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A Technical Translation of Melentiev's Graph Representation Method with Commentary

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Introduction

Online translation is akin to a game of telephone. Although useful for individual words, it results in absurd nonsense when used to translate significant blocks of text. This is why human translators are vital. The word-by-word translation offered online will not result in an understandable text; not only do many words lack equivalents across languages, but a proper translation depends as much on an understanding of language, semantics, topic, and culture, as it does on the definitions of words. In this work, the contrast between human and digital translation will be demonstrated with a translation of a Russian text on graph theory, with commentary highlighting the nuances of language that a machine misses.

This opening introduces the author of the translated work and briefly summarizes the text to be translated. Next, a commentary on the translation method and decisions precedes the full translation of the text, which is followed by select line-specific examples of translation. Finally, the translation is used to demonstrate the importance of human translation as opposed to digital services, and a final argument is presented.

Victor Alexandrovich Melentiev is a senior researcher at the Semiconductor Physics Research Institute, part of the Siberian Department of the Russian Academy of Sciences (СО РАН for its acronym in Russian) in Novosibirsk, Russia. His research focuses on the theory of fault tolerance and survivability, modeling of architectures, and investigation into and construction of optimal fault-tolerant computer systems (Лаборатории Параллельных информационных технологий НИВЦ МГУ). This paper provides a translation of Melentiev's text Аналитический подход к синтезу регулярных графов с заданными значениями порядка, степени и обхвата (Мелентьев 2010), translated here by Volkova as "An Analytical Approach to the Construction and Representation of Regular Graphs From a Given Order, Degree, and Girth". In it, Melentiev describes a new way to represent graphs in graph theory, one that maintains all the information of a visual representation, but does not rely on a geometric image.

1. The Choices, the Reasons, the Methods

I provide this section before my translation because I was inspired, somewhat ironically, by Shiltsev's 2012 translation of Lomonosov's[1] "Discovery of Venus Atmosphere in 1761". Shiltsev is a physicist, and most of his bibliography includes other scientists, not linguists. The translation itself reads very mechanically, and is full of grammatical errors which take away from the text's meaning. Furthermore, he introduces very little of the topic, the author, and his own translation methods, instead simply throwing the reader into the fray. Having experienced this, I wish to save my own audience such an adventure.

Since this is a paper, first and foremost, for mathematicians, I will define a number of linguistic terms. Then I will provide some examples of online translation errors, which I will use to explain some word choices I make in my translation. Finally, I will discuss my formatting decisions, before allowing the reader to step into Melentiev's world. Note that all examples of online translations provided come from Google translate, and reference the first translation Google offers. Each is accurate as of the date following the translation.

This project is a *technical translation*. "Broadly speaking, technical translation is the translation of materials dealing with scientific and technical subjects and using the specialized terminology of the scientific or technical field involved," (Shiftar 2016). That is, unlike a literary translation, which requires knowledge of both languages, a technical translation also requires extensive knowledge of the topic being discussed (in this case: graph theory). To avoid redundancy, I will be using the linguistic

[1] Mikhail V. Lomonosov (1711 - 1765): Russian poet and scientist, who made "substantial contributions to the natural sciences" (Langevin).

abbreviations L1 and L2 in this text. L1 is the “first language” (in this case, Russian) and L2 is the “second language” (English) (Davies 2007, 15).

Most languages have words with multiple meanings, where the correct definition must be derived from context. As an English example, consider the word “lie”: it could mean “to be horizontal”, or “to tell a falsehood”, and the audience cannot know which meaning the author intended without context. Translation websites typically translate word by word, and so will often miss such context. For example, Melentiev uses a geometric figure called the “единичный куб” to illustrate his method. Typing this term into Google Translate grants: “single cube” (2/18/2018). In fact, the figure Melentiev is using is “Q_3”, or the cube in dimension 3. A simpler, but still curious example: Google translates “ребро” as “edge” (2/18/2018), but it most commonly means “rib”. Note that for technical translations, this is especially dangerous. Consider the Russian word “множество”. Google translates this as “a bunch of” (2/18/2018), but in mathematics, a “множество” is a set. Katsberg provides more examples and numerous arguments for context awareness in “Cultural Issues Facing the Technical Translator”.

This duality of definitions is the reason for some of the specific word choices in this translation. In his text, Melentiev calls his method “синтезис”, which literally translated means “synthesis”. “Synthesis” is synonymous to “composition”, a term used to describe combining two graphs together. To spare readers this confusion, I have elected to use “construction” instead of “synthesis”, as Melentiev’s methods builds, or “constructs” a graph. Furthermore, he often calls his graphs “synthesized”, but as his process only involves one graph at a time, I often drop the descriptor, and at other times call it “the graph under construction”.

Similarly, Melentiev calls the model he constructs a system of “projections”, but this term already has a meaning in English graph theory, relating to topology. Since Melentiev himself likens his method to solving systems of equations, I call his “projections” - “parenthetical equations”, or just “equations”. I decided to keep the literal translation of “уровень” - level, which is what Melentiev calls a vertex's location in the equation. Although I believe "degree" would be a more fitting term, the paper also discusses vertex degrees, which would cause confusion.

The translation itself was written first as a text document, and then edited and inserted into a LaTex engine. LaTex is a coding language which allows for writing and managing non-standard characters (read: math symbols). This step was necessary, for although the math symbols between L1 and L2 are the same, equations will often use words such as "if, then", which must also be translated. An example of this code is for equation (2) in Melentiev's paper, under "2. Описание подхода" ("2. Method Description"): \begin{equation} 1+s\sum\limits_{i=1}^{k_e-1}(s-1)^{i-1} < n \leq 1 + s\sum_{i=1}^{k_e}(s-1)^{i-1}. \end{equation}.

Another aspect of formatting to consider is the bibliography: even in the same field, citation formats are different from country to country. Thus, although I did not translate Melentiev's sources, I did reformat them to Chicago style.

At last, I believe I have properly armed my reader to embark upon the journey that is Melentiev's "Аналитический подход к синтезу регулярных графов с заданными значениями порядка, степени и обхвата."

2. Translation

An Analytical Approach to the Construction and Representation of Regular Graphs From a Given Order, Degree, and Girth

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Abstract

This is a proposed method for analytically constructing a system - a structure with given properties. The approach is derived from representing a graph using a system of its *images*. First, the *image* of a graph is defined, and its properties are outlined. Then the approach of constructing such a system is explained: build a base image of the spanning tree of the graph and correct it using images based at different vertices. This is analogous to solving a system of equations, where the equations are the graph's multiple images. Examples of this method are provided, followed by conclusions and the significance of this construction method.

Keywords: order, diameter, girth, representations of graphs, construction of regular graphs

Introduction

Constructing computer system structures and network webs out of a list of given properties is a widely discussed topic in scientific literature. The most common method for solving such problems is generating random networks and rejecting those which do not fit the given criteria (diameter, connectivity, clustering factors, etc.). According to research on the stability of computer systems/networks after deletion of vertices from those structures [1], regular structures are extremely stable; the most stable topology of random graphs is characterized by the distribution of the vertex degrees in a way such that there are at most three discrepancies. That said, in both the theory of networks and systems, and in the basic context of graph theory, research on the problem of generating the structure of a graph from given properties and systematic methods (aside from the necessity of reordering) is severely lacking. This is, first and foremost, due to the absence of a method of describing graphs which would allow for formal analyses and transformations on the graph.

In this paper, I first introduce an analytical approach to solving problems of constructing regular graphs given order n and degree s. My approach is based on those outlined in [2, 3]: for a graph $G(V, E)$ and its parenthetical equations $P(v_0)$ with $v_0 \in V$, construct a base equation of the spanning tree of $G(V, E)$ and use that equation to determine the floor of the diameter $d(G)$ and the ceiling of the girth $g(G)$. Finally, to find the endpoints of the unknown edges of the graph (missing edges in the underlying spanning graph), compare the different equations in accordance to the desired properties of the graph, much like solving a system of equations.

1 Methodology Outline

To avoid discrepancies, I will list some common definitions as provided in [4], as well as some basic facts about paranthetical equations as I define them in this text.

Regular graph - a connected graph $G(V, E)$, in which all the vertices $v_i \in V$ have equal degree; the degree of each vertex is the degree $s(G)$ of the regular graph.

Eccentricity of a vertex - for the vertex u, eccentricity $e(u) = \max\limits_{u \in V} \delta(u, v)$, where $\delta(u, v)$ is the distance between vertices $u, v \in V$.

Diameter - the largest eccentricity of all the vertices of a connected graph: $d(G) = \max\limits_{u \in V} e(u)$.

Girth - the length of the minimal cycle in a graph.

Paranthetical equation (or just *equation*) $P(v_i)$ of a graph $G(V, E)$ - a parenthetical description of a graph with its starting point a vertex $v_i \in V$.

The method of constructing parenthetical representations of graphs, and the properties of such equations are explained well in [2, 3]. That said, because the method is a fundamental part of the argument proposed in this paper, and because it is fairly new (and thus little known), I will lay out a short explanation on assembling the equation of an undirected connected graph. I will then provide an example using Q_3, which I will use to demonstrate that the graphs constructed by this method have the same order and degree as Q_3.

Call the equation $P(w)$ of the graph $G(V, E)$ with initial vertex $w \in V$ the w-th equation of this graph, or the w-th foreshortening. To specify the number of levels in the equation, k, let us add a corresponding index: $P_k(w)$. Then $P_0(w) = w$. For example, for the first level of the equation, we get $P_1(w) = w^{\mathcal{N}(w)}$. Here the subset $\mathcal{N}(w)$, derived from vertex w, is the neighborhood of vertex w and consists of $s(w)$ vertices, where $s(w) = deg(w)$, the degree of vertex w.

Thus, the j-th vertex of the $(i-1)$-th level of the equation determines the subset of vertices $V_{ij} \subset V$ on the i-th level of the equation. Accordingly, the number of such subsets is equal to the number of vertices in the previous level of the equation. Let subset $V_{i,j} \subset V$ map to the set of vertices preceding it, $V'_{i,j} \subset V$ in the path from initial vertex v_0 to vertex $v_{i-1,j}$, $M(v_0, v_{i-1,j})$. The subset $V_{i,j}$ derives from its immediate preceding vertex $v_{i-1,j}$. Once more using the first level of the equation, $V_{1,w}$, as example, the subset of its vertices is derived from the single vertex w of the 0-th level (remember, subset $V'_{1,w}$ consists of one vertex: $V'_{1,0} = \{w\}$). Subsets of upper levels, $V'_{i+1,j}, i \geq 1$, are derived from the corresponding subsets $V'_{i,j}$ of the previous levels by adding to them the vertex $v_{i,j} \in V_{i,j} : V'_{i+1,j} = V'_{i,j} \cup v_{i,j}$, the immediate predecessor of subset $V_{i,j+1}$.

Notice that in the general case of connected graphs, including cyclic graphs, a single vertex may appear multiple times on different levels of an equation, or even on the same level (excluding the first), and the indices do not have to correspond. The lack of repetition of vertices in the first level is explained by the fact that the objects being studied in this graph are not multigraphs. Graphs with loops will also not be discussed in this work (so $v_{i-1,j} \neq v_{i,j}$). The total number of vertices in level i, call it C_i, is equal to the sum of the cardinalities of the subsets of that level $V_{i,j}$: $C_i = \sum_j |V_{i,j}|$. Call the union of the subsets $V_{i,j}$ set M_i located on level i of the equation; that is, $M_i = \bigcup_j V_{i,j}$ and $C_i \geq |M_i|$.

Vertices belonging to the subset $V_{i,j}$ can be identified by subtracting the set of its preceding vertices, $V'_{i,j}$ from the neighborhood of vertex $v_{i-1,j}$ from which the subset was derived. That is, $V_{i,j} = \mathcal{N}(v_{i-1,j}) \backslash V'_{i,j}$. This prevents repetition of vertices in paths defined by an equation by excluding those that have already appeared.

Thus, the equation for the three-level equation described in the above examples and in terms of only one vertex for one subset per level is:

$$P_3(w) = w^{\{u^{\{v^{\{t:t\in\mathcal{N}(v)\backslash V'_{3,v}, V'_{3,v}=V'_{2,u}\cup\{u\}=\{w,u\}\}}:v\in\mathcal{N}(u)\backslash V'_{2,u}, V'_{2,u}=\{w\}\}}:u\in\mathcal{N}(w)\}}.$$

Here the set of vertices in the first level consists of one subset: $V_{1,w} = \mathcal{N}(w)$ with cardinality $|V_{1,w}| = \deg(w)$. The set of vertices in the second level includes $\deg(w)$ subsets, each of which has, as an immediate predecessor, every vertex of level 1. In general, vertex v is a part of the subset $V_{2,u}$ with the immediate predecessor $u : v \in V_{2,u}, V_{2,u} = \mathcal{N}(u)\backslash V'_{1,u}, V'_{1,u} = w$. Thus the vertex set in any n-th level of the equation $P_{k(w)}$, where $n \leq k$, unifies in itself the subsets derived by subtracting from the neighborhood of each vertex in level $(n-1)$ all of its predecessors in the given equation. Note that the number of such subsets is equal to the number of vertices in level $(n-1)$. Let us demonstrate the above method on the simple example of Q_3 (fig. 1).

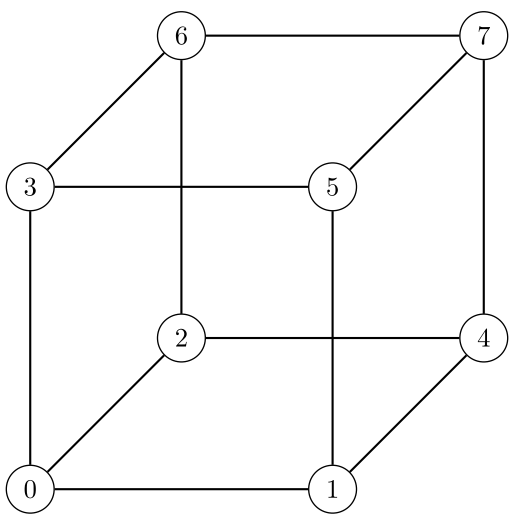

Figure 1: Q_3

Choose vertex $v_0 = 0$ as the initial vertex of the equation $P_{k(0)}$. The set of vertices adjacent to this vertex, $\mathcal{N}(0) = 1, 2, 3$ also make up the set of vertices in the first level: $M_1 = \{1, 2, 3\}$, $|M_1| = C_1 = 3$. The set of vertices in the second level M_2 unifies in itself $C_1 = 3$ subsets, which are the neighborhoods of three vertices of level 1 (sans vertex 0, the immediate predecessor to these subsets): $M_2 = M_{2,1} \cup M_{2,2} \cup M_{2,3} = \{4, 5\} \cup \{4, 6\} \cup \{5, 6\} = \{4, 5, 6\}$ while $C_2 = 6$ and $|M_2| = 3$. Notice that $M_1 \cup M_2 \neq V$, and so the equation must be expanded with another level.

The set of vertices in level 3, M_3 consists of 6 subsets, each of which consists of vertices adjacent to the corresponding 6 vertices in level 2. Each subset was modified by subtracting the sets preceding this vertex set:

$M_3 = M^1_{3,4} \cup M^1_{3,5} \cup M^2_{3,4} \cup M^1_{3,6} \cup M^2_{3,5} \cup M^2_{3,6}$. Note: The first value in the bottom index identifies the level of the equation, and the second identifies the vertex of the graph, while the top index allows to differentiate multiple examples of the same vertex on the level in consideration. Thus, the constructed equation is:

$$P_3(0) = 0^{\{1^{\{4^{\{2,7\}},5^{\{3,7\}}\}},2^{\{4^{\{1,7\}},6^{\{3,7\}}\}},3^{\{5^{\{1,7\}},6^{\{2,7\}}\}}\}}.$$

Notice that vertex 7 only appears on level 3, as it has not been included in any of the subsets of the previous levels: $M_3 = \{2,7\} \cup \{3,7\} \cup \{1,7\} \cup \{3,7\} \cup \{1,7\} \cup \{2,7\} = \{1,2,3,7\}$, $C_3 = 12$, $|M_3| = 4$. We can similarly write the same equation $P_3(0)$ in one line:

$$P_3(0) = 0\{1\{4\{2,7\},\{5\{3,7\}\},2\{4\{1,7\},6\{3,7\}\},3\{5\{1,7\},6\{2,7\}\}\}.$$

The open brackets in front of any subset point to it belonging to the previous set. At any point in the equation, the number of open brackets unpaired with a closed bracket describes how many levels deep the subset is nested into the set of preceding vertices. In sources [2, 3] it is shown that the nesting level of a subset in a set of descendants of an initial point (what we are calling the level of the equation) defines the distance from the initial point to those in the corresponding subset. Furthermore, the k-th level, which initially defines the set of all the lower level vertices in the equation of graph $G(V,E)$ up to V defines the eccentricity of the initial vertex: $e(v_0) = k$, for which $\bigcup_{i=0}^{k-1} M_i \neq V$, $\bigcup_{i=0}^{k} M_i = V$.

The equation of the graph $P_k(v_0)$ is considered *full* if it defines every vertex and every edge of the graph. Then the necessary conditions of *fullness* of an equation can be described thus: $\bigcup_{i=0}^{k} M_i = V$ and $\bigcup_{i=0}^{k} E_i = E$. Here, $E_i = \{u, v : u \in M_{i-1}, v \in M_i\}$ is the set of edges which coincide to pairs of vertices from adjacent levels in the equation. Notice that the second condition of fullness (for edges) includes in itself the first condition (for vertices). Looking at $P_3(0)$ above, it is clear that both conditions are satisfied only on the third level: $\bigcup_{i=0}^{3} M_i = V$ and $|\bigcup_{i=0}^{3} M_i| = |V| = 8$; $E_0 = \emptyset$, $E_1 = \{\{0,1\},\{0,2\},\{0,3\}\}$, $E_2 = \{\{1,4\},\{1,5\},\{2,4\}\{2,6\}\{3,5\},\{3,6\}\}$, $E_3 = \{\{4,2\},\{4,7\},\{5,3\},\{5,7\},\{4,1\},\{6,3\},\{6,7\},\{5,1\},\{6,2\}\}$, and so $E = \bigcup_{i=0}^{3} E_i = \{\{0,1\},\{0,2\},\{0,3\},\{1,4\},\{1,5\},\{2,4\},\{2,6\},\{3,5\},\{3,6\},\{4,7\},\{5,7\},\{6,7\}\}$, $|E| = 12$.

We will now combine the definition of equations provided here with their properties, proven in [2, 3]. Above it was shown that a vertex of the equation $P_k(v_0)$ not equal to the initial vertex, call it $v_j \neq v_0$, can be found on any of the $k > 0$ levels of the equation with a multiplicity $0 \leq m_{i,j} \leq C_k$. For vertices in $V_{i,j}$ on levels $0 < i \leq k$, there exist ordered vertex sets of the form $W(v_{i,j}) = v_0, v_{1,0}, \ldots, v_{i,j}$, which form simple chains from v_0 into $v_{i,j}$ with length $\delta(v_0), v_{i,j} = i$.

Let us notice and prove another property of graph equations: The number of levels $k_{\min}(v_0)$ in a minimal full projection $P_{k_{\min}}(v_0)$ of a simple connected graph $G(V,E)$ is no less than the eccentricity of the initial vertex $e(v_0)$ and is no greater than that eccentricity plus 1.

$$k_{\min}(v_0) = \begin{cases} e(v_0), & \text{if } \nexists\{u,v\} \in M_k(\delta(v_0,u) = \delta(v_0,v) = e(v_0) \quad \& \delta(u,v) = 1, \\ e(v_0)+1, & \text{if } \exists\{u,v\} \in M_k(\delta(v_0,u) = \delta(v_0,v) = e(v_0) \quad \& \delta(u,v) = 1 \end{cases}.$$

Because of this, a graph equation is always full if it is built up from any vertex up to level n where n is greater than the diameter.

By combining the above given properties and the offered method we propose:

Lemma 1. *If in the equation $P(V_0)$ of graph $G(V,E)$ $u \in V$ belongs to two subsets in one or more levels, then the girth of the graph, $g(G)$ is less than or equal to the sum of the numbers of those levels.*

Proof. Consider a graph $G(V,E)$ with equation $P(v_0)$ where v_0 is the initial vertex. Let vertex $u \in V$ belong to the i-th level, where $i > 0$. We know from the property given above that there exists a simple chain $W(v_0,u)$ with length $\delta(v_0,u) = i$. Since we have excluded the existence of multi-edge graphs for this method, vertex u cannot belong to a subset of vertices derived from a vertex that belongs to a preceding level. We index the vertices into different subsets. Note that the terminating vertex is one and the same: $u_1 = u_2 = u$, the index simply identifies which path from v_0 to u it belongs to: $W_1(v_0, u)$ or $W_2(v_0, u)$. If the intersection of sets W_1 and W_2 representing these paths contains only two vertices, that is, $W_1 \cap W_2 = \{v_0, u\}$, then the length of the simple cycle formed by these chains is maximal, and equal to the sum of the level numbers containing u: i_1 and i_2. Otherwise, if $|W_1 \cap W_2| > 2$, then there exist some (specifically $|W_1 \cap W_2| - 2 \geq 1$) vertices, not v_0, from which there are also simple paths into u_1 and u_2, which are segments of the paths W_1 and W_2. Then the point of intersection of paths W_1 and $W - 2$, v_x, located in a higher level, must be the initial vertex of two non-intersecting chains from v_x into u_1 and u_2. Then the length of the cycle formed by these chains is $(i_1 - i_x) + (i_2 - i_x) = i_1 + i_2 - 2i_x$, and so $g(G) \leq i_1 + i_2$, which is what we were trying to prove. □

2 Method Description

From the properties of an equation given in part 1, we know that vertex v_0 of a regular simple graph with unweighted edges has minimal eccentricity if level k_e of the equation $P_{ke(v_0)}$ at which the condition $\bigcup_{i=0}^{k_e} M_i = V$ is initially satisfied is the lowest possible level for the given order $n = |V|$ and degree s of the graph. Given degree s, the largest number of vertices located on the i-th level of the equation is equal to:

$$C_i(s) = s(s-1)^{i-1}. \tag{1}$$

Then, the minimum number of levels in the equation of a graph with the given conditions $\{n, s\}$ for order and degree respectively, can be found using the inequality:

$$1 + s\sum_{i=1}^{k_e - 1}(s-1)^{i-1} < n \leq 1 + s\sum_{i=1}^{k_e}(s-1)^{i-1}. \tag{2}$$

Remember that the definition of the *diameter* is the largest eccentricity of all vertices in a graph. Then a graph will contain a minimal diameter if equation (2) is satisfied for all of its equations. In the case of Q_3 above, with $n = 8$, $s = 3$, the diameter was equal to 3, which is greater than the value $k_e = 2$ given by equation (2) for a graph with $4 < n \leq 10$ and $s = 3$. This means that constructing a graph with diameter $d(G) = 2$ is less likely than constructing a Q_3 cube with the same degree and order.

We should now turn our attention to the fact that the process of constructing a graph using this method aims to not use its geometric representation, and so from here on out we will only portray the found graph traditionally as a result of our method of construction.

And so, to generate a two level equation $P_2(v_j)$ of a graph with $n = 8$, $s = 3$, and diameter $d = 2$, all of its vertices $|V| = 8$ must be placed among the two levels of the equation; let us number the vertices 0 to 7. On the 0th level of the equation $P_2(v_0)$, place vertex $v_0 = 0$; this is the root of the spanning tree which we will use to construct our equation. On the 1st level, we place three arbitrarily chosen vertices (in this case 1, 2, 3). Since these graphs do not have multiple edges, the vertices of the 1st level cannot be repeated.

Equation (1) tells us that the second level of the equation must have 6 vertices: $C_2(3) = 6$. There are only 4 vertices available to use: 4, 5, 6, 7. Thus, each of these vertices can be arranged on level 2, under the condition that some of them may be repeated and/or vertices from the previous level may be used in this level, so that $C_2(3) = 6$. Thus, $P_2(0)$ can be written:

$$p_2(0) = 0^{\{1^{\{2,3,4,5,6,7\}_2}, 2^{\{1,3,4,5,6,7\}_2}, 3^{\{1,2,4,5,6,7\}_2}\}}. \tag{3}$$

Here the set $\{v_x, v_y, ..., v_z\}_m$ is a potential subset of the neighborhood of the vertex which directly precedes this subset in the given equation. The index m identifies the number of desired vertices in the subset. At the start we include in such a subset every vertex which have fewer known neighbors than the degree s of the graph. Thus we find every vertex of the graph except for 0, that is, the initial subset is $\{1, 2, 3, 4, 5, 6, 7\}$. Since no vertex contains itself in its neighborhood, the subsets are corrected in equation $P_2(0)$.

It was noted above that the set of vertices in the second level, M_2 must contain the subset $\{4, 5, 6, 7\}$; otherwise the eccentricity of vertex 0 is greater than the value derived from (2): $e_v(0) = k_e = 2$. The last two spaces in the second level can be filled with any two vertices from $\{1, 2, 3, 4, 5, 6, 7\}$. Notice also that the total number of edges in our graph should be $|E| = ns/2 = 12$, but our equation (3) identifies only 3 edges. Thus, the 9 unknown edges must be identified, and at most 6 of those can be identified by the second level of the projection. This shows that two of the levels of the equation are incomplete if we desire a full equation, and a 3rd level is necessary if we wish to describe the graph using a single equation and not a system of equations.

From equation (3) and the lemma proved in part 1, it can be seen that if even one of the vertices of level 1 is included in level 2, then the length of its minimal cycle (its girth) won't be more than 3. But if we use only vertices from $\{4, 5, 6, 7\}$ in level 2, then the girth of such a graph would be equal to 4. We will demonstrate this by constructing the corresponding graphs, starting with girth 3. And so, to construct a graph with $g(G) = 3$, we connect vertex $v_1 = 1$ with vertex $v_2 = 2$; the choice of these vertices is arbitrary. This grants us:

$$P_2(0) = 0^{\{1^{\{2,\{4,5,6,7\}_1\}}, 2^{\{1,\{4,5,6,7\}_1\}}, 3^{\{4,5,6,7\}_2}\}}.$$

Notice that the introduced adjacency between 2 vertices of level 1 takes up two of the six positions of the 2nd level, and the remaining four positions are barely enough to fit the four vertices of the set: $\{4, 5, 6, 7\} = M_2 \backslash \{v_1, v_2\} = V \backslash (M_1 \cup M_0)$. Thus, we leave only these vertices in the lists of potential subsets of vertices in the 2nd level. Two of them must be adjacent to vertex 3, and one must be adjacent to each 1 and 2. The choice of adjacencies can be arbitrary, because vertices of the subset $\{4, 5, 6, 7\}$ are still isolated and thus completely equivalent. Let us write down the known and arbitrarily chosen adjacencies in a list of neighborhoods for each vertex of the graph under construction:

$$\begin{array}{llll} \mathcal{N}(0) = \{1, 2, 3\}, & \mathcal{N}(2) = \{0, 1, 5\}, & \mathcal{N}(4) = \{1, \{5, 6, 7\}_2\}, & \mathcal{N}(6) = \{3, \{4, 5, 7\}_2\}, \\ \mathcal{N}(1) = \{0, 2, 4\}, & \mathcal{N}(3) = \{0, 6, 7\}, & \mathcal{N}(5) = \{2, \{4, 6, 7\}_2\}, & \mathcal{N}(7) = \{3, \{4, 5, 6\}_2\}. \end{array}$$

The 2 level equation of the graph constructed in accordance to this list is:

$$P_2(0) = 0^{\{1^{\{2,4\}}, 2^{\{1,5\}}, 3^{\{6,7\}}\}}.$$

It contains in itself all 8 vertices of the graph, but it is not full, because the set of vertices in the 2nd level includes vertices with yet unknown adjacencies. The number of known edges has increased from 3 to 8, but there are still 4 unknown. Let us add a third level to the equation:

$$P_3(0) = 0^{\{1^{\{2^{\{0,5\}}, 4^{\{5,6,7\}_2}\}}, 2^{\{1^{\{0,4\}}, 5^{\{4,6,7\}_2}\}}, 3^{\{6^{\{4,5,7\}_2}, 7^{\{4,5,6\}_2}\}}\}}.$$

Using the list of neighborhoods given above, let us construct an equation of the graph with initial vertex $v_1 = 1$:

$$P_3(1) = 1^{\{0^{\{2^{\{5\}}, 3^{\{6,7\}}\}}, 2^{\{0^{\{3\}}, 5^{\{4,6,7\}_2}\}}, 4^{\{5,6,7\}_2}\}}.$$

As has already been noted, the diameter of the graph being constructed will be equal to the given diameter if the eccentricity of any of the graph's vertices is no greater than the given diameter. That is, if $d(G) = d$, then all vertices of the graph must be split amongst no more than $k = d$ levels of any v_j-th equation of the graph $P_k(v_j) : \bigcup_{i=0}^{k} M_i = V$, $k \leq d$. Thus, analyzing the equation $P_3(1)$ we notice that satisfying this condition is possible only if vertex 4 is adjacent to 6 and 7. We correct the list of adjacencies thus: $\mathcal{N}(4) = \{1, \{\cancel{5}, 6, 7\}_2\} \Rightarrow \mathcal{N}(5) = \{2, \{\cancel{4}, 6, 7\}_2\}$, which grants us the only possible solution:

$$\begin{array}{llll} \mathcal{N}(0) = \{1, 2, 3\}, & \mathcal{N}(2) = \{0, 1, 5\}, & \mathcal{N}(4) = \{1, 6, 7\}, & \mathcal{N}(6) = \{3, 4, 5\}, \\ \mathcal{N}(1) = \{0, 2, 4\}, & \mathcal{N}(3) = \{0, 6, 7\}, & \mathcal{N}(5) = \{2, 6, 7\}, & \mathcal{N}(7) = \{3, 4, 5\}. \end{array}$$

As visual proof of this solution, below are given all the minimal full equations and the geometric representation (fig. 2) of the found graph, built upon these adjacencies:

$$P_3(0) = 0^{\{1^{\{2^{\{5\}},4^{\{6,7\}}\}},2^{\{1^{\{4\}},5^{\{6,7\}}\}},3^{\{6^{\{4,5\}},7^{\{4,5\}}\}}\}}$$

$$P_3(1) = 1^{\{0^{\{2^{\{5\}},3^{\{6,7\}}\}},2^{\{0^{\{3\}},5^{\{6,7\}}\}},4^{\{6^{\{3,5\}},7^{\{3,5\}}\}}\}}$$

$$P_3(2) = 2^{\{0^{\{1^{\{4\}},3^{\{6,7\}}\}},1^{\{0^{\{3\}},4^{\{6,7\}}\}},5^{\{6^{\{3,4\}},7^{\{3,4\}}\}}\}}$$

$$P_3(3) = 3^{\{0^{\{1^{\{2,4\}},2^{\{1,5\}}\}},6^{\{4^{\{1,7\}},5^{\{2,7\}}\}},7^{\{4^{\{1,6\}},5^{\{2,6\}}\}}\}}$$

$$P_3(4) = 4^{\{1^{\{0^{\{2,3\}},2^{\{0,5\}}\}},6^{\{3^{\{0,7\}},5^{\{2,7\}}\}},7^{\{3^{\{0,6\}},5^{\{2,6\}}\}}\}}$$

$$P_3(5) = 5^{\{2^{\{0^{\{1,3\}},1^{\{0,4\}}\}},6^{\{3^{\{0,7\}},4^{\{1,7\}}\}},7^{\{3^{\{0,6\}},4^{\{1,6\}}\}}\}}$$

$$P_3(6) = 6^{\{3^{\{0^{\{1,2\}},7^{\{4,5\}}\}},4^{\{1^{\{0,2\}},7^{\{3,5\}}\}},5^{\{2^{\{0,1\}},7^{\{3,4\}}\}}\}}$$

$$P_3(7) = 7^{\{3^{\{0^{\{1,2\}},6^{\{4,5\}}\}},4^{\{1^{\{0,2\}},6^{\{3,5\}}\}},5^{\{2^{\{0,1\}},6^{\{3,4\}}\}}\}}$$

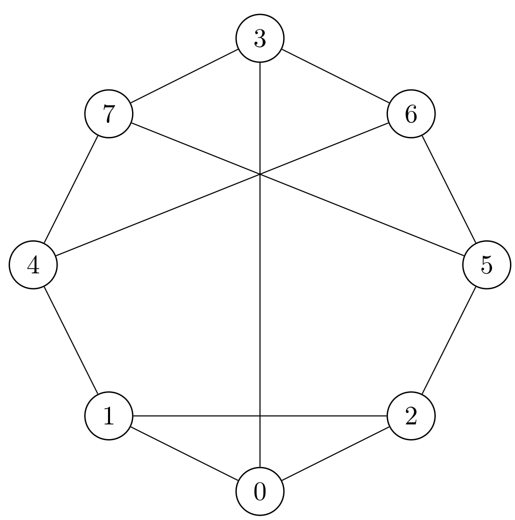

Figure 2: Regular graph with $n = 8$, $s = 3$, $g = 3$

Let us now consider the graph constructed via this method that has the same given order n, degree s, and diameter d, but a greater girth, that is, $g = 4$. Naturally, 3-cycles will be excluded in this case, and equation (3) becomes:

$$P_2(0) = 0^{\{1^{\{4,5,6,7\}},2^{\{4,5,6,7\}},3^{\{4,5,6,7\}}\}}.$$

Arbitrarily place the vertices from $\{4,5,6,7\}$ into the second level of the equation. In this case, we make vertex 1 adjacent to vertices 4 and 5, and vertex 2 with 6 and 7. Here is the equation after these changes, as well as the list of neighborhoods for each vertex:

$$P_2(0) = 0^{\{1^{\{4,5\}},2^{\{6,7\}},3^{\{4,5,6,7\}_2}\}},$$

$$\begin{array}{ll} \mathcal{N}(0) = \{1,2,3\}, & \mathcal{N}(1) = \{0,4,5\}, \\ \mathcal{N}(2) = \{0,6,7\}, & \mathcal{N}(3) = \{0,\{4,5,6,7\}_2\}, \\ \mathcal{N}(4) = \{1,\{3,5,6,7\}_2\}, & \mathcal{N}(5) = \{1,\{3,4,6,7\}_2\}, \\ \mathcal{N}(6) = \{2,\{3,4,5,7\}_2\}, & \mathcal{N}(7) = \{2,\{3,4,5,6\}_2\}. \end{array}$$

The number of edges $|E_k|$ assigned by the k-level equation $P_k(v_j)$ of regular graph $G(V,E)$ with degree s, where $k \le e(v_j)$, cannot be greater than $s\sum_{i=1}^{k}(s-1)^{i-1}$. Thus, a 2 level equation of a regular graph of degree $s=3$ can assign at most 9 edges from the total $|E = 12|$. The equation $P_2(0)$ contains within itself all 8 vertices of the graph, but identifies only 7 of its edges. Thus, for fullness of the equation, we would add an extra level; in this case we will be using a system of equations, thus fullness of the equation is guaranteed notwithstanding that the individual equations do not have this property and contain unknown edges. We limit ourselves to 2-level equations, which is enough for an analysis of eccentricities of the initial vertices, and of the girths of the corresponding subgraphs:

$$\begin{array}{ll} P_2(0) = 0^{\{1^{\{4,5\}},2^{\{6,7\}},3^{\{4,5,6,7\}_2}\}}, & P_2(1) = 1^{\{0^{\{2,3\}},4^{\{3,5,6,7\}_2},5^{\{3,4,6,7\}_2}\}}, \\ P_2(2) = 2^{\{0^{\{1,3\}},6^{\{3,4,5,7\}_2},7^{\{3,4,5,6\}_2}\}}, & P_2(3) = 3^{\{0^{\{1,2\}},\{4,5,6,7\}_2\}}, \\ P_2(4) = 4^{\{1^{\{0,5\}},\{3,5,6,7\}_2\}}, & P_2(5) = 5^{\{1^{\{0,4\}},\{3,4,6,7\}_2\}}, \\ P_2(6) = 6^{\{2^{\{0,7\}},\{3,4,5,7\}_2\}}, & P_2(7) = 7^{\{2^{\{0,6\}},\{3,4,5,6\}_2\}}. \end{array}$$

From $P_2(0)$ it can be seen that the eccentricity of the initial vertex $e(v_0)$ will not be changed by any combination of the potential adjacencies. Let's consider the question of equality between the eccentricities of all the vertices: $\forall v_j \in V, e(v_j) = d(G) = 2$. This condition can be satisfied by arranging all the vertices in no more than 2 levels of any projection. Any smaller value for the diameter can not be used because of (2). Let us also consider all the equations of the graph which ensure the necessary girth $g(G)$: in any equation in the system, the sum of the levels which contain the same vertex cannot be less than the girth. In this case, $g(G) = 4$, and the vertices of the 1st level cannot be a part of the potential subsets of the 2nd level and vice versa, because $1 + 2 = 3 < 4$. We demonstrate this by removing the corresponding vertices from our equations:

$$\begin{array}{ll} P_2(0) = 0^{\{1^{\{4,5\}},2^{\{6,7\}},3^{\{4,5,6,7\}_2}\}}, & P_2(1) = 1^{\{0^{\{2,3\}},4^{\{3,\cancel{5},6,7\}_2},5^{\{3,\cancel{4},6,7\}_2}\}}, \\ P_2(2) = 2^{\{0^{\{1,3\}},6^{\{3,4,5,\cancel{7}\}_2},7^{\{3,4,5,\cancel{6}\}_2}\}}, & P_2(3) = 3^{\{0^{\{1,2\}},\{4,5,6,7\}_2\}}, \\ P_2(4) = 4^{\{1^{\{0,5\}},\{3,\cancel{5},6,7\}_2\}}, & P_2(5) = 5^{\{1^{\{0,4\}},\{3,\cancel{4},6,7\}_2\}}, \\ P_2(6) = 6^{\{2^{\{0,7\}},\{3,4,5,\cancel{7}\}_2\}}, & P_2(7) = 7^{\{2^{\{0,6\}},\{3,4,5,\cancel{6}\}_2\}}. \end{array}$$

Let us correct the list of adjacencies as well by removing from it the "forbidden" vertices. (From here on forward, all corrections will be made without physically scratching out the incorrect values):

$$\begin{aligned}
&\mathcal{N}(0) = \{1, 2, 3\}, && \mathcal{N}(1) = \{0, 4, 5\},\\
&\mathcal{N}(2) = \{0, 6, 7\}, && \mathcal{N}(3) = \{0, \{4, 5, 6, 7\}_2\},\\
&\mathcal{N}(4) = \{1, \{3, \cancel{5}, 6, 7\}_2\}, && \mathcal{N}(5) = \{1, \{3, \cancel{4}, 6, 7\}_2\},\\
&\mathcal{N}(6) = \{2, \{3, 4, 5, \cancel{7}\}_2\}, && \mathcal{N}(7) = \{2, \{3, 4, 5, \cancel{6}\}_2\}.
\end{aligned}$$

From all the "hanging" vertices in $P_2(0)$ (vertices 3 through 7), choose a vertex of the smallest level (vertex 3). Connect it to one of the vertices from the subset of its potential neighbors: $\{4, 5, 6, 7\}_2$. The choice in this case can be arbitrary, as all of these vertices are on level 2, and are at this point hanging (that is, unconnected); this case is 4. Having connected vertices 3 and 4, correct once again the list of adjacencies:

$$\begin{aligned}
&\mathcal{N}(0) = \{1, 2, 3\}, && \mathcal{N}(1) = \{0, 4, 5\}, && \mathcal{N}(2) = \{0, 6, 7\},\\
&\mathcal{N}(3) = \{0, 4, \{5, 6, 7\}_1\}, && \mathcal{N}(4) = \{1, 3, \{6, 7\}_1\}, && \mathcal{N}(5) = \{1, \{3, 6, 7\}_2\},\\
&\mathcal{N}(6) = \{2, \{3, 4, 5\}_2\}, && \mathcal{N}(7) = \{2, \{3, 4, 5\}_2\}.
\end{aligned}$$

And equations of the graph:

$$\begin{aligned}
&P_2(0) = 0^{\{1^{\{4,5\}}, 2^{\{6,7\}}, 3^{\{4, \{5,6,7\}_1\}}\}}, && P_2(1) = 1^{\{0^{\{2,3\}}, 4^{\{3, \{6,7\}\}_1}, 5^{\{3,6,7\}_2}\}},\\
&P_2(2) = 2^{\{0^{\{1,3\}}, 6^{\{3,4,5\}_2}, 7^{\{3,4,5\}_2}\}}, && P_2(3) = 3^{\{0^{\{1,2\}}, 4^{\{1, \{6,7\}_1\}}, \{5,6,7\}_1\}},\\
&P_2(4) = 4^{\{1^{\{0,5\}}, 3^{\{0, \{5,6,7\}_1\}}, \{6,7\}_1\}}, && P_2(5) = 5^{\{1^{\{0,4\}}, \{3,6,7\}_2\}},\\
&P_2(6) = 6^{\{2^{\{0,7\}}, \{3,4,5\}_2\}}, && P_2(7) = 7^{\{2^{\{0,6\}}, \{3,4,5\}_2\}}.
\end{aligned}$$

Notice that in equation $P_2(0)$, vertex 4 is located in two different subsets of the second level, derived from vertices 1 and 3 in level 1[1].Physically, this means that vertex 4 is connected to the initial vertex $v_0 = 0$ via two paths of equal length: $\delta(0, 4) = 2$. It is clear that only one vertex in the subset $\{5, 6, 7\}_1$ will also be duplicated at this level. It is then logical to extend this condition (let two vertices appear twice on the 2nd level) to the other equations of the graph. Then, vertex 3, already twice included in the second level of $P_2(1)$ must be excluded from subset $\{3, 6, 7\}_2$, derived from vertex 5. This is equivalent to forbidding a connection between vertices 5 and 3, and instead inserting two edges, which connect vertex 5 to two vertices in $\{6, 7\}_2 = \{6, 7\}$. Considering these changes, the list of adjacencies is now:

$$\begin{aligned}
&\mathcal{N}(0) = \{1, 2, 3\}, && \mathcal{N}(1) = \{0, 4, 5\}, && \mathcal{N}(2) = \{0, 6, 7\},\\
&\mathcal{N}(3) = \{0, 4, \{6, 7\}_1\}, && \mathcal{N}(4) = \{1, 3, \{6, 7\}_1\}, && \mathcal{N}(5) = \{1, 6, 7\},\\
&\mathcal{N}(6) = \{2, \{3, 4\}_1, 5\}, && \mathcal{N}(7) = \{2, \{3, 4\}_1, 5\}.
\end{aligned}$$

And the system of equations:

$$\begin{aligned}
&P_2(0) = 0^{\{1^{\{4,5\}}, 2^{\{6,7\}}, 3^{\{4, \{6,7\}_1\}}\}}, && P_2(1) = 1^{\{0^{\{2,3\}}, 4^{\{3, \{6,7\}\}_1}, 5^{\{6,7\}}\}},\\
&P_2(2) = 2^{\{0^{\{1,3\}}, 6^{\{3,4\}_1, 5}, 7^{\{3,4\}_1, 5\}}\}}, && P_2(3) = 3^{\{0^{\{1,2\}}, 4^{\{1, \{6,7\}_1\}}, \{6,7\}_1\}},\\
&P_2(4) = 4^{\{1^{\{0,5\}}, 3^{\{0, \{6,7\}_1\}}, \{6,7\}_1\}}, && P_2(5) = 5^{\{1^{\{0,4\}}, \{6^{\{2, \{3,4\}_1\}}, 7^{\{2, \{3,4\}_1\}}\}},\\
&P_2(6) = 6^{\{2^{\{0,7\}}, \{3,4\}_1, 5^{\{1,7\}}\}}, && P_2(7) = 7^{\{2^{\{0,6\}}, \{3,4\}_1, 5^{\{1,6\}}\}}.
\end{aligned}$$

[1]This is predetermined by the initial choice of pairing off the vertices in $\{4, 5, 6, 7\}$ into two subsets derived from vertices 1 and 2 in $P_2(0)$. In another ordering it would be possible (but not necessary) for the multiplicity of one of the vertices in the set $\{4, 5, 6, 7\}$ to be 3, and that of the others to be 1. Accordingly, the choice of our system of equations with unknown edges would be different.

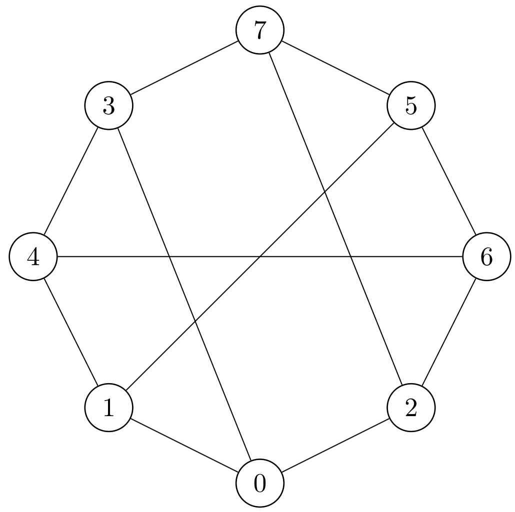

Figure 3: Regular graph with $n = 8$, $s = 3$, $g = 4$

Notice the correspondence of the known vertices and the potential vertices of the neighborhoods $\mathcal{N}(6)$ and $\mathcal{N}(7)\}$ in the new list of adjacencies: $\mathcal{N}(6) = \mathcal{N}(7) = \{2, \{3, 4\}_1, 5\}$, which implies that either arrangement of vertices will work, so add an edge connecting vertices 7 and 3[2]. Having corrected the list of adjacencies and the equations in accordance to this choice, we get the graph we were searching for (fig. 3); all the adjacencies in that graph are identified in this list:

$$\begin{aligned}
&\mathcal{N}(0) = \{1, 2, 3\}, \quad \mathcal{N}(1) = \{0, 4, 5\}, \quad \mathcal{N}(2) = \{0, 6, 7\},\\
&\mathcal{N}(3) = \{0, 4, 7\}, \quad \mathcal{N}(4) = \{1, 3, 6\}, \quad \mathcal{N}(5) = \{1, 6, 7\},\\
&\mathcal{N}(6) = \{2, 4, 5\}, \quad \mathcal{N}(7) = \{2, 3, 5\}.
\end{aligned}$$

And the system of equations that corresponds to this list is:

$$\begin{aligned}
P_2(0) &= 0^{\{1^{\{4,5\}},2^{\{6,7\}},3^{\{4,7\}}\}}, & P_2(1) &= 1^{\{0^{\{2,3\}},4^{\{3,6\}},5^{\{6,7\}}\}},\\
P_2(2) &= 2^{\{0^{\{1,3\}},6^{\{4,5\}},7^{\{3,5\}}\}}, & P_2(3) &= 3^{\{0^{\{1,2\}},4^{\{1,6\}},7^{\{2,5\}}\}},\\
P_2(4) &= 4^{\{1^{\{0,5\}},3^{\{0,7\}},6^{\{2,5\}}\}}, & P_2(5) &= 5^{\{1^{\{0,4\}},6^{\{2,4\}},7^{\{2,3\}}\}},\\
P_2(6) &= 6^{\{2^{\{0,7\}},4^{\{1,3\}},5^{\{1,7\}}\}}, & P_2(7) &= 7^{\{2^{\{0,6\}},3^{\{0,4\}},5^{\{1,6\}}\}}.
\end{aligned}$$

Unlike the little information we get from the geometric representation, the construction of equations which we have found clearly shows equality between the diameter and the eccentricities of every vertex of the graph: every vertex of this graph is listed in exactly two levels in any of its equations.

A full description of this graph can also be given using one (any of those composing the system) equation by growing it to its fullness. For example:

$$P_3(4) = 4^{\{1^{\{0^{\{2,3\}},5^{\{6,7\}}\}},3^{\{0^{\{1,2\}},7^{\{2,5\}}\}},6^{\{2^{\{0,7\}},5^{\{1,7\}}\}}\}}.$$

Let us wrap up the process of constructing graphs with a sample graph (fig. 4) of order $n = 4$, degree $s = 3$, and girth $g = 5$, which we find analogously. [3]

[2]This is not hard to prove: the addition into a system of equations of an edge, connecting vertices 3 and 6, identifies the last unknown edge, corresponding to vertices 4 and 7.

[3]Notice that every graph found here happens to be Hamiltonian. This was unintentional, and so the corresponding demand into the conditions of generalization was not hypothecated.

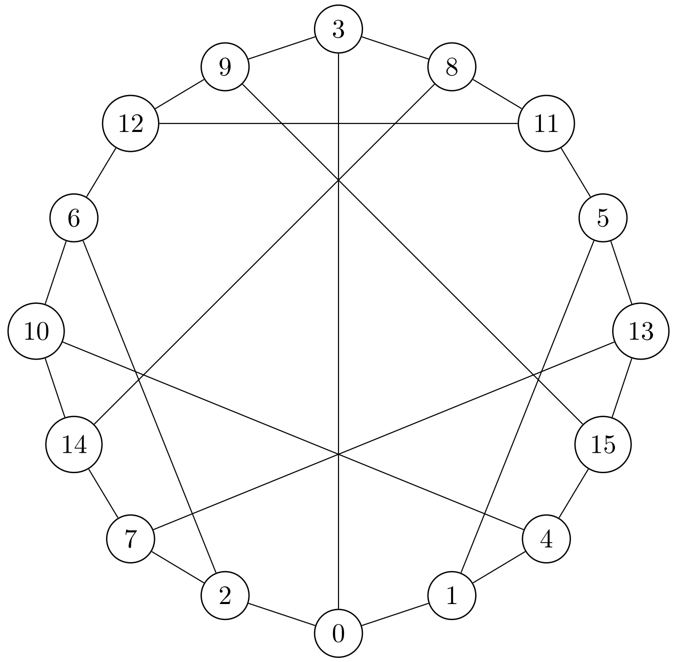

Figure 4: Graph $n = 16$, $s = 3$, $g = 5$

1. From (2), we find the minimal possible number of levels in an equation $P_k(v_j)$ which includes every vertex of the graph $G(V, E)$. This grants the minimal diameter $d(G)$.

   In this case $k = e(v_j) = d(G) = 3)$ for each $v_j) \in V$. The girth of the graph, $g(G)$ is no greater than $2d(G) - 1 \rightarrow g(G) = 5$, is given for this example.

2. Construct a k-level equation of the graph, having chosen as the initial any arbitrarily numbered vertex. The value k is found using step 1. The number of vertices on the i-th level, where $i < k$, is $C_i(s)$, and is determined using equation (1). The set of vertices on the k-th level is those vertices which complete the set V when combined with those in all the preceding levels. This can include subsets of potential vertices, so that each level has the necessary $C_i(s)$ vertices. To construct these subsets, add those vertices whose adjacencies are presently uncertain. When adding a vertex to a subset, remember that the sum of the level number where the subset is located and the minimal level where the vertex can be found must be less than or equal to the given girth.

   In this example, the arbitrary initial vertex is 0. The last (3rd) level of the equation consists of six known vertices, which with those in the previous levels makes $n = 13$. There are also six subsets of potential vertices, which results in $C_2(3) = 12$. The number of edges in this graph, $|E| = ns/2 = 24$, but the equation given by the spanning tree determines only 15 of them, so the other 9 must be identified via construction.

3. Form the initial list of adjacencies between vertices. This includes known adjacencies as well as subsets of potential neighbors.

4. Using the list of adjacencies from step 3, and considering the girth of the graph, built the other equations of the system. Make any corrections in the subsets of potential adjacencies for each newly built equation, and keep track of the corresponding changes in the list of adjacencies and previous equations.

5. The desired equation will be found once the list of adjacencies between vertices does not contain any subsets of potential neighbors. If after the construction of the last of the n equations has a subset of potential adjacencies which corresponds exactly to the unknown edges in the graph, then an adjustment

must be made in one of the subsets, and then all the other equations must be corrected in accordance to step 4. Notice that those substitutions that are incompatible with the given properties are also incompatible with the system of equations, in that the cardinalities of subsets of potential adjacencies become smaller than the number of vertices necessary to identify the corresponding adjacencies. If this happens, return to the previous step and choose and alternative vertex from the subset of potential adjacencies.

In the example, a step which does not contradict the given conditions is adding an edge between vertices 4 and 15. It is necessary to repeat this step twice more, connecting vertices 6 and 10, and 5 and 13. That is, to construct the given regular graph, it was necessary to declare three adjacencies which did not contrading the given conditions. This allowed for the 6 unknown edges to be placed.

Conclusion

Formally speaking, the core of the method proposed in this work is graph representation in parenthetical form, as an *equation*. This text explained the basic technique of constructing such equations, and pointed out an equation's most useful properties. Equations for finding the minimal possible eccentricity of the initial vertex, and the minimal value of levels in an equation of a graph with a given degree and order were given. The analytical associatiations were demonstrated between these parameters and the diameter of the graph, and between these parameters and the ultimate value for the girth of the graph.

The question of constructing a regular graph with a given order, degree, and girth is reduced to an abstract method which does not depend on a geometrical representation, but instead builds an equation of the graph with the corresponding parameters. The core equation contains in itself all the vertices of the graph, some of which are portrayed in their unaltered state, and others in subsets of potential adjacencies, which make it possible to choose various neighbors for those vertices which do not have all their adjacencies identified. The choice of vertices to be included in the subset depends on the given properties corresponding to the parameters of the graph. A proof was given for the method of excluding certain vertices from the subsets of potential adjacencies, based on the location of the subset in the equation (which level number) and the given girth of the graph.

The method of constructing such equations was illustrated by constructing two graphs of the same order and degree, but with different girths. Arguments were provided for the substitutions made to solve the system of equations. As a generalized conclusion on the process of constructing such an equation, an example was given in the form of a graph with the same degree as previous examples, but with a larger order.

Thus, this work introduces an approach to determining a system of regular graphs with given properties, which is not limited to the examples demonstrated here, which focused on generating regular graphs with a minimal diameter. This method could instead find systems of graphs which: have a hamiltonian cycle or chain, have a certain length of alternating noncrossing paths, etc. The use of this method to solve problems of scaling systems, including irregular systems, also seems doable. Furthermore, developing analytical methods of solving enumerated problems and introducing such methods into the theory and practice of building fault-tolerant systems would increase the optimality, reactivity, and predictability of the latter.

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3. Select Line-Specific Commentary

L1 - Original text	L2 - Human translation	Comments
Наиболее распространенный подход к решению этой проблемы состоит в генерации случайных сетей с последующей режекцией не отвечающих заданным критериям вариантов. В качестве критериев при этом используют такие общеизвестные показатели, как диаметр, связность, коэффициент кластеризации и т. п.	The most common method for solving such problems is generating 4 random networks and rejecting those which do not fit the given criteria (diameter, connectivity, clustering 5 factors, etc.)	*Lines 3-5* L1 word count: 38 L2 word count: 29 I chose to combine the two sentences into one, as the second one states "As examples of such criteria they use such things as..."
1. Основные положения	1 Methodology Outline	*Line 21* Google translates this phrase as "initial regulations" (2/18/2018), but "положения" can be defined as "laying out" something, in this case the facts, so I chose to translate it as "outline".
для вершины u величина e(u) = max u∈V ∂(u, v)	for the vertex u, eccentricity e(u) = max u∈V 26 δ(u, v)	*Line 26* "Величина" translates as "size", but his definition is describing the eccentricity of a vertex.
Для подмножества вершин 1-го уровня V_{1w}, порожденного единственной вершиной *w* 0-го уровня, подмножество V'_{1w} состоит из одной вершины: $V'_{10} = \{w\}$.	Once more using the first level of the equation, $V_{1,w}$, as example, the subset of its vertices is derived from the single vertex *w* of the 0-th level (remember, subset $V'_{1,w}$ consists of one vertex: $V'_{1,0} = \{w\}$).	*Lines 46-48* I chose to add the starting statement, as edits made the fact that the 1st level is being discussed lost. Melentiev also often repeats a statement in math after stating it verbally. To show that this is not new information, I put the math in parentheses where needed.
мощность	cardinalities	*Line 56* "Мощность" directly translates to "power" or "potency", which could be mistakened for the power of a graph. By context, it is actually the cardinality.

множество вершин любого *n*-го уровня проекции объединяет в себе подмножества...	the vertex set in any *n*-th level of the equation unifies in itself the subsets...	*Lines 67, 68* "Объединяет" typically translates as "connects", but since this is a group of subsets, I chose "unifies".
Для вершин из V_{ij} , расположенных на уровнях 0 < *i* <= *k*	For vertices in $V_{i,j}$ on levels 0 < *i* <= *k*	*Lines 104, 105* "Из" translates directly as "from", but I chose "in" here, as there is no movement "from" a set.
В дополнение к приведенным выше свойствам, используемым предлагаемым в работе подходом, сформулируем следующее вспомогательное утверждение.	By combining the above given properties and the offered method we propose:	*Line 112* Instead of saying "adding to the above given properties the offered method", which is not only lengthy, but is also clunky in English, I combine the two. I have also removed "следующее вспомогательное утверждение", or "the following statement", as it can be assumed by the colon.
--	Consider a graph *G(V, E)* with equation $P(v_0)$ where v_0 is the initial vertex.	*Line 115* This was a statement I added at the beginning of the proof to avoid referencing the names of the graph, equation and vertex after each time they are mentioned.
введя смежность вершины 1 с вершинами 4 и 5 и вершины 2 с вершинами 6 и 7	we make vertex 1 adjacent to vertices 4 and 5, and vertex 2 with 6 and 7	*Line 196, 197* Translating this segment word for word would result in a very long-winded statement: "we introduce an adjacency between vertex 1 and the vertices 4 and 5, and between vertex 2 and vertices 6 and 7". Not only is it too long, but the numerous "and"s make it hard to follow.

Conclusions

This project introduces Melentiev's work to English speaking mathematicians. It begins by summarizing the text to be translated. Then some linguistic terms are defined, as well as the basic methodology of translation. Finally, the translation itself is presented, followed by some select line-specific explanations of translation decisions.

From the list in section 3 it is clear that online translation leaves something to be desired. That said, translating Artificial Intelligence is becoming ever more advanced. Consider, for example, the "множество" mentioned above. Although Google does not immediately recognize it as "set" on its own, if entered in a sentence including other mathematical terms, Google's software will translate the word not as "a bunch of" but as "set". Perhaps my research could motivate software developers to include more context based algorithms, and to work with translators to develop categories of translations: literary, technical, and so on.

This is a global world, and yet so much information remains inaccessible due to language barriers. I hope that my project will begin to lower those barriers, by bringing information from one language, one culture, to another, and will inspire future projects such as this one. Although translation itself is a solitary work, the process requires communication: it is a multidisciplinary project, involving linguists and scientists; furthermore, it is nothing without representatives of multiple cultures. I thank Melentiev for granting me his work to bring across the world.

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