1-1-2007

Spinoza and Gödel: Causa Sui and Undecidable Truth

Martin Zwick
Portland State University, zwick@pdx.edu

Citation Details
Martin Zwick. "Spinoza and Gödel: Causa Sui and Undecidable Truth" [Post-print]. North American Spinoza Society Monograph Vol. 13 (2007) p. 46 - 52

SPINOZA and GÖDEL: *CAUSA SUI and UNDECIDABLE TRUTH*

Martin Zwick, Portland State University, Portland OR 97207
zwick@pdx.edu

Spinoza distinguishes between causation that is external, as in A causing B where A is external to B, and causation that is internal, where C causes itself (*causa sui*), without any involvement of anything external to C. External causation is easy to understand, but self-causation is not. This note explores an approach to self-causation based upon Gödelian undecidability[1] and draws upon ideas from an earlier study of Gödel's proof and the quantum measurement problem (Zwick, 1978)."

Spinoza often explains causation with mathematical – especially geometrical – examples, since for Spinoza physical entailment is equivalent to logical entailment. That is, the order of things is the same as the order of ideas. This might be similar, in Aristotelian terms, to saying that material cause is equivalent to formal cause, the two being the same but under different attributes, material cause being under the attribute of extension and formal cause being under the attribute of thought.

How can something be self-caused? Spinoza give at least two explanations: (1) the self-causation of C is possible if C's essence necessarily involves its existence; (2) self-causation is possible if C is the totality of what exists, so there is nothing external to C that could be its cause or the cause of things happening within it. Argument (1) is not immediately convincing. One might challenge it by denying that there *is* anything whose essence necessarily involves existence, or one might insist, with the existentialists, that existence precedes essence. Argument (2) is more plausible. In systems-theoretic terms, systems (Spinoza's modes) have environments, so events in systems always reflect at least partially the influence of the environment. Although events also reflect internal conditions and thus all systems are partially *causa sui*, external causation predominates since the environment is greater and more powerful; as Spinoza notes, no system is the equal of its environment. One might challenge this second argument by denying the coherence of speaking about the totality of everything,[2] but if this is allowed, then such an infinite system has no environment, and all that happens within it can be affected only by itself and not by anything external to it. This is Spinoza's argument.

The connection to Gödel is via the equivalence, for Spinoza, of causal and logical entailment, which is impacted by Gödel's discovery of the limitations of logical entailment. Gödel showed that in formal systems of sufficient complexity, i.e., that encompass arithmetic and thus necessarily also a notion of infinity, there are propositions ("well-formed formulae") that are not decidable. (Technically, these propositions are outside the formal system, but they obey its rules for syntactic correctness.) Gödel proved this by constructing such a proposition. This proposition – call it G – can be interpreted at two different levels: at the "base" level of number theory (NT), G is about numbers, but at the "meta" level of meta-number theory, G is about the provability of a particular proposition in NT. The particular proposition is G itself, that is, G is self-referential. Its meta-NT meaning is "G is not provable (decidable) within the formal system." This self-reference is not paradoxical, as is the idea of a class of all classes that

Martin Zwick. "Spinoza and Gödel: Causa Sui and Undecidable Truth" *North American Spinoza Society Monograph* Vol. 13 (2007) p. 46 - 52

are not members of themselves or of a barber who shaves everyone who doesn't shave himself. These other instances of self-reference are paradoxical because one runs into trouble if one asserts either their truth or their falsity. By contrast, one is blocked from asserting the falsity of G, but not from asserting its truth. If G is false one obtains a contradiction: if false, G is decidable and thus true. But if G is true, it is undecidable, and this does not lead to a contradiction, only to the separation of the ideas of truth and decidability. This separation was the revolutionary discovery of Gödel, since prior to his work it was assumed that anything that is true must be provable within the formal system in which it is a well-formed proposition. But the separation of truth from provability which follows from Gödel's proof is logically acceptable, once one gets over the shock of it. (It is like the separation of determinism from predictability implied by the mathematics of chaos.) The self-reference involved in Gödel's construction of G is not vicious or paradoxical, but instead resolves itself satisfactorily. Its resolution is simply that G must be true, since otherwise one would obtain a contradiction. This demonstration that G is true is not a proof *within* the formal system NT; it is a proof at the meta-NT level. Within NT, G is undecidable.

What has Gödel's proof to do with Spinoza's ideas about causation? If one accepts Spinoza's equivalence of physical entailment and formal (logical) entailment, then a physical system, a mode of Substance under the attribute of extension, when viewed instead under the attribute of thought is a formal system, where causation maps onto proof. That is, A causing B in a physical system (external causation) is the same as A being the proof of B in an equivalent formal system.

There are at least two differences, however, which might be noted between physical entailment and formal entailment. One difference has to do with origins; the other with time; the two are related.[3] The first is this: if one inquires about the proof of A, one regresses back to the axioms and definitions of the formal system, but there one stops, since axioms and definitions are not externally justified; formally, they are *causa sui*. By contrast, for events that occur in Substance, according to Spinoza, there is no First Cause. Substance is not only infinite but eternal; it has no "starting point." One presumes that Spinoza needs this to be true of Substance also under the attribute of thought. This suggests a question: given Spinoza's geometrical analogies, which are conceived under the attribute of thought, what corresponds to geometry's definitions and axioms under the attribute of extension? One can imagine physical entailment without a beginning, but one cannot imagine formal entailment without a beginning. Perhaps this poses a problem for Spinoza's equivalence between the order of things and the order of ideas.

The second difference arises from the fact that causal events flow in a unidimensional time, but there is no unidimensional path from the axioms to the theorems. The paths from axioms to theorems are like multiple dynamic trajectories that coexist. Or, one could regard all the theorems of a formal system as coexisting simultaneously "under the aspect of eternity." There is actually no difference here between physical and formal systems. In the physical realm also, the simultaneous existence of multiple paths of causal entailment reflects a perspective under the aspect of eternity. Such a view is found in physics in the idea of a 4-dimensional space-time in which all possible trajectories can

be represented, and in the idea of a vector field, defined in space-time, which represents all potential dynamical behaviors of a system, and in which a particular path represents a single trajectory actualized in time. The iterative graph of a discrete automaton, which displays all potential states and their transitions and not merely one particular temporal trajectory that is realized, is also a view of dynamics under the aspect of eternity.

To summarize the discussion so far: the equivalence for Spinoza of physical and formal entailment means that external causation in physical entailment is equivalent to proof in formal entailment. Just as causation is the action of laws of nature, proof is the action of rules of inference. Further, and this is the radical and critical step: the *existence* of a physical entity or condition corresponds to the *truth* of some proposition. Finally, invoking Gödel, an entity or condition (C) that is *causa sui*, i.e., that does not result from external causation, is like a proposition (G) that is true but not decidable. This schema is summarized in Table 1.

Table 1. Equivalences between the realms of extension and thought

Realm of extension	*Realm of thought*
physical entailment	formal entailment
laws of nature	rules of inference
external causation	proof
physical existence	truth
causa sui (C)	true but not provable (G)

Gödel constructed a proposition G that is true but not provable, which is equivalent to a physical condition C that is *causa sui*. Similarly, in the realm of thought, G is logically self-entailed. Its "essence" (its meta-NT meaning) requires its "existence" (its truth). However, the difficulty mentioned above must not be forgotten: axioms, which are assumed to be true, do not have a physical equivalent since Substance is eternal. So one might ask: *must* every true proposition have a physical equivalent? Given G, must some equivalent C necessarily exist? Spinoza's parallelism of physical and formal entailment plus his view that infinite Substance contains all possibilities suggests that the answer should be "yes," so G not only *could* be equivalent to some C that is *causa sui*, but some such C *must* in fact exist.

Of course, for Spinoza, it was *totality* that is *causa sui*. Neither G nor its equivalent C qualifies as such. G is a singular thing – a mode perhaps. It is far from the totality of what is. It has an environment that includes all of the theorems. But perhaps the matter is not so simple. There is a sense in which the entire formal system is "in" G. Since G says at a meta-NT level that it is not decidable, the definitions, axioms, and rules of inference of the system must somehow be included in it, at least implicitly, in order that what is decidable is fully defined. If definitions, axioms, and rules of inference are included in G, all theorems are also at least implicitly included. So from one perspective, G is just a single proposition, but from another perspective, it embodies the whole. The whole of what is decidable is infinite, and to the extent that G embodies this whole, it is an incarnation – or whisper – of the infinite. To use a mundane analogy, it is like a part of a hologram which in a limited sense includes the whole. This analogy is imperfect: *all* parts of a hologram reflect the whole, but all propositions do not. G was constructed in a

special way. Its self-reference is critical to its capacity to reflect the whole, and not all propositions exhibit such self-reference.

One might say that G is not really *causa sui* since it reflects, hence requires, the entire formal system. Still, G is not entailed in the way that theorems are entailed by axioms, definitions, and rules of inference. In this sense, G stands alone. If "the whole" is taken to be the definitions, axioms, rules of inference, and theorems of the formal system, G is not a part of this whole, but is outside of it. And yet, paradoxically, G is completely dependent on this whole – indeed is conceivable only in its context. Like a Leibnizian[4] monad, G is at once solitary and a mirror of everything outside itself. By contrast, the theorems of the system, which are decidable, are very different: as Spinozistic modes, they depend utterly on their environment. And, although G is outside the formal system, it is not isolated, not only because it presupposes and mirrors the system, but also because it is generative, i.e., has consequences in new theorems derivable from it. Equivalently, in the realm of extension, facts that are *causa sui* causally entail additional facts. Formal system and physical reality are infinite in thought and extension, respectively, yet each is still infinitely augmentable. This augmentation is implicit, so immanence implies transcendence.[5] If, instead, one internalizes the full extent of this augmentation, i.e., if one sees eternity as comprising all that is true, not merely all that is provable, then one returns to the perspective of immanence. G and its consequences are part of eternity, an infinity incommensurably greater than the lesser infinity of the merely decidable.[6]

Given such a G which presupposes the whole despite being a mere proposition, perhaps Spinoza's system could be augmented with the idea of such a possibility. If one also takes seriously the criticality of self-reference for G, and thinks about Spinoza's analysis of modes as less or more complex, and hence less or more potent and free, perhaps one might conceive of a mode being sufficiently complex, especially in its self-reference, that it reflects the whole. Like Hegel but well before him, Spinoza's project was also the subsumption (incorporation and transformation) of religion by philosophy, and like Hegel he regarded mythic religion as an inferior form, appropriate only to the masses. Perhaps, armed with Gödel's finding, Spinoza might have seen in the possibility that a part could reflect the whole a philosophical echo of the Biblical statement that man was made in the image of God. G is *causa sui* by virtue of its exploitation of the laws of entailment. This calls to mind Spinoza's idea of salvation which requires mastery of the 2nd kind of knowledge – knowledge of the laws of entailment in the equivalent realms of extension and thought. Rising to the 3rd kind of knowledge via an intuitive grasp of the whole, a system can step out of the realm of these laws, into a meta-level domain, and thereby gain freedom. This step, in turn, has consequences within the causal realm.[7]

ACKNOWLEDGEMENTS

Deep thanks to Hugo DuCoudray, Patrick Dinsmore, Doug Donkel, John Hammond, Richard Hartman, Palmer Pardington, and Kirke Wolfe, of the Portland Philosophy-Religion group for many enormously stimulating, instructive, and enjoyable discussions, without which this note would never have been written, and for the happy anticipation of many future gatherings.

REFERENCES

Gödel, Kurt, *On formally undecidable propositions of Principia Mathematica and related systems*, translated by B. Meltzer, Basic Books, New York, 1962.

Frazer-Simser, Ben, "Spinoza, Gödel, and the Incompleteness of God: Spinoza's Transcendental Arguments in the *TdIE*". Presented at the American Philosophical Association meeting, Jan. 2004. (Article gratefully received, courtesy of F-S.) http://www.apa.udel.edu/apa/publications/proceedings/v77n3/public/grpprogram-wed-thurs.asp.asp .

Nagel, E. and Newman, J., *Gödel's Proof*, New York University Press, New York, 1960.

Spinoza, Baruch, *The Ethics and Selected Letters*, translated by Samuel Shirley, Hackett Publishing Company, Indianapolis, 1982.

Zwick, Martin, "Quantum Measurement and Gödel's Proof." *Speculations in Science and Technology*, vol. 1, no. 2, 1978. http://www.sysc.pdx.edu/download/papers/godel1.pdf .

---

[1] The "ontological proof" which both Spinoza and Gödel had distinctive versions of is not of concern here. Also, the link between Spinoza and Gödel proposed in this note differs from the connections between the two developed by Frazer-Simser in his paper, "Spinoza, Gödel, and the Incompleteness of God: Spinoza's Transcendental Arguments in the *TdIE*." Fraser-Simser's argument is that Gödel's proof implies that "our system for knowing God must be inherently incomplete." Any human conception of God is incomplete just as NT is incomplete; were this not so, God's infinite status would be denied. Or, to express the argument positively, "Spinoza's God is a concept that outstrips itself."

[2] For example, the idea of a set of *all* sets – call it S – is a well-known paradoxical notion, since P(S), the power set of S (the set of all subsets of S) is necessarily larger than S, and thus cannot be included in it.

[3] There is a third difference in that formal entailment, specifically the sequence of steps in the proof of a theorem, is discrete, while physical entailment is commonly seen as continuous. Yet formal entailment *could be* continuous, as in the dynamics of differential equations, and there are those who hold that physical reality is ultimately discrete. The properties of discrete and continuous systems are quite different from one another, but, to borrow an expression of Gregory Bateson, this is a difference that does not make a difference to the essence of the argument presented in this note.

[4] The names of Spinoza and Gödel are sometimes linked in the observation that Einstein and Gödel, who were good friends, had different philosopher heroes, namely Spinoza and Leibniz, respectively. Spinoza and Leibniz have been presented, most recently in Matthew Stewart's *The Courtier & the Heretic: Leibniz, Spinoza, and the Fate of God in the Modern World* (W.W. Norton, New York, 2006), as being poles apart; perhaps the work of Gödel brings them a little closer together.

[5] This is a kind of opposite of panentheism, in which immanence is subsumed by a prior and more encompassing transcendence. Here, immanence (for Gödel, NT) is prior to transcendence (meta-NT). To deploy the notions of immanence and transcendence in another way, related but more mundane, one might say that truth is to provability as transcendence is to immanence.

[6] This is reminiscent of Cantor's hierarchy of Alephs (different gradations of infinities).

[7] The narrative here associates G with human modes, but one could alternatively adopt the conception of Fraser-Simser, who applies Gödel's proposition to conceptions of God.