Head-Wave Correlations in Layered Seabed: Theory and Modeling

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Head-wave correlations in layered seabed: Theory and modeling

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Abstract: This paper derives travel times and arrival angles of head-wave correlations from ocean ambient noise in shallow water over a layered seabed. The upcoming and surface reflected head-wave noise signal received at two receivers from the same interface are correlated, and their travel time differences give the travel times of the head-wave correlations. The arrival angle of head-wave correlations from an interface depends on sound speeds in the layers above and just below. The predictions of head-wave correlations from a seabed with two layers and the corresponding inversion results are verified with simulations. © 2021 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).

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1. Introduction

In shallow water with a fast homogeneous fluid half-space seabed, head waves are acoustic waves propagating in the water column at the critical angle with part of their propagation path in the sediment interface. In the water column, when the horizontal distance between source and receiver is relatively large, the head-wave arrival can precede other water-borne arrivals due to the greater sound speed at the bottom. Head waves can be excited in a variety of ways, including explosive sources,2,5 helicopter noise,3 and ocean surface noise,1,6–8 and they are widely used to determine sub-bottom sound speeds.

Recently, the “virtual head waves” have been observed from ocean surface generated noise9 both in simulations and experiments using vertical1,6–8 and horizontal6,10 arrays. The virtual head waves have the same phase speed as the real, acoustic head waves, but the travel times are offset, thus the term “virtual.” It has been proven theoretically from the perspective of Green’s function retrieval11–16 that the virtual head waves are produced by the cross correlations between head waves and either total reflected waves or other head waves. Therefore, the virtual head waves are more specifically described as “head-wave correlations” here. The locations of noise sources that contribute to the head-wave correlations have been derived based on the method of stationary phase.5 Although the head-wave correlations produced by a single noise source are weak, the integration over time of contributions from the stationary sources makes the head-wave correlations observable.8 The travel times and angles of arrival of the head-wave correlations have been used to invert for geometric and environmental properties in a Pekeris waveguide or in a refracting waveguide with half-space seabed.7

Most theoretical and experimental research on head-wave correlations has focused on the semi-infinite, isovelocity half-space seabed, which does not represent many realistic ocean environments. If the ocean seabed is layered, sound rays will penetrate into the ocean seabed and reflect or refract back from different interfaces. Thus, one can observe head waves not only from the water-sediment interface but also from deeper environments. Extracting information about these head-wave correlations from different interfaces is possible by cross correlation processing. The head-wave correlations from a layered seabed have been observed in simulation using controlled sources distributed near the sea surface and a horizontal receiving array.17 They appear as a sequence of signals in time at some specific angles. However, the exact travel times and angles of arrival of the head-wave correlations are not derived.

This paper studies the travel times and angles of arrival of head-wave correlations in an ocean waveguide with a layered seabed based on theory and modeling. In addition to predicting the head-wave correlations from the water-
sediment interface in previous work,6–8 the theory is extended to predict the head-wave correlations from deeper layers. Although the head-wave correlations can be extracted with four acquisition geometries (horizontal array active source, horizontal array passive surface noise, vertical array active source, and vertical array passive surface noise),6 the geometry of vertical array passive surface noise is used to simplify. Further, the travel times and angles of arrival of head-wave correlations will be combined to invert for sound speeds and layer thicknesses in the water and seabed.

2. Theory

Figure 1(a) shows a shallow water environment model with $M – 1$ sediment layers over a half-space. The noise sources $S$ generated at the sea surface are from wind and waves but potentially could be from other sources such as ship noise. For near surface sources, propagating waves are assumed only down-going from the source and both up-going and down-going at the receiver [see Fig. 1(b)]. The travel time of head waves propagating at interface 1 from source $S$ to receiver $V_j$ is the sum of travel times along the path in the water $l_w$ and interface 1 $l_{\text{int}1}$,6–8

$$t_{ij} = l_w/v_1 + l_{\text{int}1}/v_2 = \frac{2m_1H_1 - z_i \pm z_j}{v_1} + \left[ x_j - x_i - (2m_1H_1 - z_i \pm z_j)\cot \theta_1 \right] \frac{1}{v_2} = \frac{(2m_1H_1 - z_i \pm z_j)\sin \theta_1^i}{v_1} + \frac{x_j - x_i}{v_2},$$

(1)

where $m_1 \in N^+$ is the number of bounce points from interface 1 at depth $H_1$ that occur between the source $S$ and receiver $V_j$, and using Snell’s law for grazing angles, $v_2 = v_1/\cos \theta_1^i$, where $v_1$ and $v_2$ are sound speeds in the water and layer 2, $\theta_1^i$ is the grazing angle of head waves from interface 1 (superscript) at interface 1 (subscript). The signs in front of $z_i$ and $z_j$ in (1) correspond to the vertical direction of the propagating waves. At the source, there is just a down-going wave (i.e., $-z_i$), and at the receiver, there are both down-going ($+z_j$) and up-going ($-z_j$) waves. From (1), the travel time difference between receivers $V_j$ and $V_1$ ($\Delta m_1 = 0, \pm 1, \pm 2, \ldots$) is the difference of bounce numbers at interface 1 from $S$ to $V_j$ and $V_1$,

$$\delta t_{ij} = (\Delta m_1) = t_{ij} - t_{i1} = \frac{(2\Delta m_1H_1 \pm z_j - z_1)\sin \theta_1^i}{v_1}.$$

(2)

From Eq. (2), the head-wave correlations from interface 1 are periodic in time with interval $2H_1\sin \theta_1^i/v_1$, due to the bounce difference $\Delta m_1$.

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Fig. 1. (a) $M – 1$ layer over a half-space model showing the acquisition geometry. Densities ($\rho_1, \ldots, \rho_M$) and sound speeds ($v_1, \ldots, v_M$) are shown. Noise sources and receivers are denoted by stars and triangles, respectively. The noise sources $S$ are located everywhere on the surface, and the $N$ receivers form a vertical array. Head waves travel along (a) interface 1 and (b) interface 2 from source $S$ to receiver $V_j$, and $\theta_q^i$ is the grazing angle of head waves from interface $p$ at interface $q$, where $p, q = 1, 2$. 

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In addition to the top seabed layer, head waves can also travel along interface \( p \in [2, M - 1] \), as shown for \( p = 2 \) in Fig. 1(c). The travel time of head waves propagating at interface \( p \) from source \( S \) to receiver \( V_j \) is the sum of travel times along the path in the water \( l_{w} \), down-and-up through layer \( i \) \( h_{lay} \) (\( i \in [2, p] \)) and interface \( p \) \( h_{lap} \),

\[
\ell^\prime_i(m_1, \ldots, m_p) = \frac{l_w}{v_i} + \sum_{i=2}^{p} \frac{h_{lay}}{v_i} + \frac{h_{lap}}{v_{p+1}},
\]

\[
= \frac{2m_Hi \pm z_j}{\sin \theta_{lay}^\prime} + \sum_{i=2}^{p} \frac{2m_Hi}{\sin \theta_{lay}^\prime} + \left[ x_j - x_s - \frac{2m_Hi \pm z_j}{\tan \theta_{lay}^\prime} \right] - \sum_{i=2}^{p} \frac{2m_Hi}{\tan \theta_{lay}^\prime} / v_{p+1},
\]

\[
= \frac{(2m_Hi \pm z_j) \sin \theta_l^\prime}{v_1} + \sum_{i=2}^{p} \frac{2m_Hi \sin \theta_l^\prime}{v_i} + \frac{x_j - x_s}{v_{p+1}},
\]

where \( m_i \in N^+ \) is the bounce number of ray paths from \( S \) to \( V_j \) at interface \( i \).

From (3), assuming \( j = 1 \) results in (4),

\[
\ell^\prime_i(n_1, \ldots, n_q) = \sum_{i=1}^{q} \frac{2m_Hi \sin \theta_l^\prime}{v_i} + \frac{\pm z_j \sin \theta_l^\prime}{v_1} + \frac{x_j - x_s}{v_{p+1}},
\]

where \( n_i \in N^+ \) is the bounce number of ray paths from \( S \) to \( V_j \) at interface \( i \).

The up-and-down-going head waves from the water-sediment interface \( (p = 1) \) arrive at the receivers with angle \( \pm \theta_l^\prime \). It has been shown that the head-wave correlations from interface 1 between \( V_j \) and \( V_i \) are obtained by cross-correlating the head wave noise signal from \( S \) to \( V_j \) and from \( S \) to \( V_i \). In other words, these head waves from interface 1 are correlated. Similarly, the head waves from interface \( p \geq 2 \) at angle \( \pm \theta_l^\prime \) are also correlated. The travel times of head-wave correlations from interface \( p \geq 1 \) are the travel time difference of head waves from \( S \) to \( V_j \) and \( V_i \) at the same interface \( (p = q) \),

\[
\delta \ell^q_{\pm} (\Delta_{m_1}, \Delta_{m_2}, \ldots, \Delta_{m_p}) = \ell^q_i(m_1, \ldots, m_p) - \ell^q_i(n_1, \ldots, n_q)
\]

\[
= \sum_{i=1}^{p} \frac{2\Delta_{m_i}H_i \sin \theta_l^\prime}{v_i} + \frac{(\pm z_j \sin \theta_l^\prime)}{v_1} + \frac{x_j - x_s}{v_{p+1}},
\]

where \( \Delta_{m_i} = m_i - n_i = 0, \pm 1, \pm 2, \ldots, \cos \theta_l^\prime / \cos \theta_l^\prime = v_i / v_i, \cos \theta_l^\prime = v_i / v_{p+1} \). The ray geometry of head-wave correlations is shown in Fig. 2, and the travel times \( \delta \ell^2_{\pm} (\Delta_{m_1}, \Delta_{m_2}) \) can be extracted from these rays. From (5), unlike the head-wave correlations from interface 1 [Eq. (2)], the head-wave correlations from interface \( p \geq 2 \) are not periodic in time due to the bounce differences \( \Delta_{m_i}, \ldots, \Delta_{m_p} \). However, they are periodic with interval \( 2H_i \sin \theta_l^\prime / v_i, i \in [1, \ldots, p] \) when the remaining \( p - 1 \) numbers among \( \Delta_{m_i} \) are fixed.

Combining the travel times and arrival angles of head-wave correlations from different layers, it is possible to invert for the sound speeds in each layer \( (v_1, \ldots, v_M) \) and also layer thicknesses \( (H_1, \ldots, H_{M-1}) \) with observed head-wave correlation travel times \( \delta \ell^q (0)_{\text{obs}}, \delta \ell^q (1)_{\text{obs}}, \delta \ell^q (\Delta_{m_1}, \ldots, -1 = 0, H_{m_i} = 1)_{\text{obs}} \) and angles of arrival \( \theta^q_{\text{obs}} \),

\[
Y(v_1, \ldots, v_M, H_1, \ldots, H_{M-1}) = \left[ \delta \ell^q (0)_{\text{obs}} - \delta \ell^q (0) \right]^2 + \left[ \delta \ell^q (1)_{\text{obs}} - \delta \ell^q (1) \right]^2
\]

\[
+ \sum_{M-1}^{M-1} \left[ \delta \ell^q (\Delta_{m_1}, \ldots, -1 = 0, H_{m_i} = 1)_{\text{obs}} - \delta \ell^q (\Delta_{m_1}, \ldots, -1 = 0, H_{m_i} = 1) \right]^2
\]

\[
+ \lambda \sum_{M-1}^{M-1} \left[ \theta^q_{\text{obs}} - \arccos(v_i / v_{p+1}) \right]^2,
\]

where \( \lambda \) is a Lagrange multiplier. We match the interval and angle of arrival from predictions and measurements by a grid search of \( v_1, \ldots, v_M \) and \( H_1, \ldots, H_{M-1} \).

The travel time of passive fathometer returns \( (\theta = 90^\circ) \) from layered seabed is useful for understanding the modeling results in Sec. 3. The travel time between noise sources overhead to receiver \( V_j \) is

\[
T_j(m_1, \ldots, m_p) = \left\{ \begin{array}{ll}
\sum_{i=1}^{p} \frac{2m_Hi}{v_i} + \frac{z_j}{v_1}, & m_1 \geq 0, m_{i>2} \geq 1, \\
\sum_{i=1}^{p} \frac{2m_Hi}{v_i} - \frac{z_j}{v_1}, & m_1 \geq 1,
\end{array} \right.
\]

where the first and second terms correspond to waves down-going and up-going to the receiver, and \( m_1 = 0 \) is the case for direct wave. After the cross correlation processing, the difference of travel times between receivers \( V_j \) and \( V_i \) is
where \( |m_1| \leq 1 \) and \( |n_1| \leq 1 \) are shown. After the cross correlation processing, the common ray paths between \( S \) and \( V_j \) disappear, and only ray paths I and II are left.

\[
\delta T_z = (m_1, \ldots, m_p, n_1, \ldots, n_q) = T_{ij}(m_1, \ldots, m_q) - T_{ij}(n_1, \ldots, n_q)
\]

\[
= \sum_{j=1}^{p} \frac{2m_i H_i}{v_j} - \sum_{j=1}^{q} \frac{2n_i H_i}{v_j} + \frac{z_j - z_i}{v_1},
\]

(8)

3. Modeling

Simulations illustrate the head-wave correlations in an ocean waveguide with a sediment layer over a half-space, shown in Fig. 3. This layered seabed is different from the semi-infinite seabed in previous modeling. To approximate the noise field generated by breaking waves, the simulation uses sources having random amplitude and phase, which are uniformly distributed on an infinite plane close to the surface (represented as stars in Fig. 3). The vertical line array consists of 100
hydrophones with spacing of 1 m, and the first hydrophone is at depth \( z_1 = 30 \) m (triangles in Fig. 3). The wavenumber integration code OASES is used to compute the simulated array data.19,20

The travel times and angles of arrival of the head-wave correlations in a Pekeris waveguide can be estimated using measurements of ocean ambient noise on a vertical line array.6 The processing used is a generalization of the passive fathometer18 to produce all the possible correlations including auto-beam \( (C_{--}(\omega, \theta) \text{ or } C_{++}(\omega, \theta)) \) and cross-beam correlations \( (C_{--}(\omega, \theta) \text{ or } C_{++}(\omega, \theta)) \), with

\[
C_{\pm} \pm (\omega, \theta) = d_{\pm} (\omega, \theta) \delta t_{\pm} (\omega, \theta) = w_{\pm} \delta w_{\pm} , \tag{9}
\]

where \( d_{\pm} (\omega, \theta) = w_{\pm} p \) is up-going beam, \( d_{+} (\omega, \theta) = w_{+} p \) is down-going beam, \( w_{-} (\omega, \theta) = [e^{i \omega \xi_{1} \sin \theta/v_{1}}, \ldots, e^{i \omega \xi_{L} \sin \theta/v_{1}}]^T \)

and \( w_{+} (\omega, \theta) = w_{+} (\omega, \theta) \) are the steering vectors for up- and down-going wave fields, \( p(\omega) = [p_1(\omega), \ldots, p_{100}(\omega)]^T \) is the pressure vector, and superscript \( T \) and \( H \) represent the transpose and conjugate transpose. The cross-spectral density matrix \( C \) is estimated from the ensemble average \( C = (1/L) \sum_{l=1}^{L} \sum_{l} p_l p_l^H \) of \( L \) snapshots of pressure field \( p_l \). Equation (9) is transformed to the time domain,

\[
c_{\pm} \pm (\tau, \theta) = \mathcal{F}^{-1}[C_{\pm} \pm (\omega, \theta)] , \tag{10}
\]

where \( \tau \) is lag time. For four types of time domain correlations in (10), the head-wave correlations from the water-sediment interface appear as the sequences of peaks with travel times \( \delta t_{\pm} \pm (\Delta m_l) \) [Eq. (2)] at angle \( \theta_{1} = \arccos (v_1/v_2) \), while those from interface 2 arrive with travel times \( \delta t_{\pm} \pm (\Delta m_m, \Delta m_n) \) [Eq. (5)] at angle \( \theta_{2} = \arccos (v_1/v_3) \).

Figure 4(a) shows the time domain cross-beam correlation \( c_{--}(\tau, \theta) \) from the ocean waveguide with layered seabed environment illustrated in Fig. 3. As predicted by \( \delta t_{++}(\Delta m_m) \) in Eq. (2), the head-wave correlations from the first interface are periodic in the time domain with interval \( 2H_{1} \sin \theta_{1}/v_{1} = 0.11 \) s at angle \( \theta_{1} = 33.6^\circ \). The head-wave correlations from interface 2 are also observed at \( \theta_{2} = 41.4^\circ \); however, they are not periodic in time as predicted by \( \delta t_{++}(\Delta m_m, \Delta m_n) \) in Eq. (5). Due to attenuation, not all the pairs of \( (\Delta m_m, \Delta m_n) \) are observable, and among all the observed peaks, the intensity is different.

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Fig. 4. (a) The time domain cross-beam correlation \( c_{--}(\tau, \theta) \) from the ocean waveguide with layered seabed in Fig. 3. The expected locations of head-wave correlations from interface 1 (circles) and 2 (triangles), and passive fathometer returns (diamonds) are shown. (b) The ambiguity surface [Eq. (6)] of water depth, layer thickness, and sound speeds in the water and seabed with the true locations (○).
Besides the head-wave correlations, there are three peaks at $\theta = 90^\circ$. These peaks are the passive fathometer returns\textsuperscript{18} from different interfaces with travel times in Eq. (8). The leftmost one at $t = (-z_2 - z_1)/v_1 = -0.04$ s corresponds to the sum of six cases, when $m_1 = n_1$ and $m_2 = n_2$ in $\delta T_{+4}(m_1, m_2, n_1, n_2)$ [Eq. (8)], $m_1, n_1 \in [1, 2]$, $m_2, n_2 \in [0, 1, 2]$, so the six cases of $(m_1, m_2, n_1, n_2)$ are (1010, 1111, 1212, 2020, 2121, 2222). Similarly, the middle one at $t = 2H_2/t_2 + (-z_2 - z_1)/v_1 = -0.07$ s is the sum of four cases (1110, 1211, 2120, 2221), and the right one at $t = 2m_1H_1/t_1 + (-z_2 - z_1)/v_2 = 0.16$ s is the sum of four cases (1000, 2100, 2111, 2212).

Inserting the angles of arrival $\theta_{s\text{obs}}, \theta_{o\text{obs}}$ and travel times $\delta t(0)_{s\text{obs}}, \delta t(1)_{s\text{obs}}$, and $\delta t^2(0, 1)_{s\text{obs}}$ into Eq. (6), the sound speeds in the water and seabed, water depth $H_1$, and layer thickness $H_2$ in the waveguide are determined as the values giving the minimum in the ambiguity surface; see Fig. 4(b). The inversion results show good agreement with true locations.

4. Discussion and conclusion

This study derives the travel times and angles of arrival of head-wave correlations from sea surface generated noise in an ocean waveguide with a layered seabed using a vertical line array. The up- and down-going head waves from the same interface are correlated, and their travel time differences are the travel times of head-wave correlations. The head-wave correlations from the water-sediment interface are periodic in time, while those from deeper interfaces show a more complicated pattern. The angles of arrival of head-wave correlations from different interfaces are the same as those of head waves, which depend on the sound speeds in the water and seabed layer according to Snell’s law. Combining the travel times and angles of arrival of head-wave correlations from different interfaces, it is possible to invert for all the sound speeds and layer thicknesses in the water and seabed.

The simulation with noise sources and a sediment layer over a half-space seabed confirms the head-wave correlations from two interfaces and the estimation of sound speeds and layer thicknesses in the water and seabed. Besides the head-wave correlations, the passive fathometer returns are also observed at $90^\circ$.

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References and links

