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Citation Details

Chang, R., & Leung, P. T. (2024). [Pre-Print] Electromagnetic fields and radiation from localized current source interacting with a cosmic axion field in the context of massive axion electrodynamics. *Physics Letters A*, 129701.

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Electromagnetic fields and radiation from localized current source interacting with a cosmic axion field in the context of massive axion electrodynamics

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Abstract

Electromagnetic fields from localized current source interacting with an oscillatory cosmic axion field are studied, accounting for the possibility of a finite photon mass. In particular, focus will be on the dipole radiation from static current sources. The Proca theory is generalized to include the axion field and solutions to the modified EM field equations are obtained to first order in the axion coupling parameter. Analysis of the interplay between the two vanishingly small parameters – the axion mass (via the coupling constant and the oscillating frequency) and the photon mass – is carried out with reference to possible future detection of the cosmic axion and/or the photon mass via electromagnetic interaction with the axion.

Key Words: Axion electrodynamics; Proca theory; photon mass

Introduction

As most of the cosmologists would agree, one of the most intriguing mysteries in the physical Universe is the large percentage of invisible non-baryonic matter whose presence is anticipated only through the gravitational effects it provides. Over almost a century, a large number of candidates has been proposed to account for this dark matter [1]. One promising solution to this mystery refers to the illusive axion, whose existence was first proposed in the 1970's to resolve the so-called CP conservation problem in strong interaction [2]. In 1983, Sikivie first proposed to detect this particle via a possible weak electromagnetic (EM) interaction coupled with the axion, based on a generalization of the conventional Maxwell theory with a Lagrangian to include a term $\sim \Theta \mathbf{E} \cdot \mathbf{B}$ [3]. Since then, many EM experimental investigations have been carried out for the search of the axion to include observation of possible unexpected low frequency EM radiations from static current source like the neutron star [2, 4].

While these axions are expected to have very light masses with a conventional mass range expected to be from a micro eV to pico eV, more recently an ultralight class of axions has also been proposed with a mass $\sim 10^{-22}$ eV [5]. This then leads to an interesting question of the possibility for one to consider also photons with a finite mass which has an upper limit $\sim 10^{-16}$ eV [6]. Although conventionally the photon is believed to be massless, experimentally one can only set a stringent upper bound for its mass via a large variety of experiments to include both dynamic and static EM setups [7]. In fact, a scalar component of the “massive photon” has also been proposed as a possible candidate for the dark matter in the literature [8].

It is the purpose of our present work to propose a “synergistic study” of these two illusive phenomena – the cosmic axion and the photon mass – both of which have undergone tremendous investigations in the past many decades. Specifically, we shall re-investigate theoretically the

EM interaction between a static magnetic dipole and a cosmic oscillatory axion background, via a generalization of the well-known axion electrodynamics (AED) to a massive theory (m-AED) in which the photon can acquire a finite mass, arriving at an axionic Proca theory [9]. In the literature, this Proca theory has been applied recently to the study of topological superconductor in which a massive axionic Lagrangian was considered for the calculation of the photon propagator [10]. Here, our focus is on the interplay of both the axion and photon masses to investigate how these will affect the EM radiations emitted from the interaction between a localized current source and a background oscillatory axion field [4, 11, 12].

Theory (m-AED)

We start with the Lagrangian for massive axion electrodynamics (in vacuum) as follows (in Gaussian units):

$$\mathcal{L} = -\frac{1}{16\pi} F_{\beta\gamma} F^{\beta\gamma} - \frac{1}{c} J_\beta A^\beta + \frac{\mu^2}{8\pi} A_\beta A^\beta - \frac{1}{8} g_a \Theta \varepsilon^{\beta\gamma\delta\rho} F_{\beta\gamma} F_{\delta\rho}, \quad (1)$$

where $\mu = \left(\frac{\hbar}{m_\gamma c}\right)^{-1}$ is the inverse Compton wavelength for the massive photon, g_a is the small axion-photon coupling constant, and $\Theta = \theta_0 e^{-i\omega t}$ is the background cosmic harmonic axion field where we have assumed the source to be at the origin in the rest frame of the axion and θ_0 is a constant [4, 11]. Hence the dynamics of the axion is not included in Eq. (1) in this case. Note that incorporating this dynamics will only lead to an effective current which is of an order g_a smaller than the external EM source current, and hence will be a second order effect [2, 12]. Straightforward variation of the action integral with respect to the 4-potential leads to the following massive axionic field equation:

$$\partial_\alpha F^{\alpha\beta} - \frac{4\pi}{c} \mathbf{J}^\beta + \mu^2 A^\beta + 2\pi g_a \varepsilon^{\alpha\beta\gamma\delta} \partial_\alpha (\Theta F_{\gamma\delta}) = 0, \quad (2)$$

where the Lorenz gauge ($\partial_\beta A^\beta = 0$) has been fixed due to charge conservation ($\partial_\beta j^\beta = 0$) [13].

Note that this m-AED has the same gauge-fixing property as in the Proca theory as is clear from Eq. (2) since the 4-divergence of the last term is zero. The above leads to the following set of 3D axionic Proca equations:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 4\pi\rho - \mu^2\phi - 4\pi g_a \nabla\Theta \cdot \mathbf{B} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} &= 0 \\ \nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi}{c} \mathbf{J} - \mu^2 \mathbf{A} + 4\pi g_a \left[\frac{1}{c} \frac{\partial \Theta}{\partial t} \mathbf{B} + \nabla\Theta \times \mathbf{E} \right] \end{aligned} \quad (3)$$

In the following, we shall calculate the fields generated by a localized source perturbatively to first order of the axion-photon coupling constant treating the Proca solutions as the zeroth order fields. This is justified at least for ultralight axions since it is known that the coupling constant $g_a \sim 3.86 \times 10^{-19} m_a$ [14], where m_a is the mass of the axion in eV [15]. Hence we assume the following perturbative expansions:

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \mathbf{E}_0(\mathbf{r}, t) + \mathbf{E}_1(\mathbf{r}, t) + O(g_a^2), \\ \mathbf{B}(\mathbf{r}, t) &= \mathbf{B}_0(\mathbf{r}, t) + \mathbf{B}_1(\mathbf{r}, t) + O(g_a^2), \\ \mathbf{A}(\mathbf{r}, t) &= \mathbf{A}_0(\mathbf{r}, t) + \mathbf{A}_1(\mathbf{r}, t) + O(g_a^2), \\ \phi(\mathbf{r}, t) &= \phi_0(\mathbf{r}, t) + \phi_1(\mathbf{r}, t) + O(g_a^2). \end{aligned} \quad (4)$$

Following previous works as described in [4, 12], we start by ignoring the spatial gradient terms of the axion field in Eq. (3) which is obvious in our choice of frame; and in general is also justified due to the de Broglie wavelength of the axion being much greater than the oscillation

wavelength [12]. Hence with (4) in (3), we obtain each of the zeroth and first order fields satisfy the following equations:

$$\begin{aligned}
\nabla \cdot \mathbf{E}_0 &= 4\pi\rho - \mu^2\phi_0 \\
\nabla \cdot \mathbf{B}_0 &= 0 \\
\nabla \times \mathbf{E}_0 + \frac{1}{c} \frac{\partial \mathbf{B}_0}{\partial t} &= 0 \\
\nabla \times \mathbf{B}_0 - \frac{1}{c} \frac{\partial \mathbf{E}_0}{\partial t} &= \frac{4\pi}{c} \mathbf{J} - \mu^2 \mathbf{A}_0
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\nabla \cdot \mathbf{E}_1 &= -\mu^2\phi_1 \\
\nabla \cdot \mathbf{B}_1 &= 0 \\
\nabla \times \mathbf{E}_1 + \frac{1}{c} \frac{\partial \mathbf{B}_1}{\partial t} &= 0 \\
\nabla \times \mathbf{B}_1 - \frac{1}{c} \frac{\partial \mathbf{E}_1}{\partial t} &= -4\pi i g_a k \Theta \mathbf{B}_0 - \mu^2 \mathbf{A}_1
\end{aligned} \tag{6}$$

where $k \equiv \omega/c$.

The Green Dyadic solution

Next we solve the system in (5) and (6) applying the Green dyadic method assuming all fields and sources are harmonic in time. Let the characteristic frequencies for the source and the first order field be ω_0 and ω_1 , respectively. Note that the general results established here cover the special case of static current source by simply setting $\omega_0 = 0$, a case we shall focus on in the following study.

For the unperturbed system in (5), we have previously worked out the Proca Green dyadic and the results can be expressed as follows [16]:

$$\mathbf{E}_0(\mathbf{r}) = \frac{4\pi i k_0}{c} \int \mathbf{G}_e(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') d^3 x' \tag{7}$$

$$\mathbf{B}_0(\mathbf{r}) = \frac{4\pi}{c} \int \mathbf{G}_m(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') d^3 x' \tag{8}$$

with the dyadic given by:

$$\mathbf{G}_e(\mathbf{r}, \mathbf{r}') = \left(\mathbf{I} + \frac{1}{k^2} \nabla \nabla \right) G_0, \quad (9)$$

$$\mathbf{G}_m(\mathbf{r}, \mathbf{r}') = \nabla \times (\mathbf{I} G_0), \quad (10)$$

where $G_0(\mathbf{r}, \mathbf{r}') = \frac{e^{i\beta_0|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$, and $\beta_0 = \sqrt{k_0^2 - \mu^2}$, $k_0 = \omega_0/c$. Hence for a given localized current source, Eqs. (7) – (10) will yield the unperturbed Proca fields (zeroth order in g_a). Then from Eq. (6), the first order fields can be obtained as:

$$\mathbf{E}_1(\mathbf{r}) = 4\pi g_a k k_1 \theta_0 \int \mathbf{G}_{e1}(\mathbf{r}, \mathbf{r}') \mathbf{B}_0(\mathbf{r}') d^3 x', \quad (11)$$

$$\mathbf{B}_1(\mathbf{r}) = -4\pi i g_a k \theta_0 \int \mathbf{G}_{m1}(\mathbf{r}, \mathbf{r}') \mathbf{B}_0(\mathbf{r}') d^3 x', \quad (12)$$

where the dyadic \mathbf{G}_{e1} and \mathbf{G}_{m1} in (11) and (12) are the same as those in (9) and (10) with the

following wave number replacements: $k_0 \rightarrow k_1$, $\beta_0 \rightarrow \beta_1 = \sqrt{k_1^2 - \mu^2}$ where

$k_1 = \frac{\omega_1}{c}$, and $\omega_1 = \omega \pm \omega_0$. Note that the corresponding harmonic time factors associated with

the zeroth and first order fields in (7), (8) and (11), (12) are $e^{-i\omega_0 t}$ and $e^{-i\omega_1 t}$, respectively.

Calculation of fields from localized dipole source

To illustrate the above formalism, here we give two examples of calculating explicitly the electromagnetic fields generated by localized harmonic dipole sources in the context of m-AED.

(i) Magnetic dipole current source

For a point magnetic dipole moment current $\mathbf{J}(\mathbf{r}', t) = c \nabla' \times [\mathbf{m} \delta(\mathbf{r}')] e^{-i\omega_0 t}$ at the

origin, the zeroth order Proca magnetic field was obtained previously to be [16]:

$$\mathbf{B}_0(\mathbf{r}) = \mathbf{m} \cdot \nabla \nabla \left(\frac{e^{i\beta_0 r}}{r} \right) - \mathbf{m} \nabla^2 \left(\frac{e^{i\beta_0 r}}{r} \right), \quad (13)$$

which leads to Eq. (33) of Ref. [16]:

$$\mathbf{B}_0(\mathbf{r}) = \frac{8\pi}{3} \mathbf{m} \delta(\mathbf{r}) + \frac{e^{i\beta_0 r}}{r^5} \left[r^2 (-1 + i\beta_0 r + \beta_0^2 r^2) \mathbf{m} + (3 - 3i\beta_0 r - \beta_0^2 r^2) (\mathbf{r} \cdot \mathbf{m}) \mathbf{r} \right]. \quad (14)$$

Substituting (13) together with $\beta_0 \rightarrow \beta_1$ in the result in (10) into Eq. (12), we

finally obtain the first order magnetic field in m-AED in the form:

$$\mathbf{B}_1(\mathbf{r}) = -4\pi i g_a k \theta_0 \mathbf{m} \times \nabla \int G_{01}(\mathbf{r}, \mathbf{r}') \nabla'^2 \left(\frac{e^{i\beta_0 r'}}{r'} \right) d^3 x', \quad (15)$$

where $G_{01}(\mathbf{r}, \mathbf{r}') = \frac{e^{i\beta_1 |\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$. The integral in (15) can then be evaluated by

expanding $G_{01}(\mathbf{r}, \mathbf{r}')$ in terms of the spherical Bessel function, and finally leads to

the following result for the first order magnetic field:

$$\mathbf{B}_1(\mathbf{r}) = \frac{-4\pi i g_a k \theta_0}{\beta_1^2 - \beta_0^2} \left[\beta_0^2 \left(-\frac{1}{r^2} + \frac{i\beta_0}{r} \right) e^{i\beta_0 r} - \beta_1^2 \left(-\frac{1}{r^2} + \frac{i\beta_1}{r} \right) e^{i\beta_1 r} \right] \mathbf{m} \times \hat{\mathbf{r}}, \quad (16)$$

The corresponding zeroth and first order electric fields can also be obtained as:

$$\mathbf{E}_0(\mathbf{r}, \mathbf{r}') = -ik_0 \left(\frac{i\beta_0}{r} - \frac{1}{r^2} \right) e^{i\beta_0 r} \mathbf{m} \times \hat{\mathbf{r}}, \quad (17)$$

$$\mathbf{E}_1(\mathbf{r}) = \frac{4\pi g_a k k_1 \theta_0}{\beta_1^2 - \beta_0^2} \left\{ \begin{aligned} & \left[(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} \left[e^{i\beta_0 r} \left(\frac{\beta_0^2}{r} + \frac{3i\beta_0}{r^2} - \frac{3}{r^3} \right) + e^{i\beta_1 r} \left(-\frac{\beta_1^2}{r} - \frac{3i\beta_1}{r^2} + \frac{3}{r^3} \right) \right] \right] \\ & + \mathbf{m} \left[e^{i\beta_0 r} \left(-\frac{\beta_0^2}{r} - \frac{i\beta_0}{r^2} + \frac{1}{r^3} \right) + e^{i\beta_1 r} \left(\frac{\beta_1^2}{r} + \frac{i\beta_1}{r^2} - \frac{1}{r^3} \right) \right] \end{aligned} \right\}. \quad (18)$$

As a check of the above results, we apply the results in Eq. (16) and (18) to the case of massless AED and for a static current source. Thus, by setting the

following conditions: $\mu = 0$; $\omega_0 = 0$; $\beta_0 = 0$; and $\beta_1 = k_1 \equiv k$; Eq. (16) and (18)

reduce to:

$$\mathbf{B}_1(\mathbf{r}) = -4\pi i g_a k \theta_0 \left(\frac{1}{r^3} - \frac{ik}{r^2} \right) e^{ikr} \mathbf{m} \times \hat{\mathbf{r}}, \quad (19)$$

$$\mathbf{E}_1(\mathbf{r}) = 4\pi g_a \theta_0 \left\{ \left(-\frac{1}{r^3} e^{ikr} + \frac{ik}{r^2} e^{ikr} + \frac{1}{r^3} \right) [\mathbf{m} - 3(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] + \frac{k^2}{r} e^{ikr} [\mathbf{m} - (\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] \right\}. \quad (20)$$

The results in (19) and (20) reproduce the results obtained previously in the literature [4, 11].

(ii) Electric dipole current source

For a point harmonic electric dipole current of the form:

$\mathbf{J}(\mathbf{r}', t') = -i\omega_0 \mathbf{p} \delta(\mathbf{r}', t') e^{-i\omega_0 t'}$, let us first calculate the zeroth order Proca magnetic

field \mathbf{B}_0 . Previously, we have obtained the zeroth order Proca electric field for

this case in the form [16]:

$$\mathbf{E}_0(\mathbf{r}) = -\frac{4\pi}{3} \mathbf{p} \delta(\mathbf{r}) + \frac{e^{i\beta_0 r}}{r^5} \left[r^2 (-1 + i\beta_0 r + k_0^2 r^2) \mathbf{p} + (3 - 3i\beta_0 r - \beta_0^2 r^2) (\mathbf{r} \cdot \mathbf{p}) \mathbf{r} \right]. \quad (21)$$

The corresponding magnetic field can then be obtained using (8) and (10) as follows:

$$\mathbf{B}_0 = k_0 \beta_0 \left(1 - \frac{1}{i\beta_0 r} \right) \frac{e^{i\beta_0 r}}{r^2} \mathbf{r} \times \mathbf{p}. \quad (22)$$

Following similar steps as in (i), the first order corrections to the fields within m-

AED can finally be obtained from Eqs. (11) and (12) in the following form:

$$\mathbf{B}_1(\mathbf{r}) = ig_a k \theta_0 \left[(2W + \mathbf{r} \cdot \nabla W) \mathbf{p} - (\mathbf{p} \cdot \nabla W) \mathbf{r} \right], \quad (23)$$

$$\mathbf{E}_1(\mathbf{r}) = g_a k k_1 \theta_0 W \mathbf{r} \times \mathbf{p}, \quad (24)$$

where the function $W(r)$ is defined as follows:

$$W(r) = \frac{4\pi i k_0}{(\beta_1^2 - \beta_0^2) r^2} \left[\left(i\beta_0 - \frac{1}{r} \right) e^{i\beta_0 r} - \left(i\beta_1 - \frac{1}{r} \right) e^{i\beta_1 r} \right]. \quad (25)$$

Radiation from static magnetic dipole source

The most interesting radiation problem from localized dipole current source, in the background of a cosmic axion, is the one with a stationary (static) current since there is no “zeroth order contribution” (i.e. in the absence of the axion)[4, 11]. Hence detection of any radiation in this case is a signature of the axion. In this section, we are going to calculate such radiation from a static magnetic dipole using the results in Part (i) above and setting $\omega_0 = 0$. The results will be an extension of the previous results [4, 11] to accommodate for the possibility of the existence of a finite non-zero photon mass.

Let us first recall the following well-known generalized Poynting vector and energy density for the EM fields in the Proca theory [17]:

$$\mathbf{S} = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{B} + \mu^2 \phi \mathbf{A}), \quad (26)$$

$$u = \frac{1}{8\pi} \left[|\mathbf{E}|^2 + |\mathbf{B}|^2 + \mu^2 (\phi^2 + |\mathbf{A}|^2) \right]. \quad (27)$$

In the presence of the axion field, it is straightforward to show that if we still adopt the above expressions for \mathbf{S} and u , then Eq. (3) will lead to the following result:

$$\nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} = -\mathbf{J} \cdot \mathbf{E} - g_a \frac{\partial \Theta}{\partial t} \mathbf{E} \cdot \mathbf{B}, \quad (28)$$

which deviates from the general form of the Poynting theorem for time varying axion field. However, in our present perturbative approach of calculating the fields, the last term in (28) is of a higher order in the coupling constant and can thus be ignored. Hence we shall here adopt the same definitions of \mathbf{S} and u as in (26) and (27) which in the limit of zero photon mass, agree with the results adopted in the literature for the calculation of radiation power in conventional (massless) AED [2, 4, 11]. It may be relevant to mention here that the formulation of Poynting's theorem in AED has been a controversial issue in the literature [18].

Note that although the potentials here in m-AED must satisfy the Lorenz gauge as pointed out in the above, these potentials are *unique* for a given source and do not subject to further “restricted gauge transformation” (e.g. $\mathbf{A} \rightarrow \mathbf{A} + \nabla\Lambda$, with $\square\Lambda = 0$) [17]. This thus guarantees the uniqueness of the results in (26) and (27), and \mathbf{S} and u are thus measurable. In the following we shall apply the result in (26) to calculate the radiation power from a static magnetic dipole, using the dipole fields obtained above in Eqs. (16) and (18) by setting $\omega_0 = 0$, $\beta_0 = i\mu$ and $k_1 = k = \omega/c$. To begin with, we have also to calculate the first order corrections to the scalar and vector potentials as are required in Eq. (26). Using Eq. (6), it is straightforward to obtain the first order correction to the vector potential as follows:

$$\mathbf{A}_1(\mathbf{r}) = -4\pi i g_a k \theta_0 \int G_{01}(\mathbf{r}, \mathbf{r}') \mathbf{B}_0(\mathbf{r}') d^3x'. \quad (29)$$

Using Eq. (13) into (29), we finally obtain:

$$\mathbf{A}_1(\mathbf{r}) = \frac{-4\pi i g_a k \theta_0}{\beta_1^2 - \beta_0^2} \left\{ \left(\beta_0^2 e^{i\beta_0 r} - \beta_1^2 e^{i\beta_1 r} \right) \frac{1}{r} [(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{m}] \right\}. \quad (30)$$

The first order correction to the scalar potential can then be obtained from the Lorenz gauge condition leading to:

$$\phi_1 = \frac{c}{i\omega_1} \nabla \cdot \mathbf{A} \sim \frac{1}{r^2} \quad \text{as } r \rightarrow \infty. \quad (31)$$

Hence there is no contribution to the radiated power from the second term of the Poynting vector in Eq. (26) in this case, and the radiated power is exclusively determined by the far fields obtained from Eqs. (16) and (18), respectively. Thus as $r \rightarrow \infty$, we obtain the far fields from Eq. (16) and (18) as follows:

$$\mathbf{B}_1(\mathbf{r}) \rightarrow \frac{4\pi g_a k \theta_0}{\beta_1^2 - \beta_0^2} (\beta_0^3 e^{i\beta_0 r} - \beta_1^3 e^{i\beta_1 r}) \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r}, \quad (32)$$

$$\mathbf{E}_1(\mathbf{r}) \rightarrow \frac{4\pi g_a k^2 \theta_0}{\beta_1^2 - \beta_0^2} (\beta_0^2 e^{i\beta_0 r} - \beta_1^2 e^{i\beta_1 r}) \frac{[(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{m}]}{r}. \quad (33)$$

By recalling the harmonic time factor associated with the first order fields to be $e^{-i\omega_1 t} = e^{-i\omega t}$ since $\omega_0 = 0$ in this case, the first term in the parenthesis of each of (32) and (33) simply leads to a damped harmonic field (for $\beta_0 = i\mu$) which has no contribution to radiation. Furthermore, since there is no zeroth order radiation fields in this case, the radiated power will be given exclusively by the time averaged Poynting vector $\langle \mathbf{S}_1 \rangle = \frac{c}{8\pi} \text{Re}(\mathbf{E}_1 \times \mathbf{B}_1^*)$ where

$$\mathbf{B}_1(\mathbf{r}) \rightarrow -\frac{4\pi g_a k \theta_0}{\beta_1^2 + \mu^2} \beta_1^3 e^{i\beta_1 r} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r}, \quad (34)$$

$$\mathbf{E}_1(\mathbf{r}) \rightarrow -\frac{4\pi g_a k^2 \theta_0}{\beta_1^2 + \mu^2} \beta_1^2 e^{i\beta_1 r} \frac{[(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{m}]}{r}, \quad (35)$$

with $\beta_1^2 = k^2 - \mu^2 = \left(\frac{\omega}{c}\right)^2 - \mu^2$. Using (34) and (35), we finally obtain the differential and total radiated power to be:

$$\frac{dP}{d\Omega} = r^2 \langle \mathbf{S}_1 \cdot \hat{\mathbf{r}} \rangle = 2\pi c k^3 \beta_1^5 \left(\frac{m g_a \theta_0}{\beta_1^2 + \mu^2} \right)^2 \sin^2 \vartheta, \quad (36)$$

$$P = \frac{1}{3} ck^3 \beta_1^5 \left(\frac{4\pi m g_a \theta_0}{\beta_1^2 + \mu^2} \right)^2, \quad (37)$$

where \mathcal{G} is the angle between the magnetic dipole and the observer direction and m is the magnitude of the dipole moment. In the limit $\mu \rightarrow 0$, both (36) and (37) reduce to the previous results accordingly [4]. It is of interest to note that in a massive photon theory, radiation fields should in general contain three polarizations (two transverse and one longitudinal) whereas Eqs. (34) and (35) show that the magnetic dipole radiation fields are purely transverse. This is consistent with previous calculations using a general multipole expansion approach which showed explicitly that longitudinally polarized radiations are indeed present in the Proca theory for electric dipole and electric quadrupole sources; but not for magnetic dipole radiation [13]. This is mainly due to the fact that the “effective current \mathbf{j}_{eff} ” associated with a magnetic dipole is always a transverse current with $\nabla \cdot \mathbf{j}_{eff} = 0$. One can then argue as a consequence, the vector potential, the electric field, and the magnetic fields must all be transverse to the direction from the source to the observer in the far zone [19].

To study the interplay between the axion mass (m_a) and the photon mass (m_γ) in the radiated power, we first note the following relations (in appropriate units [12], [14]):

$$g_a \sim m_a; k \sim \omega \sim m_a; \theta_0 \sim m_a^{-1}; \mu \sim m_\gamma; \beta_1^2 = (k^2 - \mu^2) \sim (m_a^2 - m_\gamma^2); \quad (38)$$

hence (37) can be re-casted into the following form:

$$P = P_0 \left[1 - (m_\gamma/m_a)^2 \right]^{5/2}, \quad (39)$$

where $P_0 = \frac{16\pi^2}{3} g_a^2 \theta_0^2 m^2 ck^4$ is the radiated power from conventional AED with zero photon

mass [4]. Figure 1 shows how the axion-induced radiated power from a localized static magnetic

dipole will decrease due to the finiteness of the photon mass. Note that it is required to have $m_\gamma < m_a$ so that the propagation constant β_1 remains real.

To provide some numerical estimates for the above radiated power, we refer to a neutron star with a radius $a \sim 2$ km which can be approximated as a magnetic dipole when observed from the Earth [4], especially for axions with a small mass and hence a long oscillation wavelength $\lambda_a \sim \frac{h}{m_a c}$. With values set for $g_a \theta_0 \sim 10^{-21}$ [11] and the magnetic field $B_0 \sim 10^{13}$ G [2]; the maximum radiated power emitted in a direction perpendicular to the axis of the neutron star can be estimated from Eq. (36) to be:

$$\frac{dP}{d\Omega} = 2\pi c k^3 m^2 (g_a \theta_0)^2 \left(\frac{\beta_1^{5/2}}{\beta_1^2 + \mu^2} \right)^2 = 2\pi c k^4 m^2 (g_a \theta_0)^2 \left(\frac{\beta_1}{k} \right)^5 \rightarrow \left(\frac{dP}{d\Omega} \right)_0 \times \left[1 - \left(\frac{m_\gamma}{m_a} \right)^2 \right]^{5/2}, \quad (40)$$

where $\left(\frac{dP}{d\Omega} \right)_0 = 2\pi c k^4 m^2 (g_a \theta_0)^2$ is the radiated power from a point magnetic dipole in case of

zero photon mass. Hence if we let the magnetic moment of the neutron star $m = \frac{1}{2} B_0 a^3$ [4] and

assuming a range of the axion mass to be : $10^{-12} eV < m_a c^2 < 10^{-6} eV$, the factor $\left[1 - \left(\frac{m_\gamma}{m_a} \right)^2 \right]^{5/2}$ can

be ignored and we have:

$$\frac{dP}{d\Omega} \sim 2\pi c \left(\frac{2\pi}{\lambda_a} \right)^4 \left(\frac{1}{2} B_0 a^3 \right)^2 (g_a \theta_0)^2 \quad \text{erg / s} . \quad (41)$$

So if we take the axion mass to be $10^{-10} eV$ (hence $\lambda_a \sim 1.24 \times 10^6 cm$); and use the above quoted values for the neutron star magnetic moment, as well as for the axion coupling strength and

amplitude; Eq. (41) will yield:

$$\frac{dP}{d\Omega} \sim (2\pi)^5 3 \times 10^{10} \left(\frac{1}{1.24 \times 10^6} \right)^4 \left(\frac{1}{2} \times 10^{13} \times (2 \times 10^5)^3 \right)^2 \left(10^{-21} \right)^2 \sim 2 \times 10^5 \text{ erg/s} \sim 0.02 \text{ watt} .$$

Note that a higher value for the axion mass will yield even higher radiated power. Hence, as long as the photon mass is much smaller than the axion mass, e.g. 10^{-16} eV vs 10^{-6} eV or even 10^{-12} eV, the decrease in the above estimated power is insignificant and it may still be measurable (e.g. from space lab). However, for ultralight axions, this radiated power may become much reduced or even be extinguished. Note that the above estimate has assumed the neutron star to be a point magnetic dipole which is subject to a correction with a finite size factor as studied in our previous work in Ref. [4].

Conclusion

In this work, we have studied the effect of a finite photon mass on the axion-induced electromagnetic fields from localized current sources. In particular, we have focused on the radiation from a static current source of a magnetic dipole for which the radiation arises exclusively from the presence of an oscillatory cosmic axion background. This is achieved via an extension of the Proca theory to incorporate the axion, leading to a formulation of the massive axion electrodynamics (m-AED) theory. Such a theory has also been studied recently in the context of topological superconductor [10]. One limitation of our work is that our calculation is only up to the first order in the coupling constant and has not included back reaction on the axion field which will be a second order effect. An interesting future investigation on this effect will have to solve the Euler-Lagrange equation for the dynamics of the axion which will lead to a varying field amplitude for the axion [2].

Here it is found that for a static magnetic dipole interacting with the cosmic axion, the radiation power will in general depend on the interplay between the axion mass and the photon

mass, and will decrease if the photon is not exactly massless. In addition, it is noted that there will be a “cutoff” of this radiated power if the photon mass turns out to be greater than that of the axion. This “cutoff effect” may be understood as radiation generated with frequency matching that of the axion via an “instability effect” arises when the photon mass is below that of the axion. One application of this effect may be that in the absence of observing this kind of radiation, one can use the lower bound of the axion mass to set a lower bound for the photon mass limited by the sensitivity of the measurement. This, together with the existing knowledge of the upper bound of the photon mass, will enable one to confirm the finiteness of the photon mass. On the other hand, the detection of such radiation can also be used to set a lower limit of the axion mass assuming an upper bound for the photon mass is known (e.g. from experiments via the study of deviation from Coulomb’s law). These results should be of interest for future investigations into the mass of both the axion and the photon, and ultimately to be relevant to the probing of the dark matter universe.

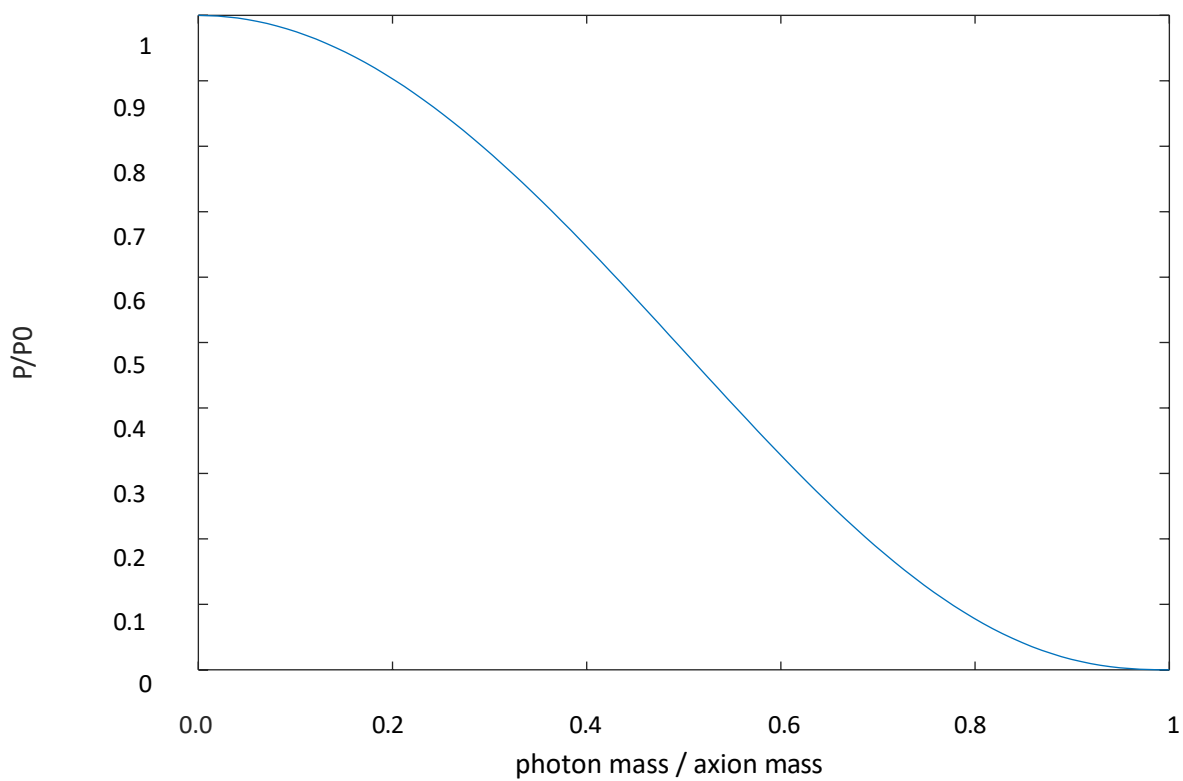


Figure 1 Normalized radiated power from a static magnetic dipole current as a function of the photon – axion mass ratio.

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- [19] Since in the Lorenz gauge, we have the vector potential $\mathbf{A} \sim \mathbf{j}_{eff}$, one can show as a consequence that \mathbf{A} is also transverse and actually satisfies the Coulomb gauge (implying for a magnetic dipole source, one can just set the scalar potential zero in the Lorenz gauge). Also, for a localized magnetic dipole, we expect in the far zone:

$\mathbf{A} \sim \mathbf{A}_0(\theta, \varphi) \frac{e^{ikr}}{r}$ (with the radial component $A_{0r} = 0$), hence the electric field $\mathbf{E} \sim \frac{\partial \mathbf{A}}{\partial t}$

being parallel to \mathbf{A} for harmonic fields must also be transverse. In addition,

$\mathbf{B} = \nabla \times \mathbf{A} \rightarrow \mathbf{B} \sim B_\theta \mathbf{e}_\theta + B_\varphi \mathbf{e}_\varphi$ to $O(1/r)$ showing \mathbf{B} is also transverse to $\hat{\mathbf{r}}$. This is true

for both the Maxwell and Proca theory.