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<https://doi.org/10.15760/etd.730>

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AN ABSTRACT OF THE THESIS OF Russell James Wayland for
the Master of Science in Mathematics presented
May 20, 1970.

Title: Distribution of the Sum of Independent Unity-
Truncated Logarithmic Series Variables.

APPROVED BY MEMBERS OF THE THESIS COMMITTEE:

Dr. Jagdish C. Ahuja, Chairman

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Dr. Eugene A. Enneking *L*

Let $X_1, X_2, \dots, X_n$ be n independent and identically
distributed random variables having the unity-truncated
logarithmic series distribution with probability function
given by

$$f(x;\theta) = \frac{\alpha\theta^x}{x} \quad x \in T$$

where $\alpha = [-\log(1-\theta) - \theta]^{-1}$, $0 < \theta < 1$, and $T = \{2, 3, \dots, \infty\}$.
Define their sum as $Z = X_1 + X_2 + \dots + X_n$.

We derive here the distribution of Z , denoted by
 $p(z;n,\theta)$, using the inversion formula for characteristic

functions, in an explicit form in terms of a linear combination of Stirling numbers of the first kind. A recurrence relation for the probability function $p(z;n,\theta)$ is obtained and is utilized to provide a short table of $p(z;n,\theta)$ for certain values of n and θ . Furthermore, some properties of $p(z;n,\theta)$ are investigated following Patil and Wani [Sankhyā, Series A, 27, (1965), 271-280].

DISTRIBUTION OF THE SUM OF INDEPENDENT UNITY-TRUNCATED
LOGARITHMIC SERIES VARIABLES

by

RUSSELL JAMES WAYLAND

A thesis submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE
in
MATHEMATICS

Portland State University
1970

TO THE OFFICE OF GRADUATE STUDIES:

The members of the Committee approve the thesis of
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ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Dr. Jagdish C. Ahuja of the Department of Mathematics, Portland State University, for suggesting the topic of this thesis, and feels highly indebted to him for his invaluable advice and comments during its preparation. He also wishes to extend his thanks to Mr. Dan Purcell for his valuable help in programming and preparation of the table, and to the Computing Center of Oregon State University for providing computing facilities. Another expression of gratitude is given by the author to Miss Jeanne Troeh for her many hours spent at the typewriter in the completion of this work.

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CHAPTER I

INTRODUCTION

Let X be a random variable having the logarithmic series distribution

$$g(x;\theta) = \frac{[-\log(1-\theta)]^{-1} \theta^x}{x} \quad x \in I$$

where $0 < \theta < 1$, and I is the set of positive integers. Then, the probability function of the unity-truncated logarithmic series distribution is given by

$$f(x;\theta) = \frac{\alpha \theta^x}{x} \quad x \in T$$

where $\alpha = [-\log(1-\theta) - \theta]^{-1}$, $0 < \theta < 1$, and $T = \{2, 3, \dots, \infty\}$.

The distribution for the sum of n independent random variables having the logarithmic series distribution $g(x;\theta)$ has been obtained by Patil and Wani [4, p. 278] which they call the first type Stirling distribution. The purpose of this thesis is to derive the distribution for their sum when the random variables have the unity-truncated logarithmic series distribution defined by $f(x;\theta)$ above.

To describe briefly the organization of the paper

we first introduce in Chapter II a few definitions and preliminaries which appear in the paper at one place or another. Chapter III contains the derivation of the distribution for the sum, denoted by $p(z;n,\theta)$, using the inversion formula for characteristic functions. Here, $p(z;n,\theta)$ is obtained in an explicit form in terms of a linear combination of Stirling numbers of the first kind. In Chapter IV, certain properties of the probability function $p(z;n,\theta)$ are investigated. In particular, we obtain two recurrence relations for $p(z;n,\theta)$, one of which is used to generate a short table for the values of $p(z;n,\theta)$ in Chapter V.

CHAPTER II

DEFINITIONS AND NOTATION

In order to allow derivations and proofs to evolve with as little interruption as possible, we state the following definitions:

Definition 1: The Stirling numbers of the first kind S_z^n are defined by Jordan [3, p. 143] as

$$(1) \quad x^{(z)} = \sum_{n=1}^z S_z^n x^n$$

where $x^{(z)} = x(x-1)(x-2)\dots(x-z+1)$ is the factorial power function, and $S_z^0 = 0$, $S_1^1 = 1$, and $S_z^n = 0$ for $n > z$.

Since the numbers S_z^n have the same sign as $(-1)^{z-n}$, we may write $|S_z^n| = (-1)^{z-n} S_z^n$.

The Stirling numbers of the first kind have been used by Jordan [3, p. 184] to define the numbers $C(m,s)$ as follows:

Definition 2: The numbers $C(m,s)$ are given by

$$(2) \quad C(m,s) = \sum_{k=m+1}^{2m-s} (-1)^{k+s} \binom{2m-s}{k} S_k^{k-m}$$

where $C(m,s) = 0$ for $m < s+1$, $C(1,0) = -1$, $C(m,0) = (-1)^m \cdot 1 \cdot 3 \cdot 5 \dots (2m-1)$, and $C(m,m-1) = (-1)^m m!$.

These numbers satisfy the recurrence relation

$$(3) \quad C(m+1, s) = -(2m-s+1)[C(m, s) + C(m, s-1)].$$

It can be readily seen with the help of (3) and (2) that the sign of $C(m, s)$ is identical with that of $(-1)^m$. Therefore, $|C(m, s)| = (-1)^m C(m, s)$. Subsequently, a corresponding definition for the absolute value of the numbers $C(m, s)$ would be

$$(4) \quad |C(m, s)| = \sum_{k=m+1}^{2m-s} (-1)^{k+s} \binom{2m-s}{k} |S_k^{k-m}|$$

and the recurrence relation (3) becomes

$$(5) \quad |C(m+1, s)| = (2m-s+1)[|C(m, s)| + |C(m, s-1)|].$$

A probability distribution which arises from the sum of n independent random variables, each having the logarithmic series distribution with the same parameter θ , has been defined by Patil [5, p. 37] in terms of the numbers $|S_z^n|$.

Definition 3: The random variable Z is said to have the first type Stirling distribution with parameters n and θ if its probability function is given by

$$(6) \quad f(z; n, \theta) = \frac{\beta^n n! |S_z^n| \theta^z}{z!}$$

for $z = n, n+1, n+2, \dots, \infty$, $n = 1, 2, 3, \dots, \infty$, where

$\beta = [-\log(1-\theta)]^{-1}$ and $0 < \theta < 1$.

The characteristic function of the probability function $f(z;n,\theta)$ is easily found to be

$$(7) \quad \psi_z(t) = \beta^n [-\log(1-\theta e^{it})]^n$$

Some of the properties, which we will examine, deal with a large class of probability distributions which is defined by G.P. Patil [5, p. 2] in the following manner.

Definition 4: A discrete random variable X is said to have a generalized power series distribution with the parameter θ and range T if its probability function is given by

$$(8) \quad p(x;\theta) = \frac{a(x)\theta^x}{g(\theta)} \quad x \in T$$

where $a(x)$ is independent of θ and is positive for $x \in T$. T is a countable subset of the set of non-negative integers (reals). The series function $g(\theta) = \sum a(x)\theta^x$, which will be summed over the entire range T , will be positive, finite, and differentiable for $\{\theta \mid 0 < \theta < R\}$ where R is the radius of convergence for the series function.

CHAPTER III

DISTRIBUTION OF THE SUM

Let $X_1, X_2, \dots, X_n$ be n independent and identically distributed random variables having the unity-truncated logarithmic series distribution with probability function given by

$$(9) \quad f(x; \theta) = \frac{\alpha \theta^x}{x} \quad x = 2, 3, \dots$$

where $\alpha = [-\log(1-\theta) - \theta]^{-1}$, and $0 < \theta < 1$.

The characteristic function of the random variable X_j is obtained as

$$\begin{aligned} (10) \quad \psi_{x_j}(t) &= \sum_{x_j=2}^{\infty} e^{itx_j} f(x_j; \theta) \\ &= \sum_{x_j=2}^{\infty} \frac{\alpha (\theta e^{it})^{x_j}}{x_j} \\ &= \alpha [-\log(1 - \theta e^{it}) - \theta e^{it}]. \end{aligned}$$

If we now let $Z = X_1 + X_2 + \dots + X_n$, then, since the X_j 's are independent, the characteristic function of Z is given by the product of characteristic functions of

the X_j 's. Hence,

$$\begin{aligned}
 (11) \quad \psi_Z(t) &= \prod_{j=1}^n \psi_{X_j}(t) \\
 &= \alpha^n [-\log(1-\theta e^{it}) - \theta e^{it}]^n.
 \end{aligned}$$

We may now find the probability function of Z by using the inversion formula for characteristic functions given by Fisz [2, p. 119]. Since it will depend on two parameters, n and θ , we shall denote it by $p(z;n,\theta)$. Thus we have

$$\begin{aligned}
 (12) \quad p(z;n,\theta) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-itz} \psi_Z(t) dt \\
 &= \frac{\alpha^n}{2\pi} \int_{-\pi}^{\pi} e^{-itz} [-\log(1-\theta e^{it}) - \theta e^{it}]^n dt
 \end{aligned}$$

which, after taking the binomial expansion of the second factor under the integral sign, becomes

$$\begin{aligned}
 (13) \quad p(z;n,\theta) &= \frac{\alpha^n}{2\pi} \int_{-\pi}^{\pi} e^{-itz} \\
 &\quad \cdot \sum_{k=0}^n \binom{n}{k} (-\theta e^{it})^k [-\log(1-\theta e^{it})]^{n-k} dt \\
 &= \frac{\alpha^n}{2\pi} \sum_{k=0}^n \binom{n}{k} (-\theta)^k \\
 &\quad \cdot \int_{-\pi}^{\pi} e^{-it(z-k)} [-\log(1-\theta e^{it})]^{n-k} dt.
 \end{aligned}$$

To evaluate the integral in (13), we observe from (7) that the quantity $\theta^{n-k} [-\log(1-\theta e^{it})]^{n-k}$ is the characteristic function of the first type Stirling distribution with parameters $n-k$ and θ . So the inversion formula for characteristic functions gives us

$$(14) \quad \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-it(z-k)} [-\log(1-\theta e^{it})]^{n-k} dt = \frac{(n-k)! |S_{z-k}^{n-k}| \theta^{z-k}}{(z-k)!}.$$

Substituting for the integral from (14) into (13), we obtain the probability function of Z in the form

$$(15) \quad p(z;n,\theta) = \frac{\alpha^n \sum_{k=0}^n \binom{n}{k} (-\theta)^k (n-k)! |S_{z-k}^{n-k}| \theta^{z-k}}{(z-k)!}$$

$$= \frac{\alpha^n n! \theta^z}{z!} \sum_{k=0}^n (-1)^k \binom{z}{k} |S_{z-k}^{n-k}|.$$

But, using (4), we find that

$$|C(z-n, z-2n)| = \sum_{j=z-n+1}^z (-1)^{j+z-2n} \binom{z}{j} |S_j^{j-z+n}|$$

which, by letting $k = z-j$, becomes

$$(16) \quad |C(z-n, z-2n)| = \sum_{k=0}^{n-1} (-1)^k \binom{z}{k} |S_{z-k}^{n-k}|$$

$$= \sum_{k=0}^n (-1)^k \binom{z}{k} |S_{z-k}^{n-k}|$$

since $|S_{z-n}^0| = 0$.

Hence, replacing the summation in (15) by (16), we have the following:

Theorem 1: Let $X_1, X_2, \dots, X_n$ be n independent and identically distributed random variables having the unity-truncated logarithmic series distribution defined by (9), and let $Z = X_1 + X_2 + \dots + X_n$. Then the distribution of Z is given by

$$(17) \quad p(z;n,\theta) = \frac{\alpha^n n!}{z!} |C(z-n, z-2n)| \theta^z$$

for $z = 2n, 2n+1, \dots, \infty$, where $\alpha = [-\log(1-\theta) - \theta]^{-1}$ and $0 < \theta < 1$.

CHAPTER IV

SOME PROPERTIES OF $p(z;n,\theta)$

Some of the more important aspects of $p(z;n,\theta)$ are examined here as properties and theorems.

Property 1: The first three moments about the mean for the probability function $p(z;n,\theta)$ with parameters n and θ are

$$\mu_1 = \frac{n\alpha\theta^2}{1-\theta}$$

$$\mu_2 = \frac{n\alpha\theta^2}{(1-\theta)^2} [2 - \theta - \alpha\theta^2]$$

$$\mu_3 = \frac{n\alpha\theta^2}{(1-\theta)^3} [4 - 3\theta + \theta^2 - 6\alpha\theta^2 + 3\alpha\theta^3 + 2\alpha^2\theta^4].$$

Proof: Using the characteristic function for the unity-truncated logarithmic series distribution given by (10), we immediately have

$$\mu'_1 = \mu = (-i) \left. \frac{d\psi_x(t)}{dt} \right|_{t=0} = \frac{\alpha\theta^2}{1-\theta}$$

$$\mu'_2 = (-i)^2 \left. \frac{d^2\psi_x(t)}{dt^2} \right|_{t=0} = \frac{\alpha\theta^2}{(1-\theta)^2} [2 - \theta]$$

$$\mu_3' = (-i)^3 \left[\frac{d^3 \psi_x(t)}{dt^3} \right]_{t=0} = \frac{\alpha \theta^2}{(1-\theta)^3} [4 - 3\theta + \theta^2].$$

These give

$$\mu_2 = \mu_2' - \mu^2$$

$$= \frac{\alpha \theta}{(1-\theta)^2} [2 - \theta - \alpha \theta^2]$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu + 2\mu^3$$

$$= \frac{\alpha \theta^2}{(1-\theta)^3} [4 - 3\theta + \theta^2 - 6\alpha \theta^2 + 3\alpha \theta^3 + 2\alpha^2 \theta^4].$$

Since Z is the sum of n independent random variables, its first three moments about the mean may be obtained, following Burington and May [1, p. 37], as the sum of the moments about the mean of the X_j 's and this completes the proof.

Property 2: The probability function $p(z;n,\theta)$ with parameters n and θ satisfies the recurrence relation

$$(18) \quad p(z+1;n,\theta) = \frac{\theta |C(z-n+1, z-2n+1)|}{(z+1) |C(z-n, z-2n)|} p(z;n,\theta)$$

Proof: Follows easily by considering the ratio of $p(z+1;n,\theta)$ and $p(z;n,\theta)$.

Since the numbers $C(m,s)$ grow too big too soon

with increase in their arguments, the recurrence relation (18) is not of much use. The next property provides a recurrence relation independent of numbers $C(m,s)$ and useful for tabulation purposes.

Property 3: The probability function $p(z;n,\theta)$ with parameters n and θ enjoys the recurrence relation

$$(19) \quad p(z+1;n,\theta) = \frac{\theta}{z+1} [\alpha n \theta p(z-1;n-1,\theta) + z p(z;n,\theta)]$$

$$\text{where } p(z;n,\theta) = \frac{\alpha \theta^2}{z} \text{ for } n = 1 \text{ and } p(z;n,\theta) = \frac{\alpha^n \theta^{2n}}{2^n}$$

for $z = 2n$.

Proof: From (17), we have

$$p(z+1;n,\theta) = \frac{\alpha^n n!}{(z+1)!} |C(z-n+1, z-2n+1)| \theta^{z+1}$$

which, using (5), may be written as

$$(20) \quad p(z+1;n,\theta) = \frac{\alpha^n n!}{(z+1)!} [z |C(z-n, z-2n+1)| \theta^{z+1} + z |C(z-n, z-2n)| \theta^{z+1}]$$

$$\text{But } p(z-1;n-1,\theta) = \frac{\alpha^{n-1} (n-1)!}{(z-1)!} |C(z-n, z-2n+1)| \theta^{z-1}$$

$$\text{and } p(z;n,\theta) = \frac{\alpha^n n!}{z!} |C(z-n, z-2n)| \theta^z. \text{ Hence, we obtain (20) as}$$

$$p(z+1;n,\theta) = \frac{\theta}{z+1} [\alpha n \theta p(z-1;n-1,\theta) + z p(z;n,\theta)].$$

Also, for $n = 1$, we get

$$\begin{aligned} p(z;1,\theta) &= \frac{\alpha}{z!} |C(z-1,z-2)| \theta^z \\ &= \frac{\alpha \theta^z}{z}, \end{aligned}$$

and for $z = 2n$, we have

$$\begin{aligned} p(2n;n,\theta) &= \frac{\alpha^n n!}{(2n)!} |C(n,0)| \theta^{2n} \\ &= \frac{\alpha^n n!}{(2n)!} \cdot 1 \cdot 3 \cdot 5 \dots (2n-1) \theta^{2n} \\ &= \frac{\alpha^n \theta^{2n}}{2^n}, \end{aligned}$$

which completes the proof.

Theorem 2: Let $Z_1, Z_2, \dots, Z_k$ be k independent random variables having the probability function

$$p(z_j; n_j, \theta) = \frac{\alpha^{n_j} n_j!}{z_j!} |C(z_j - n_j, z_j - 2n_j)| \theta^{z_j}$$

where, for $j = 1, 2, \dots, k$, $z_j = 2n_j, 2n_j+1, \dots, \infty$,

$= [-\log(1-\theta) - \theta]^{-1}$, and $0 < \theta < 1$. Let $Y = Z_1 + Z_2 +$

... + Z_k , then the distribution of Y is given by

$$p(y; \sum_{j=1}^k, \theta).$$

Proof: The characteristic function of Z_j , using (11), is

$$\psi_{Z_j}(t) = \alpha^{n_j} [-\log(1 - \theta e^{it}) - \theta e^{it}]^{n_j}.$$

Since the Z_j 's are assumed to be independent, the characteristic function of Y is obtained as

$$\begin{aligned} \psi_Y(t) &= \prod_{j=1}^k \psi_{Z_j}(t) \\ &= \alpha^{\sum_{j=1}^k n_j} [-\log(1 - \theta e^{it}) - \theta e^{it}]^{\sum_{j=1}^k n_j} \end{aligned}$$

which, by the uniqueness theorem for characteristic functions, establishes the proof.

Theorem 3: Let X and Y be two independent random variables having the distribution $p(x; n, \theta)$ and $p(y; m, \lambda)$ respectively. Let $Z = X + Y$ and $\beta = \theta/\lambda$. Then the conditional distribution of X given $Z = z$ is a generalized power series distribution with the series function

$$(21) \quad g(\beta) = \sum_{x=2n}^{z-2m} h(x, z) \beta^x$$

where

$$h(x,z) = \frac{|C(x-n, x-2n)|}{x!} \cdot \frac{|C(z-x-m, z-x-2m)|}{(z-x)!}$$

Proof: Using the definition of conditional distribution, we have

$$(22) \quad P(X=x|X+Y=z) =$$

$$\frac{\frac{\alpha^n n! |C(x-n, x-2n)| \theta^x}{x!} \cdot \frac{\alpha^m m! |C(z-x-m, z-x-2m)| \lambda^{z-x}}{(z-x)!}}{\sum_{x=2n}^{z-2m} \frac{\alpha^n n! |C(x-n, x-2n)| \theta^x}{x!} \cdot \frac{\alpha^m m! |C(z-x-m, z-x-2m)| \lambda^{z-x}}{(z-x)!}}$$

Since $\theta = \beta\lambda$, we may express (22) as

$$P(X=x|X+Y=z) = \frac{\frac{|C(x-n, x-2n)|}{x!} \cdot \frac{|C(z-x-m, z-x-2m)|}{(z-x)!} \beta^x}{\sum_{x=2n}^{z-2m} \frac{|C(x-n, x-2n)|}{x!} \cdot \frac{|C(z-x-m, z-x-2m)|}{(z-x)!} \beta^x}$$

which can be easily recognized as the generalized power series distribution with the series function given by (21).

Theorem 4: Let X and Y be two independent discrete random variables with their ranges containing $2n$ and $2m$ respectively. Let $X + Y = Z$. If the conditional

distribution of X given $Z = z$ is given to be a generalized power series distribution with the series function (21) for $0 < \beta < \infty$ for every $z > 2n + 2m$, then each X and Y has the distribution $p(x;n,\theta)$ and $p(y;m,\lambda)$ respectively so that $\theta = \beta\lambda$.

Proof: Let $f(x)$ and $g(y)$ be the probability functions of X and Y respectively. We then have

$$(23) \quad \frac{f(x) g(z-x)}{\sum_{x=2n}^{z-2m} f(x) g(z-x)} = \frac{\frac{|C(x-n, x-2n)|}{x!} \cdot \frac{|C(z-x-m, z-x-2m)|}{(z-x)!} \beta^x}{\sum_{x=2n}^{z-2m} \frac{|C(x-n, x-2n)|}{x!} \cdot \frac{|C(z-x-m, z-x-2m)|}{(z-x)!} \beta^x}.$$

Also,

$$(24) \quad \frac{f(x-1) g(z-x+1)}{\sum_{x=2n}^{z-2m} f(x) g(z-x)} = \frac{\frac{|C(x-n-1, x-2n-1)|}{(x-1)!} \cdot \frac{|C(z-x-m+1, z-x-2m+1)|}{(z-x+1)!} \beta^x}{\sum_{x=2n}^{z-2m} \frac{|C(x-n, x-2n)|}{x!} \cdot \frac{|C(z-x-m, z-x-2m)|}{(z-x)!} \beta^x}.$$

Dividing (23) by (24) gives

$$(25) \quad \frac{f(x) g(z-x)}{f(x-1) g(z-x+1)} = \frac{(z-x+1)}{x} \cdot \frac{|C(x-n, x-2n)| |C(z-x-m, z-x-2m)|}{|C(x-n-1, x-2n-1)| |C(z-x-m+1, z-x-2m+1)|} \beta$$

Taking $z - x = 2m$ in (25), we get

$$\frac{f(x) g(2m)}{f(x-1) g(2m+1)} = \frac{(2m+1) |C(x-n, x-2n)| |C(m, 0)|}{x |C(x-n-1, x-2n-1)| |C(m+1, 1)|} \beta$$

which may be written as

$$\frac{f(x)}{f(x-1)} = \frac{(x-1)! |C(x-n, x-2n)|}{x! |C(x-n-1, x-2n-1)|} \theta$$

$$\text{where } \theta = \frac{(2m+1) g(2m+1) |C(m, 0)|}{g(2m) |C(m+1, 1)|} \beta$$

This gives

$$f(x) = k \frac{|C(x-n, x-2n)|}{x!} \theta^x$$

where k is a constant. Since $f(x) > 0$ and $\sum_{x=2n}^{\infty} f(x) = 1$, we obtain $k = \alpha^n n!$. Hence, $f(x) = p(x; n, \theta)$. A similar approach would show that $g(y) = p(y; m, \lambda)$.

CHAPTER V

TABULATION OF $p(z;n,\theta)$

In this concluding chapter, a short table for the values of $p(z;n,\theta)$ is presented. The computations were obtained using the Oregon State University terminal and are accurate to four decimal places. In order to construct this table, the recurrence relation (19) was used along with the corresponding initial values given in Property 3. Although the program was originally completed for $n = 1(1)25$, the table here is furnished only for $n = 1(1)15$ and $\theta = 0.1(0.1)0.9$. This is primarily due to the fact that, for $n = 16(1)25$ and θ large, the values of $p(z;n,\theta)$ do not approach zero quickly for large z , and hence the table would be quite extensive.

THETA (θ)

n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	2	.9327	.8642	.7940	.7219	.6472	.5691	.4861	.3953	.2888
	3	.0622	.1152	.1588	.1925	.2157	.2276	.2269	.2108	.1733
	4	.0047	.0173	.0357	.0577	.0809	.1024	.1191	.1265	.1169
	5	.0004	.0028	.0086	.0185	.0324	.0492	.0667	.0810	.0842
	6	.0000	.0005	.0021	.0062	.0135	.0246	.0389	.0540	.0632
	7	.0000	.0001	.0006	.0021	.0058	.0126	.0233	.0370	.0488
	8	.0000	.0000	.0001	.0007	.0025	.0066	.0143	.0259	.0384
	9	.0000	.0000	.0000	.0003	.0011	.0035	.0089	.0184	.0307
	10	.0000	.0000	.0000	.0001	.0005	.0019	.0056	.0133	.0249
	11	.0000	.0000	.0000	.0000	.0002	.0010	.0036	.0096	.0203
	12	.0000	.0000	.0000	.0000	.0001	.0006	.0023	.0071	.0168
	13	.0000	.0000	.0000	.0000	.0000	.0003	.0015	.0052	.0139
	14	.0000	.0000	.0000	.0000	.0000	.0002	.0010	.0039	.0117
	15	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0029	.0098
	16	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0022	.0083
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0016	.0070
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0059
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0009	.0051
	20	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0007	.0043
	21	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0037
	22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004	.0032
	23	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0027
	24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0024
	25	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0020
	26	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0018
	27	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0015
	28	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0013
	29	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0012
	30	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0010
	31	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0009
	32	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0008
	33	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0007
	34	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	35	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	36	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	37	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	39	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	40	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	41	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	42	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	43	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	44	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	45	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	46	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

		THETA (θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
3	38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0044
	39	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0040
	40	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0036
	41	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0032
	42	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0028
	43	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0025
	44	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0023
	45	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0020
	46	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0018
	47	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0016
	48	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0015
	49	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0013
	50	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0012
	51	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0010
	52	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0009
	53	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0008
	54	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0007
	55	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0007
	56	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	57	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	58	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	59	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	60	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	61	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
62	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	
63	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	
64	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	
65	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	
66	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	
67	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	
68	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
69	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
70	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
71	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
72	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
73	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
4	8	.7569	.5577	.3975	.2715	.1754	.1049	.0559	.0244	.0070
	9	.2018	.2074	.3180	.2896	.2339	.1678	.1043	.0521	.0167
	10	.0353	.1041	.1669	.2027	.2047	.1762	.1277	.0730	.0263
	11	.0051	.0303	.0728	.1179	.1488	.1537	.1300	.0849	.0344
	12	.0007	.0079	.0286	.0618	.0976	.1210	.1193	.0890	.0406
	13	.0001	.0020	.0106	.0304	.0599	.0892	.1027	.0875	.0449
	14	.0000	.0005	.0037	.0143	.0353	.0630	.0845	.0824	.0475
	15	.0000	.0001	.0013	.0065	.0201	.0431	.0675	.0751	.0488
	16	.0000	.0000	.0004	.0029	.0112	.0288	.0526	.0670	.0489
17	.0000	.0000	.0001	.0013	.0061	.0189	.0403	.0587	.0482	

		THETA (Θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
4	65	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	66	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	67	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	68	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	69	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	70	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	71	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	72	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	73	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	74	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	75	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	76	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	77	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	78	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
79	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
80	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
81	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
82	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	
5	10	.7060	.4819	.3156	.1960	.1135	.0597	.0272	.0097	.0020
	11	.2353	.3213	.3156	.2613	.1892	.1194	.0634	.0258	.0060
	12	.0490	.1339	.1972	.2178	.1971	.1492	.0924	.0429	.0113
	13	.0082	.0448	.0991	.1450	.0650	.1500	.1083	.0575	.0170
	14	.0012	.0132	.0438	.0861	.1217	.1327	.1118	.0679	.0226
	15	.0002	.0036	.0179	.0468	.0827	.1082	.1063	.0737	.0276
	16	.0000	.0009	.0069	.0240	.0530	.0832	.0955	.0757	.0319
	17	.0000	.0002	.0025	.0118	.0326	.0614	.0822	.0744	.0353
	18	.0000	.0001	.0009	.0056	.0194	.0439	.0685	.0710	.0378
	19	.0000	.0000	.0003	.0026	.0113	.0306	.0558	.0660	.0396
	20	.0000	.0000	.0001	.0012	.0064	.0209	.0445	.0602	.0406
	21	.0000	.0000	.0000	.0005	.0036	.0141	.0349	.0540	.0410
	22	.0000	.0000	.0000	.0002	.0020	.0094	.0271	.0478	.0409
	23	.0000	.0000	.0000	.0001	.0011	.0062	.0208	.0419	.0403
	24	.0000	.0000	.0000	.0000	.0006	.0040	.0158	.0364	.0393
	25	.0000	.0000	.0000	.0000	.0003	.0026	.0119	.0313	.0381
	26	.0000	.0000	.0000	.0000	.0002	.0017	.0089	.0268	.0367
	27	.0000	.0000	.0000	.0000	.0001	.0011	.0066	.0228	.0351
	28	.0000	.0000	.0000	.0000	.0000	.0007	.0049	.0193	.0334
	29	.0000	.0000	.0000	.0000	.0000	.0004	.0036	.0162	.0316
	30	.0000	.0000	.0000	.0000	.0000	.0003	.0026	.0136	.0299
	31	.0000	.0000	.0000	.0000	.0000	.0002	.0019	.0114	.0281
	32	.0000	.0000	.0000	.0000	.0000	.0001	.0014	.0095	.0263
	33	.0000	.0000	.0000	.0000	.0000	.0001	.0010	.0079	.0246
	34	.0000	.0000	.0000	.0000	.0000	.0000	.0007	.0065	.0229
	35	.0000	.0000	.0000	.0000	.0000	.0000	.0005	.0054	.0213
	36	.0000	.0000	.0000	.0000	.0000	.0000	.0004	.0045	.0198
	37	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0037	.0184

[illegible]

THETA (θ)

n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
7	104	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	105	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	106	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	107	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	108	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	109	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
8	16	.5729	.3110	.1580	.0737	.0308	.0110	.0031	.0006	.0000
	17	.3056	.3318	.2538	.1573	.0821	.0352	.0116	.0025	.0002
	18	.0942	.2046	.2338	.1940	.1265	.0651	.0251	.0063	.0006
	19	.0220	.0957	.1640	.1815	.1479	.0914	.0411	.0117	.0014
	20	.0043	.0377	.0969	.1430	.1457	.1080	.0567	.0185	.0024
	21	.0008	.0132	.0509	.1001	.1276	.1135	.0695	.0259	.0038
	22	.0001	.0042	.0245	.0644	.1025	.1094	.0782	.0333	.0055
	23	.0000	.0013	.0111	.0387	.0771	.0988	.0824	.0401	.0074
	24	.0000	.0004	.0048	.0222	.0551	.0847	.0825	.0459	.0095
	25	.0000	.0001	.0020	.0122	.0378	.0697	.0792	.0504	.0118
	26	.0000	.0000	.0008	.0064	.0251	.0555	.0735	.0534	.0141
	27	.0000	.0000	.0003	.0033	.0162	.0429	.0663	.0551	.0163
	28	.0000	.0000	.0001	.0017	.0102	.0324	.0585	.0555	.0185
	29	.0000	.0000	.0000	.0008	.0063	.0240	.0505	.0548	.0205
	30	.0000	.0000	.0000	.0004	.0038	.0175	.0429	.0532	.0224
	31	.0000	.0000	.0000	.0002	.0023	.0125	.0358	.0508	.0241
	32	.0000	.0000	.0000	.0001	.0013	.0088	.0295	.0479	.0255
	33	.0000	.0000	.0000	.0000	.0008	.0062	.0241	.0446	.0268
	34	.0000	.0000	.0000	.0000	.0004	.0043	.0194	.0411	.0277
	35	.0000	.0000	.0000	.0000	.0003	.0029	.0155	.0376	.0285
	36	.0000	.0000	.0000	.0000	.0001	.0020	.0123	.0340	.0291
	37	.0000	.0000	.0000	.0000	.0001	.0013	.0097	.0306	.0294
	38	.0000	.0000	.0000	.0000	.0000	.0009	.0076	.0273	.0295
	39	.0000	.0000	.0000	.0000	.0000	.0006	.0059	.0242	.0295
	40	.0000	.0000	.0000	.0000	.0000	.0004	.0045	.0214	.0293
	41	.0000	.0000	.0000	.0000	.0000	.0003	.0035	.0188	.0289
	42	.0000	.0000	.0000	.0000	.0000	.0002	.0027	.0164	.0284
	43	.0000	.0000	.0000	.0000	.0000	.0001	.0020	.0143	.0278
	44	.0000	.0000	.0000	.0000	.0000	.0001	.0015	.0124	.0271
	45	.0000	.0000	.0000	.0000	.0000	.0000	.0012	.0107	.0264
	46	.0000	.0000	.0000	.0000	.0000	.0000	.0009	.0092	.0255
	47	.0000	.0000	.0000	.0000	.0000	.0000	.0007	.0079	.0246
	48	.0000	.0000	.0000	.0000	.0000	.0000	.0005	.0068	.0237
	49	.0000	.0000	.0000	.0000	.0000	.0000	.0004	.0058	.0227
	50	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0049	.0218
	51	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0042	.0208
	52	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0035	.0198
	53	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0030	.0188
	54	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0025	.0179
	55	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0021	.0169

		THETA (θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
8	103	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	104	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	105	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	106	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	107	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	108	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	109	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	110	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	111	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	112	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	113	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	114	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	115	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	116	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	117	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
9	18	.5344	.2688	.1254	.0532	.0199	.0063	.0015	.0002	.0000
	19	.3206	.3225	.2258	.1277	.0597	.0225	.0064	.0011	.0001
	20	.1096	.2204	.2314	.1746	.1021	.0462	.0152	.0031	.0002
	21	.0281	.1129	.1778	.1778	.1307	.0710	.0273	.0063	.0005
	22	.0060	.0482	.1139	.1527	.1395	.0910	.0408	.0108	.0010
	23	.0011	.0182	.0643	.1150	.1313	.1027	.0538	.0163	.0017
	24	.0002	.0062	.0331	.0789	.1127	.1058	.0646	.0224	.0027
	25	.0000	.0020	.0159	.0504	.0900	.1014	.0723	.0286	.0039
	26	.0000	.0006	.0072	.0305	.0680	.0919	.0764	.0346	.0052
	27	.0000	.0002	.0031	.0176	.0490	.0796	.0772	.0399	.0068
	28	.0000	.0000	.0013	.0098	.0341	.0663	.0750	.0444	.0085
	29	.0000	.0000	.0005	.0053	.0229	.0536	.0707	.0478	.0103
	30	.0000	.0000	.0002	.0028	.0150	.0422	.0649	.0501	.0122
	31	.0000	.0000	.0001	.0014	.0096	.0324	.0582	.0514	.0141
	32	.0000	.0000	.0000	.0007	.0061	.0244	.0512	.0517	.0159
	33	.0000	.0000	.0000	.0004	.0037	.0181	.0443	.0510	.0177
	34	.0000	.0000	.0000	.0002	.0023	.0132	.0377	.0496	.0193
	35	.0000	.0000	.0000	.0001	.0014	.0095	.0316	.0476	.0209
	36	.0000	.0000	.0000	.0000	.0008	.0068	.0263	.0452	.0223
	37	.0000	.0000	.0000	.0000	.0005	.0048	.0216	.0424	.0235
	38	.0000	.0000	.0000	.0000	.0003	.0033	.0175	.0394	.0246
	39	.0000	.0000	.0000	.0000	.0002	.0023	.0141	.0363	.0255
	40	.0000	.0000	.0000	.0000	.0001	.0016	.0113	.0332	.0262
	41	.0000	.0000	.0000	.0000	.0001	.0011	.0090	.0301	.0267
	42	.0000	.0000	.0000	.0000	.0000	.0007	.0071	.0271	.0271
	43	.0000	.0000	.0000	.0000	.0000	.0005	.0055	.0243	.0273
	44	.0000	.0000	.0000	.0000	.0000	.0003	.0043	.0217	.0274
	45	.0000	.0000	.0000	.0000	.0000	.0002	.0034	.0192	.0273
	46	.0000	.0000	.0000	.0000	.0000	.0001	.0026	.0169	.0271
	47	.0000	.0000	.0000	.0000	.0000	.0001	.0020	.0149	.0268
	48	.0000	.0000	.0000	.0000	.0000	.0001	.0015	.0130	.0264

THETA (θ)

n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
9	96	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0014
	97	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0013
	98	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0012
	99	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0011
	100	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0010
	101	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0009
	102	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0009
	103	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0008
	104	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0007
	105	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0007
	106	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	107	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	108	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	109	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	110	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	111	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	112	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	113	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	114	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	115	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	116	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	117	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	118	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	119	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	120	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	121	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	122	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	123	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	124	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	125	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
10	20	.4985	.2323	.0996	.0384	.0129	.0036	.0007	.0001	.0000
	21	.3323	.3097	.1992	.1024	.0430	.0143	.0034	.0005	.0000
	22	.1246	.2323	.2241	.1537	.0806	.0321	.0090	.0015	.0001
	23	.0347	.1292	.1870	.1710	.1121	.0535	.0176	.0033	.0002
	24	.0080	.0595	.1291	.1574	.1289	.0739	.0283	.0061	.0004
	25	.0016	.0240	.0780	.1269	.1299	.0894	.0406	.0099	.0008
	26	.0003	.0088	.0427	.0926	.1185	.0979	.0511	.0144	.0013
	27	.0000	.0030	.0217	.0626	.1002	.0993	.0604	.0195	.0019
	28	.0000	.0009	.0103	.0399	.0797	.0948	.0673	.0248	.0027
	29	.0000	.0003	.0047	.0242	.0604	.0861	.0714	.0300	.0037
	30	.0000	.0001	.0021	.0140	.0439	.0751	.0726	.0349	.0049
	31	.0000	.0000	.0009	.0079	.0308	.0633	.0714	.0392	.0062
	32	.0000	.0000	.0004	.0043	.0210	.0518	.0818	.0428	.0076
	33	.0000	.0000	.0001	.0023	.0140	.0413	.0634	.0455	.0091
	34	.0000	.0000	.0001	.0012	.0091	.0322	.0577	.0473	.0106
	35	.0000	.0000	.0000	.0006	.0058	.0247	.0515	.0483	.0122

[illegible]

		THETA (θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
10	131	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	132	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	133	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	134	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
11	22	.4649	.2007	.0791	.0277	.0083	.0020	.0004	.0000	.0000
	23	.3410	.2944	.1740	.0813	.0306	.0089	.0018	.0002	.0000
	24	.1392	.2404	.2131	.1329	.0624	.0219	.0053	.0007	.0000
	25	.0418	.1445	.1921	.1596	.0938	.0394	.0111	.0017	.0001
	26	.0103	.0713	.1421	.1575	.1157	.0583	.0191	.0034	.0002
	27	.0022	.0306	.0916	.1353	.1242	.0751	.0287	.0058	.0003
	28	.0004	.0118	.0531	.1047	.1202	.0872	.0389	.0089	.0006
	29	.0001	.0042	.0285	.0747	.1072	.0934	.0486	.0127	.0009
	30	.0000	.0014	.0143	.0500	.0897	.0937	.0569	.0170	.0014
	31	.0000	.0004	.0068	.0317	.0711	.0892	.0631	.0216	.0020
	32	.0000	.0001	.0031	.0193	.0540	.0813	.0671	.0262	.0027
	33	.0000	.0000	.0014	.0113	.0395	.0713	.0687	.0307	.0035
	34	.0000	.0000	.0006	.0064	.0280	.0606	.0681	.0348	.0045
	35	.0000	.0000	.0002	.0035	.0193	.0501	.0657	.0383	.0056
	36	.0000	.0000	.0001	.0019	.0130	.0404	.0618	.0413	.0068
	37	.0000	.0000	.0000	.0010	.0085	.0320	.0570	.0435	.0080
	38	.0000	.0000	.0000	.0005	.0055	.0248	.0516	.0450	.0093
	39	.0000	.0000	.0000	.0003	.0035	.0189	.0459	.0457	.0107
	40	.0000	.0000	.0000	.0001	.0022	.0142	.0403	.0458	.0121
	41	.0000	.0000	.0000	.0001	.0014	.0105	.0348	.0453	.0134
	42	.0000	.0000	.0000	.0000	.0008	.0077	.0298	.0443	.0147
	43	.0000	.0000	.0000	.0000	.0005	.0056	.0252	.0428	.0160
	44	.0000	.0000	.0000	.0000	.0003	.0040	.0211	.0409	.0172
	45	.0000	.0000	.0000	.0000	.0002	.0029	.0175	.0388	.0184
	46	.0000	.0000	.0000	.0000	.0001	.0020	.0144	.0365	.0194
	47	.0000	.0000	.0000	.0000	.0001	.0014	.0117	.0340	.0204
	48	.0000	.0000	.0000	.0000	.0000	.0010	.0095	.0315	.0213
	49	.0000	.0000	.0000	.0000	.0000	.0007	.0077	.0290	.0220
	50	.0000	.0000	.0000	.0000	.0000	.0005	.0061	.0265	.0227
	51	.0000	.0000	.0000	.0000	.0000	.0003	.0049	.0241	.0232
	52	.0000	.0000	.0000	.0000	.0000	.0002	.0039	.0218	.0236
	53	.0000	.0000	.0000	.0000	.0000	.0001	.0030	.0197	.0239
	54	.0000	.0000	.0000	.0000	.0000	.0001	.0024	.0176	.0241
	55	.0000	.0000	.0000	.0000	.0000	.0001	.0019	.0157	.0242
	56	.0000	.0000	.0000	.0000	.0000	.0000	.0014	.0140	.0242
	57	.0000	.0000	.0000	.0000	.0000	.0000	.0011	.0124	.0241
	58	.0000	.0000	.0000	.0000	.0000	.0000	.0009	.0109	.0239
	59	.0000	.0000	.0000	.0000	.0000	.0000	.0007	.0096	.0237
	60	.0000	.0000	.0000	.0000	.0000	.0000	.0005	.0084	.0233
	61	.0000	.0000	.0000	.0000	.0000	.0000	.0004	.0074	.0229
	62	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0064	.0225
	63	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0056	.0220

		THETA (θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
11	111	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0014
	112	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0013
	113	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0012
	114	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0011
	115	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0010
	116	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0009
	117	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0009
	118	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0008
	119	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0007
	120	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0007
	121	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	122	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0006
	123	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	124	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	125	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	126	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	127	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	128	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	129	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	130	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	131	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	132	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	133	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	134	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	135	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	136	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	137	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	138	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	139	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	140	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	141	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	142	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
12	24	.4337	.1735	.0628	.0200	.0054	.0012	.0002	.0000	.0000
	25	.3469	.2775	.1507	.0641	.0216	.0055	.0010	.0001	.0000
	26	.1532	.2452	.1997	.1132	.0477	.0147	.0030	.0003	.0000
	27	.0494	.1582	.1932	.1460	.0769	.0284	.0068	.0009	.0000
	28	.0130	.0833	.1526	.1538	.1013	.0449	.0126	.0018	.0001
	29	.0030	.0380	.1044	.1402	.1154	.0614	.0200	.0033	.0001
	30	.0006	.0155	.0640	.1147	.1180	.0753	.0287	.0053	.0002
	31	.0001	.0058	.0361	.0862	.1108	.0849	.0377	.0080	.0004
	32	.0000	.0020	.0190	.0605	.0972	.0893	.0463	.0113	.0007
	33	.0000	.0007	.0094	.0401	.0806	.0889	.0537	.0150	.0010
	34	.0000	.0002	.0045	.0254	.0638	.0844	.0595	.0189	.0014
	35	.0000	.0001	.0020	.0154	.0485	.0770	.0634	.0230	.0019
	36	.0000	.0000	.0009	.0091	.0356	.0679	.0652	.0271	.0026
	37	.0000	.0000	.0004	.0052	.0254	.0581	.0651	.0309	.0033

[illegible]

		THETA (θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
12	132	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0005
	133	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	134	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	135	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0004
	136	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	137	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	138	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	139	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	140	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	141	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	142	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	143	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	144	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	145	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	146	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	147	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	148	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	149	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	150	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
13	26	.4045	.1499	.0499	.0144	.0035	.0007	.0001	.0000	.0000
	27	.3506	.3598	.1296	.0501	.0151	.0034	.0005	.0000	.0000
	28	.1665	.2468	.1847	.0952	.0360	.0097	.0017	.0002	.0000
	29	.0574	.1702	.1911	.1313	.0620	.0201	.0041	.0004	.0000
	30	.0161	.0953	.1604	.1470	.0868	.0338	.0081	.0009	.0000
	31	.0039	.0460	.1161	.1418	.1046	.0489	.0136	.0018	.0001
	32	.0008	.0198	.0750	.1222	.1127	.0633	.0206	.0031	.0001
	33	.0002	.0078	.0444	.0964	.1112	.0749	.0284	.0049	.0002
	34	.0000	.0029	.0245	.0708	.1020	.0825	.0365	.0072	.0003
	35	.0000	.0010	.0127	.0490	.0883	.0857	.0442	.0100	.0005
	36	.0000	.0003	.0063	.0323	.0727	.0847	.0510	.0132	.0007
	37	.0000	.0001	.0030	.0204	.0574	.0802	.0564	.0167	.0010
	38	.0000	.0000	.0014	.0124	.0437	.0733	.0601	.0203	.0014
	39	.0000	.0000	.0006	.0073	.0323	.0649	.0621	.0240	.0019
	40	.0000	.0000	.0003	.0042	.0232	.0559	.0624	.0275	.0024
	41	.0000	.0000	.0001	.0024	.0162	.0470	.0612	.0309	.0030
	42	.0000	.0000	.0000	.0013	.0111	.0387	.0587	.0338	.0038
	43	.0000	.0000	.0000	.0007	.0075	.0312	.0553	.0364	.0045
	44	.0000	.0000	.0000	.0004	.0049	.0247	.0511	.0385	.0054
	45	.0000	.0000	.0000	.0002	.0032	.0193	.0465	.0400	.0063
	46	.0000	.0000	.0000	.0001	.0021	.0148	.0418	.0411	.0073
	47	.0000	.0000	.0000	.0000	.0013	.0113	.0370	.0416	.0083
	48	.0000	.0000	.0000	.0000	.0008	.0085	.0324	.0416	.0093
	49	.0000	.0000	.0000	.0000	.0005	.0063	.0281	.0412	.0104
	50	.0000	.0000	.0000	.0000	.0003	.0046	.0241	.0403	.0115
	51	.0000	.0000	.0000	.0000	.0002	.0034	.0204	.0392	.0125
	52	.0000	.0000	.0000	.0000	.0001	.0024	.0172	.0377	.0136

[illegible]

[illegible]

		THETA (θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
13	147	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003
	148	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	149	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	150	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	151	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	152	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	153	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	154	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	155	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	156	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	157	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
14	28	.3773	.1295	.0396	.0104	.0023	.0004	.0000	.0000	.0000
	29	.3521	.2418	.1108	.0389	.0106	.0021	.0003	.0000	.0000
	30	.1790	.2458	.1690	.0792	.0268	.0064	.0010	.0001	.0000
	31	.0657	.1804	.1861	.1162	.0492	.0141	.0025	.0002	.0000
	32	.0195	.1070	.1655	.1379	.0730	.0250	.0051	.0005	.0000
	33	.0050	.0545	.1264	.1403	.0928	.0382	.0091	.0010	.0000
	34	.0011	.0247	.0859	.1271	.1051	.0519	.0144	.0018	.0000
	35	.0002	.0102	.0532	.1051	.1086	.0643	.0208	.0029	.0001
	36	.0000	.0039	.0306	.0806	.1043	.0740	.0280	.0045	.0001
	37	.0000	.0014	.0166	.0582	.0939	.0801	.0353	.0065	.0002
	38	.0000	.0005	.0085	.0398	.0804	.0823	.0424	.0089	.0004
	39	.0000	.0002	.0042	.0261	.0659	.0809	.0486	.0117	.0005
	40	.0000	.0000	.0020	.0165	.0519	.0765	.0536	.0147	.0007
	41	.0000	.0000	.0009	.0000	.0396	.0700	.0572	.0180	.0010
	42	.0000	.0000	.0004	.0059	.0293	.0622	.0593	.0213	.0014
	43	.0000	.0000	.0002	.0034	.0212	.0539	.0599	.0246	.0018
	44	.0000	.0000	.0001	.0019	.0149	.0456	.0592	.0277	.0022
	45	.0000	.0000	.0000	.0011	.0103	.0378	.0572	.0306	.0028
	46	.0000	.0000	.0000	.0006	.0070	.0307	.0543	.0332	.0034
	47	.0000	.0000	.0000	.0003	.0047	.0246	.0507	.0355	.0041
	48	.0000	.0000	.0000	.0002	.0031	.0194	.0466	.0372	.0048
	49	.0000	.0000	.0000	.0001	.0020	.0151	.0422	.0386	.0056
	50	.0000	.0000	.0000	.0000	.0013	.0115	.0378	.0395	.0065
	51	.0000	.0000	.0000	.0000	.0008	.0088	.0334	.0399	.0074
	52	.0000	.0000	.0000	.0000	.0005	.0066	.0292	.0399	.0083
	53	.0000	.0000	.0000	.0000	.0003	.0049	.0253	.0395	.0092
	54	.0000	.0000	.0000	.0000	.0002	.0036	.0217	.0387	.0102
	55	.0000	.0000	.0000	.0000	.0001	.0026	.0185	.0377	.0111
	56	.0000	.0000	.0000	.0000	.0001	.0019	.0156	.0363	.0121
	57	.0000	.0000	.0000	.0000	.0000	.0014	.0131	.0348	.0130
	58	.0000	.0000	.0000	.0000	.0000	.0010	.0109	.0331	.0139
	59	.0000	.0000	.0000	.0000	.0000	.0007	.0090	.0313	.0148
	60	.0000	.0000	.0000	.0000	.0000	.0005	.0074	.0294	.0156
	61	.0000	.0000	.0000	.0000	.0000	.0003	.0060	.0274	.0164
	62	.0000	.0000	.0000	.0000	.0000	.0002	.0049	.0255	.0172

		THETA (θ)								
n	z	.1	.2	.3	.4	.5	.6	.7	.8	.9
14	157	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	158	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	159	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	160	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	161	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	162	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	163	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	164	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	165	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	166	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
15	30	.3519	.1119	.0314	.0075	.0015	.0002	.0000	.0000	.0000
	31	.3519	.2239	.0943	.0301	.0073	.0013	.0001	.0000	.0000
	32	.1906	.2425	.1532	.0653	.0198	.0041	.0005	.0000	.0000
	33	.0742	.1888	.1789	.1016	.0386	.0097	.0014	.0001	.0000
	34	.0232	.1183	.1681	.1273	.0604	.0182	.0032	.0002	.0000
	35	.0062	.0633	.1350	.1363	.0808	.0292	.0059	.0005	.0000
	36	.0015	.0301	.0962	.1295	.0960	.0417	.0099	.0010	.0000
	37	.0003	.0130	.0623	.1119	.1037	.0540	.0150	.0017	.0000
	38	.0001	.0052	.0374	.0895	.1037	.0648	.0209	.0027	.0001
	39	.0000	.0020	.0211	.0672	.0973	.0730	.0275	.0041	.0001
	40	.0000	.0007	.0112	.0478	.0865	.0778	.0342	.0059	.0002
	41	.0000	.0002	.0057	.0324	.0734	.0792	.0406	.0079	.0003
	42	.0000	.0001	.0028	.0211	.0598	.0775	.0464	.0104	.0004
	43	.0000	.0000	.0013	.0133	.0471	.0732	.0511	.0131	.0005
	44	.0000	.0000	.0006	.0081	.0359	.0671	.0546	.0159	.0007
	45	.0000	.0000	.0003	.0048	.0267	.0598	.0568	.0189	.0010
	46	.0000	.0000	.0001	.0028	.0194	.0520	.0577	.0220	.0013
	47	.0000	.0000	.0000	.0016	.0137	.0443	.0573	.0249	.0017
	48	.0000	.0000	.0000	.0009	.0095	.0370	.0558	.0277	.0021
	49	.0000	.0000	.0000	.0005	.0065	.0303	.0533	.0303	.0026
	50	.0000	.0000	.0000	.0003	.0044	.0244	.0502	.0326	.0031
	51	.0000	.0000	.0000	.0001	.0029	.0194	.0465	.0345	.0037
	52	.0000	.0000	.0000	.0001	.0019	.0152	.0425	.0361	.0043
	53	.0000	.0000	.0000	.0000	.0012	.0118	.0384	.0373	.0050
	54	.0000	.0000	.0000	.0000	.0008	.0090	.0343	.0380	.0058
	55	.0000	.0000	.0000	.0000	.0005	.0068	.0303	.0384	.0066
	56	.0000	.0000	.0000	.0000	.0003	.0051	.0265	.0384	.0074
	57	.0000	.0000	.0000	.0000	.0002	.0038	.0229	.0380	.0082
	58	.0000	.0000	.0000	.0000	.0001	.0028	.0197	.0373	.0091
	59	.0000	.0000	.0000	.0000	.0001	.0020	.0168	.0363	.0099
	60	.0000	.0000	.0000	.0000	.0000	.0015	.0142	.0351	.0108
	61	.0000	.0000	.0000	.0000	.0000	.0011	.0119	.0337	.0117
	62	.0000	.0000	.0000	.0000	.0000	.0008	.0100	.0321	.0125
	63	.0000	.0000	.0000	.0000	.0000	.0005	.0083	.0305	.0133
	64	.0000	.0000	.0000	.0000	.0000	.0004	.0068	.0287	.0141
	65	.0000	.0000	.0000	.0000	.0000	.0003	.0056	.0269	.0149

[illegible]

[illegible]

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