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BAYESIAN INQUIRY: AN APPROACH TO THE USE OF EXPERTS

by

KING G. YEE

A thesis submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY
in
SYSTEMS SCIENCE

Portland State University
1976

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TO THE OFFICE OF GRADUATE STUDIES AND RESEARCH:

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Subjective information is a valuable resource; however, decision-makers often ignore it because of difficulties in eliciting it from assessors. This thesis is on Bayesian inquiry and it presents an approach to eliciting subjective information from assessors. Based on the concepts of cascaded inference and Bayesian statistics, the approach is designed to reveal to the decision-maker the way in which the assessor considers his options and the reasons he has for selecting particular alternatives. Unlike previous works on cascaded inferences, the approach here focuses on incoherency. Specifically, it employs the use of additional information to revise and check the estimates. The reassessment may be done directly or indirectly. The indirect procedure uses a second order probability or type II distribution. An algorithm utilizing this approach is also presented. The methodology is applicable to any number of assessors. Procedures for aggregating and deriving surrogate distributions are also proposed.

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CHAPTER I

INTRODUCTION

Subjective information is a valuable resource often ignored or used in such a manner that its value is reduced to little or no consequence; hence there is a need to develop a formal procedure for its use. There are many situations in which traditional and direct methods for obtaining information cannot be used. For example, the president of a large corporation cannot spend all his time examining empirical data such as sales, production rates, inventory figures, etc. His aides provide him information for decision making. Similarly, in assessing a political situation, a politician or statesman will rely on his own experiences and information from other sources.

MAIN PURPOSE

A Bayesian approach to inquiry makes use of empirical and subjective information. The purpose of this thesis is to mathematically formalize and develop an algorithm for structuring inquiry. The proposed approach is based on cascading principles so that inferences may be derived indirectly. A feedback step utilizing additional information is used to check and/or revise the estimates.

SUMMARY OF REMAINING CHAPTERS

A summary of the thesis is as follows:

Chapter II

A review of personalistic and non-personalistic interpretation of probability is presented. Each of the three schools of probability theory is briefly discussed separately. This is followed by a discussion on the differences between Bayesian and non-Bayesian approaches to inference. The final section in this chapter concerns information concepts relating to Bayesian and non-Bayesian inquiry.

Chapter III

Most of the research in Bayesian information processing and inference is of recent origin. Edwards (1960) designed an optimization model using Bayes' theorem in a man-computer interacting mode. However, this approach to the use of Bayes' theorem poses many unavoidable difficulties, that is, in $P(H|D) = P(H)P(D|H)/P(D)$, the requirement that D be measurable is difficult to fulfill in most situations. Dodson (1961) introduced a non-mathematical model of Bayes' theorem which circumvented this requirement. This model has formed the basis for today's research in cascaded inference. Included in this chapter is a review of a number of findings that are relevant to the proposed methodology.

Chapter IV

General systems theories range from the very formal to the informal types. The aim of this chapter is to point out the position and role of Bayesian inquiry as an instrument or tool in general systems.

Chapter V

A framework for Bayesian inquiry is introduced. Next, formal mathematical models are developed and an algorithm for using them is discussed. This algorithm is then applied to an actual problem. The results are presented along with an analysis of the advantages and disadvantages of the method.

Chapter VI

In the concluding chapter some of the problems encountered in the research and development of the thesis are listed with suggestions for further research.

SIGNIFICANCE

The significance of this study lies in the provision of a formal and methodologically sound framework for using experts. This study clarifies the heuristic art of inquiry and is a significant step toward disciplining Delphi. Once this disciplined exercise has been completed, it should open up the Delphi technique to analytical studies in such areas as comparative social and psychological controls.

CHAPTER II

BACKGROUND: BAYESIAN INFERENCE

PROBABILITY CONCEPTS

The purpose of this chapter is to review the interpretations of probability in Bayesian and non-Bayesian approaches to inference. This discussion will provide the foundation for the remainder of the thesis.

The most widely held theory of probability is the empirical, objective or frequency concept. This interpretation identifies probability as the observed behaviour of repetitive events. The objectivist interpretation of probability is summarized by Fisher[1]

> ...probability is the most elementary of statistical concepts. It is a parameter which specifies a simple dichotomy in an infinite hypothetical population, and it represents neither more nor less than the frequency ratio which we imagine such a population to exhibit.

In the necessary or logical concept, probability statements are <u>not</u> empirical statements. Instead, probability is a <u>logical</u> relationship between a proposition and a body of evidence. For a given statement S and a body of evidence E, there is <u>one and only one</u> degree of belief P, which S may have given the evidence E.[2] Jeffreys summarized

[1] R. A. Fisher, <u>Statistical Methods for Research Workers</u>, 13th ed. (New York: Hafner Publishing Company, Inc., 1959), p. 9.

[2] Henry E. Kyburg, Jr. and Howard E. Smokler, eds., <u>Studies in Subjective Probability</u> (New York: John Wiley and Sons, Inc., 1964), p. 5.

this interpretation,[3]

> When we make an inference beyond the observational data, we express a logical relation between the data and the inference . . . It [the relation] assesses the support for the inference, given the data, . . . This relation between a set of data and a conclusion is called probability.

This interpretation of probability as a direct extension of logic has never been and is not active in shaping statistical opinion.[4]

The subjective concept is distinguished from the necessary concept by its denial that there is one and only one probability which represents a relation between a statement and a body of evidence. For a subjectivist, probability values represent the degree of beliefs that an individual has in a given statement. This value is not uniquely determined and may differ from person to person. This concept is often labeled as the personalistic interpretation of probability.

The subjective view of probability was originated by Jacob Bernoulli and systematically developed by Laplace. Outstanding works by de Finetti,[5] Good[6] and Savage have contributed much to the development and acceptance of this theory. The personalistic interpretation is

[3]Harold Jeffreys, Scientific Inference, 2nd ed. (Cambridge: Cambridge University Press, 1957), p. 22.

[4]L. J. Savage, "The Foundations of Statistics Reconsidered," in Studies in Subjective Probability, eds. Henry E. Kyburg, Jr. and Howard E. Smokler (New York: John Wiley and Sons, Inc., 1964), p. 176.

[5]Bruno de Finnetti, "Foresight: Its Logical Laws, Its Subjective Sources," in Studies in Subjective Probability, eds. Henry E. Kyburg, Jr. and Howard E. Smokler (New York: John Wiley and Sons, Inc., 1964), p. 93-158.

[6]I. J. Good, The Estimation of Probabilities (An Essay on Modern Bayesian Methods), Research Monograph No. 30 (Cambridge: The M.I.T. Press, 1965).

summarized by Savage.[7]

> Personalistic views hold that probability measures the confidence that a particular individual has in the truth of a particular proposition, for example, the proposition that it will rain tomorrow. These views postulate that the individual concerned is in some ways 'reasonable' but they do not deny the possibility that two reasonable individuals faced with the same evidence may have different degrees of confidence in the truth of the same proposition.

Although individuals may have different degrees of belief for a proposition probability assignments to the set of alternatives must be coherent as well as consistent[8], and furthermore this set or body of beliefs must be rational.

Coherence of a body of beliefs may be explained in terms of bets. For a person obeying the postulate of coherence, it is impossible to set up a series of wagers which ensures that the bettor will lose regardless of the outcome. Anyone engaging in such a gamble would be acting irrationally or incoherently.

Although differences exist among the three schools of probability, there is an important commonality among them.

> Considering the confusion about the foundations of statistics, it is surprising and certainly gratifying, to find that almost everyone is agreed on what the purely mathematical properties of probability are. Virtually all controversy therefore centers on questions of interpreting the generally accepted axiomatic concept of probability, that is, of determining the extramathematical properties of probability.[9]

[7]L. J. Savage, The Foundation of Statistics (New York: John Wiley Sons, Inc., 1954), p. 3.

[8]A number of scoring rules have been developed toward fulfillment of the consistency requirements; see R. L. Winkler, "Scoring Rules and the Evaluation of Probability Assessors," Journal of the American Statistical Association 64 (1969): p. 1073-1078.

[9]Savage, The Foundation of Statistics, p. 2.

BAYESIAN AND NON-BAYESIAN APPROACHES TO INFERENCE

A Bayesian statistician contends that probability values represent a degree of belief. And Bayes' rule provides the formal mechanism for revising probabilities in the light of new information. The probability P(H) of a certain proposition H is revised to P(H|D) when the event D is observed,

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

The terms P(D|H) and P(H) in the numerator are called the likelihood function and prior distribution. P(H|D) is the posterior distribution.

There is no disagreement on the mathematics of Bayes' theorem. The debate is on the interpretation and usage of information in the theorem. For non-Bayesian analyses, the use of prior information is uncontroversial only if the prior information is substantiated by empirical evidence. With Bayesian theory, any and all available information, both subjective and empirical, has relevance in statistical inference. All available information is incorporated *formally* in the analysis of the prior distribution. This is in marked contrast with non-Bayesian analyses where subjective information is generally used informally and often arbitrarily. Formal techniques for establishing prior distributions must be based on available sample evidence.

Bayesian inference can be based on prior subjective and sample information. There is no need to justify inferences in terms of

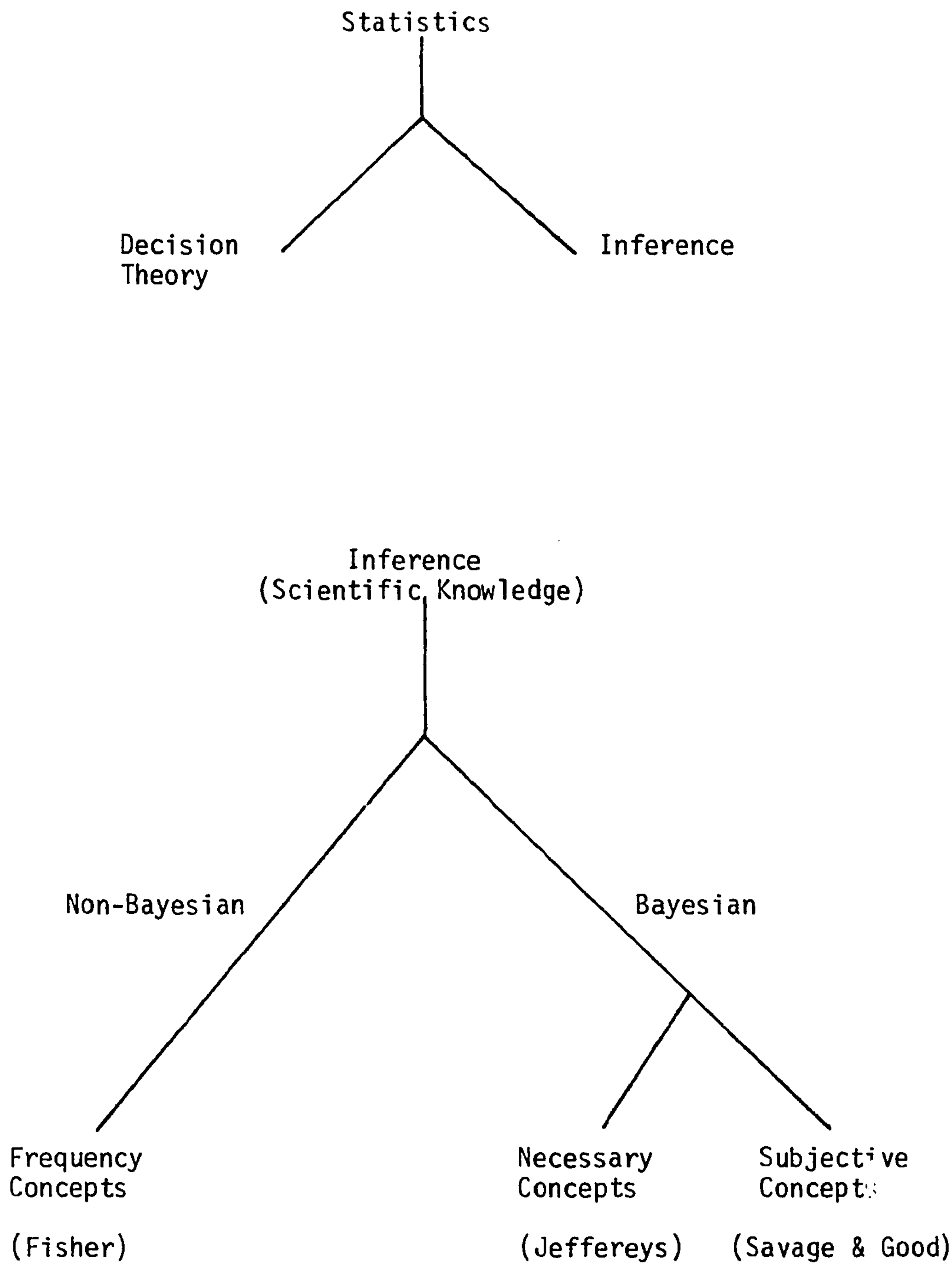


Figure 1. Bayesian and Non-Bayesian Inference

behavior in repeated samples. This is not to say that the results from repeated samples are not of interest; on the contrary, the better the sample evidence, the better the Bayesian estimation.

Another feature of Bayes' theorem deserves comment. It is a learning model and invaluable in accomplishing what Jeffreys and others consider a major objective of science, namely, learning from experience.[10]

Under the informationless state, i.e., diffuse or uniform prior distribution, Bayesian and classical procedures give identical results. If the prior distribution is not diffused, the results will be quite different.

There are differences amongst Bayesians themselves, as Good notes.

> Several different kinds of Bayesians exist, but it seems to me that the essential defining property of a Bayesian is that he regards it as meaningful to talk about the probability P(H|E) of a hypothesis H, given evidence E. Consequently, he will make more use of Bayes' theorem than a non-Bayesian will. Bayes' theorem itself is a trival consequence of the product axiom of probability, and it is not a belief in this theorem that makes a person a Bayesian. Rather it is a readiness to incorporate intuitive probability into statistical theory and practice, and into the philosophy of science and of the behavior of human, animals, and automata, and in an understanding of all forms of communication, and everything.
>
> The mathematics used by a Bayesian can be interpreted without agreeing with his philosophy. . . An extreme Bayesian believes that every intuitive probability is precise, whereas less extreme Bayesian regard intuitive probabilities as only partially ordered so that each probability merely lies in some interval of values. . . One is more or less a Bayesian depending on the precision with which one is prepared to make intuitive probability estimates. . .[11]

[10]Arnold Zellner, "The Bayesian Approach and Alternatives in Econometrics, " in Frontiers of Quantitative Economics, ed. Michael D. Intriligator (London: North-Holland Publishing Company, 1969), p. 180.

[11]Good, The Estimation of Probabilities (An Essay on Modern Bayesian Methods), p. 8-10.

SUBJECTIVE INFORMATION

The motivation for using any and all information in a Bayesian approach is clear. However, the concept of information in Bayesian analysis needs clarification. Information is a loose term and may be viewed as the evidence which could lead to a reduction of uncertainty in a decision situation or a change in belief. The former may be considered quantitative and the latter qualitative information (see Table I).[12] From a Bayesian viewpoint, the change in belief is a more general notion than a reduction of uncertainty. Reduction of uncertainty is a special case of change in belief. Information concepts based on relative frequency is a well developed theory and is attributed to Shannon and Weaver. In contract, subjective information theory is relatively undeveloped.

For the Bayesian approach to qualify as an instrument of inquiring systems, it may be necessary to consider as a prerequisite an unambiguous definition of subjective information. Following Jamison[13] and Roby,[14] this presents no difficulty. Let b be a person's belief about a set of m mutually exclusive and collectively exhaustive possible states of nature, $e = \{e_1, e_2, \ldots, e_m\}$. Define an $m - 1$

[12]Dean Jamison, "Bayesian Information Usage," in Information and Inference, eds. Jaakko Hintikka and Patrick Suppes (Holland: D. Reidel Publishing Company, 1970), p. 29.

[13]Ibid., p. 30-31.

[14]R. Roby, "Belief States and the Uses of Evidence," Behavioral Science 10 (1965): p. 255-270.

TABLE I

THEORIES OF INFORMATION

Concept of Information	Concept of Probability: Relative Frequency	Concept of Probability: Subjective
Change in Belief	CR	CS
Reduction of Uncertainty	RR	RS

CR: Change in belief as reflected in the change in relative frequency.

RR: Reduction of uncertainty as reflected in the change in relative frequency.

CS: Change in belief as reflected in the change in subjective probabilities.

RS: Reduction of uncertainty as reflected in the change in subjective probabilities.

dimensional simplex $\mathcal{E}$ in an m dimensional space in the following manner: $\mathcal{E} = \left\{ b \,\middle|\, \sum_{i}^{m} b_i = 1,\ 0 \leq b_i \leq 1, \text{ for } 1 \leq i \leq m \right\}$. An intuitive interpretation of the vector $b = \{b_1, \ldots, b_m\}$ is a probability distribution over the state of nature with b_i the probability of the i^{th} state, $P(e_i)$, hence $b = \{P(e_1), \ldots, P(e_m)\}$. $\mathcal{E}$ is the set of all possible probability distributions over the m states of nature.

Let $b^{\bullet}$ be a person's initial belief before he receives some information IF, and b' his belief afterwards. The amount of relevant information IF received between $b^{\bullet}$ and b' may be expressed by,

$$\inf (IF) = \left| b^{\bullet} - b' \right| = \sum_{i}^{m} (b_i^{\bullet} - b_i')^2$$

or better still,

$$\inf (IF) = \frac{m\sqrt{m - 1}}{2\,(m - 1)} \left| b^{\circ} - b' \right|$$

This measure of information, like Shannon's, is sensitive to m.

Thus, subjective information can be discussed in a clear and formal way. Furthermore, the Bayesian measure seems to be the only method available for quantifying the otherwise non-quantifiable aspects of information.

CHAPTER III

APPRAISAL OF EXISTING BAYESIAN INFERENCE MODELS

INTRODUCTION

Bayesian information processing is in its infancy, due partly to recent acceptance of the subjective or Bayesian view of probability. Edwards' (1962) Probabilistic Information Processing (PIP)[15] was a major contribution. It was one of the first attempts to apply Bayes' theorem in a man-computer interacting mode. The use of the standard Bayes' model has a limitation: it requires that the data set be available. Reality is too rich to permit this simplification.

Dodson (1961) presented a modified Bayes theorem (MBT) based on the notion of expectation.[16] Gettys and Willke (1969)[17] published a mathematical representation of Dodson's model which relaxed the certainty requirement of the Bayes' theorem. Gettys (1969) in an

[15] W. Edwards, "Dynamic Decision Theory and Probabilistic Information Processing," Human Factors 4 (1962), p. 59-73.

[16] J. D. Dodson, "Simulation system design for a TEAS simulation research facility," (Los Angeles: Planning Research Corporation, November 1961, No. AFCRL-1112, PRC R-194).

[17] Charles F. Gettys and T. A. Willke, "The Application of Bayes's Theorem When the True Data State is Uncertain," Organizational Behavior and Human Performance 4 (1969), p. 125-141.

unpublished work[18] derived another model of Dodson's MBT for independent multiple inputs.[19] A review of these contributions is discussed in the following sections.

PROBABILISTIC INFORMATION PROCESSING SYSTEMS (PIP)

Edwards (1962) introduced the notion of PIP because of his concern about the optimal use of information in military and business situations. The motivation behind designing this system was to relieve human information processors from the routine calculations involved in Bayes' theorem. In this model, men are taught to estimate the probability that a data set D would be observed given a specific hypothesis, i.e., $P(D|H)$. The program then integrates these estimates, $P(D|H)$ across the data and across the hypotheses by means of Bayes' theorem. The resulting output is a set of a posteriori probabilities, $P(H|D)$. The model permits the talents of both man and machine to complement each other and to be used to the best advantage. Bayes' theorem is an optimal way of aggregating information, whether from one source or from many sources. The usefulness of the model is limited because D is assumed to be known. This requirement is obviously too restrictive. Tversky and Kahneman (1974)[20] report that man may be as poor at

[18]Charles F. Gettys, "A case where Dodson's MBT is appropriate," (Mimeo copy, University of Oklahoma, 1969).

[19]The interpretation of the unpublished works by Dodson and Gettys is based on the publication by Gettys and Willke, because the former remains inaccessible.

[20]A.Tversky and D. Kahneman, "Judgment under Uncertainty: Heuristics and Biases," Science 185 (1974): p. 1124-1131.

estimating P(D|H) values as he is at estimating a posteriori probabilities. This problem, however, may be alleviated with formal training.[21]

MODIFIED BAYES' THEOREM

In Dodson's model, uncertainty concerning the data or primary event is incorporated into the posterior probability through the notion of expectation. The uncertainty in the data is denoted by w.

$$\text{Expectation } (H_\alpha) = \sum_i \psi(E_i)P(H_\alpha | E_i) \qquad (1)$$

In this model, the elements in the "knowledge" state $E = \{E_1, \ldots, E_m\}$ are assumed to be mutually exclusive and one of these must occur. Each E_i is a "posterior" probability, i.e. $\psi(E_i) = P(E_i | w)$. The above equation (1) may be expanded,

$$P(H_\alpha | E_i) = \frac{P(H_\alpha)P(E_i | H_\alpha)}{\sum_j P(H_j)P(E_i | H_j)} \qquad (2)$$

$$\text{Since, } P(E_i) = \sum_j P(H_j)P(E_i | H_j)$$

[21]A study by Robert L. Winkler ("The Assessment of Prior Distributions in Bayesian Analysis," Journal of American Statistical Association (September 1967), p. 777-795.) demonstrated that with preliminary training in probability assessment, the results showed marked improvement.

Therefore,

$$\text{Expectation } (H_\alpha) = \sum_i \psi(E_i) \frac{P(H_\alpha)P(E_i|H_\alpha)}{\sum_j P(H_j)P(E_i|H_j)} \tag{3}$$

Thus uncertainty is expressed in the form of $\psi(E_i)$.

A more explicit form of MBT was derived by Gettys and Willke (1969). Since w is assumed to have occurred, but may not have been observed, E_i is conditional on w.

$$\psi(E_i) = P(E_i|w) \tag{4}$$

Also

$$\text{Expectation } (H_\alpha) = P(H_\alpha|w) \tag{5}$$

Assuming Markovian conditional independence, the explicit form of Dodson's MBT is,

$$P(H_\alpha|w) = P(H_\alpha) \sum_i \frac{P(E_i|w)P(E_i|H_\alpha)}{P(E_i)} \tag{6}$$

The above model was generalized for multi-inputs, $w^1, \ldots, w^k$, where each w^k leads to a distinctive set of knowledge states $\{E_i^k\}$.

$$P(H_\alpha|w^1, \ldots, w^k) = P(H_\alpha|w^1, \ldots, w^{k-1}) \sum_{i=1} \frac{P(E_i^k|w^k)P(E_i^k|H_\alpha)}{P(E_i^k)} \tag{7}$$

RELEVANT FINDINGS

A number of experimental findings illustrating the justification in support of the proposed methodology are summarized below. One such set of findings by Gettys et al.[22] indicates that results can be improved by decomposing multi-state inferences into a series of single-stage inferences and then combining them with an appropriate algorithm. A second set is due to Youssef who concludes:

> Subjective cascaded (multi-step) inference was less conservative than non-cascaded (one-step) inference at all diagnostic levels. These results support the generality of the hypothesis that unless diagnosticity is very low, cascaded inference is more nearly optimal than its non-cascaded controls.[23]

Conservatism in probablistic inferences (single-stage) was repeatedly found in many earlier studies; however, this is not a valid assumption for cascaded inference.[24] Finally, Winkler,[25] in his experiments with questions regarding contemplation of future samples and hypothetical lotteries, concludes that it is often useful to consider the judgment

[22] Charles Gettys et al., "Multiple State Probabilistic Information Processing," Organizational Behavior and Human Performance 10 (December 1973): p. 374-378.

[23] Zakhour I. Youssef, "The Effects of Cascaded Inference on the Subjective Value of Information," Organizational Behavior and Human Performance 10 (December 1973): p. 359-363.

[24] David A. Schum, "Concluding Comments," Organizational Behavior and Human Performance 10 (December 1973): p. 427.

[25] Robert L. Winkler, "The Quantification of Judgment: Some Experimental Results," Journal of American Statistical Association, Proceedings (1967): p. 386-394.

of a number of experts rather than one. Experimental findings in Bayesian information processing have also emphasized the advantage of using expertise.[26] In a later finding, Winkler concluded that it was not unreasonable to ask an assessor to give probability estimates. Also, with training and experience, inconsistencies in their estimates were significantly reduced.

Beach, in a study comparing man as probabilistic information processor with the normative Bayes' model, concluded that:

> . . . Ss [assessors] possess a rule for revising subjective probabilities that they apply to whatever subjective probabilities they have at the moment. . . . As has been amply demonstrated, the Ss [assessor's] revision rule is essentially Bayes' theorem. That is to say, Ss' revision can be predicted with a good deal of precision using Bayes' theorem as the model.[27]

Another useful finding is in the form of empirical results found in Delphi methodology;[28, 29, 30] and in personality theory and social

[26]C. Stael von Holstein, Assessment and Evaluation of Subjective Probability Distribution (Stockholm: The Economic Research Institute, 1970), p. 22.

[27]L. R. Beach, "Accuracy and Consistency in the Revision of Subjective Probabilities," IEEE Transactions on Human Factors in Electronics 7 (March 1966): p. 29-36.

[28]An excellent collection of articles on Delphi methodology may be found in The Delphi Method: Techniques and Applications, edited by Harold A. Linstone and M. Turoff (New York: Addison-Wesley Publishing Co., 1975).

[29]Joseph P. Martino, "The Lognormality of Delphi Estimates," Technological Forecasting and Social Change 1 (1970): p. 355-358.

[30]Norman C. Dalkey, The Delphi Method: An Experimental Study of Group Opinion (Santa Monica: Rand Corporation, RM-5888-Pr, June 1969).

psychology.[31] Studies in these areas demonstrated the lognormal characteristics of human responses. The theoretical justification in these findings comes from combining the well-known Weber-Fechner law[32] and the theory that individual judgment can be stated as a simple additive combination of informational inputs.[33, 34]

[31]D. Cartwright, "Risk Taking by Individual and Groups: An Assessment of Research Employing Choice Dilemmas," Journal of Personality and Social Psychology 20 (1971): p. 361-378.

[32]M. F. M. Osborne, "Brownian Motion in the Stock Market," in The Random Character of Stock Market Prices, ed. P. H. Cootner (Cambridge: The M. I. T. Press, 1964), p. 100-128.

[33]N. H. Anderson, "A Simple Model for Information Integration," in Theories of Cognitive Consistency: A Sourcebook, eds. R. P. Abelson et al. (Chicago: Rand McNally, 1968), p. 731-743.

[34]Devendra Sahal, "On the Lognormality of Bayesian Information," Portland State University, Portland, Oregon, 1974.

CHAPTER IV

RELEVANCE TO GENERAL SYSTEMS THEORY

INTRODUCTION

General systems theory may be viewed as an attempt to stimulate, organize, understand and control "systems" and their components. The term systems is generally defined as a composition of elements which are related and form a whole.

An important trend in general systems theory is the development of methods which permit the construction of conceptual systems where interactions between elements are sufficiently, but not completely, incorporated.[35] According to Klir, there is need to incorporate probabilistic concepts in systems methodologies. The existing theories of Klir, Mesarovic and Wymore are generally inadequate because of this lack of probabilistic consideration, which is a sufficient justification for the present study. Wymore has recognized the incompleteness in his theory and the need for development in the direction of probability theory. To quote Wymore himself:

[35] George J. Klir, ed., Trends in General Systems Theory (New York: John Wiley and Sons, Inc., 1972), p. 6.

> It is quite clear that in the definition or imposition of measures of effectiveness on various sets of systems, probability measures will play an important part, but these will be extremely arbitrary, based not only on empirical data but also on subjective appraisals as well as on the desirability of outcomes.[36]

Klir's approach to general systems is based on the identification or classification of problems according to their fundamental systemic traits.[37] His approach, however, presupposes the availability of empirical information so that probability considerations are based on a traditional frequency interpretation. Although Sutherland's (1973)[38] approach is neither theoretical nor methodological like those of Klir, Mesarovic, and Wymore, his approach explicitly shows the importance and role of Bayesian analysis as a heuristic instrument in general systems.

RELEVANCE TO GENERAL SYSTEMS

The role of Bayesian analysis in general systems is perhaps best illustrated by Sutherland's concepts of analytical ideal-types. In this approach, it is presumed that all systems problems possess inherent

[36]Wayne Wymore, "A Wattled Theory of Systems," in Trends in General Systems Theory, ed., George Klir (New York: John Wiley and Sons, Inc., 1972), p. 291.

[37]George J. Klir, An Approach to General Systems Theory (New York: Van Nostrand Reinhold Company, 1969).

[38]John W. Sutherland, A General Systems Philosophy for Social and Behavioral Sciences (New York: George Braziller, Inc., 1973).

properties which can be classified as either deterministic, moderately stochastic, severely stochastic or indeterminate.[39]

Associated with each of the analytical-types are instrumental categories which are expected to be the most effective and efficient in dealing with problems that are classified by the four ideal-types (see Table II). By using instruments that are constantly congruent to the properties of the system, that is, using those instruments for which the problem fits at that stage of the analysis, it becomes possible to minimize potential errors.

Normatively, the analytic process proceeds from indeterminacy to determinacy. In practice, a system or problem is expected to respond to analytical efforts. Just how far the analysis can go from indeterminacy to determinacy depends on the inherent properties of the problem. However, by adhering to systems congruence, the analytic process will approach optimal efficiency (see Table II).

A Bayesian approach is in no way opposed to any of the existing general systems theories; rather, it complements them. This is illustrated in Table III.

[39]Ibid.

TABLE II

ANALYSIS PROCESS

ANALYTICAL STATE	INDETERMINACY	SEVERELY STOCHASTIC		MODERATELY STOCHASTIC	DETERMINACY
ANALYTICAL MODE →	Development of Heuristics →	Development of Deductive Hypotheses →	Validation and Refinement →	Employment of Inductive Models →	Development of Optimization Algorithm

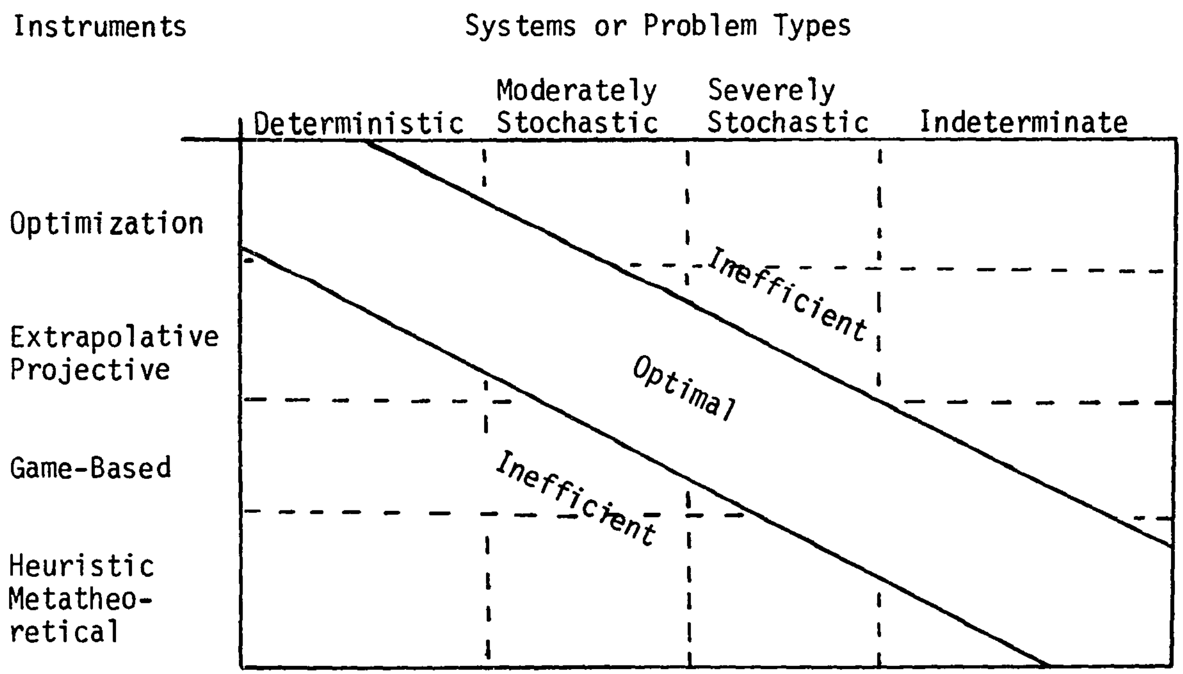


TABLE III

BAYESIAN INQUIRY IN SYSTEMS THEORETIC PERSPECTIVE

SYSTEM OR PROBLEM TYPE	ANALYTICAL PRECEDENTS AND PROCEDURES	ASSOCIATED ANALYTICAL INSTRUMENTS
(1) Deterministic		
...data bases and causal relationships are highly specific and accurate with respect to the phenomenon at hand.	There is expected to be one and only one 'probable' event...generally a simple replicate of present and/or past events (or parametric value). Hence we search for one-answer projection or transform functions which 'fit' the temporal and/or cross-sectional data base available to us.	-Finite -State System Analysis Models -Linear Programming and Max-Min Models -Optimization Models -Regression, correlation, time series and spectral analysis techniques, with error treated exogenously (if at all).
(2) Moderately Stochastic		
...basic causal relationships are	Here we are concerned with the possibility of	-Range estimation techniques (e.g., probabilistic projection models).

probably a-priori known (and accurate), but data base is incomplete... hence the parametric uncertainty.	a single state variable or parameter assuming some value within a pre-specified, manageable range.	-Numerical approximation techniques (e.g., Taylor Series). -Finite State systems analysis techniques. -Shock Models (e.g., those semi-deterministic econometric constructs which treat error nonspecifically).
(3) Severely Stochastic		
...data bases might be fairly good but causal models are ill-defined or entity is inherently capable of assuming any one of some set of pre-definable states.	Here we might consider a range of significantly different events which might occur, each of which will lead to highly differentiated 'futures.' Empirical investigation will be used to 'converge' on one or another of the futures.	-Game-based Models -Stochastic systems analysis techniques. -Adaptive or dynamic (usually Bayesian based) programming algorithms.

(4) Indeterminate

...there is no relevant data base and the inherent causal relationships for the phenomenon at hand are a-priori unallegorizable. Typical examples are found in the areas of futureology, i.e., technological forecasting and technological assessments.

Here, lacking pre-specified alternative outcomes, futures must be deduced by references to any generalized, empirically-unvalidated theoretical constructs which might exist.

The usual strategy is to gradually narrow the range of alternatives so that the indeterminate a-priori state may gradually be transformed into a more actionable stochastic situation.

With so many structural and relational (dynamic) unknowns, the analyst can only use the most gross analytical instruments:

- -deductive analysis leading to the generation of broadly-defined possible future 'states' (qualitative or categorical alternatives).
- -stochastic simulation methods.
- -Bayesian analysis as a learning or heuristic instrument.

CHAPTER V

A BAYESIAN APPROACH

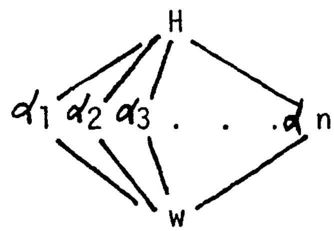
INTRODUCTION

In the previous chapters, we reviewed and presented the needs for a Bayesian approach to inquiring systems. In this chapter, we develop a Bayesian model and an algorithm for structuring inquiry. The algorithm is demonstrated with an example. This is followed with a discussion of the results along with an analysis of the advantages and disadvantages in the proposed method.

Current methodologies for the utilization of experts--ranging from a brainstorming session to more sophisticated approaches--all tend to mask the expert's use of data from the user or decision-maker. This masking robs the decision-maker the basic instrumentality he seeks--the manner by which the expert arrives at his opinion. The approach presented here is designed to take advantage of the expert's knowledge and reveal to the decision-maker the way in which the expert approaches his options and reasons he has for selecting particular alternatives.

The proposed model is built on cascading principles,[40] that is, the difficulty in assessing the connections--the causal relationships--between an immeasurable primary event and a target set[41] is made easier by decomposing the problem and using intermediate states. Unlike previous works on cascaded inferences,[42, 43] the approach here focuses on incoherency.[44] Specifically, the approach utilizes

[40]The sequential nature of cascading can be described by an inference tree.



The tree is structured such that one (node) knowledge state is below another only if the inference from the node is required to make an inference about the higher node. Thus, the inference at any stage is based on the inference about the knowledge states subordinate to it. By assuming the occurrence of the primary event w, we can begin the inference process using the information on α, that is, $P(\alpha_i|w)$. Each $P(\alpha_i|w)$ is used as inputs to the next stage, $P(H_j|\alpha_i)$.

[41]A primary event w is termed immeasurable if we cannot accurately derive P(w) by conventional methods. The term "target set" is used here to mean a set of hypotheses or alternatives about which information and judgments are sought from experts.

[42]Dodson, "Simulation systems design for a TEAS simulation research facility."

[43]Gettys and Willke, "The Application of Bayes's Theorem when Data State is Undertain," p. 125-141.

[44]The term "incoherency" relates to the mathematical errors made in expressing the estimates. The term "consistency" relates to estimates made by an expert that correspond to his inner beliefs.

additional information concerning the elements in either the knowledge state set or the target set to improve the coherency of the estimates. The reassessment may be done directly or indirectly using a type II distribution. A type II probability distribution represents the uncertainty about the initial estimates.

Like the more sophisticated techniques, such as Delphi, the approach here also seeks to get judgments and opinions from several experts. However, in addition to this capability, the method structures the inference paths for the expert and formalizes the inquiry process. This provides more information in a form that allows easy identification of the salient factors of agreement and disagreement among the experts. It thus can identify the "gray area," i.e., the uncertainty aspect of the problem. Such capabilities provide the decision-maker with a better chance of making the "best" decision.

AN APPROACH

The proposed approach is as follows. Let $H = \{H_1, H_2, \ldots, H_m\}$ be a set of target hypotheses. Following Savage,[45] Dodson[46] and others, we consider the expectation of H as a conditional probability of a given primary event w, representing any and all information.

[45]Savage, The Foundation of Statistics.

[46]Dodson, "Simulation systems design for a TEAS simulation research facility."

$$E(H_j) = P(H_j|w), \text{ for each } j. \tag{8}$$

Since w is immeasurable, inference may be made easier using α, where $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a set of mutually exclusive and exhaustive, subjective knowledge states due to w (fig. 2).

By assuming Markovian properties, the above system (fig. 2) may be represented,

$$P(H_j|w) = \sum_{j} P(H_j|\alpha_i)P(\alpha_i|w), \text{ for each } j. \tag{9}$$

Obviously, included in w, <u>inter alia</u>, is the feeling or belief about the elements within the set under consideration.[47] The above equation may be viewed as follows: (1) the term $P(H_j|\alpha_i)$ may be viewed as a prior decision rule and (2) once the judgments on the $P(\alpha_i|w)$ are found, hence given by the assessor, then $P(H_j|w)$ becomes the current, a posteriori, decision rule.

Suppose additional information from a primary event r concerning the elements in α becomes available, then a reassessment of α may be considered (fig.3).

[47]Meadows, Forrester and others refer to this as subjective causality or assumed causality.

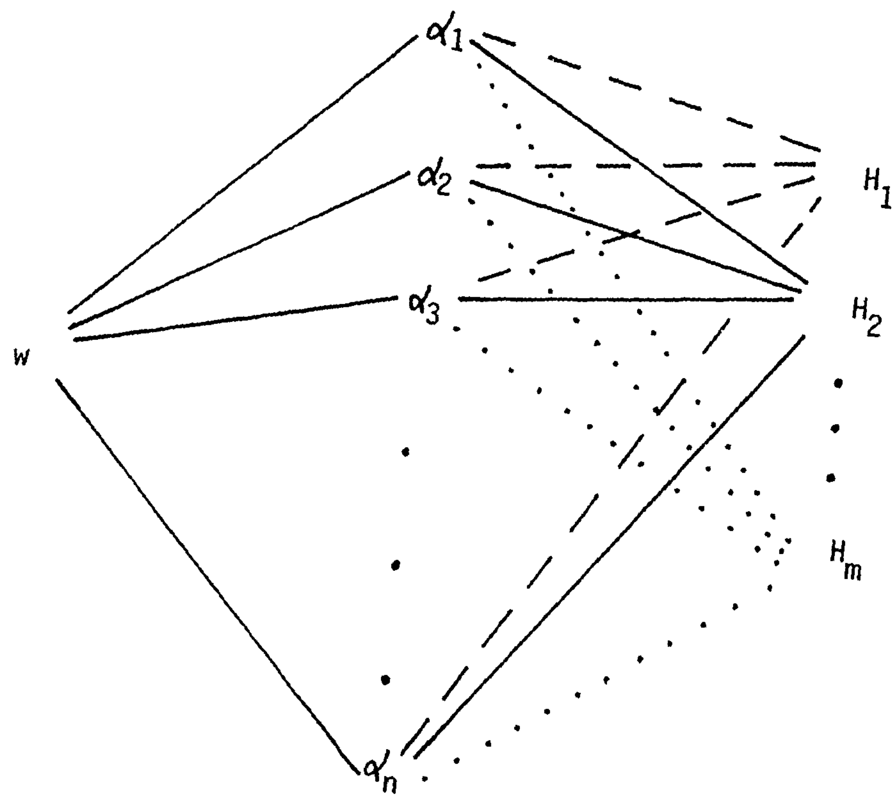
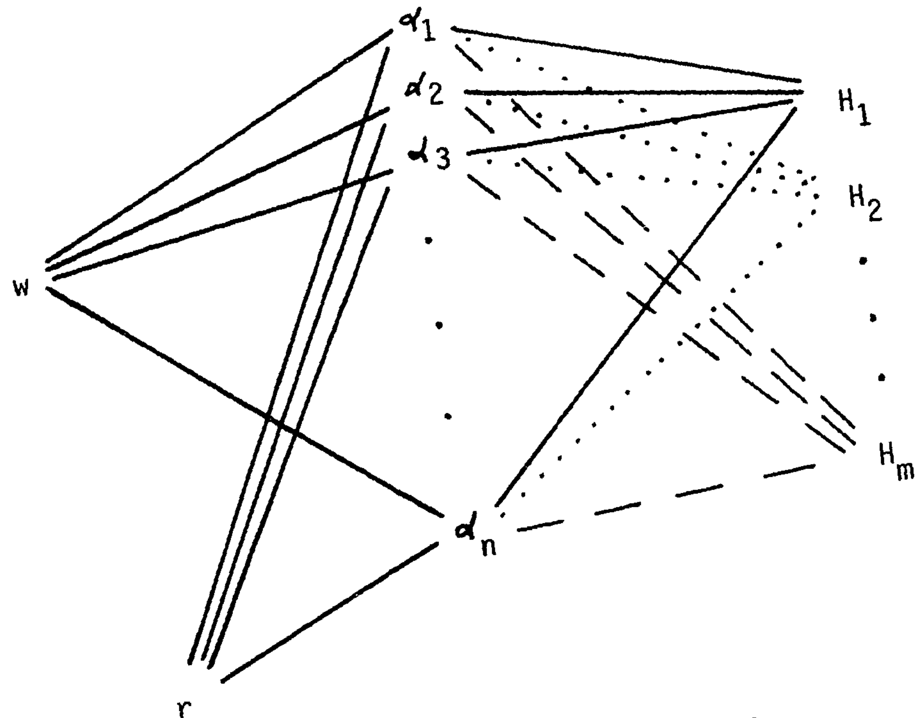


Figure 2. Cascaded Inference



<u>Figure</u> 3. Additional information.

This system with Markovian properties may be represented by,

$$P(H_j | r,w) = \sum_i P(H_j | \alpha_i ,w)P(\alpha_i | r,w)$$

Since

$$P(H_j | \alpha_i ,w) = \frac{P(H_j | w)P(\alpha_i | H_j ,w)}{\sum_j P(H_j | w)P(\alpha_i | H_j ,w)}$$

We have

$$P(H_j | r,w) = \sum_i \frac{P(H_j | w)P(\alpha_i | H_j ,w)}{\sum_j P(H_j | w)P(\alpha_i | H_j ,w)} P(\alpha_i | r,w), \text{ for each } j. \qquad (10)$$

The change in the probability of any item in the set is simply,

$$\Delta P(\alpha_i) = P(\alpha_i | r,w) - P(\alpha_i | w), \text{ for each } i. \qquad (11)$$

$$\Delta P(H_j) = \sum_i P(\alpha_i | r,w)P(H_j | \alpha_i ,w) - P(\alpha_i | w)P(H_j | \alpha_i), \text{ for each } j. \qquad (12)$$

This formulation (equation 10) may be easily contrasted with Turoff's cross impact model[48] for deriving coherent estimates. However, the above formulation takes into account the combined effects of all the α_i's on H.

Suppose new information from a primary event s becomes available and concerns only H, then the coherency of the estimates on H may be assessed directly. Let $\beta = \{\beta_1, \beta_2, \ldots, \beta_l\}$ be the subjective knowledge states due to s (fig. 4).

[48] Turoff assumed the probability of an event to be a function of the remaining n-1 probabilities, $P_i = P_i(p_1, p_2, \ldots, p_n)$, where p_i is the probability of event i. The changes in the estimates due to additional new information are expressed by the difference equation

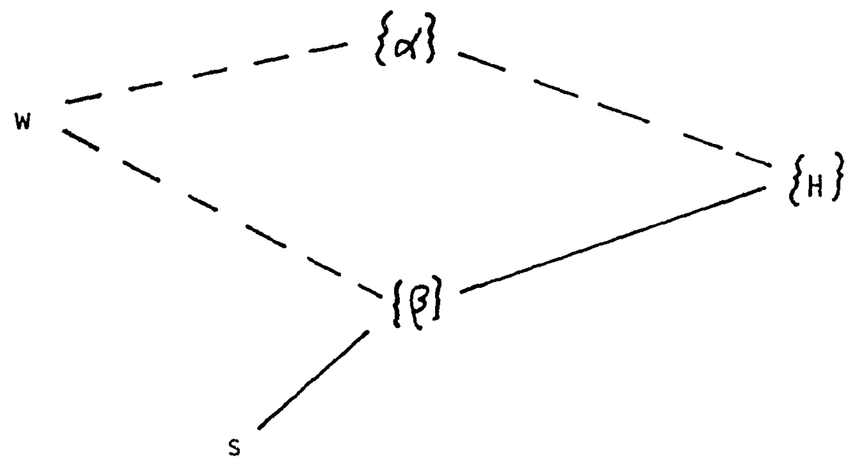
$$\partial P_i = \sum_{i \neq k} \partial P_i / \partial P_k + \partial P_i / \partial \beta$$

where β is the collective impact not included in the defined set. Solving the difference equation yields,

$$P_i = 1/(1 + \exp(-\gamma_i - \sum_{i \neq k} c_{ik} P_k)),$$

where c_{ik} represents the cross impact factors, and γ_i represents the "residual" term containing the effect of higher order interactions among the p_k probabilities.

By expressing each event (hypotheses) in terms of intermediate states, as in the proposed formulations, the changes in the estimates due to additional information are derived using the effect of new information on the α_i rather than on the H_i's directly. A complete discussion on the implication for cross impact analysis is found in Sahal and Yee, "Cross Impact Analysis: An Alternative Formulation," Portland State University, Portland, Oregon 1975.



<u>Figure</u> 4. Inference on H.

The above system may be expressed by,

$$P(H_j|s,w) = \sum_k P(H_j|\beta_k,w)P(\beta_k|s,w)$$

$$= \sum_k \frac{P(H_j|w)P(\beta_k|H_j,w)}{\sum_j P(H_j|w)P(\beta_k|H_j,w)} P(\beta_k|s,w), \text{ for each j.} \tag{13}$$

The change in the decision rule is simply,

$$\Delta P(H_j) = \sum_k P(H_j|\beta_k,w)P(\beta_k|s,w) - \sum_i P(H_j|\alpha_i)P(\alpha_i|w) \tag{14}$$

By taking into account any new knowledge, the coherency of the estimates on H is addressed in the above formulation.

In this section, a model was proposed for deriving a quantitative relationship between a target set and an immeasurable primary event. Intermediaries, hence knowledge states, were used to assist the inference process. Crucial to the approach is the use of new subjective knowledge to derive coherent estimates.

ELICITING ESTIMATES

An expert may be employed to furnish the necessary inputs to the model. The decision to consult an expert, or more than one expert, would depend on the problem and the decision-maker's confidence in the judgments of experts. Although there is strong evidence to support the assumption that knowledgeable individuals can make useful estimates based on incomplete information, a well developed theory for selecting and sorting out better experts from poorer ones is not available. Here, we limit the discussion to the topics of accuracy and honesty.[49] A complete review would lead far from the focus of this thesis.

The accuracy of an expert may be tested by a calibration process.[50, 51] The expert is gauged by a series of performances. He is asked to give midrange estimates for a large number of variables. If the true values fall in the midrange of half of the assessments and an equal number in the upper and lower quartiles, the expert is said to be externally validated. Testing an expert on issues similar or related to the actual problem can provide the decision-maker with a means to assess the accuracy of the expert. The testing procedure is illustrated in the empirical example below.

Scoring rules involve the computation of a score based on the expert's stated estimates and on the event that actually occurs. Used in this manner, they provide another formal means of evaluating expert's past performances, and so serve as a screening mechanism for the selection of experts. Furthermore, scoring rules are useful in

[49]A complete review is found in Norman C. Dalkey, "Toward a Theory of Group Estimation," in The Delphi Method: Techniques and Applications, eds. Harold A. Linstone and M. Turoff (New York: Addison-Wesley Publishing Co., 1975), p. 236-261.

[50]Peter A. Morris, "Bayesian Expert Resolution" (Ph.D. dissertation, Stanford University, 1971).

[51]Howard Raiffa, "Assessments of Probabilities," Harvard University, 1969.

the sense that they provide motivation for honesty of response by encouraging experts to consider the situation at hand carefully before reporting their judgments.[52] For example, each expert is induced to think that his personal future, i.e., wealth, reputation, etc., is affected by his performance and it is to his advantage to report his judgments in an honest fashion. These testing procedures should help in sorting out the better experts.[53]

If the decision is to employ a group of experts, then their individual distributions may be combined into a single distribution. However, care should be taken to avoid possible incoherencies due to the aggregating method. As Dalkey has indicated,[54]

> . . . when group probability estimates are manipulated, care should be taken to assure that the manipulations are compatible with the original aggregation. For example, if group estimates are multiplied, then some multiplicative aggregation such as the geometric mean would be appropriate. If group estimates are to be added, weighted means might be appropriated.

Thus the difficulties encountered in aggregating might outweigh the advantages of group process. In these situations, one expert should be picked. The selection may be made on the basis of tests of accuracy, honesty and past performances in related exercises, where conditions are comparable. Another criteria in the selection process is the judgments of peers.

[52]Robert L. Winkler, "Rewarding Expertise in Probability Assessment," Indiana University, 1975.

[53]If the experts are also stakeholders, hence decision-makers, selecting and sorting may be complicated by additional factors such as politics, position within the group and investments.

[54]Norman C. Dalkey, "An Impossibility Theorem for Group Probability Function" (The Rand Corporation, P-4862, June 1972), p. 5.

PROPOSED ALGORITHM

In this section, we introduce a procedure for using the above model. The discussion presented here is brief. Details are postponed until the next section where the algorithm is illustrated by an example. Prior to introducing the problem to the assessors, the decision-maker must select elements for the knowledge states and target sets. He has to balance the desire for greater accuracy, hence more elements and finer partition of the set, against the lower cost and simpler calculations obtained in using fewer elements and a coarse partition.

Initial estimates for $P(H_j|\alpha_i)$ and $P(\alpha_i|w)$ are obtained from each assessor. Their estimates for each $P(H_j|w)$ are then calculated. Next, depending on the availability of additional information and desire to check for incoherency, the assessors may be asked to reassess either α, H, or both. The reassessment may be accomplished directly or indirectly using type II distribution.[55] The additional information may be collected from self evaluation or feedback. It is up to the the decision-maker to choose the alternative that is both feasible and suitable for his particular problem. In either case, the final results are then calculated using these revised estimates.

[55]Additional details on the use of type II probability to Bayesian inquiry are found in Sahal and Yee, "Delphi: An Investigation from a Bayesian Viewpoint," Technological Forecasting and Social Change 7 (1975), p. 165-178.

AN EMPIRICAL EXAMPLE

In adopting the conceptual model to a "real world" problem, we relaxed the mutually exclusive and exhaustive assumptions on the knowledge states. This was necessary because the experts used had diverse backgrounds and came from different academic disciplines. Consequently, they examine the problem from different perspectives. At the moment there is no formal unified methodology for structuring interdisciplinary inquiry. Therefore, it was not possible to structure a formal set of mutually exclusive and exhaustive knowledge states with which all the members of the panel could agree.[56] The approach presented is capable of eliciting information from experts in diverse fields. Due to a time constraint, type II distribution was not used. The experts were asked to reassess their estimates directly.

A second example, in this instance a hypothetical problem with formal characteristics illustrating the use of type II distribution, is found in appendix C.

[56]Although it is technically possible to derive a set of mutually exclusive and exhaustive knowledge states for a given panel, the cost and time constraints in "real world" situations would tend to discourage such an endeavor. And should the makeup of the panel change, a new set of knowledge states would have to be rederived.

If the elements in the alpha set are time dependent, they may well be mutually exclusive.

PROBLEM

Oregon, unlike most states, is faced with the problem of people moving into the state, particularly into the Willamette Basin Area. This is a problem that is continuously being discussed by members of the state legislature and citizen groups. Assume that we are faced with the task of investigating a set of alternative actions to ensure a "livable Oregon" in the future (25 years from now), that is, a situation where population is balanced with environmental factors. Since the available information, i.e., settlement patterns, migration trends, etc., is scattered and incomplete, and because of the complexity of the problem, direct analysis was ruled out. However, an indirect method, in this case a cascaded process, may be used to conduct the investigation.

OBJECTIVE

Due to continued in-migration and the resulting demographic changes in Oregon, i.e., w, we want to obtain judgments on the following set of alternatives.[57]

H_1 Lesiglation to attract industry, capital and revenue to the state.

H_2 Legislation to discourage in-migration, i.e., residence requirements for public education and state-sponsored social services.

H_3 Research to find ways and means to preserve or improve the quality of life with the expected growth.

H_4 No action until the in-migration problem is understood fully.

[57]This problem was developed with the help of Paul Molnar, who is a social anthropologist at Portland State University.

Since it is not possible to derive $P(H_j|w)$ directly, the knowledge space was partitioned into ten states to facilitate the inference of H.

α_1 Continued suburban sprawl, similar to what we have today.

α_2 Urbanization, with people moving back into the cities due to various factors such as energy shortages and the convenience of downtown.

α_3 New population centers developing near local energy sources.

α_4 Changes in attitudes, such as the continued acceptances of birth control and emphasis on zero population growth, reducing the projected growth by as much as 15%.

α_5 Minimal net in-migration because of taxes and environmental control measures such as limiting density and land usages.

α_6 Technological breakthrough so that energy will be a factor contributing to growth.

α_7 In-migration problem being short term, because most of the people moving into Oregon are either retired or will be retiring shortly, and so will die in about 10 years.

α_8 An increase in pressure for more social services for both the young and old.

α_9 Population growth stabilizing due to limited resources, jobs, etc.

α_{10} Oregon's becoming a mecca for new development similar to California in the 1950 to 1960's.

The estimates for $P(\alpha_i|w)$ may be derived by any suitable methods. A statistical ranking technique[58] was used here because the procedure is free of prior commitment to a particular distribution. Secondly,

[58] Lee H. Smith, "Ranking Procedures and Subjective Probability Distribution," Management Science 14 (December 1976): P. 236-249. Further details are found in M. G. Kendall, "Rank and Measures," Biometrika 49 (1963): p. 133-137.

it is much easier to rank relative probabilities than to assign absolute probabilities per se. Third, little or no knowledge of probability theory is required. The procedure is as follows:

a. Each assessor is asked to rank the alphas (α_i's) in ascending order (from 1 to 10), that is, he is asked to make a forecast on the alphas for the prescribed time period and to arrange them from the least probable to the most probable.

b. Using the arranged alphas, the assessor is asked to consider them in successive pairs and rank their differences. He is asked to compare his judgments on the difference between successive pairs and rank them.

c. Finally, he is asked to give two probability values--the least and most probable of the alphas.

Further details on the procedure are discussed in appendix B.

Using the quantified rankings and the probability values, a distribution is constructed for the alpha set. This may be done by computer (see appendix B). A similar procedure may be used to obtain conditional estimates for each $P(H_j | \alpha_i)$. Since there are only four alternatives, the assessors were asked to give them directly. The values for $P(H_j|w)$ can now be calculated.

$$\begin{pmatrix} P(H_1|w) \\ P(H_2|w) \\ P(H_3|w) \\ \\ P(H_4|w) \end{pmatrix} = \begin{pmatrix} P(H_1|\alpha_1) & . & . & . & P(H_1|\alpha_{10}) \\ P(H_2|\alpha_1) & . & & . & . \\ . & . & & . & . \\ . & . & & . & . \\ P(H_4|\alpha_1) & . & . & . & P(H_4|\alpha_{10}) \end{pmatrix} \begin{pmatrix} P(\alpha_1|w) \\ . \\ . \\ . \\ P(\alpha_{10}|w) \end{pmatrix} \quad (15)$$

A reassessment of α may be desired to improve the coherency of the estimates,

$$P(H_j|r,w) = \sum_i \frac{P(H_j|w)P(\alpha_i|H_j,w)}{\sum_j P(H_j|w)P(\alpha_i|H_j,w)} P(\alpha_i|r,w), \text{ for each } j \tag{16}$$

where r is new knowledge concerning the potential interactions among the α_i's. This may be done with the following procedure.

1. From the previous round, we have a 4x1 matrix

$$P_{4x1} = \left(P(H_j|w)\right) \tag{17}$$

2. Steps a to c may be used to get estimates for

$$A_{4x10} = \left(P(\alpha_i|H_j,w)\right) \tag{18}$$

3. Let

$$B_{10x1} = A'_{10x4}P_{4x1} = \left(b_{1,j}\right) \tag{19}$$

Expand the B matrix to a diagonal matrix C, with diagonal elements,

$$c_{i,j} = \frac{1}{b_{1,j}} \quad \text{, for all } i=j$$

4. Expand P to a diagonal matrix,

$$D_{4x4} = \begin{pmatrix} P(H_1|w) & 0 & 0 & 0 \\ 0 & P(H_2|w) & 0 & 0 \\ 0 & 0 & P(H_3|w) & 0 \\ 0 & 0 & 0 & P(H_4|w) \end{pmatrix} \quad (20)$$

5. Then

$$D_{4x4}A_{4x10}C_{10x10} = F_{4x10} = \left(P(H_j|\alpha_i, w) \right) \quad (21)$$

6. The estimates for $P(\alpha_i|r,w)$ are derived using steps a to c.

$$G_{10x1} = P(\alpha_i|r,w) \quad (22)$$

7. The results for $P(H_j|r,w)$ are calculated,

$$F_{4x10}G_{10x1} = R_{4x1} = \begin{pmatrix} P(H_1|r,w) \\ \vdots \\ P(H_4|r,w) \end{pmatrix} \quad (23)$$

SELECTING EXPERTS

The ideal expert is a knowledgeable demographer who is also active in either state government or in a citizen group concerned with the future of Oregon. But due to a limitation of resources and limited access to a number of qualified experts, the criteria used in selecting the panel were based on knowledge about the in-migration problem and interest in participating in the exercise. There were eight members on the panel.

1. R. D. is a professor in Systems Science, whose interest includes modeling and simulation and resource conservation.

2. G. B. is a graduate student in Systems Science with research interest in policy science.

3. R. L. is a graduate student in geography.

4. P. M. is a former professor of anthropology, who is presently engaged in general systems research.

5. T. P. is a team leader in the language arts department at a Beaverton school.

6. D. S. is the controller at Portland Student Services, a non-profit corporation providing housing for students attending Portland State University.

7. E. W. is a professional social worker.

8. W. W. is a graduate student in Systems Science. His dissertation topic is the reliability of the power system at a major utility company in Oregon.

Since one of the purposes of the exercise was to compare the results with the outcomes from conventional Delphi, the assessors were required to answer two sets of questionnaires in each round. Most of the panel members had never participated in a forecasting study. Because of the inexperience, both the model and the Delphi procedures were explained before the start of the exercise and each round was individually administered. After the completion of the exercise, members of the group were tested for their accuracy and knowledge of Oregon. The external test was a set of questions on Oregon, i.e., economic activities, natural resources, geography and population. Their answers were checked for accuracy and a score was assigned to each assessor (table V).

TABLE IV

DELPHI RESULTS

	Round 1				Round 2			
	H_1	H_2	H_3	H_4	H_1	H_2	H_3	H_4
Assessors								
G. B.	.5	.3	.2	.0	.5	.3	.2	.0
R. L.	.5	.1	.3	.1	.5	.1	.3	.1
P. M.	.4	.5	.01	.09	.4	.5	.01	.09
T. P.	.6	.05	.25	.1	.55	.05	.3	.1
D. S.	.4	.3	.2	.1	.4	.25	.25	.1
E. W.	.1	.6	.0	.3	.3	.5	.0	.2
W. W.	.1	.05	.4	.45	.2	.1	.4	.3

TABLE V

MODEL RESULTS

Assessors	score	H_1	H_2	H_3	H_4	H_1	H_2	H_3	H_4
G. B.	5	.412	.243	.272	.074	.411	.238	.278	.072
R. L.	7	.380	.271	.235	.149	.326	.250	.268	.156
P. M.	7	.327	.368	.066	.240	.303	.387	.066	.243
T. P.	5	.059	.104	.582	.161	.064	.126	.613	.198
D. S.	10	.348	.263	.275	.115	.337	.264	.282	.117
E. W.	6	.662	.252	.056	.030	.695	.210	.060	.035
W. W.	5	.403	.096	.273	.229	.380	.090	.278	.252

The direct estimates from Delphi and the calculated estimates from the model are shown in tables IV and V.[59]

AGGREGATION METHOD

The aggregation method used will depend on many factors. Most important is the reasonableness of the independence assumption. If we can demonstrate that experts based their estimates on independent information, then a multiplicative aggregation method may be appropriate.[60] This seems to be a rather large assumption. A more realistic assumption is that the estimates were based, at least in part, on the same information, i.e., similar training, experiences, etc.[61]

In this example, because of the criteria used in selecting the panel, the second assumption seems to be reasonable. A weighted means method known as "Opinion Pool" or "Weighted-Average" was used.[62] The median values from Delphi and the aggregated results are shown along with estimates from the "best" expert in table VI. The weights were determined from the scores achieved in the accuracy test.[63]

[59]The panel started with eight members, but one resigned because of prior commitments. A list of the computer outputs are shown in appendix B.

[60]If we assume independence, and that human responses are skewed and fit a lognormal distribution as indicated by Blackman and others, then one aggregating method is simply to multiply the individual distributions together. This assumption was used in the hypothetical example in appendix C.

[61]Robert L. Winkler, "The Consensus of Subjective Probability Distribution," Management Science 15 (October 1968): p. 61-75.

[62]M. Stone, "The Opinion Pool," Annals of Mathematical Statistics 32 (1961): p. 1139-1342.

[63]In the example, the weights were determined from the number of correct answers in the accuracy test.

TABLE VI

FINAL RESULTS

	H_1	H_2	H_3	H_4
Delphi-median				
round 1	.4	.3	.2	.1
round 2	.4	.25	.25	.1
Model-median				
round 1	.380	.252	.272	.149
round 2	.227	.238	.278	.156
<u>AGGREGRATION</u>				
equal weights				
round 1	.370	.228	.251	.142
round 2	.360	.224	.264	.153
scaled weights				
round 1	.372	.241	.240	.141
round 2	.360	.236	.253	.151
<u>BEST EXPERT</u>				
round 1	.348	.263	.275	.115
round 2	.337	.264	.282	.117

ANALYSIS

The exercise has compared and contrasted a conventional Delphi and a structured inquiry process. There are several interesting and important findings. First, let us consider the results in table VI. Both methods led to the same choice for the most and least probable alternatives, H_1 and H_4. The advantage in cascading is that we can now examine judgments on the knowledge states and causal effects which led to the predicted values. From the data in appendix B , we can see that the knowledge states α_1, α_8, α_{10} are considered to be the most likely. These are also dominant in the conditional responses; in fact the "pool" judgment is 11 "votes" for H_1, 5 "votes" each for H_2 and H_3, and none for H_4. Thus, cascading enables the decision-maker to gain additional information that otherwise would be lost or not available in a conventional Delphi. This is important because: (1) it identifies the specific items or information (alphas) which experts see as the leading causal factors, and (2) <u>it enables the decision-maker, by using the dominant alphas, to relate the specific problem under consideration to other situations and decisions</u>. Thus, in our example, the dominant alphas--α_1, α_8, α_{10} -- permit the decision-maker to consider these factors in relation to other policy areas such as land use planning, pollution control, etc. The alphas identified by the expert pool to be most important in demography can thus be related to the alpha selection in other problem areas as well.

There are some differences in the results using both methods. Tables IV and V showed a shift in the results from assessors T. P. and E. W. This may be explained by examining the conditional responses. Using the defined set of knowledge states, T. P. judged H_3 to be the most probable, and E. W. judged H_1 to be the most likely. These shifts indicate that T. P. and E. W. may have considered different causal factors other than those in the defined knowledge states. An additional round could be designed to get the assessors to reveal them.

The different patterns in assigning the alphas between ranks 3 to 8 indicate a higher degree of uncertainty or a lack of sufficient knowledge to make judgments among the experts. This is reflected in the results for H_2 and H_3, and constitutes a "gray area" in the solution space. This ability of the model to specify the areas of uncertainty is important because it indicates which alphas are open to question or are unclear. Since there is a fuzziness about these alphas, even for experts, <u>it indicates a need to obtain more information or to restate the alphas</u>. In either case, the tool leads to the <u>delimitation of the area of uncertainty and the decision-maker now has the option of initiating further efforts</u>, such as restating the alphas for an additional round or revising the alphas and conducting another study, before commitment to a decision.

Finally and most important, the dominant alphas can be used as potential signals or indicators. By monitoring the social (α_1, α_8), technological (α_{10}) and economic (α_1, α_8, α_{10}) sectors for these signals, the decision-maker will be able to forecast with greater accuracy which of the alternatives will be enacted.

The benefit of more information is not without certain disadvantages. The proposed approach took longer to complete than Delphi. It took an average of about 25 minutes longer for each round. The added time taxed the patience of some members of the panel. For some situations, such added cost may not be worth the added benefit of more information.

There is also the possibility of presenting an alpha set that some members may consider to be either incomplete or inadequate for the particular situation. However, a possible solution to this problem is to have experts participate in the selection of the knowledge states.

In this section, we demonstrated the feasibility of the model. While this exercise did not illustrate the case where additional new information was given to the experts, the approach is capable of handling this situation through an additional round. On the basis of the exercise, we cannot claim that the model improves the accuracy of the forecast. We can, however, see the advantages of the model. The proposed approach provides more information as well as greater insight into the basis for the forecast. The method also shows how the expert operates, revealing to the decision-maker the processes of decision as the alphas shift in the various rounds. This permits some insight into the manner in which experts consider the problem.

Additional insights may be drawn by analyzing the expert's professional discipline and background. Their selected alpha patterns may be representative of the opinions of a particular group of a sector of the population affected by the decisional alternatives. A decision-maker having this information is better equipped to handle his problem. For example, a corporate manager knowing the attitudes of those concerned with a particular policy will select subordinates who will support his decision. A politician having this added information can prepare more effectively to win the support of a particular group or groups to implement a selected alternative.

CHAPTER VI

CONCLUSION AND EXTENSION

SUMMARY

An approach to Bayesian inquiry was presented. The proposed methodology is based on the concepts of cascaded inference. A feedback step was used to allow individuals to revise their initial estimates. The reassessment may be done either directly or indirectly. The indirect procedure uses a second order probability distribution to measure the imprecision or fuzziness in the initial estimates.

The proposed method is an improvement over existing methods based on the use of Bayes' theorem as an inference model. A key feature of the proposed method is its applicability to problems where the primary events are either unobservable or unknown, hence immeasurable. The need for Bayesian techniques in general systems is obvious. The proposed methodology is a step toward fulfilling that need.

An algorithm utilizing the formal models was presented and demonstrated with examples. A computer program was written and used in working the exercises. An obvious extension is to program it for an interactive computer. This would allow the assessors to interact directly with the decision-maker through a console. Assessed distributions could also be programmed for display on the terminal to speed up the process.

FURTHER RESEARCH

There are certain limitations in the model. The problem of inconsistency requires further research. It is diffuclt to discriminate between "improbable" events with very small probability values in the order of 10^{-2} or higher. One possible approach is to construct a model of the problem where these events are related to another set of events which are easier to assess.

The use of Fuzzy Sets as an instrument for expressing impression in estimating uncertain events is another topic that should be explored. There appears to be a relationship between Zadeh's Fuzzy Sets[64] and I. J. Good's concept of second order probability.[65] If the suspicion is verified, then a set of axiomatic principles may be developed similar to the Kolmogorov axioms of probability. With such a foundation, the potential of Fuzzy Sets as an instrument for measuring and expressing imprecision is virtually unlimited. This would be especially useful for handling complex systems, e.g., biological and behavioral systems.

Finally, the approach shows the advantage of structuring an inquiry process through the use of knowledge states and provides some parameter for generating and selecting them. The model itself, however, does not provide guidelines for generating either the minimal nor the maximal number of knowledge states or decisional alternatives.

[64] L. A. Zadeh, "Fuzzy Sets," Informaton and Control 8 (1965): p. 338-353.

[65] I. J. Good, "Subjective Probability as the Measure of a Non-Measurable set," in Logic, Methodology and Philosophy, eds., Ernest Nagel, Patrick Suppes, and Alfred Tarski (Stanford: Stanford University Press, 1962): p. 319-329.

ADDITIONAL PROBLEMS

The model may be further extended to include time-dependent primary events $\{w(t)\}$. Further research is required, however, before such a model can be developed. In such situations, the redundant and/or marginal effects over time must be considered, that is $\{w(t)\}$ may not be a disjoint set. Also the problem of non-stationarity will have to be investigated.

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APPENDIX A

SOME DEFINITIONS AND TERMS

Inference: The concept of inference is basic to Bayesian analysis. Savage in his book, Statistical Inference, states the following: "By inference I mean roughly how we find things out--whether with a view to using new knowledge as a basis for explicit action or not--and how it comes to pass that we often acquire practically identical opinions in the light of evidence. Statistical inference is not the whole of inference but a special kind."[66]

Another definition is due to Jamison. "We might distinguish between inductive and deductive inferences in the following way: Deductive inferences refer to the implications of coherence for a given set of belief, whereas inductive inferences follow from conditions for 'rational' change in belief."[67]

Throughout this thesis I used the following definition, which is more concise than the above: Inference is a process using empirical evidences, experiences, etc., to draw a conclusion.

[66]L. J. Savage, Statistical Inference (Great Britain: Spottiswoode Ballantyne & Co., Ltd., 1962), p. 11.

[67]D. Jamison, "Bayesian Information Usage," p. 29.

Cascaded Inference: It is an indirect approach to inference and is often called hierarchical, cascaded or multiple-stage inference (see figure 5). In multiple-stage inference, the process is decomposed into a series of steps or stages, where at each step the assessor focuses only on that portion of the hierarchy. The output of the previous step or stages becomes the input to the next stage.

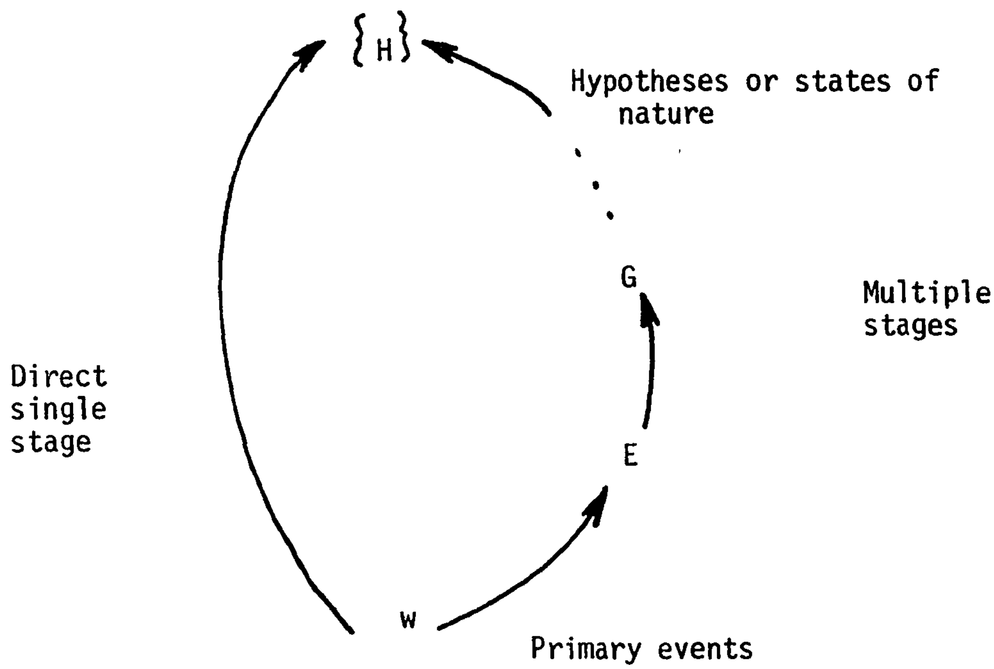


Figure 5. Inference

APPENDIX B

COMPUTER PROGRAM

QUESTIONNAIRES

INPUT DATA

CALCULATED RESULTS

```
1  REM           ******************************
3  REM           *      A BAYESIAN MODEL      *
5  REM           *                            *
7  REM           *      HEWLETT-PACKARD       *
9  REM           *       BASIC 2000/F         *
11  REM          *        JAN. 1976           *
13  REM          ******************************
15  DIM F[4,10],C[10,10],G[10],T[4,10],U[10,4]
17  DIM H[4,10],P[4],R[9],A[10],B[10],Y[10],N[9]
19  R[1]=280
21  R[2]=595
23  R[3]=955
25  R[4]=1375
27  R[5]=1879
29  R[6]=2509
31  R[7]=3349
33  R[8]=4609
35  R[9]=7129
37  MAT A=ZER
39  MAT B=ZER
41  MAT U=ZER
43  MAT T=ZER
45  MAT P=ZER
47  N2=1
49  N1=1
51  MAT  READ Y[10]
53  MAT  READ N[9]
55  READ A1,A2
57  FOR I=1 TO 9
59  K=N[I]
61  A[I]=R[K]
63  NEXT I
65  S=0
67  FOR J=1 TO 9
69  S=S+A[J]
71  NEXT J
73  R=(A2-A1)/S
75  FOR L=1 TO 9
77  K=L+1
79  B[K]=A[L]*R
81  NEXT L
83  A[1]=A1
85  FOR I=2 TO 10
87  J=I-1
89  A[I]=A[J]+B[I]
91  NEXT I
93  S=0
```

```
95  FOR J=1 TO 1Ø
97  S=A[J]+S
99  NEXT J
101  R=1/S
103  FOR K=1 TO 1Ø
105  A[K]=A[K]*R
107  NEXT K
109  FOR I=1 TO 1Ø
111  J=Y[I]
113  B[I]=A[J]
115  NEXT I
117  IF N1=1 THEN 147
119  IF N1=999 THEN 131
121  FOR I=1 TO 1Ø
123  G[I]=B[I]
125  NEXT I
127  N1=999
129  GOTO 51
131  I=N2
133  FOR J=1 TO 1Ø
135  T[I,J]=B[J]
137  U[J,I]=B[J]
139  NEXT J
141  N2=N2+1
143  IF N2<5 THEN 51
145  GOTO 179
147  N1=2
149  PRINT " **** INITIAL ALPHAS ****"
151  MAT  PRINT B
153  MAT  READ H
155  FOR I=1 TO 4
157  FOR J=1 TO 1Ø
159  P[I]=P[I]+H[I,J]*B[J]
161  NEXT J
163  NEXT I
165  PRINT " "
167  PRINT "FIRST ROUND RESULTS"
169  PRINT P[1],P[2],P[3],P[4]
171  S=P[1]+P[2]+P[3]+P[4]
173  PRINT "CHECK ON SUM   "S
175  PRINT " "
177  GOTO 51
179  REM          ****** CHECK COHERENCY ******
181  FOR J=1 TO 1Ø
183  S=Ø
185  FOR I=1 TO 4
```

```
187  S=U[J,I]*P[I]+S
189  NEXT I
191  B[J]=S
193  NEXT J
195  MAT C=ZER
197  FOR I=1 TO 10
199  C[I,I]=1/B[I]
201  NEXT I
203  MAT D=ZER
205  FOR I=1 TO 4
207  D[I,I]=P[I]
209  NEXT I
211  S=0
213  FOR K=1 TO 4
215  FOR I=1 TO 10
217  FOR J=1 TO 4
219  S=D[K,J]*T[J,I]+S
221  NEXT J
223  H[K,I]=S
225  S=0
227  NEXT I
229  NEXT K
231  S=0
233  FOR K=1 TO 4
235  FOR I=1 TO 10
237  FOR J=1 TO 10
239  S=H[K,J]*C[J,I]+S
241  NEXT J
243  F[K,I]=S
245  S=0
247  NEXT I
249  NEXT K
251  FOR K=1 TO 4
253  S=0
255  FOR I=1 TO 10
257  S=S+F[K,I]*G[I]
259  NEXT I
261  A[K]=S
263  NEXT K
265  PRINT " **** REVISED ALPHAS ****"
267  MAT  PRINT G
269  PRINT " "
271  PRINT "SECOND ROUND RESULTS"
273  PRINT A[1],A[2],A[3],A[4]
275  S=A[1]+A[2]+A[3]+A[4]
277  PRINT " CHECK ON SUM   "S
279  REM          ***** INPUT INITIAL ESTIMATES *****
```

QUESTIONNAIRES

INPUT DATA

CALCULATED RESULTS

On the basis of your experiences and knowledge about Oregon, in particular the in-migration problem and the resulting demographic changes, please give your opinions on the following alternative actions being considered to ensure a future "livable Oregon," that is a situation where population is balanced with environmental factors.

H_1 Legislation to attract industry, capital and revenue to the state.

H_2 Legislation to discourage in-migration, i.e., residence requirements for public education and state-sponsored social services.

H_3 Research to find ways and means to preserve or improve the quality of life with the expected growth.

H_4 No action until the in-migration problem is understood fully.

ROUND 1

Please give your opinions on the above alternatives, that is, your probability estimates for each of the actions to ensure a "livable Oregon."

$P(H_1)$	$P(H_2)$	$P(H_3)$	$P(H_4)$
____	____	____	____

- -

ROUND 2

Please reconsider your initial estimates for H_1, H_2, H_3, and H_4. Listed below are the median values given by the panel.

	$P(H_1)$	$P(H_2)$	$P(H_3)$	$P(H_4)$
Your initial estimates:	____	____	____	____
Median:	____	____	____	____
Your revised estimates:	____	____	____	____

Due to the continuance of in-migration and resulting demographic changes, the following ten states may occur in Oregon.

α_1 Continued suburban sprawl, similar to what we have today.

α_2 Urbanization, with people moving back into the cities due to various factors such as energy shortages and the convenience of downtown.

α_3 New population centers developing near local energy sources.

α_4 Changes in attitudes, such as the continued acceptances of birth control and emphasis on zero population growth, reducing the projected growth by as much as 15%.

α_5 Minimal net in-migration because of taxes and environmental control measures such as limiting density and land usages.

α_6 Technological breakthrough so that energy will be a factor contributing to growth.

α_7 In-migration problem being short term, because most of the people moving into Oregon are either retired or will be retiring shortly, and so will die in about 10 years.

α_3 An increase in pressure for more social services for both the young and old.

α_9 Population growth stabilizing due to limited resources, jobs, etc.

α_{10} Oregon's becoming a mecca for new development similar to California in the 1950 to 1960's.

ROUND 1

Please give your opinions on the defined set of knowledge states.

I. Please rank the α_i's (from 1 to 10) in ascending order of occurrence, that is, rank the states in the order that you feel will most likely occur.

States: 1 2 3 4 5 6 7 8 9 10

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

II. Arrange your ranked α_i's, starting with the least likely (rank of 1) to the most likely (rank of 10). Now consider the arranged α_i's in successive pairs and rank their differences. The ranking is from 1 to 9, with a rank value 1 denoting the smallest difference and a value 9 for the largest difference.

Ranked States: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Ranking of Differences, Pair-wise: ___ ___ ___ ___ ___ ___ ___ ___ ___

III. Give probability values for your least and most likely estimates, that is, the α_i's you assigned ranks 1 and 10 in I. ___ ___

IV. Please give conditional probability estimates for each H_j.

"If α_i occurs, what are your estimates for H_j?"

	$P(H_1 \mid \alpha_i)$	$P(H_2 \mid \alpha_i)$	$P(H_3 \mid \alpha_i)$	$P(H_4 \mid \alpha_i)$
1	______	______	______	______
2	______	______	______	______
3	______	______	______	______
4	______	______	______	______
5	______	______	______	______
6	______	______	______	______
7	______	______	______	______
8	______	______	______	______
9	______	______	______	______
10	______	______	______	______

ROUND 2

In light of what you have learned or did not consider earlier, please reconsider the α_i's.

I. Please rank the α_i's (from 1 tp 10) in ascending order of occurrence, that is, rank the states in the order that you feel will most likely occur.

States:	1	2	3	4	5	6	7	8	9	10
	___	___	___	___	___	___	___	___	___	___

II. Arrange your ranked α_i's, starting with the least likely (rank of 1) to the most likely (rank of 10). Now consider the arranged α_i's in successive pairs and rank their differences. The ranking is from 1 to 9, with a rank value 1 denoting the smallest difference and a value 9 for the largest difference.

Ranked States: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Ranking of Differences, Pair-wise: ___ ___ ___ ___ ___ ___ ___ ___ ___

III. Give probability values for your least and most likely estimates, that is, the α_i's you assigned ranks 1 and 10 in I. ___ ___

Consider the conditionality of the α_i's, that is, given that alternative H_j is enacted, please rank the α_i's in ascending order of occurrence.

A. If H_1 (Legislation to attract industry, capital and revenue to the state) is enacted, how would you rank the α_i's?

I. Please rank the α_i's (from 1 to 10) in ascending order of occurrence, that is, rank the states in the order that you feel will most likely occur.

States:	1	2	3	4	5	6	7	8	9	10
	___	___	___	___	___	___	___	___	___	___

II. Arrange your ranked α_i's, starting with the least likely (rank of 1) to the most likely (rank of 10). Now consider the arranged α_i's in successive pairs and rank their differences. The ranking is from 1 to 9, with a rank value 1 denoting the smallest difference and a value 9 for the largest difference.

Ranked States: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Ranking of Differences, Pair-wise: ___ ___ ___ ___ ___ ___ ___ ___ ___

III. Give probability values for your least and most likely estimates, that is, the α_i's you assigned ranks 1 and 10 in I. ___ ___

B. If H_2 (Legislation to discourage in-migration, i.e., residence requirements for public education and state sponsored social services) is enacted, how would you rank the α_i's.

I. Please rank the α_i's (from 1 to 10) in ascending order of occurrence, that is, rank the states in the order that you feel will most likely occur.

States:	1	2	3	4	5	6	7	8	9	10
	___	___	___	___	___	___	___	___	___	___

II. Arrange your ranked α_i's, starting with the least likely (rank of 1) to the most likely (rank of 10). Now consider the arranged α_i's in successive pairs and rank their differences. The ranking is from 1 to 9, with a rank value 1 denoting the smallest difference and a value 9 for the largest difference.

Ranked States: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Ranking of Differences, Pair-wise: ___ ___ ___ ___ ___ ___ ___ ___ ___

III. Give probability values for your least and most likely estimates, that is, the α_i's you assigned ranks 1 and 10 in I. ___ ___

C. If H_3 (Initiate research to find ways and means to preserve or improve the quality of life with the expected growth) is enacted, how would you rank the α_i's?

I. Please rank the α_i's (from 1 to 10) in ascending order of occurrence, that is, rank the states in the order that you feel will most likely occur.

States:	1	2	3	4	5	6	7	8	9	10
	___	___	___	___	___	___	___	___	___	___

II. Arrange your ranked α_i's, starting with the least likely (rank of 1) to the most likely (rank of 10). Now consider the arranged α_i's in successive pairs and rank their differences. The ranking is from 1 to 9, with a rank value 1 denoting the smallest difference and a value 9 for the largest difference.

Ranked States: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Ranking of Differences, Pair-wise: ___ ___ ___ ___ ___ ___ ___ ___ ___

III. Give probability values for your least and most likely estimates, that is, the α_i's you assigned ranks 1 and 10 in I. ___ ___

D. If H_4 (No action until the in-migration problem is understood fully) is enacted, how would you rank the α_i's?

I. Please rank the α_i's (from 1 to 10) in ascending order of occurrence, that is, rank the states in the order that you feel will most likely occur.

States:	1	2	3	4	5	6	7	8	9	10
	___	___	___	___	___	___	___	___	___	___

II. Arrange your ranked α_i's, starting with the least likely (rank of 1) to the most likely (rank of 10). Now consider the arranged α_i's in successive pairs and rank their differences. The ranking is from 1 to 9, with a rank value 1 denoting the smallest difference and a value 9 for the largest difference.

Ranked States: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Ranking of Differences, Pair-wise: ___ ___ ___ ___ ___ ___ ___ ___ ___

III. Give probability values for your least and most likely estimates, that is, the α_i's you assigned ranks 1 and 10 in I. ___ ___

Input format:

Round 1: a. Initial alpha estimates.

b. Initial H estimates.

Round 2: a. Revised alpha estimates.

b. Conditional alpha estimates.

** G. B. **

```
681  REM          ***** INPUT INITIAL ESTIMATES *****
686  DATA 1Ø,1,6,7,3,4,2,5,8,9
691  DATA 9,8,7,4,5,6,2,1,3
696  DATA .Ø8,.9
7Ø1  DATA .2,.25,.7,Ø,.5,.8,.8,.7,.5,Ø
7Ø6  DATA .Ø5,.25,Ø,.6,.3,Ø,.Ø5,.2,Ø,.8
711  DATA .5,.25,.25,.4,.2,.2,.1,.1,.5,Ø
716  DATA .25,.25,.Ø5,Ø,Ø,Ø,.Ø5,Ø,Ø,.2
721  REM          **** INPUT REVISED ESTIMATES ****
726  DATA 1Ø,2,7,6,3,4,1,8,5,9
731  DATA 9,7,4,2,8,6,5,1,3
736  DATA .Ø8,.9
741  DATA 1Ø,8,7,4,2,3,1,5,6,9
746  DATA 9,8,2,4,7,3,5,6,1
751  DATA .Ø6,.85
756  DATA 6,3,8,4,9,5,2,1Ø,7,1
761  DATA 8,7,9,4,7,5,1,2,3
766  DATA .Ø3,.9
771  DATA 9,5,8,6,2,3,1,7,4,1Ø
776  DATA 9,8,5,7,4,6,3,2,1
781  DATA .Ø7,.8
786  DATA 1Ø,8,4,7,5,3,1,9,6,2
791  DATA 8,6,1,3,4,7,5,9,2
796  DATA .Ø3,.9
```

** R. L. **

```
696  REM          **** INPUT INITIAL ESTIMATES *****
7Ø1  DATA 5,9,6,4,2,7,1,1Ø,3,8
7Ø6  DATA 7,6,5,8,1,4,3,9,2
711  DATA .1,.9
716  DATA .7,.1,.5,.5,.3,.6,.25,.1,.1,.7
721  DATA .1,.2,.3,.2,.6,.1,.25,.5,.7,.Ø5
726  DATA .1,.5,.15,.1,.Ø5,.2,.25,.3,.1,.2
731  DATA .1,.4,.Ø5,.2,.Ø5,.1,.25,.1,.1,.Ø5
736  REM          **** INPUT REVISED ESTIMATES ****
741  DATA 3,9,8,2,7,5,1,1Ø,6,4
746  DATA 2,6,1,3,9,5,7,8,4
751  DATA .1,.9
756  DATA 9,2,8,5,6,7,4,3,1,1Ø
761  DATA 1,6,8,3,7,9,5,4,2
766  DATA .4,.6
771  DATA 3,1Ø,4,6,7,1,8,9,5,2
776  DATA 2,5,6,9,7,8,4,1,3
781  DATA .35,.65
786  DATA 3,9,8,2,7,4,1,1Ø,5,6
791  DATA 1,5,2,6,3,7,8,9,4
796  DATA .1,.9
8Ø1  DATA 3,9,7,2,8,6,1,1Ø,4,5
8Ø6  DATA 2,5,7,9,1,8,6,3,4
811  DATA .15,.85
```

** P. M. **

```
550  REM          ***** INPUT INITIAL ESTIMATES *****
600  DATA 10,2,7,1,4,5,3,9,6,8
610  DATA 9,2,7,8,1,4,3,5,6
615  DATA .05,.5
620  DATA .07,.2,.4,.1,.05,.5,.1,.7,.2,.5
625  DATA .6,.7,.4,.2,.8,.1,.4,.1,.2,.3
630  DATA .03,.01,.1,.1,.05,.2,.1,.01,.05,.05
635  DATA .3,.09,.1,.6,.1,.2,.4,.19,.55,.15
640  REM          ***** INPUT REVISED ESTIMATES *****
645  DATA 10,6,7,2,4,1,3,9,5,8
650  DATA 9,3,8,2,7,6,5,4,1
655  DATA .03,.63
660  DATA 10,5,7,1,2,6,4,9,3,8
665  DATA 1,4,5,7,3,8,6,9,2
670  DATA .03,.5
675  DATA 10,5,4,3,8,1,2,9,7,6
680  DATA 9,2,3,8,7,5,6,4,1
685  DATA .01,.65
690  DATA 10,9,7,3,2,8,4,1,5,6
695  DATA 2,3,6,9,4,8,7,5,1
700  DATA .25,.3
705  DATA 10,7,6,2,3,1,4,9,5,8
710  DATA 2,7,5,6,8,9,4,3,1
715  DATA .05,.6
```

** T. P. **

```
550  REM          ***** INPUT INITIAL ESTIMATES *****
600  DATA 9,2,1,3,7,8,1,10,4,5
605  DATA 2,3,4,5,8,9,3,2,1
610  DATA .05,.9
615  DATA 0,0,.3,0,0,.2,0,.1,0,0
620  DATA .1,.2,0,0,0,0,0,.3,0,.4
625  DATA .7,.6,.2,0,1,.7,0,.5,0,.6
630  DATA .2,.2,.5,1,0,.1,1,.1,1,0
640  REM    ****** INPUT REVISED ESTIMATES *****
645  DATA 10,3,4,2,5,6,1,8,7,9
650  DATA 3,2,2,5,3,6,8,2,1
655  DATA .05,1
660  DATA 10,7,5,3,2,8,1,9,4,6
665  DATA 4,2,3,6,5,5,8,3,1
670  DATA 0,1
675  DATA 10,8,5,2,3,6,1,9,7,4
680  DATA 1,3,5,6,2,8,1,4,2
685  DATA 0,.7
690  DATA 10,4,3,2,5,6,1,8,3,9
695  DATA 1,2,3,3,2,4,9,1,1
700  DATA .05,.95
704  DATA 10,2,7,3,4,5,1,8,6,9
705  DATA 3,4,5,2,1,2,7,2,1
710  DATA .05,1
```

** D. S. **

```
550  REM          ***** INPUT INITIAL ESTIMATES *****
600  DATA 1Ø,8,6,2,5,4,1,9,7,3
605  DATA 6,2,1,9,8,7,5,3,4
61Ø  DATA .Ø5,.75
615  DATA .65,.2,.6,.5,.15,.3,.35,.1,.3,.4
62Ø  DATA .1,.2,.1,.Ø5,.4,.15,.35,.6,.3,.1
625  DATA .2,.4,.25,.3,.4,.15,.15,.2,.3,.25
639  DATA .Ø5,.2,.Ø5,.15,.Ø5,.4,.15,.1,.1,.25
64Ø  REM          ***** INPUT REVISED ESTIMATES ******
645  DATA 1Ø,8,7,2,5,3,1,9,6,4
65Ø  DATA 4,7,9,8,5,6,2,3,1
655  DATA .Ø5,.65
66Ø  DATA 9,3,7,4,5,6,1,8,2,1Ø
665  DATA 9,6,1,2,7,8,5,4,3
67Ø  DATA .Ø5,.7
675  DATA 6,9,4,3,8,2,1,1Ø,7,5
68Ø  DATA 7,2,5,6,3,4,1,8,9
685  DATA .Ø5,.95
69Ø  DATA 4,9,7,5,6,3,1,1Ø,8,2
695  DATA 2,1,9,6,8,7,3,5,4
7ØØ  DATA .Ø5,.7
7Ø5  DATA 1Ø,7,5,2,3,6,1,8,4,9
71Ø  DATA 4,9,7,1,3,8,6,5,2
715  DATA .Ø5,.95
```

** E. W. **

```
55Ø  REM          ***** INPUT INITIAL ESTIMATES *****
6ØØ  DATA 5,1,8,6,4,9,2,7,3,1Ø
6Ø5  DATA 1,7,5,9,6,4,8,3,2
61Ø  DATA .Ø2,.8
615  DATA .9,.57,.5,.3,.3,.9,.3,.9,.8,.6
62Ø  DATA .Ø7,.4,.47,.4,.6,.Ø7,.4,.Ø6,.1,.3
621  DATA .Ø2,.Ø2,.Ø2,.2,.Ø6,.Ø2,.2,.Ø3,.Ø6,.Ø6
625  DATA .Ø1,.Ø1,.Ø1,.1,.Ø4,.Ø1,.1,.Ø1,.Ø4,.Ø4
64Ø  REM            ****** INPUT REVISED ESTIMATES *****
645  DATA 7,5,8,1,2,9,4,6,3,1Ø
65Ø  DATA 3,2,4,5,7,8,6,9,1
655  DATA .1,.9
66Ø  DATA 1Ø,6,7,3,1,5,2,8,4,9
665  DATA 7,3,9,8,5,6,4,2,1
67Ø  DATA .1,.9
675  DATA 6,4,6,8,5,3,1,9,2,7
68Ø  DATA 9,6,2,5,4,3,8,7,1
685  DATA .ØØ5,.95
69Ø  DATA 9,1,8,3,5,6,2,7,4,1Ø
695  DATA 9,6,8,4,7,3,2,5,1
7ØØ  DATA .1,.9
71Ø  DATA 9,6,7,2,1,8,3,5,4,1Ø
715  DATA 1,2,5,6,3,4,7,8,9
72Ø  DATA .1,.9
```

** W. W. **

```
550  REM          ***** INPUT INITIAL ESTIMATES *****
600  DATA 3,7,9,10,5,4,2,6,8,1
605  DATA 9,8,2,6,1,7,6,3,4
610  DATA .01,.99
615  DATA .2,.6,.5,.5,0,.5,.2,.3,.5,1
620  DATA .1,0,0,0,.7,0,.05,0,.1,0
625  DATA .5,.35,.5,0,.1,.5,.1,.4,.1,0
630  DATA .2,.05,0,.5,.2,0,.65,.3,.3,0
640  REM         ****** INPUT REVISED ESTIMATES *****
645  DATA 3,7,10,9,2,6,5,4,8,1
650  DATA 9,8,2,1,7,3,6,4,5
655  DATA .01,.95
660  DATA 4,6,9,10,3,5,1,8,7,2
665  DATA 3,9,1,5,6,7,2,8,4
700  DATA .01,.8
705  DATA 3,6,9,10,5,4,1,7,8,2
710  DATA 1,9,2,6,8,5,7,3,4
715  DATA .01,.99
720  DATA 3,7,9,4,5,10,2,6,8,1
725  DATA 5,9,3,7,1,8,2,4,6
730  DATA .1,.95
735  DATA 4,7,8,10,5,2,3,6,9,1
740  DATA 9,2,8,4,1,7,5,3,6
745  DATA .1,.8
```

Calculated Results

** G. B. **

**** INITIAL ALPHAS ****

.140174

1.24599E-02

.115741

.129869

7.85583E-02

.097417

5.26044E-02

.10516

.13322

.134797

FIRST ROUND RESULTS

.411686	.243112	.271666	7.35351E-02

CHECK ON SUM 1.

**** REVISED ALPHAS ****

.149499

5.61036E-02

.130797

.115728

7.62167E-02

8.44746E-02

1.32888E-02

.142081

.088048

.143763

SECOND ROUND RESULTS

.411436	.238223	.277985	7.23549E-02

CHECK ON SUM 1

** R. L. **

**** INITIAL ALPHAS ****

.1Ø5432

.17307

.1Ø7377

7.34228E-Ø2

.Ø42948

.116926

1.96891E-Ø2

.1772Ø2

6.Ø3731E-Ø2

.123559

FIRST ROUND RESULTS

.379721	.271479	.234892	.148522

CHECK ON SUM 1.Ø3461

**** REVISED ALPHAS ****

4.741Ø3E-Ø2

.1927Ø7

.155914

2.73813E-Ø2

.129179

5.72692E-Ø2

2.26315E-Ø2

.2Ø3683

.114179

4.96455E-Ø2

SECOND ROUND RESULTS

.325867	.249846	.268236	.156Ø51

CHECK ON SUM 1.

** P. M. **

```
**** INITIAL ALPHAS ****
.156279

5.98388E-02

.123144

1.56279E-02

8.42977E-02

.112881

6.35287E-02

.140719

.114617

.129067

FIRST ROUND RESULTS
.326696        .367891        6.58992E-02    .239514
CHECK ON SUM   1.

**** REVISED ALPHAS ****
.154214

.115081

.131328

5.35092E-02

8.95403E-02

7.34351E-03

5.96936E-02

.152401

9.33934E-02

.143496

SECOND ROUND RESULTS
.303434        .38764         6.63591E-02    .243167
CHECK ON SUM   1.
```

** T. P. **

**** INITIAL ALPHAS ****
.190589

1.66576E-02

1.07429E-02

2.61509E-02

.175181

.184674

1.07429E-02

.193372

3.98193E-02

5.84978E-02

FIRST ROUND RESULTS

.059495	.103801	.581794	.161339

CHECK ON SUM .906429

**** REVISED ALPHAS ****
.207742

3.39686E-02

4.30209E-02

2.49164E-02

7.16078E-02

8.61371E-02

1.03871E-02

.19443

.124309

.203482

SECOND ROUND RESULTS

6.39216E-02	.125501	.612834	.197743

CHECK ON SUM 1

** D. S. **

**** INITIAL ALPHAS ****

.181835

.164399

.125279

3.Ø8969E-Ø2

.Ø9Ø79

3.74444E-Ø2

1.21223E-Ø2

.171546

.15Ø339

3.53492E-Ø2

FIRST ROUND RESULTS

.347179	.262875	.274715	.115231

CHECK ON SUM 1.

**** REVISED ALPHAS ****

.154499

.146733

.142992

2.Ø53Ø7E-Ø2

.1154

4.15896E-Ø2

1.18845E-Ø2

.152738

.127215

8.64176E-Ø2

SECOND ROUND RESULTS

.336512	.264211	.282127	.11715

CHECK ON SUM 1.

** E. W. **

**** INITIAL ALPHAS ****

.105385

4.63631E-03

.173095

.125388

4.85488E-02

.180709

6.86860E-03

.13635

3.35635E-02

.185452

FIRST ROUND RESULTS

.661757	.252441	5.58724E-02	2.99301E-02

CHECK ON SUM 1

**** REVISED ALPHAS ****

.130138

.063738

.151072

2.36546E-02

3.16229E-02

.210555

4.80601E-02

9.16813E-02

3.65874E-02

.212891

SECOND ROUND RESULTS

.694567	.21019	6.00078E-02	3.52356E-02

CHECK ON SUM 1

** W. W. **

**** INITIAL ALPHAS ****
8.07877E-02

.126208

.149576

.158851

.101727

8.48015E-02

4.96959E-02

.103616

.143133

1.60456E-03

FIRST ROUND RESULTS

.402691	9.60857E-02	.272657	.228566

CHECK ON SUM 1

**** REVISED ALPHAS ****
8.43541E-02

.120823

.161404

.148173

5.18991E-02

.114098

9.05156E-02

8.85439E-02

.13849

1.69899E-03

SECOND ROUND RESULTS

.37944	8.99711E-02	.278095	.252494

CHECK ON SUM 1

APPENDIX C

A HYPOTHETICAL EXAMPLE

For the purpose of illustrating the formal features not used in the migration problem (chapter V), a hypothetical example is presented. In this exercise, we demonstrate the use of type II probability in the revision phase of feedback and show that aggregation can be performed in each stage, which generally would not be done in a real problem. We also assumed the lognormal properties of human responses and thereby used a multiplicative aggregation procedure.

Commercial fishing is a major industry in the Western states. The viability of the industry affects both domestic and foreign economic policies. In recent years, commercial fishing has gone through a series of economic difficulties. Because of its importance a policy analyst has been asked to investigate probable changes (hypotheses) in existing policies to ensure the viability of the industry.

Since it was not possible to derive a distribution for the primary event w, a direct line of reasoning was ruled out. Therefore the analyst decided to assess the situation by means of an indirect method, in this case a cascaded process. Such a structured (cascaded) process permits the analyst to gather information on intermediate events (knowledge states) which he feels are relevant to the issue. In addition to providing more information, this process aids the assessors to focus on specific events.

Thus the cascading process structures the inference path for the assessors and provides the kind of information desired by the analyst.

Suppose that two experts have been asked to assess the following hypotheses (assumed to be mutually exclusive, and exhaustive):

H_1: Diplomatic actions to extend U. S. fishing zone to 50 miles.

H_2: Aids in the form of federal subsidy to the fishing industry.

Three causal states (knowledge states) are assumed. These are taken to be mutually exclusive and one of them occurs.

α_1: Foreign competitions have taken big catches off the U. S. coast.

α_2: Recent court actions giving unlimited fishing rights to native Americans will sharply disrupt the natural reproduction of the species.

α_3: The market price has not kept up with the rising cost of the industry.

It is assumed that the above causal states were the results of the following primary event:

w: Some empirical records are available, i.e., mortality rates of the species, feeding ranges, etc., but they are incomplete. Some experimental data on hybrids is available, e.g. the hybrid type, "super salmon," developed at the University of Washington.

The assessors were asked to give estimates on $P(H_1|w)$ and $P(H_2|w)$. Since w is incomplete and additional substantive data is unavailable, Bayes' theorem is not applicable. The proposed algorithm circumvents this problem by using the estimates $P(\alpha|w)$ as input.

STAGE 1

Each assessor is asked to estimate $P(\alpha|w)$, hence $P(\alpha_1|w)$, $P(\alpha_2|w)$ and $P(\alpha_3|w)$. Their responses are shown in figures 6, 7, 8 and 9 (not drawn exactly to scale). Figure 6 is assessor 1's responses for $P_1(\alpha|w)$ along with the underlying distributions for each of the $P_1(\alpha_i|w)$. The responses for assessor 2 are shown in figure 7.

Each assessor then receives the other expert's initial PDF. He is asked to give a credibility function of his initial estimates. The credibility function is a "measure of preciseness" for each estimate $P(\alpha_i|w)$. Each assessor's credibility function, CRDF, is shown in figures 8 and 9. The higher the credibility value, the smaller the fuzzy interval, hence a sharper and more precise estimate of the corresponding $P(\alpha_i|w)$.

A revised PDF is derived from the credibility estimate.[68]

$$\text{revised } P_k(\alpha|w) = C\cdot \text{CRDF}(b)b,\ k = 1,2 \tag{24}$$

where $b = P_k(\alpha|w)$; and C is a constant. The revised PDFs for assessors 1 and 2 are shown in table VII. A high credibility value implies a high degree of precision or less fuzziness, hence the interval (x to y) is narrow.[69] A low CRDF value indicates fuzziness and the corresponding

[68] In this discrete example, we use the following

$$P_k(\alpha_i|w) = \frac{\text{CRDF}(b_i)b_i}{\sum\limits_{j=1} \text{CRDF}(b_j)b_j},\quad k = 1,2 \tag{25}$$

[69] There is an underlying distribution for each $P(\alpha_i)$ where x to y is a likelihood interval. If the assessor feels that there is an increase in precision, then the new interval x' to y' will be less than x to y, hence $|x'-y'| < |x-y|$. On the other hand, an increase in fuzziness will have $|x'-y'| > |x-y|$.

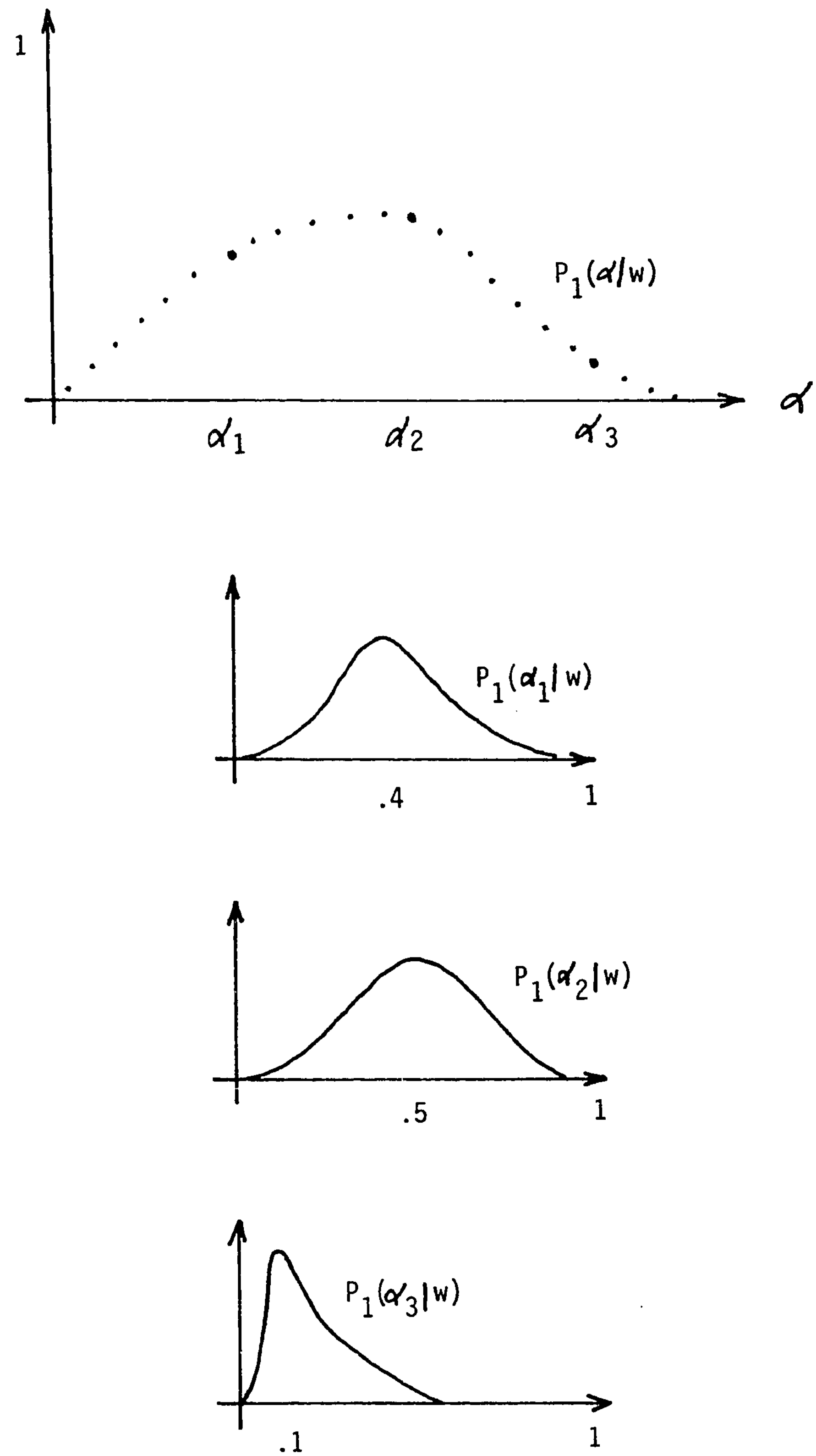


Figure 6. Initial estimates (Assessor 1).

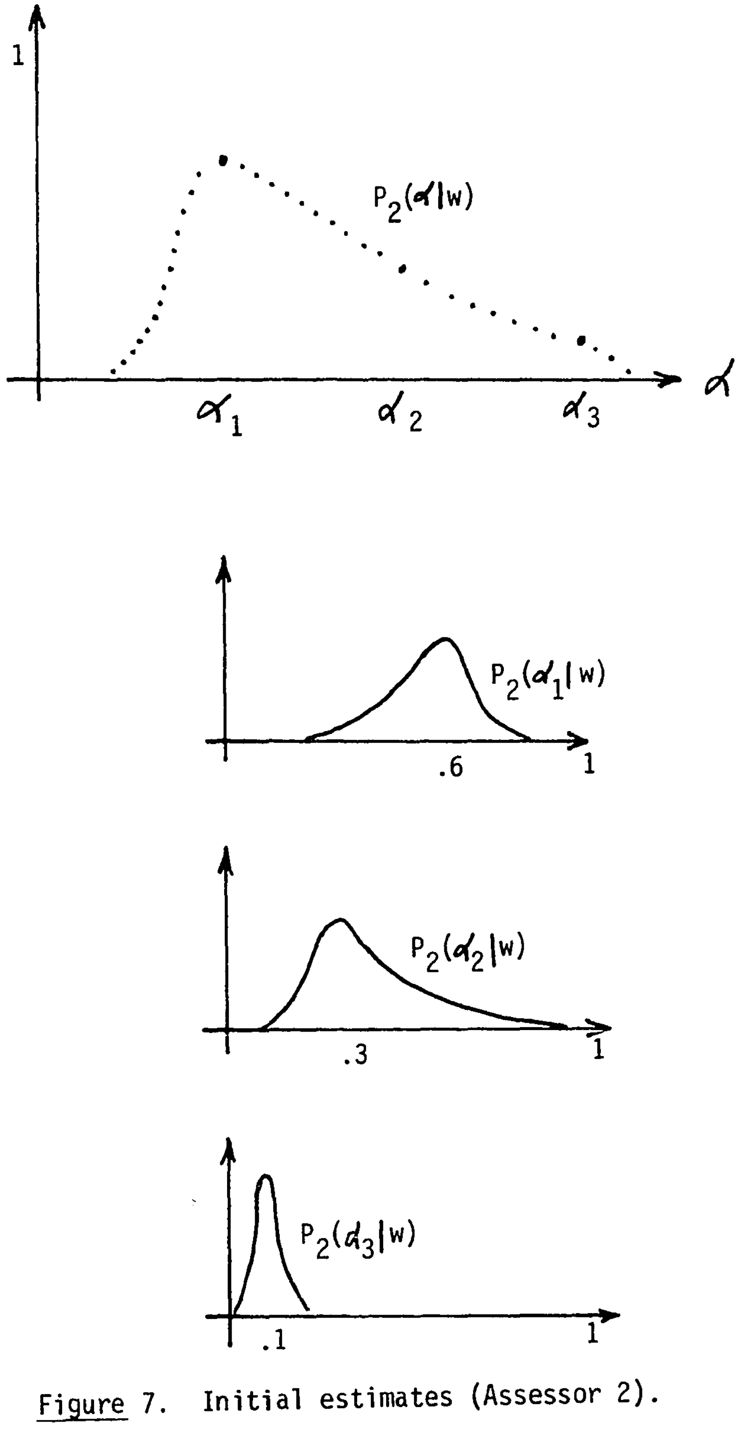


Figure 7. Initial estimates (Assessor 2).

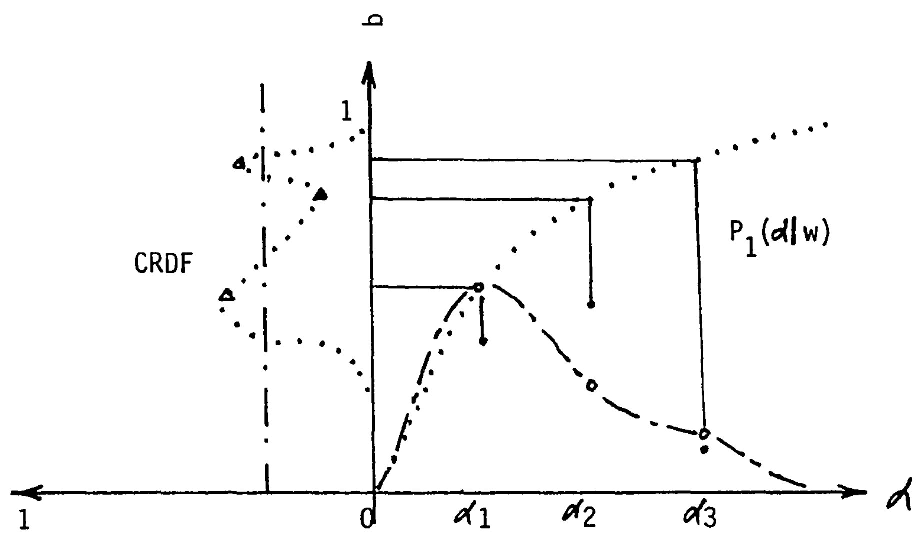
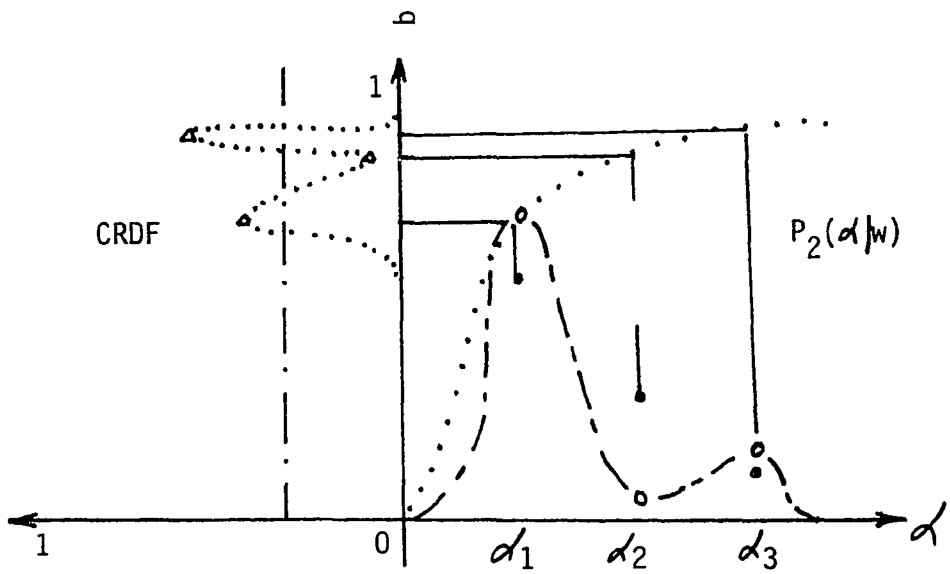


Figure 8. Revised estimates (Assessor 1).



<u>Figure</u> 9. Revised estimates (Assessor 2).

TABLE VII

REVISED STAGE 1 ESTIMATES

	Assessor 1			Assessor 2		
	Initial	CRDF	Revised	Initial	CRDF	Revised
$P(\alpha_1 \mid w)$	.4	.43	.619	.6	.40	.764
$P(\alpha_2 \mid w)$	.5	.14	.252	.3	.07	.067
$P(\alpha_3 \mid w)$	.1	.36	.129	.1	.53	.169

<u>Aggregated Estimates</u>

$P_a(\alpha_1 \mid w) = .925$

$P_a(\alpha_2 \mid w) = .033$

$P_a(\alpha_3 \mid w) = .043$

distribution is diffuse, hence the slightest adjustment due to CRDF is reflected in the revised PDF. For example, assessor 2's initial estimate of α_2 is $P_2(\alpha_2|w) = b_2 = .5$, with CRDF $(b_2) = .07$ and this leads to a revised $b_2 = .06$. Thus, the relative change on α_2 is maximum due to the corresponding CRDF estimate, which is the lowest of the three values.

The two revised PDFs may now be aggregated.[70]

$$P_a(\alpha|w) = C\cdot\prod_i P_i(\alpha|w) \tag{26}$$

These results are necessary and are the inputs for the next stage (see Table VII).

STAGE 2

Each assessor is now asked to estimate the unknown $P(H_1|\alpha_i)$ and $P(H_2|\alpha_i)$. As in stage 1, each assessor is asked to give an initial PDF, $P_k(H_j|\alpha)$. Since he is working with only two alternatives, a credibility measure is not needed. Each assessor is asked to reassess his previous estimated directly. Their initial and revised results are shown in tables VIII and IX. Their aggregated results are listed in table X.

[70]Since there are only 3 points, the following approximation procedure was used,

$$P_a(\alpha_i|w) = \frac{P_1(\alpha_i|w)P_2(\alpha_i|w)}{\sum_i P_1(\alpha_i|w)P_2(\alpha_i|w)}\text{, for each } i \tag{27}$$

TABLE VIII

STAGE 2 ESTIMATES (ASSESSOR 1)

Given:	$P(H_1\|\alpha_i)$	$P(H_2\|\alpha_i)$
α_1	.429	.571
α_2	.714	.286
α_3	.791	.209

TABLE IX

STAGE 2 ESTIMATES (ASSESSOR 2)

Given:	$P(H_1\|\alpha_i)$	$P(H_2\|\alpha_i)$
α_1	.451	.545
α_2	.600	.400
α_3	.666	.333

TABLE X

AGGREGATED RESULTS

$P_a(H_1|\alpha_1) = .383$ $\quad$ $P_a(H_2|\alpha_1) = .616$

$P_a(H_1|\alpha_2) = .789$ $\quad$ $P_a(H_2|\alpha_2) = .227$

$P_a(H_1|\alpha_3) = .882$ $\quad$ $P_a(H_3|\alpha_3) = .117$

Using the aggregated results from stages 1 and 2, the estimates for $P(H_j|w)$ may now be calculated by equation (15). The results are,

$$P(H_1|w) = .418$$

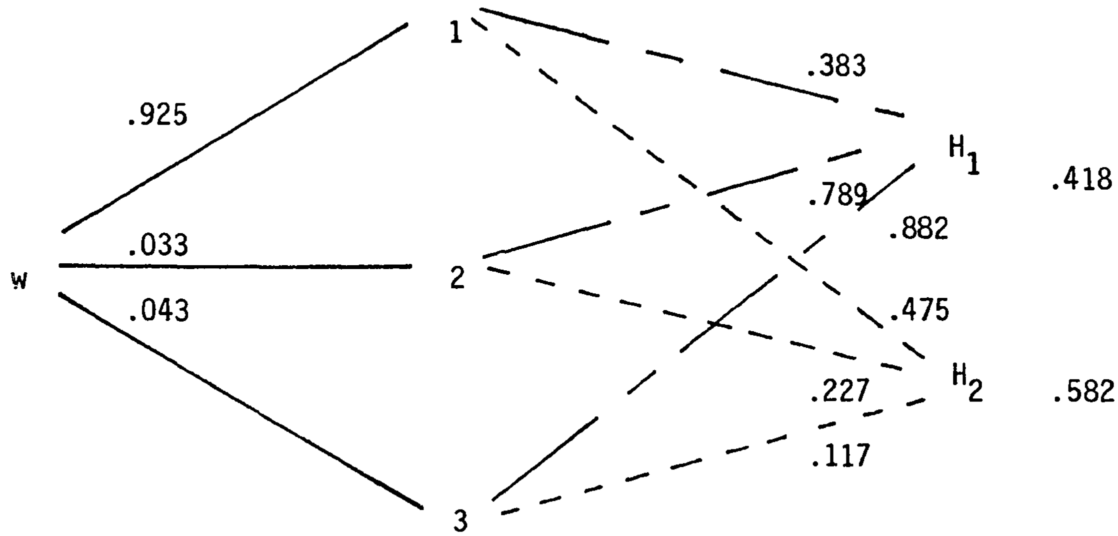
$$P(H_2|w) = .582$$

A trace of the inference tree (figure 10) shows that both assessors consider α_1, foreign competition, to be the most important factor which could effectively lead to changes in existing policies.

Table VII shows that α_1 is dominant over α_2 and α_3. The effect of α_1 on hypotheses H_1 and H_2 is shown in tables VIII and IX. A conventional approach, such as Delphi, to the above problem would not have revealed as much information as the proposed approach.

These estimates should be of considerable use to the decision-maker. Generally, he will use this information along with other criteria to evaluate the advantages and disadvantages of each alternative. For instance, option H_1 is to extend the territorial zone to 50 miles. This will relieve some of the economic pressure on the fishing industry, but the U. S. early warning defense system was designed to operate at the present three-mile limit. On the other hand, the second option H_2 to provide federal subsidies to the industry is only a short term solution. These and other ramifications to both domestic and foreign policies must be assessed by the decision-maker.

Alternatively, the decision-maker could extend the inquiry by means of an additional stage. The results from the previous exercise become the starting point of the extended inquiry. In the above



$$P(H_j|w) = \sum_i P_a(\alpha_i|w)P_a(H_j|\alpha_i)$$

The subscript 'a' denotes aggregated estimates.

<u>Figure</u> 10. Inference tree.

example, the incremental addition could be a set of knowledge states which might include the knowledge of foreign trade policies, the U. S. economy, dietary preference, etc. The benefit of more information will not be without cost.

APPENDIX D

A RULE FOR THE NUMBER OF STAGES TO BE EMPLOYED

Cascading improves the accuracy of subjective estimates. There remains the question of the number of stages to be employed. This is a problem for the decision-maker (DM), since the value of the estimates, hence information, can only be judged by DM himself. This problem may be analyzed as follows:

Consider the following situation,

$$\begin{tikzcd} & H^2 & & H^3 & & \\ w \arrow[r] & \{\alpha\} \arrow[u] \arrow[rr] & & \{\gamma\} \arrow[u] & \cdots \arrow[r] & {} \end{tikzcd}$$

where the two-stage process, $w \longrightarrow \{\alpha\} \longrightarrow H^2$, has just been completed. The value of this information, $\{H^2\}$, to DM is

$$\text{EXPECTED} \quad V(H^2) \tag{28}$$

DM must decide whether to employ the third stage, $\{\gamma\}$, that is, $w \longrightarrow \{\alpha\} \longrightarrow \{\gamma\} \longrightarrow \{H^3\}$.

One approach to this problem is to consider the net worth if another stage is employed. Let

$$\text{EXPECTED} \quad V(H^3) \tag{29}$$

be the expected value that can be obtained with the added stage. The net gain due to the third stage is

$$\text{Net Gain} = \text{EXPECTED } V(H^3) - \text{EXPECTED } V(H^2) - C(\gamma) \tag{30}$$

where $C(\gamma)$ is the cost of adding the third stage.

MULTIPLE-STAGE

A three-stage process is a direct extension of the two-stage process developed in chapter V. In a two-stage, the equation is

$$P(H_\alpha \mid w) = \sum_{i}^{I} P(\alpha_i \mid w) P(H_\alpha \mid \alpha_i) \tag{31}$$

For three stages, the term $P(H_\alpha \mid \alpha_i)$ is expanded to

$$P(H_\alpha \mid \alpha_i) = \sum_{j}^{J} P(\gamma_j \mid \alpha_i) P(H_\alpha \mid \gamma_j) \tag{32}$$

Similarly, this step may be expanded for any number of stages.