in Table 3.1, and Figure 3-3 shows the corresponding density function graphs. The cases correspond to four unimodal shapes and one bimodal shape. The unimodal distributions cover two right-skewed distributions, one symmetric and one left-skewed distribution. The skewness in Table 3.1 refers to the third standardized moment. The reliability increases as we move from right to left-skewed distributions.

Table 3.1: Summary of Parameters and Statistics

Case	Shape	Distribution (Parameters)	Skew-ness	True Mean	True 95th Percent-ile	True Buffer Index	True Modi-fied Buffer Index	True Relative Width
A	Right Skewed	Lognormal ($\mu = 0.7034$, σ = 0.4868)	1.6896	337.6003	667.8292	0.9781	1.2269	1.3300
B	Right Skewed	Lognormal ($\mu = 0.7034$, $\sigma = 0.3245$)	1.0367	859.2416	1390.1334	0.6178	0.7053	0.8559
C	Symmetric	Normal ($\mu_N = 700, \sigma_N = 220$)	0	700	1061.8678	0.5169	0.5169	0.8055
D	Left Skewed	Skewed Normal ($\xi = 1250$, $\omega = 400, \alpha = -2.5$)	-0.5757	953.6731	1342.7043	0.4079	0.3658	0.6921
E	Bimodal	Normal $\mu_{BN} = [700,1200], \sigma_{BN} = [150,110], p = [0.8,0.2]$	0.6399	800	1274.2831	0.5928	0.7040	0.9000

We consider the following sample sizes: 100, 300, 500, 1000, and 2000. The travel time samples were generated from the above distributions using a simple Monte Carlo simulation. The number of bootstrap samples was fixed at 1000. Efron and Tibshirani (1993) recommend at least 300. We generated 500 sets of travel time samples of each size. The confidence intervals were then calculated for each set. Then we evaluated the: (i) Width: the average width of the 500 confidence intervals, and (ii) Coverage: the proportion of times the confidence intervals capture the true reliability index. This study focuses on 95% confidence intervals, as they are most widely used in transportation applications. Therefore, we expect the coverage to be around 95%.

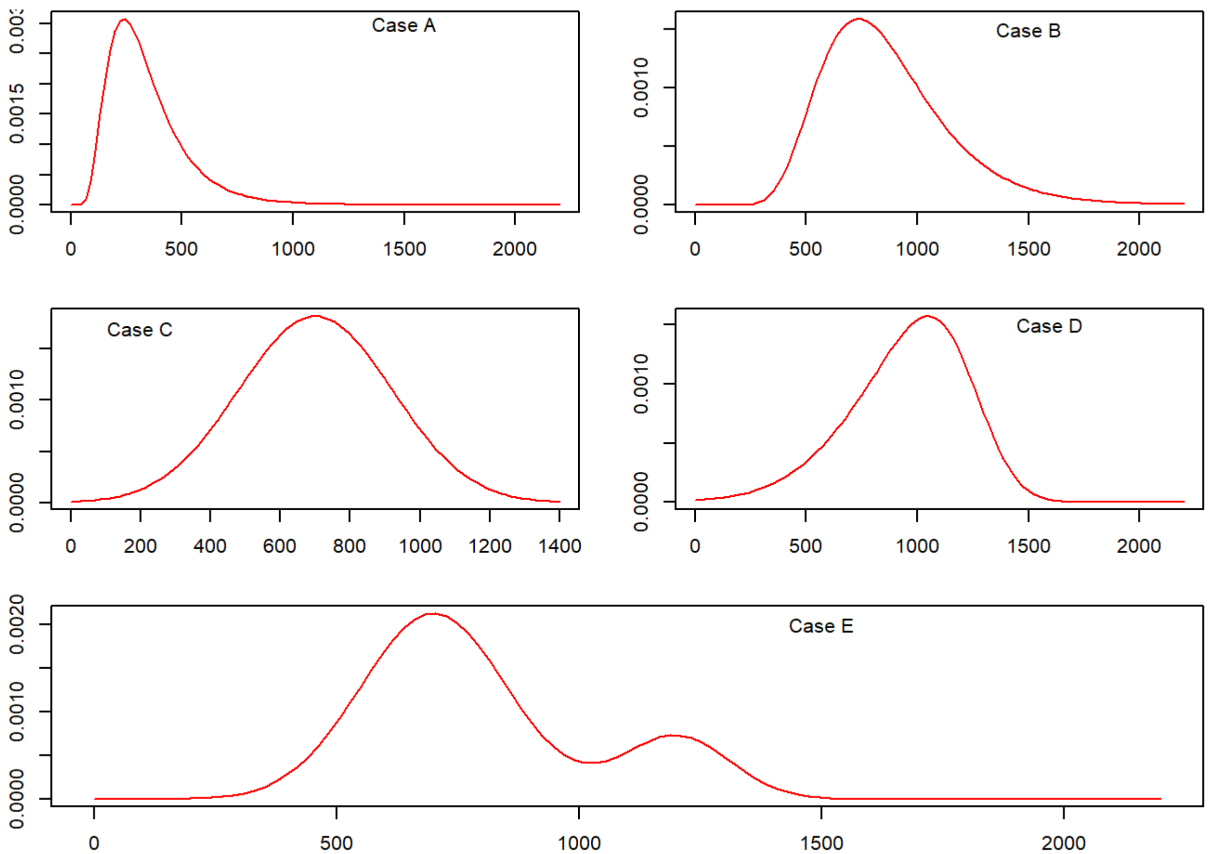

Figure 3-3: Density function graphs

3.1 COMPUTATIONAL RESULTS FOR BUFFER INDEX AND MODIFIED BUFFER INDEX

Among the procedures tested, Simple Bootstrap performs the worst with significantly lower coverage. Figure 3-4 and Figure 3-5 show the 500 confidence intervals generated for a sample size of 1000 using Standard Normal with Asymptotic Standard Errors and Simple Bootstrap, respectively. The red lines correspond to confidence interval which does not contain the true value. The figures demonstrate that Standard Normal with Asymptotic Standard Errors based confidence interval is better at capturing the true value as Figure 3-4 has fewer red lines than Figure 3-5

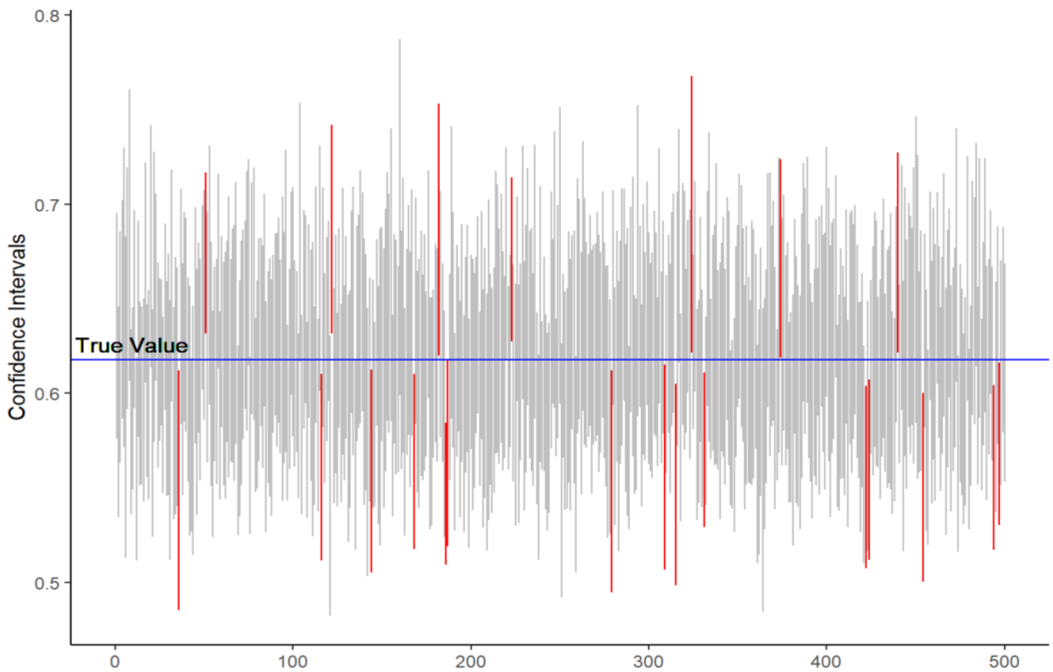

Figure 3-4: 500 Buffer Index Confidence Intervals for sample size of 1000 (Case B) using Standard Normal with Asymptotic Standard Errors

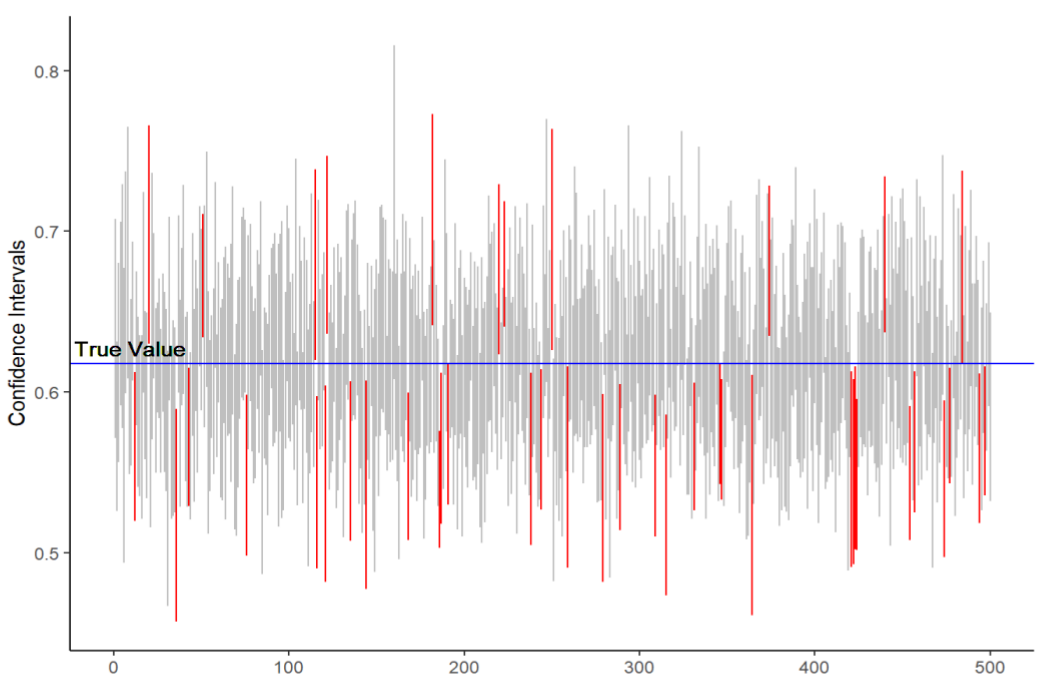

Figure 3-5: 500 Buffer Index Confidence Intervals for sample size of 1000 (Case B) using Simple Bootstrap

Table 3.2 to Table 3.6 present the width and coverage for the five population distributions for different sample sizes for the buffer index. Table ***3.7*** to Table ***3.11*** provide the same information for the modified buffer index. When the coverage is lower than 95%, the corresponding entries have been shown in red. The entries with the lowest width have been highlighted in yellow. The width of the confidence interval depends on the skewness. The width decreases from right skew to left skew and then increases for the bimodal distribution. As the buffer index decreases or as the travel time reliability increases, the confidence interval width also decreases.

Studentized Bootstrap provides confidence intervals with the lowest width for the buffer index. However, the coverage is lower than 95% for both buffer index and modified buffer index. In general, Percentile Bootstrap, Standard Normal with Bootstrapped Standard Errors, Standard Normal with Log-transformed Buffer Index, BCa, and Standard Normal with Asymptotic Standard Errors perform the best. For these best-performing confidence interval procedures, there is no noticeable increase in coverage with sample size. However, the width of the confidence interval decreases with sample size. The width of the Percentile Bootstrap, Standard Normal with Bootstrapped Standard Errors, BCa, and the Standard Normal with Log-transformed Buffer Index are similar to the width of Standard Normal with Asymptotic Standard Errors confidence intervals for sample sizes higher than 100. In general, we can see that width of the confidence intervals decreases proportionally to the inverse square root of the sample size.

Table 3.2: Buffer Index - Width and Coverage for Case A: Lognormal, Skewness = 1.6896

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.772	0.958	0.408	0.97	0.310	0.966	0.208	0.956	0.146	0.966
Simple Bootstrap	0.669	0.834	0.373	0.882	0.291	0.874	0.199	0.896	0.141	0.918
Percentile Bootstrap	0.669	0.972	0.373	0.964	0.291	0.972	0.199	0.95	0.141	0.964
Normal with Bootstrapped SE	0.684	0.956	0.376	0.946	0.295	0.948	0.201	0.946	0.141	0.958
Normal with Log-transformed Buffer index	0.680	0.958	0.377	0.946	0.296	0.958	0.200	0.946	0.141	0.96
BCa Bootstrap	0.720	0.938	0.381	0.948	0.290	0.952	0.203	0.956	0.142	0.948
Studentized Bootstrap	0.679	0.878	0.373	0.876	0.294	0.908	0.199	0.9	0.140	0.908

Table 3.3: Buffer Index - Width and Coverage for Case B: Lognormal, Skewness = 1.0367

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.424	0.954	0.228	0.97	0.174	0.966	0.118	0.96	0.083	0.962
Simple Bootstrap	0.374	0.842	0.211	0.884	0.165	0.876	0.113	0.894	0.080	0.924
Percentile Bootstrap	0.375	0.968	0.211	0.962	0.165	0.972	0.113	0.946	0.080	0.97
Normal with Bootstrapped SE	0.382	0.948	0.213	0.946	0.167	0.944	0.114	0.942	0.081	0.956
Normal with Log-transformed Buffer index	0.382	0.958	0.213	0.946	0.168	0.954	0.114	0.938	0.081	0.96
BCa Bootstrap	0.402	0.942	0.216	0.952	0.164	0.954	0.116	0.952	0.081	0.946
Studentized Bootstrap	0.382	0.874	0.212	0.884	0.167	0.91	0.114	0.9	0.080	0.914

Table 3.4: Buffer Index - Width and Coverage for Case C: Symmetric Normal, Skewness = 0

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.269	0.954	0.148	0.966	0.114	0.962	0.078	0.952	0.055	0.964
Simple Bootstrap	0.250	0.89	0.143	0.906	0.111	0.922	0.077	0.912	0.054	0.936
Percentile Bootstrap	0.251	0.966	0.142	0.96	0.111	0.976	0.077	0.952	0.054	0.968
Normal with Bootstrapped SE	0.252	0.948	0.143	0.952	0.112	0.954	0.077	0.938	0.054	0.966
Normal with Log-transformed Buffer index	0.254	0.958	0.143	0.952	0.112	0.96	0.077	0.932	0.054	0.962
BCa Bootstrap	0.259	0.938	0.144	0.962	0.110	0.954	0.078	0.952	0.054	0.95
Studentized Bootstrap	0.240	0.884	0.140	0.91	0.111	0.916	0.077	0.904	0.054	0.94

Table 3.5: Buffer Index - Width and Coverage for Case D: Left Skewed Normal, Skewness = -0.5757

Sample Size	100		300		500		1000		2000	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.193	0.96	0.107	0.96	0.082	0.954	0.057	0.948	0.040	0.948
Simple Bootstrap	0.184	0.92	0.105	0.938	0.082	0.93	0.057	0.922	0.040	0.938
Percentile Bootstrap	0.184	0.972	0.105	0.972	0.081	0.966	0.057	0.952	0.040	0.962
Normal with Bootstrapped SE	0.184	0.958	0.105	0.96	0.082	0.956	0.057	0.936	0.040	0.954
Normal with Log-transformed Buffer index	0.187	0.954	0.105	0.958	0.082	0.954	0.057	0.936	0.040	0.946
BCa Bootstrap	0.188	0.954	0.106	0.956	0.082	0.958	0.057	0.94	0.040	0.932
Studentized Bootstrap	0.174	0.914	0.102	0.94	0.081	0.936	0.057	0.92	0.040	0.934

Table 3.6: Buffer Index - Width and Coverage for Case E: Bimodal, Skewness = 0.6399

Sample Size	100		300		500		1000		2000	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.207	0.96	0.115	0.956	0.087	0.962	0.061	0.972	0.043	0.952
Simple Bootstrap	0.202	0.924	0.114	0.932	0.087	0.94	0.061	0.964	0.043	0.944
Percentile Bootstrap	0.202	0.974	0.114	0.958	0.087	0.962	0.061	0.972	0.043	0.96
Normal with Bootstrapped SE	0.202	0.968	0.114	0.944	0.087	0.956	0.061	0.974	0.043	0.95
Normal with Log-transformed Buffer index	0.207	0.97	0.115	0.952	0.088	0.962	0.061	0.976	0.043	0.95
BCa Bootstrap	0.204	0.956	0.113	0.956	0.087	0.948	0.061	0.946	0.043	0.952
Studentized Bootstrap	0.185	0.902	0.111	0.916	0.086	0.934	0.061	0.96	0.043	0.944

Table 3.7: Modified Buffer Index - Width and Coverage for Case A: Lognormal, Skewness = 1.6896

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	1.119	0.964	0.598	0.964	0.458	0.968	0.311	0.962	0.218	0.964
Simple Bootstrap	1.010	0.876	0.563	0.902	0.438	0.908	0.301	0.916	0.212	0.932
Percentile Bootstrap	1.014	0.964	0.562	0.968	0.438	0.974	0.301	0.95	0.212	0.974
Normal with Bootstrapped SE	1.029	0.954	0.564	0.954	0.441	0.958	0.302	0.95	0.213	0.954
Normal with Log-transformed Buffer index	1.033	0.948	0.567	0.96	0.444	0.962	0.302	0.942	0.213	0.96
BCa Bootstrap	1.089	0.952	0.576	0.952	0.446	0.962	0.303	0.938	0.213	0.956
Studentized Bootstrap	1.102	0.906	0.571	0.932	0.443	0.922	0.302	0.91	0.210	0.928

Table 3.8: Modified Buffer Index - Width and Coverage for Case B: Lognormal, Skewness = 1.0367

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.555	0.962	0.302	0.964	0.231	0.966	0.158	0.962	0.111	0.966
Simple Bootstrap	0.506	0.868	0.286	0.908	0.223	0.91	0.153	0.916	0.108	0.936
Percentile Bootstrap	0.508	0.964	0.286	0.968	0.223	0.974	0.153	0.950	0.108	0.974
Normal with Bootstrapped SE	0.514	0.958	0.286	0.954	0.224	0.956	0.154	0.950	0.109	0.956
Normal with Log-transformed Buffer index	0.519	0.946	0.288	0.958	0.226	0.962	0.154	0.942	0.109	0.96
BCa Bootstrap	0.538	0.952	0.291	0.952	0.226	0.962	0.154	0.938	0.109	0.956
Studentized Bootstrap	0.551	0.906	0.291	0.932	0.227	0.928	0.154	0.914	0.107	0.928

Table 3.9: Modified Buffer Index - Width and Coverage for Case C: Symmetric Normal, Skewness = 0

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.343	0.968	0.189	0.96	0.146	0.97	0.100	0.958	0.071	0.958
Simple Bootstrap	0.324	0.88	0.184	0.912	0.144	0.916	0.100	0.932	0.070	0.934
Percentile Bootstrap	0.325	0.976	0.185	0.97	0.144	0.98	0.100	0.952	0.070	0.97
Normal with Bootstrapped SE	0.327	0.96	0.184	0.946	0.144	0.962	0.100	0.944	0.070	0.956
Normal with Log-transformed Buffer index	0.332	0.956	0.186	0.95	0.145	0.964	0.100	0.948	0.070	0.956
BCa Bootstrap	0.335	0.954	0.186	0.964	0.145	0.964	0.100	0.946	0.070	0.952
Studentized Bootstrap	0.333	0.912	0.186	0.914	0.146	0.936	0.101	0.928	0.070	0.938

Table 3.10: Modified Buffer Index - Width and Coverage for Case D: Left Skewed Normal, Skewness = -0.5757

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.233	0.95	0.128	0.956	0.099	0.964	0.069	0.95	0.048	0.944
Simple Bootstrap	0.224	0.884	0.127	0.93	0.098	0.936	0.068	0.924	0.048	0.93
Percentile Bootstrap	0.224	0.97	0.127	0.96	0.098	0.962	0.068	0.962	0.048	0.952
Normal with Bootstrapped SE	0.225	0.942	0.127	0.944	0.098	0.952	0.068	0.95	0.048	0.942
Normal with Log-transformed Buffer index	0.231	0.94	0.128	0.952	0.098	0.954	0.068	0.95	0.048	0.942
BCa Bootstrap	0.231	0.932	0.127	0.948	0.098	0.958	0.068	0.938	0.048	0.936
Studentized Bootstrap	0.227	0.876	0.128	0.93	0.098	0.924	0.069	0.916	0.048	0.922

Table 3.11: Buffer Index - Width and Coverage for Case E: Bimodal, Skewness = 0.6399

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.297	0.966	0.164	0.952	0.124	0.968	0.085	0.944	0.060	0.96
Simple Bootstrap	0.279	0.902	0.159	0.914	0.120	0.948	0.085	0.958	0.060	0.93
Percentile Bootstrap	0.278	0.966	0.159	0.966	0.120	0.966	0.085	0.976	0.060	0.96
Normal with Bootstrapped SE	0.280	0.946	0.159	0.948	0.120	0.958	0.085	0.958	0.059	0.96
Normal with Log-transformed Buffer index	0.287	0.956	0.160	0.952	0.121	0.962	0.085	0.964	0.060	0.948
BCa Bootstrap	0.283	0.944	0.159	0.944	0.120	0.956	0.086	0.972	0.059	0.952
Studentized Bootstrap	0.267	0.902	0.157	0.904	0.118	0.93	0.085	0.944	0.060	0.926

For the buffer index, Standard Normal with Asymptotic Standard Errors consistently delivers higher than 95% coverage for all sample sizes for all right-skewed, symmetric, and bimodal cases tested. Percentile bootstrap fails to meet the 95% bar in one of the 20 right-skewed, symmetric, and bimodal cases tested. BCa, Standard Normal with Bootstrapped Standard Errors and Standard Normal with Log-transformed Buffer Index fails to hit the 95% coverage mark in 7, 10, and 5 of the 20 right-skewed, symmetric, and bimodal cases. Standard Normal with Asymptotic Standard Error narrowly misses the 95% coverage for the less common left-skewed case for two sample sizes. For the modified buffer index, Standard Normal with Asymptotic Standard Errors consistently delivers higher than 95% coverage for all sample sizes for all right-skewed and symmetric cases tested. For the less common left-skewed and bimodal data, Standard Normal with Asymptotic Standard Error narrowly misses the 95% coverage for two cases. Percentile bootstrap delivers 95% coverage for all cases tested.

While percentile bootstrap provides marginally better (smaller) confidence interval widths, we recommend the Standard Normal with Asymptotic Standard Errors for higher than 100 samples as it consistently delivers the required coverage for the more common right-skewed and symmetric shapes. Moreover, we can develop one-sample hypothesis tests described in the next chapter with the knowledge of the asymptotic normal distribution.

3.2 COMPUTATIONAL RESULTS FOR RELATIVE WIDTH OF TRAVEL TIME DISTRIBUTIONS

Table 3.12 to
Table 3.16 present the width and coverage for the five population distributions for different sample sizes for the relative width of travel time distributions. Similar to the buffer index results, the confidence interval width decreases with relative width or increase in reliability. There are no significant differences in the width of the confidence interval among all confidence interval procedures. As expected, the width of the confidence interval decreases with sample size.

For the more common right skewed distribution, Percentile Bootstrap and Standard Normal with Asymptotic Standard Errors consistently delivers more than 95% coverage. For the less common left skewed and bimodal distribution, Standard Normal with Asymptotic Standard Errors achieves 95% coverage for all cases tests. Percentile Bootstrap fails to achieve 95% coverage in four out of the ten cases tested.

Table 3.12: Relative Width - Width and Coverage for Case A: Lognormal, Skewness = 1.6896

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.711	0.970	0.393	0.960	0.301	0.966	0.208	0.950	0.145	0.958
Simple Bootstrap	0.662	0.904	0.377	0.934	0.288	0.920	0.204	0.926	0.141	0.932
Percentile Bootstrap	0.662	0.966	0.377	0.956	0.289	0.962	0.203	0.952	0.141	0.956
Normal with Bootstrapped SE	0.669	0.962	0.378	0.956	0.288	0.940	0.204	0.946	0.142	0.954
Normal with Log-transformed Buffer index	0.667	0.958	0.380	0.950	0.289	0.950	0.204	0.948	0.141	0.956
BCa Bootstrap	0.686	0.944	0.380	0.950	0.291	0.948	0.204	0.940	0.141	0.950
Studentized Bootstrap	0.662	0.888	0.375	0.936	0.284	0.906	0.202	0.916	0.140	0.926

Table 3.13: Relative Width - Width and Coverage for Case B: Lognormal, Skewness = 1.0367

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.394	0.972	0.221	0.962	0.170	0.968	0.117	0.950	0.081	0.966
Simple Bootstrap	0.364	0.906	0.210	0.928	0.160	0.922	0.113	0.924	0.079	0.932
Percentile Bootstrap	0.365	0.978	0.209	0.954	0.161	0.966	0.113	0.950	0.079	0.962
Normal with Bootstrapped SE	0.367	0.956	0.210	0.960	0.160	0.952	0.114	0.938	0.079	0.956
Normal with Log-transformed Buffer index	0.368	0.958	0.212	0.960	0.161	0.954	0.114	0.942	0.079	0.956
BCa Bootstrap	0.373	0.944	0.211	0.958	0.161	0.950	0.113	0.940	0.079	0.950
Studentized Bootstrap	0.359	0.886	0.205	0.928	0.155	0.902	0.111	0.910	0.077	0.930

Table 3.14: Relative Width - Width and Coverage for Case C: Symmetric Normal, Skewness = 0

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.368	0.974	0.211	0.976	0.163	0.984	0.110	0.970	0.074	0.972
Simple Bootstrap	0.320	0.908	0.185	0.930	0.141	0.936	0.100	0.904	0.070	0.934
Percentile Bootstrap	0.321	0.966	0.185	0.958	0.141	0.964	0.100	0.950	0.070	0.960
Normal with Bootstrapped SE	0.321	0.950	0.185	0.954	0.141	0.964	0.100	0.940	0.070	0.946
Normal with Log-transformed Buffer index	0.323	0.954	0.186	0.958	0.142	0.966	0.100	0.938	0.070	0.950
BCa Bootstrap	0.324	0.940	0.186	0.958	0.142	0.956	0.100	0.940	0.070	0.942
Studentized Bootstrap	0.291	0.868	0.166	0.886	0.125	0.892	0.092	0.870	0.066	0.908

Table 3.15: Relative Width - Width and Coverage for Case D: Left Skewed Normal, Skewness = -0.5757

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.350	0.978	0.203	0.982	0.159	0.984	0.104	0.980	0.070	0.974
Simple Bootstrap	0.294	0.874	0.166	0.912	0.128	0.898	0.091	0.926	0.064	0.942
Percentile Bootstrap	0.294	0.946	0.166	0.948	0.129	0.938	0.091	0.962	0.064	0.972
Normal with Bootstrapped SE	0.297	0.938	0.168	0.940	0.129	0.920	0.091	0.944	0.064	0.964
Normal with Log-transformed Buffer index	0.301	0.940	0.168	0.938	0.129	0.916	0.091	0.944	0.064	0.960
BCa Bootstrap	0.303	0.940	0.168	0.930	0.129	0.930	0.091	0.950	0.064	0.954
Studentized Bootstrap	0.270	0.832	0.142	0.842	0.107	0.840	0.081	0.878	0.059	0.910

Table 3.16: Relative Width - Width and Coverage for Case E: Bimodal, Skewness = 0.6399

Sample Size	**100**		**300**		**500**		**1000**		**2000**	
Confidence Interval Procedures	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage	Width	Cove-rage
Normal with Asymptotic Standard Error	0.316	0.976	0.174	0.990	0.134	0.968	0.090	0.968	0.062	0.952
Simple Bootstrap	0.300	0.930	0.160	0.942	0.122	0.930	0.085	0.912	0.059	0.936
Percentile Bootstrap	0.299	0.972	0.160	0.976	0.122	0.964	0.085	0.950	0.059	0.948
Normal with Bootstrapped SE	0.300	0.970	0.159	0.974	0.122	0.950	0.085	0.938	0.059	0.948
Normal with Log-transformed Buffer index	0.311	0.976	0.161	0.980	0.123	0.956	0.085	0.938	0.059	0.940
BCa Bootstrap	0.298	0.960	0.159	0.972	0.122	0.944	0.084	0.930	0.059	0.928
Studentized Bootstrap	0.284	0.912	0.146	0.908	0.111	0.898	0.080	0.896	0.057	0.922

4.0 HYPOTHESIS TEST

This chapter presents the one-sample hypothesis test procedures for the three travel time reliability metrics - buffer index, modified buffer index, and relative width. Using Theorem 3, 4, and 5, and the limiting standard error result (equations 2.13, 2.15, and 2.17), we construct hypothesis testing procedures for the three reliability indices studied in this project. Table ***4.1*** presents the test statistic, rejection region, and p-values for the lower-tailed, two-tailed, and upper-tailed hypothesis tests. Use equations (2.2), (2.4), and (2.6) for $\hat{\beta}$ for the buffer index, modified buffer index, and relative widths respectively. Note that $\hat{\sigma}_{\hat{\beta}} = \sqrt{\frac{\hat{\sigma}_B^2}{n}}$ and use equations (2.13), (2.15), and (2.17) for $\hat{\sigma}_B^2$ for the buffer index, modified buffer index, and relative widths respectively.

Table 4.1: One-Sample Hypothesis Test

Hypothesis	Test Statistic	Rejection region for large n	P-value
$H_0: \beta = \beta_0$ $H_A: \beta < \beta_0$	$b = \dfrac{\hat{\beta} - \beta_0}{\hat{\sigma}_{\hat{\beta}}}$	$b \leq z_\alpha$	$\Phi(b)$
$H_0: \beta = \beta_0$ $H_A: \beta \neq \beta_0$		$\lvert b \rvert \geq z_{\frac{\alpha}{2}}$	$\lvert b \rvert \geq z_{\frac{\alpha}{2}}$
$H_0: \beta = \beta_0$ $H_A: \beta > \beta_0$		$b \geq z_\alpha$	$1 - \Phi(b)$

In Table ***4.1***, β_0 corresponds to the null value or the prior belief about the buffer index or the modified buffer index or the relative width. For example, if on a roadway segment, the travel time reliability is known to be 0.7, then the null hypothesis becomes $H_0: \beta = 0.7$. If one suspects or want to test the fact that the travel time reliability has worsened, then one will choose the alternative hypothesis of $H_A: \beta > 0.7$. The test statistic will be $b = \frac{\hat{\beta} - 0.7}{\hat{\sigma}_{\hat{\beta}}}$. If one is conducting the test at 5% significance, then $\alpha = 0.05, z_\alpha = 1.645$ and therefore one will reject the null if $b \geq 1.645$ or fail to reject the null otherwise.

Table ***4.2*** gives the power of the above asymptotic test for the upper tailed hypothesis tests of buffer index. The hypothesis test studied is $H_0: \beta = 0.6178 \text{ against } H_A: \beta > 0.6178$ at 5% level of significance. The null value is 0.6178. The first row of the table corresponds to the case where the travel time samples are generated from the lognormal distribution with a true value of β of 0.6178. We generated 1000 sets of samples of sizes 100, 300, 500, 1000, 2000 and identified the proportion of times the null is rejected. In this case, the proportion of times the null is rejected corresponds to the probability of type 1 error which is 0.05. The probabilities estimated by simulation 0.03, 0.03, 0.028, and 0.041 are lower than 0.05, indicating the reliability of this test.

The second row of Table ***4.2*** is when the travel times are generated from a population with a slightly higher β. At higher samples, the hypothesis test correctly rejects the null hypothesis with a probability of 0.92. In general, as expected, as the true buffer index of the population deviates from the null value of 0.6178, the power of the test increases. Also, the probability of correctly rejecting the null hypothesis increases with sample size. Table ***4.3*** gives the power of the upper tailed hypothesis tests of the modified buffer index of $H_0{:}\beta = 0.7053$ against $H_A{:}\beta > 0.7053$ at 5% level of significance. Table ***4.4*** presents the power of the upper tailed hypothesis tests of the relative width. The hypothesis test studied is $H_0{:}\beta = 0.8559$ against $H_A{:}\beta > 0.8559$ at 5% level of significance. As seen in Table ***4.3*** and Table ***4.4***, the same trends are observed for the modified buffer index and relative width respectively.

The results also highlight the importance of sample sizes. Samples of size 2000 or higher can be easily obtained in high traffic corridors in a week or less time. Even assuming a low sampling rate of 1%, with an AADT of 50,000, it is possible to obtain 2000 observations in only four days (assuming 80% of the daily traffic takes place between 6 AM and 7 PM). With technologies based on license plate readings and matchings, it is possible to achieve sample rates close to 2000 observations in one day in most roadways with AADT higher than 2500.

Table 4.2: Buffer Index - Probability of Rejection for Upper-tailed Hypothesis Test

Population for generating travel time samples	Sample Size				
	100	300	500	1000	2000
Lognormal $\mu = 6.70, \sigma = 0.32, \beta = 0.6178$	0.030	0.030	0.028	0.041	0.052
Lognormal $\mu = 6.70, \sigma = 0.35, \beta = 0.6877$	0.125	0.272	0.387	0.694	0.926
Lognormal $\mu = 6.70, \sigma = 0.37, \beta = 0.7232$	0.175	0.455	0.667	0.913	0.995
Lognormal $\mu = 6.70, \sigma = 0.38, \beta = 0.7589$	0.271	0.619	0.834	0.987	1.000

Table 4.3: Modified Buffer Index - Probability of Rejection for Upper-tailed Hypothesis Test

Population for generating travel time samples	Sample Size				
	100	300	500	1000	2000
Lognormal $\mu = 6.70, \sigma = 0.32, \beta = 0.7053$	0.028	0.029	0.032	0.037	0.048
Lognormal $\mu = 6.70, \sigma = 0.35, \beta = 0.7988$	0.101	0.252	0.369	0.668	0.925
Lognormal $\mu = 6.70, \sigma = 0.37, \beta = 0.8474$	0.151	0.417	0.659	0.905	0.995
Lognormal $\mu = 6.70, \sigma = 0.38, \beta = 0.8974$	0.231	0.615	0.824	0.988	1.000

Table 4.4: Relative Width - Probability of Rejection for Upper-tailed Hypothesis Test

Population for generating travel time samples	Sample Size				
	100	300	500	1000	2000
Lognormal $\mu = 6.70, \sigma = 0.32, \beta = 0.8559$	0.03	0.035	0.024	0.03	0.038
Lognormal $\mu = 6.70, \sigma = 0.35, \beta = 0.9471$	0.16	0.403	0.581	0.904	0.995
Lognormal $\mu = 6.70, \sigma = 0.37, \beta = 0.9933$	0.269	0.676	0.895	0.991	1

Lognormal $\mu = 6.70, \sigma = 0.38, \beta = 1.04$	0.41	0.874	0.985	1	1

5.0 CONCLUSIONS

This research focuses on conducting statistical inferences and attaching statistical guarantees on the travel time reliability measures - buffer index, modified buffer index, and relative widths. The multivariate delta method is used to show that the asymptotic distribution of the buffer index, modified buffer index, and relative width is normal. A formula for the standard error of the three reliability metrics is derived. This result is used to arrive at a confidence interval formula for the reliability metrics. The asymptotic normality-based confidence interval does not impose any shape requirement on travel time distributions and thus is widely applicable. The performance of the Standard Normal with Asymptotic Standard Error confidence interval is compared against six other bootstrapping confidence intervals on travel time data obtained from a corridor in the Portland Metropolitan region, USA. For the buffer index, the Standard Normal with Asymptotic Standard Error confidence interval provides consistent coverage over 95% for common right-skewed, symmetric, and bimodal travel time distribution shapes. For the modified buffer index, Standard Normal with Asymptotic Standard Errors consistently delivers higher than 95% coverage for all sample sizes for all right-skewed and symmetric cases tested. For the travel time relative widths, Standard Normal with Asymptotic Standard Errors consistently achieves 95% for all cases tested. The asymptotic normality result is used to derive upper-tailed, lower-tailed, and two-tailed one-sample hypothesis tests for the buffer index, modified buffer index, and relative widths. Simulation results show that the power of the hypothesis test increases rapidly with sample size, which allows researchers and practitioners to easily test the impact of factors related to traffic or environmental conditions with relatively small sample sizes.

This research can be extended in multiple directions. One potential direction for future research is to derive confidence intervals and hypothesis testing for other travel time reliability measures, such as the planning time index. Another direction for research is to derive two-sample confidence interval and hypothesis test procedures, which can be used for conducting before and after travel time reliability evaluation studies. There is also scope for developing methodologies to compare entire travel time distributions rather than relying on reliability metrics which only focus on part of the travel time distributions.The developed methods can also be used to arrive at practical estimates of changes in traffic or travel times which result in a statistically significantly lower travel time reliability metric.

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