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Connecting the “Real-World” to the Math Classroom: Implementing Professional Development for Mathematical Modeling

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Connecting the "Real-World" to the Math Classroom: Implementing Professional Development for Mathematical Modeling

Abstract

This qualitative research study focused on the changes in classroom pedagogy and content of rural mathematics teachers who engaged in a year-long professional development project focused on mathematical modeling. During a 2-week summer institute, teachers solved mathematical modeling problems as learners and then went through an iterative design process of creating, testing and refining lessons for classroom implementation. The lessons were implemented during the academic year. Results of this study indicate that teachers developed a willingness to move from traditional lecture and replication as the main form of pedagogy. Instead they incorporated more group tasks, alternate assessments, and created their own mathematical modeling problems that were relevant to their students' lives.

Keywords

Mathematics, Mathematical Modeling, Emergent Bilinguals

Cover Page Footnote

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Introduction

The introduction of the Common Core State Standards for Mathematics (CCSSM) in 2010 witnessed an increased emphasis on mathematical modeling as it is stated both as a content and mathematical practice standard. Although it has been over a decade since the introduction of these standards, classroom implementation of mathematical modeling still remains a challenge since "most teachers who are now required to teach mathematical modeling, or, in some cases, to teach mathematics through mathematical modeling have not themselves experienced modeling nor have studied it in a systematic way" (Phillips, 2016, p. 249). However, developing new content and styles of pedagogy that they have never seen before takes time and significant effort on the part of practicing classroom teachers who have limited time and/or resources for trying innovative practices. Thus, teachers fall back on what they know and have done in the past because they perceive those things to be less time consuming and more successfully implemented (Corrêa, 2021). One way of helping teachers to incorporate new content such as mathematical modeling is through professional development.

The research featured in this paper was part of a small, funded professional development project, in the rural, western United States in which mathematics teachers were engaged in utilizing and creating mathematical modeling problems and enacting them in their classes through highly interactive classroom environments. Due to a high number of emergent bilingual students in the geographical area serviced by this grant, particular emphasis was on fostering both language and content development. The professional development comprised a two-part summer institute augmented with three additional workshops throughout the year. Gutierrez's (2009) theoretical framework on equity was used to examine teachers' implementation of mathematical modeling activities as a result of participation in a year-long professional development project. This research study sought to address the question: *How did classroom pedagogy and content change as a result of this professional development*? This question was examined through a qualitative study that comprised pre-and-post surveys, individual and focus group interviews.

Review of Relevant Literature

For many students, mathematics can be seen as dry, boring, and irrelevant. This is particularly true when mathematics is taught by focusing on a series of computations and with an emphasis on exams (Venezky, 2018). When this is the focus of pedagogy, students often have little understanding of the link between mathematics lessons and future applications of mathematics content. As Wees (2011) noted, "too little of the mathematics we learn is situated in a useful context for students, and as a result, our mathematics curriculum is unmotivated in the eyes of the people learning it" (para. 3). As a result, students have a limited perspective on the role of mathematics in the world (Lim, 2016), yet mathematics permeates their daily lives. Thus, one of the eight Common Core Standards for Mathematical Practice, "Model with Mathematics," states that "mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (National Governors association Center for Best Practices and Council of Chief State School Officers, 2010, p. 7).

Mathematical Modeling

One suggested way to achieve the goal outlined in the "Model with Mathematics" Standard for Mathematical Practice is through the use of mathematical modeling problems. The Guidelines for Assessment and Instruction in Mathematical Modeling Education (2019) defined mathematical modeling as "a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (Garfunkel, et al, 2019, p. 8). This allows students to incorporate authentic "real-world" problems of interest with high cognitive demand. It is thought that these types of problems increase proficiency as well as engage students in mathematics (Corrêa, 2021). When engaging in the modeling process, students address authentic real-world situations by problematizing and translating a problem situation into mathematics, working it out mathematically, translating it back into the original context, testing the result, revising, and then communicating their findings to others (Bonotto, 2013; Lesh & Doerr, 2003). These problems are open-ended with multiple solution paths and can be tied to students' everyday lives.

In order to access the underlying mathematics in mathematical modeling problems, students have to navigate the general language of the problem situation first and the academic language of mathematics to be able to interpret the problem and extract the abstract mathematics in order to find a solution. This can be challenging for many students and in particular for emergent bilingual students who are learning both new content and language. Emergent bilingual students generally find word problems difficult to comprehend (Bernado, 2005, Martiniello & Wolf, 2012). However, with the right kind of comprehension strategies, emergent bilingual students can successfully engage in problem solving (Orosco & Abdulrahim, 2018). Thus, "it is critical to set both content and language objectives," (Hill & Flynn, 2006, p. 22), when teaching emergent bilingual students. Thus, it can be said that with the right kind of scaffolding, emergent bilingual students can learn both new content and language while engaging in mathematical modeling (Anhault, 2014). Unfortunately, teachers who may already find it challenging to incorporate these types of problems both mathematically and pedagogically may be particularly ill-equipped to address the needs of emergent bilingual students (Mitchell, 2019).

Emergent Bilingual Learners

Emergent bilingual learners are a growing population of students in US schools, with percentages in public schools ranging from 0.8% in West Virginia to 19.2% in California in 2017 (Hussar et al., 2020). Historically, emergent bilinguals are funneled into lower-level courses, typically taught with very procedural methods and with limited access to interesting mathematics with high cognitive demand that have relevance to the world around them (RAND, 2002). As a result, this population includes a disproportionately high portion of students who are not reaching proficiency levels in mathematics (National Center for Education Statistics, 2020). However, research suggests that emergent bilinguals can benefit from engaging in real-life complex mathematical tasks. However, most emergent bilinguals have unique real life experiences which tend to be foreign to their teachers'. Thus, many teachers find it challenging to facilitate real-life complex tasks that are relevant to the lives of emergent bilingual students.

Since conceptual understanding helps students succeed in mathematics it is imperative that emergent bilinguals are exposed to problems that are more complex than can be learned through rote memorization and series of procedural computations. Mathematical modeling including tasks with high cognitive demand has been reserved for students in advanced classes that have disproportionately lower numbers of emergent bilinguals. Thus, teaching mathematical modeling in all classes may provide access for many students to advanced mathematics, leading to engagement and meaningful mathematical learning (Scott-Wilson, et al., 2017), increased confidence in mathematical abilities and improved attitudes towards mathematics (English & Watters, 2004; Lesh & Doer, 2003). However, for modeling tasks to benefit all students, particularly emergent bilinguals, they must be implemented with pedagogical strategies that support students' learning.

Pedagogy that Supports the Teaching of Mathematical Modeling

A study by Wethall (2011) on the impact of mathematical modeling on student learning and attitudes showed how students actively engaged with each other and as they focused on solving the problems. According to Wethall (2011), "this can be partially attributed to the context of mathematical modeling problems that allowed students entry into mathematical problem solving and led them to discuss possible solution strategies with peers (p.55)" which is especially true of emergent bilingual students. Studies show that high concentrations of emergent bilinguals, when immersed in classrooms where they are engaged in small group work involving authentic problem solving and innovative use of technology, two important components of mathematical modeling, showed significant gains on mathematics achievement, not only in computation, but also problem solving and language acquisition (Trautman & Howe, 2004). Thus, mathematical modeling, by its nature, is especially well poised to help emergent bilingual children succeed in mathematics when pedagogy supports communication and use of technology.

Technology

Technology use is integral to mathematical modeling. Not only does it broaden "the range of approaches that can be taken to solve certain mathematical problems but also the types of situations that can be investigated by providing the possibility of using solution strategies which would not be accessible otherwise" (Greefrath & Reiß, 2013, p. 446). In addition, technology enables students to work on more realistic applications of mathematics with real data (Teague, et al., 2016). Perhaps, most importantly, research has shown that emergent bilingual students benefit from the use of technology in the mathematics classroom (Kim & Chang 2010; Lopez, 2010).

Technology also may play a role in developing dynamic thinking in students solving mathematical tasks (Arzello, et al., 2012). This dynamic thinking allows students to alter and refine their solution paths during the modeling process when faced with new data or examining the reasonableness of their models. While the focus of instruction must remain on mathematics, rather than technology (Office of Superintendent of Public Instruction [OSPI], 2008), technology can be used in pedagogically valuable ways to support or refute conjectures during the modeling process. Thus, technology is used as a tool for students to generate, examine, and prove conjectures. This is a pedagogical shift from a view of mathematics as something students observe to something that they do and invent. This shift requires supporting teachers as they change their fundamental ideas about mathematics, pedagogy, assessment, student learning, and use of classroom time (Lampert, 1998; Wiske, 1990).

Professional Development for Supporting Mathematical Modeling

Effective professional development engages teachers in investigating differences in student learning and comprehension of mathematics (Borko, 2004), considers content, pedagogy, and assessment of mathematics (Wilson, et al., 1996), has regular opportunities for discussion and sharing strategies (Featherstone, et al., 1993), is job embedded and available just in time (Darling-Hammond & Rothman, 2015) and develops technological competence in how to use technology for reasoning and problem solving in mathematics (Grassl & Mingus, 1997). There are also longstanding recommendations that adult and professional learning opportunities be directly related to participants' current responsibilities, involve solving meaningful problems, require collaboration with colleagues, engage learners in organizing needed information, and involve planning for specific applications (Chickering & Ehrmann, 1997; Grabinger, et al., 1997). It was with these recommendations that we developed the teacher professional workshop described herein.

Theoretical Framework

We use Gutierrez's (2009) theoretical framework on equity to examine teachers' implementation of mathematical modeling activities as a result of participation in a year-long professional development project. According to Gutierrez's (2009) "equity is ultimately about the distribution of power--power in the classroom, power in future schooling, power in one's everyday life and power in a global society" (p. 5). In this framework, equity is conceptualized in terms of four dimensions along two axes. The Dominant axis has *Access* on one end and *Achievement* on the other and the two often operate in tension with one another. Access is about available resources which allow students to participate in mathematics. This could include such things as curriculum, teacher quality, availability of technology, and classroom culture that is conducive to learning (Gutierrez, 2009). On the other end of the dominant axis is *Achievement* which relates to class participation, standardized test scores, and participation in courses that lead to a solid base in mathematics. The dominant axis "measures how well students can play the game called mathematics" (Gutierrez, 2009, p. 6). The second axis, termed the critical axis, situates *Power* on one end and *Identity* on the other. Examples of power include who gets to talk in a classroom, who has opportunities to engage in interesting mathematics that can be used to explore and solve societal problems, and more. Identity involves attention to race, gender, ethnicity and how these are valued in settings such as classrooms. The critical axis attends to students' lived realities and resources to help them become citizens, capable of changing the system. Gutierrez (2009) suggests that equity is possible only if all four dimensions are considered.

Using Gutierrez's equity framework, we sought to examine changes in teachers' classroom practice when incorporating mathematical modeling and the pedagogical strategies that support its successful application. In addition, we wanted to examine the impact of these strategies on emergent bilingual students. In order to address these questions, we reviewed prior research and examined teachers' data from the year-long professional development project, and drilled down to specific case studies of participating teachers, to get a clearer picture of factors which contributed to successful implementation of project experiences.

Design of Professional Development

We conducted a two-week summer professional development institute with three academic-year follow up days (Figure 1) structured as follows:

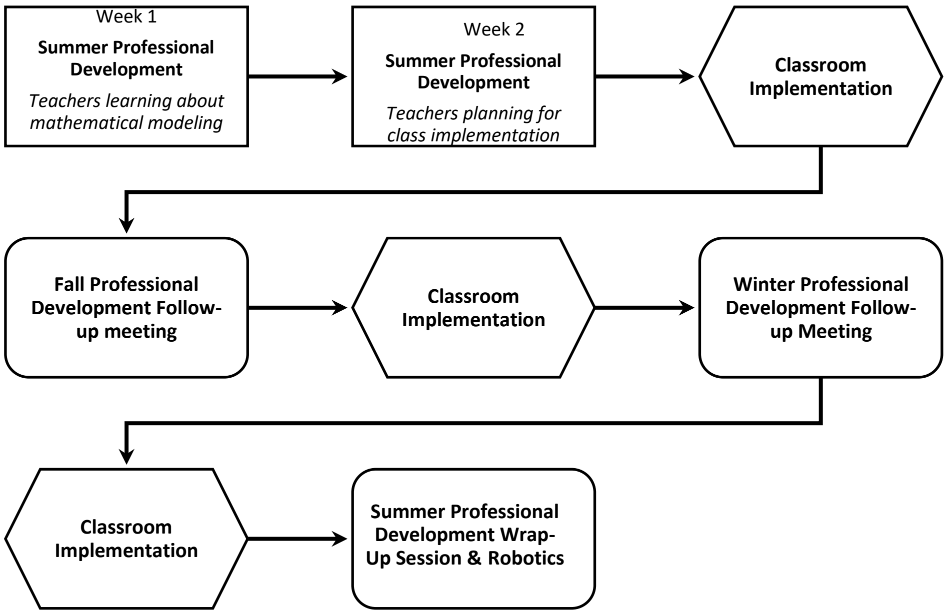

Figure 1: Summary of the Professional Development Structure

Week 1: Teachers as Learners. In order to address all aspects of Gutierrez' (2009) equity framework as well as the research on professional development, we immersed teachers in a series of boundary spanning, culturally relevant, and "real-world" mathematical modeling problems using multiple and different types of pedagogy. Solving these types of realistic problems involves building and analyzing models which requires the modeler to think mathematically and apply sophisticated mathematical knowledge and skills (Verschaffel, et al., 2000). These problems are open-ended with multiple solution paths which can be intimidating to many teachers to both create and to teach when compared to typical mathematics problems with one right answer. Thus, we designed the workshop so that teacher participants experienced solving these types of problems as learners while the workshop facilitators used whole-group, small-group, and individual instruction. At the completion of each problem-solving activity, small group and whole class discussions focused on decomposing the mathematics embedded in the problem, examining cross-disciplinary connections to other subject areas and societal concerns, scaling problems to grade levels, pedagogical aspects of group work, and formative assessment of student learning when using group work. We also engaged teachers in discussion of the merits of utilizing a collaborative learning environment to foster communication and enable emergent bilingual students to engage in interesting authentic mathematical applications. In this way, we addressed all the dimensions of Gutierrez's (2009) equity framework as well as best practices for professional development of mathematics teachers.

Week 2: Lesson Creation: Utilizing what they learned in week one, teachers began creating culturally relevant lessons and mathematical modeling projects for use in their classrooms. They went through an iterative design process where they created, tested, refined, and then presented their classroom implementation ideas to their peers. This process was meant to build a professional learning community and a corpus of usable problems for teachers to share. *Academic Year:* We conducted two supportive one-day professional development sessions during the year (October, March) and a 3-day workshop (June). This third session included evaluating the project, showcasing accomplishments and discussing implementation and pedagogical best practices of mathematical modeling.

Participants

The teachers in this professional development were from 14 different rural school districts. Seven of the school districts were classified as High-needs Local Education Agencies (HLEA). Most of these teachers are isolated geographically as their schools are surrounded by farms and mountains and a significant part of their student population are either emergent bilinguals or Native American. For the professional development workshop, the teachers met with us at a centrally located district, with the teachers living farthest driving 5 hours to the site. Of the 25 teachers who took part in this professional development, 3 were elementary school teachers, while 15 were middle school teachers and 7 were high school teachers. Approximately one-third (36%) had 0 to 5 years of experience and a little less than one-third (28%) had over 15 years of experience. Twenty percent (20%) of the teachers had 6 to 10 years and 15% reported 11 to 15 years of teaching experience.

Methodology

Data for this qualitative study consisted of a survey on teacher understanding and application as well as interviews. To improve the response rate, surveys were administered during face-to-face meetings. The pre-survey was administered on the first day of the summer institute before teachers engaged in any professional development activities. This was meant to determine baseline knowledge of the teachers. The post survey was administered at the end of the wrap-up session after completing all the session activities. The survey had five items that collected teacher demographics, nine items asking respondents to rate their expertise and comfort level with content and pedagogical strategies on a scale of 1-10, and five open-ended items. Means of the scale ratings of the teachers' expertise and comfort levels for both the pre and post survey were calculated and the difference between the pre and post mean for each item was calculated. To triangulate data from the surveys and explore changes in teachers' understanding and application of the knowledge and skills learned, 60- minute interviews were conducted in May and June at the teachers' schools. Interview responses were transcribed and analyzed using Quantitative Narrative Analysis (QNA) (Franzosi, 2010). QNA is a methodological approach to text responses that allow researchers to structure the information in ways that make it possible to perform a statistical analysis of the information.

Results

Teachers' post-survey responses indicated a trend towards increased understanding and application (Table 1). For example, teachers indicated they were more comfortable and gained expertise adapting lessons for cultural relevance with Native American/Latino/rural students, and using technology to produce mathematical models. This speaks to both axes of Gutierrez (2009) equity framework by addressing access to quality teachers and technology on the dominant axis and identity and power on the critical axis.

Table 1: Pre/Post Means on Teachers' Changes in Understanding & Application Survey

Please read each statement and on a scale from 1 (lowest) to 10 (highest), rate your expertise and comfort level with:	**Pre Mean**	**Post Mean**	**Delta**
Adapting lessons for cultural relevance with Native Americans	5.4	7.8	+2.4
Using problems that involve mathematical modeling	6.6	5.4	-1.2
Using technology tools to produce mathematical models	5.1	7.4	+2.3
Adapting lessons for cultural relevance with Latino populations	4.0	6.8	+2.8
Adapting lessons to rural Local community	3.9	6.4	+2.5

The open-ended questions on the pre- and post-survey provided further insight into changes in participants' use of the strategies for emergent bilinguals and the incorporation of cultural/local relevance. In the pre-survey, some teachers reported using strategies to encourage and support communication between and among students; the most common strategy reported was *group work*; both large and small groups. Partner work was also used by teachers and described in several ways: *Think-Pair-Share*, *Elbow Partners*, and *Turn-and-Talk* being the most often used collaborative learning strategies. Teachers also reported peer evaluation of work, some using rubrics to assist students, and to encourage communication about mathematics related work.

Post survey responses included additional strategies for emergent bilinguals and for fostering discourse which addressed the power end of the critical axis (Gutierrez, 2009). One teacher shared,

> *When creating groups, I like to make sure these students [Emergent Bilinguals] are in groups that I know will work with them. Also, I try to make sure that in the project these students will still have an access point into the problem so that they feel they are contributing to the group.*

Responses illustrated the teachers' application of the ideas from the workshops: "*As much as possible, I try to assign complex problems that require students to problem solve and work together to come up with an answer*." Another teacher shared, *"I really believe hands-on and realistic are key in improving my students' engagement."* In an interview, one teacher reflected on his experience implementing one of the professional development activities in his classroom:

> *The problems have so many entry points; essentially this will be a math lesson on graphing. But this guy who can't or doesn't care about a graph can put [the robot] together and now he is super important in his group because he has a skill that they need. Someone else might get how to do the coding. Then there is the whole trial and error part about getting the robot to go where you need it to go. There are so many places that students can use skills to be part of the solution. Then when it is all over, they need to figure out how to model that solution with a graph and an equation.*

Some teachers adapted mathematical modeling activities from the workshops for their classrooms such as using water flow to illustrate rate of change, paper plates and paper clips for a zip line activity that illustrates gravity and friction, and an introduction to fractions and measurement using small robots. Teachers also collaborated together to design activities accessible to all students that provided an entryway to mathematical modeling such as 1) modeling the mathematics used to calculate perimeter by developing an engineering plan prior to the activity, 2) modeling overpopulation by growing beans and learning how limited space eventually leads to death, 3) modeling deforestation and studying the effects on organisms due to loss of habitat by building a Mexican forest with real vegetation and plastic animals/birds that live in the Mexican forests.

In an interview, one teacher described how he applied what he learned as a participant. He teaches in a small rural school approximately two hours from the site of the workshop in an area which is surrounded by orchards. Most of the students are emergent bilinguals whose parents work in the orchards or packing shed. He had been a Math Specialist for grades K-5 for six years and worked with struggling math students and special education students. He had his students work in groups and discuss their learning which "was particularly helpful to my ELL students" and which addresses the power end of the critical axis outlined in Gutierrez (2009). The sharing of ideas and the group discussions also allowed individual students to "pick-up skills or knowledge from group members that they did not have in a non-threatening way." He implemented pedagogical strategies he learned in the workshop where students examined the work of other groups and received feedback. He used this as a formative assessment and allowed students to re-address their solutions and make any changes they thought necessary to their work. He shared, "The changes I made in how the learning is structured, is directly inspired by the workshops."

This teacher also explained how he brought local relevance into the math lessons by talking about the three bridges that surround their town. "I wanted to use something that they see every day. Every day the bus goes over the bridges and I got them to think about it." Children in his 3rd and 4th grade math class built bridges with toothpicks and marshmallows which provided an opportunity to talk about right and obtuse angles as well as applied mathematical modeling. He excitedly proclaimed, "Every kid wanted to participate. They didn't hang back!" He explained his reaction when he saw the increase in his students' test scores in mathematics: "I

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was impressed because my … students made 28 point gains but the average gain for one year is usually 16 points for 2nd grade. I saw the same pattern for 1st graders. I asked myself 'What did I do differently this year?' and I did five to six exercises this year using the workshop's techniques." The interviewee went on to explain that the large increase in math achievement was realized by his students that were labeled "low performing" and the students designated as special education students. These gains in student achievement speak to the dominant access of Gutierrez's (2009) equity framework.

Discussion

Teachers immersed in the professional development described in this project developed a willingness to move from traditional lecture and replication as their main form of pedagogy to attempting to incorporate more group tasks, alternate assessments, and creating their own mathematical modeling problems that were relevant to their students' lives. The survey results as well as data collected regarding teachers' classroom implementation showed they intentionally integrated locally relevant mathematical modeling problems which improved student motivation and engagement as well as applications of mathematical modeling in their classroom. Although they were successful in making the lessons locally relevant, they had limited success in creating culturally relevant problems (Hyland, 2009; Aguirre et al., 2013; Neri, 2019), as most of them did not come from the same cultural communities as their students. Since the research team was not located in the local community, it was challenging to help the teachers find culturally relevant material due to lack of cultural awareness of the local community.

The study found that teachers demonstrated ideas learned in the professional development project through incorporating group work, developing problems with multiple entry points so that all students could access the complex mathematics, and creating assessments that included group discussions of student work. All of these things fostered more collaboration and communication among students. Participating teachers felt that these techniques were instrumental in the increases they saw in their emergent bilingual students' test scores and proficiency in mathematical language acquisition. The teachers discovered the benefits of intentionally grouping students for effective learning and for lowering the affective filter for emergent bilingual students.

It should be noted that although these results are promising, the author is not suggesting it can be easily replicated. These study results were directly related to these particular participants and their teaching situations. However, this study does support similar results (Wethall, 2011; Sofroniou & Poutos, 2016; English & Watters, 2004; Lesh & Doer 2003) which show that moving away from traditional lecture and incorporating group tasks as well as focusing on mathematical modeling can improve students' mathematical understanding and connect mathematics to students' everyday lives. The study data also supports research that shows that more opportunities to have structured academic talk (Francis et al., 2006) can benefit both the mathematical success as well as the language acquisition for emergent bilingual students.

Conclusion

Professional development projects such as the one featured in this paper can be a good way to introduce classroom teachers to important mathematics content such as mathematical modeling that they may not have received in teacher preparation programs (Phillips, 2016). New mathematical knowledge combined with innovative pedagogical practices and alternate assessment strategies can be very intimidating to implement all at once. Professional development projects where teachers have opportunities to try their learning out in their classroom (Darling-Hammond & Rothman, 2015)) and then discuss challenges and successes with other teachers and reflect on their learning may ease teachers' fears of trying new strategies.

Since mathematical modeling is relatively new to most practicing teachers and culturally relevant mathematics instruction is a challenge to implement, there is a need for research that focuses on the role of mathematical modeling as a vehicle for culturally relevant mathematics. In particular, how can mathematics teachers facilitate mathematical modeling in ways that foster culturally relevant mathematics instruction so that *all* students can "play the game called mathematics" (Gutierrez, 2009, p. 6)?

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