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# Selected Topics of the Past Thirty Years in Ocean Acoustics

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# Selected Topics of the Past Thirty Years in Ocean Acoustics

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This paper reviews some of the highlights of selected topics in ocean acoustics during the thirty years that have passed since the founding of the *Journal of Theoretical and Computational Acoustics*. Advances in computational methods and computers helped to make computational ocean acoustics a vibrant area of research during that period. The parabolic equation method provides an unrivaled combination of accuracy and efficiency for propagation problems in which the bathymetry, sound speed, and other environmental parameters vary in the horizontal directions. The extension of this approach to cases involving layers that support shear waves has been an active area of research throughout the thirty year period. Interest in basin-scale and global-scale propagation was stimulated by the Heard Island Feasibility Test for monitoring climate change in terms of changes in travel time that occur as the temperature of the ocean rises. Diminishing ice cover in the Arctic, which is one of the consequences of climate change, has stimulated renewed interest in Arctic acoustics during the past decade. Reverberation is a challenging problem that was the topic of a major research program during the beginning of the thirty year period. An innovative approach for making it feasible to solve such problems was applied to data for reverberation from the seafloor and from schools of fish, and some of the findings were featured in *Science* and *Nature*. Source localization is one of the core problems in ocean acoustics. When applied on a 2-D array of receivers, an approach based on the eigenvectors of the covariance matrix is capable of separating the signals from different sources from each other, determining when this partitioning step is successful, and tracking sources that cross each other in bearing; one of the advantages of this approach is that it does not require environmental information or solutions of the wave equation. Geoacoustic inversion for estimating the layer structure, wave speeds, density, and other parameters of ocean bottoms has also been a topic of interest throughout the thirty year period.

*Keywords*: Range dependence; seismo-acoustics; parabolic equation method; global-scale acoustics; basin-scale acoustics; ocean acoustic tomography; acoustic thermometry; climate change; Heard Island Feasibility Test; Arctic acoustics; reverberation; source localization; matched-field processing; geoacoustic inversion.



## 1. Introduction

In the decades leading up to the founding of the *Journal of Theoretical and Computational Acoustics* (JTCA), pioneering work in several areas of ocean acoustics, many of which are covered in *Computational Ocean Acoustics*,[1] laid the foundation for the past thirty years of research in that field. Throughout that period, advances that were taking place in computational methods and computers made ocean acoustics a rich and rapidly evolving area of research. The impact of those advances is exemplified by progress in how fast problems in ocean acoustics can be solved with the parabolic equation method.[1–3] Due to the combination of improvements in the efficiency of the algorithms and in the speed of computers (split about evenly between the two), the time required to run parabolic equation models decreased by about a factor of a million during a thirty year period beginning in the early 1980s.[4] There were also major improvements in the accuracy and capability of parabolic equation techniques during that period. It would be difficult to do justice to the entire field of ocean acoustics with a single review paper, but some of the highlights of the past thirty years are discussed here for selected topics in that field.

## 2. Range-Dependent Seismo-Acoustics

Propagation problems in ocean acoustics are referred to as 'range dependent' when there are horizontal variations in the bathymetry, sound speed, and/or other properties of the environment. The parabolic equation method[1–3] provides an unmatched combination of accuracy and efficiency for solving range-dependent problems. This approach is based on factoring the operator in the frequency-domain wave equation into a product of operators corresponding to outgoing (in the horizontal directions) and incoming operators, assuming that outgoing energy dominates incoming energy when range dependence is gradual, and obtaining solutions by solving an outgoing wave equation that neglects backscattering. For the 2-D case in which coupling of energy between planes of constant azimuth is negligible and may be ignored,[5] the implementation of this approach involves approximating the square root of a depth operator. The field is marched outward in range by applying a series of depth operators at each range step. For many years, horizontal variations in the properties of the medium were taken into account with the 'naive' approach of simply updating the environmental parameters that appear in the depth operators.

The accurate treatment of range dependence started to become a topic of interest in the ocean acoustics community shortly before the foundation of JTCA. After a series of range-dependent benchmark problems was proposed, it came to light that the naive approach for handling range dependence often results in significant amplitude errors,[6,7] even for cases in which range dependence is gradual. It was initially believed that such errors are due to the neglect of backscatter, but this hypothesis was ruled out when accurate solutions were obtained with a ray-based model that accounts only for outgoing waves.[8] This revelation provided hope that the accuracy of the parabolic equation method (and other models that neglect backscatter) could be improved for range-dependent problems. A short time later, accurate solutions were obtained for problems involving a sloping interface between the ocean and sediment by implementing the parabolic wave equation in a rotated coordinate system that parallels the interface.[9] Later on, approaches based on conservation of energy[10,11] and single scattering[12] were developed for accurately handling more general types of range dependence. During the same period, the first successful seismo-acoustic parabolic equation models that account for shear waves in the sediment were developed.[13–15]

Extending the parabolic equation method to accurately handle range-dependent problems in seismo-acoustics, such as the example appearing in Fig. 1, has been an active area of research during the entire history of JTCA. An extension of the rotated coordinates approach that accounts for changes in slope has been developed for the seismo-acoustic case.[16] The energy-conservation approach is very effective for the acoustic case, but this approach has proven to be of limited use for the seismo-acoustic case.[17] The single-scattering approach was initially extended to the purely solid case using an iterative approach,[18] but the most effective implementation to date does not require iterations.[19] This approach is based on an approximation that has a simple physical interpretation. In the exact single-scattering solution, the conditions at a vertical interface between two range-independent regions include conservation of normal and tangential stress and normal and tangential displacement. The approximate solution corresponds to an average of two solutions that each conserves two of those four quantities.[19] This approach has proven to be accurate for problems involving sloping solid-solid interfaces and sloping solid boundaries; the latter case is handled by placing an artificial solid layer with low wave speeds and density on the other side of the boundary.[20]

Various approaches have been proposed and tested for handling a sloping fluid-solid interface,[21–26] which has proven to be the most challenging type of range dependence to handle accurately with the parabolic equation method. The most promising approach to date involves modeling the fluid as a solid with low shear speed (so that a sloping fluid-solid interface is approximated in terms of a sloping solid-solid interface) and applying the single-scattering approximation.[26] As currently implemented, this approach requires relatively fine depth grid spacing to account for shear waves that correspond to short wavelengths. When a fluid layer is modeled as a solid with low shear speed, the tangential displacement is continuous across an interface between that layer and another solid layer, but the tangential displacement is not continuous across a fluid-solid interface. The mismatch in interface conditions is what causes waves with short wavelengths to be excited in a boundary layer near the interface.[26] It may be possible to avoid this complication by using slip conditions at the interface between the solid layer that represents the sediment and the low-speed solid

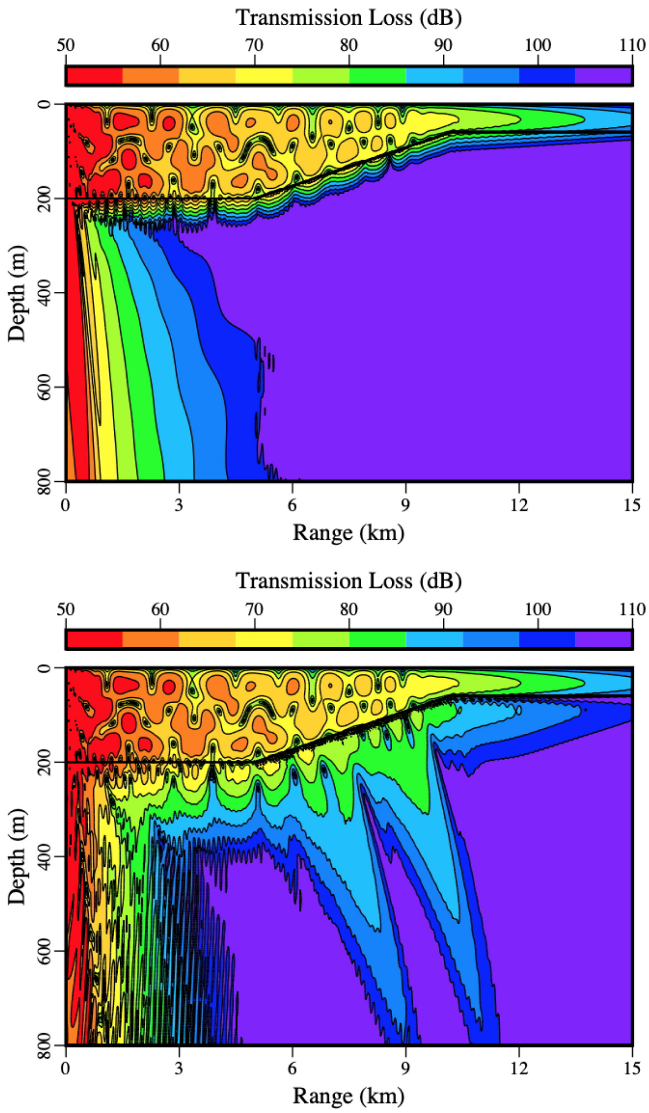


Fig. 1. Solution generated with the parabolic equation method for a seismo-acoustics problem in which modes in the water column couple into shear wave beams in the elastic basement at cutoff. There is no sign of the shear wave beams in the display of the dilatation $\Delta$ in the basement (top), but they are prominent in the display of the normal stress $\sigma_{zz}$ in the basement (bottom).

layer that represents the water column. The tangential displacement is not continuous at this type of interface, which is more compatible with modeling a fluid layer in the limit of low shear speed. Additional contributions to extending the parabolic equation method to accurately handling range-dependent problems in seismo-acoustics include improvements to the self-starter,[27,28] an accurate and efficient approach for generating initial conditions, and rational approximations to the square root of the operator that provide greater accuracy and stability.[29]

## 3. Mega-Meter Propagation Ranges

The fact that sound travels long distances in the ocean is exemplified by a 1960 experiment in which an explosive charge was used as an acoustic source off the west coast of Australia, with multiple arrivals being detected several hours later near the antipode in the north Atlantic Ocean.[30–32] Much of the initial interest in mega-meter problems in ocean acoustics (all the way up to basin-scale and global-scale problems) was rooted in tomography.[33,34]

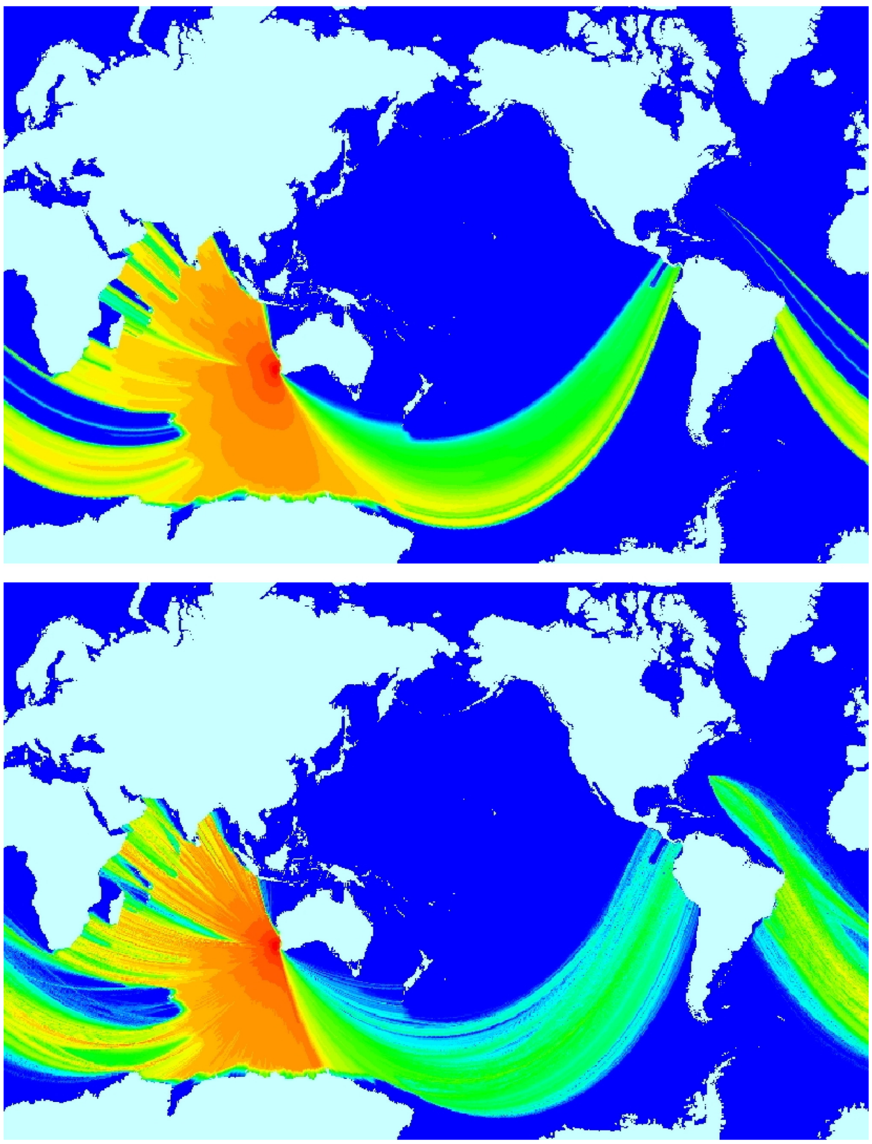

Fig. 2. Adiabatic mode solutions that neglect (top) and account for (bottom) azimuthal coupling for the first mode at 1 Hz for a source off the coast of Australia. Reproduced from Ref. 51 with the permission of the Acoustical Society of America.

Just before the founding of JTCA, the Heard Island Feasibility Test[35–37] stimulated further interest in this area. For that experiment, there were more than a dozen receiving stations located at mega-meter ranges from a 57-Hz source that was deployed off the coast of Heard Island in the Indian Ocean. Signals were received at stations off the coast of California,[38–40] near Ascension Island in the Atlantic Ocean,[41–44] near Christmas Island in the Indian Ocean,[45] near the Gulf Stream in the Atlantic Ocean,[46] off the coast of South Africa,[47] and over the Krylov Seamount off the west coast of Africa.[48]

For one of the longest propagation paths to a vertical array of receivers off the coast of California, a recent improvement in the efficiency of the parabolic equation method[49] was used to generate results consistent with the observed distribution of modes.[38,39] Propagation along this path took several hours, but a 57-Hz calculation with the 2-D parabolic equation model was completed in several minutes.[38] The effects of azimuthal coupling have been taken into account for some mega-meter problems, initially by solving horizontal wave equations for the coefficients of the adiabatic mode solution,[50,51] as in the example appearing in Fig. 2, and then later with a 3-D parabolic equation model that includes mode coupling effects.[52,53] The Heard Island Feasibility Test was the foundation for long-term efforts in basin-scale thermometry.[54,55] Other topics of interest in mega-meter propagation during the past thirty years include efforts to understand arrival patterns of time series received in the sound channel,[56,57] long-range source localization,[58,59] and issues related to the Comprehensive Nuclear Test Ban Treaty.[60–64]

## 4. Arctic Acoustics

Interest in Arctic acoustics started developing in the 1970s.[65–73] Although the level of interest in this area declined for a few decades after the founding of JTCA, there was a groundbreaking experiment during that period. The feasibility of monitoring climate change with long-range acoustic transmissions through the Arctic was tested during an experiment in which coherent transmissions across the Arctic basin were received at stations located 1000 and 2600 km from the source.[74–76] Additional topics of interest during that period include other propagation problems,[77–79] ambient noise,[80–82] and other topics.[83,84] After interest in Arctic acoustics started increasing again, the parabolic equation method, which had been an indispensable tool in ocean acoustics since the 1970s, was finally extended to handle ice cover.[22,23]

The renewed interest in Arctic acoustics was triggered by changing conditions in the Arctic,[85] which was one of the motivations for an experiment involving multiple sea trips in 2016 and 2017 for deploying and recovering acoustic sources and receivers to the north of Alaska.[86–91] In data that were obtained at one of the receivers between October 2016 and March 2017, transmission loss due to scattering from the rough water-ice interface increased as the ice formed and developed keels and other features during fracturing, drifting, ridging, and rafting events.[87] Some of these types of features appear in the photo in Fig. 3. Similar variations in scattering loss were observed on an array that kept recording until September 2017 and documented that the received level continued to decrease well into April 2017

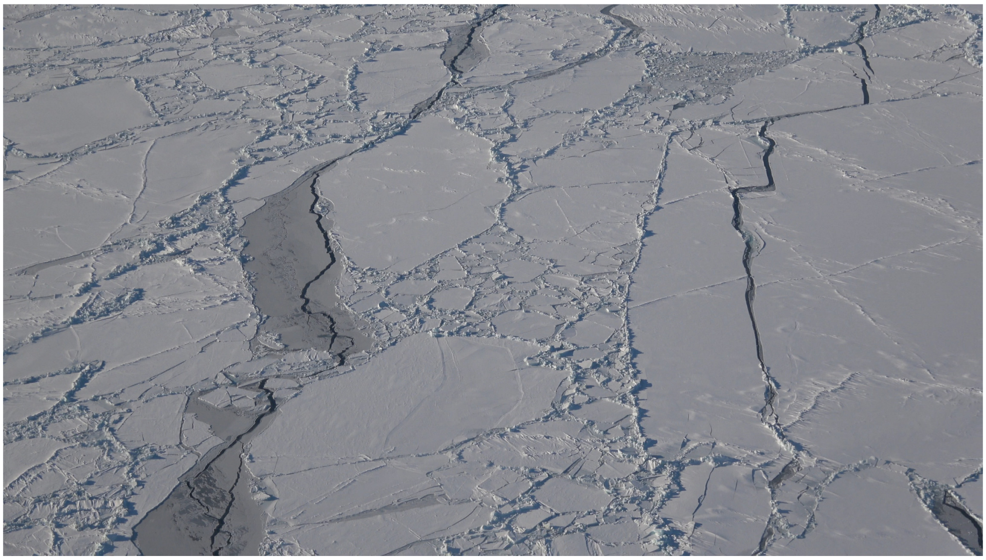

Fig. 3. A photo that shows the aftermath of fracturing, drifting, and ridging events.[87]

before gradually increasing back to the starting level.[88] Acoustic modeling was used to illustrate how the roughness of the water-ice interface[87,88] and oceanographic effects[88] can account for the observed seasonal variations.

One of the advantages of recording over long periods of time is that there is a better chance of capturing strong signals from nearby ice fracturing, marine mammal vocalizations, and other events.[87] Audio recordings of such events are available for download in the supplementary material of Ref. 87. Appearing in Fig. 4 is a spectrogram of a recording of ice floes rubbing together that is rich with features that raise questions about how such

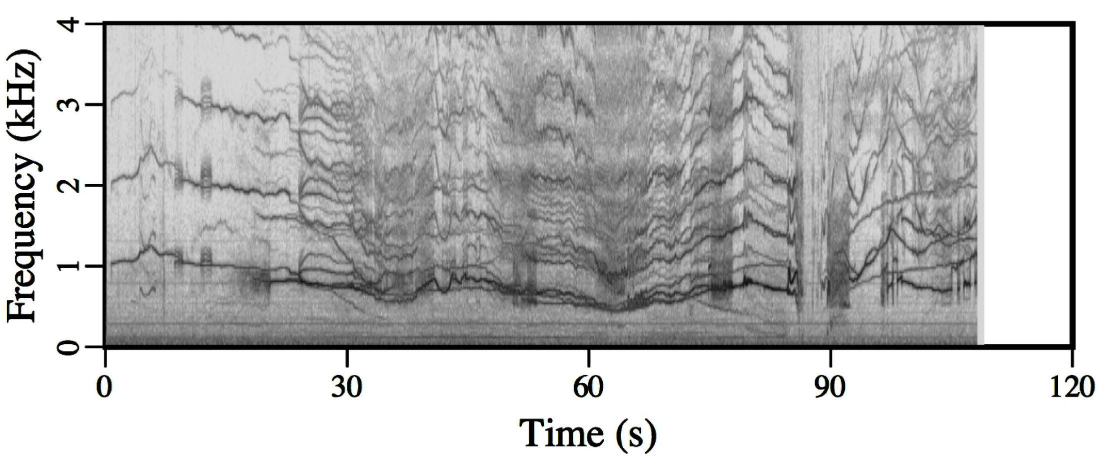


Fig. 4. Spectrogram of sounds that are generated when moving ice floes rub together.[87] This audio recording is available for download in the supplementary material of Ref. 87.

sounds are generated.[87] According to an analysis of $SH$ waves in ice plates,[73] only odd harmonics should be excited when ice floes rub together, but even and odd harmonics are excited in the spectrogram in Fig. 4. The variations in the harmonics of the spectrogram are too rapid to be explained in terms of the resonances of an isolated floe, which would be expected to vary on a much longer time scale. It is possible that the variations are related to (1) coupling between the vibrations in the two plates of ice that occurs at moving contact points and (2) variations in ambient quantities (e.g., stresses, deformations, amount of air trapped under the ice, distribution of cracks) near the contact points as the plates move relative to each other.

## 5. Reverberation from the Seafloor and Fish Schools

Many acoustic scattering problems are extremely difficult to solve. Even relatively simple cases, such as scattering from a compact object in free space, can be challenging. During the past thirty years, there has been considerable interest in reverberation problems in which energy scatters from many distributed features on the seafloor and/or from many distributed objects within the water column. For such problems, it would be challenging to solve for the scattered field corresponding to just one such feature or object, but a great deal of progress has been made by breaking the problem into propagation and scattering components and stressing the former. A reverberation problem may be approximated in terms of a propagation problem from the source to the scattering features or objects, scattering problems at the features or objects, and a propagation problem from the receiver to the scattering features or objects. It is necessary to apply an approximation in order to decouple the propagation and scattering components of the problem from each other. After applying an additional approximation, it becomes a routine matter to solve reverberation problems that would otherwise be intractable.

Since there is typically a much greater dynamic range in the propagated field than in the scattered field, it is possible to obtain approximate solutions by neglecting variations in the scattered field, which is treated as a constant. The propagated field may be obtained by applying a parabolic equation model to compute the incident field throughout the region of interest (e.g., over the seafloor interface). For a receiver that is located at the same position as the source, the same calculation may be used for the propagation back to the receiver. This approximation is compatible with what is typically known about ocean environments. The sound speed profiles, bathymetry, and other parameters of an ocean environment are often known well enough for obtaining fairly good solutions for the propagated field. On the other hand, the environmental information that would be required to model the individual scattering events would likely be highly uncertain. This approximation has been used to analyze data from towed arrays of acoustic receivers that provide bearing (with left-right ambiguity) and time of flight of backscattered returns for problems in reverberation from the seafloor[92–94] and from schools of fish.[95–98] This approach was found to be reliable for identifying features on the seafloor that give rise to strong returns, and it was used to detect synchronized behavior in the formation of massive schools of fish.

## 6. Source Localization

The location of a submerged acoustic source may in some cases be determined with matched-field processing,[99] an approach in which acoustic data from an array of hydrophones is compared with replica fields (solutions of the wave equation). As was the case for Arctic acoustics, there was a peak in activity in this area in the years leading up to the foundation of JTCA.[99–108] It would not be practical to include here a comprehensive review of matched-field processing during the past thirty years. The coverage instead focuses on approaches based on the strategy of introducing additional information in order to improve the chances of successfully localizing a source. Various alternative strategies that appear promising have also been considered.[109–115]

One of the first challenges to be identified in matched-field processing was that this approach may break down when there are uncertainties in bathymetry, sound speed, and other environmental parameters.[103,104] In some cases, this problem may be overcome by including environmental parameters (the additional information) along with the source location in the set of unknowns.[116] This optimization problem would probably be regarded as intractable if not for a parameter hierarchy in which source location outranks environmental parameters; the acoustic field is typically much more sensitive to variations in the location of the source than to variations in the environmental parameters (within typical bounds of uncertainty for these types of parameters). Due to the parameter hierarchy, there may exist many points in the parameter space for which there is good agreement between the data and the replica field. This non-uniqueness can be an advantage when (1) good matches occur for many sets of environmental parameters but only for the correct source location and (2) the primary objective is to determine the source location. In that case, it is not necessary to determine the correct environmental parameters, which is often an extremely challenging inverse problem; it suffices to merely 'tweak' the environmental parameters in a process known as 'focalization,' which has been tested with promising results for cases in which there are uncertainties due to internal waves[117] and sediment parameters.[118]

A common strategy for attempting to achieve improved performance in signal processing is to replace the Bartlett processor with a different processor, such as the Capon processor.[119] These processors are defined in Eqs. (1) and (3) of Ref. 106, where the Capon processor is referred to as the maximum likelihood processor (the same terminology is used in Ref. 120). Since the Bartlett and Capon processors are essentially just different ways for comparing data with replica fields, it seems unlikely that either processor would provide more than marginal improvement over the other. A type of additional information that is not used in the Bartlett and Capon processors is that signals from different sources tend to partition into different eigenvectors of the covariance matrix. The MUSIC processor[120] is based on this additional information, but it does not fully exploit it. Once the energy from different sources is partitioned into different eigenvectors, a logical strategy would be to keep the eigenvectors apart, regard them as high signal-to-noise data for the corresponding sources, and process them separately; this is how the multi-valued Bartlett (MVB) processor works.[121] Possibly due to the tradition of striving to develop improved processors that are single-valued functions for comparing data with replica fields (as are the Bartlett and Capon processors), the MUSIC processor is based on reassembling a subset of the eigenvectors to define a single-valued function. With the MVB processor, isolated eigenvectors are compared with replica fields to obtain multiple ambiguity surfaces. For problems in which the data are obtained with a vertical line array, this approach has provided promising results for problems involving multiple sources buried in noise. As sources move through a waveguide, there may be many points at which the partitioning breaks down, but the tracks of the sources may be revealed if the partitioning is favorable at a series of isolated points.[121]

For the case of a 2-D array consisting of vertical and horizontal subarrays, it is possible to determine the points at which the partitioning is favorable without generating replica fields,[122] which is the most challenging aspect of matched-field processing (it requires information about environmental parameters that may not be available and calculation times that may be prohibitive). When the partitioning is favorable, there is a high correlation between vertical subarrays of the corresponding eigenvector; this allows one to determine when the signal from a single source has been isolated into a single eigenvector without having to generate replica fields. The potential of this approach is illustrated for an example appearing in Fig. 5 that involves a moving source that crosses the bearings of four fixed sources.[122] With a single horizontal array, it is impossible to track sources that cross each other in bearing. With the MVB processor on a 2-D array, it is possible to continuously track the moving source, even when it crosses the bearings of the other sources. This application of the MVB processor does not require solutions of the wave equation; plane wave replica fields are all that is required. This test case and others suggest that the application of the MVB processor to data from a 2-D array may prove to be the most powerful combination of hardware and processing for localizing acoustic sources that has been developed to date. Its capabilities include determining when the signal from a single source has been isolated and tracking such sources without the need to generate replica fields or obtain environmental information.

Several additional ideas have been proposed for exploiting additional information in match-field processing. The signal received from a relatively strong 'guide source' that is located nearly in line with a relatively weak target source may provide additional information that can be used to account for distorting effects caused by uncertainties in the environmental parameters.[123] When a source moves through a complex ocean environment, there may be highly complex variations in the acoustic field on an array of receivers that contain additional information (relative to the case of a fixed source) about the location of the source.[124] Relative to the narrow-band case, broad-band data may contain additional information that can be used to improve the performance of matched-field processing.[125–128] If an estimate of the noise component of the covariance matrix is available, this additional information may be useful for improving the performance of matched-field processing when the signal from a source is obscured by the noise.[129–131] The performance of matched-field processing can also be improved by updating unknown parameters as additional information becomes available.[132] Recent interest in compressive sensing in acoustics[133] was inspired by

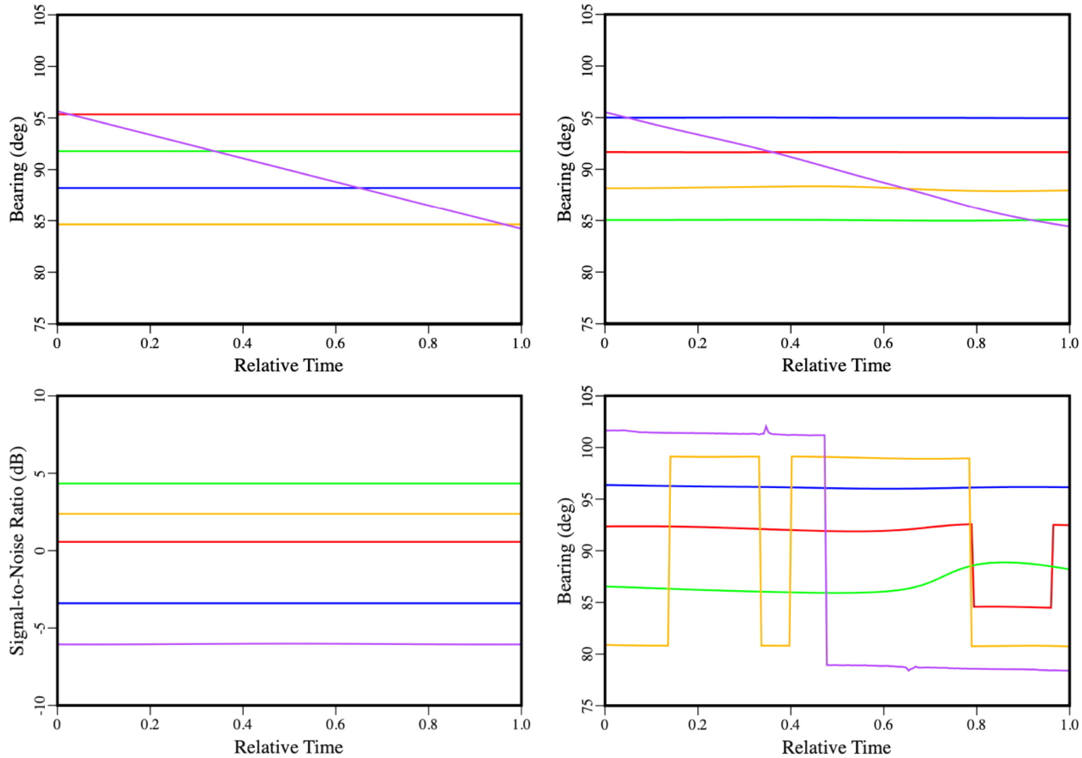


Fig. 5. Results for an example involving four fixed sources and one moving source.[122] Left column: (red = source 1, green = source 2, blue = source 3, orange = source 4, purple = source 5) Right column: (red = eigenvector 1, green = eigenvector 2, blue = eigenvector 3, orange = eigenvector 4, purple = eigenvector 5). Bearings (top left) and SNR (bottom left) for the sources. Estimates of the bearings of the sources obtained with the MVB processor for a rectangular array (top right) and with a horizontal subarray (bottom right). The tracks of all of the sources are recovered fairly well with the rectangular array. The tracking breaks down with a horizontal subarray. The small errors in the recovered bearings are due to the fact that the received fields may be approximated locally in terms of plane waves but are not exactly plane waves.



the discovery that signals may often be reconstructed with sampling that is much sparser than the Nyquist rate.[134,135] This approach has been applied to beamforming[136] and source localization.[137–139]

In addition to developing effective matched-field processing approaches, it is essential to obtain data of the highest possible quality. A common approach for obtaining high-quality data for matched-field processing is to use a vertical line array that is anchored on the seafloor and suspended by a float. The deployment of such an array is time consuming and limited to certain ranges of bathymetry, and there is a possibility of losing hardware and/or data with this approach. As illustrated in Fig. 6, an alternative approach would be to deploy a vertical line array from a surface vessel using motion compensation technology to cancel out the effects of surface motion.[140] Such a system could be rapidly deployed in a wide range of environments and would allow immediate access to data. There has recently been an interest in deploying arrays near fixed platforms (such as oil rigs and

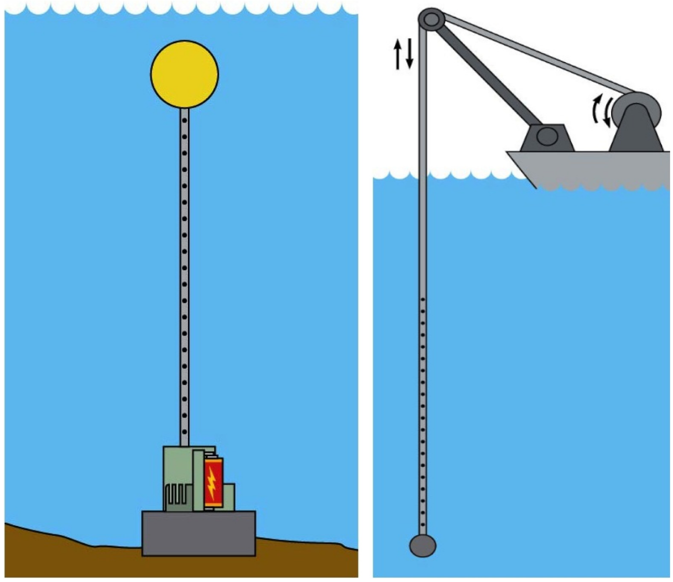

Fig. 6. Vertical arrays are often deployed by suspending with a float and anchoring on the seafloor (left). With motion-compensation technology, it might be possible to achieve a stable deployment from a surface platform (right).[140]

at-sea wind turbines) and using them as acoustic observatories. Deploying a 2-D array at such an observatory would allow for a wide range of processing and capability.

## 7. Geoacoustic Inversion

The sound speed in the water column may be obtained directly from CTD or XBT data. It is much harder to make direct measurements of the parameters of ocean sediments, which consist of layers of thicknesses that may vary in the horizontal directions with wave speeds and other parameters that may vary in all directions within each layer. As an alternative to direct measurement, the parameters of ocean sediments may be determined by geoacoustic inversion, which is a type of matched-field processing in which the parameters of the sediment replace the location of the source as the unknowns. Before the founding of JTCA, there had been some work in this area,[141–145] which exploded in interest in the decades that followed. Since a comprehensive review paper on geoacoustic inversion was recently published in JTCA,[146] we refer the reader to some of the key contributions of the past thirty years[147–166] and limit the discussion here to a simple (but powerful) approach that is widely applicable for improving the efficiency of geoacoustic inversion techniques.[167]

The basic components of geoacoustic inversion include (1) a cost function that quantifies the match between the data and the replica field and (2) a parameter space that defines the sediment, which may consist of multiple layers of unknown thicknesses, wave speeds, attenuations, and density. Let us consider a case in which the ocean bottom consists of a sediment layer over a half-space basement, the environment is range independent, and there are no shear waves. The sound speed is assumed to increase linearly with depth in the sediment layer. A natural choice would be to assign two of the parameters to the values of the sound speed at the top and bottom of the sediment layer, but this would not be an effective choice. At lower frequencies, the cost function would be sensitive to the average sound speed in the layer but not to the wave speed gradient. At higher frequencies (at which energy may penetrate only a short distance into the sediment), the cost function would be sensitive only to the sound speed at the top of the sediment layer.

When a parameter search is conducted in a poorly chosen coordinate system, a lot of time may be wasted searching through long valleys in which the cost function has only small variations along the axes of the valleys. A simple way to improve efficiency is to work in a rotated coordinate system; the eigenvectors of the covariance of the gradient of the cost function define the axes of the rotated coordinate system that are optimally aligned (over the parameter space) with the longest valleys.[167] This is a simple approach for determining the underlying parameters (which are linear combinations of the original parameters) to which the cost function is most sensitive. This selection of coordinates allows one to (1) focus on the most resolvable parameters for a given experimental configuration (e.g., frequency, array geometry, array location relative to the source) and (2) avoid wasting time attempting to determine parameters that would be difficult to resolve under that configuration (i.e., getting trapped in long valleys). Appearing in Fig. 7 are results for an example involving

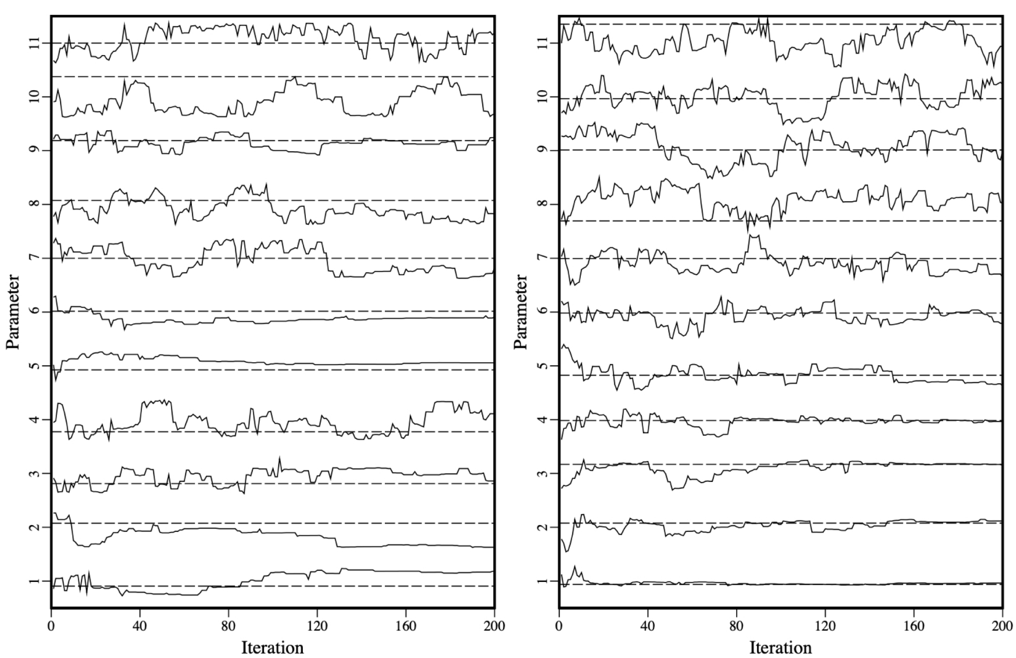


Fig. 7. Parameter searches in the original (left) and rotated (right) coordinates for a problem involving an ocean bottom with two sediment layers over a basement that is defined in terms of 11 parameters. None of the original parameters converge to the correct values. Four of the rotated parameters converge to the correct values. Reproduced from Ref. 167 with the permission of the Acoustical Society of America.

an ocean bottom with two sediment layers over a basement that is defined in terms of 11 parameters.[167] None of the original parameters converge to the correct values. Four of the rotated parameters converge to the correct values.

## 8. Discussion

In addition to the topics discussed in the preceding sections, there has been notable progress in several other areas of ocean acoustics during the past thirty years. There was a great deal of activity in phase conjugation and time reversal,[168] including experiments based on this concept.[169] During a period of growth in the average number of tropical storms and hurricanes per year (which is correlated with increasing ocean temperatures), an approach based on ocean acoustics was derived for quantifying hurricane destructive power.[170] Biot theory has been a topic of interest since the 1950s.[171] Although no experimental evidence exists of the slow wave in an ocean sediment, there was a wave of interest in Biot theory in ocean acoustics in the 1990s that resulted in (1) the discovery that the number of equations in Biot's original formulation can be reduced from four to three for the 2-D case (i.e., there was a redundant equation) and (2) extending the parabolic equation method to porous media.[172] Although Biot theory may be of limited relevance in ocean acoustics, the development of other models for ocean sediments has been an active area of research.[173–175] In recent years, interest in machine learning has been growing rapidly in acoustics,[176] and this area of research is expected to remain active for years to come.

## Acknowledgments

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## References

1. F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, *Computational Ocean Acoustics* (American Institute of Physics, New York, 1994).
2. F. D. Tappert, "The parabolic approximation method," in *Wave Propagation and Underwater Acoustics*, edited by J. B. Keller and J. S. Papadakis, Lecture Notes in Physics, Vol. 70 (Springer, New York, 1977).
3. M. D. Collins and W. L. Siegmann, *Parabolic Wave Equations with Applications* (Springer, Berlin, 2019).
4. W. M. Sanders and M. D. Collins, "Nonuniform depth grids in parabolic equation solutions," *J. Acoust. Soc. Am.* **133** (2012) 1953–1958.
5. J. S. Perkins and R. N. Baer, "An approximation of the three-dimensional parabolic-equation method for acoustic propagation," *J. Acoust. Soc. Am.* **72** (1982) 515–522.
6. F. B. Jensen and C. M. Ferla, "Numerical solutions of range-dependent benchmark problems," *J. Acoust. Soc. Am.* **87** (1990) 1499–1510.
7. M. D. Collins, "Benchmark calculations for higher-order parabolic equations," *J. Acoust. Soc. Am.* **87** (1990) 1535–1538.
8. E. K. Westwood, "Ray model solutions to the benchmark wedge problems," *J. Acoust. Soc. Am.* **87** (1990) 1539–1545.

9. M. D. Collins, "The rotated parabolic equation and sloping ocean bottoms," *J. Acoust. Soc. Am.* **87** (1990) 1035–1037.
10. M. B. Porter, F. B. Jensen, and C. M. Ferla, "The problem of energy conservation in one-way models," *J. Acoust. Soc. Am.* **89** (1991) 1058–1067.
11. M. D. Collins and E. K. Westwood, "A higher-order energy-conserving parabolic equation for range-dependent ocean depth, sound speed, and density," *J. Acoust. Soc. Am.* **89** (1991) 1068–1075.
12. M. D. Collins and R. B. Evans, "A two-way parabolic equation for acoustic backscattering in the ocean," *J. Acoust. Soc. Am.* **91** (1992) 1357–1368.
13. M. D. Collins, "A higher-order parabolic equation for wave propagation in an ocean overlying an elastic bottom," *J. Acoust. Soc. Am.* **86** (1989) 1459–1464.
14. B. T. R. Wetton and G. H. Brooke, "One-way wave equations for seismoacoustic propagation in elastic waveguides," *J. Acoust. Soc. Am.* **87** (1990) 624–632.
15. M. D. Collins, "Higher-order Padé approximations for accurate and stable elastic parabolic equations with application to interface wave propagation," *J. Acoust. Soc. Am.* **89** (1991) 1050–1057.
16. D. A. Outing, W. L. Siegmann, M. D. Collins, and E. K. Westwood, "Generalization of the rotated parabolic equation to variable slopes," *J. Acoust. Soc. Am.* **120** (2006) 3534–3538.
17. M. D. Collins, "An energy-conserving parabolic equation for elastic media," *J. Acoust. Soc. Am.* **94** (1993) 975–982.
18. M. D. Collins, "A two-way parabolic equation for elastic media," *J. Acoust. Soc. Am.* **93** (1993) 1815–1825.
19. M. D. Collins, "A single-scattering correction for the seismo-acoustic parabolic equation," *J. Acoust. Soc. Am.* **131** (2012) 2638–2642.
20. K. Woolfe, M. D. Collins, D. C. Calvo, and W. L. Siegmann, "Seismo-acoustic benchmark problems involving sloping solid-solid interfaces and variable topography," *J. Comp. Acoust.* **24** (2016) 1650019.
21. M. D. Collins and W. L. Siegmann, "Treatment of a sloping fluid-solid interface and sediment layering with the seismo-acoustic parabolic equation," *J. Acoust. Soc. Am.* **137** (2015) 492–497.
22. M. D. Collins, "Treatment of ice cover and other thin elastic layers with the parabolic equation method," *J. Acoust. Soc. Am.* **137** (2015) 1557–1563.
23. J. M. Collis, S. D. Frank, A. M. Metzler, and K. S. Preston, "Elastic parabolic equation and normal mode solutions for seismo-acoustic propagation in underwater environments with ice covers," *J. Acoust. Soc. Am.* **139** (2016) 2672–2682.
24. K. Woolfe, M. D. Collins, D. C. Calvo, and W. L. Siegmann, "Seismo-acoustic benchmark problems involving sloping fluid-solid interfaces," *J. Comp. Acoust.* **24** (2016) 1650022.
25. J. M. Fialkowski, M. D. Collins, D. C. Calvo, and A. Turgut, "Range-dependent seismo-acoustic propagation in the marginal ice zone," *J. Theor. Comp. Acoust.* **26** (2018) 1850013.
26. M. D. Collins and A. Ramamurti, "Parabolic equation modeling of Scholte waves and other effects along sloping fluid-solid interfaces," *J. Theor. Comp. Acoust.* **29** (2021) 2050025.
27. M. D. Collins, "A self-starter for the parabolic equation method," *J. Acoust. Soc. Am.* **92** (1992) 2069–2074.
28. M. D. Collins, "The stabilized self-starter," *J. Acoust. Soc. Am.* **106** (1999) 1724–1726.
29. F. A. Milinazzo, C. A. Zala, and G. H. Brooke, "Rational square-root approximations for parabolic equation algorithms," *J. Acoust. Soc. Am.* **101** (1997) 760–766.
30. R. C. Shockley, J. Northrup, and P. G. Hansen, "SOFAR propagation paths from Australia to Bermuda: Comparison of signal speed algorithms and experiments," *J. Acoust. Soc. Am.* **71** (1982) 51–60.

31. W. H. Munk, W. C. O'Reilly, and J. L. Reid, "Australia-Bermuda sound propagation (1960) revisited," *J. Phys. Oceanogr.* **18** (1988) 1876–1898.
32. K. D. Heaney, W. A. Kuperman, and B. E. McDonald, "Perth-Bermuda sound propagation (1960): Adiabatic mode interpretation," *J. Acoust. Soc. Am.* **90** (1991) 2586–2594.
33. W. H. Munk and C. Wunsch, "Ocean acoustic tomography: A scheme for large scale monitoring," *Deep-Sea Res.* **26A** (1979) 123–161.
34. J. L. Spiesberger and K. Metzger, "Basin-scale tomography: A new tool for studying weather and climate," *J. Geophys. Res.* **96** (1991) 4869–4889.
35. A. Baggeroer and W. Munk, "The Heard Island Feasibility Test," *Phys. Today* (September 1992) 22–30.
36. W. H. Munk, R. C. Spindel, A. Baggeroer, and T. G. Birdsall, "The Heard Island Feasibility Test," *J. Acoust. Soc. Am.* **96** (1994) 2330–2342.
37. T. G. Birdsall, K. Metzger, and M. A. Dzieciuch, "Signals, signal processing, and general results," *J. Acoust. Soc. Am.* **96** (1994) 2343–2352.
38. B. E. McDonald, M. D. Collins, W. A. Kuperman, and K. D. Heaney, "Comparison of data and model predictions for Heard Island acoustic transmissions," *J. Acoust. Soc. Am.* **96** (1994) 2357–2370.
39. A. B. Baggeroer, B. Sperry, K. Lashkari, C. S. Chiu, J. H. Miller, P. N. Mikhalevsky, and K. von der Heydt, "Vertical array receptions of the Heard Island transmissions," *J. Acoust. Soc. Am.* **96** (1994) 2395–2413.
40. G. J. Heard and N. R. Chapman, "Heard Island Feasibility Test: Analysis of Pacific path data obtained with a horizontal line array," *J. Acoust. Soc. Am.* **96** (1994) 2389–2394.
41. A. Forbes and W. Munk, "Doppler-inferred launch angles of global acoustic ray paths," *J. Acoust. Soc. Am.* **96** (1994) 2425–2427.
42. T. M. Georges, L. R. Boden, and D. R. Palmer, "Features of the Heard Island signals received at Ascension," *J. Acoust. Soc. Am.* **96** (1994) 2441–2447.
43. E. C. Shang, Y. Y. Wang, and T. M. Georges, "Dispersion and repopulation of Heard-Ascension modes," *J. Acoust. Soc. Am.* **96** (1994) 2371–2379.
44. D. R. Palmer, T. M. Georges, J. J. Wilson, L. D. Weiner, J. A. Paisley, R. Mathiesen, R. R. Pleshek, and R. R. Mabe, "Reception at Ascension of the Heard Island Feasibility Test transmissions," *J. Acoust. Soc. Am.* **96** (1994) 2432–2440.
45. M. Dzieciuch and W. Munk, "Differential Doppler as a diagnostic," *J. Acoust. Soc. Am.* **96** (1994) 2414–2424.
46. I. A. Fraser and P. D. Morash, "Observation of the Heard Island signals near the Gulf Stream," *J. Acoust. Soc. Am.* **96** (1994) 2448–2457.
47. G. B. Brundrit and L. Krige, "Heard Island signals through the Agulhas retroreflection region," *J. Acoust. Soc. Am.* **96** (1994) 2464–2468.
48. S. V. Burenkov, A. N. Gavrilov, A. Y. Uporin, and A. V. Furduev, "Heard Island Feasibility Test: Long-range sound transmission from Heard Island to Krylov underwater mountain," *J. Acoust. Soc. Am.* **96** (1994) 2458–2463.
49. M. D. Collins, "A split-step Padé solution for the parabolic equation method," *J. Acoust. Soc. Am.* **93** (1993) 1736–1742.
50. M. D. Collins, "The adiabatic mode parabolic equation," *J. Acoust. Soc. Am.* **94** (1993) 2269–2278.
51. M. D. Collins, B. E. McDonald, K. D. Heaney, and W. A. Kuperman, "Three-dimensional effects in global acoustics," *J. Acoust. Soc. Am.* **97** (1995) 1567–1575.
52. K. D. Heaney and R. L. Campbell, "Three-dimensional parabolic equation modeling of mesoscale eddy deflection," *J. Acoust. Soc. Am.* **139** (2016) 918–926.

53. K. D. Heaney, M. Prior, and R. L. Campbell, "Bathymetric diffraction of basin-scale hydroacoustic signals," *J. Acoust. Soc. Am.* **141** (2017) 878–885.
54. P. F. Worcester, B. D. Cornuelle, M. A. Dzieciuch, W. H. Munk, B. M. Howe, J. A. Mercer, R. C. Spindel, J. A. Colosi, Kurt Metzger, Theodore G. Birdsall, and A. B. Baggeroer, "A test of basin-scale acoustic thermometry using a large-aperture vertical array at 3250-km range in the eastern North Pacific Ocean," *J. Acoust. Soc. Am.* **105** (1999) 3185–3201.
55. B. D. Dushaw, P. F. Worcester, W. H. Munk, R. C. Spindel, J. A. Mercer, B. M. Howe, K. Metzger, T. G. Birdsall, R. K. Andrew, M. A. Dzieciuch, B. D. Cornuelle, and D. Menemenlis, "A decade of acoustic thermometry in the North Pacific Ocean," *J. Geophys. Res.* **114** (2009) C07021.
56. T. F. Duda, S. M. Flatté, J. A. Colosi, B. D.Cornuelle, J. A.Hildebrand, W. S. Hodgkiss, P. F. Worcester, B. M. Howe, J. A. Mercer, and R. C. Spindel, "Measured wave-front fluctuations in 1000-km pulse propagation in the Pacific Ocean," *J. Acoust. Soc. Am.* **92** (1992) 939–955.
57. P. F. Worcester, B. D. Cornuelle, J. A. Hildebrand, W. S. Hodgkiss, T. F. Duda, J. Boyd, B. M. Howe, J. A. Mercer, and R. C. Spindel, "A comparison of measured and predicted broadband acoustic arrival patterns in travel time-depth coordinates at 1000-km range," *J. Acoust. Soc. Am.* **95** (1994) 3118–3128.
58. K. D. Heaney and W. A. Kuperman, "Very long-range source localization with a small vertical array," *J. Acoust. Soc. Am.* **104** (1998) 2149–2159.
59. W. A. Kuperman, G. L. D'Spain, and K. D. Heaney, "Long range source localization from single hydrophone spectrograms," *J. Acoust. Soc. Am.* **109** (2001) 1935–1943.
60. G. L. D'Spain, L. W. Berger, W. A. Kuperman, J. L. Stevens, and G. E. Baker "Normal mode composition of earthquake $T$ phases," *Pure Appl. Geophys.* **158** (2001) 475–512.
61. N. R. Chapman and R. Marrett, "The directionality of acoustic $T$-phase signals from small magnitude submarine earthquakes," *J. Acoust. Soc. Am.* **119** (2006) 3669–3675.
62. M. K. Prior, O. Meless, P. Bittner, and H. Sugioka, "Long-range detection and location of shallow underwater explosions using deep-sound-channel hydrophones," *IEEE J. Ocean. Eng.* **36** (2011) 703–715.
63. K. D. Heaney, R. L. Campbell, and M. Snellen, "Long range acoustic measurements of an undersea volcano," *J. Acoust. Soc. Am.* **134** (2013) 3299–3306.
64. D. Metz, A. B. Watts, I. Grevemeyer, M. Rodgers, and M. Paulatto, "Ultra-long-range hydroacoustic observations of submarine volcanic activity at Monowai, Kermadec Arc," *Geophys. Res. Lett.* **43** (2016) 1529–1536.
65. O. I. Diachok, "Effects of sea-ice ridges on sound propagation in the Arctic Ocean," *J. Acoust. Soc. Am.* **59** (1976) 1110–1120.
66. M. Schulkin, G. R. Garrison, and T. Wen, "Acoustic variability due to layered finestructure in the Arctic," *J. Acoust. Soc. Am.* **66** (1979) 235–249.
67. I. Dyer, A. B. Baggeroer, J. D. Zittel, and R. J. Williams, "Acoustic backscattering from the basin and margins of the Arctic Ocean," *J. Geophys. Res.* **87** (1982) 9477–9488.
68. A. E. Hay, "Remote acoustic imaging of the plume from a submarine in an Arctic fjord," *Science* **225** (1984) 1154–1156.
69. N. C. Makris and I. Dyer, "Environmental correlates of pack ice noise," *J. Acoust. Soc. Am.* **79** (1986) 1434–1440.
70. A. J. Langley, "Acoustic emission from the Arctic ice sheet," *J. Acoust. Soc. Am.* **85** (1989) 692–701.
71. P. Zakarauskas and J. M. Thorleifson, "Directionality of ice cracking events," *J. Acoust. Soc. Am.* **89** (1991) 722–734.
72. N. C. Makris and I. Dyer, "Environmental correlates of Arctic ice-edge noise," *J. Acoust. Soc. Am.* **90** (1991) 3288–3298.

73. Y. Xie and D. M. Farmer, "The sound of ice break-up and floe interaction," *J. Acoust. Soc. Am.* **91** (1992) 1423–1428.
74. P. N. Mikhalevsky, A. N. Gavrilov, and A. B. Baggeroer, "The transarctic acoustic propagation experiment and climate monitoring in the Arctic," *IEEE J. Ocean. Eng.* **24** (1999) 183–201.
75. A. N. Gavrilov and P. N. Mikhalevsky, "Low-frequency acoustic propagation loss in the Arctic Ocean: Results of the Arctic climate observations using underwater sound experiment," *J. Acoust. Soc. Am.* **119** (2006) 3694–3706.
76. H. C. Song, P. N. Mikhalevsky, and A. B. Baggeroer, "Transarctic acoustic telemetry," *J. Acoust. Soc. Am.* **136** (2014) 1491–1494.
77. K. LePage and H. Schmidt, "Modeling of low-frequency transmission loss in the central Arctic," *J. Acoust. Soc. Am.* **96** (1994) 1783–1795.
78. K. L. Williams and D. E. Funk, "High-frequency forward scattering from the arctic canopy: Experiment and high-frequency modeling," *J. Acoust. Soc. Am.* **96** (1994) 2956–2964.
79. P. Alexander, A. Duncan, N. Bose, and D. Smith, "Modelling acoustic transmission loss due to sea ice cover," *Acoust. Aust.* **41** (2013) 79–87.
80. E. H. Roth, J. A. Hildebrand, and S. M. Wiggins, "Underwater ambient noise on the Chukchi Sea continental slope from 2006–2009," *J. Acoust. Soc. Am.* **131** (2012) 104–110.
81. G. B. Kinda, Y. Simard, C. Gervais, J. I. Mars, and L. Fortier, "Under-ice ambient noise in Eastern Beaufort Sea, Canadian Arctic, and its relation to environmental forcing," *J. Acoust. Soc. Am.* **134** (2013) 77–87.
82. G. B. Kinda, Y. Simard, C. Gervaise, J. I. Mars, and L. Fortier, "Arctic underwater noise transients from sea ice deformation: Characteristics, annual time series, and forcing in Beaufort Sea," *J. Acoust. Soc. Am.* **138** (2015) 2034–2045.
83. S. E. Dosso and G. H. Brooke, "Measurement of seismo-acoustic ocean-bottom properties in the high Arctic," *J. Acoust. Soc. Am.* **98** (1995) 1657–1666.
84. C. Stamoulis and I. Dyer, "Acoustically derived ice-fracture velocity in central Arctic pack ice," *J. Acoust. Soc. Am.* **108** (2000) 96–104.
85. J. Stroeve, M. M. Holland, W. Meier, T. Scambos, and M. Serreze, "Arctic sea ice decline: Faster than forecast," *Geophys. Res. Lett.* **34** (2007) L09501.
86. M. Badiey, L. Wan, S. Pecknold, and A. Turgut, "Azimuthal and temporal sound fluctuations on the Chukchi Continental Shelf during the Canada Basin Acoustic Propagation Experiment 2017," *J. Acoust. Soc. Am.* **146** (2019) EL530–EL536.
87. M. D. Collins, A. Turgut, R. Menis, and J. A. Schindall, "Acoustic recordings and modeling under seasonally varying sea ice," *Sci. Rep.* **9** (2019) 8323.
88. M. S. Ballard, M. Badiey, J. D. Sagers, J. A. Colosi, A. Turgut, S. Pecknold, Y. T. Lin, A. Proshutinsky, R. Krishfield, P. F. Worcester, and M. A. Dzieciuch, "Temporal and spatial dependence of a yearlong record of sound propagation from the Canada Basin to the Chukchi Shelf," *J. Acoust. Soc. Am.* **148** (2020) 1663–1680.
89. T. F. Duda, W. G. Zhang, and Y. T. Lin, "Effects of Pacific Summer Water layer variations and ice cover on Beaufort Sea underwater sound ducting," *J. Acoust. Soc. Am.* **149** (2021) 2117–2136.
90. J. Bonnel, G. B. Kinda, and D. P. Zitterbart, "Low-frequency ocean ambient noise on the Chukchi Shelf in the changing Arctic," *J. Acoust. Soc. Am.* **149** (2021) 4061–4072.
91. M. S. Ballard and J. D. Sagers, "Clustering analysis of a yearlong record of ambient sound on the Chukchi Shelf in the 40 Hz to 4 kHz frequency range," *J. Acoust. Soc. Am.* **150** (2021) 1597–1608.
92. N. C. Makris and J. M. Berkson, "Long-range backscatter from the Mid-Atlantic Ridge," *J. Acoust. Soc. Am.* **95** (1994) 1865–1881.

93. N. C. Makris, L. Z. Avelino, and R. Menis, "Deterministic reverberation from ocean ridges," *J. Acoust. Soc. Am.* **97** (1995) 3547–3574.
94. P. Ratilal, Y. Lai, D. T. Symonds, L. A. Ruhlmann, J. R. Preston, E. K. Scheer, M. T. Garr, C. W. Holland, J. A. Goff, and N. C. Makris, "Long range acoustic imaging of the continental shelf environment: The Acoustic Clutter Reconnaissance Experiment 2001," *J. Acoust. Soc. Am.* **117** (2005) 1977–1998.
95. N. C. Makris, P. Ratilal, D. T. Symonds, S. Jagannathan, S. Lee, and R. W. Nero, "Fish population and behavior revealed by instantaneous continental shelf-scale imaging," *Science* **311** (2006) 660–663.
96. A. Galinde, N. Donabed, M. Andrews, S. Lee, N. C. Makris, and P. Ratilal, "Range-dependent waveguide scattering model calibrated for bottom reverberation in continental shelf environments," *J. Acoust. Soc. Am.* **123** (2008) 1270–1281.
97. N. C. Makris, P. Ratilal, S. Jagannathan, Z. Gong, M. Andrews, I. Bertsatos, O. R. Godø, R. W. Nero, and J. M. Jech, "Critical population density triggers rapid formation of vast oceanic fish shoals," *Science* **323** (2009) 1734–1737.
98. D. Wang, H. Garcia, W. Huang, D. D. Tran, A. D. Jain, D. H. Yi, Z. Gong, J. M. Jech, O. R. Godø, N. C. Makris, and P. Ratilal, "Vast assembly of vocal marine mammals from diverse species on fish spawning ground," *Nature* **531** (2016) 366–370.
99. H. P. Bucker, "Use of calculated sound fields and matched-field detection to locate sound sources in shallow water," *J. Acoust. Soc. Am.* **59** (1976) 368–373.
100. M. B. Porter, R. L. Dicus, and R. G. Fizell, "Simulations of matched-field processing in a deep-water Pacific environment," *IEEE J. Ocean. Eng.* **12** (1987) 173–181.
101. A. B. Baggeroer, W. A. Kuperman, and H. Schmidt, "Matched field processing: Source localization in correlated noise as an optimum parameter estimation problem," *J. Acoust. Soc. Am.* **83** (1988) 571–587.
102. J. M. Ozard, "Matched-field processing in shallow water for range, depth, and bearing estimation: Results of experiment and simulation," *J. Acoust. Soc. Am.* **86** (1989) 744–753.
103. A. Tolstoy, "Sensitivity of matched field processing to sound-speed profile mismatch for vertical arrays in a deep water Pacific environment," *J. Acoust. Soc. Am.* **85** (1989) 2394–2404.
104. R. M. Hamson and R. M. Heitmeyer, "Environmental and system effects on source localization in shallow water by the matched-field processing of a vertical array," *J. Acoust. Soc. Am.* **86** (1989) 1950–1959.
105. J. S. Perkins and W. A. Kuperman, "Environmental signal processing: Three-dimensional matched-field processing with a vertical array," *J. Acoust. Soc. Am.* **87** (1990) 1553–1556.
106. H. Schmidt, A. B. Baggeroer, W. A. Kuperman, and E. K. Scheer, "Environmentally tolerant beamforming for high-resolution matched field processing: Deterministic mismatch," *J. Acoust. Soc. Am.* **88** (1990) 1851–1862.
107. A. Tolstoy, *Matched Field Processing for Underwater Acoustics* (World Scientific, Singapore, 1992).
108. A. B. Baggeroer, W. A. Kuperman, and P. N. Mikhalevsky, "An overview of matched field methods in ocean acoustics," *IEEE J. Ocean. Eng.* **18** (1993) 401–424.
109. L. T. Fialkowski, M. D. Collins, W. A. Kuperman, J. S. Perkins, L. J. Kelly, A. Larsson, J. A. Fawcett, and L. H. Hall, "Matched-field processing using measured replica fields," *J. Acoust. Soc. Am.* **107** (2000) 739–746.
110. P. Hursky, W. S. Hodgkiss, and W. A. Kuperman, "Matched field processing with data-derived modes," *J. Acoust. Soc. Am.* **109** (2001) 1355–1366.
111. S. E. Dosso and M. J. Wilmut, "Uncertainty estimation in simultaneous Bayesian tracking and environmental inversion," *J. Acoust. Soc. Am.* **124** (2008) 82–97.

112. K. L. Gemba, W. S. Hodgkiss, and P. Gerstoft, "Adaptive and compressive matched field processing," *J. Acoust. Soc. Am.* **141** (2017) 92–103.
113. K. L. Gemba, S. Nannuru, P. Gerstoft, and W. S. Hodgkiss, "Multi-frequency sparse Bayesian learning for robust matched field processing," *J. Acoust. Soc. Am.* **141** (2017) 3411–3420.
114. H. Niu, E. Reeves, and P. Gerstoft, "Source localization in an ocean waveguide using supervised machine learning," *J. Acoust. Soc. Am.* **142** (2017) 1176–1188.
115. Z. Huang, J. Xu, Z. Gong, H. Wang, and Y. Yan, "Source localization using deep neural networks in a shallow water environment," *J. Acoust. Soc. Am.* **143** (2018) 2922–2932.
116. M. D. Collins and W. A. Kuperman, "Focalization: Environmental focusing and source localization," *J. Acoust. Soc. Am.* **90** (1991) 1410–1422.
117. R. N. Baer and M. D. Collins, "Source localization in the presence of internal waves," *J. Acoust. Soc. Am.* **118** (2005) 3117–3121.
118. R. N. Baer and M. D. Collins, "Source localization in the presence of gross sediment uncertainties," *J. Acoust. Soc. Am.* **120** (2006) 870–874.
119. H. L. Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory* (Wiley-Interscience, New York, 2002).
120. R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Ant. Prop.* **AP-34** (1986) 276–280.
121. M. D. Collins, L. T. Fialkowski, W. A. Kuperman, and J. S. Perkins, "The multivalued Bartlett processor and source tracking," *J. Acoust. Soc. Am.* **97** (1995) 235–241.
122. M. D. Collins and J. F. Lingevitch, "Multi-valued eigen-processing for isolating multiple sources with a rectangular array," *IEEE Access* **9** (2021) 8990–8996.
123. M. Siderius, D. R. Jackson, D. Rouseff, and R. Porter, "Multipath compensation in shallow water environments using a virtual receiver," *J. Acoust. Soc. Am.* **102** (1997) 3439–3449.
124. M. D. Collins, L. T. Fialkowski, W. A. Kuperman, and J. S. Perkins, "Environmental source tracking," *J. Acoust. Soc. Am.* **94** (1993) 3335–3341.
125. E. K. Westwood, "Broadband matched-field source localization," *J. Acoust. Soc. Am.* **91** (1992) 2777–2789.
126. Z. H. Michalopoulou and M. B. Porter, "Matched-field processing for broad-band source localization," *IEEE J. Ocean. Eng.* **21** (1996) 384–392.
127. N. O. Booth, P. A. Baxley, J. A. Rice, P. W. Schey, W. S. Hodgkiss, G. L. D'Spain, and J. J. Murray, "Source localization with broad-band matched-field processing in shallow water," *IEEE J. Ocean. Eng.* **21** (1996) 402–412.
128. C. Soares and S. M. Jesus, "Broadband matched-field processing: Coherent and incoherent approaches," *J. Acoust. Soc. Am.* **113** (2003) 2587–2598.
129. M. D. Collins, N. C. Makris, and L. T. Fialkowski, "Noise cancellation and source localization," *J. Acoust. Soc. Am.* **96** (1994) 1773–1776.
130. L. T. Fialkowski, M. D. Collins, J. S. Perkins, and W. A. Kuperman, "Source localization in noisy and uncertain ocean environments," *J. Acoust. Soc. Am.* **101** (1997) 3539–3545.
131. M. D. Collins, R. N. Baer, and H. J. Simpson, "Experimental testing of the noise-canceling processor," *J. Acoust. Soc. Am.* **130** (2011) 1217–1221.
132. C. Yardim, Z. H. Michalopoulou, and P. Gerstoft, "An overview of sequential Bayesian filtering in ocean acoustics," *IEEE J. Ocean. Eng.* **36** (2011) 73–91.
133. P. Gerstoft, C. F. Mecklenbräuker, W. Seong, and M. Bianco, "Introduction to compressive sensing in acoustics," *J. Acoust. Soc. Am.* **143** (2018) 3731–3736.
134. E. J. Candès, J. K. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Commun. Pure Appl. Math.* **59** (2006) 1207–1233.
135. E. J. Candès and M. B. Wakin, "An introduction to compressive sensing," *IEEE Signal Proc. Mag.* **25** (2008) 21–30.

136. A. Xenaki, P. Gerstoft, and K. Mosegaard, "Compressive beamforming," *J. Acoust. Soc. Am.* **136** (2014) 260–271.
137. K. L. Gemba, W. S. Hodgkiss, and P. Gerstoft, "Adaptive and compressive matched field processing," *J. Acoust. Soc. Am.* **141** (2017) 92–103.
138. K. L. Gemba, S. Nannuru, P. Gerstoft, and W. S. Hodgkiss, "Multi-frequency sparse Bayesian learning for robust matched field processing," *J. Acoust. Soc. Am.* **141** (2017) 3411–3420.
139. K. L. Gemba, S. Nannuru, and P. Gerstoft, "Robust ocean acoustic localization with sparse Bayesian learning," *IEEE J. Sel. Top. Signal Process.* **13** (2019) 49–60.
140. M. D. Collins, "Applications of a motion compensation stabilized vertical array of hydrophones," *IEEE Access* **7** (2019) 79433–79437.
141. N. R. Chapman, "Modeling ocean-bottom reflection loss measurements with the plane-wave reflection coefficient," *J. Acoust. Soc. Am.* **73** (1983) 1601–1607.
142. D. C. Stickler, "Inverse scattering in a stratified medium," *J. Acoust. Soc. Am.* **74** (1983) 994–1005.
143. E. C. Shang, H. P. Wang, and Z. Y. Huang, "Waveguide characterization and source localization in shallow water waveguides using the Prony method," *J. Acoust. Soc. Am.* **83** (1988) 103–108.
144. J. F. Lynch, S. D. Rajan, and G. V. Frisk, "A comparison of broadband and narrow-band modal inversions for bottom geoacoustic properties at a site near Corpus Christi, Texas," *J. Acoust. Soc. Am.* **89** (1991) 648–665.
145. M. D. Collins, W. A. Kuperman, and H. Schmidt, "Nonlinear inversion for ocean-bottom properties," *J. Acoust. Soc. Am.* **92** (1992) 2770–2783.
146. N. R. Chapman and E. C. Shang, "Review of geoacoustic inversion in underwater acoustics," *J. Theor. Comp. Acoust.* **29** (2021) 2130004.
147. S. E. Dosso, M. L. Yeremy, J. M. Ozard, and N. R. Chapman, "Estimation of ocean-bottom properties by matched-field inversion of acoustic field data," *IEEE J. Ocean. Eng.* **18** (1993) 232–239.
148. P. Gerstoft and C. F. Mecklenbräuker, "Ocean acoustic inversion with estimation of *a posteriori* probability distributions," *J. Acoust. Soc. Am.* **104** (1998) 808–819.
149. M. Siderius, P. Gerstoft, and P. Nielsen, "Broadband geoacoustic inversion from sparse data using genetic algorithms," *J. Comp. Acoust.* **6** (1998) 117–134.
150. N. M. Carbone, G. B. Deane, and M. J. Buckingham, "Estimating the compressional and shear wave speeds of a shallow water seabed from the vertical coherence of ambient noise in the water column," *J. Acoust. Soc. Am.* **103** (1998) 801–813.
151. M. Siderius and J. P. Hermand, "Yellow Shark Spring 1995: Inversion results from sparse broadband acoustic measurements over a highly range-dependent soft clay layer," *J. Acoust. Soc. Am.* **106** (1999) 637–1651.
152. C. F. Mecklenbräuker and P. Gerstoft, "Objective functions for ocean acoustic inversion derived by likelihood methods," *J. Comp. Acoust.* **8** (2000) 259–270.
153. M. Snellen, D. Simons, M. Siderius, J. Sellschopp, and P. L. Nielsen, "An evaluation of the accuracy of shallow water matched field inversion results," *J. Acoust. Soc. Am.* **109** (2001) 514–527.
154. M. Siderius, P. L. Nielsen, J. Sellschopp, M. Snellen, and D. Simons, "Experimental study of geo-acoustic inversion uncertainty due to ocean sound-speed fluctuations," *J. Acoust. Soc. Am.* **110** (2001) 769–781.
155. M. Siderius, P. L. Nielsen, and P. Gerstoft, "Range-dependent seabed characterization by inversion of acoustic data from a towed receiver array," *J. Acoust. Soc. Am.* **112** (2002) 1523–1535.
156. C. H. Harrison and M. Siderius, "Effective parameters for matched field geoacoustic inversion in range-dependent environments," *IEEE J. Ocean. Eng.* **28** (2003) 432–445.

157. D. J. Battle, P. Gerstoft, W. A. Kuperman, W. S. Hodgkiss, and M. Siderius, "Geoacoustic inversion of tow-ship noise via near-field–matched-field processing," *IEEE J. Ocean. Eng.* **28** (2003) 454–467.
158. M. R. Fallat, P. L. Nielsen, S. E. Dosso, and M. Siderius, "Geoacoustic characterization of a range-dependent ocean environment using towed array data," *IEEE J. Ocean. Eng.* **30** (2005) 198–206.
159. M. Siderius, C. H. Harrison, and M. B. Porter, "A passive fathometer technique for imaging seabed layering using ambient noise," *J. Acoust. Soc. Am.* **120** (2006) 1315–1323.
160. T. C. Yang, K. Yoo, and L. T. Fialkowski, "Subbottom profiling using a ship towed line array and geoacoustic inversion," *J. Acoust. Soc. Am.* **122** (2007) 3338–3352.
161. C. H. Harrison and M. Siderius, "Bottom profiling by correlating beam-steered noise sequences," *J. Acoust. Soc. Am.* **123** (2008) 1282–1296.
162. P. Gerstoft, W. S. Hodgkiss, M. Siderius, C. F. Huang, and C. H. Harrison, "Passive fathometer processing," *J. Acoust. Soc. Am.* **123** (2008) 1297–1305.
163. J. Dettmer, S. E. Dosso, and C. W. Holland, "Trans-dimensional geoacoustic inversion," *J. Acoust. Soc. Am.* **128** (2010) 3393–3405.
164. M. Siderius, H. Song, P. Gerstoft, W. S. Hodgkiss, and C. H. Harrison, "Adaptive passive fathometer processing," *J. Acoust. Soc. Am.* **127** (2010) 2193–2200.
165. J. E. Quijano, S. E. Dosso, J. Dettmer, L. M. Zurk, M. Siderius, and C. H. Harrison, "Bayesian geoacoustic inversion using wind-driven ambient noise," *J. Acoust. Soc. Am.* **131** (2012) 2658–2667.
166. S. E. Crocker, P. L. Nielsen, J. H. Miller, and M. Siderius, "Geoacoustic inversion of ship radiated noise in shallow water using data from a single hydrophone," *J. Acoust. Soc. Am.* **136** (2014) EL362–EL368.
167. M. D. Collins and L. Fishman, "Efficient navigation of parameter landscapes," *J. Acoust. Soc. Am.* **98** (1995) 1637–1644.
168. M. Fink, "Time reversal mirrors," in *Acoustical Imaging*, edited by J. P. Jones, Vol. 21 (Plenum, New York, 1994).
169. W. A. Kuperman, W. S. Hodgkiss, H. C. Song, T. Akal, C. Ferla, and D. R. Jackson, "Phase conjugation in the ocean: Experimental demonstration of an acoustic time-reversal mirror," *J. Acoust. Soc. Am.* **103** (1998) 25–40.
170. J. D. Wilson and N. C. Makris, "Quantifying hurricane destructive power, wind speed, and air-sea material exchange with natural undersea sound," *Geophys. Res. Lett.* **35** (2008) L10603.
171. M. A. Biot, "Theory of propagation of elastic waves in a fluid-saturated porous solid," *J. Acoust. Soc. Am.* **28** (1956) 168–191.
172. M. D. Collins, W. A. Kuperman, and W. L. Siegmann, "A parabolic equation for poro-elastic media," *J. Acoust. Soc. Am.* **98** (1995) 1645–1656.
173. M. J. Buckingham, "Theory of compressional and shear waves in fluidlike marine sediments," *J. Acoust. Soc. Am.* **98** (1998) 288–299.
174. M. J. Buckingham, "Wave propagation, stress relaxation, and grain-to-grain shearing in saturated, unconsolidated marine sediments," *J. Acoust. Soc. Am.* **108** (2000) 2796–2815.
175. M. J. Buckingham, "Compressional and shear wave properties of marine sediments: Comparisons between theory and data," *J. Acoust. Soc. Am.* **117** (2005) 137–152.
176. M. J. Bianco, P. Gerstoft, J. Traer, E. Ozanich, M. A. Roch, S. Gannot, and C. A. Deledalle, "Machine learning in acoustics: Theory and applications," *J. Acoust. Soc. Am.* **146** (2019) 3590–3628.