Carbon Emissions Reductions in Last Mile and Grocery Deliveries Utilizing Autonomous Vehicles

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ABSTRACT

New driverless air and ground vehicles are being launched and tested to deliver products or services in the areas of retail, groceries, and healthcare. This research focuses on the efficiency of autonomous (driverless) delivery vehicles in terms of vehicle-miles, energy consumption, and carbon emissions. Drones or UAVs, sidewalk autonomous delivery robots (SADRs), and road autonomous delivery robots (RADRs) vehicles carbon emissions are compared against emissions from an electric van (e-van), a conventional internal combustion engine van, and driving to a store utilizing electric and conventional vehicles. The impacts of vehicle capacity, range, and time constraints are analyzed as well as the impacts of number of deliveries, service time, area of service, and depot-service area distance. Novel results are found regarding the efficiency of each vehicle type and tradeoffs between driving to a store and store delivery as a function of order size and type of vehicle driven by consumers.

KEYWORDS: carbon emissions, autonomous vehicles, ground robot, air drone, energy, travel distance, delivery industry, grocery shopping

Please Cite as:

1. Introduction

Autonomous delivery robots (ADRs) that travel on sidewalks and roads are being tested in several US cities by startups. Even large delivery companies like FedEx (FedEx, 2019) and online retailers like Amazon are also testing this technology (CNBC, 2019). Amazon is also testing a drone prototype that can deliver packages under five pounds to customers in less than 30 minutes; 75% to 90% of Amazon deliveries are less than five pounds (Forbes, 2019). Although most deployments are still at the pilot level, air drones and ground ADRs may soon be able to meet the growing delivery demands caused by e-commerce, which is growing at a double-digit annual rate (USCB, 2020). According to a recent survey, a large majority of people in the US believe that delivery robots will be in use within the next five years (USPS, 2018). According to this USPS study, customers highly value the ability to receive deliveries when and where recipients choose. A significant increase in e-commerce is also expected during the current COVID-19 pandemic.

Even modest improvements in delivery efficiency can result in large reductions in carbon emissions given the size and growth of the delivery industry. In the US the number of parcels delivered in 2018 was approximated 13 billion; worldwide the number of parcels delivered in 2018 was 87 billion which doubles the worldwide number of parcels delivered in 2014 (Buchholz, 2019). E-commerce is also changing consumer habits and shopping trips, for example in the US 21% of shoppers are buying groceries online at least once a month or more often (FMI, 2019). In the US there are approximately 120 million households with an average number of 2.5 grocery trips per week per household (FMI, 2019). A growing share of grocery e-commerce and store delivery is going to impact the carbon emissions of billions of annual grocery trips.

This research analyses the delivery efficiency of autonomous vehicles in terms of energy consumption, carbon emissions, and travel, a topic that has not been studied in previous publications. Novel contributions include the comparison of conventional and autonomous technologies as well as the impact of electric vehicles (EV) and vehicles with internal combustion engines (ICE). A novel approach and formulation to estimate the number of feasible deliveries per scenario is developed as well as the introduction of the ratio between grocery store order size and delivery order size to analyze the efficiency of in-store vs home delivery.

This research is organized as follows: Section 2 presents a literature review. Section 3 summarizes vehicle characteristics. Section 4 presents the formulation, the methodology, and scenarios utilized in the analysis. Section 5 discusses energy consumption results for delivery services by vehicle type, varying delivery area, depot distance, delivery time duration, and number of customers served. Section 6 analyzes on-road, sidewalk, and air distance traveled by different vehicles. Section 7 presents and discusses CO2 emissions and Section 8 analyzes emissions tradeoffs when comparing delivery services and store customer trips. Section 9 ends with conclusions and a discussion of future research opportunities.

2. Literature Review

For last mile deliveries, studies have compared the carbon footprints of conventional and online retailing (Edwards et al., 2010), the impact of collection and delivery points (Song et al., 2013), and the effectiveness of lockers to reduce carbon emissions (Iwan et al., 2016). Vehicle
electrification has also been studied as potential and cost-effective way to reduce urban freight emissions (Feng and Figliozzi, 2012; Lee et al., 2013; Davis and Figliozzi, 2013).

The potential of autonomous vehicles for passenger transportation has been studied extensively, for example in Fagnant and Kockelman (2015). By comparison, significantly less work focuses on the potential of autonomous vehicles in the freight sector. The work of Slowik and Sharpe (2018) focuses on the potential of autonomous technology to reduce fuel use and emissions for heavy-duty freight vehicles. This study focuses on long haul transportation and finds that potential benefits of autonomous trucking could be substantial in terms of fuel consumption and emissions.

There are scant studies focusing on urban deliveries or short-haul freight trips. Vleeshouwer et al. (2017) simulated bakery deliveries utilizing a robot but the occupation of the robot was low and not feasible from an economic viewpoint. These authors suggest that robots can be feasible if companies scale up or cooperate to increase robot utilization. The other studies (Jennings and Figliozzi, 2019 and 2020) analyzed the regulation and characteristics of sidewalk and road ADRs (respectively) in the USA and studied potential time and cost savings. When compared to a conventional human-driven delivery van, sidewalk ADRs can reduce cost, time, and vehicle travel in some instances. Road delivery robots are also more economical when delivery routes are relatively short. However, due to their limited range, vehicle miles tend to increase in most scenarios. Other researchers have analyzed the shortcomings of current regulations for delivery robots (Hoffman and Prause, 2018). There are still many uncertainties related to ADRs’ regulation and technological evolution. The rate and speed of adoption of RARs will greatly depend on costs and ease of entry into the delivery market as discussed by studies focusing on the adoption of autonomous trucks by freight organizations (Talebian and Mishra, 2018; Simpson et al. 2019).

Drone deliveries have also been compared against van or truck delivery systems in terms of carbon emissions. Goodchild and Toy (2017) using GIS-based simulations suggest that distance from depot and customers per route have a major impact on CO₂ emission levels when comparing trucks and drones. Figliozzi (2017) derives analytical formulas to compare operational and lifecycle emissions and energy consumptions of drones, diesel van, electric vans, and tricycles. Figliozzi (2017) shows that it is possible to find emissions breakeven points as a function of customers in a route, efficiency of the vehicles, distance to customers, density of delivery area, and drone size/payload. Air drones are more CO₂ efficient for small payloads and single deliveries in rural areas but less efficient for large payloads or when many customers are grouped in dense urban areas. Drones consume less energy and emissions per package-km than delivery vans. Kirschstein (2020) provide a more detailed energy consumption model including takeoff, level flight, hovering, and landing; similar results were obtained, i.e. drones requires more energy in urban areas with high customer density and less energy in rural settings low customer density that trucks. Park et al. (2018) in South Korea concluded that the global warming potential (GWP) per 1 km delivery by drone was one-sixth that of motorcycle delivery; it is also found that drones are more efficient (i.e. have less environmental impact) in rural areas. Stolaroff et al. (2018) analyzed emissions of short-range multi-copters that require a network of intermediary support waystations. It was estimated that warehouses contribute significantly to life-cycle emissions and also agreed with previous research regarding the relative efficiency of drones when delivering small packages. Other researchers have also studied how wind and temperature affect drone range and operations (Chauhan et al. 2020; Glick et al., 2020; Kirschstein, 2020). Drone logistics is a fast-evolving field; an up-to-date overview of logistic issues associated to drone deliveries and modelling opportunities is presented by Roca-Riu and Menendez (2019).
3. Vehicle Characteristics

ADRs are electric powered ground vehicles that can deliver items or packages to customers without the intervention of a delivery person. ADRs can be divided into two types. Sidewalk autonomous delivery robots (SADRs) are pedestrian sized robots that only utilize sidewalks or pedestrian paths. On-road or simply road autonomous delivery robots (RADRs) are vehicles that travel on roadways shared with conventional vehicles. ADRs use sensors and navigation technology that allow them to travel on roads and sidewalks without a driver or on-site delivery staff. The ground vehicles studied in this research have already been utilized in previous studies. Starship Technologies is producing the SADR that has been deployed in most locations and have received ample media coverage (Jennings and Figliozzi, 2019). Among the companies prototyping RADRs there are two that stand out: NURO and uDelv (Jennings and Figliozzi, 2020). The chosen high-performance drone, MD4-3000, has already been analyzed by Figliozzi (2017) where it was compared against the performance of tricycles, a typical delivery diesel van (Dodge RAM) utilized in the US, and an electric van (Renault Kangoo) utilized in Europe. The Dodge RAM is the only internal combustion engine (ICE) vehicle in Table 1.

Table 1 presents a summary of vehicle characteristics. The drone has the lowest tare and payload but the SADR has the lowest range and speed. The RADRs have less limitations regarding payload and range but these are still substantially lower than the payload and range of a conventional van. The electric van has relatively low energy consumption but reasonable performance in terms of payload and range for short and medium route lengths.

SADRs have limited range and can be complemented by specialized vans (see Figure 1, left), usually called “mothership” vans, that can be utilized to drop off and pick up several SADRs. The mothership van is not an autonomous vehicle and requires a driver. Henceforward specialized vans that carry SADRs will be denoted simply as motherships. If SADRs are serving customers nearby, i.e. the service area is not far away, a mothership is not necessary which simplifies the operation of the sidewalk deliver robot and considerably reduces road distance traveled, energy consumption, and emissions. Unlike SADRs, even small RADRs are designed to share roadways with conventional motorized vehicles. Hence RADRs are not dependent on a mothership; a NURO RADR is shown in Figure 1 (right).
### TABLE 1 - Key Vehicle Characteristics

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Tare (kg)</th>
<th>Max. Speed (kph)</th>
<th>Payload (kg)</th>
<th>Range (km)</th>
<th>Energy consumption (wh/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starship</td>
<td>18.1</td>
<td>6.4</td>
<td>18.1</td>
<td>6.4</td>
<td>24.7</td>
</tr>
<tr>
<td>Nuro</td>
<td>680</td>
<td>56</td>
<td>110</td>
<td>16.1</td>
<td>139.6</td>
</tr>
<tr>
<td>uDelv</td>
<td>1890</td>
<td>97</td>
<td>590</td>
<td>97</td>
<td>193.9</td>
</tr>
<tr>
<td>MD4-3000</td>
<td>10.2</td>
<td>72</td>
<td>5.0</td>
<td>36</td>
<td>21.6</td>
</tr>
<tr>
<td>Renault Kangoo EV</td>
<td>1300-1430</td>
<td>160</td>
<td>650-800</td>
<td>120*</td>
<td>205</td>
</tr>
<tr>
<td>Dodge RAM</td>
<td>2170</td>
<td>180</td>
<td>1890</td>
<td>695</td>
<td>1,016</td>
</tr>
</tbody>
</table>

* 120 km under extreme winter conditions, 199 km under temperate conditions

---

4. Methodology

In this section the methodology used for comparing the travel, energy, and emissions performance is presented. The methodology is based on continuous approximations. As indicated by Daganzo et al. (2012) these types of analytical approximations are appropriate to address big picture questions because they are parsimonious, tractable, and yet realistic when the main tradeoffs are included. This type of modeling approach has been successfully used in the past by many authors to model urban deliveries and logistic problems (Franceschetti et al., 2017; Ansari et al., 2018).

4.1 Formulation

The constraints analyzed in this research include range, duration, capacity, and number of customers served. The notation used is presented below.

- \( n \) = Number of stops or delivery locations
- \( m \) = Number of vehicles required to deliver to \( n \) customers
- \( a \) = Area (units length squared) of the service area (SA) where \( n \) customers reside
- \( \delta = n/a \), customer density
- \( d \) = Service area to depot distance
- \( R \) = Effective vehicle range
- \( Q \) = Vehicle capacity in number of customers
\( v \) = Average speed of the vehicle in the long haul
\( v_t \) = Average speed of the vehicle in the local service area
\( T \) = Maximum duration of shift or tour (same for all vehicle types)
\( t_0 \) = Time to deliver at a customer including time to pick up a parcel
\( t_u \) = Unload time
\( t = t_0 + t_u \) = Total time vehicle is idle (i.e. not traveling) during a delivery
\( k_l \) = Routing parameter
\( e \) = Energy consumption per unit distance (wh/km)
\( \kappa \) = Carbon emissions per unit of energy (CO\(_2\) kg/wh)
\( \varepsilon \) = ratio between store order size and delivery order size

By utilizing continuous approximations to estimate the average distance \( l(n, m) \) traveled by a fleet of vehicles (Daganzo et al. (2012); Figliozzi, 2008) it is possible to write the following fleet size and route duration constraints, (1) and (2) respectively:

\[
m \geq \frac{k_l \sqrt{an}}{R-2d} \quad (1)
\]

\[
T \geq \left( \frac{2d}{v} + \frac{k_l}{m v_t} \sqrt{an} \right) + \frac{t n}{m} \quad (2)
\]

It is assumed that all the vehicles depart the depot at the beginning of the time period \( T \). Hence, the fleet size \( m \) is equal to the number of tours.

In some scenarios, the constraint is to serve a maximum number of customers \( n \) within the allowed time \( T \) with a given fleet size. For this purpose, a novel equation is developed. The equation that provides this number is obtained by solving for \( n \) in equation (2). The result is the following expression (3):

\[
n = \left\lfloor \frac{2(\tau -2d)(\frac{t}{m}) + (\frac{k_l \sqrt{a}}{m v_t})^2}{2 (\frac{t}{m})^2} - \left(4 \left( \frac{\tau -2d}{v} \right) \frac{t}{m} \frac{k_l \sqrt{a}}{m v_t} \right)^2 + (\frac{k_l \sqrt{a}}{m v_t})^4 \right\rfloor^{1/2} \quad (3)
\]

The floor function is used in equation (3) to avoid a fractional number of customers. The derivation of the formula is presented in the Appendix. The capacity constraint is simply stated as \( m \geq \lfloor n/Q \rfloor \) (4).

The above formulas (1) to (4) assume that when several vehicles are utilized, \( m > 1 \), the service area is partitioned into homogenous areas as described by Daganzo (1984) and illustrated in Figure 2 assuming \( m = 4 \).

The energy consumed \( E \) and carbon emissions \( K \) produced by \( m \) vehicles are calculated utilizing the travel distance multiplied by the corresponding energy and emission factors for each vehicle type with \( m \) that satisfy the range, capacity, time, and number of delivery constraints in equations (1) to (4).

\[
K = \kappa \ E = \kappa \ e \left( 2dm + k_l \sqrt{an} \right) \quad (5)
\]
Figure 2. Schematic of service area partition into homogenous subareas, e.g. $m = 4$.

### 4.2 Data, Assumptions, and Scenario Design

Vehicle efficiencies are analyzed utilizing two approaches: (a) estimating efficiency of each vehicle when delivery area size, delivery time, and distance from the depot vary and (b) estimating the efficiency of the vehicles when customer density or number of deliveries change. In the latter approach the number of customers served varies from $n = 25$ to $n = 200$ and this interval includes the average number of package deliveries per conventional vehicle that is approximately 120. Vehicle specifications are drawn from manufacturer websites or press releases and parameters are representative of delivery conditions in urban and suburban areas.

The Starship SADR is assumed herein because it complies with US regulation and it is the most extensively tested SADR at the time of this writing. The Starship SADR range is approximately 4 miles and a Starship is designed to carry up to three grocery bags or up to six small/medium packages – as a reference most Amazon packages parcels are less than five pounds or 2.3 kg (Forbes, 2019). SADRs are complemented by a mothership van (see Figure 1). The mothership drop-offs or picks-up up to 8 SADRs per tour (Daimler, 2017). The same formulas and constraints presented earlier in this section can be directly applied to the mothership but assuming that $n$ represents the number of SADRs and that the capacity of the mothership is eight SADRs. When the depot is at the center of the SA ($d = 0$), a mothership is not utilized. The drone always services one customer at the time hence the calculations of energy and emissions are straightforward since customers are not grouped in routes and distances are estimated as the crow flies (i.e. Euclidian). For ground vehicles, the Manhattan metric is assumed which is reflected in the value of the routing parameter $k_1$.

The energy consumption coefficient per unit distance, denoted $e$, for each vehicle was estimated utilizing available battery and range information for the vehicles or manufacturers data. The values assumed for SADRs and RADRs are reasonable also according to other studies.
analyzing small ground robot energy consumption, for example Broderick et al. (2014). The following values are utilized in the numerical analysis: SADRs consume 24.7 wh/km, NUROS consume 140 wh/km, uDelvs consume 194 wh/km, electric vans (E-vans) consume 205 wh/km, and the electric mothership consume of 427 wh/km (currently the mothership can use a conventional internal combustion engine but an electric engine is assumed to facilitate future comparisons). The E-van and conventional ICE van values were obtained from the performance of the electric Renault Kangoo van and Dodge RAM (Figliozzi, 2017). The value for the mothership is obtained from the Mercedes Benz Sprinter cargo van (MBS, 2018) that was modified to accommodate the SADRs shown in Figure 1 and assuming that the mothership utilizes an electric engine.

The literature review indicates that urban areas are complex environments for autonomous vehicles with many deliveries/stops and interactions with pedestrians and cyclists (Kristoffersson et al., 2018). Hence, it is likely that autonomous vehicles will be designed with high safety standards and would require extra time to park, unload/load, and avoid conflicts with pedestrians and/or cyclists. Hence additional minutes and a lower average speed are assumed for autonomous vehicles. The following average delivery times \( t \) per customer are assumed: five minutes for autonomous SADRs, Nuros, Udelvs, and drone; 3 minutes for the E-van and conventional van with a driver. The following average local or service area speeds \( v \) are assumed: SADR: 2 km/h, Nuro and Udelv 10 km/h, and mothership or E-van 20 km/h. For the long-haul segment connecting the depot and the service area an average speed of \( v = 40 \) km/h is assumed.

5. Energy Consumption Results

There are large differences among SADRs, RADRs, drones and EV/conventional vans in terms of capabilities. Hence, for a few combinations of parameters some vehicles cannot be feasibly deployed due to range, capacity, or time constraints. When it is not feasible to deploy a vehicle a table cell is filled out with “NA” which stands for “Not Available”. It should be noted that continuous approximations work best when the number of customers per vehicle is four or higher (Daganzo, 1984). In the scenarios presented only the SADR may serve less than four customers per vehicle and when this is the case the value has an accompanying asterisk and note in the corresponding table. However, it is important to point out that when this is the case the SADR is already not the most efficient vehicle and the value in the table is slightly underestimating the energy consumed per customer and the general trends and insights are not affected.

5.1 One vehicle efficiency

The first set of results study the efficiency of each vehicle varying key parameters such as delivery area size, distance to depot, and delivery time window. One vehicle of each type is deployed and energy consumption efficiencies are compared across vehicles. The baseline scenario assumes a service area \( a = 1 \) km\(^2\), a depot at the center of the service area \( d = 0 \), and a delivery time window \( T = 8 \) hours. The number of customers is estimated utilizing equation (3).

5.1.1 Service Area (SA)

Table 2 reports the results in terms of energy consumption per customer served as service area increases from \( a = 1 \) km\(^2\) to \( a = 30 \) km\(^2\). It is assumed that the depot is at the center of the SA.
(d = 0) and service time is T = 8 hours. A different color and bold are utilized to pinpoint the lowest value per row. The SADR is most energy efficient option for smaller service areas because a SADR can serve multiple customers (up to six) and have a low energy consumption per unit distance. The drone becomes the most energy efficient for large service areas (i.e. low-density scenarios) where the SADR serves less than six customers per vehicle.

**TABLE 2. Impact of a on Energy Consumption per Customer (wh/customer)**

<table>
<thead>
<tr>
<th>Service Area a (in km²)</th>
<th>Vehicle Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SADR</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>38</td>
</tr>
<tr>
<td>20</td>
<td>*76</td>
</tr>
<tr>
<td>30</td>
<td>*131</td>
</tr>
</tbody>
</table>

* Three or less customers per SADR

5.1.2 Depot - Service Area Distance

Table 3 reports the results in terms of energy consumption per customer served as depot-SA distance increases from d = 0 km to d = 10 kms. It is assumed that the service area is constant and equal to a = 1 km² and service time is T = 8 hours. The E-van is the most efficient vehicle when the depot is not at the center of the service area. Since SADRs are severely range constrained they must be complemented by a mothership even for relatively low d values. Hence, the SADR is less efficient than all the other ground delivery vehicles when a mothership must be utilized (even assuming an electric powered mothership).

**TABLE 3. Impact of d on Energy Consumption per Customer**

<table>
<thead>
<tr>
<th>Distance Depot – SA d (in kms)</th>
<th>Vehicle Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SADR+MS*</td>
</tr>
<tr>
<td>0.0</td>
<td>3</td>
</tr>
<tr>
<td>2.5</td>
<td>72</td>
</tr>
<tr>
<td>5.0</td>
<td>117</td>
</tr>
<tr>
<td>7.5</td>
<td>161</td>
</tr>
<tr>
<td>10.0</td>
<td>206</td>
</tr>
</tbody>
</table>

* Mothership not utilized when d = 0.0

5.1.3 Delivery Time Duration

Table 4 reports the results in terms of energy consumption per customer served as delivery time duration decreases from T = 8 hours to T = 1/2 hour. It is assumed that the service area is constant and equal to a = 1 km² and distance d = 0 km. A different pattern emerges from Table 4. The SADR is the most efficient vehicle until the duration of the delivery time window is T = 2 hours. For smaller values of T the SADR is too slow to feasibly deliver in a limited time frame. Given the low average travel speed of SADRs, 2 km/h in sidewalks, it is not possible to deliver to customers in a ½ time frame taking into account travel time and service
time. The other ground vehicles also see a reduction of efficiency as the delivery duration decreases since it reduces the number of feasible deliveries according to equation (3). The drone becomes most efficient when delivery durations are short and fewer customers can be grouped in the route of a ground vehicle.

**TABLE 4. Impact of $T$ on Energy Consumption per Customer**

<table>
<thead>
<tr>
<th>Time to deliver $T$ (in hours)</th>
<th>Vehicle Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SADR</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>*17</td>
</tr>
<tr>
<td>0.5</td>
<td>NA</td>
</tr>
</tbody>
</table>

* Three or less customers per SADR

**5.2 Fleet Efficiency to Serve $n$ Customers**

The second set of results study the efficiency a fleet of vehicles that must service $n$ customers. More than one vehicle of each type may be deployed to meet capacity, time, or range constraints. The baseline scenario assumes a service area $a = 1$ km$^2$, a depot service area distance $d = 2.5$ kms, and a delivery time window $T = 8$ hours. Table 5 reports the results in terms of energy consumption per customer served as the number of customers served increases from $n = 25$ to $n = 200$. More than one vehicle of each type may be deployed to satisfy the range, capacity, or time constraints (see Table 6). The energy efficiency of all the ground vehicles increases when the number of deliveries or the delivery density increases as routes become more efficient by including more customers per vehicle. However, efficiency is reduced when more vehicles are required. For $n = 25$ to $n = 100$ NURO and Udelv are most efficient but the electric van is the most efficient vehicle for $n = 200$.

**TABLE 5. Impact of $n$ on Energy Consumption per Customer**

<table>
<thead>
<tr>
<th>Customers Served ($n$)</th>
<th>Vehicle Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SADR+ MS</td>
</tr>
<tr>
<td>25</td>
<td>127</td>
</tr>
<tr>
<td>50</td>
<td>114</td>
</tr>
<tr>
<td>100</td>
<td>84</td>
</tr>
<tr>
<td>200</td>
<td>67</td>
</tr>
</tbody>
</table>
TABLE 6. Required Fleet Size Varying $n$

<table>
<thead>
<tr>
<th>Customers Served ($n$)</th>
<th>SADR</th>
<th>NURO</th>
<th>Udelv</th>
<th>E-van</th>
<th>Drone</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>50</td>
<td>9</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>17</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>200</td>
<td>34</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

6. Distance Traveled

Changes in commercial vehicle on-road distance traveled are important to analyze autonomous delivery vehicles potential safety impacts and congestion relief. Table 7 shows the distance traveled by the different vehicles per customer served as the number of customers served increases from $n = 25$ to $n = 200$. The baseline scenario assumes area size $a = 1 \text{ km}^2$, distance $d = 2.5 \text{ km}$, and $T = 8 \text{ hours}$.

When comparing values, it is important to highlight that drones do not share the roads used by all the other ground vehicles. However, even for relatively short distances between the depot and the service area, drones travel a significantly longer distance and this is related to the inefficiency of their routes that only accommodate one customer per round trip from the depot.

SADRs travel on the sidewalk and the mothership travels on the road, hence, the distance for these two complementary vehicles are disaggregated. It was shown that SADRs can be energy efficient when delivering from the depot and without a mothership. Table 7 shows that when a mothership is utilized to carry the SADRs there could be an increase of both on-road and sidewalk travel when comparing against the other ground vehicles. There is a clear increase of MS on-road vehicle travel for $n \geq 50$. RADRs and the E-van have the same on-road distance traveled as long as the necessary fleets have the same size. RADRs could be more energy efficient for lower values of $n$ but generate more miles and potentially more congestion when more vehicles are needed. Regarding safety and crashes, the NURO is smaller and designed to collapse and reduce damage in case of crashes (Verger, 2018). Future research efforts should analyze in detail potential externalities and conflicts with pedestrians caused by SADR travel on sidewalks.

Potential congestion impacts of RADRs are also a function of their size, the uDelv is similar in size to the E-van; however, the NURO is considerably smaller, roughly $1/2$ the size of a small E-van and likely to contribute less to congestion on a per unit distance traveled basis. Regarding motherships, it is important to highlight that the Mercedes Benz Sprinter van, which is used as a prototype to carry the eight SADRs, is 70% and 130% longer than the Udelv and NURO vehicles respectively. Regarding vehicle width, which is important for parking in congested areas, two and $1/2$ NUROs can potentially occupy the same parking space utilized by one mothership van. Considering that the mothership van may require additional space to unload and load the SADRs, SADRs may not be efficient in terms of curb utilization if a mothership is required.
TABLE 7. Per Customer Distance Traveled Varying $n$

<table>
<thead>
<tr>
<th>Customers Served $(n)$</th>
<th>SADR*</th>
<th>MS</th>
<th>NURO</th>
<th>Udelv</th>
<th>E-van</th>
<th>Drone**</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.19</td>
<td>0.29</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>5.75</td>
</tr>
<tr>
<td>50</td>
<td>0.14</td>
<td>0.26</td>
<td>0.34</td>
<td>0.24</td>
<td>0.24</td>
<td>5.75</td>
</tr>
<tr>
<td>100</td>
<td>0.10</td>
<td>0.19</td>
<td>0.25</td>
<td>0.15</td>
<td>0.15</td>
<td>5.75</td>
</tr>
<tr>
<td>200</td>
<td>0.07</td>
<td>0.15</td>
<td>0.22</td>
<td>0.12</td>
<td>0.09</td>
<td>5.75</td>
</tr>
</tbody>
</table>

* sidewalk travel, ** air travel

7. Carbon Emissions

Relative emissions by vehicle type can be easily estimated by taking into account that each unit of energy used by an ICE diesel vehicle generates 22.5 more CO$_2$ equivalent emissions than the emissions generated by using a similar unit of energy sourced from the electric grid in Oregon (Figliozzi, 2017). The 22.5 value may change with the electricity generation profile of each city or country, i.e. it is a function of the percentage of clean or dirty sources used, and the previous value is applicable to the state of Oregon in the US. The 22.5 value used in this research accounts for diesel vehicle emissions including well-to-tank (WTT) and tank-to-wheel (TTW) emissions.

The E-van consumes almost five times less energy per unit distance than the conventional ICE van (205 wh/km vs. 1016 wh/km) and including emissions then the E-van generates approximately 112 times less CO$_2$ emissions per unit distance traveled. Table 8 shows the carbon emissions of the electric vehicles as a percentage of the carbon emissions generated by the ICE Dodge RAM. It is clear that, regardless of the autonomous vehicle type utilized, the reduction in carbon emissions is always likely to be highly significant. On average, any autonomous vehicle emits less than 2% of the emissions emitted by utilizing an ICE diesel van. But as shown by the 0.9% emissions of the E-van, the main cause of this impressive reduction in emissions is the electrification of the engines.

TABLE 8. Per Customer CO$_2$ Emissions Varying $n$ – baseline ICE Van

<table>
<thead>
<tr>
<th>Customers Served $(n)$</th>
<th>SADR+</th>
<th>MS</th>
<th>NURO</th>
<th>Udelv</th>
<th>E-van</th>
<th>Drone</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1.4%</td>
<td>0.6%</td>
<td>0.8%</td>
<td>0.9%</td>
<td>1.4%</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2.1%</td>
<td>0.9%</td>
<td>0.8%</td>
<td>0.9%</td>
<td>2.3%</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>2.5%</td>
<td>1.0%</td>
<td>0.8%</td>
<td>0.9%</td>
<td>3.7%</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>3.1%</td>
<td>1.4%</td>
<td>1.1%</td>
<td>0.9%</td>
<td>5.8%</td>
<td></td>
</tr>
</tbody>
</table>

The numbers in Table 8 will be affected by the energy sources utilized to generate electricity. For example, in the US the national average CO$_2$ rate is 947.2 lbs/MWh but there is a high degree of variability across regions. The lowest rate is found in New York and upstate region (NYUP) with an average CO$_2$ rate of 253.1 lbs/MWh and the highest rate in the Mid-west region (MROE) with an average CO$_2$ rate of 1,678 lbs/MWh. In the NYUP region the main sources of electricity generation are hydro and nuclear (account for 66% of the total) whereas in the MROE region the main energy source is coal that accounts for 64% of the total generation (EPA, 2020).
In the Pacific Norwest region, where Oregon is located, the average CO₂ rate of 639 lbs/MWh (EPA, 2020). Even assuming the dirtiest electricity generation profile in the US, CO2 emissions from a conventional van are several times higher than operational emissions from electric vehicles.

8. Grocery Store Delivery vs Customer Driving

Since electrification is a key factor to reduce carbon emissions, a question arises regarding the benefit of autonomous delivery vehicles when compared to the emissions of an electric and/or ICE passenger car. The issue of traffic generated when delivering from the supermarket or driving from home has been studied in the past (Cairns, 2005; Halldórsson et al., 2010) as well as the emissions reductions (Wygonik and Goodchild, 2012). It was concluded that with enough customers per route, supermarket delivery is more efficient than individuals driving to the store. However, previous research efforts did not include several key factors that affect the relative efficiency of store delivery vs customer driving to a store: (a) the growth of passenger EVs, (b) logistics sprawl, and (c) the number of items purchased and delivered.

In recent years there has been a growth of passenger electric vehicles in the US, mainly driven by the advent of the Tesla Model 3 (TM3). According to the EPA the energy efficiency of the TM3 is 28 kwh/100 miles or 174 wh per km (EPA, 2018); the TM3 is one of the most energy efficient passenger vehicles in the market. The Tesla Model 3 is now the most popular EV in the US and the third best-selling car in the state of California where the TM3 accounts for almost 60% of EV sales (Dow, 2020).

Urban logistics sprawl refers to increases in depot-service area distances, i.e. the relocation of depots away from customer locations. This phenomenon has been observed in many urban areas in different continents (Dablanc and Rakotonarivo, 2010; Aljohani and Thompson, 2016). Furthermore, the advent of e-grocery home delivery may increase distance delivery distances because delivery areas are not necessarily distributed around traditional industrial or wholesale land use areas (Keeling, 2019). In this paper, higher levels of logistics sprawl are represented by higher values of \(d\), the depot-service area distance. Next subsections analyze driving to store efficiency of ICE and EV vehicles with different levels of customer delivery density and logistics sprawl.

Research have shown that consumers have a strong preference for nearby grocery stores. Utilizing 1-week trip data from GPS devices and travel diaries, Liu et al. (2015) found that 64% of the grocery store trips are in the 0-1 mile range. In this section it is assumed that a store is at the center of a circular delivery region of 1 km² and that customers can be served by an autonomous vehicle, E-van, conventional van, or drive to the store. Customers are distributed uniformly across the circular region. The delivery depot can be located at the store (\(d = 0\)) or at a distance \(d = 2.5\) or \(d = 10\) kms to simulate logistics sprawl.

The relative efficiency of driving to the store vs store delivery is a function of the number of products purchased at the store or delivered to each customer. Previous studies in multiple countries have shown that the mean number of products purchased in grocery trips is approximately nine with a long right tail that reaches up to 60 products (Sorensen et al., 2017). The average number of items in an online grocery basket is likely to have a similar value (Suel et al., 2015) but the number of products purchased online is influenced by the structure of shipping fees (Lewis, 2006). When free shipping is available after reaching a minimum shopping cart value, customers tend to take advantage of this pricing feature which increases average order size. Subscription services like Amazon prime incentivize more frequent and
smaller orders since shipping is free (Belavine et al., 2017). To facilitate comparisons between customer store shopping and store delivery, the ratio $\epsilon$ between store order size and delivery order size for carbon emissions breakeven condition is estimated.

### 8.1 Breakeven Results for ICE Vehicles

The 2016 average fuel efficiency of vehicles in the US was approximately 24.7 miles per gallon according to an EPA report (2017). Utilizing this ICE efficiency, the numbers in Table 9 can be interpreted as follows: to generate the same level of carbon emissions the store order size of a customer driving an ICE is 4,064 ($\epsilon = 4064$) times larger than the order size delivered by a SADR when the depot is at the store ($d = 0$) and the SADR fleet delivers to $n = 25$ customers.

Given that the mean number of products purchased in grocery trips is approximately nine with a right tail of up to 60 products (Sorensen et al., 2017), deliveries utilizing any autonomous vehicle generate less emissions even with a long depot-delivery area distance. On the other hand, delivering with a conventional van could be more environmentally friendly only when the depot is at or close to the store and many customers are served (large $n$). Bold and a different color are utilized in Table 9 to highlight when delivering from the store is more efficient assuming an average order size of nine products.

### TABLE 9. ICE Order Size Ratio $\epsilon$ for CO$_2$ Breakeven Condition

<table>
<thead>
<tr>
<th>Distance Depot – SA (in kms)</th>
<th>Customers Served (n)</th>
<th>Vehicle Type</th>
<th>SADR + MS*</th>
<th>NURO</th>
<th>Udelv</th>
<th>E-van</th>
<th>Drone</th>
<th>Conv. Van</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>4,064</td>
<td>718</td>
<td>517</td>
<td>489</td>
<td>1,197</td>
<td>4.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>5,748</td>
<td>1,016</td>
<td>731</td>
<td>692</td>
<td>1,197</td>
<td>6.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>8,129</td>
<td>1,436</td>
<td>1,034</td>
<td>978</td>
<td>1,197</td>
<td>8.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>11,496</td>
<td>2,031</td>
<td>1,462</td>
<td>1,383</td>
<td>1,197</td>
<td>12.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d = 2.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>25</td>
<td>153</td>
<td>354</td>
<td>255</td>
<td>241</td>
<td>157</td>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>171</td>
<td>413</td>
<td>423</td>
<td>400</td>
<td>157</td>
<td>3.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>233</td>
<td>564</td>
<td>682</td>
<td>645</td>
<td>157</td>
<td>5.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>290</td>
<td>637</td>
<td>846</td>
<td>1,014</td>
<td>157</td>
<td>9.1</td>
<td></td>
<td></td>
</tr>
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<td>$d = 10$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>101</td>
<td>95</td>
<td>43</td>
<td>0.9</td>
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</tr>
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<td>53</td>
<td>149</td>
<td>187</td>
<td>177</td>
<td>43</td>
<td>1.6</td>
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<td></td>
</tr>
<tr>
<td>100</td>
<td>71</td>
<td>200</td>
<td>338</td>
<td>319</td>
<td>43</td>
<td>2.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>86</td>
<td>208</td>
<td>373</td>
<td>563</td>
<td>43</td>
<td>5.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* MS required when $d > 0$

Bold when delivering from the store is more efficient

### 8.2 Breakeven Results for Tesla 3 EV

Table 10 shows the breakeven order size $\epsilon$ when a customer drives to the store utilizing a Tesla model 3. Given that the mean number of products purchased in grocery trips is approximately nine with a right tail of up to 60 products (Sorensen et al., 2017), deliveries utilizing a conventional van are no longer more environmentally friendly.
Even electric autonomous vehicles may not be more environmentally friendly than a TM3 when there is logistics sprawl ($d > 0.0$). With free shipping and small order sizes, customers ordering 1 or 2 items per delivery, driving a TM3 to the store is on average more environmentally friendly than home delivery. Hence, it is not self-evident anymore that store delivery is more environmentally friendly than driving to the store unless all the key factors are stated: delivery vehicle type, customer vehicle type, depot-service area distance, in-store order size, and delivery order size. Bold and a different color are utilized in Table 10 to highlight when delivering from the store is more efficient assuming an average order size of nine products.

### TABLE 10. Tesla 3S Ratio Order Size Ratio $\varepsilon$ for CO$_2$ Breakeven Condition

<table>
<thead>
<tr>
<th>Distance Depot – SA (in kms)</th>
<th>Customers Served (n)</th>
<th>SADR +MS*</th>
<th>NURO</th>
<th>Udelv</th>
<th>E-van</th>
<th>Drone</th>
<th>Conv. Van</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>25</td>
<td>35</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>10</td>
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<td>11</td>
<td>0.11</td>
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<tr>
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<td>25</td>
<td>1.3</td>
<td>3.0</td>
<td>2.2</td>
<td>2.1</td>
<td>1.3</td>
<td>0.02</td>
</tr>
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<td>1.5</td>
<td>3.5</td>
<td>3.6</td>
<td>3.4</td>
<td>1.3</td>
<td>0.03</td>
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<td>2.0</td>
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<td>5.8</td>
<td>5.5</td>
<td>1.3</td>
<td>0.05</td>
</tr>
<tr>
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<td>2.5</td>
<td>5.4</td>
<td>7.2</td>
<td>8.7</td>
<td>1.3</td>
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</tr>
<tr>
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<td>1.2</td>
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<td>0.8</td>
<td>0.4</td>
<td>0.01</td>
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<td>0.4</td>
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<td>1.5</td>
<td>0.4</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.6</td>
<td>1.7</td>
<td>2.9</td>
<td>2.7</td>
<td>0.4</td>
<td>0.02</td>
</tr>
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<td>200</td>
<td>0.7</td>
<td>1.8</td>
<td>3.2</td>
<td>4.8</td>
<td>0.4</td>
<td>0.04</td>
</tr>
</tbody>
</table>

* MS required when $d > 0$

Bold when delivering from the store is more efficient

#### 9. Conclusions

This research has evaluated the potential of air and ground autonomous delivery robots to reduce CO$_2$ emissions in the delivery industry. Results show that these new autonomous vehicle types have the potential to reduce energy consumption and a vast potential to reduce CO$_2$ emissions when replacing ICE delivery vans. In many instances autonomous delivery vehicles are even more efficient than E-vans currently in the market.

In terms of energy and emissions efficiency there is no vehicle type that dominates across the board. Sidewalk autonomous delivery robots (SADRs) can greatly reduce carbon emissions with respect to ICE vans and other delivery vehicles when a mothership is not required, i.e. in scenarios where the delivery area surrounds the depot. Drones are more efficient in time constrained and low-density delivery scenarios. Road autonomous delivery robots (RADRs) are more efficient than E-vans when delivering to relatively low number of customers.

Autonomous delivery technologies can have a major role to reduce the carbon footprint of package deliveries as well as store deliveries. The analysis of the carbon footprint of grocery in-store shopping and store delivery showed that many factors must be considered: delivery vehicle type, customer vehicle type, depot-service area distance, in-store order size, and
delivery order size. With the advent of passenger EV, it is no longer true that store deliveries are always more efficient. Delivery density and logistics sprawl, passenger and delivery vehicle type, and the location of the delivery depot are also key factors.

New air and ground autonomous vehicle types may reduce carbon emissions significantly but they may not necessarily reduce on-road travel. For some autonomous vehicles the reduction of on-road travel is also accompanied by additional travel on sidewalks (SADRs) or air travel (drones). Pedestrian safety and sidewalk congestion (SADRs) or air safety and congestion (drones) could be major shortcomings of these new delivery vehicles. Distance traveled may increase significantly when autonomous vehicles are range and/or time constrained.

In summary, autonomous electric delivery vehicles can significantly reduce carbon emissions; policy makers and regulators should seriously consider their benefits. However, policy makers and freight planners should also consider tradeoffs and potential unintended consequences of new services utilizing autonomous delivery vehicles. The large-scale adoption of autonomous delivery vehicles can bring about important changes to the labor market as well as a realignment of supply chains and the growth of e-commerce fulfillment centers and dark stores (Hübner et al., 2016). Potential cost savings brought about by driverless technologies can accelerate the growth of e-commerce as well as package and grocery deliveries. Unintended consequences may also be positive and include the delivery of products with less human contact and proximity which is appealing during pandemics. In addition, future research efforts can evaluate the impact of these new vehicles and technologies on parking and curb utilization, safety, and congestion. This research focused mainly on operational emissions; future research efforts may also consider lifecycle emissions.

Acknowledgements

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Appendix

The time constraint is represented by the following equation:

\[ \tau = \left( \frac{2d}{v} + \frac{k_l}{mv_l} \sqrt{\alpha n} \right) + \frac{tn}{m} \]

Rearranging terms

\[ \tau - \frac{2d}{v} - \frac{tn}{m} = \frac{k_l \sqrt{\alpha n}}{mv_l} \]

Squaring both sides

\[ \left( \frac{k_l \sqrt{\alpha n}}{mv_l} \right)^2 = \left( \tau - \frac{2d}{v} - \frac{t}{m n} \right)^2 \]

Introducing the following notation to simplify notation

\[ y_a = \left( \tau - \frac{2d}{v} \right) \]

\[ y_{cm} = \left( \frac{t}{m} \right) \]

\[ y_{lm} = \left( \frac{k_l \sqrt{\alpha}}{mv_l} \right) \]

Replacing

\[ n y_{lm}^2 = (y_a - y_{cm} n)^2 \]

\[ y_{lm}^2 n = y_a^2 - 2 y_a y_{cm} n + y_{cm}^2 n^2 \]

\[ (y_{cm}^2) n^2 - (2 y_a y_{cm} + y_{lm}^2) n + (y_a^2) = 0 \]

\[ n^2 - \frac{(2 y_a y_{cm} + y_{lm}^2)}{(y_{cm}^2)} n + \frac{(y_a^2)}{(y_{cm}^2)} = 0 \]

Solving the second order equation
\[ n = \frac{2 y_a y_{cm} + y_{lm}^2}{2 (y_{cm}^2)} \pm \left( \frac{1}{4} \left( \frac{2 y_a y_{cm} + y_{lm}^2}{y_{cm}^2} \right)^2 - \left( \frac{y_a}{y_{cm}} \right)^2 \right)^{1/2} \]

Only the negative root is feasible

\[ n = \frac{1}{2 y_{cm}} \left[ (2 y_a y_{cm} + y_{lm}^2) - (4 y_a y_{cm} y_{lm}^2 + y_{lm}^4)^{1/2} \right] \]

Replacing the previously defined \( y \) terms

\[ n = \frac{1}{2 \left( \frac{t}{m} \right)^2} \left[ \left( 2 \left( \frac{\tau - 2d}{v} \right) \left( \frac{t}{m} \right) + \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^2 \right) \right.

\[ \left. - \left( 4 \left( \frac{\tau - 2d}{v} \right) \left( \frac{t}{m} \right) \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^2 + \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^4 \right)^{1/2} \right] \]

\[ n = \frac{m^2}{2 t^2} \left[ 2 \left( \frac{\tau - 2d}{v} \right) \left( \frac{t}{m} \right) + \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^2 \right]

\[ - \left( 4 \left( \frac{\tau - 2d}{v} \right) \left( \frac{t}{m} \right) \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^2 + \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^4 \right)^{1/2} \]

Or

\[ n = \left( \frac{2 \left( \frac{\tau - 2d}{v} \right) \left( \frac{t}{m} \right) + \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^2}{2 \left( \frac{t}{m} \right)^2} \right) - \left( \frac{4 \left( \frac{\tau - 2d}{v} \right) \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^2 + \left( \frac{k_\tau \sqrt{a}}{mv_l} \right)^4}{4 \left( \frac{t}{m} \right)^4} \right)^{1/2} \]