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Selective reflection of light at a solid-gas interface and its application

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SELECTIVE REFLECTION OF LIGHT AT A SOLID-GAS
INTERFACE AND ITS APPLICATION

by
FUMIHIDE TAKEDA

A dissertation submitted in partial fulfillment of
the requirements for the degree of


DOCTOR OF PHILOSOPHY
in
ENVIRONMENTAL SCIENCE
AND RESOURCES - PHYSICS

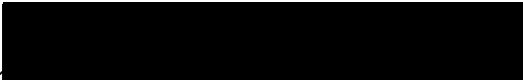
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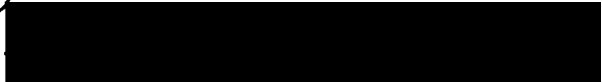
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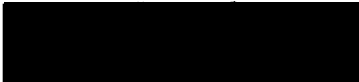

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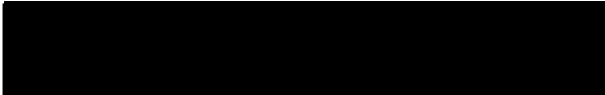

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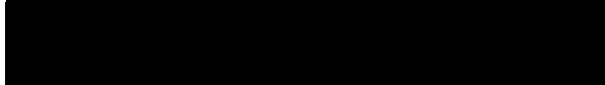
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APPROVED BY MEMBERS OF THE DISSERTATION COMMITTEE:


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In order to study the detailed spectral intensity distribution of light reflected from a solid-gas interface, the extinction theorem in optics is extended to include an absorbing medium and the thermal motion of the gas molecules near the interface. The theoretical spectral intensity distribution in the region of anomalous dispersion is found to be strongly modified compared to that predicted by existing theory.

An important consequence of this theory in the line shapes of the reflected light is the possibility of using recently developed saturation spectroscopic techniques to study atoms and molecules near surfaces.

In order to investigate the feasibility of these new techniques for obtaining solid-gas molecule interaction potentials, models of solid-gas interfaces were studied with and without interactions of the type $1/z^p$ ($p = 2,3,4$), where z is the distance between gas molecules and solid surface. A marked difference in the line shapes of the reflected light among the possible interactions suggests that the forms of interaction at the interface can be measured using known techniques.

Furthermore, the possibility of measuring the flow of gas near walls where currently available laser-Doppler anemometers can not spatially reach is investigated. It is shown that the shift and width of the numerically calculated line shape of the reflected light in our model flow is directly related to the mean and the fluctuating velocity fields respectively.

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CHAPTER I

1. INTRODUCTION

In order to understand surface phenomena in relation to lubrication, corrosion, catalysis, emissive and adsorptive properties, and adhesion, it is appropriate to first study the static characteristics in terms of the chemical identity of atoms present on the surface, their geometrical arrangement, and their electronic states. Experimental probes used to investigate surfaces are photons, electrons, ions, atoms, molecules, external fields, and their various combinations (J. R. Schrieffer, et. al., 1975). Although the methods developed for using these probes are ingenious, they have limitations in practice due to their technical and physical sensitivity to the surface involved. The response of the surface to an externally applied probe has the following general energy resolutions: They are of the order of $1 - 10^{-1}$ eV for LEED, $10^{-1} - 10^{-2}$ eV for ion beam spectroscopy (atomic or molecular beam spectroscopy), and $0.2 - 0.3$ eV for photoemission spectroscopy.

For detailed study of surface properties, however, the above energy resolutions are probably inadequate for resolving the energy change in electronic states in an adsorbed atom. In fact, adsorption on a surface can occur with binding energy of the order of 10^{-2} eV (physisorption).

Now consider a solid in contact with gas. For the gas (atoms or molecules), the detailed structure of the characteristic frequency is in general strongly modified by Doppler broadening. However, J. L. Cojan (1954) and J. P. Woerdman and M. F. H. Schuurmans (1974) observed an increased resolution (due to a partial absence of Doppler broadening) in light reflected normal to the solid-gas interface when its frequency was in the anomalous dispersion region. As a consequence, the dynamic response of the atom's own internal state to the surface may be detected through the anomalous dispersion of the reflected light. This particular reflection of light has been known as "selective reflection of light" which was first investigated many years ago by R. W. Wood (See the text of R. W. Wood, 1934). However, a satisfactory theory of the phenomenon has not been reported to date.

In this dissertation, we report on a theoretical analysis of optical reflection and refraction related to anomalous dispersion at a solid-gas interface. An important result of this study predicts a large increase (due to near absence of Doppler broadening) of the previously observed resolution. Therefore we place emphasis on its possible application as a new surface probe for environmentally related interface phenomena, such as, a model interface between a thin metal layer deposited on glass and Na vapor (or any other atom or molecule).

Another possible application will be in the field of

flow measurements near a solid surface. In our daily environment, we encounter not only natural flows (such as, wind, river and ocean currents, etc.) but also artificially created flows (such as, flows in laboratory systems, in power plants, etc.). Both types of flows are very complex and important to our environment. In an effort to improve our environmental resources or protect them from destruction, the close interplay between the flow measurements and theory has been successfully applied to the modeling of some complex flows. (See summer lecture notes of B. E. Launder, K. Hanjalic, H. A. Dwyer and F. J. Durst, 1978 and 1979.) In flow measuring techniques, the optical method has advanced with the development of Laser Doppler Anemometry (L.D.A.) systems. (See the text of F. Durst, A. Melling and J. H. Whitelaw, 1976). Until recently, technical problems have limited their use mainly to artificially created flows. However, a recent development in the mechanical flow meter (propeller type) will make possible accurate measurements of natural flows (F. Takeda and R. Takeda, 1979).

In modeling the flow of fluids in the environment, measurements of flow within the boundary layer have an essential role in predicting any complex flow. This is due to the difficulty of modeling turbulent flows on digital computers. The lack of physical information on the rapidly changing velocity field within the boundary layer causes not only a severe numerical problem, but also a substantial increase of computer storage and computing time. Unfortun-

ately, currently available L.D.A. techniques cannot reach the region of gas flow very close to the boundary. However, this region can be explored by using reflected light under conditions of anomalous dispersion.

In order to theoretically determine the unique properties of reflected light in the region of anomalous dispersion, we must note that the penetration depth of the incident light is very often of the order of the mean free path of the gaseous atom. In such a case, we cannot use the well known continuity conditions at the interface for the solutions of the macroscopic Maxwell's wave equations. This formal method is only applicable for the bulk gas (of known optical properties), not that within the penetration depth. Even if we use the familiar "surface impedance" method to solve this optical reflection and refraction problem, the precise optical properties at the interface are still required. Thus it would be more appropriate and may sometimes be imperative to use microscopic field descriptions at the beginning, and then to appropriately average them. These averaged quantities will be the macroscopic field quantities of interest.

This approach can in principle be used for any system. However, for simplicity, let us consider the case when light enters from a vacuum into a gaseous medium. Even though we will observe only the refracted light in the medium, we will consider that the incident light enters the gaseous medium without modification of phase, amplitude, phase velocity or

propagation direction. However, due to scattering of the incident light by the gaseous atoms, extinction of the incident light occurs everywhere in the medium. As a result, the refracted light becomes the macroscopic description of all the microscopic fields and propagates with the phase velocity of the medium. This extinction of the incident light, which has a spatially non-local nature, is known as the "Ewald-Oseen extinction theorem" in classical molecular optics. (See, for instance, the text of M. Born and E. Wolf, 1975a.) This unique relationship between the incident light and refracted light may be regarded as a spatially non-local version of the usual boundary condition used in order to obtain the unique response of the medium to the incident light. This point of view, based on classical molecular optics, has been successfully applied by E. Lalor and E. Wolf (1972) to derive the well known reflection and refraction law of electromagnetic waves at a geometrically sharp interface between a vacuum and a semi-infinite, linear, isotropic, homogeneous, non-absorbing dielectric. Here the words "geometrically sharp" mean that the optical properties at the interface are the same everywhere in the medium.

However, when the absorption of the medium is strong, the resulting field of the light in the medium can change drastically over the mean free path of the gaseous atoms near the interface. This requires a special averaging process of the scattered field, as described in Chapter IV.

A generalization of the Ewald-Oseen extinction theor-

em based on macroscopic Maxwell's equations was initiated by J. J. Sein (1970) and has been carried out by J. deGeode and P. Muzar (1972), D. N. Pattanayak and E. Wolf (1972), and E. Wolf (1976). They considered an interface between a vacuum and a simple generalized material as a geometrically sharp one where the electromagnetic wave's properties are the same as that of the bulk material. Their results cannot be directly applied to our case because the optical properties at the interface are different from those of the bulk medium. However, using mathematical techniques similar to those used by the above investigators, we derive a formal solution of the macroscopic Maxwell's wave equations in the form of a spatially non-local representation. As a result, our formal solutions contain interface effects due to the physical boundary. Thus in Chapter II, we first formulate various formal solutions for the response of a simple generalized material, including physical surface effects, to incident radiation whose source is located in the medium enclosing the material. The medium enclosing the material is not necessarily a vacuum and may be replaced by any other material, for example, a simple dielectric. The formal solutions contain two different spatial points. One is the observation point. The other is an induced source point in the material, from which the incident radiation is scattered.

In Chapter III, we remove this restriction and derive fully non-local (in space) representations. Then we can express the spatially non-local nature of the physical

extinction of the incident light in mathematical form. Furthermore, by using this extinction theorem as a non-local boundary condition, we obtain Fresnel's formulas of anomalous dispersion from the set of formal solutions obtained in Chapter II.

In Chapter IV, using the results of Chapter III, we explain the increased resolution of the reflected light in the frequency region of anomalous dispersion which was mentioned at the beginning of this chapter. We also obtain numerically calculated theoretical line shapes of the reflected light, including the case where the incident angle is larger than the critical angle.

In Chapter V, we use the phenomena predicted in Chapter IV and apply recently developed laser-saturation spectroscopic techniques to a model interface of a thin metal layer deposited on glass with Na vapor as the gas. The last chapter explores possible gas flow measurement techniques. The computer program used for numerical calculations and the results are presented in the appendices.

CHAPTER II

SOLUTIONS OF MAXWELL'S WAVE EQUATIONS AND THE EXTINCTION THEOREM; PARTIALLY NON-LOCAL (IN SPACE) REPRESENTATIONS

1. INTRODUCTION

In order to establish the basis for the treatment of the extinction theorem, we derive formal solutions of the macroscopic Maxwell's wave equations for various model systems. These systems are composed of two media including a transition region where the molecular environment is different from that in the bulk. The formal solutions are said to be partially non-local in space. Here the words "partially non-local in space" mean that the form of the formal solutions is expressed in terms of different spatial points. One is an observation point in either medium and the other is a point in the material medium. In the second section we will consider the system of a material bounded by a vacuum with an electromagnetic wave incident on the material from a source point in the vacuum. Our results on reflection and refraction contain physical surface effects due to the transition region in the material. In the limit of the vanishingly small thickness of the transition region, our results can be reduced to those which have appeared in the literature. The remainder of this chapter is organized

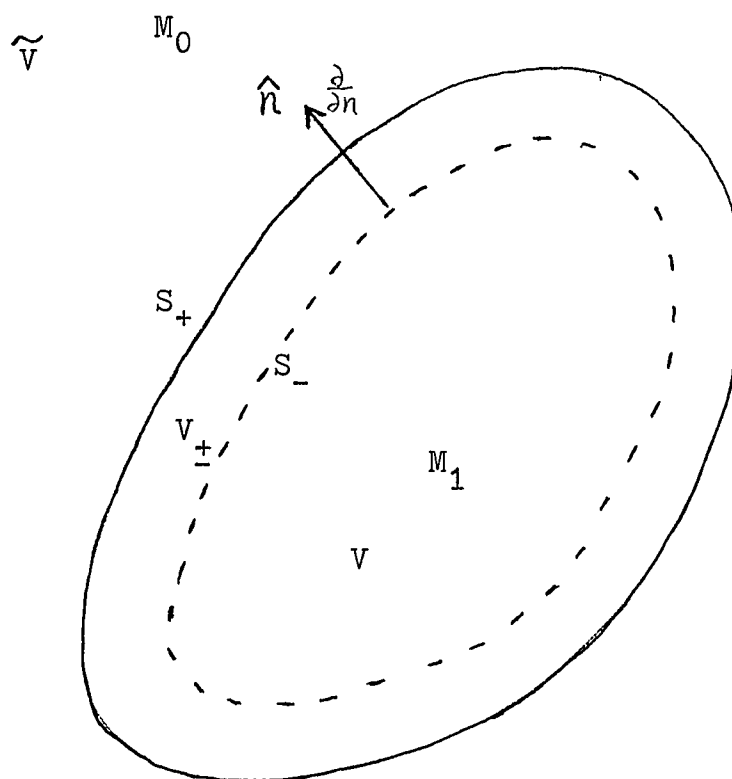
as follows: In section 3, we replace the region of vacuum by another simple non-absorbing dielectric and place the source of the incident wave in it. At the interface between the two media, the transition region is also assumed. As a result of this replacement, we will not only be able to examine our formal solutions from various physical viewpoints, but will also be able to apply them to various important real systems, such as, a solid-gas interface. This case will be discussed in detail in later chapters (see Chapters IV, V and VI). In section 4, as a simple application of the extinction theorem, a "local effective field" is included in our formal solution in order to derive the Lorentz-Lorenz formula. The detailed analysis of the extinction theorem such as the precise phase relation between the incident and induced waves will be discussed in the third chapter.

2. MATERIAL BOUNDED BY A VACUUM

We choose the boundary surface S_+ to be the vacuum boundary at which atoms (or molecules) gradually start to form a material medium. This transition region of volume V_{\pm} extends to another boundary S_- inside the material, where the material can be regarded as linear, isotropic, and homogeneous. We denote by V this material region bounded by the surface S_- . We also denote by \tilde{V} the vacuum space surrounding the surface S_+ (see Fig. 2.1).

In this system, electromagnetic radiation is incident

MATERIAL BOUNDED BY A MEDIUM M_0



$\frac{\partial}{\partial n}$ denotes differentiation along the outward normal to S_- .

Fig. 2.1

on the material from the vacuum. In the material it induces microscopic charges, currents, electric and magnetic dipole moments, etc. Their appropriately averaged field quantities obey the macroscopic Maxwell equations. In terms of the usual notations, they are;

$$\begin{aligned}\nabla \cdot \vec{E}(\vec{r}, t) &= 4\pi \langle \rho(\vec{r}, t) \rangle, \\ \nabla \times \vec{E}(\vec{r}, t) &= -\frac{1}{c} \frac{\partial}{\partial t} \vec{B}(\vec{r}, t), \\ \nabla \cdot \vec{B}(\vec{r}, t) &= 0, \\ \nabla \times \vec{B}(\vec{r}, t) &= \frac{4\pi}{c} \langle \rho(\vec{r}, t) \vec{v}(\vec{r}, t) \rangle + \frac{1}{c} \frac{\partial}{\partial t} \vec{E}(\vec{r}, t).\end{aligned}$$

Here $\langle \rho \rangle$ and $\langle \rho \vec{v} \rangle$ are a macroscopic average of the microscopic charge density of free and bound charges and their current densities in the material respectively. In the region V , the macroscopic quantities are obtained by an appropriate spatial average which is described in the text of J. D. Jackson, 1975, but in the transition region V_{\pm} a different averaging process is used (see Chapter IV).

In the material, we assume the following relation for $\langle \rho \rangle$,

$$\langle \rho(\vec{r}, t) \rangle = \rho(\vec{r}, t) - \nabla \cdot \vec{P}(\vec{r}, t),$$

where $\rho(\vec{r}, t)$ is the macroscopic charge density of free electrons and $\vec{P}(\vec{r}, t)$ is the macroscopic polarization of molecular electric dipole moments. Similarly we assume the following expression for $\langle \rho \vec{v} \rangle$,

$$\langle \rho(\vec{r}, t) \vec{v}(\vec{r}, t) \rangle = \vec{j}(\vec{r}, t) + \frac{\partial}{\partial t} \vec{P}(\vec{r}, t) + c \nabla \times \vec{M}(\vec{r}, t),$$

where $\vec{j}(\vec{r}, t)$ is the macroscopic current density of free electrons and $\vec{M}(\vec{r}, t)$ is the macroscopic magnetization of the molecular magnetic moments (intrinsic magnetic moments

can be added),

Noting that the source current density, \vec{J} , is in the vacuum, we write the Maxwell's equation for the entire region in terms of a single equation,

$$\nabla \times \vec{H}(\vec{r}, t) = \frac{4\pi}{c} \vec{J}(\vec{r}, t) + \frac{4\pi}{c} \left(\frac{\partial}{\partial t} \vec{P}(\vec{r}, t) + \vec{J}(\vec{r}, t) \right) + \frac{1}{c} \frac{\partial}{\partial t} \vec{E}(\vec{r}, t).$$

Then the wave equations for the entire region are;

$$\nabla \times (\nabla \times \vec{E}(\vec{r}, t)) + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}, t) = - \frac{4\pi}{c^2} \left(\frac{\partial}{\partial t} \vec{J}(\vec{r}, t) + \frac{\partial}{\partial t} \vec{J}(\vec{r}, t) + \frac{\partial^2}{\partial t^2} \vec{P}(\vec{r}, t) \right) - \frac{4\pi}{c} \nabla \times \frac{\partial}{\partial t} \vec{M}(\vec{r}, t), \quad (2.1a)$$

$$\nabla \times (\nabla \times \vec{H}(\vec{r}, t)) + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{H}(\vec{r}, t) = \frac{4\pi}{c} \nabla \times \left(\vec{J}(\vec{r}, t) + \vec{J}(\vec{r}, t) + \frac{\partial}{\partial t} \vec{P}(\vec{r}, t) \right) - \frac{4\pi}{c^2} \frac{\partial^2}{\partial t^2} \vec{M}(\vec{r}, t). \quad (2.1b)$$

When the material is dispersive, we must transform the field quantities from local forms to non-local forms (in time). This is done by using their Fourier transforms with respect to time. The Fourier integral for the electric field $\vec{E}(\vec{r}, t)$ is expressed as

$$\vec{E}(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \vec{E}(\vec{r}, \omega) e^{-i\omega t},$$

where its Fourier vector amplitude $\vec{E}(\vec{r}, \omega)$ is given as

$$\vec{E}(\vec{r}, \omega) = \int_{-\infty}^{+\infty} dt \vec{E}(\vec{r}, t) e^{i\omega t}.$$

Similar expressions for the other field quantities also hold. Thus, in terms of Fourier vector amplitudes of angular frequency ω , the above wave equations become;

$$\nabla \times (\nabla \times \vec{E}(\vec{r}, \omega)) - \frac{\omega^2}{c^2} \vec{E}(\vec{r}, \omega) = 4\pi \left(i \frac{\omega}{c^2} \vec{J}(\vec{r}, \omega) + i \frac{\omega}{c^2} \vec{J}(\vec{r}, \omega) + \frac{\omega^2}{c^2} \vec{P}(\vec{r}, \omega) + i \frac{\omega}{c} \nabla \times \vec{M}(\vec{r}, \omega) \right), \quad (2.2a)$$

$$\nabla \times (\nabla \times \vec{H}(\vec{r}, \omega)) - \frac{\omega^2}{c^2} \vec{H}(\vec{r}, \omega) = 4\pi \left(\frac{1}{c} \nabla \times \vec{J}(\vec{r}, \omega) + \frac{1}{c} \nabla \times \vec{J}(\vec{r}, \omega) - i \frac{\omega}{c} \nabla \times \vec{P}(\vec{r}, \omega) + \frac{\omega^2}{c^2} \vec{M}(\vec{r}, \omega) \right). \quad (2.2b)$$

We assume that the source $\vec{J}(\vec{r}, \omega)$ is far away from the boundary of the material. Then the formal solutions of the above wave equations, Eqs. (2.2a) and (2.2b), can be obtained by using the transverse dyadic Green's function. The Green's function is defined, noting that the free space wave number is $k_0 = \frac{\omega}{c}$, as

$$\mathcal{G}_T(\vec{r}|\vec{r}'; k_0) = g(\vec{r}|\vec{r}'; k_0) \mathcal{U} - \frac{1}{k_0^2} \nabla \nabla' g(\vec{r}|\vec{r}'; k_0), \quad (2.3)$$

where \mathcal{U} is the unit dyadic and $g(\vec{r}|\vec{r}'; k_0) = \frac{e^{ik_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$ is an outgoing free space Green's function. In Eq. (2.3), the second term on the right expresses the longitudinal part of the dyadic Green's function (see the text of P. M. Morse and H. Feshbach, 1953). The two Green's functions obey the following equations respectively,

$$[\nabla \times \nabla \times + k_0^2] \mathcal{G}_T(\vec{r}|\vec{r}'; k_0) = -4\pi \delta(\vec{r}-\vec{r}') \mathcal{U}, \quad (2.4a)$$

$$[\nabla^2 + k_0^2] g(\vec{r}|\vec{r}'; k_0) = -4\pi \delta(\vec{r}-\vec{r}'), \quad (2.4b)$$

where $\delta(\vec{r}-\vec{r}')$ is the Dirac delta function. We note that for the case of $\vec{r} \neq \vec{r}'$, the symmetry property of Eq. (2.4a) with respect to \vec{r} and \vec{r}' gives the following relation;

$$\begin{aligned} \mathcal{G}_T(\vec{r}|\vec{r}'; k_0) &= \frac{1}{k_0^2} \nabla' \times \nabla' \times \mathcal{G}_T(\vec{r}|\vec{r}'; k_0) \\ &= \frac{1}{k_0^2} \nabla' \times \nabla' \times \left[\mathcal{U} g(\vec{r}|\vec{r}'; k_0) - \frac{1}{k_0^2} \nabla \nabla' g(\vec{r}|\vec{r}'; k_0) \right] \\ &= \frac{1}{k_0^2} \nabla' \times \nabla' \times \mathcal{U} g(\vec{r}|\vec{r}'; k_0), \quad (\vec{r} \neq \vec{r}'). \end{aligned} \quad (2.5)$$

The formal solution of Eq. (2.2a) can now be obtained by using the following well known method. First we take the dot product of $\mathcal{G}_T(\vec{r}|\vec{r}'; k_0)$ with every term of Eq. (2.2a) from its left, similarly we take the dot product of $\vec{E}(\vec{r}, \omega)$ with every term of Eq. (2.4a) from its right, then we add the resulting equations noting that by definition $\vec{J}(\vec{r}, \omega)$ vanishes in V_{\pm} and that $\delta(\vec{r}-\vec{r}')=0$ for $\vec{r} \neq \vec{r}'$. Integrating (with respect to \vec{r}) the resulting equation over the volume, $\tilde{V}+V_{\pm}$, we find the solution to be,

$$\begin{aligned} &\underbrace{\frac{ik_0}{c} \int_{\tilde{V}} d^3r \mathcal{G}_T(\vec{r}|\vec{r}'; k_0) \cdot \vec{J}(\vec{r}, \omega)}_{\text{I}} + \underbrace{\int_{V_{\pm}} d^3r \mathcal{G}_T(\vec{r}|\vec{r}'; k_0) \cdot \left(i \frac{k_0}{c} \vec{J}(\vec{r}, \omega) + k_0^2 \vec{E}(\vec{r}, \omega) + i k_0 \nabla \times \vec{E}(\vec{r}, \omega) \right)}_{\text{II}} + \\ &\underbrace{\frac{1}{4\pi} \int_{\tilde{V}+V_{\pm}} d^3r \left\{ (\nabla \times (\nabla \times \mathcal{G}_T(\vec{r}|\vec{r}'; k_0))) \cdot \vec{E}(\vec{r}, \omega) - \mathcal{G}_T(\vec{r}|\vec{r}'; k_0) \cdot (\nabla \times (\nabla \times \vec{E}(\vec{r}, \omega))) \right\}}_{\text{III}} = \begin{cases} \vec{E}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}, \\ 0, & \text{if } \vec{r}' \in V, \end{cases} \quad (2.6) \end{aligned}$$

where $\vec{r}' = \vec{r}'$ on the lefthand side if $\vec{r}' \in \tilde{V}$. Also $\vec{r}' = \vec{r}'$ if $\vec{r}' \in V$. On the left-hand side of Eq. (2.6), the first, second and third terms are denoted respectively by I, II, and III for later reference. Term I is related with the source. Term II contains the effect of the transition region. Term III will become physically more tractable if we transform it into a surface integral over S_- . This can be done by noting that the integrand of term III can be written,

$$(\nabla \times (\nabla \times \vec{A})) \cdot \vec{E} - \vec{A} \cdot (\nabla \times (\nabla \times \vec{E})) = \nabla \cdot [(\nabla \times \vec{A}) \times \vec{E} + \vec{A} \times (\nabla \times \vec{E})],$$

where we have used the vector relations,

$$\nabla \cdot (\vec{A} \times (\nabla \times \vec{E})) = (\nabla \times \vec{A}) \cdot (\nabla \times \vec{E}) - \vec{A} \cdot (\nabla \times (\nabla \times \vec{E})),$$

$$\nabla \cdot [(\nabla \times \vec{A}) \times \vec{E}] = (\nabla \times (\nabla \times \vec{A})) \cdot \vec{E} - (\nabla \times \vec{A}) \cdot (\nabla \times \vec{E}).$$

Then term III leads to

$$\begin{aligned} \text{III} &= \frac{1}{4\pi} \int_{S_-} dS (\hat{n}) \cdot \left[-\vec{E}(\vec{r}, \omega) \times (\nabla \times \vec{A}(\vec{r}|\vec{r}'; k_0)) + \vec{A}(\vec{r}|\vec{r}'; k_0) \times (\nabla \times \vec{E}(\vec{r}, \omega)) \right] \\ &= \frac{1}{4\pi} \int_{S_-} dS \left\{ (\hat{n} \times \vec{E}(\vec{r}, \omega)) \cdot (\nabla \times \vec{A}(\vec{r}|\vec{r}'; k_0)) + \vec{A}(\vec{r}|\vec{r}'; k_0) \cdot (\hat{n} \times (\nabla \times \vec{E}(\vec{r}, \omega))) \right\}, \end{aligned}$$

where \hat{n} is the outward normal unit vector to the closed surface S_- . Using the relation established in Eq. (2.5), term III can now be written as

$$\text{III} = \frac{1}{4\pi k_0} \nabla \times \nabla \times \int_{S_-} dS \left\{ (\hat{n} \times \vec{E}(\vec{r}, \omega)) \cdot (\nabla \times \vec{U}g(\vec{r}|\vec{r}'; k_0)) + \vec{U}g(\vec{r}|\vec{r}'; k_0) \cdot (\hat{n} \times (\nabla \times \vec{E}(\vec{r}, \omega))) \right\}. \quad (2.7)$$

Any Cartesian component of the integrand in Eq. (2.7) can be written

$$\left\{ (\hat{n} \times \vec{E}) \cdot (\nabla \times \vec{U}g) + \vec{U}g \cdot (\hat{n} \times (\nabla \times \vec{E})) \right\}_{\alpha} = g n_{\beta} \nabla_{\alpha} E_{\beta} - g n_{\beta} \nabla_{\beta} E_{\alpha} + E_{\alpha} n_{\beta} (\nabla_{\beta} g) - n_{\beta} E_{\beta} (\nabla_{\alpha} g), \quad (2.8)$$

where α and β are any of the Cartesian co-ordinates x , y and z , and the Einstein summation convention is used. ∇_{β} denotes the partial differentiation with respect to the co-ordinate β . The last term in Eq. (2.8) may be written as,

$$n_{\alpha} E_{\beta} (\nabla_{\beta} g) = n_{\alpha} (\nabla_{\beta} E_{\beta} g) - n_{\alpha} g \nabla_{\beta} E_{\beta}$$

$$= n_{\alpha} (\nabla_{\beta} E_{\beta} g) - n_{\alpha} g \left\{ 4\pi \left(\frac{1}{i\omega} \nabla \cdot \vec{J} - \nabla \cdot \vec{P} \right) \right\},$$

where we have used the continuity equation for the macroscopic charge density $\rho(\vec{r}, \omega)$ of free electrons, the current density $\vec{J}(\vec{r}, \omega)$ and also the polarization vector field $\vec{P}(\vec{r}, \omega)$ defined through the relation,

$$\langle \rho(\vec{r}, \omega) \rangle = \rho(\vec{r}, \omega) - \nabla \cdot \vec{P}(\vec{r}, \omega).$$

Thus Eq. (2.7) leads to

$$\begin{aligned} \text{III} = & \frac{1}{4\pi k_0^2} \nabla \times \nabla \times \int_{S_-} dS \left\{ \vec{E}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k_0) - g(\vec{r}|\vec{r}'; k_0) \frac{\partial}{\partial n} \vec{E}(\vec{r}, \omega) \right\} \\ & + \frac{1}{4\pi k_0^2} \nabla \times \nabla \int_{S_-} dS \left\{ g(\vec{r}|\vec{r}'; k_0) \nabla (\hat{n} \cdot \vec{E}(\vec{r}, \omega)) - \hat{n} (\nabla \cdot \vec{E}(\vec{r}, \omega)) g(\vec{r}|\vec{r}'; k_0) \right\} \\ & + \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} dS \frac{\hat{n}}{i\omega} g(\vec{r}|\vec{r}'; k_0) \nabla \cdot \vec{J}(\vec{r}, \omega) - \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} dS \hat{n} g(\vec{r}|\vec{r}'; k_0) \nabla \cdot \vec{P}(\vec{r}, \omega). \end{aligned}$$

The second term in the above equation vanishes. To see this, consider the following surface integral,

$$\begin{aligned} \int_{S_-} dS \hat{n} \{ \nabla \cdot (g \vec{E}) \} &= - \int_{\vec{V} + V_{\pm}} d^3r \{ \nabla \cdot (\nabla \cdot (g \vec{E})) \} \\ &= - \int_{\vec{V} + V_{\pm}} d^3r \{ (\nabla \cdot \nabla)(g \vec{E}) + \nabla \times (\nabla \times g \vec{E}) \} \\ &= \int_{S_-} dS \{ \hat{n} \cdot \nabla (g \vec{E}) + \hat{n} \times (\nabla \times g \vec{E}) \} \\ &= \int_{S_-} dS \{ \nabla (\hat{n} \cdot g \vec{E}) \} \\ &= \int_{S_-} dS \{ g \nabla (\hat{n} \cdot \vec{E}) + \hat{n} \cdot \vec{E} \nabla g \}. \end{aligned}$$

We then find the second term to be,

$$\begin{aligned} & \frac{1}{4\pi k_0^2} \nabla \times \nabla \times \int_{S_-} dS \left\{ g(\vec{r}|\vec{r}'; k_0) \nabla (\hat{n} \cdot \vec{E}(\vec{r}, \omega)) - g(\vec{r}|\vec{r}'; k_0) \nabla (\hat{n} \cdot \vec{E}(\vec{r}, \omega)) - \hat{n} \cdot \vec{E}(\vec{r}, \omega) \nabla g(\vec{r}|\vec{r}'; k_0) \right\} \\ &= \frac{1}{4\pi k_0^2} \nabla \times \nabla \times \int_{S_-} dS \{ \hat{n} \cdot \vec{E}(\vec{r}, \omega) \nabla g(\vec{r}|\vec{r}'; k_0) \} = 0. \end{aligned}$$

Using Eq. (2.5), we rewrite term II in Eq. (2.6), in terms of the free space Green's function $g(\vec{r}|\vec{r}'; k_0)$. Then Eq. (2.6) becomes,

$$\begin{aligned} & \frac{ik_0}{c} \int_{\vec{V}} d^3r \left\{ \frac{\text{I}}{g(\vec{r}|\vec{r}'; k_0) \cdot \vec{J}(\vec{r}, \omega)} + \frac{\text{II}}{k_0^2 \nabla \times \nabla \times \int_{V_{\pm}} d^3r g(\vec{r}|\vec{r}'; k_0) \left(i \frac{k_0}{c} \vec{J}(\vec{r}, \omega) + k_0^2 \vec{P}(\vec{r}, \omega) + i k_0 \nabla \times \vec{P}(\vec{r}, \omega) \right)} \right\} \\ & + \frac{1}{4\pi k_0^2} \nabla \times \nabla \times \int_{S_-} dS \left\{ \vec{E}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k_0) - g(\vec{r}|\vec{r}'; k_0) \frac{\partial}{\partial n} \vec{E}(\vec{r}, \omega) \right\} + \frac{\text{IV}}{k_0^2 \nabla \times \nabla \times \int_{S_-} dS \left\{ \frac{\hat{n}}{i\omega} g(\vec{r}|\vec{r}'; k_0) \nabla \cdot \vec{J}(\vec{r}, \omega) \right\}} \\ & - \frac{\text{V}}{k_0^2 \nabla \times \nabla \times \int_{S_-} dS \left\{ \hat{n} g(\vec{r}|\vec{r}'; k_0) \nabla \cdot \vec{P}(\vec{r}, \omega) \right\}} = \begin{cases} \vec{E}(\vec{r}', \omega), & \text{if } \vec{r}' \in \vec{V}. \\ 0, & \text{if } \vec{r}' \in V. \end{cases} \quad (2.9) \end{aligned}$$

The process of obtaining the formal solution of Eq. (2.2b) is exactly the same as that of Eq. (2.2a). The result is,

$$\begin{aligned}
 & \frac{1}{c} \int_{\tilde{V}} d^3r \left(\vec{G}(\vec{r}|\vec{r}'; k_0) \cdot (\nabla \times \vec{J}(\vec{r}, \omega)) + \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r g(\vec{r}|\vec{r}'; k_0) \left(\frac{1}{c} \nabla \times \vec{J}(\vec{r}, \omega) - i k_0 \nabla \times \vec{P}(\vec{r}, \omega) + k_0 \vec{M}(\vec{r}, \omega) \right) \right) \\
 & \quad \text{----- I' ----- II' -----} \\
 & + \frac{1}{4\pi k_0^2} \nabla' \times \nabla' \times \int_{S_-} dS \left\{ \vec{H}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k_0) - g(\vec{r}|\vec{r}'; k_0) \frac{\partial}{\partial n} \vec{H}(\vec{r}, \omega) \right\} \\
 & \quad \text{----- III' -----} \\
 & - \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{S_-} dS \left\{ g(\vec{r}|\vec{r}'; k_0) \hat{n} \cdot \nabla \cdot \vec{M}(\vec{r}, \omega) \right\} = \begin{cases} \vec{H}(\vec{r}, \omega) & , \text{ if } \vec{r}' \in \tilde{V}. \\ 0 & , \text{ if } \vec{r}' \in V. \end{cases} \quad (2.10) \\
 & \quad \text{----- IV' -----}
 \end{aligned}$$

In the volume integral, II, of Eq. (2.9), the magnetization term can be written as

$$\begin{aligned}
 & \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \left\{ i k_0 g(\vec{r}|\vec{r}'; k_0) \nabla \times \vec{M}(\vec{r}, \omega) \right\} \\
 & = \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r i k_0 \left\{ \nabla \times (\vec{M}(\vec{r}, \omega) g(\vec{r}|\vec{r}'; k_0)) - (\nabla g(\vec{r}|\vec{r}'; k_0)) \times \vec{M}(\vec{r}, \omega) \right\} \\
 & = \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{S_-} dS \left\{ i k_0 g(\vec{r}|\vec{r}'; k_0) (-\hat{n}) \times \vec{M}(\vec{r}, \omega) \right\} - \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \left\{ i k_0 [\nabla g(\vec{r}|\vec{r}'; k_0)] \times \vec{M}(\vec{r}, \omega) \right\}. \quad (2.11)
 \end{aligned}$$

Similarly the current density and the polarization vector terms in II' of Eq. (2.10) become respectively,

$$\begin{aligned}
 & \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \left\{ \frac{1}{c} g(\vec{r}|\vec{r}'; k_0) \nabla \times \vec{J}(\vec{r}, \omega) \right\} \\
 & = \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{S_-} dS \left\{ \frac{1}{c} g(\vec{r}|\vec{r}'; k_0) (-\hat{n}) \times \vec{J}(\vec{r}, \omega) \right\} - \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \left\{ [\nabla g(\vec{r}|\vec{r}'; k_0)] \times \frac{1}{c} \vec{J}(\vec{r}, \omega) \right\} \quad (2.12)
 \end{aligned}$$

and

$$\begin{aligned}
 & \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \left\{ (-i k_0) g(\vec{r}|\vec{r}'; k_0) \nabla \times \vec{P}(\vec{r}, \omega) \right\} \\
 & = \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{S_-} dS \left\{ i k_0 g(\vec{r}|\vec{r}'; k_0) \hat{n} \times \vec{P}(\vec{r}, \omega) \right\} + \frac{1}{k_0^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \left\{ i k_0 [\nabla g(\vec{r}|\vec{r}'; k_0)] \times \vec{P}(\vec{r}, \omega) \right\} \quad (2.13)
 \end{aligned}$$

Thus the final forms of the formal solutions, Eqs. (2.9)

and (2.10) are;

$$\begin{aligned}
\vec{E}^{(0)}(\vec{r}, \omega) + \frac{1}{4\pi k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ \vec{E}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k_0) - g(\vec{r}|\vec{r}'; k_0) \frac{\partial}{\partial n} \vec{E}(\vec{r}, \omega) \right\} \\
- \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ g(\vec{r}|\vec{r}'; k_0) \hat{n} \cdot \nabla \vec{E}(\vec{r}, \omega) \right\} + \frac{1}{k_0^2} \nabla \times \nabla \times \int_{V_{\pm}} d\vec{r}' \left\{ k_0^2 g(\vec{r}|\vec{r}'; k_0) \vec{E}(\vec{r}', \omega) \right\} \\
- \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ \frac{1}{\omega} g(\vec{r}|\vec{r}'; k_0) \hat{n} \cdot \nabla \vec{J}(\vec{r}, \omega) \right\} + \frac{1}{k_0^2} \nabla \times \nabla \times \int_{V_{\pm}} d\vec{r}' \left\{ \frac{1}{c} g(\vec{r}|\vec{r}'; k_0) \vec{J}(\vec{r}', \omega) \right\} \\
- \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ i k_0 g(\vec{r}|\vec{r}'; k_0) \hat{n} \times \vec{M}(\vec{r}, \omega) \right\} - \frac{1}{k_0^2} \nabla \times \nabla \times \int_{V_{\pm}} d\vec{r}' \left\{ i k_0 [\nabla g(\vec{r}|\vec{r}'; k_0) \times \vec{M}(\vec{r}', \omega)] \right\} = \begin{cases} \vec{E}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}, \\ 0, & \text{if } \vec{r}' \in V, \end{cases} \quad (2.14)
\end{aligned}$$

and

$$\begin{aligned}
\vec{H}^{(0)}(\vec{r}, \omega) + \frac{1}{4\pi k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ \vec{H}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k_0) - g(\vec{r}|\vec{r}'; k_0) \frac{\partial}{\partial n} \vec{H}(\vec{r}, \omega) \right\} \\
- \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ g(\vec{r}|\vec{r}'; k_0) \hat{n} \cdot \nabla \vec{H}(\vec{r}, \omega) \right\} + \frac{1}{k_0^2} \nabla \times \nabla \times \int_{V_{\pm}} d\vec{r}' \left\{ k_0^2 g(\vec{r}|\vec{r}'; k_0) \vec{H}(\vec{r}', \omega) \right\} \\
- \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ \frac{1}{c} g(\vec{r}|\vec{r}'; k_0) \hat{n} \times \vec{J}(\vec{r}, \omega) \right\} - \frac{1}{k_0^2} \nabla \times \nabla \times \int_{V_{\pm}} d\vec{r}' \left\{ \frac{1}{c} [\nabla g(\vec{r}|\vec{r}'; k_0) \times \vec{J}(\vec{r}', \omega)] \right\} \\
+ \frac{1}{k_0^2} \nabla \times \nabla \times \int_{S_-} d\vec{s} \left\{ i k_0 g(\vec{r}|\vec{r}'; k_0) \hat{n} \times \vec{P}(\vec{r}, \omega) \right\} + \frac{1}{k_0^2} \nabla \times \nabla \times \int_{V_{\pm}} d\vec{r}' \left\{ i k_0 [\nabla g(\vec{r}|\vec{r}'; k_0) \times \vec{P}(\vec{r}', \omega)] \right\} = \begin{cases} \vec{H}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}, \\ 0, & \text{if } \vec{r}' \in V, \end{cases} \quad (2.15)
\end{aligned}$$

where the incident waves, $\vec{E}^{(i)}(\vec{r}, \omega)$ and $\vec{H}^{(i)}(\vec{r}, \omega)$, are defined as

$$\vec{E}^{(i)}(\vec{r}, \omega) = \frac{i k_0}{c} \int_{\tilde{V}} d\vec{r}' \Gamma(\vec{r}|\vec{r}'; k_0) \cdot \vec{J}(\vec{r}', \omega)$$

and

$$\vec{H}^{(i)}(\vec{r}, \omega) = \frac{1}{c} \int_{\tilde{V}} d\vec{r}' \Gamma(\vec{r}|\vec{r}'; k_0) \cdot (\nabla \times \vec{J}(\vec{r}', \omega)).$$

In these formal solutions, Eqs. (2.14) and (2.15), the physical surface effects appear as the volume integrals in the transition region, V_{\pm} . An example of the use of the volume integrals for anomalous dispersion will be fully discussed in the next chapter.

If the boundary of the material can be assumed to be a geometrically sharp boundary S and if the material can be regarded as a perfect conductor, we can intuitively see that by taking the limit as the thickness of transition volume V_{\pm} approaches zero, all the volume integral terms in Eqs. (2.14) and (2.15) except for those containing the current density $\vec{J}(\vec{r}, \omega)$ vanish. Furthermore all the surface integrals in Eqs. (2.14) and (2.15) must vanish. In this

case, Eqs. (2.14) and (2.15) reduce to

$$\begin{aligned}\vec{E}^{(i)}(\vec{r}, \omega) + \frac{1}{k_0^2} \nabla \times \nabla \times \int_S dS \left\{ i \frac{k_0}{c} g(\vec{r} | \vec{r}'; k_0) \vec{J}^{(s)}(\vec{r}', \omega) \right\} &= \begin{cases} \vec{E}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}, \\ 0, & \text{if } \vec{r}' \in V, \end{cases} \\ \vec{H}^{(i)}(\vec{r}, \omega) + \frac{1}{k_0^2} \nabla \times \nabla \times \int_S dS \left\{ i k_0 \nabla g(\vec{r} | \vec{r}'; k_0) \times \vec{J}^{(s)}(\vec{r}', \omega) \right\} &= \begin{cases} \vec{H}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}, \\ 0, & \text{if } \vec{r}' \in V, \end{cases}\end{aligned}$$

where the surface current density is denoted by $\vec{J}^{(s)}(\vec{r}, \omega)$.

It can be defined by integrating the volume current density of $\vec{J}(\vec{r}, \omega)$ over a vanishingly small thickness along the normal to the surface S . In deriving these surface current contributions, we can also use the following expression (see the text of M. Born and E. Wolf, 1975b),

$$\vec{J}(\vec{r}, \omega) = \vec{J}^{(s)}(\vec{r}, \omega) | \nabla F(\vec{r}) | \delta(F), \quad (2.16)$$

where $F(\vec{r})=0$ defines the boundary surface S . The condition, $F(\vec{r}) > 0$, refers to the material region and $\delta(F)$ is the Dirac delta function.

If the material is not a perfect conductor, all the volume integrals in Eqs. (2.14) and (2.15) vanish with the surface integrals at S^- remaining.

We can compare our results with those which have appeared in the literature. In the work of D. N. Pattanayak and E. Wolf (1972) and E. Wolf (1976), they did not introduce the transition region V_{\pm} so that their results are for the sharp boundary. In order to get their equation similar to our Eq. (2.6), they took the volume integral over the volume \tilde{V} so that their boundary surface corresponds to our surface S^+ . The field quantities at S^+ are then for the vacuum. In order to relate the surface integrals at S^+

to those at S^- just inside of the sharp boundary of the material, they used the following well known continuity conditions;

$$\hat{n} \times [\vec{E}^{(+)}(\vec{r}, \omega) - \vec{E}^{(-)}(\vec{r}, \omega)] = 0 ,$$

$$\hat{n} \times [\vec{H}^{(+)}(\vec{r}, \omega) - \vec{H}^{(-)}(\vec{r}, \omega)] = \begin{cases} \frac{4\pi}{c} \vec{J}^{(s)}(\vec{r}, \omega), & \text{if the conductor is perfect,} \\ 0, & \text{otherwise,} \end{cases}$$

where the subscripts (+) and (-) refer to surfaces S^+ and S^- respectively. Therefore, although the method is different, our results, Eqs. (2.14) and (2.15) in the limit of the sharp boundary agree exactly with those of the above investigators.

On the other hand, J. deGeode and P. Mazur (1972) considered the transition region, V_{\pm} , for conductors, although their results are for a sharp boundary. In their approach to the sharp boundary, they took a vanishingly small thickness of the transition region so that the surface current appears from the singularity of $c \nabla \times \vec{M}(\vec{r}, \omega)$ at the sharp boundary. Namely, they expressed the magnetization vector, $\vec{M}(\vec{r}, \omega)$, for the sharp boundary, as

$$\vec{M}(\vec{r}, \omega) = U(F) \vec{M}(\vec{r}, \omega) ,$$

where $U(F)$ is the Heaviside unit step function defined as,

$$U(F) = \begin{cases} 0, & \text{if } F(\vec{r}) < 0. \\ 1, & \text{if } F(\vec{r}) > 0. \end{cases}$$

Here $F(\vec{r})=0$ defines the boundary surface. Then they found the following,

$$c \nabla \times \vec{M}(\vec{r}, \omega) = U(F) c \nabla \times \vec{M}(\vec{r}, \omega) + c \delta(F) (\nabla F(\vec{r})) \times \vec{M}(\vec{r}, \omega) .$$

Using the outward normal unit vector, $\hat{n} = -\frac{\nabla F(\vec{r})}{|\nabla F(\vec{r})|}$, this can be written as

$$(\nabla \times \vec{M}(\vec{r}, \omega) = U(F) (\nabla \times \vec{M}(\vec{r}, \omega) - \delta(F) |\nabla F(\vec{r})| \hat{n} \times \vec{M}(\vec{r}, \omega).$$

The second term of this equation is related to the surface current density. It corresponds to the first term in Eq. (2.11) of our model.

3. MATERIAL BOUNDED BY ANOTHER MATERIAL MEDIUM

In this section, let us consider the solution of Maxwell's wave equations for the case where the previous vacuum region is replaced by a material medium. This medium will be referred to as M_0 and the bounded material as M_1 . We assume that the medium M_0 is a linear, isotropic, homogeneous, and non-absorbing dielectric. The source, $\vec{J}(\vec{r}, \omega)$, is in the medium M_0 . Since M_0 is no longer a vacuum, the previous Maxwell's equations are modified by adding an induced macroscopic charge density and a current density of the medium M_0 ,

$$\begin{aligned} \nabla \cdot \vec{E}(\vec{r}, \omega) &= \begin{cases} 4\pi \langle \rho_0(\vec{r}, \omega) \rangle + \frac{4\pi}{i\omega} \nabla \cdot \vec{J}(\vec{r}, \omega), & \text{in } M_0, \\ 4\pi \langle \rho(\vec{r}, \omega) \rangle & , \text{ in } M_1, \end{cases} \\ \nabla \times \vec{E}(\vec{r}, \omega) &= ik_0 \vec{B}(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) &= 0 \\ \nabla \times \vec{B}(\vec{r}, \omega) &= \begin{cases} \frac{4\pi}{c} (\langle \rho_0(\vec{r}, \omega) \vec{v}_0(\vec{r}, \omega) \rangle + \vec{J}(\vec{r}, \omega)) - ik_0 \vec{E}(\vec{r}, \omega), & \text{in } M_0, \\ \frac{4\pi}{c} (\langle \rho(\vec{r}, \omega) \vec{v}(\vec{r}, \omega) \rangle) - ik_0 \vec{E}(\vec{r}, \omega) & , \text{ in } M_1, \end{cases} \end{aligned}$$

where the subscript "0" refers to the medium M_0 in which the source $\vec{J}(\vec{r}, \omega)$ is located (k_0 still stands for the free space wave number).

From the assumptions made for the media M_0 and M_1 , we have their macroscopic charge densities,

$$\langle \rho_o(\vec{r}, \omega) \rangle = -\nabla \cdot \vec{P}_o(\vec{r}, \omega)$$

and

$$\langle \rho(\vec{r}, \omega) \rangle = \rho(\vec{r}, \omega) - \nabla \cdot \vec{P}(\vec{r}, \omega)$$

for M_0 and M_1 respectively. Here we note that there are no free charges in M_0 except for those contributing to the source term $\vec{J}(\vec{r}, \omega)$. Similarly, their macroscopic current densitites can be written as

$$\langle \rho_o(\vec{r}, \omega) \vec{U}_o(\vec{r}, \omega) \rangle = -i\omega \vec{P}_o(\vec{r}, \omega) + (\nabla \times \vec{M}_o(\vec{r}, \omega))$$

for the medium M_0 , and

$$\langle \rho(\vec{r}, \omega) \vec{U}(\vec{r}, \omega) \rangle = \vec{J}(\vec{r}, \omega) - i\omega \vec{P}(\vec{r}, \omega) + (\nabla \times \vec{M}(\vec{r}, \omega))$$

for the medium M_1 .

Since the medium M_0 is linear, isotropic and homogeneous, we assume the following relations,

$$\begin{aligned} \vec{D}(\vec{r}, \omega) &= \vec{E}(\vec{r}, \omega) + 4\pi \vec{P}_o(\vec{r}, \omega) = (1 + 4\pi \eta_o(\omega)) \vec{E}(\vec{r}, \omega) = \epsilon_o(\omega) \vec{E}(\vec{r}, \omega), \\ \vec{B}(\vec{r}, \omega) &= \vec{H}(\vec{r}, \omega) + 4\pi \vec{M}_o(\vec{r}, \omega) = (1 + 4\pi \chi_o(\omega)) \vec{H}(\vec{r}, \omega) = \mu_o(\omega) \vec{H}(\vec{r}, \omega), \end{aligned}$$

where $\eta_o(\omega)$ and $\epsilon_o(\omega)$ are the electric susceptibility and the dielectric constant respectively and $\chi_o(\omega)$ and $\mu_o(\omega)$ are the magnetic susceptibility and the magnetic permeability respectively. Furthermore, the medium M_0 is non-absorbing so that $\epsilon_o(\omega)$ and $\mu_o(\omega)$ are real.

We assume that the transition region V_{\pm} starts at the boundary surface S^+ where the material parameters are those for the bulk medium M_0 and ends at the boundary surface S^- in the medium M_1 . We may reconstruct the medium M_1 by first filling the region $V+V_{\pm}$ with the medium M_0 . Next we superpose "virtually induced field quantities" on the medium M_0 in the region, $V+V_{\pm}$. This construction can be done by re-

writing the induced fields in M_1 , $\vec{P}(\vec{r}, \omega)$ and $\vec{M}(\vec{r}, \omega)$, in terms of the material parameters of M_0 , $\eta_0(\omega)$ and $\chi_0(\omega)$, and the adjustable virtual field quantities. These induced fields are

$$\vec{P}(\vec{r}, \omega) = \vec{P}_{o1}(\vec{r}, \omega) + \vec{P}(\vec{r}, \omega) - \vec{P}_{o1}(\vec{r}, \omega) = \vec{P}_{o1}(\vec{r}, \omega) + \vec{P}_v(\vec{r}, \omega)$$

and

$$\vec{M}(\vec{r}, \omega) = \vec{M}_{o1}(\vec{r}, \omega) + \vec{M}(\vec{r}, \omega) - \vec{M}_{o1}(\vec{r}, \omega) = \vec{M}_{o1}(\vec{r}, \omega) + \vec{M}_v(\vec{r}, \omega),$$

where $\vec{P}_{o1}(\vec{r}, \omega) = \eta_0(\omega) \vec{E}(\vec{r}, \omega)$ and $\vec{M}_{o1}(\vec{r}, \omega) = \chi_0(\omega) \vec{H}(\vec{r}, \omega)$ in terms of the mean fields \vec{E} and \vec{H} of M_1 , and $\vec{P}_v(\vec{r}, \omega)$ and $\vec{M}_v(\vec{r}, \omega)$ are the virtual polarization and the virtual magnetization fields respectively. The virtual fields in the transition region V_{\pm} change continuously from zero at $S+$ to their bulk quantities at $S-$.

Using the virtually induced fields, we obtain the following wave equations for the entire region, $\tilde{V} + V_{\pm} + V$,

$$\nabla \times [\nabla \times \vec{E}(\vec{r}, \omega)] - k_0^2 \epsilon_0 \mu_0 \vec{E}(\vec{r}, \omega) = ik_0 \mu_0 \left\{ \frac{4\pi}{c} \vec{J}(\vec{r}, \omega) + \frac{4\pi}{c} \vec{J}(\vec{r}, \omega) - 4\pi ik_0 \vec{P}_v(\vec{r}, \omega) + 4\pi \nabla \times \vec{M}_v(\vec{r}, \omega) \right\} \quad (2.17a)$$

$$\nabla \times [\nabla \times \vec{H}(\vec{r}, \omega)] - k_0^2 \epsilon_0 \mu_0 \vec{H}(\vec{r}, \omega) = \frac{4\pi}{c} \nabla \times \vec{J}(\vec{r}, \omega) + \frac{4\pi}{c} \nabla \times \vec{J}(\vec{r}, \omega) - ik_0 4\pi \nabla \times \vec{P}_v(\vec{r}, \omega) + k_0 \vec{M}_v(\vec{r}, \omega) \quad (2.17b)$$

Using the transverse dyadic Green's function $\vec{G}_T(\vec{r}; \vec{r}'; k)$ in M_0 and exactly the same method as in section 2, we obtain the formal solutions of the above wave equations, (2.17a) and (2.17b), in their partially non-local forms in space. The results are, noting that the wave number in the medium M_0 is $k = k_0 \sqrt{\epsilon_0(\omega)}$,

$$\begin{aligned} & \vec{E}(\vec{r}, \omega) + \frac{1}{4\pi k^2} \nabla \times \nabla \times \left\{ \vec{E}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}; \vec{r}'; k) - g(\vec{r}; \vec{r}'; k) \frac{\partial}{\partial n} \vec{E}(\vec{r}, \omega) \right\} \\ & - \frac{1}{k^2} \nabla \times \nabla \times \left\{ g(\vec{r}; \vec{r}'; k) \hat{n} \cdot \nabla \cdot \vec{P}(\vec{r}, \omega) \right\} + \frac{\mu_0}{k^2} \nabla \times \nabla \times \left\{ \vec{P}_v(\vec{r}, \omega) \right\} \\ & - \frac{1}{k^2} \nabla \times \nabla \times \left\{ \frac{ik_0}{c} g(\vec{r}; \vec{r}'; k) \hat{n} \cdot \nabla \cdot \vec{J}(\vec{r}, \omega) \right\} + \frac{\mu_0}{k^2} \nabla \times \nabla \times \left\{ \vec{J}(\vec{r}, \omega) \right\} \\ & - \frac{\mu_0}{k^2} \nabla \times \nabla \times \left\{ ik_0 g(\vec{r}; \vec{r}'; k) \hat{n} \times \vec{M}_v(\vec{r}, \omega) \right\} - \frac{\mu_0}{k^2} \nabla \times \nabla \times \left\{ \vec{M}_v(\vec{r}, \omega) \right\} = \begin{cases} \vec{E}(\vec{r}, \omega), & \text{if } \vec{r}' \in \tilde{V}. \\ 0, & \text{if } \vec{r}' \in V. \end{cases} \quad (2.18a) \end{aligned}$$

and

$$\begin{aligned}
 \vec{H}^{(i)}(\vec{r}', \omega) + \frac{1}{4\pi k^2} \nabla \times \nabla \times \int_{S_-} dS \{ \vec{H}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k) - g(\vec{r}|\vec{r}'; k) \frac{\partial}{\partial n} \vec{H}(\vec{r}, \omega) \} \\
 - \frac{1}{k^2} \nabla \times \nabla \times \int_{S_-} dS \{ g(\vec{r}|\vec{r}'; k) \hat{n} \cdot \vec{M}(\vec{r}, \omega) \} + \frac{1}{k^2} \nabla \times \nabla \times \int_{V_\pm} dV \{ k_0^2 g(\vec{r}|\vec{r}'; k) \vec{M}(\vec{r}, \omega) \} \\
 - \frac{1}{k^2} \nabla \times \nabla \times \int_{S_-} dS \{ g(\vec{r}|\vec{r}'; k) \hat{n} \times \vec{J}(\vec{r}, \omega) \} - \frac{1}{k^2} \nabla \times \nabla \times \int_{V_\pm} dV \{ \frac{1}{c} [\nabla g(\vec{r}|\vec{r}'; k)] \times \vec{J}(\vec{r}, \omega) \} \\
 + \frac{1}{k^2} \nabla \times \nabla \times \int_{S_-} dS \{ i k_0 g(\vec{r}|\vec{r}'; k) \hat{n} \times \vec{P}_V(\vec{r}, \omega) \} + \frac{1}{k^2} \nabla \times \nabla \times \int_{V_\pm} dV \{ i k_0 [\nabla g(\vec{r}|\vec{r}'; k)] \times \vec{P}_V(\vec{r}, \omega) \} = \begin{cases} \vec{H}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}. \\ 0, & \text{if } \vec{r}' \in V. \end{cases} \quad (2.18b)
 \end{aligned}$$

The incident waves with the wave number k in the medium M_0 are,

$$\vec{E}^{(i)}(\vec{r}, \omega) = i \frac{k_0 \mu_0(\omega)}{c} \int_{\tilde{V}} dV \{ \mathcal{G}(\vec{r}|\vec{r}'; k) \cdot \vec{J}(\vec{r}, \omega) \}$$

and

$$\vec{H}^{(i)}(\vec{r}, \omega) = \int_{\tilde{V}} dV \{ \mathcal{G}(\vec{r}|\vec{r}'; k) \cdot (\nabla \times \vec{J}(\vec{r}, \omega)) \}.$$

When the observation point \vec{r}' is in V , the formal solutions, Eqs. (2.18a) and (2.18b), express the new form of the extinction theorem. These solutions will reduce to Eqs. (2.14) and (2.15), if the medium M_0 is replaced by vacuum ($\mu_0(\omega) = \epsilon_0(\omega) = 1$).

4. A SIMPLE APPLICATION OF THE EXTINCTION THEOREM AND THE LORENTZ-LORENZ FORMULA.

In this section, we first look for a formal solution of the macroscopic Maxwell's wave equations, which contain a "local effective field". We then use the extinction theorem to express the local effective field in terms of mean fields. We also use this expression to obtain the Lorentz-Lorenz formula.

Let us assume that the medium M_0 is a vacuum where the source of incident radiation is located, and that the

radiation enters into the material M_1 through the transition region V_{\pm} . The medium M_1 is assumed to be a linear, isotropic, homogeneous, absorbing and non-magnetic dielectric. When an electromagnetic wave is incident on the dielectric, it induces microscopic charges, currents, etc., inside the dielectric.

By the assumption, $\vec{J}(\vec{r}, \omega) = \vec{M}(\vec{r}, \omega) = 0$ in M_1 , the macroscopic Maxwell wave equation in the entire region $\tilde{V} + V_{\pm} + V$, can be written as

$$\nabla \times (\nabla \times \vec{E}(\vec{r}, \omega)) - \frac{\omega^2}{c^2} \vec{E}(\vec{r}, \omega) = \frac{4\pi}{c^2} \omega^2 \vec{P}(\vec{r}, \omega) + 4\pi i \frac{\omega}{c^2} \vec{J}(\vec{r}, \omega), \quad (2.19)$$

Using Eq. (2.4a), we again look for a formal solution of Eq. (2.19). In following the method used to derive Eq. (2.6), we now carry the integration over the entire region, $\tilde{V} + V_{\pm} + V$, to obtain

$$\begin{aligned} \vec{E}(\vec{r}', \omega) = & \underbrace{\frac{ik_0}{c} \int_{\tilde{V}} d^3r \mathcal{G}(\vec{r}|\vec{r}'; k_0) \cdot \vec{J}(\vec{r}, \omega)}_{\text{I}} + \underbrace{k_0^2 \int_{V_{\pm}+V} d^3r \mathcal{G}(\vec{r}|\vec{r}'; k_0) \cdot \vec{P}(\vec{r}, \omega)}_{\text{II}} \\ & + \underbrace{\frac{1}{4\pi} \int_{\tilde{V}+V_{\pm}+V} d^3r \{ [\nabla \times (\nabla \times \mathcal{G})] \cdot \vec{E}(\vec{r}, \omega) - \mathcal{G} \cdot [\nabla \times (\nabla \times \vec{E})] \}}_{\text{III}}. \end{aligned} \quad (2.20)$$

Term I represents the incident electric field $\vec{E}^{(i)}(\vec{r}', \omega)$. Term II represents the total scattered field at \vec{r}' due to all the induced dipoles in the region $V + V_{\pm}$. Since term III can be expressed in the form of a surface integral by Gauss' theorem, it vanishes upon integration over all space.

Therefore, we find the mean electric field for $\vec{r}' \in V$ to be

$$\vec{E}(\vec{r}', \omega) = \vec{E}^{(i)}(\vec{r}', \omega) + k_0^2 \int_{V_{\pm}+V} d^3r \{ \mathcal{G}(\vec{r}|\vec{r}'; k_0) \cdot \vec{P}(\vec{r}, \omega) \}. \quad (2.21)$$

The local effective field, polarizing a molecule at $\vec{r}'_<$, can be obtained by removing the polarized molecule at $\vec{r}'_<$ from the above volume integral. The effective field is then found to be

$$\vec{E}(\vec{r}'_<, \omega) = \underbrace{\vec{E}^{(i)}(\vec{r}'_<, \omega)}_{\text{I}} + k_o^2 \underbrace{\int_{V_{\pm}+V} d^3r \{g(\vec{r}|\vec{r}'_<; k_o) \cdot \vec{P}(\vec{r}, \omega)\}}_{\text{II}}, \quad (2.22)$$

where the prime " / " on the integral represents the exclusion of a small volume in the neighborhood of $\vec{r}'_<$. Term II in Eq. (2.22) thus expresses the total scattered field at $\vec{r}'_<$ due to all the induced dipoles in M_1 except for the dipole at $\vec{r}'_<$.

We now use the extinction theorem to simplify term II in Eq. (2.22). Using Eq. (2.5), we first rewrite it as

$$\text{II} = \int_{V_{\pm}+V} d^3r \{ \nabla \times [\nabla \times g(\vec{r}|\vec{r}'_<; k_o) \vec{P}(\vec{r}, \omega)] \}. \quad (2.23)$$

We then use the following well-known mathematical identity,

$$\int_{V_{\pm}+V} d^3r \{ \nabla \times [\nabla \times g(\vec{r}|\vec{r}'_<; k_o) \vec{P}(\vec{r}, \omega)] \} = \nabla \times \nabla \times \int_{V_{\pm}+V} d^3r \{ g(\vec{r}|\vec{r}'_<; k_o) \vec{P}(\vec{r}, \omega) \} - \frac{8\pi}{3} \vec{P}(\vec{r}'_<, \omega)$$

to get the second term into the form;

$$\text{II} = \nabla \times \nabla \times \int_{V_{\pm}} d^3r \{ g(\vec{r}|\vec{r}'_<; k_o) \vec{P}(\vec{r}, \omega) \} + \nabla \times \nabla \times \int_V d^3r \{ g(\vec{r}|\vec{r}'_<; k_o) \vec{P}(\vec{r}, \omega) \} - \frac{8\pi}{3} \vec{P}(\vec{r}'_<, \omega). \quad (2.24)$$

In order to express the second term on the right side of Eq. (2.24) in terms of a surface integral, we use the following set of wave equations for the bulk region V;

$$\nabla^2 \vec{P}(\vec{r}, \omega) + k_o^2 \epsilon(\omega) \vec{P}(\vec{r}, \omega) = 0, \quad (2.25a)$$

where $\epsilon(\omega)$ is the dielectric constant for the bulk material

M_1 , and

$$\nabla^2 g(\vec{r}|\vec{r}'; k_0) + k_0^2 g(\vec{r}|\vec{r}'; k_0) = -4\pi \delta(\vec{r} - \vec{r}'). \quad (2.25b)$$

From these two equations, we obtain for a point \vec{r}' in V ,

$$\int_V d^3r \{ g(\vec{r}|\vec{r}'; k_0) \vec{P}(\vec{r}, \omega) \} = \frac{4\pi}{k_0^2 (\epsilon(\omega) - 1)} \vec{P}(\vec{r}', \omega) - \frac{1}{k_0^2 (\epsilon(\omega) - 1)} \int_{S_-} dS \{ g(\vec{r}|\vec{r}'; k_0) \frac{\partial}{\partial n} \vec{P}(\vec{r}, \omega) - \vec{P}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k_0) \}. \quad (2.26)$$

First inserting Eq. (2.26) into Eq. (2.24) and then assuming that $\vec{P}(\vec{r}', \omega)$ is transverse, i.e.,

$$\nabla \times (\nabla \times \vec{P}(\vec{r}', \omega)) = k_0^2 \epsilon(\omega) \vec{P}(\vec{r}', \omega)$$

with the relation,

$$\vec{P}(\vec{r}', \omega) = \frac{\epsilon(\omega) - 1}{4\pi} \vec{E}(\vec{r}', \omega). \quad (2.27)$$

Eq. (2.24) can be put in a final form,

$$\begin{aligned} \Pi = \nabla \times \nabla \times \int_{V_\pm} d^3r \{ g(\vec{r}|\vec{r}'; k_0) \vec{P}(\vec{r}, \omega) \} + \frac{1}{4\pi k_0^2} \nabla \times \nabla \times \int_{S_-} dS \{ \vec{E}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k_0) - g(\vec{r}|\vec{r}'; k_0) \frac{\partial}{\partial n} \vec{E}(\vec{r}, \omega) \} \\ + \vec{E}(\vec{r}', \omega) + \frac{4\pi}{3} \vec{P}(\vec{r}', \omega). \end{aligned} \quad (2.28)$$

Here the extinction theorem, Eq. (2.14), requires that the first two terms must be extinguished everywhere in V , by the incident wave, $\vec{E}^{(i)}(\vec{r}, \omega)$. The extinction theorem also claims that the first two terms in Eq. (2.28) must represent the waves which propagate with the vacuum phase velocity. Thus, the effective field will satisfy the well known equation:

$$\vec{E}'(\vec{r}', \omega) = \vec{E}(\vec{r}', \omega) + \frac{4\pi}{3} \vec{P}(\vec{r}', \omega) \quad (\vec{r}' \in V). \quad (2.29)$$

We now derive the Lorentz-Lorenz formula. To do this we note that it is the effective field which polarizes the molecule at \vec{r}' . Assuming a linear relationship, the induced

dipole moment is given by

$$\vec{p}(\vec{r}', \omega) = \alpha'(\omega) \vec{E}'(\vec{r}', \omega),$$

where $\alpha'(\omega)$ is the molecular polarizability. Using the mean polarizability, $\alpha(\omega)$, we obtain the macroscopic polarization field,

$$\vec{P}(\vec{r}, \omega) = N \alpha(\omega) \vec{E}'(\vec{r}', \omega), \quad (2.30)$$

where N is the number of dipoles per unit volume. In the transition region of M_1 , N may be a function of the position \vec{r} , but it is constant in the region V . Inserting Eq. (2.29) into Eq. (2.30), we obtain

$$\vec{P}(\vec{r}', \omega) = N \alpha(\omega) \left[\vec{E}(\vec{r}', \omega) + \frac{4\pi}{3} \vec{P}(\vec{r}', \omega) \right] \quad (\vec{r}' \in V). \quad (2.31)$$

Using the relation of Eq. (2.27), this leads to

$$\left[1 - \frac{\epsilon(\omega) + 2}{\epsilon(\omega) - 1} \frac{4\pi}{3} N \alpha(\omega) \right] \vec{P}(\vec{r}', \omega) = 0, \quad (2.32a)$$

so that we obtain the well known Lorentz-Lorenz formula,

$$\frac{4\pi}{3} N \alpha(\omega) = \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2}. \quad (2.32b)$$

We now note that if the absorption in the dielectric M_1 is so strong that the field quantities, \vec{E} and \vec{P} , of Eq. (2.28) virtually vanish at the surface S^- (also in the bulk region V), the formal solution of Eq. (2.22) leads to

$$0 = \vec{E}^{(i)}(\vec{r}', \omega) + \nabla \times \nabla \times \int_{V_{\pm}} d^3r \{ g(\vec{r} | \vec{r}'; k_0) \vec{P}(\vec{r}, \omega) \} \quad (\vec{r}' \in V). \quad (2.33)$$

For the case where the vacuum is replaced with the medium M_0 of dielectric constant $\epsilon_0(\omega)$, as in section 3, we can follow the analysis above. To do this, we first note that the virtual polarization is related to the mean electric

field through the following relation,

$$\vec{P}_v(\vec{r}', \omega) = \eta_v(\omega) \vec{E}(\vec{r}', \omega) \quad (2.34a)$$

with the virtual electric susceptibility, $\eta_v(\omega)$, defined as

$$\eta_v(\omega) = \frac{1}{4\pi} (\epsilon(\omega) - \epsilon_o(\omega)) \quad (2.34b)$$

and in terms of the effective electric field,

$$\vec{P}_v(\vec{r}', \omega) = N \alpha_v(\omega) \vec{E}'(\vec{r}', \omega), \quad (2.34c)$$

where $\alpha_v(\omega)$ is the virtual mean polarizability. We then obtain expressions similar to Eqs. (2.29), (2.32b) and (2.33):

$$\vec{E}'(\vec{r}', \omega) = \vec{E}(\vec{r}', \omega) + \frac{4\pi}{3} \frac{\vec{P}_v(\vec{r}', \omega)}{\epsilon_o(\omega)}, \quad (2.35a)$$

$$\frac{4\pi}{3} N \frac{\alpha_v(\omega)}{\epsilon_o(\omega)} = \frac{\epsilon(\omega) - \epsilon_o(\omega)}{\epsilon(\omega) + 2\epsilon_o(\omega)}, \quad (2.35b)$$

and

$$0 = \vec{E}(\vec{r}, \omega) + \frac{k_o^2}{k^2} \nabla \times \nabla \times \int_{V_{\pm}} d^3r' \{ g(\vec{r}|\vec{r}'; k) \vec{P}_v(\vec{r}', \omega) \} \quad (k = k_o \sqrt{\epsilon_o(\omega)}). \quad (2.35c)$$

5. SUMMARY OF THE GENERALIZED EXTINCTION THEOREM

Eqs. (2.18a) and (2.18b) formally express the generalized extinction theorem, if the observation point \vec{r}' belongs to the bulk material region of M_1 . Our results are more general than those which have appeared in the literature. The boundary used is physical, not necessarily geometrically sharp and the medium M_0 is not necessarily a vacuum. If we must consider a strong surface effect, such modifications become important.

When the entire space is filled with a medium M_0

without M_1 , the virtually induced fields, \vec{P}_v and \vec{M}_v , in Eqs. (2.18a) and (2.18b) are zero. Then the extinction theorem can be read as the mathematical statement of Huygens' principle for the propagation of the incident waves (see also J. J. Sein, 1970). On the other hand, if M_1 is present somewhere in space ($\vec{P}_v \neq 0, \vec{M}_v \neq 0$), the reflection and refraction of the incident waves occur at the boundary of M_1 . Then the extinction theorem does not reflect Huygens' principle for the propagation of the refracted wave, instead it describes the propagation of the incident waves which enter the medium M_1 without modification of phase, phase velocity, amplitude or propagation direction. The propagation of the incident wave in M_1 is virtual, since it will be extinguished everywhere in the bulk region of M_1 by the virtual scattered wave which is a part of the scattered wave due to \vec{P}_v and \vec{M}_v .

The extinction of the incident wave can be looked upon as the destructive interference between the incident wave and a part of the scattered wave in the material M_1 . Then a definite phase relation between the two waves should be required. The precise analysis of the phase relation becomes possible when we remove a locality in space from our formal solutions by expressing them in terms of plane wave expansions. This will be discussed in the next chapter.

CHAPTER III

THE EXTINCTION THEOREM IN ANOMALOUS DISPERSION WITH PLANE BOUNDARIES

1. INTRODUCTION

In this chapter, we will treat anomalous dispersion. For this purpose, let us closely examine the role of the generalized extinction theorem as a boundary condition in order to determine the unique response of a simple material to an incident radiation field. The extinction of the incident wave everywhere in the bulk region of the material medium can be looked upon as a result of the destructive interference with a part of the induced wave. To find the precise phase relation for the interference, we need an expression of such form that the phases can be defined at every observation point in the material medium. We thereby assume that every harmonic field quantity involved can be expressed as a linear superposition of plane waves (inhomogeneous plane waves if absorption is involved), propagating in all directions in space. This linear superposition of plane waves is known as a "plane wave mode expansion". Such a mode expansion has been successfully applied by E. Lalor and E. Wolf (1972), in deriving Fresnel's law at the plane interface between a vacuum and a linear, homogeneous, isotropic, non-magnetic and non-absorbing dielectric filling a

semi-infinite space. However, in their analysis, anomalous dispersion was not treated.

Since the refractive index is complex in the frequency region of anomalous dispersion, the plane wave mode expansions of the field quantities in the absorbing material must be obtained. Using these expansions, we will investigate optical effects of two different surface boundaries: the physical boundary, which will be discussed further in Chapter IV, V, and VI, and the geometrically sharp boundary.

2. FULLY NON-LOCAL REPRESENTATIONS (IN SPACE AND TIME) OF FORMAL SOLUTIONS FOR MAXWELL'S WAVE EQUATIONS

Let us assume that the media M_0 and M_1 are isotropic, homogeneous, linear and non-magnetic. M_1 is absorbing and occupies the half space $z > 0$ with M_0 occupying the region $z < 0$. Therefore the surface S_+ where the transition region V_+ starts is taken to be the x-y plane, $z=0$. Since $\vec{J}(\vec{r}, \omega) = \vec{M}_v(\vec{r}, \omega) = 0$ and $\mu_0(\omega) = 1$ in the present case, Eqs.

(2.18a) and (2.18b) lead to,

$$\begin{aligned} \vec{E}^{(i)}(\vec{r}, \omega) + \frac{1}{4\pi k^2} \nabla \times \nabla \times \int_{S_-} dS \left\{ \vec{E}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k) - g(\vec{r}|\vec{r}'; k) \frac{\partial}{\partial n} \vec{E}(\vec{r}, \omega) \right\} - \frac{1}{k^2} \nabla \times \nabla \times \int_{S_-} dS \{ g(\vec{r}|\vec{r}'; k) \hat{n} \times \vec{P}_v(\vec{r}, \omega) \} \\ + \frac{1}{k^2} \nabla \times \nabla \times \int_{V_+} d^3r \{ k_0^2 g(\vec{r}|\vec{r}'; k) \vec{P}_v(\vec{r}, \omega) \} = \begin{cases} \vec{E}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}, \\ 0, & \text{if } \vec{r}' \in V, \end{cases} \quad (3.1a) \end{aligned}$$

and

$$\begin{aligned} \vec{H}^{(i)}(\vec{r}, \omega) + \frac{1}{4\pi k^2} \nabla \times \nabla \times \int_{S_-} dS \left\{ \vec{H}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k) - g(\vec{r}|\vec{r}'; k) \frac{\partial}{\partial n} \vec{H}(\vec{r}, \omega) \right\} + \frac{1}{k^2} \nabla \times \nabla \times \int_{S_-} dS \{ ik_0 g(\vec{r}|\vec{r}'; k) \hat{n} \times \vec{P}_v(\vec{r}, \omega) \} \\ + \frac{1}{k^2} \nabla \times \nabla \times \int_{V_+} d^3r \{ ik_0 [\nabla g(\vec{r}|\vec{r}'; k)] \times \vec{P}_v(\vec{r}, \omega) \} = \begin{cases} \vec{H}(\vec{r}', \omega), & \text{if } \vec{r}' \in \tilde{V}, \\ 0, & \text{if } \vec{r}' \in V. \end{cases} \quad (3.1b) \end{aligned}$$

We now consider anomalous dispersion. We can place

the surface S_- , defined in section 3 of Chapter II, at the plane $z=L$ where the radiation field virtually vanishes due to the strong resonance absorption of the dielectric. Then from Eq. (3.1a), the total electric and magnetic fields outside the medium M_1 can be written as

$$\vec{E}^{(out)}(\vec{r}', \omega) = \vec{E}^{(i)}(\vec{r}', \omega) + \frac{1}{k^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \{ k_0^2 g(\vec{r}|\vec{r}'; k) \vec{P}_v(\vec{r}, \omega) \} \quad (3.2a)$$

and

$$\vec{H}^{(out)}(\vec{r}', \omega) = \vec{H}^{(i)}(\vec{r}', \omega) + \frac{1}{k^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \{ i k_0 [\nabla g(\vec{r}|\vec{r}'; k)] \times \vec{P}_v(\vec{r}, \omega) \}. \quad (3.2b)$$

Similarly the extinction theorems become, in V ,

$$0 = \vec{E}^{(i)}(\vec{r}', \omega) + \frac{1}{k^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \{ k_0^2 g(\vec{r}|\vec{r}'; k) \vec{P}_v(\vec{r}, \omega) \} \quad (3.3a)$$

and

$$0 = \vec{H}^{(i)}(\vec{r}', \omega) + \frac{1}{k^2} \nabla' \times \nabla' \times \int_{V_{\pm}} d^3r \{ i k_0 [\nabla g(\vec{r}|\vec{r}'; k)] \times \vec{P}_v(\vec{r}, \omega) \}. \quad (3.3b)$$

We then introduce mode expansions, where each mode is a single plane wave with a specific vector amplitude (an expansion coefficient). Noting that the wave number in the medium M_0 is $k = k_0 n_0(\omega)$, the incident waves can be expressed in terms of their expansion coefficients, $\vec{E}^{(i)}(p, q, \omega)$ and $\vec{H}^{(i)}(p, q, \omega)$, as (see E. Lalor, 1968 and J. R. Shewell and E. Wolf, 1968),

$$\vec{E}^{(i)}(\vec{r}', \omega) = \iint_{\Delta} dp dq \vec{E}^{(i)}(p, q, \omega) \exp\{ik(px' + qy' + mz')\}, \quad (3.4a)$$

$$\vec{H}^{(i)}(\vec{r}', \omega) = \iint_{\Delta} dp dq \vec{H}^{(i)}(p, q, \omega) \exp\{ik(px' + qy' + mz')\}, \quad (3.4b)$$

where the variables p and q are real and the integration domain, $p^2 + q^2 \leq 1$, is denoted by Δ . In this domain, we have a real m ,

$$m = +\sqrt{1 - (p^2 + q^2)} \quad , \quad (p^2 + q^2 \leq 1),$$

so that p , q , and m represent the direction cosines of the propagation direction of the plane wave (i.e., mode). Each mode will be simply referred to by its direction cosines, (p, q, m) . We also express the total electric fields, $\vec{E}^{(out)}(\vec{r}', \omega)$, and the reflected field (the scattered field), $\vec{E}^{(r)}(\vec{r}', \omega)$, in terms of plane wave mode expansions,

$$\vec{E}^{(out)}(\vec{r}', \omega) = \iint dp dq \vec{E}^{(out)}(p, q, \omega) \exp\{ik(px' + qy' - mz')\} \quad (z' > 0) \quad (3.5a)$$

and

$$\vec{E}^{(r)}(\vec{r}', \omega) = \iint dp dq \vec{E}^{(r)}(p, q, \omega) \exp\{ik(px' + qy' - mz')\} \quad (z' < 0). \quad (3.5b)$$

Similarly, for the magnetic fields, we have

$$\vec{H}^{(out)}(\vec{r}', \omega) = \iint dp dq \vec{H}^{(out)}(p, q, \omega) \exp\{ik(px' + qy' - mz')\} \quad (z' > 0) \quad (3.6a)$$

and

$$\vec{H}^{(r)}(\vec{r}', \omega) = \iint dp dq \vec{H}^{(r)}(p, q, \omega) \exp\{ik(px' + qy' - mz')\} \quad (z' < 0). \quad (3.6b)$$

The outgoing Green's function $g(\vec{r}|\vec{r}'; k)$ can also be expressed as a bundle of plane waves propagating in all directions in space (Weyl, 1919, see also the text of J. A. Stratton, 1941),

$$g(\vec{r}|\vec{r}'; k) = \frac{ik}{2\pi} \iint_{-\infty}^{+\infty} dp dq \frac{1}{m} \exp\{ik[p(x'-x) + q(y'-y) + m(z-z')]\}. \quad (3.7)$$

We now assume that the incident wave in Eq. (3.2a) with real m induces fields in M_1 . The induced fields are in general composed of two parts: one is the wave which will be extinguished by the incident wave, whereas the other is the refracted wave. Let us first derive the mode expansions for the mean refracted electric field $\vec{E}(\vec{r}, \omega)$. For simplicity, we assume that $\vec{E}(\vec{r}, \omega)$ in the transition region, V_+ , obeys the wave equation:

$$\nabla^2 \vec{E}(\vec{r}, \omega) + k_0^2 n^2(\omega) \vec{E}(\vec{r}, \omega) = 0, \quad (z \geq 0) \quad (3.8)$$

where $n(\omega) = \sqrt{\epsilon(\omega)}$ is the complex refractive index of M_1 , $n(\omega) = n_r(\omega) + i n_i(\omega)$. We will see in Chapter V that a surface effect in anomalous dispersion can be treated by such a simplified treatment. Any component of a solution of Eq. (3.8), $\vec{E}(\vec{r}, \omega)$, can be written in the form of

$$E_\alpha(\vec{r}, \omega) = \lambda_\alpha X(x) Y(y) Z(z) \Omega(\omega), \quad (\alpha = x, y, \text{ or } z),$$

where the components satisfy

$$\frac{d^2 X}{dx^2} = -k_o^2 n_x^2 X, \quad \frac{d^2 Y}{dy^2} = -k_o^2 n_y^2 Y \quad \text{and} \quad \frac{d^2 Z}{dz^2} = -k_o^2 n_z^2 Z, \quad (n^2 = n_x^2 + n_y^2 + n_z^2)$$

and λ_α is the α component of the polarization vector $\hat{\lambda} = \frac{\vec{E}(\vec{r}, \omega)}{|\vec{E}(\vec{r}, \omega)|}$.

The boundary is the x-y plane at $z=0$ so that there is complete homogeneity in the x-y plane. The plane of constant amplitude is then parallel to the x-y plane, i.e., absorption occurs in the z direction. Thus n_x and n_y are real, whereas n_z is complex,

$$n_z = n_z^r + i n_z^i$$

Then the solution of Eq. (3.8) is given by

$$E_\alpha(\vec{r}, \omega) = E_\alpha^0 \exp\{i k_o (n_x x + n_y y + n_z^r z) - k_o n_z^i z\}, \quad (\alpha = x, y, \text{ or } z). \quad (3.9)$$

We must note in Eq. (3.9) that the planes of constant phase and constant amplitude associated with a wave in an absorbing medium do not in general coincide. In order to avoid any possible confusion arising from this distinction, we define the propagation direction of the wave by the normal to the plane of constant phase. Then the vector $\vec{n} = (n_x, n_y, n_z^r)$ defines the propagation direction. In terms of spherical

co-ordinates with the polar axis along the z direction,

$$n_x = |\vec{n}| \sin \theta' \cos \phi', n_y = |\vec{n}| \sin \theta' \sin \phi' \text{ and } n_z = |\vec{n}| \cos \theta', \quad (3.10)$$

where $|\vec{n}| = \sqrt{n_x^2 + n_y^2 + n_z^2}$.

Since Eq. (3.8) is linear, any linear superposition of the plane waves, Eq. (3.9), of different \vec{n} , i.e., of different propagation directions, provided that $n(\omega)$ is constant, also satisfies Eq. (3.8). Thus, if the mean electric field $\vec{E}(\vec{r}, \omega)$ inside the absorbing medium is propagating in the positive z direction, it can be expressed in terms of its expansion coefficient, $\vec{E}'(\theta', \phi', \omega)$, as

$$\vec{E}(\vec{r}, \omega) = \int d\Omega' \vec{E}'(\theta', \phi', \omega) \exp \{ i k_0 |\vec{n}| (\sin \theta' \cos \phi' x + \sin \theta' \sin \phi' y + \cos \theta' z) - k_0 n_z z \} \\ \text{where } d\Omega' = \sin \theta' d\theta' d\phi'. \quad (3.11)$$

Let us now change the variables of the integration, from the polar angles (θ', ϕ') to the direction cosines (p', q') defined by

$$p' = \sin \theta' \cos \phi', q' = \sin \theta' \sin \phi' \text{ and } m' = \cos \theta'. \quad (3.12)$$

Here m' can also be written as

$$m' = \sqrt{1 - (p'^2 + q'^2)}.$$

If p' , q' and m' are real, they represent the direction cosines of the propagating wave (the refracted mode). They are not arbitrary, for the refracted mode depends on the incident mode. We may also have a situation of total internal reflection in which m' is purely imaginary. In order to extend our expression above to include such a possibility, we choose p' and q' to be real and allow them to vary from positive to

negative infinity. Thus we can write Eq. (3.11) as

$$\vec{E}(\vec{r}, \omega) = \iint_{-\infty}^{+\infty} d\rho d\eta d\xi \vec{E}(\rho, \eta, \xi, \omega) \exp\{ik_0 |\vec{r}| (\rho x + \eta y + \xi z) - k_0 n_z^i z\}, \quad (z \geq 0). \quad (3.13)$$

Since the virtual polarization field is linearly related to $\vec{E}(\vec{r}, \omega)$ through Eq. (2.34a), we express the virtual field in terms of its expansion coefficient, $\vec{Q}(\rho, \eta, \xi, \omega)$ as

$$\vec{P}_v(\vec{r}, \omega) = \iint_{-\infty}^{+\infty} d\rho d\eta d\xi \vec{Q}(\rho, \eta, \xi, \omega) \exp\{ik_0 |\vec{r}| (\rho x + \eta y + \xi z) - k_0 n_z^i z\} \quad (z \geq 0). \quad (3.14)$$

The magnetic field in M_1 obeys the equation similar to Eq. (3.8) so that we can also express it in terms of the plane wave mode expansion,

$$\vec{H}(\vec{r}, \omega) = \iint_{-\infty}^{+\infty} d\rho d\eta d\xi \vec{h}(\rho, \eta, \xi, \omega) \exp\{ik_0 |\vec{r}| (\rho x + \eta y + \xi z) - k_0 n_z^i z\} \quad (z \geq 0). \quad (3.15)$$

In order to apply the extinction theorem, we first evaluate the volume integral of Eq. (3.3a) using Eqs. (3.7) and (3.14),

$$k_0^2 \int_{V_{\pm}} d^3r g(\vec{r}|\vec{r}'; k) \vec{P}_v(\vec{r}, \omega) = - \iint_{-\infty}^{+\infty} d\rho d\eta d\xi \iint_{-\infty}^{+\infty} d\rho' d\eta' d\xi' \vec{Q}(\rho', \eta', \xi', \omega) \left(\frac{n_0}{|\vec{r}|}\right)^2 \delta(\rho - \frac{n_0}{|\vec{r}|} \rho') \delta(\eta - \frac{n_0}{|\vec{r}|} \eta') \delta(\xi - \frac{n_0}{|\vec{r}|} \xi') \frac{\exp\{ik(\rho x' + \eta y' + \xi z')\}}{|\vec{r}| m' + i n_z^i - n_0 m}.$$

In this integral, the Dirac delta functions lead to Snell's law (see Eq. (3.18)). We also note that waves are propagating with the phase velocities of the medium M_0 . We can then write Eq. (3.3a) as

$$0 = \iint_{\Delta} d\rho d\eta d\xi \vec{E}^{(i)}(\rho, \eta, \xi, \omega) \exp\{ik(\rho x + \eta y + \xi z)\} + \iint_{-\infty}^{+\infty} d\rho d\eta d\xi \hat{S} \times [\hat{S} \times \vec{Q}(\frac{n_0}{|\vec{r}|} \rho, \frac{n_0}{|\vec{r}|} \eta, \frac{n_0}{|\vec{r}|} \xi, \omega)] \left(\frac{n_0}{|\vec{r}|}\right)^2 \frac{2\pi}{n_0 m} \frac{\exp\{ik(\rho x + \eta y + \xi z)\}}{|\vec{r}| m' + i n_z^i - n_0 m}$$

where the integration domain, Δ , is referred to the region $p^2 + q^2 \leq 1$ and the unit vector \hat{S} is (p, q, m) . Here, it is also required by the extinction of the incident plane waves everywhere in the region V , that the induced plane waves must

propagate with the same phase velocities as those of the incident plane waves. We thus obtain

$$0 = \iint_{\tilde{\Delta}} dp dq \exp\{ik(px' + qy' + mz')\} \left\{ \vec{E}^{(i)}(p, q, \omega) + \hat{S} \times \hat{S} \times \vec{Q}\left(\frac{n_o}{|n|} p, \frac{n_o}{|n|} q, \omega\right) \left(\frac{n_o}{|n|}\right)^2 \frac{2\pi}{n_o m} \frac{1}{|n| m' + i n_z^i - n_o m} \right\} \\ + \iint_{\tilde{\Delta}} dp dq \exp\{ik(px' + qy' + mz')\} \left\{ \hat{S} \times \hat{S} \times \vec{Q}\left(\frac{n_o}{|n|} p, \frac{n_o}{|n|} q, \omega\right) \left(\frac{n_o}{|n|}\right)^2 \frac{2\pi}{n_o m} \frac{1}{|n| m' + i n_z^i - n_o m} \right\} \quad (3.16)$$

where the integration domain, $\tilde{\Delta}$, is referred to as $p^2 + q^2 > 1$. Since the expressions of $\exp\{ik(px' + qy' + mz')\}$ for each different mode are linearly independent, the amplitudes must satisfy, for $p^2 + q^2 \leq 1$,

$$0 = \vec{E}^{(i)}(p, q, \omega) + \hat{S} \times \left[\hat{S} \times \vec{Q}\left(\frac{n_o}{|n|} p, \frac{n_o}{|n|} q, \omega\right) \left(\frac{n_o}{|n|}\right)^2 \right] \frac{2\pi}{n_o m} \frac{1}{|n| m' + i n_z^i - n_o m}, \quad (3.17a)$$

whereas, for $p^2 + q^2 > 1$,

$$\vec{Q}\left(\frac{n_o}{|n|} p, \frac{n_o}{|n|} q, \omega\right) = 0. \quad (3.17b)$$

Eq. (3.17a) expresses the spatially non-local form of the extinction theorem. Here we also recall that because of the delta functions, $\delta(p' - \frac{n_o}{|n|} p)$ and $\delta(q' - \frac{n_o}{|n|} q)$, the incident mode (p, q, m) is uniquely related to the refracted mode (p', q', m') in $\vec{Q}(p', q', \omega)$ through the following relations;

$$p' = \frac{n_o}{|n|} p, \quad q' = \frac{n_o}{|n|} q \quad \text{and} \quad m' = \sqrt{1 - \left(\frac{n_o}{|n|}\right)^2 (p^2 + q^2)}. \quad (3.18)$$

These are just Snell's law (see the next section).

We can now express Eq. (3.2a) in terms of the mode expansions. Noting that the second term of Eq. (3.2a) is the reflected field, $\vec{E}^{(r)}(\vec{r}', \omega)$, we obtain, for the incident

mode (p, q, m) satisfying $p^2 + q^2 \leq 1$,

$$\vec{e}^{(out)}(p, q, \omega) = \vec{e}^{(i)}(p, q, \omega) + \vec{e}^{(r)}(p, q, \omega), \quad (3.19a)$$

with

$$\vec{e}^{(r)}(p, q, \omega) = \hat{S}^{(r)} \times \hat{S}^{(r)} \times \vec{Q}\left(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} q, \omega\right) \frac{2\pi}{n_o m} \frac{1}{(|\vec{n}| m' + i n_z^i + n_o m)}, \quad (3.19b)$$

where the real unit vector $\hat{S}^{(r)}$ is the reflected mode $(p, q, -m)$.

In order to find the relationship between $\vec{e}(p', q', \omega)$ of the refracted field and $\vec{Q}(p', q', \omega)$ of the virtual polarization, we use the definition of the virtual polarization, Eqs. (2.34a) and (2.34b). We then obtain the relation,

$$\vec{Q}(p', q', \omega) = \frac{1}{4\pi} (n^2(\omega) - n_o^2(\omega)) \vec{e}(p', q', \omega). \quad (3.20)$$

Using this relation, we can express Eqs. (3.17a) and (3.19b) in terms of the vector amplitude of the mean electric field. They are

$$\vec{e}^{(i)}(p, q, \omega) + \frac{1}{2} \hat{S} \times \hat{S} \times \vec{e}\left(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} q, \omega\right) \left(\frac{n_o}{|\vec{n}|}\right)^2 \frac{n^2 - n_o^2}{n_o m (|\vec{n}| m' + i n_z^i - n_o m)} = 0 \quad (3.21a)$$

and

$$\vec{e}^{(r)}(p, q, \omega) = \frac{1}{2} \hat{S}^{(r)} \times \hat{S}^{(r)} \times \vec{e}\left(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} q, \omega\right) \left(\frac{n_o}{|\vec{n}|}\right)^2 \frac{n^2 - n_o^2}{n_o m (|\vec{n}| m' + i n_z^i + n_o m)}, \quad (3.21b)$$

where $p^2 + q^2 \leq 1$.

We now derive similar expressions for the magnetic field. After calculations similar to those for the electric fields, Eqs. (3.2b) and (3.3b) can be written as, for $p^2 + q^2 \leq 1$,

$$\vec{h}^{(out)}(p, q, \omega) = \vec{h}^{(i)}(p, q, \omega) + \vec{h}^{(r)}(p, q, \omega) \quad (3.22a)$$

and

$$0 = \vec{h}^{(0)}(\vec{p}, \vec{q}, \omega) + \hat{S} \times \hat{S} \times \left[\hat{S} \times \vec{Q} \left(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} q, \omega \right) \left(\frac{n_o}{|\vec{n}|} \right)^2 \right] \frac{2\pi}{m(|\vec{n}|m' + i n_z^i - n_o m)} , \quad (3.22b)$$

where

$$\vec{h}^{(r)} = \hat{S}^{(r)} \times \hat{S}^{(r)} \times \left[\hat{S}^{(r)} \times \vec{Q} \left(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} q, \omega \right) \left(\frac{n_o}{|\vec{n}|} \right)^2 \right] \frac{2\pi}{m(|\vec{n}|m' + i n_z^i + n_o m)} . \quad (3.22c)$$

For later convenience, we relate $[\hat{S} \times \vec{Q}]$ of Eq. (3.22b) and $[\hat{S}^{(r)} \times \vec{Q}]$ of Eq. (3.22c) to the magnetic vector amplitude, i.e., the expansion coefficient, \vec{h} , defined in Eq. (3.15). In order to do this, let us consider Maxwell's equation in V_+ ,

$$\nabla \times \vec{H}(\vec{r}, \omega) = -i k_o n^2(\omega) \vec{E}(\vec{r}, \omega) . \quad (3.23)$$

Here we recall that $\mu_o(\omega) = \mu(\omega) = 1$ and $n(\omega)$ is the complex refractive index, $n(\omega) = \sqrt{\epsilon(\omega)}$. Then, substituting the plane wave mode expansions of Eq. (3.13) and (3.15) into Eq. (3.23), we obtain,

$$i k_o (|\vec{n}| \hat{S}' + i n_z^i \hat{z}) \times \vec{h}(\vec{p}', \vec{q}', \omega) = -i k_o n^2 \vec{E}(\vec{p}', \vec{q}', \omega) , \quad (3.24)$$

where the unit vector \hat{S}' is (p', q', m') . Taking the vector product of the unit vector $\hat{S} = (p, q, m)$ and Eq. (3.24), then using Eq. (3.20), we obtain

$$- [|\vec{n}| (\hat{S} \cdot \hat{S}') + i n_z^i (\hat{S} \cdot \hat{z})] \vec{h} + |\vec{n}| (\hat{S} \cdot \vec{h}) \hat{S}' + i n_z^i (\hat{S} \cdot \vec{h}) \hat{z} = -n^2 \frac{4\pi}{n^2 - n_o^2} \hat{S} \times \vec{Q}(\vec{p}', \vec{q}', \omega) . \quad (3.25)$$

We can obtain similar results for the expression, $[\hat{S}^{(r)} \times \vec{Q}]$, if we replace \hat{S}' with $\hat{S}^{(r)}$.

In the next section, we use the precise phase relations among the various vector amplitudes obtained here as basic equations to derive Fresnel's formula. We also note that these basic equations are fully non-local representations

(in space) of our formal solutions for anomalous dispersion.

3. A NEW DERIVATION OF SNELL'S LAW AND FRESNEL'S FORMULA

The extinction of the incident wave everywhere in the bulk medium M_1 (in V), requires that the polarization field in V_+ must produce an induced wave (the scattered wave) in V , a part of which propagates with the same phase velocity and propagation direction as that of incident wave, but with the opposite phase (see Eqs. (3.16) and (3.17a)). We then obtain the relationship between the incident and the refracted mode to be (see Eq. (3.18)),

$$p' = \frac{n_0}{|\vec{n}|} p, \quad q' = \frac{n_0}{|\vec{n}|} q, \quad m' = \sqrt{1 - \left(\frac{n_0}{|\vec{n}|}\right)^2 (p^2 + q^2)} \quad \text{and} \quad m = \sqrt{1 - (p^2 + q^2)}, \quad (3.26)$$

where m is real because of $p^2 + q^2 \leq 1$. We note that m' is real if p and q satisfy

$$p^2 + q^2 \leq \left(\frac{|\vec{n}|}{n_0}\right)^2. \quad (3.27)$$

However, when $|\vec{n}| < n_0$, m' becomes imaginary if p and q satisfy

$$\left(\frac{|\vec{n}|}{n_0}\right)^2 < p^2 + q^2 \leq 1. \quad (3.28)$$

Here the imaginary m' is denoted by

$$m' = i m'_i = \sqrt{\left(\frac{n_0}{|\vec{n}|}\right)^2 (p^2 + q^2) - 1}.$$

This imaginary m' is related to the case of total internal reflection which will be discussed later.

In order to derive Snell's law, as usually expressed, we consider the relationship between the three real unit vectors, $\hat{S}=(p,q,m)$, $\hat{S}'=(p',q',m')$ and \hat{z} . They are expressed in terms of spherical co-ordinates (see Eq. (3.12)), respectively, as

$$\hat{S}=(p,q,m)=(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

and

$$\hat{S}'=(p',q',m')=(\sin\theta'\cos\phi', \sin\theta'\sin\phi', \cos\theta')$$

The vanishing triple scalar product, $[\hat{S} \cdot (\hat{S}' \times \hat{z})] = 0$, shows that the unit vectors, \hat{S} , \hat{S}' and \hat{z} are coplanar. We will call this plane the plane of incidence. We choose this plane such that the angles, ϕ and ϕ' , are equal to zero. Denoting by θ and θ' the incident and the refracted angle respectively, and using one of the necessary conditions for the extinction of the incident wave, i.e., Eq. (3.26), we obtain Snell's law,

$$|\vec{n}| \sin\theta' = n_0 \sin\theta$$

where $|\vec{n}|$ is regarded as the refractive index of the absorbing medium.

The refractive index $|\vec{n}|$ can be expressed in terms of the real and imaginary part of the complex refractive index, n_r and n_i , as well as the refractive index of the medium M_0 , n_0 , and the angle of incidence, θ . In order to do this, we must first note the relations,

$$n^2 = n_r^2 - n_i^2 + 2in_r n_i \quad (3.29a)$$

and

$$\begin{aligned} n^2 &= n_x^2 + n_y^2 + n_z^2 \quad (n_z = n_z^r + in_z^i) \\ &= |\vec{n}|^2 - (n_z^i)^2 + 2in_z^i n_z^r. \end{aligned} \quad (3.29b)$$

Then, by comparing Eq. (3.29a) with Eq. (3.29b), we obtain

$$n_r^2 - n_i^2 = |\vec{n}|^2 - (n_z^i)^2 \quad (3.30a)$$

and

$$n_z^i = \frac{n_r n_i}{n_z^r} = \frac{n_r n_i}{|\vec{n}| \cos \theta'} \quad (3.30b)$$

Substituting Eq. (3.30b) into Eq. (3.30a), we find

$$|\vec{n}|^4 (\cos^2 \theta' - |\vec{n}|^2 \cos^2 \theta' (n_r^2 - n_i^2) - n_r^2 n_i^2 = 0$$

in terms of the refracted angle θ' .

Using Snell's law, it can be further written in terms of the incident angle θ ,

$$|\vec{n}|^4 - |\vec{n}|^2 (n_r^2 - n_i^2 + n_o^2 \sin^2 \theta) + n_o^2 \sin^2 \theta (n_r^2 - n_i^2) - n_r^2 n_i^2 = 0.$$

Solving this, we obtain

$$|\vec{n}| = \left\{ \frac{1}{2} (n_r^2 - n_i^2 + n_o^2 \sin^2 \theta) + \frac{1}{2} [(n_r^2 - n_i^2 - n_o^2 \sin^2 \theta)^2 + 4 n_r^2 n_i^2]^{1/2} \right\}^{1/2} \quad (3.31)$$

We now consider the case where m' becomes imaginary.

In this case the propagation direction of the wave in Eq.

(3.13) is along a new vector,

$$(|\vec{n}| p', |\vec{n}| q', n_z^i), \quad (3.32)$$

where $n_z^i = n_r n_i / (|\vec{n}| m_i')$ (see Eq. (3.30b)). We therefore define the new propagation direction vector for this case, as

$$\begin{aligned} \vec{n}'' &= (n_x'', n_y'', n_z'') \\ &= |\vec{n}''| (\sin \theta'' \cos \phi'', \sin \theta'' \sin \phi'', \cos \theta'') \\ &= |\vec{n}''| (p'', q'', m''), \end{aligned} \quad (3.33)$$

where the real unit vector is $\hat{S}'' = (p'', q'', m'')$.

The relation between Eqs. (3.32) and (3.33) is

$$p'' = \frac{|\vec{n}|}{|\vec{n}''|} p', \quad q'' = \frac{|\vec{n}|}{|\vec{n}''|} q', \quad \text{and} \quad m'' = \frac{1}{|\vec{n}''|} \cdot \frac{n_r n_i}{|\vec{n}| m_i'} \quad (m'' = \sqrt{1 - (p''^2 + q''^2)}). \quad (3.34)$$

The imaginary m' , corresponds to the case of total internal reflection in the absorbing medium. However, because of the

imaginary part of the complex refractive index $n(\omega)$, we do not have the usual surface waves in total internal reflection. Instead, we have the wave propagating along the new direction of \vec{n}'' and Snell's law in terms of the refractive index $|\vec{n}''|$. Noting that the plane of incidence is specified by $\phi''=0$ and that θ'' is the refracted angle, we obtain Snell's law,

$$n_o \sin \theta = |\vec{n}''| \sin \theta''. \quad (3.35)$$

In order to express the refractive index, $|\vec{n}''|$, in terms of n_r , n_i , n_o , $|\vec{n}|$ and θ , we first note the relation,

$$m'' = \sqrt{1 - (p''^2 + \xi''^2)} = \sqrt{1 - \sin^2 \theta''}.$$

Using Snell's law, Eq. (3.35), we then find

$$m'' = \frac{1}{|\vec{n}''|} (|\vec{n}''|^2 - n_o^2 \sin^2 \theta)^\frac{1}{2}. \quad (3.36)$$

On the other hand, using Eq. (3.34), we obtain

$$\begin{aligned} m'' &= \frac{1}{|\vec{n}''|} \frac{n_r n_i}{m'} \\ &= \frac{1}{|\vec{n}''|} n_r n_i (n_o^2 \sin^2 \theta - |\vec{n}|^2)^\frac{1}{2}. \end{aligned} \quad (3.37)$$

Equating Eqs. (3.36) and (3.37), we have

$$|\vec{n}''|^2 - n_o^2 \sin^2 \theta = n_r^2 n_i^2 / (n_o^2 \sin^2 \theta - |\vec{n}|^2),$$

so that $|\vec{n}''|$ can be written as

$$|\vec{n}''| = \{n_r^2 n_i^2 / (n_o^2 \sin^2 \theta - |\vec{n}|^2) + n_o^2 \sin^2 \theta\}^\frac{1}{2}. \quad (3.38)$$

To derive Fresnel's formula, we define the polarization states for the plane waves; one is perpendicular and the other is parallel to the plane of incidence. We denote them by unit vectors $\hat{\lambda}_\perp$ and $\hat{\lambda}_\parallel$, respectively. Since the electromagnetic waves are inhomogeneous in the absorbing medium M_1 , the

relationships between the $\hat{\lambda}_{\parallel}$ states and the propagation directions (for instance, $\hat{S}' = (p', q', m')$) are complicated. But the behavior of the perpendicular components of fields at refraction is simple. Namely, if the polarization state for the incident electric vector amplitudes, $\vec{e}^{(i)}(p, q, \omega)$, is in the $\hat{\lambda}_{\perp}$ state, the reflected and refracted vector amplitudes, $\vec{e}^{(r)}(p, q, \omega)$ and $\vec{e}(p', q', \omega)$, are also in the $\hat{\lambda}_{\perp}$ state. Thus, for the incident plane wave in the $\hat{\lambda}_{\parallel}$ state, we deal with its magnetic field which is in the $\hat{\lambda}_{\perp}$ polarization state.

Assuming that $\vec{e}^{(i)}(p, q, \omega)$ is in the $\hat{\lambda}_{\perp}$, taking first the dot product of $\hat{\lambda}_{\perp}$ with Eqs. (3.21a) and (3.21b) from their left sides and then using the following relations,

$$\hat{\lambda}_{\perp} \cdot \hat{S} = \hat{\lambda}_{\perp} \cdot \hat{S}^{(r)} = 0$$

and

$$\hat{S} \cdot \vec{e}(p', q', \omega) = \hat{S}^{(r)} \cdot \vec{e}^{(r)}(p, q, \omega) = 0,$$

we obtain

$$\hat{\lambda}_{\perp} \cdot \vec{e}^{(i)}(p, q, \omega) = \frac{1}{2} \frac{n^2 - n_o^2}{n_o m (|\vec{m}| m' + i n_z^i - n_o m)} \hat{\lambda}_{\perp} \cdot \vec{e}(\frac{n_o p}{|\vec{m}|}, \frac{n_o q}{|\vec{m}|}, \omega) (\frac{n_o}{|\vec{m}|})^2 \quad (3.39a)$$

and

$$\hat{\lambda}_{\perp} \cdot \vec{e}^{(r)}(p, q, \omega) = -\frac{1}{2} \frac{n^2 - n_o^2}{n_o m (|\vec{m}| m' + i n_z^i - n_o m)} \hat{\lambda}_{\perp} \cdot \vec{e}(\frac{n_o p}{|\vec{m}|}, \frac{n_o q}{|\vec{m}|}, \omega) (\frac{n_o}{|\vec{m}|})^2, \quad (3.39b)$$

where $p^2 + q^2 \leq 1$ and $n_z^i = \frac{n_r n_i}{|\vec{m}| m'}$. We note that m' is either real or pure imaginary depending upon the incident direction cosines (see Eqs. (3.27) and (3.28)).

Thus, for the case where the polarization state of $\vec{e}^{(i)}(p, q, \omega)$ is perpendicular to the plane of incidence, we obtain the transmission and the reflection amplitude, $b_{T_{\perp}}$

and $\delta_{R_{\perp}}$ respectively, as

$$\delta_{T_{\perp}} = \frac{\hat{\lambda}_{\perp} \cdot \vec{E}^{(i)}(p, q, \omega)}{\hat{\lambda}_{\perp} \cdot \vec{E}^{(r)}(p, q, \omega)} = \frac{2n_0 m (|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} - n_0 m)}{n^2 - n_0^2}$$

and

$$\delta_{R_{\perp}} = \frac{\hat{\lambda}_{\perp} \cdot \vec{E}^{(o)}(p, q, \omega)}{\hat{\lambda}_{\perp} \cdot \vec{E}^{(i)}(p, q, \omega)} = \frac{|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} - n_0 m}{|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} + n_0 m},$$

where the subscript, \perp , refers to the perpendicular polarization state of $\vec{E}^{(i)}$. These can be expressed in terms of the incident angle θ , if we note the following relation,

$$(|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'})^2 = n^2 - n_0^2 \sin^2 \theta, \quad (3.40)$$

where $n(\omega)$ is the complex refractive index for the medium M_1 and n_0 is the refractive index for M_0 . We then have

$$\delta_{T_{\perp}} = \frac{2n_0 \cos \theta}{n_0 \cos \theta + \sqrt{n^2 - n_0^2 \sin^2 \theta}} \quad (3.41a)$$

and

$$\delta_{R_{\perp}} = \frac{n_0 \cos \theta - \sqrt{n^2 - n_0^2 \sin^2 \theta}}{n_0 \cos \theta + \sqrt{n^2 - n_0^2 \sin^2 \theta}}. \quad (3.41b)$$

We now assume that the polarization of $\vec{E}^{(i)}(p, q, \omega)$ is parallel to the plane of incidence. The polarization state of $\vec{h}^{(i)}(p, q, \omega)$ is now perpendicular. Taking the dot product of $\hat{\lambda}_{\perp}$ with Eqs. (3.22b) and (3.22c) from their lefthand sides and then using the relation of Eq. (3.25), we obtain

$$\hat{\lambda}_{\perp} \cdot \vec{h}^{(i)}(p, q, \omega) = \frac{1}{2} \frac{n^2 - n_0^2}{n^2} \frac{|\vec{n}|(\hat{s} \cdot \hat{s}') + i \frac{n_r n_i}{|\vec{n}| m'} (\hat{s} \cdot \hat{z})}{m(|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} - n_0 m)} \hat{\lambda}_{\perp} \cdot \vec{h}(\frac{n_0}{|\vec{n}|} p, \frac{n_0}{|\vec{n}|} q, \omega) (\frac{n_0}{|\vec{n}|})^2$$

$$(3.42a)$$

and

$$\hat{\lambda}_{\perp} \cdot \vec{h}^{(r)}(p, q, \omega) = \frac{1}{2} \frac{n^2 - n_o^2}{n^2} \frac{|\vec{n}| (\hat{S}^{(i)} \cdot \hat{S}') + i \frac{n_r n_i}{|\vec{n}| m'} (\hat{S}^{(i)} \cdot \hat{z})}{m (|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} + n_o m)} \hat{\lambda}_{\perp} \cdot \vec{h}(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} q, \omega) (\frac{n_o}{|\vec{n}|})^2. \quad (3.42b)$$

Using Eq. (3.40) and noting that $\hat{S} \cdot \hat{S}' = \cos(\theta' - \theta)$ and $\hat{S} \cdot \hat{z} = \cos \theta$, the transmission and reflection amplitudes, $\Pi_{T//}$ and $\Pi_{R//}$, are

$$\begin{aligned} \Pi_{T//} &= \frac{\hat{\lambda}_{\perp} \cdot \vec{h}(p', q', \omega)}{\hat{\lambda}_{\perp} \cdot \vec{h}^{(i)}(p, q, \omega)} = \frac{2 n^2 m}{n^2 - n_o^2} \frac{|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} - n_o m}{|\vec{n}| (\hat{S} \cdot \hat{S}') + i \frac{n_r n_i}{|\vec{n}| m'} (\hat{S} \cdot \hat{z})} \\ &= \frac{2 n^2 \cos \theta}{n^2 \cos \theta + n_o \sqrt{n^2 - n_o^2} \sin^2 \theta} \end{aligned} \quad (3.43a)$$

and

$$\begin{aligned} \Pi_{R//} &= \frac{\hat{\lambda}_{\perp} \cdot \vec{h}^{(r)}(p, q, \omega)}{\hat{\lambda}_{\perp} \cdot \vec{h}^{(i)}(p, q, \omega)} = \frac{(|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} - n_o m) [|\vec{n}| (\hat{S}^{(i)} \cdot \hat{S}') + i \frac{n_r n_i}{|\vec{n}| m'} (\hat{S}^{(i)} \cdot \hat{z})]}{(|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} + n_o m) [|\vec{n}| (\hat{S} \cdot \hat{S}') + i \frac{n_r n_i}{|\vec{n}| m'} (\hat{S} \cdot \hat{z})]} \\ &= \frac{n^2 \cos \theta - n_o \sqrt{n^2 - n_o^2} \sin^2 \theta}{n^2 \cos \theta + n_o \sqrt{n^2 - n_o^2} \sin^2 \theta}. \end{aligned} \quad (3.43b)$$

Here the subscript, //, refers to the parallel polarization state of the incident vector amplitude of $\vec{E}^{(i)}(p, q, \omega)$.

The incident electromagnetic wave, $\vec{E}^{(i)}(\vec{r}, \omega)$ with $\vec{H}^{(i)}(\vec{r}, \omega)$, which is expressed in terms of a bundle of plane waves at various incident angles, can also be related uniquely to the corresponding reflected and refracted waves. If the polarization state of $\vec{E}^{(i)}(\vec{r}, \omega)$ is perpendicular to the

plane of incidence, using the reflection amplitude of Eq.

(3.41b), the reflected field of Eq. (3.5b) can be written as

$$\vec{E}^{(r)}(\vec{r}', \omega) = \iint_{\Delta} dp dq \vec{E}^{(i)}(p, q, \omega) b_{RL}(p, q, \omega) \exp\{ik(px' + qy' - mz')\}, \quad (z' < 0), \quad (3.44)$$

where the domain, Δ , stands for the region of $p^2 + q^2 \leq 1$. Similarly, using the refraction amplitude of Eq. (3.41a), the refracted field of Eq. (3.13) can be written as

$$\vec{E}(\vec{r}, \omega) = \iint_{\Delta'} dp dq' \vec{E}^{(i)}(p, q, \omega) \left(\frac{|\vec{m}|}{n_0}\right)^2 \delta_{TL}(p, q, \omega) \exp\{ik(px + q'y + mz) - k_0 \frac{n_r n_i}{|\vec{m}| m'} z\}, \quad (z \geq 0), \quad (3.45)$$

where

$$p = \frac{|\vec{m}|}{n_0} p', \quad q = \frac{|\vec{m}|}{n_0} q'$$

and the domain, Δ' , stands for the region of $p'^2 + q'^2 = \left(\frac{n_0}{|\vec{m}|}\right)^2 (p^2 + q^2) \leq \left(\frac{n_0}{|\vec{m}|}\right)^2$. We note that the domain, Δ' , includes the case of an imaginary m' ($m' = im'_i$).

If the polarization state of $\vec{E}^{(i)}(\vec{r}', \omega)$ is parallel to the plane of incidence, the state of $\vec{H}^{(i)}(\vec{r}', \omega)$ is perpendicular. We now look for the reflected and refracted waves in terms of the magnetic fields. They are, respectively

$$\vec{H}^{(r)}(\vec{r}', \omega) = \iint_{\Delta} dp dq \vec{H}^{(i)}(p, q, \omega) \Pi_{RL}(p, q, \omega) \exp\{ik(px' + qy' - mz')\}, \quad (z' < 0), \quad (3.46a)$$

and

$$\vec{H}(\vec{r}, \omega) = \iint_{\Delta'} dp dq' \vec{H}^{(i)}(p, q, \omega) \left(\frac{|\vec{m}|}{n_0}\right)^2 \Pi_{TL}(p, q, \omega) \exp\{ik(px + q'y + mz) - k_0 \frac{n_r n_i}{|\vec{m}| m'} z\}. \quad (3.46b)$$

The reflected and refracted electromagnetic waves become unique when the functional form of the incident vector amplitudes, $\vec{E}^{(i)}(p, q, \omega)$ and $\vec{H}^{(i)}(p, q, \omega)$, are given. For instance, if the incident electromagnetic wave is a plane wave of a fixed mode, (p_0, q_0, m_0) , i.e., with an incident angle θ_0 , we can write the vector amplitudes, $\vec{E}^{(i)}(p, q, \omega)$ and $\vec{H}^{(i)}(p, q, \omega)$, using Dirac delta functions. They are

$$\vec{E}^{(i)}(p, z, \omega) = \vec{E}^{(i)}(\omega) \delta(p-p_0) \delta(z-z_0) \quad (3.47a)$$

and

$$\vec{H}^{(i)}(p, z, \omega) = \vec{H}^{(i)}(\omega) \delta(p-p_0) \delta(z-z_0) . \quad (3.47b)$$

For this case, we summarize Fresnel's formulas and the reflection coefficients in the following way. If $\vec{E}^{(i)}(\omega)$ is in the state, $\hat{\lambda}_\perp$, then we find

$$\sigma_{T_\perp}(\theta_0, \omega) = \frac{2n_0 \cos \theta_0}{n_0 \cos \theta_0 + \sqrt{n^2 - n_0^2 \sin^2 \theta_0}} , \quad (3.48a)$$

$$\sigma_{R_\perp}(\theta_0, \omega) = \frac{n_0 \cos \theta_0 - \sqrt{n^2 - n_0^2 \sin^2 \theta_0}}{n_0 \cos \theta_0 + \sqrt{n^2 - n_0^2 \sin^2 \theta_0}} \quad (3.48b)$$

and

$$R_\perp = |\sigma_{R_\perp}|^2 = \left| \frac{n_0 \cos \theta_0 - \sqrt{n^2 - n_0^2 \sin^2 \theta_0}}{n_0 \cos \theta_0 + \sqrt{n^2 - n_0^2 \sin^2 \theta_0}} \right|^2 . \quad (3.48c)$$

On the other hand, if $\vec{E}^{(i)}(\omega)$ is in the state, $\hat{\lambda}_\parallel$, then

$$\pi_{T_\parallel}(\theta_0, \omega) = \frac{2n^2 \cos \theta_0}{n^2 \cos \theta_0 + n_0 \sqrt{n^2 - n_0^2 \sin^2 \theta_0}} , \quad (3.49a)$$

$$\pi_{R_\parallel}(\theta_0, \omega) = \frac{n^2 \cos \theta_0 - n_0 \sqrt{n^2 - n_0^2 \sin^2 \theta_0}}{n^2 \cos \theta_0 + n_0 \sqrt{n^2 - n_0^2 \sin^2 \theta_0}} \quad (3.49b)$$

and

$$R_\parallel = |\pi_{R_\parallel}(\theta_0, \omega)|^2 = \left| \frac{n^2 \cos \theta_0 - n_0 \sqrt{n^2 - n_0^2 \sin^2 \theta_0}}{n^2 \cos \theta_0 + n_0 \sqrt{n^2 - n_0^2 \sin^2 \theta_0}} \right|^2 . \quad (3.49c)$$

In the expressions above, the complex refractive index, $n(\omega)$, is for the transition region just inside the surface of the absorbing material. The complex refractive index may be different from that of the bulk material depending upon the degree of the surface effect. The detailed analysis of this surface effect on the refractive index will be discussed

in the following two chapters. We note that we still have the same Fresnel formulas, even when the angle of incidence satisfies the condition for total internal reflection,

$$1 \geq \sin^2 \theta_0 > \left(\frac{n_1}{n_0}\right)^2, \quad (n_1 < n_0).$$

4. A DERIVATION OF FRESNEL'S FORMULA FOR THE CASE OF A SHARP BOUNDARY

In this section, we consider the limiting case of a vanishingly small thickness of the transition region, V_+ . We redenote by S the plane boundary, S_- , (x - y plane) in that limit and place it at $z=0$. Then our formal solutions at $\vec{r}'=(x', y', z')$, Eqs. (3.1a) and (3.1b), can be written as

$$\vec{E}^{(i)}(\vec{r}', \omega) + \frac{1}{4\pi k^2} \nabla' \times \nabla' \times \int_S dS \left\{ \vec{E}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k) - g(\vec{r}|\vec{r}'; k) \frac{\partial}{\partial n} \vec{E}(\vec{r}, \omega) \right\} = \begin{cases} \vec{E}(\vec{r}', \omega), & z' < 0, \\ 0, & z' > 0, \end{cases} \quad (3.50a)$$

and

$$\vec{H}^{(i)}(\vec{r}', \omega) + \frac{1}{4\pi k^2} \nabla' \times \nabla' \times \int_S dS \left\{ \vec{H}(\vec{r}, \omega) \frac{\partial}{\partial n} g(\vec{r}|\vec{r}'; k) - g(\vec{r}|\vec{r}'; k) \frac{\partial}{\partial n} \vec{H}(\vec{r}, \omega) \right\} = \begin{cases} \vec{H}(\vec{r}', \omega), & z' < 0, \\ 0, & z' > 0. \end{cases} \quad (3.50b)$$

Using plane wave mode expansions and denoting (p, q, m) and $(p, q, -m)$ by \hat{S} and $\hat{S}^{(r)}$ respectively, we can express Eq. (3.50a) in the fully non-local form in space;

$$\vec{E}^{(i)}(p, q, \omega) + \frac{1}{2} \left[1 + \frac{1}{n_0 m} (|\vec{r}| |\vec{m}'| + i \frac{n_r n_i}{|\vec{r}| |\vec{m}|}) \right] \hat{S} \times [\hat{S} \times \vec{E}(\frac{n_0}{|\vec{r}|} p, \frac{n_0}{|\vec{r}|} q, \omega) (\frac{n_0}{|\vec{r}|})^2] = 0, \quad (3.51a)$$

$$\vec{E}^{(out)}(p, q, \omega) = \vec{E}^{(i)}(p, q, \omega) + \vec{E}^{(r)}(p, q, \omega),$$

where

$$\vec{E}^{(r)}(p, \hat{z}, \omega) = -\frac{1}{2} \left[1 - \frac{1}{n_o m} (\vec{n} | m' + i \frac{n_r n_i}{|\vec{n}| m'}) \right] \hat{S}^{(r)} \times \left[\hat{S}^{(r)} \times \vec{E}(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} \hat{z}, \omega) (\frac{n_o}{|\vec{n}|})^2 \right] \quad (3.51b)$$

Here we note that Eqs. (3.50a) and (3.50b) can be reduced to Eqs. (3.21a) and (3.21b) respectively if we use the following identity obtained from Eq. (3.40),

$$(n^2 - n_o^2) / (\vec{n} | m' + i \frac{n_r n_i}{|\vec{n}| m'} \mp n_o m) = \vec{n} | m' + i \frac{n_r n_i}{|\vec{n}| m'} \pm n_o m. \quad (3.52)$$

Similarly, noting that $\hat{n} = -\hat{z}$, Eq. (3.50b) can be written as

$$\begin{aligned} \vec{h}^{(r)}(p, \hat{z}, \omega) + \frac{1}{2} \left[1 + \frac{1}{n_o m} (\vec{n} | m' + i \frac{n_r n_i}{|\vec{n}| m'}) \right] \hat{S} \times \left[\hat{S} \times \vec{h}(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} \hat{z}, \omega) (\frac{n_o}{|\vec{n}|})^2 \right] \\ + \frac{2\pi}{n_o m} \hat{S} \times \left\{ \hat{S} \times \left[(-\hat{z}) \times \vec{Q}(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} \hat{z}, \omega) (\frac{n_o}{|\vec{n}|})^2 \right] \right\} = 0 \end{aligned} \quad (3.53a)$$

and

$$\vec{h}^{(out)}(p, \hat{z}, \omega) = \vec{h}^{(i)}(p, \hat{z}, \omega) + \vec{h}^{(r)}(p, \hat{z}, \omega),$$

where

$$\begin{aligned} \vec{h}^{(r)}(p, \hat{z}, \omega) = -\frac{1}{2} \left[1 - \frac{1}{n_o m} (\vec{n} | m' + i \frac{n_r n_i}{|\vec{n}| m'}) \right] \hat{S}^{(r)} \times \left[\hat{S}^{(r)} \times \vec{h}(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} \hat{z}, \omega) (\frac{n_o}{|\vec{n}|})^2 \right] \\ + \frac{2\pi}{n_o m} \hat{S}^{(r)} \times \left\{ \hat{S}^{(r)} \times (-\hat{z}) \times \vec{Q}(\frac{n_o}{|\vec{n}|} p, \frac{n_o}{|\vec{n}|} \hat{z}, \omega) (\frac{n_o}{|\vec{n}|})^2 \right\}. \end{aligned} \quad (3.53b)$$

Here we can relate the terms, $-\hat{z} \times \vec{Q}$, in Eqs. (3.53a) and (3.53b) to the magnetic vector amplitude \vec{h} through a relation similar to Eq. (3.25).

In order to obtain Fresnel's formulas, we will follow the procedure of the previous section. Then, because of the relation of Eq. (3.52), we find exactly the same expressions for the reflection and the refraction amplitudes as before. When the incident wave is assumed to be a plane wave of a

fixed mode, (p_o, q_o, m_o) , i.e., for the incident angle θ_o , we obtain the same expressions for Fresnell's formulas and the reflection coefficients, (see Eqs. (3.48) and (3.49)) with the complex refractive index $n(\omega)$ for the bulk material, as before. Then, these results agree exactly with the results which are obtained by the usual treatment, for instance, in the text of L. D. Landau and E. M. Lifshitz (1960).

Finally replacing the medium M_0 with vacuum and M_1 with non-absorbing dielectric, i.e.,

$n_o=1$ and n_i (the imaginary part of $n(\omega)$)=0 , our model reduces to that studied by E. Lalor and E. Wolf (1972).

CHAPTER IV

SELECTIVE REFLECTION OF LIGHT FROM A SOLID-GAS INTERFACE

1. INTRODUCTION

We now consider a system consisting of Na vapor contained in a quartz bulb a part of which consists of a slightly prismatic glass plate (see the text of R. W. Wood, 1934). We wish to study the selective reflection of light from the vapor at the prismatic plate. This plate is made prismatic to prevent interference effects of the light reflected from the inner and the outer surface of the plate. However, in order to simplify the theoretical treatment of the system, we consider it to consist of a Na vapor which fills the semi-infinite half space ($z \geq 0$) and a glass material which occupies the other semi-infinite half space ($z < 0$).

J. P. Woerdman and M. F. H. Schuurmans (1974) used a tunable laser beam to experimentally study the selective reflection by using a system very similar to the above idealized system. An incident beam with a narrow spectral width was introduced from the glass side. They obtained the line shapes of the reflected light, when the incident beam was swept across the strong resonance lines, $D_1(^2S_{1/2} - ^2P_{1/2}, 5896\text{\AA})$ and $D_2(^2S_{1/2} - ^2P_{3/2}, 5890\text{\AA})$. Various incident angles less than the critical angle were used. In their experiments, the gas

temperature was approximately 600K so that the density of atoms in the gas was about 10^{15}cm^{-3} .

They observed the following phenomena. A hyperfine doublet splitting (1772MHz) in each of the Na D lines was observed when the light was reflected normally. The resolution decreased as the incident angle increased until the splitting was completely masked by the Doppler broadening. We note that the hyperfine splitting of 1772 MHz is approximately equal to the Doppler half-width (2000 MHz) expected under their experimental conditions (see Fig. 4.1).

We will develop a theory to explain this phenomena based on the following model. Because of the strong resonance absorption of Na atoms, the penetration depth of the incident light is expected to be less than the kinetic mean free path of the Na atoms. Thus only the gaseous atoms near the wall affect the reflection. The motion of atoms near the wall may be isotropic. However, those atoms which are moving in a direction nearly perpendicular to the wall will spend only a short time in the penetration region. On the other hand, those atoms moving nearly parallel to the wall will remain in the region for at least the mean free time before being deflected out at their next collision. Therefore the atoms contributing significantly to the reflected light are those which are moving nearly parallel to the wall. As a result, when the angle of incidence is increased, the Doppler broadening of the reflected light becomes greater.

In order to apply the extinction theorem to our model,

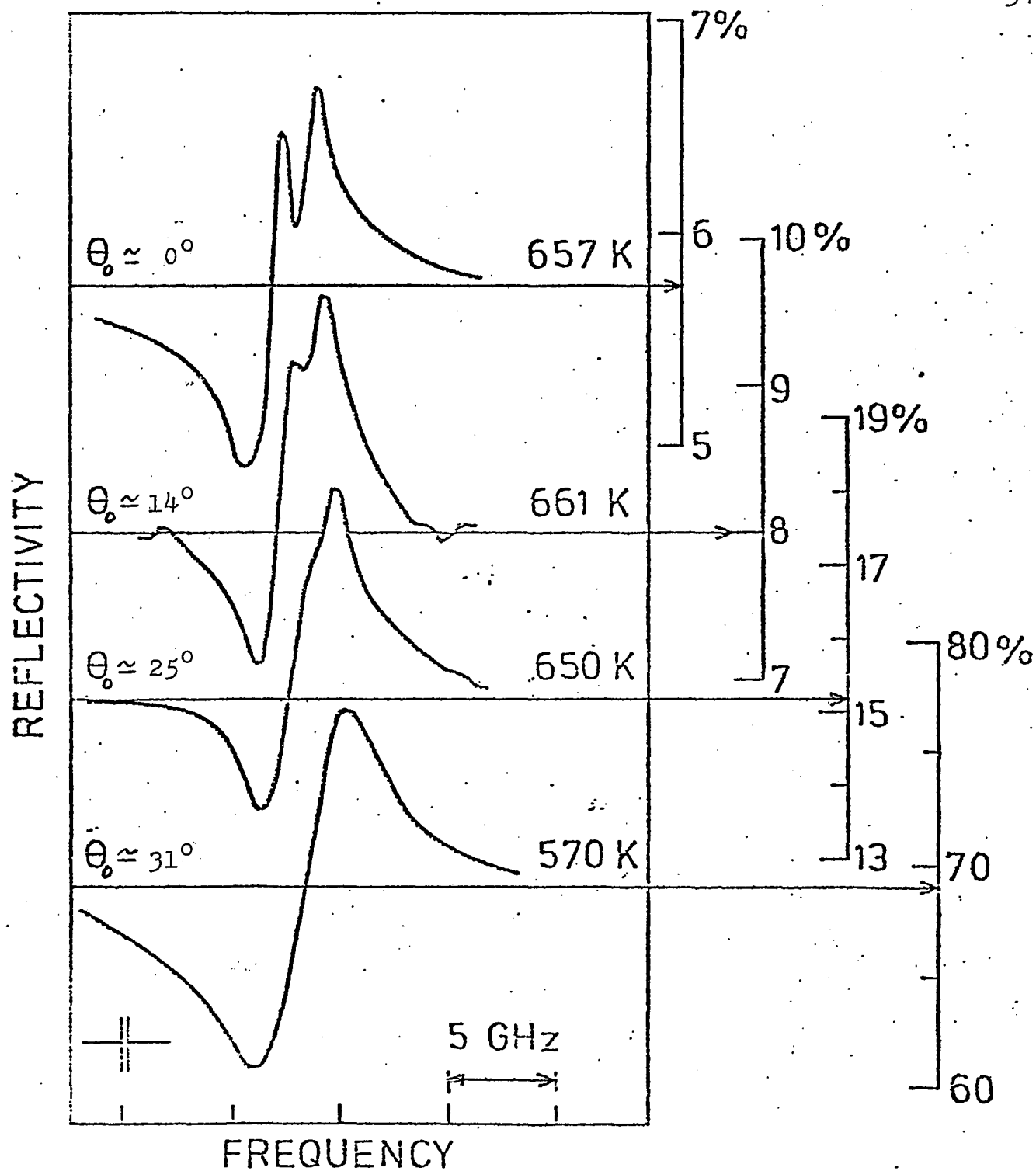


Fig. 4.1 Measured reflectivity of a gehlenite glass-sodium vapor interface near the D_2 line for various incident angles θ_0 's less than the critical angle (J. P. Woerdman and M. F. H. Schuurmans, 1974).

we separate our system into regions by placing the hypothetical surface \mathcal{S}^+ at the wall and \mathcal{S}^- at a distance deep enough that all the field quantities vanish. This is because of the strong resonance absorption within the transition region V_+ . In this region V_+ , there are atoms which have been directly reflected from the wall. Since the polarizability depends strongly on frequencies near a resonance, the induced dipole moment depends on the past history of the atoms, including those reflected from the wall. To obtain the macroscopic fields, we must average the fields over the past behavior of the atoms. This averaging will be discussed in section 2. The rest of the chapter is then organized as follows. In section 3, expressions of the line shape of the reflected light used in the numerical calculations are given. The computer program used for these calculations and the results are given in Appendix A.

2. THE MACROSCOPIC POLARIZATION FIELD WITHIN THE PENETRATION LAYER

Let us consider the system of the glass and Na vapor which was introduced in section 1. If the vapor is at room temperature without strong incident light, the majority of the Na atoms are in the ground state. In the presence of incident light some of the atoms will be raised to excited states by absorbing incident photons. These excited states have finite lifetimes which produce the "natural line width" of the spectral lines emitted by the excited atoms. Also,

since all the vapor atoms are in thermal motion, and an excited atom may pass near an unexcited atom, then there is a finite probability that the excitation energy will be transferred from the first to the second atom if their states can be radiatively connected. This results in a reduction of the duration of the coherent oscillation of light emitted by the excited atom and in a corresponding broadening of the emitted light. Line broadening of this type, as if due to "optical collisions", is known as "resonance broadening" and the line width is proportional to the density of Na atoms at low vapor pressure. In the response of the gas system to incident light, all of the above mechanism contribute to the frequency dependence of the induced polarization.

We can treat the polarization classically by replacing the excited Na atom with a damped harmonic oscillator of natural angular frequency ω_{or} . We will denote by γ_c the damping constant of the oscillator which is proportional to the natural line width, and by m_e and e the mass and charge of the electron respectively. We find the equation of motion of the x component of the displacement vector of the oscillating electron with oscillator strength f_{or} under the influence of an effective electric field, $e_{ox} e^{-i\omega t}$, to be

$$\ddot{x}(t) + \gamma_c \dot{x}(t) + \omega_{or}^2 x(t) = \frac{e}{m_e} f_{or} e_{ox} e^{-i\omega t} \quad (4-1a)$$

with a solution,

$$\chi(t) = C_1 e^{-i(\omega_{oe} - i\frac{\gamma_e}{2})t} + C_2 e^{i(\omega_{oe} + i\frac{\gamma_e}{2})t} + \frac{e}{m_e} f_{oe} \frac{e_{ox} e^{-i\omega t}}{\omega_{oe}^2 - \omega^2 - i\omega\gamma_e}, \quad (4-1b)$$

where C_1 and C_2 are constants. The polarization is proportional to $x(t)$. In order to include effects of optical collisions, we must consider what happens immediately after the collisions (see a review article by S. Y. Chen and M. Takeo, 1957). The optical effect of collisions with a wall can be treated in the same way.

We assume that the collision duration is very short and the collision is "strong", when the energy transfer is likely to occur through resonance. After a strong collision the atom has no memory concerning its orientation or other distribution properties before the collision. Suppose that a collision takes place at time t_x . It is assumed, because of the random nature of collisions, that the averages of $x(t_x)$ and $\dot{x}(t_x)$ over the collisions are zero. This is known as the "Lorentz assumption". The solution of Eq. (4.1a), i.e., Eq. (4.1b), under this assumption becomes

$$\begin{aligned} \chi(t) = & \frac{e}{m_e} f_{oe} \cdot \frac{e_{ox}}{2\omega_{oe}} \frac{e^{-i\omega t}}{\omega_{oe} - \omega - i\frac{\gamma_e}{2}} \left\{ 1 - e^{-i(\omega_{oe} - \omega - i\frac{\gamma_e}{2})(t-t_x)} \right\} \\ & + \frac{e}{m_e} f_{oe} \cdot \frac{e_{ox}}{2\omega_{oe}} \frac{e^{-i\omega t}}{\omega_{oe} + \omega + i\frac{\gamma_e}{2}} \left\{ 1 - e^{i(\omega_{oe} + \omega + i\frac{\gamma_e}{2})(t-t_x)} \right\}. \end{aligned} \quad (4.2)$$

For an effective electric field, $e_{ox}(t)$, this solution can be written as

$$\chi(t) = \frac{e}{m} f_{oe} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \frac{e_{ox}(\omega)}{2\omega_0} \left\{ \frac{1 - e^{-i(\omega_{oe} - \omega - i\frac{\gamma_e}{2})(t-t_x)}}{\omega_{oe} - \omega - i\gamma_e/2} - \frac{1 - e^{-i(\omega + \omega_{oe} + i\frac{\gamma_e}{2})(t-t_x)}}{\omega_{oe} + \omega + i\gamma_e/2} \right\}, \quad (4.3)$$

where $e_{ox}(t)$ is expressed in a Fourier integral form,

$$e_{ox}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e_{ox}(\omega) e^{-i\omega t}. \quad (4.4)$$

Denoting the electric dipole moment induced in the atom as $p_x(t) = e\chi(t)$ and the electric polarizability as $\alpha_e(t)$, then using the convolution theorem, the solution can be further rewritten as a linear response form of

$$p_x(t) = \int_{t_x}^t d\tau \alpha_e(t-\tau) e_{ox}(\tau), \quad (4.5a)$$

$$\text{where} \quad \alpha_e(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \alpha_e(\omega) \quad (4.5b)$$

$$\text{with} \quad \alpha_e(\omega) = \frac{e^2}{m_e} \frac{f_{oe}}{\omega_{oe}^2 - \omega^2 - i\omega\gamma_e}. \quad (4.5c)$$

Therefore, if the effective field $\vec{e}(\vec{r}_a, t)$ is nearly identical to the mean electric field $\vec{E}(\vec{r}_a, t)$ at \vec{r}_a as in our model case, we obtain

$$\vec{p}(\vec{r}_a, t) = \int_{t_x}^t d\tau \alpha(t-\tau) \vec{E}(\vec{r}_a, \tau), \quad (4.6)$$

where we recall that t_x is the time of the last collision. We will write the subscript "x" as "c" if the last collision is with another atom and as "w" if the last collision is with the wall.

This polarized atom moves with a thermal velocity $\vec{v}_a = (v_x^a, v_y^a, v_z^a)$, so that the position of the atom at time t can be written as

$$\vec{r}_a(t) = \vec{r}_a(t_x) + \vec{v}_a \times (t - t_x), \quad \left(\begin{array}{l} \vec{r}_a(t_x) = (x_c, y_c, z_c) \\ \vec{r}_a(t) = (x_a, y_a, z_a) \end{array} \right). \quad (4.7)$$

The time of the last collision, t_c , distributes randomly. Therefore we must average Eq. (4.6) over the distribution of t_c . If collisions are statistically independent, the probability of having had a collision with another atom in the time dt_c between $t_c - dt_c$ and t_c before the present time t is

$$\frac{1}{\tau_c} e^{-(t-t_c)/\tau_c} dt_c ,$$

where τ_c is the mean free time for the atomic optical collisions. If $v_z^a < 0$, the atom is moving toward the wall so that the last collision could have occurred at any time in the past, i.e., t_c could be $-\infty$. However, if $v_z^a > 0$, the atom is moving away from the wall so that t_c is limited by the distance to the wall z_a and its thermal velocity v_z^a ; it becomes

$$t_c \geq t - \frac{z_a}{v_z^a} , \quad (v_z^a > 0) .$$

Therefore, the probability, for an atom moving away from the wall, and having its last collision with another atom, is given by

$$\int_{t - z_a/v_z^a}^t \frac{d\tau}{\tau_c} e^{-(t-\tau)/\tau_c} , \quad (v_z^a > 0) .$$

The complement of this is the probability of finding the atom, which was reflected from the wall, at the present position without any further collision. Such a wall collision must have occurred at

$$t_w = t - z_a/v_z^a , \quad (v_z^a > 0) .$$

Therefore, the induced dipole moment averaged over

the last collision is

$$\begin{aligned} \langle \vec{p}(\vec{r}_a(t), t) \rangle_{t_x} = & \left[U(-v_z^a) \int_{-\infty}^t \frac{dt_c}{\tau_c} e^{-(t-t_c)/\tau_c} \delta_{t_x, t_c} + U(v_z^a) \left(e^{-z_a/v_z^a \tau_c} \delta_{t_x, t_c} \right. \right. \\ & \left. \left. + \int_{t - z_a/v_z^a}^t \frac{dt_c}{\tau_c} e^{-(t-t_c)/\tau_c} \delta_{t_x, t_c} \right) \right] \times \int_{t_x}^t d\tau \alpha(t-\tau) \vec{E}(\vec{r}_a(\tau), \tau), \quad (4.8) \end{aligned}$$

where δ_{t_x, t_c} is the Kronecker delta and $U(v_z)$ is the Heaviside unit step function. In Eq. (4.8), since $\alpha(t-\tau)$ is non-local, we must consider an additional change of \vec{E} at the atom due to its motion.

The field in Eq. (4.8) is the mean electric field at the moving atom. We express it in the Fourier integral form;

$$\vec{E}(\vec{r}_a(t), t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \vec{E}(\vec{r}_a(t), \omega). \quad (4.9)$$

Using Eqs. (3.13) and (4.7), $\vec{E}(\vec{r}_a(t), \omega)$ can be written as

$$\begin{aligned} \vec{E}(\vec{r}_a(t), \omega) = & \iint dp dq \left(\frac{n_0}{|\vec{m}|} \right)^2 \vec{E} \left(\frac{n_0 p}{|\vec{m}|}, \frac{n_0 q}{|\vec{m}|}, \omega \right) \exp \left\{ ik(p x_c + q y_c + \frac{|\vec{m}|}{n_0} m' z_c) - k \frac{n_r n_i}{|\vec{m}| n_0} \frac{z_c}{m'} \right\} \times \\ & \exp \left\{ ik(p v_x^a + q v_y^a + \frac{|\vec{m}|}{n_0} m' v_z^a)(t - t_x) - k \frac{n_r n_i}{|\vec{m}| n_0} \frac{v_z^a}{m'} (t - t_x) \right\}, \quad (n^2(\omega); \text{const}) \end{aligned} \quad (4.10)$$

where k is $k_0 n_0$ with the refractive index of the glass as n_0 . Thus, noting Eq. (4.5b) and assuming that the induced dipole has a single resonance at frequency $\omega_{0\ell}$, Eq. (4.8) becomes

$$\begin{aligned} \langle \vec{p}(\vec{r}_a(t), t) \rangle_{t_x} = & \left\langle \iint dp dq \int_{t_x - \infty}^t d\tau \int_{-\infty}^{+\infty} \frac{d\omega''}{2\pi} \frac{e^2}{m_e} \frac{f_{0\ell}}{\omega_{0\ell}^2 - \omega''^2 - i\omega''\gamma_0} \exp[-i\omega''(t-\tau)] \left(\frac{d\omega}{2\pi} \left(\frac{n_0}{|\vec{m}|} \right)^2 \vec{E} \left(\frac{n_0 p}{|\vec{m}|}, \frac{n_0 q}{|\vec{m}|}, \omega \right) \exp\{ikr_c\} \right. \right. \\ & \left. \left. \times \exp\{ik(A+iB)(t-t_x)\} \exp\{i\omega\tau\} \right) \right\rangle_{t_x}, \quad (4.11) \end{aligned}$$

where $r_c = px_c + qy_c + \frac{|\vec{m}|}{n_0} m' z_c + i \frac{n_r n_i}{|\vec{m}| n_0} \frac{z_c}{m'}$, $A = pv_x^a + qv_y^a + \frac{|\vec{m}|}{n_0} m' v_z^a$ and $B = \frac{n_r n_i}{|\vec{m}| n_0} \frac{v_z^a}{m'}$.

Evaluating the integrals with respect to time \hat{t} as well as frequency ω'' (with the help of contour integrals), Eq. (4.11) leads to

$$\langle \vec{P}(\vec{r}_A(t), t) \rangle_{t_x} = -\frac{e^2 f_{oe}}{m_e 2\omega_{oe}} \iint d\rho dq \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left(\frac{n_o}{|\vec{n}|} \right)^2 \vec{E} \left(\frac{n_o}{|\vec{n}|} \rho, \frac{n_o}{|\vec{n}|} q, \omega \right) \exp\{ikr_c\} \times$$

$$\left\langle \exp\{i\omega t_x\} \left\{ \frac{\exp\{i\omega'(t-t_x)\} - \exp\{-i\omega_e(t-t_x)\}}{\omega' - \omega_e} - \frac{\exp\{i\omega'(t-t_x)\} - \exp\{i\omega_e(t-t_x)\}}{\omega' + \omega_e} \right\} \right\rangle_{t_x}, \quad (4.12)$$

where $\omega' = \omega - kA - ikB$ and $\omega_e = \omega_{oe} - i\frac{\gamma_e}{2}$. For the optical region, we can neglect the off-resonance term (the second term of Eq. (4.12)). Using Eq. (4.7), Eq. (4.12) can be written as

$$\langle \vec{P}(\vec{r}_A(t), t) \rangle_{t_x} = -\frac{e^2 f_{oe}}{m_e 2\omega_{oe}} \iint d\rho dq \exp\{ik(\rho x_A + q y_A + \frac{|\vec{n}|}{n_o} m' z_A) - k \frac{n_r n_i}{|\vec{n}| m'} z_A\} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \exp\{-i\omega t\} \times$$

$$\left(\frac{n_o}{|\vec{n}|} \right)^2 \vec{E} \left(\frac{n_o}{|\vec{n}|} \rho, \frac{n_o}{|\vec{n}|} q, \omega \right) \left\langle \frac{1 - \exp\{i(\omega - \omega_{oe} - kA - i(kB - \frac{\gamma_e}{2}))(t-t_x)\}}{\omega - \omega_{oe} - kA - i(kB - \frac{\gamma_e}{2})} \right\rangle_{t_x}. \quad (4.13)$$

If the incident electric field is a plane wave of a fixed mode, (p_o, q_o, m_o) , Eq. (4.13) can now be written using Eq. (3.47a) as

$$\langle \vec{P}(\vec{r}_A(t), t) \rangle_{t_x} = -\frac{e^2 f_{oe}}{2m_e \omega_{oe}} \exp\{ik(\rho_o x_A + q_o y_A + \frac{|\vec{n}|}{n_o} m' z_A) - k \frac{n_r n_i}{|\vec{n}| n_o m'} z_A\} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \vec{E}^{(i)}(\omega) \delta_{T_L}(\rho_o, q_o, \omega) \times$$

$$\left\{ \left[U(-v_z^a) \int_{-\infty}^t \frac{dt_c}{\tau_c} e^{-\frac{(t-t_c)}{\tau_c}} \delta_{t_x, t_c} + U(v_z^a) \left(e^{-\frac{z_A}{v_z^a \tau_c}} \delta_{t_x, t_x} + \int_{t - \frac{z_A}{v_z^a}}^t \frac{dt_c}{\tau_c} e^{-\frac{(t-t_c)}{\tau_c}} \delta_{t_x, t_c} \right) \right] \times \right.$$

$$\left. \left[1 - \exp\{i(\omega - \omega_{oe} - kA - i(kB - \frac{\gamma_e}{2}))(t-t_x)\} \right] \right\}, \quad (4.14)$$

where the superscript "o" on A^o and B^o refers to the fixed incident mode (p_o, q_o, m_o) . Evaluating the time integral above, we finally obtain

$$\begin{aligned} \langle \vec{p}(\vec{r}_a(t), t) \rangle_{t_x} = & -i \frac{e^2}{2m_e} \frac{f_{oe}}{\omega_{oe}} \exp\left\{ik(p_0 x_a + \vec{e}_0 y_a + \frac{|\vec{r}_a|}{n_0} m' z_a) - k \frac{n_r n_i}{|\vec{r}_a| n_0} \frac{z_a}{m'}\right\} \left\{ \frac{d\omega}{2\pi} e^{-i\omega t} \frac{\vec{E}^{(i)}(\omega) \delta_{T_L}(p_0 \vec{e}_0, \omega)}{(k B_0 - \frac{1}{2c} - \frac{\gamma_0}{2}) + i(\omega - \omega_{oe} - k A^0)} \right. \\ & \left. \times \left[U(v_z^a) + U(v_z^a) \left(1 - \exp\left\{i[(\omega - \omega_{oe} - k A^0) - i(k B_0^0 - \frac{\gamma_0}{2} - \frac{1}{2c})] \frac{z_a}{v_z^a}\right\} \right) \right] \right\}. \quad (4.15) \end{aligned}$$

In Eq. (4.15), let us consider the expression, i.e.,

$$U(v_z^a) + U(v_z^a) \left(1 - \exp\left\{i[(\omega - \omega_{oe} - k A^0) - i(k B_0^0 - \frac{\gamma_0}{2} - \frac{1}{2c})] \frac{z_a}{v_z^a}\right\} \right).$$

The exponential term is due to the wall or surface effect.

Only those atoms near the wall with positive v_z^a will be influenced by the presence of the wall and the wall effect will vanish for small v_z^a . For large v_z^a , namely, if

$$\frac{z_a}{v_z^a} \left(\frac{\gamma_0}{2} + \frac{1}{2c} - k B^0 \right) \ll 1, \quad \left(\frac{\gamma_0}{2} \ll \frac{1}{2c} \text{ and } v_z^a > 0 \right), \quad (4.16)$$

the exponential is nearly unity under a resonance condition, $\omega \simeq \omega_{oe} + k A^0$. As a result, there are no effective induced waves from those dipoles satisfying Eq. (4.16). We thus approximate the exponential term by a step function (for a treatment without this approximation see F. Takeda and M. Takeo, 1980);

$$\exp\left\{i[(\omega - \omega_{oe} - k A^0) - i(k B_0^0 - \frac{\gamma_0}{2} - \frac{1}{2c})] \frac{z_a}{v_z^a}\right\} = U(v_z^a - v_L),$$

where the upper limit (a truncation constant) may be determined from Eq. (4.16). Noting that z_a is of the order of the penetration depth of the refracted light, v_L may be written as

$$v_L \simeq \left(k_0 \frac{n_r n_i}{|\vec{r}_a| (\cos \theta_0)} \right)^{-1} \tau_c^{-1}. \quad (4.17)$$

The induced electric dipole moment in V_{\pm} is a microscopic field quantity. In order to derive the corresponding macroscopic polarization field, we define the probability density distribution function for the Na vapor atoms, consisting of N atoms in V_{\pm} , as

$$\rho(\vec{x}_1, \dots, \vec{x}_a, \dots, \vec{x}_N)$$

where \vec{X}_a is (\vec{r}_a, \vec{v}_a) . Noting that the macroscopic polarization field is equal to the density of the dipole moments, we may write the macroscopic field for a single resonance line, ℓ , as

$$\vec{P}(\vec{r}, t) = \left(\sum_{a=1}^N \delta(\vec{r} - \vec{r}_a(t)) \langle \vec{P}(\vec{r}_a(t), t) \rangle_{t_x} \right) \rho(\vec{X}_1, \dots, \vec{X}_N) d\vec{X}_1 \dots d\vec{X}_N. \quad (4.18)$$

Since the atoms are identical, the probability distribution function is symmetric with respect to any permutation of the \vec{X} s, namely,

$$\rho(\vec{X}_1, \dots, \vec{X}_a, \vec{X}_b, \dots, \vec{X}_N) = \rho(\vec{X}_N, \dots, \vec{X}_1, \vec{X}_a, \dots, \vec{X}_b).$$

Using this symmetry condition and carrying out the integrations over the \vec{r}_a 's, we can rewrite Eq. (4.18) as

$$\begin{aligned} \vec{P}(\vec{r}, t) &= \int N \langle \vec{P}(\vec{r}, t) \rangle_{t_x} \rho(\vec{r}, \vec{v}; \vec{X}_2, \dots, \vec{X}_N) d\vec{v} d\vec{X}_2 \dots d\vec{X}_N \\ &= \int \langle \vec{P}(\vec{r}, t) \rangle_{t_x} f(\vec{r}, \vec{v}) d\vec{v}, \end{aligned} \quad (4.19)$$

where $f(\vec{r}, \vec{v})$ is the "reduced one particle distribution function" (see the text of R. Balescu, 1975),

$$f(\vec{r}, \vec{v}) = N \int \rho(\vec{r}, \vec{v}; \vec{X}_2, \dots, \vec{X}_N) d\vec{X}_2 \dots d\vec{X}_N.$$

We note that $f(\vec{r}, \vec{v})$ for the ideal gas is given by a Gaussian (Maxwellian) distribution,

$$f(\vec{r}, \vec{v}) = \frac{N}{V_{\pm}} \left(\frac{2}{\pi} \right)^3 \frac{1}{\Delta v_x} \frac{1}{\Delta v_y} \frac{1}{\Delta v_z} \exp \left\{ -\frac{4v_x^2}{\Delta v_x^2} - \frac{4v_y^2}{\Delta v_y^2} - \frac{4v_z^2}{\Delta v_z^2} \right\}. \quad (4.20)$$

Here V_{\pm} is the volume of the transition region and $\Delta v_x, \Delta v_y$ and Δv_z are all equal to

$$\Delta U = 2\sqrt{2} \frac{k_B T}{M}, \quad (4.21)$$

where M is the mass of a Na atom, k_B is the Boltzman constant and T is the gas temperature.

The Fourier component of Eq. (4.19) is given by

$$\begin{aligned} \vec{P}(\vec{r}, \omega) = & -i \frac{e^2}{2m_e \omega_{ce}} \left\{ \int d^3v f(\vec{r}, \vec{v}) \frac{[U(-v_z) + U(v_z)(1 - U(v_z - v_L))]}{(kB^0 - \frac{1}{\tau_c} - \frac{v_z}{2}) + i(\omega - \omega_{ce} - kA^0)} \right. \\ & \times \vec{E}^{(i)}(\omega) \delta_{\perp}(p_0, q_0, \omega) \exp\left\{ik(p_0 x + q_0 y + \frac{|\vec{n}|}{n_0} m' z) - k \frac{n_r n_i}{|\vec{n}| n_0} \frac{z}{m'}\right\}. \end{aligned} \quad (4.22)$$

Using the relation,

$$\vec{P}(\vec{r}, \omega) = \eta_{\ell}(p_0, q_0, \omega) \vec{E}(\vec{r}, \omega),$$

we find that the electric susceptibility η_{ℓ} , for a single resonance line ℓ , is

$$\eta_{\ell}(p_0, q_0, \omega) = -i \frac{e^2}{2m_e \omega_{ce}} \int d^3v f(\vec{r}, \vec{v}) \frac{[U(-v_z) + U(v_z)(1 - U(v_z - v_L))]}{(kB^0 - \frac{1}{\tau_c} - \frac{v_z}{2}) + i(\omega - \omega_{ce} - kA^0)}. \quad (4.23)$$

In the following, we specify the plane of incidence (the x-z plane) by setting $\phi_0 = 0$ (see section 3 in Chapter III). A^0 and B^0 can then be written as

$$kA^0 = k \sin \theta_0 v_x + k_0 |\vec{n}| \cos \theta'_0 v_z \quad (4.24a)$$

and

$$kB^0 = k_0 \frac{n_r n_i}{|\vec{n}| \cos \theta'_0} v_z, \quad (4.24b)$$

where the angles, θ_0 and θ'_0 , are the incident and the refracted angle respectively. For later convenience in calculating numerically the line shape of the reflected light, we express A^0 and B^0 in terms of θ_0 , n_r , n_i and n_0 . To do this, we take the square root of both sides of Eq. (3.40). We then find

$$|\vec{n}| m' + i \frac{n_r n_i}{|\vec{n}| m'} = A(\theta_0) + i B(\theta_0), \quad (m' = \cos \theta'_0), \quad (4.25a)$$

where $A(\theta_0)$ and $B(\theta_0)$ are

$$\left. \begin{array}{l} A(\theta_0) \\ B(\theta_0) \end{array} \right\} = \frac{1}{\sqrt{2}} \left\{ \pm (n_r^2 - n_i^2 - n_o^2 \sin^2 \theta_0) + [(n_r^2 - n_i^2 - n_o^2 \sin^2 \theta_0)^2 + 4(n_r n_i)^2]^{\frac{1}{2}} \right\}^{\frac{1}{2}}. \quad (4.25a)$$

Thus Eqs. (4.24a) and (4.24b) can be written as

$$kA^0 = k \sin \theta_0 v_z + k_0 A(\theta_0) v_z, \quad (k = n_0 k_0), \quad (4.26a)$$

and

$$kB^0 = k_0 B(\theta_0) v_z, \quad (k_0 = \frac{2\pi}{\lambda}). \quad (4.26b)$$

In order to compare our results with the experimentally observed line shapes, we express the electric susceptibility in terms of the frequency ν instead of the angular frequency ω . The real and the imaginary part of $\eta_e(\theta_0, \nu)$ can be written as

$$\eta_{e,r}(\theta_0, \nu) = -\frac{e^2 f_{oe}}{2m_e (2\pi)^2} \frac{1}{v_{oe}} \cdot \frac{N}{V_{\pm}} \int_{-\infty}^{+\infty} dv_z \frac{2}{\sqrt{\pi}} \frac{1}{\Delta v} \exp\left\{-\frac{4v_z^2}{\Delta v^2}\right\} \left\{ \int_{-\infty}^{v_L} dv_z \frac{2}{\sqrt{\pi}} \frac{1}{\Delta v} \exp\left\{-\frac{4v_z^2}{\Delta v^2}\right\} \times \right. \\ \left. \frac{\nu - \nu_e}{(\nu - \nu_e)^2 + (B(\theta_0)v_z/\lambda - 1/2\pi\tau_c - \bar{\nu}_0/2)^2} \right\}, \quad (\bar{\nu}_0 = \frac{\nu_c}{2\pi}), \quad (4.27a)$$

$$\eta_{e,i}(\theta_0, \nu) = -\frac{e^2 f_{oe}}{2m (2\pi)^2} \frac{1}{v_{oe}} \cdot \frac{N}{V_{\pm}} \int_{-\infty}^{+\infty} dv_z \frac{2}{\sqrt{\pi}} \frac{1}{\Delta v} \exp\left\{-\frac{4v_z^2}{\Delta v^2}\right\} \left\{ \int_{-\infty}^{v_L} dv_z \frac{2}{\sqrt{\pi}} \frac{1}{\Delta v} \exp\left\{-\frac{4v_z^2}{\Delta v^2}\right\} \times \right. \\ \left. \frac{B(\theta_0)v_z/\lambda - 1/2\pi\tau_c - \bar{\nu}_0/2}{(\nu - \nu_e)^2 + (B(\theta_0)v_z/\lambda - 1/2\pi\tau_c - \bar{\nu}_0/2)^2} \right\}. \quad (4.27b)$$

Here the upper limit for the v_z integration is (see Eq.

(4.17))

$$v_L \simeq \frac{\lambda}{B(\theta_0)} \frac{1}{2\pi\tau_c}. \quad (4.28)$$

Further, ν_e is the Doppler shifted resonance frequency given by

$$\nu_e = \nu_{oe} + \nu n_0 \frac{v_z}{c} \sin \theta_0 + \nu A(\theta_0) \frac{v_z}{c}.$$

This can be approximated for the present case, where the

incident frequency ν is near the natural resonance frequency $\nu_{0\ell}$, as

$$\nu_{\ell} = \nu_{0\ell} \left(1 + n_0 \frac{\nu_{\ell}}{c} \sin \theta_0 + A(\theta_0) \frac{\nu_{\ell}^2}{c^2} \right). \quad (4.29)$$

The electric susceptibility $\eta_{\ell}(\theta_0, \nu)$ is related to the complex refractive index $n(\omega)$ (for the single resonance line, ℓ) through the following relation,

$$n(\theta_0, \nu) = \sqrt{1 + 4\pi \eta_{\ell}(\theta_0, \nu)}. \quad (4.30)$$

Since $n = n^r + i n^i$ and $\eta_{\ell} = \eta_{\ell}^r + i \eta_{\ell}^i$, from Eq. (4.30) we obtain

$$n_r^2 - n_i^2 = 1 + 4\pi \eta_{\ell}^r$$

and

$$n_r n_i = 2\pi \eta_{\ell}^i.$$

We can then express $A(\theta_0)$ and $B(\theta_0)$ in terms of $\eta_{\ell}^r(\theta_0, \nu)$ and $\eta_{\ell}^i(\theta_0, \nu)$. They are

$$\left. \begin{array}{l} A(\theta_0) \\ B(\theta_0) \end{array} \right\} = \frac{1}{\sqrt{2}} \left\{ \pm (1 + 4\pi \eta_{\ell}^r - n_0^2 \sin^2 \theta_0) + \left[(1 + 4\pi \eta_{\ell}^r - n_0^2 \sin^2 \theta_0)^2 + 4 \cdot (2\pi \eta_{\ell}^i)^2 \right]^{1/2} \right\}^{1/2}. \quad (4.31)$$

Eqs. (4.27a) and (4.27b) can be solved on a digital computer by using an iteration technique (see Appendix A).

In order to see more clearly the behavior of the electric susceptibility, we change the integration variables in Eqs. (4.27a) and (4.27b) using the following transformations:

$$F_{\ell} = \nu - \nu_{0\ell}, \quad (4.32a)$$

$$X_{\ell} = \nu_{0\ell} n_0 \sin \theta_0 \frac{\nu_{\ell}}{c} \quad (4.32b)$$

and

$$Z_{\ell} = \nu_{0\ell} A(\theta_0) \frac{\nu_{\ell}^2}{c^2}. \quad (4.32c)$$

Then Eq. (4.27a) can be written as

$$\eta_{\ell}^r(\theta_0, \bar{F}_{\ell}) = \frac{-\frac{e^2}{2m_e} \frac{f_{\ell\ell}}{(2\pi)^2} \frac{1}{\nu_{\ell\ell}} \frac{N}{V_{\pm}} \int_{-\infty}^{+\infty} dX_{\ell} \frac{2}{\sqrt{\pi}} \frac{1}{\Delta X_{\ell}} \exp\left\{-\frac{4X_{\ell}^2}{\Delta X_{\ell}^2}\right\} \int_{-\infty}^L dZ_{\ell} \frac{2}{\sqrt{\pi}} \frac{1}{\Delta Z_{\ell}} \exp\left\{-\frac{4Z_{\ell}^2}{\Delta Z_{\ell}^2}\right\} \times}{(\bar{F}_{\ell} - X_{\ell} - Z_{\ell})^2 + (B(\theta_0) \cdot Z_{\ell} / A(\theta_0) - \Gamma_0/2 - 1/2\pi\tau_c)^2}, \quad (4.33)$$

where $\Delta X_{\ell} = \nu_{\ell\ell} n_0 \sin \theta_0 \frac{\Delta v}{c}$, $\Delta Z_{\ell} = \nu_{\ell\ell} A(\theta_0) \frac{\Delta v}{c}$ and $L = \frac{A(\theta_0)}{B(\theta_0)} \cdot \frac{1}{2\pi\tau_c}$. Similarly, Eq. (4.27b) becomes

$$\eta_{\ell}^i(\theta_0, \bar{F}_{\ell}) = \frac{-\frac{e^2}{2m_e} \frac{f_{\ell\ell}}{(2\pi)^2} \frac{1}{\nu_{\ell\ell}} \frac{N}{V_{\pm}} \int_{-\infty}^{+\infty} dX_{\ell} \frac{2}{\sqrt{\pi}} \frac{1}{\Delta X_{\ell}} \exp\left\{-\frac{4X_{\ell}^2}{\Delta X_{\ell}^2}\right\} \int_{-\infty}^L dZ_{\ell} \frac{2}{\sqrt{\pi}} \frac{1}{\Delta Z_{\ell}} \exp\left\{-\frac{4Z_{\ell}^2}{\Delta Z_{\ell}^2}\right\} \times}{(\bar{F}_{\ell} - X_{\ell} - Z_{\ell})^2 + (B(\theta_0) \cdot Z_{\ell} / A(\theta_0) - \Gamma_0/2 - 1/2\pi\tau_c)^2}. \quad (4.34)$$

For the case where the angle of incidence is normal ($\theta_0=0$), $X_{\ell}=0$ and there is no integration over X_{ℓ} . In this case, we have a partial absence of Doppler broadening in $\eta_{\ell}^r(\theta_0, \bar{F}_{\ell})$ and $\eta_{\ell}^i(\theta_0, \bar{F}_{\ell})$ due to the restricted selection of dipoles, i.e., the restricted upper limit on Z_{ℓ} . When the incident angle increases, the Doppler broadening due to the integral over X_{ℓ} becomes large and the contribution from the Z_{ℓ} integration becomes less important. Using the expressions for the reflectivity, Eqs. (3.48c) and (3.49c), for an incident monochromatic plane wave, the numerically calculated line shapes of the reflected light are similar to those observed experimentally (see Appendix A).

We recall that the damped harmonic oscillator in our model has two isolated resonances, $\ell=1$ and $\ell=2$. In this case the electric susceptibility $\eta(\theta_0, \nu)$ can be written as a sum,

$$\eta(\theta_0, \nu) = \sum_{\ell=1}^2 \eta_{\ell}(\theta_0, \bar{H}_{\ell}). \quad (4.35)$$

In the next section, in order to compare our results with the experimentally observed line shapes, we derive expressions for the line shape of the reflected light when incident light with a relatively narrow line width sweeps across the two resonances, $\ell=1$ and $\ell=2$.

3. THE LINE SHAPE OF THE REFLECTED LIGHT

We assume that the spectral distribution $\vec{\xi}^{(i)}(\nu)$ of the incident light is Gaussian, with a maximum amplitude ξ_0 . Then

$$\vec{\xi}^{(i)}(\nu) = \hat{\lambda} \xi_0 \exp\left\{-\frac{2(\nu - \nu^{(i)})^2}{\Delta\nu^2}\right\}, \quad (4.36)$$

where $\hat{\lambda}$ is the polarization unit vector and $\nu^{(i)}$ is the central frequency. For the case of perpendicular polarization $\hat{\lambda}_{\perp}$, (see section 3 of Chapter III), the reflectivity can be written as

$$R_{\perp}(\theta_0, \nu^{(i)}) = \frac{\left| \int_{-\infty}^{+\infty} d\nu \vec{\xi}^{(i)}(\nu) \delta_{R_{\perp}}(\theta_0, \nu) \right|^2}{\left| \int_{-\infty}^{+\infty} d\nu \vec{\xi}^{(i)}(\nu) \right|^2}. \quad (4.37)$$

Here we note from Eq. (3.45b) that the reflection amplitude, $\delta_{R_{\perp}}(\theta_0, \nu)$, is given as

$$\delta_{R\perp}(\theta_o, \nu) = \frac{n_o \cos \theta_o - \sqrt{n^2(\nu) - n_o^2 \sin^2 \theta_o}}{n_o \cos \theta_o + \sqrt{n^2(\nu) - n_o^2 \sin^2 \theta_o}} . \quad (4.38)$$

Using Eqs. (3.40) and (4.25a), we can express Eq. (4.38) in terms of the real and imaginary parts of the susceptibility, $\eta^r(\theta_o, \nu)$ and $\eta^i(\theta_o, \nu)$,

$$\delta_{R\perp}(\theta_o, \nu) = \frac{n_o \cos \theta_o - A(\theta_o, \nu) - i B(\theta_o, \nu)}{n_o \cos \theta_o + A(\theta_o, \nu) + i B(\theta_o, \nu)} , \quad (4.39a)$$

where

$$\left. \begin{matrix} A(\theta_o, \nu) \\ B(\theta_o, \nu) \end{matrix} \right\} = \frac{1}{\sqrt{2}} \left\{ \pm (1 + 4\pi \eta^r(\theta_o, \nu) - n_o^2 \sin^2 \theta_o) + [(1 + 4\pi \eta^r(\theta_o, \nu) - n_o^2 \sin^2 \theta_o)^2 + 4(2\pi \eta^i(\theta_o, \nu))^2]^{\frac{1}{2}} \right\}^{\frac{1}{2}} . \quad (4.39b)$$

For incident light with parallel polarization, we obtain the reflectivity using Eq. (3.49b),

$$R_{//}(\theta_o, \nu^{(i)}) = \frac{\left| \int_{-\infty}^{+\infty} d\nu \vec{\mathcal{H}}^{(i)}(\nu) \Pi_{R//}(\theta_o, \nu) \right|^2}{\left| \int_{-\infty}^{+\infty} d\nu \vec{\mathcal{H}}^{(i)}(\nu) \right|^2} , \quad (4.40a)$$

where the functional form of $\vec{\mathcal{H}}^{(i)}$ is also assumed to be Gaussian,

$$\vec{\mathcal{H}}^{(i)}(\nu) = \hat{\lambda}_\perp \mathcal{H}_o \exp \left\{ -\frac{2(\nu - \nu^{(i)})^2}{\Delta \nu^2} \right\} . \quad (4.40b)$$

We can also write $\Pi_{R//}(\theta_o, \nu)$ in terms of $\eta^r(\theta_o, \nu)$ and $\eta^i(\theta_o, \nu)$ as

$$\Pi_{R//} = \frac{(1 + 4\pi \eta^r(\theta_o, \nu)) \cos \theta_o + i 4\pi \eta^i(\theta_o, \nu) (\cos \theta_o - n_o A(\theta_o, \nu) - i n_o B(\theta_o, \nu))}{(1 + 4\pi \eta^r(\theta_o, \nu)) \cos \theta_o + i 4\pi \eta^i(\theta_o, \nu) (\cos \theta_o + n_o A(\theta_o, \nu) + n_o i B(\theta_o, \nu))} , \quad (4.41)$$

where $A(\theta_0, \nu)$ and $B(\theta_0, \nu)$ are given by Eq. (4.39b).

Line shapes can be numerically calculated from Eqs. (4.37) and (4.40a) with the help of a digital computer (see Appendix A).

CHAPTER V

APPLICATION OF SATURATION SPECTROSCOPY TO SURFACE INVESTIGATION

1. INTRODUCTION

When a tunable laser with a narrow line width is used as a light source, it can selectively excite atoms whose Doppler shifted resonance frequencies are within the narrow line width. The atoms with these particular thermal velocities are excited and temporarily removed from those atoms in the ground state. If the beam is sufficiently intense so that the rate of excitation is faster than that of de-excitation, these excited atoms produce a hole in the velocity distribution of the atoms in the ground state. This phenomenon has been referred to as "hole burning" (W. R. Bennett, Jr., 1962). The shape of this hole burned into the distribution can be observed by scanning a second and weak tunable laser beam over the entire Doppler profile. This spectroscopic technique is known as "saturation spectroscopy" (see a review article of W. Lange, J. Luther and A. Steudel, 1974, and T. W. Hänsch, A. L. Schawlow and G. W. Series, 1979).

Since the line shape of the reflected beam probing this "hole" is nearly free from Doppler broadening, we may be able to use this line shape for surface investigations.

The line shape of interest is that of light reflected from an environmentally related model interface where interactions between a solid wall and gaseous atoms (or molecules) create optical properties different from that of the bulk gas. Interaction potentials between a solid surface and gaseous atoms (or molecules) have yet to be studied, both experimentally and theoretically (see A. Shih and V. A. Parsegian, 1975; also review articles by T. Takaishi, 1975, and by S. Nir, 1976). For this purpose, we will investigate the experimental feasibility of the tunable dye laser as a new surface probe.

2. REFLECTION AT A SOLID-GAS INTERFACE WITHOUT INTERACTIONS

In order to investigate the validity of saturation spectroscopy for surface investigations, we use the previously studied model interface of glass and Na vapor. For this purpose, we split an intense laser beam with a narrow line width into a strong and a weak beam by a beam splitter. Then let both the strong and the weak beam be incident on the solid-gas interface at the angles, θ_{10} and θ_{20} , respectively. When θ_{10} and θ_{20} are larger than the critical angle of the system, the refracted angles are about 89 degrees. If the beams are incident from opposite sides of the normal, the two refracted beams are propagating nearly antiparallel to each other. On the other hand, if the beams are incident on the same side, the refracted beams result in nearly para-

lled propagation (see Fig. 5.1).

Suppose that there are N_o^0 atoms in the ground state "o" per unit volume and N_l^0 atoms in the excited state "l" per unit volume. In the absence of incident radiation, collisions maintain their thermal equilibrium distributions. We now assume that the intense laser beam is tuned to the resonance line " λ_o ". For the moment, we assume that the weak beam is cut off. Then the radiative transition between the atomic states due to the strong incident beam with an incident frequency ν will be induced at a rate that is not negligible compared with the collision rate τ^{-1} . Thus the gas is no longer in thermal equilibrium, and a new equilibrium between the two states "o" and "l" must be set up. Under this new equilibrium, the probability, W'_{ol} , of finding a moving atom near the wall, which was initially in the "o" state, in the excited state "l" in time τ may be written as (R. Karplus and J. Schwinger, 1948 and the text of C. H. Townes and A. L. Schawlow, 1955),

$$W'_{ol} = \frac{G^2}{2} \frac{(\frac{1}{2}\pi\tau_c)^2}{(\nu - \nu'_{ol})^2 + (\frac{1}{2}\pi\tau_c)^2(1+G^2)}, \quad (5.1)$$

where $G^2 = e^2 f_{ol} |\vec{E}|^2 \tau_c \tau / (4m_e \nu_{ol} h)$, with the electron mass m_e , Planck's constant h , and $\nu'_{ol} = \nu_{ol} (1 + n_o \frac{v_x}{c} \sin \theta_{lo})$ (see Eq. (4.29)). τ_c is the mean free time for the atomic optical collisions. \vec{E} is the mean electric field at the atom. Recall that when the incident angle is large, a significant contribution to the reflected light comes from those atoms which are moving parallel to the wall. Hereaf-

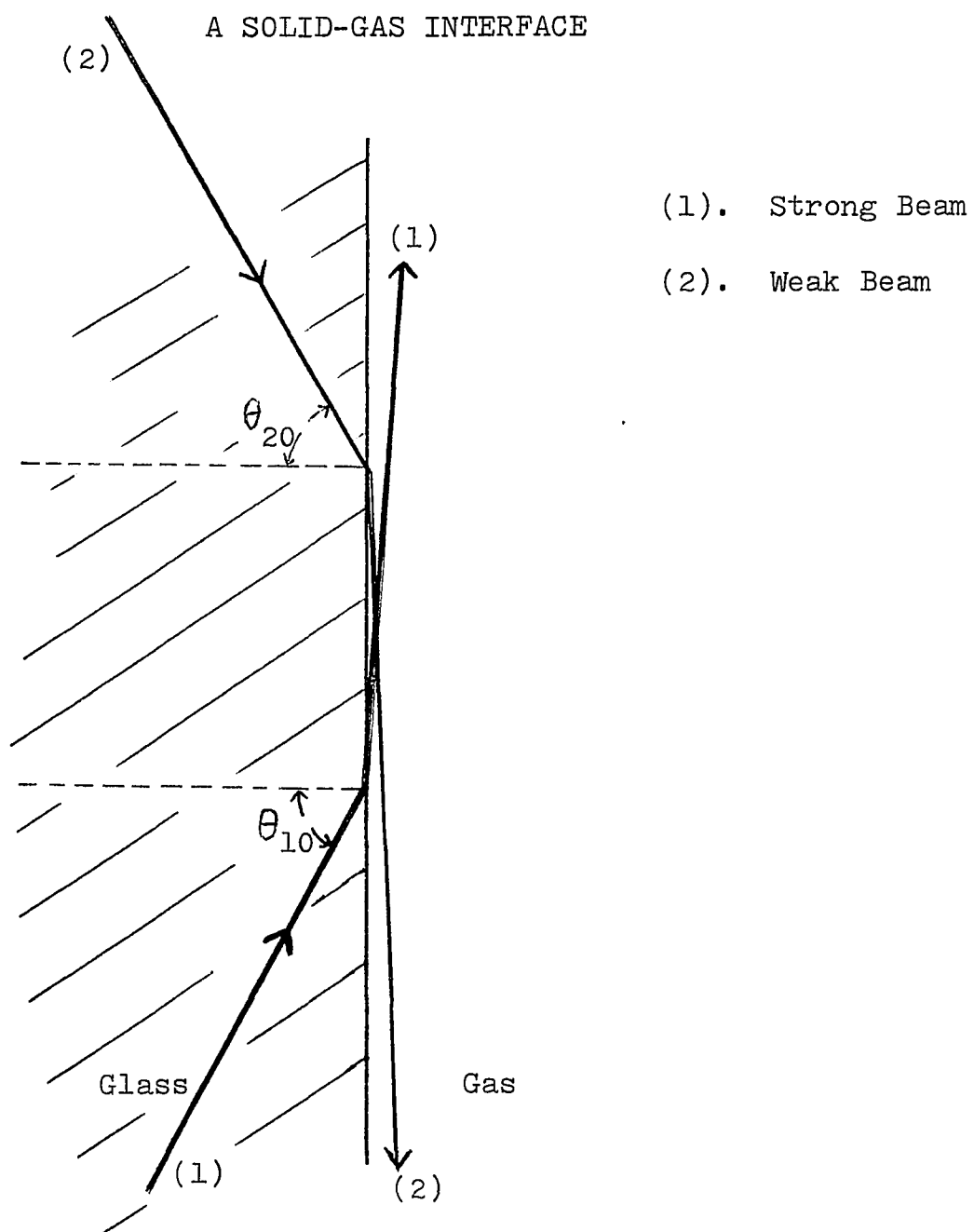


Fig. 5.1

ter we suppress the subscript "x" on the parallel velocity component v_x .

The strong beam can resonantly interact with the atoms if their Doppler shifted resonance frequencies are within the laser's narrow line width $\delta\nu^{(i)}$. Thus, denoting by v^0 the velocity of the atom whose Doppler shifted resonance frequency is equal to the incident frequency, $\nu^{(i)}$, at the intensity peak, the restriction on v for the resonance can be expressed as

$$\nu^0 - \frac{1}{2}\delta\nu^0 \leq \nu \leq \nu^0 + \frac{1}{2}\delta\nu^0, \quad (5.2)$$

where $\nu^0 = c(\nu^{(i)} - \nu_{02})/\nu_{02}$ and $\delta\nu^0 = c\delta\nu^{(i)}/\nu_{02}$.

Using the unit step function, $U(\nu)$, the probability, W_{02} , of finding the atom in the "2" state, which is selectively excited from the ground state "0" in time τ by the strong beam, may be written as

$$W_{02}(\theta_{10}, \nu, \nu) = \left[U(\nu - \nu^0 + \frac{1}{2}\delta\nu^0) - U(\nu - \nu^0 - \frac{1}{2}\delta\nu^0) \right] \frac{\epsilon^2}{2} \frac{(\frac{1}{2\pi}\tau_c)^2}{(\nu - \nu_{02})^2 + (\frac{1}{2\pi}\tau_c)^2 (1 + \epsilon^2)}. \quad (5.3)$$

This transition creates a hole in the velocity distribution of atoms in the ground state. Thus the new equilibrium velocity distribution function with this "burned hole" may be written as

$$f_o^c(\nu) = f_o^0(\nu) (1 - W_{02}), \quad (5.4)$$

where $f_o^0(\nu)$ is the reduced one particle distribution function similar to Eq. (4.25),

$$f_o^0(\nu) = \frac{N}{V_{\pm}\sqrt{\pi}} \frac{1}{\Delta\nu} \exp\left\{-\frac{4\nu^2}{\Delta\nu^2}\right\} \quad (5.5)$$

and N is the average number of atoms in V_{\pm} .

Here we note that Eq. (5.4) does not include another possible source to increase the ground state population in the hole. Because of gas-kinetic collisions, the velocities of the atoms will be redistributed. Thus the number of atoms in the ground state, all of which are in the narrow velocity distribution corresponding to the hole, will change. However, if the optical excitation rate, $P_{0 \rightarrow \ell} (=P_{\ell \rightarrow 0})$, is large, we can ignore this change. This can be justified in the following way. Consider the rate equation for the ground state population N_0 . Atoms are transferred by collisions from the ground to the excited state at the probability rate $\tau_{0\ell}^{-1}$, or conversely at the rate $\tau_{\ell 0}^{-1}$. For the gas in equilibrium with no incident radiation, we have

$$\frac{N_{\ell}^e}{\tau_{\ell 0}} - \frac{N_0^e}{\tau_{0\ell}} = 0.$$

Then, in the hole, the rate equation of the ground state with incident radiation can be written as

$$\frac{dN_0}{dt} = \frac{\tilde{N}_0}{\tau_k} + \frac{N_{\ell}}{\tau_{\ell 0}} - \frac{N_0}{\tau_{0\ell}} + N_{\ell}P_{\ell \rightarrow 0} - N_0P_{0 \rightarrow \ell}, \quad (P_{\ell \rightarrow 0} = P_{0 \rightarrow \ell}), \quad (5.6)$$

where τ_k^{-1} is the probability rate of transferring atoms from outside to inside the hole by gas-kinetic collisions. \tilde{N}_0 is related to the population in the ground state outside the hole, which can only come into the hole by the collisions within time τ_k . In steady state, $\tilde{N}_0 \simeq N_{\ell}$ and we obtain

$$N_{\ell} \left(\frac{1}{\tau_k} + \frac{1}{\tau_{\ell 0}} + P_{\ell \rightarrow 0} \right) - N_0 \left(\frac{1}{\tau_{0\ell}} + P_{0 \rightarrow \ell} \right) = 0. \quad (5.7)$$

If the optical excitation is large so that $P_{\ell \rightarrow 0} \gg \tau_k^{-1}$, the

steady state condition Eq. (5.7) gives the "burned hole" predicted by Eq. (5.4), under the same condition, influences from the atoms in the region outside of V_{\pm} on the population change in N_0 can be ignored.

Using Eq. (5.4), the macroscopic polarization vector field $\vec{P}(\vec{r}, \nu)$, which is induced by the weak probing beam with the incident angle θ_{20} , can be expressed by an expression similar to Eq. (4.22). The electric susceptibility $\eta_{\ell}(\theta_{20}, \nu)$ is then given by an expression similar to Eq. (4.23). The real and the imaginary part are given by (see Eqs. (4.27a) and (4.27b));

$$\eta_{\ell}^r(\theta_{20}, \nu) = -\frac{e^2}{2m_e} \frac{f_{\ell\ell}}{(2\pi)^2} \frac{1}{V_{\ell\ell}} \cdot \frac{N}{V_{\pm}} \int_{-\infty}^{+\infty} d\nu' \frac{1}{\sqrt{\pi}} \frac{1}{\Delta\nu'} \exp\left\{-\frac{4\nu'^2}{\Delta\nu'^2}\right\} \left[1 - W_{\ell\ell}(\nu, \nu', \theta_{10})\right] \times \frac{\nu - \nu_{\ell\ell}''}{(\nu - \nu_{\ell\ell}'')^2 + (\frac{1}{2\pi}\Gamma_{\ell} + \Gamma_0/2)^2} \quad (5.8a)$$

and

$$\eta_{\ell}^i(\theta_{20}, \nu) = -\frac{e^2}{2m_e} \frac{f_{\ell\ell}}{(2\pi)^2} \frac{1}{V_{\ell\ell}} \cdot \frac{N}{V_{\pm}} \int_{-\infty}^{+\infty} d\nu' \frac{1}{\sqrt{\pi}} \frac{1}{\Delta\nu'} \exp\left\{-\frac{4\nu'^2}{\Delta\nu'^2}\right\} \left[1 - W_{\ell\ell}(\nu, \nu', \theta_{10})\right] \times \frac{-(\frac{1}{2\pi}\Gamma_{\ell} + \Gamma_0/2)}{(\nu - \nu_{\ell\ell}'')^2 + (\frac{1}{2\pi}\Gamma_{\ell} + \Gamma_0/2)^2} \quad , \quad (5.8b)$$

where $\nu_{\ell\ell}'' = \nu_{\ell\ell} (1 + n_0 \frac{v}{c} \sin \theta_{20})$ and $\Delta\nu = 2\sqrt{2k_B T/M}$.

In order to numerically calculate the line shape of the reflected weak beam, we assume that the spectral distribution of the incident weak and the strong beam is Gaussian (like Eq. (4.36) where $\Delta\nu = \delta\nu^{(i)}$), and that $\delta\nu^{(i)}$ is the order of Γ_c^{-1} . The two beams are incident from opposite sides of the normal with $\theta_{10} = \theta_{20} = \theta_0$,

and the Doppler shifted resonance frequency ν'_{o2} in W_{o2} is $\nu'_{o2} = \nu_{o2} (1 - n_o \frac{v}{c} \sin \theta_o)$. We now substitute the above $\eta_k^r(\theta_o, \nu)$ and $\eta_k^i(\theta_o, \nu)$ into Eq. (4.31), and then use the line shape expressions, Eqs. (4.37) and (4.40a) from section 3 of Chapter IV to carry out the calculation.

We scan the incident frequency, $\nu^{(i)}$, of the incident beam over the entire Doppler profile. Since the propagation directions of the two refracted beams are anti-parallel along the surface, the weak beam can detect the hole burned out by the strong beam only when the incident frequency $\nu^{(i)}$ is tuned to the unshifted natural resonance frequency ν_{o2} . Thus the atoms which are responsible for the hole detected by the weak beam are almost at rest. Therefore the line shape of the reflected weak beam from the "burned hole" is nearly free from Doppler broadening. When the saturation effect is relatively weak, the entire pressure broadened profile may be obtained. A result of the above model calculation is shown in Fig. 5.2. We note that a profile of the "burned hole" may also be obtained by scanning the weak beam over the entire Doppler broadened profile, and that the line width of the weak beam is not an important factor in detecting the "burned hole" (see also Appendix B).

3. REFLECTIONS AT A SOLID-GAS INTERFACE WITH INTERACTIONS

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-30.	0.933322	I
-28.	0.933047	I
-26.	0.932802	I
-24.	0.932588	I
-22.	0.932409	I
-20.	0.932270	I
-18.	0.932176	I
-16.	0.932138	I
-14.	0.932174	I
-12.	0.932313	I
-10.	0.932603	I
-8.	0.933094	I
-6.	0.933795	I
-4.	0.934621	I
-2.	0.935271	I
0.	0.935484	I
2.	0.935269	I
4.	0.934644	I
6.	0.933899	I
8.	0.933323	I
10.	0.932980	I
12.	0.932844	I
14.	0.932856	I
16.	0.932968	I
18.	0.933148	I
20.	0.933379	I
22.	0.933651	I
24.	0.933957	I
26.	0.934294	I
28.	0.934659	I
30.	0.935049	I

Fig. 5.2 See page 153
for experimental condi-
tions.

* THE NARROW STRONG & WEAK BEAMS ARE SIMULTANEOUSLY SCANNED OVER NA F=2 LINE(F=J+1, J=1/2&1=3/2) *

* THE SATURATED ATOMS DETECTED BY THE WEAK BEAM ARE NEARLY AT REST *

* THE PEAK FREQ IS THEREFORE THAT OF THE UNSHIFTED LINE *

In this section we will study the line shape of light reflected from a probing beam at the interface between a solid and gaseous atoms, where atoms are physically adsorbed on the solid wall. As a model system, the interface between a thin metal layer deposited on a glass surface and Na vapor as the gas, will be considered. In order to simplify this model interface, we first separate it into various regions by placing the hypothetical surfaces; S_+ at the glass wall ($z=0$), S_0 at the thin metal layer surface ($z=d_0$) where the molecular environment can be regarded as those of the bulk metal, and S_- at a distance ($z=d$) where all the field quantities vanish because of the strong resonance absorption within the transition region V_{\pm} (see Fig. 5.3). Thus V_{\pm} has two regions. The first region $V_{\pm}^{(1)}$ starts at S_+ and ends at S_0 , and the second region $V_{\pm}^{(2)}$ starts at S_0 and ends at S_- .

In order to further simplify our model, we assume that the thickness (d_0) of the metal is much smaller than the wavelength of the incident light so that interference (due to the thickness of the metal) in the reflected beam can be ignored. Therefore in the simplified model we can use the extinction theorem, Eq. (3.3a), and thus the same line shape expressions are as those in section 3 of Chapter IV. However, the van der Waals' interaction between the metal and Na atoms may create a different environment for the gaseous atoms near the surface.

As mentioned in section 1, the surface potential is

A THIN METAL LAYER(DEPOSITED ON A GLASS)-GAS
INTERFACE

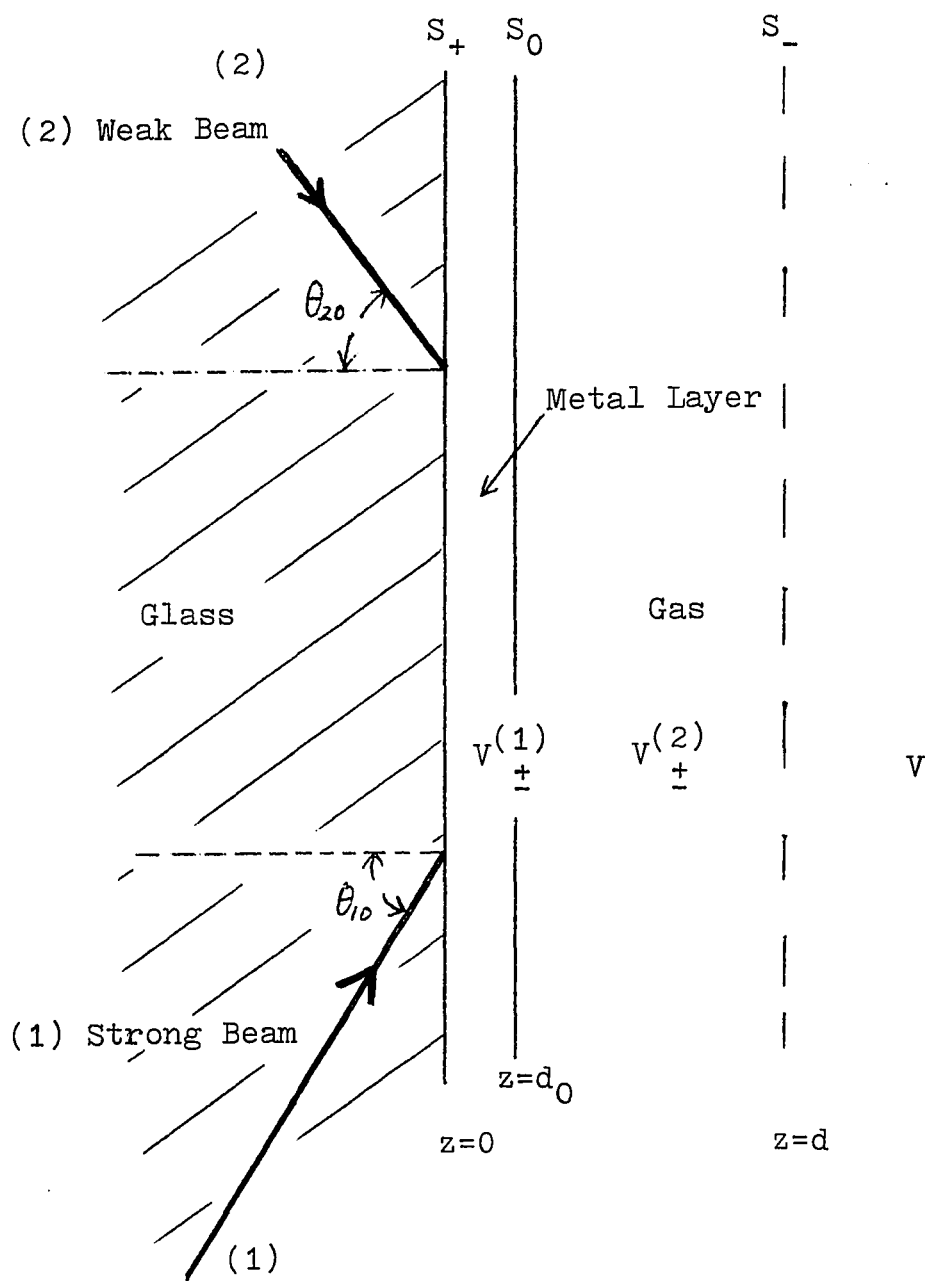


Fig.5.3

not known. We assume here that it has the form of an inverse power of the normal distance z from the surfaces with an infinite repulsive part at a small distance, δ . Then the change of the resonance frequency, $\Delta\nu_{0\ell}$, due to the potential is given by

$$\Delta\nu_{0\ell} = \alpha_p / z^p, \quad (p=2,3,4, \text{ etc.}), \quad (5.9)$$

where α_p is a constant. In the quasi-statistical approach to line shape theory, the influence of the frequency change due to the interaction on the line shape may be investigated by first finding the macroscopic polarization vector $\vec{P}(\vec{r}, \omega)$ in $V_{\pm}^{(2)}$, and then the electric susceptibility similar to Eqs. (5.8a) and (5.8b). Because of the interactions, the reduced one particle distribution function for those atoms which are in the ground state is now given by

$$f_0(z, \nu) = \frac{N}{V_{\pm}^{(2)}} \exp\left\{-\frac{1}{k_B T} \frac{C_p}{z^p}\right\} \frac{2}{\sqrt{\pi}} \frac{1}{\Delta\nu} \exp\left\{-\frac{4\nu^2}{\Delta\nu^2}\right\} \{1 - W_{0\ell}\}, \quad (5.10a)$$

with

$$W_{0\ell}(\theta_{10}, \nu, z, \nu) = \left[U(\nu - \nu_{+}^{(i)} + \frac{1}{2}\delta\nu^{(i)}) - U(\nu - \nu_{-}^{(i)} + \frac{1}{2}\delta\nu^{(i)}) \right] \frac{G^2}{2} \frac{(1/2\pi \tau_c)^2}{(\nu - \nu'_{0\ell} + \Delta\nu_{0\ell})^2 + (1/2\pi \tau_c)^2 (1 + G^2)}, \quad (5.10b)$$

where $\nu'_{0\ell} = \nu_{0\ell}(1 + n_{0\ell}^{(i)} \sin\theta_{10})$, $\Delta\nu_{0\ell} = \alpha_p / z^p$, $\Delta\nu = 2\sqrt{2k_B T/M}$ and C_p is constant in the interaction potential energy, $-C_p/z^p$, for the atoms in the ground state.

Using this reduced one particle distribution function, an electric susceptibility similar to Eqs. (5.8a) and (5.8b) can be obtained. When the incident frequency $\nu^{(i)}$ is tuned to a shifted natural resonance frequency, $\nu'_{0\ell} - \Delta\nu_{0\ell}$, where the frequency shift $\Delta\nu_{0\ell}$ due to the interaction is much

larger than that due to the Doppler effect, the electric susceptibility is that of a region close to the surface.

Its real and imaginary parts may be written as

$$\eta_{\epsilon}^r(\theta_{20}, \nu) = -\frac{e^2}{2m_e(2\pi)^2} \frac{f_{oe}}{\nu_{oe}} \frac{1}{V_{\pm}^{(2)}} \frac{N}{V_{\pm}^{(2)}} \exp\left\{+\frac{1}{k_B T} \frac{C_p}{z^p}\right\} \int_{-\infty}^{+\infty} d\nu \frac{2}{\sqrt{\pi}} \frac{1}{\Delta\nu} \left(\exp\left\{-\frac{4\nu^2}{\Delta\nu^2}\right\}\right) [1 - W_{oe}(\theta_{10}, \nu, z, \nu)]$$

$$\times \frac{\nu - \nu_{oe}'' + \Delta\nu_{oe}}{(\nu - \nu_{oe}'' + \Delta\nu_{oe})^2 + (1/2\pi\tau_c + \Gamma_o/2)^2} \quad (5.11a)$$

and

$$\eta_{\epsilon}^i(\theta_{20}, \nu) = -\frac{e^2}{2m_e(2\pi)^2} \frac{f_{oe}}{\nu_{oe}} \frac{1}{V_{\pm}^{(2)}} \frac{N}{V_{\pm}^{(2)}} \exp\left\{+\frac{1}{k_B T} \frac{C_p}{z^p}\right\} \int_{-\infty}^{+\infty} d\nu \frac{2}{\sqrt{\pi}} \frac{1}{\Delta\nu} \left(\exp\left\{-\frac{4\nu^2}{\Delta\nu^2}\right\}\right) [1 - W_{oe}(\theta_{10}, \nu, z, \nu)]$$

$$\times \frac{-(1/2\pi\tau_c + \Gamma_o/2)}{(\nu - \nu_{oe}'' + \Delta\nu_{oe})^2 + (1/2\pi\tau_c + \Gamma_o/2)^2}, \quad (5.11b)$$

where $\nu_{oe}'' = \nu_{oe}(1 + n_{oe} \frac{\nu}{\nu_{oe}} \sin\theta_{20})$.

We now derive the electric susceptibility for the region away from the surface by a distance on the order of 10^{-6} cm. In this region, i.e., between z and $z+\Delta z$, the number of atoms whose frequency shifts, $\Delta\nu_{oe}$, are within the line width, $\delta\nu^{(i)}$, of incident strong beam is significant. Then, the real part of the electric susceptibility may be

re-expressed as

$$\eta_{\epsilon}^r(\theta_{20}, \nu) = -\frac{e^2}{2m_e(2\pi)^2} \frac{f_{oe}}{\nu_{oe}} \frac{1}{V_{\pm}^{(2)}} \frac{N}{V_{\pm}^{(2)}} \int_{-\infty}^{+\infty} d\nu \frac{2}{\sqrt{\pi}} \frac{1}{\Delta\nu} \left(\exp\left\{-\frac{4\nu^2}{\Delta\nu^2}\right\}\right) \frac{\nu - \nu_{oe}'' + \Delta\nu_{oe}}{(\nu - \nu_{oe}'' + \Delta\nu_{oe})^2 + (1/2\pi\tau_c + \Gamma_o/2)^2}$$

$$\times \left[1 - \frac{1}{C_N} \int_{\Delta z}^{\Delta z} dz \exp\left\{\frac{1}{k_B T} \frac{C_p}{z^p}\right\} W_{oe}(\theta_{10}, \nu, z, \nu)\right], \quad (5.12)$$

with the normalization constant,

$$C_N = \int dz \exp\left\{\frac{1}{k_B T} \frac{C_p}{z^p}\right\}.$$

Similarly, the imaginary part may be re-expressed as

$$\eta_{\epsilon}^i(\theta_{20}, \nu) = -\frac{e^2}{2m_e} \frac{f_{oe}}{(2\pi)^2} \frac{1}{\nu_{oe}} \cdot \frac{N}{V_{\pm}^{(2)}} \int_{-\infty}^{+\infty} d\nu \frac{2}{N\pi} \frac{1}{\Delta\nu} \left(\exp\left\{-\frac{4\nu^2}{\Delta\nu^2}\right\} \right) \frac{-(1/2\pi\tau_c + \Gamma_0/2)}{(\nu - \nu_{oe}'' + \Delta\nu)_{oe}^2 + (1/2\pi\tau_c + \Gamma_0/2)^2} \\ \times \left[1 - \frac{1}{C_N} \int_{-\infty}^{+\infty} dz \exp\left\{\frac{1}{k_B T} \frac{C_p}{z^p}\right\} W_{oe}(\theta_{10}, \nu, z, \nu) \right]. \quad (5.13)$$

In order to numerically calculate the line shape of the reflected weak beam, we will follow the procedure outlined in section 2. Some of the numerically calculated results for the case where the refracted weak beam and strong beams are counter-propagating with the incident angles $\theta_{10} = \theta_{10} = \theta_0$, are given in Appendix B.

When the strong and weak beams are simultaneously scanned, the intensity distributions of the weak beam reflected from our model interfaces are also obtained. The intensity distributions among the possible interactions are given in Fig. 5.4.

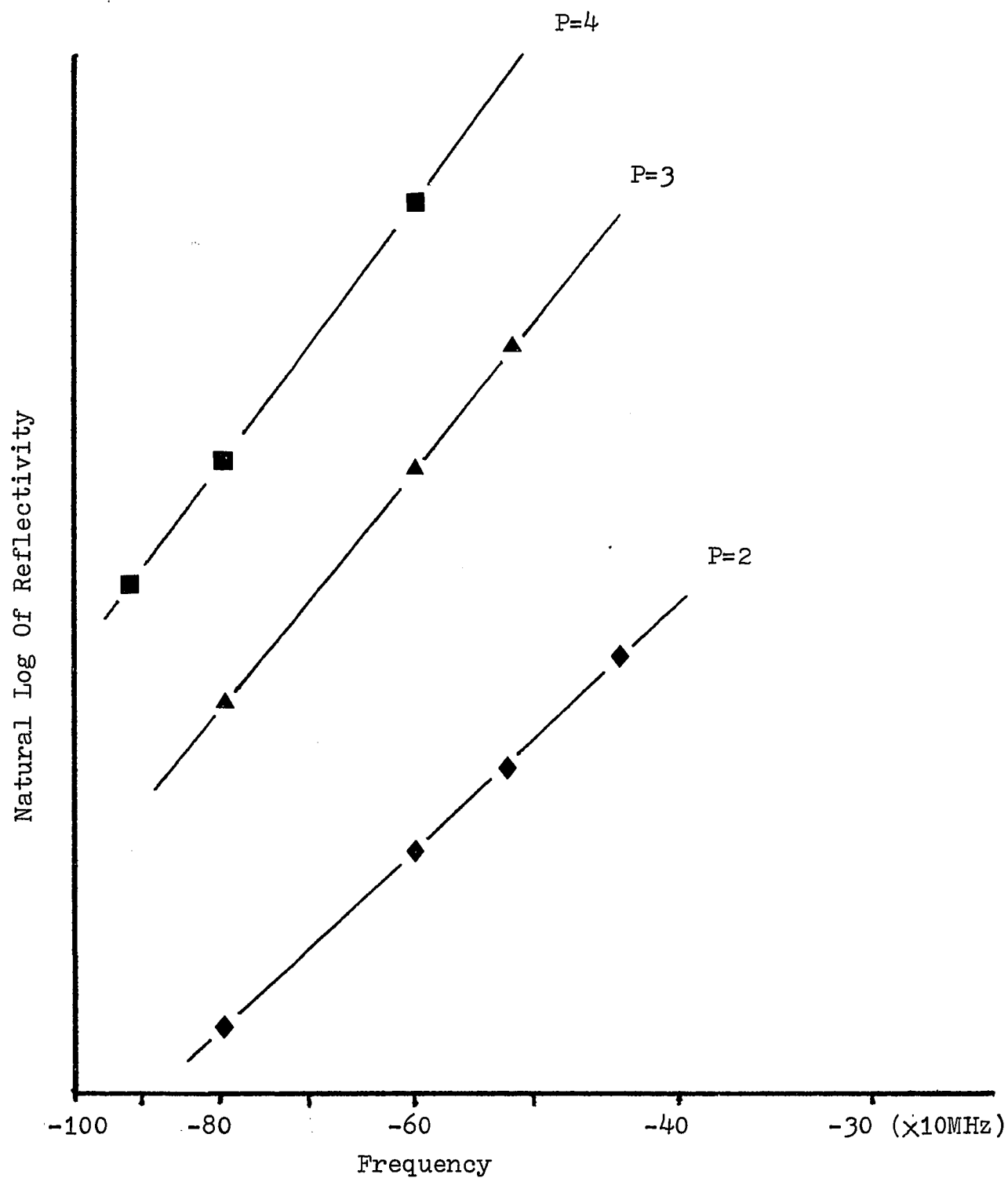


Fig. 5.4 The unshifted natural resonance line is at frequency 0. For detailed experimental conditions, see Appendix B.

CHAPTER VI

APPLICATION OF TUNABLE LASER TO GAS FLOW MEASUREMENTS NEAR A WALL

1. INTRODUCTION

Consider gas flow in a region near a solid wall. This region is divided into two parts: the region V_{\pm} which is within a distance of the mean free path of the gaseous atoms (or molecules) from the wall, and the remainder of the bulk gas region, V . When the gas flow in V is laminar, the flow field may be obtained by solving the Navier-Stokes equation numerically with the usual boundary conditions. When the flow becomes turbulent, attempts to obtain the complex flow fields numerically may be made by applying the Navier-Stokes equation with appropriate turbulent boundary conditions. However, these calculations will be successful only if the rapidly changing flow fields within the boundary layer are known.

The physical flow information near the wall, such as: mean flow velocity, Reynolds' stress, turbulent kinetic energy, etc., may be obtained using a Laser Doppler Anemometer. However, it cannot be used right up to the wall because of physical limitations due to the effective

probe volume at the beam intersection (typically $100\mu\text{m} \times 100\mu\text{m} \times 200\mu\text{m}$) and the diameter of seeding particles, $1\mu\text{m}$, (see the text of F. Durst, et. al., 1976). Also, when the gas becomes rarefied, the boundary conditions cannot be obtained by L.D.A.. Thus we need a new experimental technique to obtain the macroscopic flow field (appropriately averaged) in V_{\pm} . Not only would we obtain the boundary conditions but also physical information for the flow field just outside V_{\pm} . In fact, any observation of turbulent kinetic energy in this region V_{\pm} , may help in understanding the production and dissipation process of turbulent kinetic energy in the region just outside V_{\pm} , i.e., the boundary layer of the flow in V .

Thus, the purpose of this chapter is to investigate a possible application of a tunable laser to macroscopic gas flow measurements in the region V_{\pm} using the saturation spectroscopic technique which was previously studied.

2. RESPONSE OF THE WEAK BEAM TO GAS FLOW

We will consider the same interface between the glass and Na vapor as before using the saturation spectroscopic technique studied in section 2 of Chapter V. We assume that within the transition region V_{\pm} , there is a macroscopic instantaneous flow velocity, $u(z,t)$, at a distance z from the wall, and parallel to the wall. In order to define $u(z,t)$, we use the time dependent reduced one particle distribution

function

$$f_o^o(\vec{r}, v, t) = \left(\frac{M}{2\pi k_B T} \right)^{1/2} \frac{N}{V_{\pm}} \exp \left\{ - \frac{[v - u(z, t)]^2}{2 k_B T} \right\}, \quad (6.1)$$

where M is the mass of the gaseous atom (or molecule) and N is the average number in V_{\pm} . We may then define $u(z, t)$ through the mass flow,

$$\rho u(z, t) = \int_{V_{\pm}} d^3r' \int_{-\infty}^{+\infty} dv' \delta(\vec{r}' - \vec{r}) M v' f_o^o(\vec{r}', v', t),$$

where the mass density is defined as

$$\rho = \int_{V_{\pm}} d^3r' \int_{-\infty}^{+\infty} dv' M \delta(\vec{r}' - \vec{r}) f_o^o(\vec{r}', v', t) = \frac{NM}{V_{\pm}}.$$

In obtaining a mean of the instantaneous $u(z, t)$, the time average over a long period of time may be changed to an ensemble average over realizations of $u(z, t)$. Denoting the ensemble average by $\langle \rangle$, we have

$$\langle u(z, t) \rangle = U_o(z) \text{ and } \langle u(z, t) u(z, t) \rangle \neq 0.$$

We therefore write $u(z, t)$ as

$$u(z, t) = U_o(z) + \tilde{u}(z, t),$$

where $\tilde{u}(z, t)$ is a velocity of flow fluctuating around $U_o(z)$.

We now study the response of the probing beam to a fluctuating velocity field $\tilde{u}(z, t)$ in terms of the numerically calculated line shape of the reflected light around the burned hole. For simplicity, let us assume the following hypothetical flow conditions within V_{\pm} : $U_o(z)$ and $u(z, t)$ are independent of z , and are so small that $U_o \ll \Delta v$, and $\tilde{u}(z, t) \ll \Delta v$ ($\Delta v = 2\sqrt{2k_B T/M}$). The distribution of \tilde{u} is assumed to be Gaussian,

$$f(\tilde{u}) = \frac{2}{\sqrt{\pi}} \frac{1}{\Delta \tilde{u}} \exp\left\{-\frac{4\tilde{u}^2}{\Delta \tilde{u}^2}\right\}, \quad (6.2)$$

where the Doppler shift corresponding to the effective fluctuation, $\Delta \tilde{u}$, is larger than the line width of the incident beam, $\delta \nu^{(i)}$. The magnitude of $\delta \nu^{(i)}$ will then become a physical limitation on the resolution.

In order to numerically calculate the line shape of the reflected probing beam, we must first obtain the real and the imaginary part of the electric susceptibility similar to those of Chapter V. Using the same notations as Eqs. (5.8a) and (5.8b) of Chapter V, they can be written

$$\begin{aligned} \text{as} \quad \eta_{\ell}^r(\theta_{20}, \nu) = & -\frac{e^2}{2m_e} \frac{f_{0\ell}}{(2\pi)^2} \frac{1}{\nu_{0\ell}} \cdot \frac{N}{V_{\perp}} \int_{-\infty}^{+\infty} d\tilde{u} \frac{2}{\sqrt{\pi}} \cdot \frac{1}{\Delta \tilde{u}} \exp\left\{-\frac{4\tilde{u}^2}{\Delta \tilde{u}^2}\right\} \int_{-\infty}^{+\infty} d\nu \frac{2}{\sqrt{\pi}} \frac{1}{\Delta \nu} \exp\left\{-\frac{4(\nu - \tilde{u} - U_0)^2}{\Delta \nu^2}\right\} \\ & \times \left[1 - W_{0\ell}(\nu, U_0, \tilde{u}, \nu, \theta_{10})\right] \frac{\nu - \nu_{0\ell}''}{(\nu - \nu_{0\ell}'')^2 + (1/2\pi \tau_c + \Gamma_0/2)^2} \quad (6.3a) \end{aligned}$$

and

$$\begin{aligned} \eta_{\ell}^i(\theta_{20}, \nu) = & -\frac{e^2}{2m_e} \frac{f_{0\ell}}{(2\pi)^2} \frac{1}{\nu_{0\ell}} \cdot \frac{N}{V_{\perp}} \int_{-\infty}^{+\infty} d\tilde{u} \frac{2}{\sqrt{\pi}} \cdot \frac{1}{\Delta \tilde{u}} \exp\left\{-\frac{4\tilde{u}^2}{\Delta \tilde{u}^2}\right\} \int_{-\infty}^{+\infty} d\nu \frac{2}{\sqrt{\pi}} \frac{1}{\Delta \nu} \exp\left\{-\frac{4(\nu - \tilde{u} - U_0)^2}{\Delta \nu^2}\right\} \\ & \times \left[1 - W_{0\ell}(\nu, U_0, \tilde{u}, \nu, \theta_{10})\right] \frac{-(1/2\pi \tau_c + \Gamma_0/2)}{(\nu - \nu_{0\ell}'')^2 + (1/2\pi \tau_c + \Gamma_0/2)^2}, \quad (6.3b) \end{aligned}$$

where $\nu_{0\ell}'' = \nu_{0\ell} (1 + \frac{\nu - U_0 - \tilde{u}}{c} n_0 \sin \theta_{20})$. Here $W_{0\ell}(\nu, U_0, \tilde{u}, \nu, \theta_{10})$ is given by

$$W_{0\ell}(\nu, U_0, \tilde{u}, \nu, \theta_{10}) = \left[U(\nu - \nu_{0\ell}^{(i)} + \frac{1}{2}\delta \nu^{(i)}) - U(\nu - \nu_{0\ell}^{(i)} - \frac{1}{2}\delta \nu^{(i)}) \right] \frac{G^2}{2} \frac{(1/2\pi \tau_c)^2}{(\nu - \nu_{0\ell}'')^2 + (1/2\pi \tau_c)^2 (1 + G^2)},$$

where we recall that $\nu_{0\ell}' = \nu_{0\ell} (1 + \frac{\nu - U_0 - \tilde{u}}{c} n_0 \sin \theta_{10})$, and

$G^2 = e^2 f_{0\ell} |\vec{E}|^2 \hat{\tau}_c \hat{\tau} / (4m_e \nu_{0\ell} h)$ (see section 2 in Chapter V). We

then follow the procedure outlined in sections 2 and 3 of Chapter V to obtain the line shapes of the reflected weak

beam. Some of these results are given in Appendix C.

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APPENDIX A

Honeywell 66-20 computer at P.S.U. was used for the numerical calculations. The following programs were written in terms of Fortran. The control command words for the TSS terminal and batch interface were preceded by \$ signs in the listed programs.

```

1  $$JOUT,MONI,T
2  $      IDENT  RPHYA001,SELEC REF
3  $      OPTION  FORTRAN
4  $      FORTRAN
5  C      CALCULATION OF THE INTENSITY DISTRIBUTION OF THE REFLECTED
6  C      LIGHT FROM A GLASS- GAS  INTERFACE
7  C
8  C      SIMPSON'S METHOD IS USED IN NUMERICAL INTEGRATIONS
9  C
10 C      TO OBTAIN THE WALL EFFECT, OPTICAL PARAMETER OF BULK GAS
11 C      IS USED AS AN INITIALIZATION IN THE ITERATIONS
12 C
13 C      WHEN ISINGLE=3 (SEE BELOW), THE CANGES OF THE UPPER LIMITS
14 C      FOR +Z COMPONENT OF THE THERMAL VELOCITY FOR EACH LINE
15 C      ARE OBTAINED BY USING THE BULK OPTICAL PARAMETER DUE
16 C      TO THE TWO RESONANCE LINES (EXCEPT WHEN "ANGINC" IS LARGER THAN
17 C      A CRITICAL ANGLE)
18 C
19 C      THIS METHOD SAVES A CONSIDERABLE AMOUNT OF C.P. TIME TO
20 C      OBTAIN THE SAME RESULTS
21 C
22 C
23 C      *CONTROL INTEGERS*
24 C
25 C      FOR IPROGM=0, INCIDENT LIGHT WILL BE WHITE
26 C
27 C      FOR IPROGM=1, INCIDENT LIGHT WILL BE A NARROW GAUSSIAN BEAM
28 C      AND SCANNED OVER NA D2 LINES
29 C
30 C      FOR ISINGLE=1, ONLY THE LINE(F=2) WILL BE EXAMINED
31 C      WHERE F=J+I, (J=1/2 & I=3/2)
32 C
33 C      FOR ISINGLE=2, ONLY THE LINE(F=1) WILL BE EXAMINED
34 C      WHERE J=-1/2 & I=3/2
35 C
36 C      FOR ISINGLE=3, BOTH LINES WILL BE EXAMINED
37 C
38 C      IPOLARI=-1; PARALLEL POLARIZATION
39 C      IPOLARI=0 ; PLANE POLARIZED LIGHT AT 45 DEGREE
40 C      IPOLARI=+1; PARPENDICULAR POLARIZATION
41 C
42 C      *CONSTANTS*
43 C
44 C      REWAVE = RESONANCE WAVE LENGTH OF NA D2 LINE
45 C      GASDEN = DENCITY OF GAS VAPOR (PER CUBIC CM)
46 C      MOLWEI = MOLCULAR WEIGHT OF ATOM
47 C      HWI    = HALF OF 1/E POINTS WIDTH OF INCIDENT LIGHT(*10MHZ)
48 C      SP     = HYPERFINE SPILITTING OF NA ATOM AT ITS GRAND STATE
49 C              (*10MHZ)
50 C      SORIND = REFRACTIVE INDEX OF GLASS
51 C      OS1    =OSCIRATOR STRENGTH FOR F=2 LINE
52 C      OS2    =OSCIRATOR STRENGTH FOR F=1 LINE
53 C      TEMPER = GASTEMPERATURE
54 C      XLIGHT = VELOCITY OF LIGHT
55 C      PLANK  = FLANK CONSTANT

```

```

C   ELMASS = ELECTRON MASS
C   GASCON = GAS CONSTANT
C   XLOSHM = LOSHMIDT'S NUMBER
C   AANGLE = ANGLE OF INCIDENCE (DEGREE)
C   ANGINC = ANGLE OF INCIDECE (RADIAN)
C   EFDOP  = 1/E POINTS WIDTH OF DOPPLER BROADENED NA D2 LINE
C   PAI    = PAI
      DIMENSION Z(58),REINDT(58),REFRET(58),REFCMT(58),REFT(58),
&   DEPTH(58),UPPER1(10),UPPER2(10),REIND(58),DIERE(58),DIECM(58),
&   REF(58),ANGRET(58),ANGREN(58),DEPTHN(58),BET(5)
      GASDEN=1.05E+15
      XXE=ALOG(2.)
      HWI=20./SQRT(XXE)
      TEMPER=619.
      POLARI=+1
      ISINGLE=3
      IPRGM=1
      REWAVE=5.890E-5
      XMOLWE=22.9898
      SP    =177.2
      SORIND=1.63
      OS1=0.4166301
      IF(ISINGLE.EQ.2)OS1=0.
      OS2=0.2500363
      IF(ISINGLE.EQ.1)OS2=0.
      XLIGHT=2.99793E+10
      PLANK  =1.05459E-27
      ELMASS=9.10953E-28
      GASCON=8.31441E+7
      PAI    =3.14159
      XLOSHM=2.68675E+19
      DOP    =2.*GASCON*TEMPER/XMOLWE
      EFCDOP=2.*SQRT(DOP)/REWAVE
      XMHZ   =10.E+6
      XEM7=1.E-7
C   HALF OF HALF-VALUE WIDTH OF RESONACE BROADENED LINES
C   HWN1 IS FOR F=2 & HWN2 IS FOR F=1
      HWN1=0.8142*XEM7*GASDEN/XMHZ/2.
      HWN2=0.5543*XEM7*GASDEN/XMHZ/2.
      EFDOP =EFCDOP/XMHZ
C
C   CONST=CONSTANT APPERING IN SUSCEPTIBILITY
C
      CONST1=PLANK*GASDEN*REWAVE
      CONST2=137.036*ELMASS*SQRT(PAI)*4.*EFCDOP*PAI**2
      CONST =CONST1/CONST2
      IPOLA=IFIX(POLARI)
898  FORMAT(1H ,3X,"POLARIZATION IS PARALLEL")
897  FORMAT(1H ,3X,"POLARIZATION IS PERPENDICURAR")
896  FORMAT(1H ,3X,"PLANE POLARIZED LIGHT AT 45 DEGREE")
C
C   VAPOR=VAPOR PRESSURE IN UNIT OF MMHG
C
      VAPOR =760.*TEMPER*GASDEN/XLOSHM/273.2

```

```

900  FORMAT(1H ,3X,"RESONANCE WAVE LENGTH OF NA D2 LINE",/
&      23X,"=",E10.3,2X,"(CM)",/1H ,3X,
&      "SPILITTING OF HYPERFINE DOUBLET AT GRAND STATE",/
&      23X,"=",F6.1,4X,"(*10MHZ)",/1H ,3X, "DENSITY OF",
&      " VAPOR",3X,"=",E9.3,1X,"(PER CUBIC CM)",/1H ,
&      3X,"VAPOR PRESSURE",5X,"=",F7.3,3X,"(MMHG)",/1H ,
&      3X,"TEMPERATURE OF GAS",1X,"=",F5.0,/1H ,3X,
&      "HALF OF 1/E POINTS' WIDTH OF INCIDENT LIGHT",/23X,
&      "=",F6.1,5X,"(*10MHZ)",/1H ,3X,"1/E POINTS' DOPPLER",
&      " WIDTH",/23X,"=",F6.1,4X,"(*10MHZ)",
&      /1H ,3X,"HALF OF HALF WIDTH OF",
&      " PRESSURE BROADENED LINE 1",/
&      23X,"=",F5.1,5X,"(*10MHZ)",/1H ,3X,"HALF OF HALF WIDTH",
&      " OF PRESSURE BROADENED LINE 2",/23X,"=",F5.1,5X,"(*10MHZ)",/)

XREF=1.E-6
ZERO=0.
NA=6
FNA=FLOAT(NA)
EFHWI=1.6*HWI
HA=2.*EFHWI/FNA
LLA=15
DO 300 LLAN=10,55,LLA
LANGLE=LLAN-10
AANGLE=FLOAT(LANGLE)
ANGINC=AANGLE*PAI/180.
SORSIN=SORIND*SIN(ANGINC)
DOPS=EFDOP*SORSIN
TLOW=-0.8*EFDOP
THIG=0.8*EFDOP
FNNT=0.8*EFDOP/4.
IF(IPROGM.EQ.0)FNNT=0.8*EFDOP/2.
NTT=IFIX(FNNT)
FNT=2.*FLOAT(NTT)
NT=IFIX(FNT)
HT=1.6*EFDOP/FNT
IF(LANGLE.EQ.0)GO TO 4
FND1=DOPS*0.8/4.
ND1=IFIX(FND1)
FND=FLOAT(ND1)*2.
ND=IFIX(FND)
HD=1.6*DOPS/FND
C  XUPFAC IS A TRANCATION FACTOR FOR THE UPPER LIMIT IN THE
C  AVERAGE OVER Z COMPONENT OF THERMAL VELOCITY
4  XUPFAC=1.2
WRITE(6,69)
69  FORMAT(1H1,///4X,"< UPPER LIMIT FOR VELOCITY AVERAGE >")
IFREQ=270
FSC=30.
RATIO=FLOAT(IFREQ)/FSC+0.5
RATIO3=3.*RATIO
IC=IFIX(RATIO3)
DO 200 IZ=1,IC
IIZ=IZ-1
C  Z(IZ) IS THE FREQ(OR CENTRAL FREQ) OF INCIDENT LIGHT W.R.T

```

```

C   THE CENTRAL FREQ OF THE LINE OF F=2
    Z(IZ)=-FLOAT(IFREQ)+FLOAT(IIZ)*FSC
    SLOW=Z(IZ)-EFHWI
C   SIMPSON'S METHOD STARTS TO TAKE A CONVOLUTION INTEGRAL W.R.T
C   A NARROW SPECTRAL LINE WIDTH OF INCIDENT LIGHT
    SB=0.
    SBG=0.
    SSUSR=0.
    SSUSC=0.
    SBT=0.
    SUSRT=0.
    SUSCT=0.
    IF(IPROGM.EQ.0)NA=0
    DO 600 JB=1,NA+1,2
    IF(IPROGM.EQ.0)GO TO 308
    JBB=JB-1
    HB=HA*FLOAT(JBB)
    XB=SLOW+HB
    SMB=0.
    SMBG=0.
    SMSUSR=0.
    SMSUSC=0.
    SMBT=0.
    SMSURT=0.
    SMSUCT=0.
    LLBB=2
308 IF(IPROGM.EQ.0)LLBB=1
    DO 650 LLB=1,LLBB
    IF(IPROGM.EQ.0)GO TO 301
    LB=LLB-1
    FL=FLOAT(LB)
    XA=XB+HA*FL
    CALL GAUSIA(AIGAUS,Z(IZ),XA,HWI,PAI)
    GAUS=AIGAUS
301 IF(IPROGM.EQ.0)XA=Z(IZ)
    ILINE=1
    CALL SUSCEP(RESUM,CMSUM,NT,XA,HT,TLOW,SP,CONST,
&   OS1,OS2,EFDOP,HWN1,HWN2,ILINE)
    RESUM21=RESUM
    CMSUM21=CMSUM
    ILINE=2
    CALL SUSCEP(RESUM,CMSUM,NT,XA,HT,TLOW,SP,CONST,
&   OS1,OS2,EFDOP,HWN1,HWN2,ILINE)
    RESUM22=RESUM
    CMSUM22=CMSUM
    RESUM2=RESUM21+RESUM22
    CMSUM2=CMSUM21+CMSUM22
    IF(ISINGLE.EQ.3)GO TO 400
    CALL REFINDEX(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM22,
&   CMSUM22,SORIND,ANGINC,PAI)
    REIND22=REINDX
    REFRE22=REFREX
    REFCM22=REFCMX
    ALFA22=ALFAX

```

```

BETA22=BETAX
XANGLE22=SORSIN/REIND22
ANGREF22=ARSIN(XANGLE22)
ANGRE22=180.*ANGREF22/PAI
CALL  REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM21,
&    CMSUM21,SORIND,ANGINC,PAI)
REIND21=REINDX
REFRE21=REFREX
REFCM21=REFCMX
ALFA21=ALFAX
BETA21=BETAX
XANGLE21=SORSIN/REIND21
ANGREF21=ARSIN(XANGLE21)
ANGLE21=180.*ANGREF21/PAI
400 CALL  REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM2,
&    CMSUM2,SORIND,ANGINC,PAI)
REIND2=REINDX
REFRE2=REFREX
REFCM2=REFCMX
ALFA2=ALFAX
BETA2=BETAX
XANGLE=SORSIN/REIND2
ANGREF=ARSIN(XANGLE)
ANGRE2=180.*ANGREF/PAI
CALL  REFLEC(REFLFX,ALFA2,BETA2,ANGINC,ANGREF,POLARI,
&    SORIND)
REF2=REFLFX
IF(1SINGLE.EQ.3)GO TO 401
IF(LANGLE.EQ.0)CN1=CONST/ALFA21
IF(LANGLE.EQ.0)CN2=CONST/ALFA22
IF(LANGLE.GT.0)CN1=CONST/ALFA21/DOPS
IF(LANGLE.GT.0)CN2=CONST/ALFA22/DOPS
CRITAN=80.
IF(ANGRE2.GT.CRITAN)CN=CONST/SORSIN
IF(ANGRE2.GT.CRITAN)GO TO 11
IF(BETA21.LE.XREF)TUP1=THIG
IF(BETA22.LE.XREF)TUP2=THIG
IF(BETA21.GT.XREF)TUP1=XUPFAC*ALFA21/BETA21
IF(BETA22.GT.XREF)TUP2=HWN2*XUPFAC*ALFA22/HWN1/BETA22
TLOW1=TLOW*ALFA21
TLOW2=TLOW*ALFA22
GO TO 402
401 IF(LANGLE.EQ.0)CN1=CONST/ALFA2
IF(LANGLE.EQ.0)CN2=CONST/ALFA2
IF(LANGLE.GT.0)CN1=CONST/ALFA2/DOPS
IF(LANGLE.GT.0)CN2=CONST/ALFA2/DOPS
CRITAN=80.
IF(ANGRE2.GT.CRITAN)CN=CONST/SORSIN
IF(ANGRE2.GT.CRITAN)GO TO 11
IF(BETA2.LE.XREF)TUP1=THIG
IF(BETA2.LE.XREF)TUP2=THIG
IF(BETA2.GT.XREF)TUP1=XUPFAC*ALFA2/BETA2
IF(BETA2.GT.XREF)TUP2=HWN2*XUPFAC*ALFA2/HWN1/BETA2
TLOW1=TLOW*ALFA2

```

```

1      TLOW2=TLOW1
2
3 402   TWIDT1=TUP1-TLOW1
4       TWIDT2=TUP2-TLOW2
5       TH1=TWIDT1/4.
6       IF(LANGLE.GT.0)TH1=TWIDT1/8.
7       NNTH1=IFIX(TH1)
8       FNTH1=2.*FLOAT(NNTH1)
9       NTH1=IFIX(FNTH1)
10      THX1=TWIDT1/FNTH1
11      TH2=TWIDT2/THX1/2.
12      NNTH2=IFIX(TH2)
13      FNTH2=2.*FLOAT(NNTH2)
14      NTH2=IFIX(FNTH2)
15      IF(ISINGLE.EQ.3)ALFA21=ALFA2
16      IF(ISINGLE.EQ.3)ALFA22=ALFA2
17      IF(ISINGLE.EQ.3)BETA21=BETA2
18      IF(ISINGLE.EQ.3)BETA22=BETA2
19      IF(ISINGLE.EQ.3)ANGRE21=ANGRE2
20      IF(ISINGLE.EQ.3)ANGRE22=ANGRE2
21      IF(TUP1-THIG)10,10,20
22
23 10    ILINE=1
24      CALL SUSCER(RESUM,CMSUM,NTH1,NTH2,XA,THX1,TLOW1,SP,CN1,
25      & OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFA21,BETA21,
26      & LANGLE,ANGINC,ANGRE21,ND,HD,ILINE)
27      GO TO 21
28
29 20    ILINE=1
30      CALL SUSCER(RESUM,CMSUM,NT,NT,XA,HT,TLOW,SP,CN1,
31      & OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFA21,BETA21,
32      & LANGLE,ANGINC,ANGRE21,ND,HD,ILINE)
33
34 21    SUSRE11=RESUM
35      SUSCM11=CMSUM
36      IF(TUP2-THIG)30,30,40
37
38 30    ILINE=2
39      CALL SUSCER(RESUM,CMSUM,NTH1,NTH2,XA,THX1,TLOW2,SP,CN2,
40      & OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFA22,BETA22,
41      & LANGLE,ANGINC,ANGRE22,ND,HD,ILINE)
42      GO TO 41
43
44 40    ILINE=2
45      CALL SUSCER(RESUM,CMSUM,NT,NT,XA,HT,TLOW,SP,CN2,
46      & OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFA22,BETA22,
47      & LANGLE,ANGINC,ANGRE22,ND,HD,ILINE)
48
49 41    SUSRE12=RESUM
50      SUSCM12=CMSUM
51      CALL REFINO(REINDX,REFREX,REFCMX,ALFAX,BETAX,SUSRE11,SUSCM11,
52      & SORIND,ANGINC,PAI)
53      REIND11=REINDX
54      DIECM11=REFCMX
55      DIERE11=REFREX
56      ALFAN11=ALFAX
57      BETAN11=BETAX
58      CALL REFINO(REINDX,REFREX,REFCMX,ALFAX,BETAX,SUSRE12,SUSCM12,
59      & SORIND,ANGINC,PAI)
60      REIND12=REINDX
61      DIECM12=REFCMX

```



```

DIERE12=REFREX
ALFAN12=ALFAX
BETAN12=BETAX
IF (ISINGLE.EQ.3) GO TO 403
IF (BETAN11.LT.XREF) TUP1=TUP1
IF (BETAN12.LT.XREF) TUP2=TUP2
IF (BETAN11.GE.XREF) TUP1=XUPFAC*ALFAN11/BETAN11
IF (BETAN12.GE.XREF) TUP2=HWN2*XUPFAC*ALFAN12/BETAN12/HWN1
IF (TUP1.GE.THIG.AND.TUP2.GE.THIG) GO TO 185
IF (TUP1.GE.THIG.AND.TUP2.LT.THIG) GO TO 71
ILINE=1

```

```

C THIS ITERATION STARTS FOR ISINGLE=1

```

```

DO 199 N=1,10

```

```

UPPER1(N)=XUPFAC*ALFAN11/BETAN11

```

```

IF (N.EQ.1) GO TO 190

```

```

DIF1=UPPER1(N-1)-UPPER1(N)

```

```

ADIF1=ABS(DIF1)

```

```

XADIF1=ADIF1-1.

```

```

IF (N.EQ.4) GO TO 190

```

```

IF (XADIF1.LT.ZERO) GO TO 71

```

```

IF (N.GE.3) UPPER1(N)=0.5*(UPPER1(N-1)+UPPER1(N))

```

```

NN=N-1

```

```

WRITE(6,191) NN,UPPER1(NN),N,UPPER1(N),DIF1,Z(IZ)

```

```

191 FORMAT(1H,4X,'<UPPER1(',I2,')=' ,F8.2,'>',2X,'<UPPER1(',
& I2,')=' ,F8.2,'>',2X,F5.0,2X,F5.0)

```

```

190 XLOWER1=-THIG*ALFAN11

```

```

WIDTH=UPPER1(N)-XLOWER1

```

```

YNT=WIDTH/2.

```

```

IF (LANGE.GT.0) YNT=WIDTH/4.

```

```

NNT=IFIX(YNT)

```

```

YNTT=2.*FLOAT(NNT)

```

```

NNTT=IFIX(YNTT)

```

```

YHT=WIDTH/YNTT

```

```

IF (LANGE.EQ.0) CN1=CONST/ALFAN11

```

```

IF (LANGE.GT.0) CN1=CONST/ALFAN11/DOPS

```

```

CALL SUSCER(RESUM,CMSUM,NNTT,NT2,XA,YHT,XLOWER1,SP,CN1,
& OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFAN11,BETAN11,LANGE,ANGINC,
& ANGREG21,ND,HD,ILINE)

```

```

RESUM11=RESUM

```

```

CMSUM11=CMSUM

```

```

CALL REFINO(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM11,CMSUM11,
& SORIND,ANGINC,PAI)

```

```

REIND11=REINDX

```

```

REFRE11=REFREX

```

```

REFCM11=REFCMX

```

```

ALFAN11=ALFAX

```

```

BETAN11=BETAX

```

```

199 CONTINUE

```

```

C THIS ITERATION STARTS FOR ISINGLE=2

```

```

C

```

```

71 ILINE=2

```

```

IF (TUP2.GE.THIG) GO TO 187

```

```

DO 75 N=1,10

```

```

1  UPPER2(N)=HWN2*XUPFAC*ALFAN12/BETAN12/HWN1
2  IF(N.EQ.1) GO TO 76
3  DIF2=UPPER2(N-1)-UPPER2(N)
4  ADIF2=ABS(DIF2)
5  XADIF2=ADIF2-1.
6  IF(N.EQ.4) GO TO 76
7  IF(XADIF2.LT.ZERO) GO TO 187
8  IF(N.GE.3) UPPER2(N)=0.5*(UPPER2(N-1)+UPPER2(N))
9  NN=N-1
10 WRITE(6,77) NN,UPPER2(NN),N,UPPER2(N),DIF2,XA
11 77  FORMAT(1H,4X,"<UPPER2(",I2,")=",F8.2,">",2X,"<UPPER2(",
12 & I2,")=",F8.2,">",2X,F5.0,2X,F5.0)
13 76  XLOWER2=-THIG*ALFAN12
14  WIDTH2=UPPER2(N)-XLOWER2
15  YNTT2=WIDTH2/2.
16  IF(LANGLE.GT.0) YNTT2=WIDTH2/4.
17  NNT2=IFIX(YNTT2)
18  FNT2=2.*FLOAT(NNT2)
19  NT2=IFIX(FNT2)
20  YHT=WIDTH2/FNT2
21  IF(LANGLE.EQ.0) CN2=CONST/ALFAN12
22  IF(LANGLE.GT.0) CN2=CONST/ALFAN12/DOPS
23  CALL SUSCER(RESUM,CMSUM,NNT2,NT2,XA,YHT,XLOWER2,SP,CN2,
24 & OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFAN12,BETAN12,LANGLE,ANGINC,
25 &  ANGREG2,ND,HD,ILINE)
26  RESUM12=RESUM
27  CMSUM12=CMSUM
28  CALL REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM12,CMSUM12,
29 &  SORIND,ANGINC,PAI)
30  REIND12=REINDX
31  REFRE12=REFREX
32  REFCM12=REFCMX
33  ALFAN12=ALFAX
34  BETAN12=BETAX
35 75  CONTINUE
36  WRITE(6,192)(UPPER1(N),UPPER2(N),N=1,10)
37 192  FORMAT(4X,"<UPPER1=",F10.2,">",2X,"<UPPER2=",F10.2,">",/)
38  GO TO 187
39 11  ALFAN1=ALFA2
40  C  THIS ITERATION STARTS WHEN THE INCIDENT ANGLE IS
41  C  LARGER THAN A CRITICAL ANGLE
42  DO 13 NCO=1,5
43  BET(NCO)=BETA2
44  IF(NCO.EQ.1) GO TO 14
45  DBET=1.-BET(NCO)/BET(NCO-1)
46  ADBET=ABS(DBET)
47  REFBET=.01
48  IF(ADBET.LE.REFBET) GO TO 15
49 14  CALL SUSCER(RESUM,CMSUM,NT,NT,XA,HT,TLOW,SP,CN,
50 &  OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFAN1,BET(NCO),
51 &  LANGLE,ANGINC,ANGREG2,ND,HD,ILINE)
52  RESUM1=RESUM
53  CMSUM1=CMSUM
54  CALL REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM1,

```

```

&    CMSUM1,SORIND,ANGINC,PAI)
    ALFAN1=ALFAX
    BETAN1=BETAX
    REIND1=REINDX
    REFRE1=REFREX
    REFCM1=REFCMX
13    CONTINUE
15    BETAN1=BET(NCO)
    GO TO 186
185   IF(LANGLE.EQ.0)CN1=CONST/ALFAN11
    IF(LANGLE.EQ.0)CN2=CONST/ALFAN12
    IF(LANGLE.GT.0)CN1=CONST/ALFAN11/DOPS
    IF(LANGLE.GT.0)CN2=CONST/ALFAN12/DOPS
    ILINE=1
    CALL SUSCER(RESUM,CMSUM,NT,NT,XA,HT,TLOW,SP,CN1,
&    OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFAN11,BETAN11,
&    LANGLE,ANGINC,ANGRE21,ND,HD,ILINE)
    RESUM11=RESUM
    CMSUM11=CMSUM
    ILINE=2
    CALL SUSCER(RESUM,CMSUM,NT,NT,XA,HT,TLOW,SP,CN2,
&    OS1,OS2,EFDOP,HWN1,HWN2,DOPS,ALFAN12,BETAN12,
&    LANGLE,ANGINC,ANGRE22,ND,HD,ILINE)
    RESUM12=RESUM
    CMSUM12=CMSUM
    GO TO 187
403   CNN1=ALFA2/ALFAN11
    CNN2=ALFA2/ALFAN12
    RESUM1=CNN1*SUSRE11+CNN2*SUSRE12
    CMSUM1=CNN1*SUSCM11+CNN2*SUSCM12
    GO TO 404
187   RESUM1=RESUM11+RESUM12
    CMSUM1=CMSUM11+CMSUM12
404   CALL REFINO(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM1,CMSUM1,
&    SORIND,ANGINC,PAI)
    REIND1=REINDX
    DIECM1=REFCMX
    DIERE1=REFREX
    ALFAN1=ALFAX
    BETAN1=BETAX
186   XANGLE=SORSIN/REIND1
    ANGREF=ARSIN(XANGLE)
    CALL REFLEC(REFLFX,ALFAN1,BETAN1,ANGINC,ANGREF,POLARI,SORIND)
    REF1=REFLFX
    IF(IPROGM.EQ.0)GO TO 650
    IF(JBB.EQ.0)GO TO 602
    IF(JBB.EQ.NA)GO TO 603
    IF(LB.EQ.0)MLB=2
    IF(LB.EQ.1)MLB=4
    GO TO 604
602   IF(LB.EQ.0)MLB=1
    IF(LB.EQ.1)MLB=4
    GO TO 604
603   IF(LB.EQ.0)MLB=1

```

```

1  IF(LB.EQ.1)MLB=0
2
3  604  FML=FLOAT(MLB)
4      IF(JBB.EQ.NA.AND.LB.EQ.1) GO TO 605
5      SMSUSR=SMSUSR+RESUM1
6      SMSUSC=SMSUSC+CMSUM1
7      SMSURT=SMSURT+RESUM2
8      SMSUCT=SMSUCT+CMSUM2
9  605  SMB=SMB+FML*GAUS*REF1
10     SMBG=SMBG+FML*GAUS
11     SMBT=SMBT+FML*GAUS*REF2
12  650  CONTINUE
13     IF(IPROGM.EQ.0)GO TO 600
14     SB=SB+SMB
15     SBT=SBT+SMBT
16     SBG=SBG+SMBG
17     SSUSR=SSUSR+SMSUSR
18     SSUSC=SSUSC+SMSUSC
19     SUSRT=SUSRT+SMSURT
20     SUSCT=SUSCT+SMSUCT
21  600  CONTINUE
22     IF(IPROGM.EQ.0)GO TO 303
23     SUMB=HA*SB/3.
24     SUMBT=HA*SBT/3.
25     SUMBG=HA*SBG/3.
26     REF(IZ)=SUMB/SUMBG
27     ARESUM=SSUSR/(FNA+1.)
28     ACMSUM=SSUSC/(FNA+1.)
29     REFT(IZ)=SUMBT/SUMBG
30     ARESUT=SUSRT/(FNA+1.)
31     ACMSUT=SUSCT/(FNA+1.)
32  303  IF(IPROGM.EQ.0)ARESUM=RESUM1
33       IF(IPROGM.EQ.0)ACMSUM=CMSUM1
34       IF(IPROGM.EQ.0)REFT(IZ)=REF2
35       IF(IPROGM.EQ.0)REF(IZ)=REF1
36  C    CALCULATION OF THE AVERAGE REFRACTIVE INDEX AND DIELECTRIC
37  C    CONSTANT
38       CALL REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,ARESUM,
39       & ACMSUM,SORIND,ANGINC,PAI)
40       REIND(IZ)=REINDX
41       DIECM(IZ)=REFCMX
42       DIERE(IZ)=REFREX
43       ALFAN=ALFAX
44       BETAN=BETAX
45       DEPTHN(IZ)=REWAVE/BETAN/PAI/2.
46       ANGLE2=SORSIN/REIND(IZ)
47       ANGREN(IZ)=180.*ARSIN(ANGLE2)/PAI
48       IF(IPROGM.EQ.0)ARESUT=RESUM2
49       IF(IPROGM.EQ.0)ACMSUT=CMSUM2
50       CALL REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,ARESUT,
51       & ACMSUT,SORIND,ANGINC,PAI)
52       REINDT(IZ)=REINDX
53       REFCMT(IZ)=REFCMX
54       REFRET(IZ)=REFREX
55       ALFAT=ALFAX

```

```

1      BETAT=BETAX
2      DEPTH(IZ)=REWAVE/BETAT/PAI/2.
3
4      ANGLE2=SORSIN/REINDT(IZ)
5      ANGRET(IZ)=180.*ARSIN(ANGLE2)/PAI
6
7      200  CONTINUE
8
9      IF(LANGLE.GT.0)GO TO 912
10     WRITE(6,908)
11     WRITE(6,894)
12     IF(IPROGM.EQ.0)WRITE(6,960)
13     IF(IPROGM.EQ.1)WRITE(6,970)
14     IF(IPOLA.LT.0)WRITE(6,898)
15     IF(IPOLA.GT.0)WRITE(6,897)
16     IF(IPOLA.EQ.0)WRITE(6,896)
17     WRITE(6,900)REWAVE,SP,GASDEN,VAPOR,TEMPER,HWI,EFDOP,HWN1,HWN2
18     WRITE(6,907)AANGLE
19     908  FORMAT(1H1,///23X,"<*** EXPERIMENTAL CONDITIONS ***>",/)
20     WRITE(6,909)
21     WRITE(6,910)(Z(IZ),REFRET(IZ),REFCMT(IZ),REINDT(IZ),DEPTH(IZ),
22     &    ANGRET(IZ),REFT(IZ),IZ=1,IC)
23     910  FORMAT(1H ,4X,F5.0,6X,F6.4,2X,F6.4,4X,F6.4,1X,E8.1,2X,F4.1,
24     &    3X,F6.4)
25     WRITE(6,901)AANGLE
26     WRITE(6,955)
27     CALL GRAPH(IPROGM,ISINGLE,0,IC,IFREQ,FSC,REFRET)
28     WRITE(6,901)AANGLE
29     WRITE(6,956)
30     CALL GRAPH(IPROGM,ISINGLE,0,IC,IFREQ,FSC,REFCMT)
31     WRITE(6,901)AANGLE
32     WRITE(6,957)
33     CALL GRAPH(IPROGM,ISINGLE,0,IC,IFREQ,FSC,REINDT)
34     WRITE(6,901)AANGLE
35     WRITE(6,958)
36     CALL GRAPH(IPROGM,ISINGLE,0,IC,IFREQ,FSC,REFT)
37     912  WRITE(6,908)
38     WRITE(6,895)
39     IF(IPROGM.EQ.0)WRITE(6,960)
40     IF(IPROGM.EQ.1)WRITE(6,970)
41     IF(IPOLA.LT.0)WRITE(6,898)
42     IF(IPOLA.GT.0)WRITE(6,897)
43     IF(IPOLA.EQ.0)WRITE(6,896)
44     WRITE(6,900)REWAVE,SP,GASDEN,VAPOR,TEMPER,HWI,EFDOP,HWN1,HWN2
45     907  FORMAT(1H0,23X,"** ANGLE OF INCIDENCE",1X,"=",F4.1,"DEGREE **",/)
46     WRITE(6,909)
47     901  FORMAT(1H1,///23X,"** ANGLE OF INCIDENCE",1X,"=",F4.1,"DEGREE ",
48     &    "**",/)
49     909  FORMAT(1H0,"INCIDENT FREQ",1X,"DIELECTRIC CONST",2X,"REFRACT",
50     &    2X,"PENET",2X,"REFRCT",2X,"REFLECTIVITY",/2X,"(*10MHZ)",
51     &    6X,"REAL",4X,"IMAG",5X,"INDEX",4X,"DEPTH",4X,"ANGLE",/)
52     WRITE(6,910)(Z(IZ),DIERE(IZ),DIECM(IZ),REIND(IZ),DEPTHN(IZ),
53     &    ANGREN(IZ),REF(IZ),IZ=1,IC)
54     WRITE(6,901)AANGLE
55     WRITE(6,955)
56     CALL GRAPH(IPROGM,ISINGLE,1,IC,IFREQ,FSC,DIERE)
57     WRITE(6,901)AANGLE

```

```

1      WRITE(6,956)
2      CALL GRAPH(IPROGM,ISINGLE,1,IC,IFREQ,FSC,DIECM)
3      WRITE(6,901)AANGLE
4      WRITE(6,957)
5      CALL GRAPH(IPROGM,ISINGLE,1,IC,IFREQ,FSC,REIND)
6      WRITE(6,901)AANGLE
7      WRITE(6,958)
8      CALL GRAPH(IPROGM,ISINGLE,1,IC,IFREQ,FSC,REF)
9
10     895  FORMAT(1H0,20X,"<* WALL EFFECTS ARE TAKEN INTO ACCOUNT *>")
11     894  FORMAT(1H0,20X,"<* WALL EFFECTS ARE NOT TAKEN INTO ACCOUNT *>")
12     955  FORMAT(1H0,"<FREQ>",34X,"<REAL PART OF THE DIELECTRIC",
13         & "CONST>")
14     956  FORMAT(1H0,"<FREQ>",34X,"<IMAG PART OF THE DIELECTRIC",
15         & "CONST>")
16     957  FORMAT(1H0,"<FREQ>",34X,"<REFRACTIVE INDEX>")
17     958  FORMAT(1H0,"<FREQ>",34X,"<REFLECTIVITY>")
18     960  FORMAT(4X,"WHITE LIGHT IS INCIDENT")
19     970  FORMAT(4X,"GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS IN",
20         & "CIDENT & SCANNED")
21     300  CONTINUE
22     STOP
23     END

```

C SUBROUTINE TO CALCULATE SUSCEPTIBILITY FOR THE BULK GAS

```

24     SUBROUTINE SUSCEP(RESUM,CMSUM,N,Z,H,XALLX,SP,CONST,
25     & OS1,OS2,D1V,HWN1,HWN2,ILINE)
26     ZERO=0.
27     IF(OS1.EQ.ZERO.AND.ILINE.EQ.1)GO TO 12
28     IF(OS2.EQ.ZERO.AND.ILINE.EQ.2)GO TO 12
29     IF(ILINE.EQ.1)HWNX=HWN1
30     IF(ILINE.EQ.2)HWNX=HWN2
31     IF(ILINE.EQ.1)OSX=OS1
32     IF(ILINE.EQ.2)OSX=OS2
33     IF(ILINE.EQ.2)Z=Z-SP
34     SRAA=0.
35     SCAA=0.
36     DO 10 J=1,N+1,2
37     JL=J-1
38     D=H*FLOAT(JL)+XALLX
39     SRAAA=0.
40     SCAAA=0.
41     DO 5 L=1,2
42     M=L-1
43     DM=FLOAT(M)
44     XA=D+H*DM
45     A11=4.*(XA**2)/(D1V**2)
46     A1=-A11
47     B1V=OSX*EXP(A1)
48     FAA=(XA-Z)**2+HWNX**2
49     FA=B1V/FAA
50     REFA=(XA-Z)*FA
51     CMFA=HWNX*FA
52     IF(JL.EQ.0) GO TO 2

```

```

1      IF(JL.EQ.N) GO TO 3
2      IF(M.EQ.0) ML=2
3      IF(M.EQ.1) ML=4
4      GO TO 4
5      2      IF(M.EQ.0) ML=1
6      IF(M.EQ.1) ML=4
7      GO TO 4
8      3      IF(M.EQ.0) ML=1
9      IF(M.EQ.1) ML=0
10     4      DML=FLOAT(ML)
11     SRAAA=SRAAA+DML*REFA
12     SCAAA=SCAAA+DML*CMFA
13     5      CONTINUE
14     SRAA=SRAA+SRAAA
15     SCAA=SCAA+SCAAA
16     10     CONTINUE
17     RESUM=CONST*H*SRAA/3.
18     CMSUM=CONST*H*SCAA/3.
19     IF(ILINE.EQ.2) Z=Z+SP
20     GO TO 13
21     12     RESUM=0.
22     CMSU=0.
23     13     RETURN
24     END

```

C SUBROUTINE TO GRAPH DATA

```

1      SUBROUTINE GRAPH(IPROGM,ISINGLE,IWALL,IC,IFREQ,FSC,VALUE)
2      DIMENSION PLOT(83),VALUE(IC)
3      DATA DASH/'-'/,BAR/'I'/,X/'*'/,BLANK/' '/
4      DO 5 I=1,IC
5      IF(I.EQ.1) YMAX=VALUE(I)
6      IF(I.EQ.1) YMIN=VALUE(I)
7      IF(I.GE.2) GO TO 2
8      GO TO 5
9      2      BIG=VALUE(I)-YMAX
10     SMALL=VALUE(I)-YMIN
11     ZERO=0.
12     IF(BIG.GE.ZERO) YMAX=VALUE(I)
13     IF(SMALL.LT.ZERO) YMIN=VALUE(I)
14     5      CONTINUE
15     DIF=YMAX-YMIN
16     DO 10 K=1,83
17     PLOT(K)=DASH
18     10     CONTINUE
19     WRITE(6,20) PLOT
20     FORMAT(1X, '(*10MHZ)', /1H0, 15X, 83A1)
21     DO 30 K=1,83
22     PLOT(K)=BLANK
23     30     CONTINUE
24     PLOT(1)=BAR
25     DO 60 KZ=1,IC
26     KZZ=KZ-1
27     Z=-FLOAT(IFREQ)+FLOAT(KZZ)*FSC

```

```
YDIF=VALUE(KZ)-YMIN
```

```
YN=YDIF*80./DIF+2.5
```

```
NY=IFIX(YN)
```

```
PLOT(NY)=X
```

```
WRITE(6,40)Z,VALUE(KZ),PLOT
```

```
40  FORMAT(1X,F5.0,2X,F6.4,1X,B3A1)
```

```
PLOT(NY)=BLANK
```

```
60  CONTINUE
```

```
IF(IPROGM.EQ.0)WRITE(6,71)
```

```
IF(IPROGM.EQ.1)WRITE(6,72)
```

```
IF(ISINGLE.EQ.3)WRITE(6,61)
```

```
IF(ISINGLE.EQ.1)WRITE(6,64)
```

```
IF(ISINGLE.EQ.2)WRITE(6,65)
```

```
IF(IWALL.EQ.1)WRITE(6,62)
```

```
IF(IWALL.EQ.0)WRITE(6,63)
```

```
64  FORMAT(1H0,"* THE CENTRAL FREQ OF F=2 LINE IS AT 0. & ",
```

```
& " F=1 LINE IS NOT PRESENT")
```

```
65  FORMAT(1H0,"* THE CENTRAL FREQ OF F=1 LINE IS AT 177.2 & "
```

```
& " F=2 LINE IS NOT PRESENT")
```

```
61  FORMAT(1H0,"* THE CENTRAL FREQ OF ONE OF D2 LINES ",
```

```
& "(F=2) IS AT FREQ=0, WHEREAS THE OTHER (F=1) IS AT 177.2 ",
```

```
& " *")
```

```
62  FORMAT(1H0,"* THE WALL EFFECT IS TAKEN INTO ACCOUNT *")
```

```
63  FORMAT(1H0,"* THE WALL EFFECT IS NOT TAKEN INTO ",
```

```
& "ACCOUNT *")
```

```
71  FORMAT(1H0,"* WHITE LIGHT IS INCIDENT *")
```

```
72  FORMAT(1H0,"* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCI",
```

```
& "DENT & SCANNED *")
```

```
RETURN
```

```
END
```

```
C  SUBROUTINE TO CALCULATE SUSCEPTIBILITY FOR THE GAS NEAR THE SURFACE
```

```
      SUBROUTINE SUSCER(RESUM,CMSUM,N1,N2,Z,H,XLOWAL,SP,
```

```
&  C,OS1,OS2,DP,HWN1,HWN2,DOPS,ALFAZT,BETAZT,
```

```
&  LANGLE,ANGINC,ANGREF,NY,HY,ILINE)
```

```
C  SRR=0. AND SCR=0. ARE INITIALIZATIONS TO TAKE VELOCITY AVERAGE
```

```
C  W.R.T THE X COMPONENT OF THE DOPPLER SHIFTED FREQ
```

```
ZERO=0.
```

```
IF(OS1.EQ.ZERO.AND.ILINE.EQ.1)GO TO 33
```

```
IF(OS2.EQ.ZERO.AND.ILINE.EQ.2)GO TO 33
```

```
CRITAN=80.
```

```
IF(ILINE.EQ.1)OSX=OS1
```

```
IF(ILINE.EQ.1)HWNX=HWN1
```

```
IF(ILINE.EQ.2)OSX=OS2
```

```
IF(ILINE.EQ.2)HWNX=HWN2
```

```
IF(ILINE.EQ.2.AND.ANGREF.LT.CRITAN)Z=Z-SP
```

```
IF(ILINE.EQ.1)N=N1
```

```
IF(ILINE.EQ.2)N=N2
```

```
IF(LANGLE.EQ.0)GO TO 1
```

```
SRR=0.
```

```
SCR=0.
```

```
1  IF(LANGLE.EQ.0)NY=0
```

```
DO 20 IY=1,NY+1,2
```


IF(LANGLE.EQ.0) GO TO 2

IYY=IY-1

DY=HY*FLOAT(IYY)-0.8*DOPS

SRRY=0.

SCRY=0.

LL2=2

2 IF(LANGLE.EQ.0)LL2=1

DO 15 LL=1,LL2

IF(LANGLE.EQ.0)GO TO 5

MM=LL-1

FMM=FLOAT(MM)

Y1=DY+HY*FMM

AY1=-4.*(Y1**2)/(DOPS**2)

EAY1=EXP(AY1)

5 IF(LANGLE.EQ.0)Y1=0.

IF(LANGLE.EQ.0)EAY1=1.

SREAB=0.

SCMAB=0.

IF(ANGREF.GT.CRITAN)N=1

DO 16 LA=1,N+1,2

IF(ANGREF.GT.CRITAN)GO TO 3

LLA=LA-1

DAL=H*FLOAT(LLA)+XLOWAL

SRRAB=0.

SCRAB=0.

II2=2

3 IF(ANGREF.GT.CRITAN)II2=1

DO 17 II=1,II2

IF(ANGREF.GT.CRITAN)GO TO 6

NN=II-1

FNN=FLOAT(NN)

AL=DAL+H*FNN

IF(LANGLE.EQ.0)Y1=0.

DOPC=DP*ALFAZT

ALL1=4.*(AL**2)/DOPC**2

IF(ALL1.GT.60)GO TO 42

EAL1=EXP(-ALL1)

GO TO 7

42 EAL1=0.

GO TO 7

6 EAL1=1.

AL=0.

ALZ1=AL+Y1-Z

ALZ2=AL+Y1-Z+SP

ALW=BETAZT*AL/ALFAZT

ALW1=ALW-HWN1

ALW2=ALW-HWN2

ALZ12=ALZ1**2

ALZ22=ALZ2**2

ALW12=ALW1**2

ALW22=ALW2**2

ALX1=ALZ12+ALW12

ALX2=ALZ22+ALW22

IF(LANGLE.EQ.0)EAY1=1.

```

RFAL1=EAY1*EAL1*OS1*ALZ1/ALX1
RFAL2=EAY1*EAL1*OS2*ALZ2/ALX2
CFAL1=-EAY1*EAL1*ALW1*OS1/ALX1
CFAL2=-EAY1*EAL1*ALW2*OS2/ALX2
GO TO 8
7  ALZ1=AL+Y1-Z
   ALW=BETAZT*AL/ALFAZT
   ALWX=ALW-HWNX
   ALZ12=ALZ1**2
   ALW12=ALWX**2
   ALX1=ALZ12+ALW12
   IF(LANGLE.EQ.0)EAY1=1.
   RFAL1=EAY1*EAL1*OSX*ALZ1/ALX1
   CFAL1=-EAY1*EAL1*ALWX*OSX/ALX1
   IF(LLA.EQ.0)GO TO 12
   IF(LLA.EQ.N)GO TO 13
   IF(NN.EQ.0)NC=2
   IF(NN.EQ.1)NC=4
   GO TO 14
12  IF(NN.EQ.0)NC=1
   IF(NN.EQ.1)NC=4
   GO TO 14
13  IF(NN.EQ.0)NC=1
   IF(NN.EQ.1)NC=0
14  FNC=FLOAT(NC)
   SRRAB=SRRAB+FNC*RFAL1
   SCRAB=SCRAB+FNC*CFAL1
17  CONTINUE
   SREAB=SRRAB+SREAB
   SCMAE=SCRAB+SCRAB
16  CONTINUE
   FUNREY=H*SREAB/3.
   FUNCMY=H*SCMAE/3.
8   IF(ANGREF.GT.CRITAN)FUNREY=RFAL1+RFAL2
   IF(ANGREF.GT.CRITAN)FUNCMY=CFAL1+CFAL2
   IF(LANGLE.EQ.0)GO TO 31
   IF(IYY.EQ.0)GO TO 22
   IF(IYY.EQ.NY)GO TO 23
   IF(MM.EQ.0)MC=2
   IF(MM.EQ.1)MC=4
   GO TO 24
22  IF(MM.EQ.0)MC=1
   IF(MM.EQ.1)MC=4
   GO TO 24
23  IF(MM.EQ.0)MC=1
   IF(MM.EQ.1)MC=0
24  FMC=FLOAT(MC)
   SRRY=SRRY+FMC*FUNREY
   SCRY=SCRY+FMC*FUNCMY
15  CONTINUE
   SRR=SRR+SRRY
   SCR=SCR+SCRY
20  CONTINUE
   RESUM=C*HY*SRR/3.

```

```
CMSUM=C*HY*SCR/3.
```

```
GO TO 32
```

```
31 RESUM=C*FUNREY
```

```
CMSUM=C*FUNCMY
```

```
32 IF (ILINE.EQ.2.AND.ANGREF.LT.CRITAN)Z=Z+SP
```

```
GO TO 34
```

```
33 RESUM=0.
```

```
CMSUM=0.
```

```
34 RETURN
```

```
END
```

```
C SUBROUTINE TO CALCULATE REFRACTIVE INDEX
```

```
SUBROUTINE REFINDEX(REINDEX,REFREX,REFCMX,ALFAX,BETAX,  
& SUSREX,SUSCMX,SOINDEX,ANGINC,PAI)
```

```
REFREX=1.+4.*PAI*SUSREX
```

```
REFCMX=2.*PAI*SUSCMX
```

```
PART1=(SOINDEX*SIN(ANGINC))*2
```

```
A=REFREX-PART1
```

```
B=2.*REFCMX
```

```
C=REFREX+PART1
```

```
D=0.
```

```
AB=ABS(A)
```

```
IF (AB-B)60,61,62
```

```
60 X=A/B
```

```
XRT=1.+X**2
```

```
ALFA=B*(SQRT(XRT)+X)/2.
```

```
ALFAX=SQRT(ALFA)
```

```
BETA=B*(SQRT(XRT)-X)/2.
```

```
E=SQRT(XRT)-X
```

```
IF (E.EQ.D)BETAX=0.
```

```
BETAX=SQRT(BETA)
```

```
PARTX=(C+B*XRT)/2.
```

```
REINDEX=SQRT(PARTX)
```

```
GO TO 63
```

```
61 ALFA=B*(1.+SQRT(2.))/2.
```

```
ALFAX=SQRT(ALFA)
```

```
BETA=B*(SQRT(2.)-1.)/2.
```

```
BETAX=SQRT(BETA)
```

```
PARTE=(C+B*(1.+SQRT(2.)))/2.
```

```
REINDEX=SQRT(PARTE)
```

```
GO TO 63
```

```
62 Y=B/A
```

```
YRT=1.+Y**2
```

```
ALFA=A*(1.+SQRT(YRT))/2.
```

```
IF (A.LT.D)ALFA=AB*(SQRT(YRT)-1.)/2.
```

```
ALFAX=SQRT(ALFA)
```

```
BETA=A*(SQRT(YRT)-1.)/2.
```

```
IF (A.LT.D)BETA=AB*(SQRT(YRT)+1.)/2.
```

```
F=SQRT(YRT)-1.
```

```
IF (F.EQ.D)BETAX=0.
```

```
BETAX=SQRT(BETA)
```

```
PARTY=(C+A*YRT)/2.
```

```
REINDEX=SQRT(PARTY)
```

```
63 IF (REINDEX**2-PART1)64,65,65
```

64 XC1=REFCMX**2/(PART1-REINDX**2)

XC2=XC1+PART1

XC=SQRT(XC2)

REINDX=XC

65 RETURN

END

C SUBROUTINE TO DEFINE A FUNCTIONAL FORM OF REFLECTIVITY

SUBROUTINE REFLEC(REFLFX,ALFAX,BETAX,ANGINC,
& ANGREF,POLARI,SOINDX)

AC=COS(ANGINC)

AS=SIN(ANGINC)

RNUMAX=(SOINDX*AC-ALFAX)**2+BETAX**2

RDENO=(SOINDX*AC+ALFAX)**2+BETAX**2

SA=SOINDX*AS

XA=(ALFAX**2-BETAX**2+SA**2)*AC

XAN=XA-SOINDX*ALFAX

XB=2.*ALFAX*BETAX

XBN=XB-SOINDX*BETAX

RNPER=XAN**2+XBN**2

XAD=XA+SOINDX*ALFAX

XBD=XB+SOINDX*BETAX

RDPER=XAD**2+XBD**2

IF(POLARI)701,702,703

701 REFLFX=RNPER/RDPER

GO TO 705

703 REFLFX=RNUMAX/RDENO

GO TO 705

702 REFLFX=.5*(RNPER/RDPER+RNUMAX/RDENO)

705 RETURN

END

C SUBROUTINE TO FIND THE SQUARE OF THE INCIDENT LIGHT AMPLITUDE

SUBROUTINE GAUSIA(AIGAU,XINC,AAZ,HWI,PAI)

AZ=AAZ-XINC

AIG=-(AZ**2)/HWI/HWI

AIGAU=EXP(AIG)

RETURN

END

\$ EXECUTE

\$ LIMITS 40,47K,5000

\$ SYSOUT 06,ORG

\$ ENDJOB

<*** EXPERIMENTAL CONDITIONS ***>

<* WALL EFFECTS ARE NOT TAKEN INTO ACCOUNT *>

GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED

POLARIZATION IS PERPENDICULAR

RESONANCE WAVE LENGTH OF NA D2 LINE

= 0.589E-04 (CM)

SPLITTING OF HYPERFINE DOUBLET AT GRAND STATE

= 177.2 (*10MHZ)

DENSITY OF VAPOR = 0.105E 16 (PER CUBIC CM)

VAPOR PRESSURE = 0.067 (MMHG)

TEMPERATURE OF GAS = 619.

HALF OF 1/E POINTS' WIDTH OF INCIDENT LIGHT

= 24.0 (*10MHZ)

1/E POINTS' DOPPLER WIDTH

= 227.2 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 1

= 4.3 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 2

= 2.9 (*10MHZ)

** ANGLE OF INCIDENCE = 0. DEGREE **

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRACT ANGLE	REFLECTIVITY
-270.	1.0186	0.0002	1.0093	0.5E-01	0.	0.0553
-240.	1.0214	0.0002	1.0107	0.4E-01	0.	0.0550
-210.	1.0258	0.0008	1.0128	0.1E-01	0.	0.0546
-180.	1.0306	0.0024	1.0152	0.4E-02	0.	0.0540
-150.	1.0351	0.0053	1.0174	0.2E-02	0.	0.0535
-120.	1.0378	0.0095	1.0188	0.1E-02	0.	0.0532
-90.	1.0377	0.0144	1.0188	0.7E-03	0.	0.0532
-60.	1.0335	0.0195	1.0169	0.5E-03	0.	0.0536
-30.	1.0256	0.0238	1.0133	0.4E-03	0.	0.0545
0.	1.0151	0.0262	1.0082	0.4E-03	0.	0.0557
30.	1.0044	0.0266	1.0029	0.4E-03	0.	0.0569
60.	0.9949	0.0252	0.9981	0.4E-03	0.	0.0581
90.	0.9884	0.0234	0.9948	0.4E-03	0.	0.0588
120.	0.9835	0.0215	0.9922	0.4E-03	0.	0.0592
150.	0.9790	0.0195	0.9898	0.5E-03	0.	0.0598
180.	0.9744	0.0174	0.9874	0.5E-03	0.	0.0604
210.	0.9709	0.0147	0.9856	0.6E-03	0.	0.0608
240.	0.9688	0.0118	0.9844	0.8E-03	0.	0.0610
270.	0.9682	0.0084	0.9841	0.1E-02	0.	0.0612
300.	0.9693	0.0055	0.9846	0.2E-02	0.	0.0610
330.	0.9718	0.0030	0.9858	0.3E-02	0.	0.0606
360.	0.9755	0.0013	0.9877	0.7E-02	0.	0.0603
390.	0.9790	0.0004	0.9894	0.2E-01	0.	0.0598
420.	0.9820	0.0001	0.9909	0.6E-01	0.	0.0594
450.	0.9840	0.0001	0.9920	0.8E-01	0.	0.0592
480.	0.9855	0.0001	0.9927	0.1E 00	0.	0.0590
510.	0.9867	0.0001	0.9933	0.1E 00	0.	0.0589
540.	0.9877	0.0001	0.9938	0.1E 00	0.	0.0588

** ANGLE OF INCIDENCE = 0. DEGREE **

<FREQ>
(*10MHZ)

<REAL PART OF THE DIELECTRICCONST>

-270.	1.0186	I
-240.	1.0214	I
-210.	1.0258	I
-180.	1.0306	I
-150.	1.0351	I
-120.	1.0378	I
-90.	1.0377	I
-60.	1.0335	I
-30.	1.0256	I
0.	1.0151	I
30.	1.0044	I
60.	0.9949	I
90.	0.9884	I
120.	0.9835	I
150.	0.9790	I
180.	0.9744	I
210.	0.9709	I
240.	0.9688	I
270.	0.9682	I
300.	0.9693	I
330.	0.9718	I
360.	0.9755	I
390.	0.9790	I
420.	0.9820	I
450.	0.9840	I
480.	0.9855	I
510.	0.9867	I
540.	0.9877	I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS NOT TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE = 0. DEGREE **

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRICCONST>

-270.	0.0002	I *
-240.	0.0002	I *
-210.	0.0008	I *
-180.	0.0024	I *
-150.	0.0053	I *
-120.	0.0095	I *
-90.	0.0144	I *
-60.	0.0195	I *
-30.	0.0238	I *
0.	0.0262	I *
30.	0.0266	I *
60.	0.0252	I *
90.	0.0234	I *
120.	0.0215	I *
150.	0.0195	I *
180.	0.0174	I *
210.	0.0147	I *
240.	0.0118	I *
270.	0.0084	I *
300.	0.0055	I *
330.	0.0030	I *
360.	0.0013	I *
390.	0.0004	I *
420.	0.0001	I *
450.	0.0001	I *
480.	0.0001	I *
510.	0.0001	I *
540.	0.0001	I *

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** ANGLE OF INCIDENCE = 0. DEGREE **

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-270.	0.0553	I
-240.	0.0550	I
-210.	0.0546	I
-180.	0.0540	I
-150.	0.0535	I
-120.	0.0532	I
-90.	0.0532	I
-60.	0.0536	I
-30.	0.0545	I
0.	0.0557	I
30.	0.0569	I
60.	0.0581	I
90.	0.0588	I
120.	0.0592	I
150.	0.0598	I
180.	0.0604	I
210.	0.0608	I
240.	0.0610	I
270.	0.0612	I
300.	0.0610	I
330.	0.0606	I
360.	0.0603	I
390.	0.0598	I
420.	0.0594	I
450.	0.0592	I
480.	0.0590	I
510.	0.0589	I
540.	0.0588	I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

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<*** EXPERIMENTAL CONDITIONS ***>

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-270.	1.0187	0.0002	1.0093	0.5E-01	0.	0.0553
-240.	1.0215	0.0002	1.0107	0.4E-01	0.	0.0550
-210.	1.0259	0.0008	1.0129	0.1E-01	0.	0.0546
-180.	1.0307	0.0025	1.0153	0.4E-02	0.	0.0539
-150.	1.0348	0.0055	1.0173	0.2E-02	0.	0.0535
-120.	1.0359	0.0095	1.0179	0.1E-02	0.	0.0534
-90.	1.0336	0.0143	1.0169	0.7E-03	0.	0.0536
-60.	1.0269	0.0192	1.0137	0.5E-03	0.	0.0543
-30.	1.0156	0.0235	1.0083	0.4E-03	0.	0.0555
0.	0.9979	0.0258	0.9996	0.4E-03	0.	0.0574
30.	0.9828	0.0209	0.9918	0.4E-03	0.	0.0599
60.	0.9783	0.0142	0.9893	0.7E-03	0.	0.0604
90.	0.9837	0.0099	0.9919	0.9E-03	0.	0.0588
120.	0.9884	0.0121	0.9943	0.8E-03	0.	0.0585
150.	0.9849	0.0143	0.9926	0.7E-03	0.	0.0590
180.	0.9763	0.0149	0.9883	0.6E-03	0.	0.0600
210.	0.9658	0.0128	0.9829	0.7E-03	0.	0.0614
240.	0.9603	0.0096	0.9800	0.1E-02	0.	0.0624
270.	0.9604	0.0061	0.9800	0.2E-02	0.	0.0622
300.	0.9661	0.0042	0.9829	0.2E-02	0.	0.0612
330.	0.9715	0.0029	0.9856	0.3E-02	0.	0.0606
360.	0.9757	0.0013	0.9878	0.7E-02	0.	0.0602
390.	0.9791	0.0004	0.9895	0.2E-01	0.	0.0597
420.	0.9820	0.0001	0.9910	0.6E-01	0.	0.0594
450.	0.9840	0.0001	0.9920	0.8E-01	0.	0.0592
480.	0.9855	0.0001	0.9927	0.1E 00	0.	0.0590
510.	0.9867	0.0001	0.9933	0.1E 00	0.	0.0589
540.	0.9877	0.0001	0.9938	0.1E 00	0.	0.0588

★ ★ ANGLE OF INCIDENCE = 0. DEGREE ★ ★

<FREQ>
(★10MHZ)

<REAL PART OF THE DIELECTRICCONST>

-270.	1.0187	I
-240.	1.0215	I
-210.	1.0259	I
-180.	1.0307	I
-150.	1.0348	I
-120.	1.0359	I
-90.	1.0336	I
-60.	1.0269	I
-30.	1.0156	I
0.	0.9979	I
30.	0.9828	I
60.	0.9783	I
90.	0.9837	I
120.	0.9884	I
150.	0.9849	I
180.	0.9763	I
210.	0.9658	I
240.	0.9603	I★
270.	0.9604	I★
300.	0.9661	I
330.	0.9715	I
360.	0.9757	I
390.	0.9791	I
420.	0.9820	I
450.	0.9840	I
480.	0.9855	I
510.	0.9867	I
540.	0.9877	I

★ GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED ★

★ THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 ★

★ THE WALL EFFECT IS TAKEN INTO ACCOUNT ★

** ANGLE OF INCIDENCE = 0. DEGREE **

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRICCONST>

-270.	0.0002	I*
-240.	0.0002	I *
-210.	0.0008	I *
-180.	0.0025	I *
-150.	0.0055	I *
-120.	0.0095	I *
-90.	0.0143	I *
-60.	0.0192	I *
-30.	0.0235	I *
0.	0.0258	I *
30.	0.0209	I *
60.	0.0142	I *
90.	0.0099	I *
120.	0.0121	I *
150.	0.0143	I *
180.	0.0149	I *
210.	0.0128	I *
240.	0.0096	I *
270.	0.0061	I *
300.	0.0042	I *
330.	0.0029	I *
360.	0.0013	I *
390.	0.0004	I *
420.	0.0001	I *
450.	0.0001	I *
480.	0.0001	I *
510.	0.0001	I *
540.	0.0001	I *

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE = 0. DEGREE **

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-270.	0.0553	I
-240.	0.0550	I
-210.	0.0546	I
-180.	0.0539	I
-150.	0.0535	I
-120.	0.0534	I
-90.	0.0536	I
-60.	0.0543	I
-30.	0.0555	I
0.	0.0574	I
30.	0.0599	I
60.	0.0604	I
90.	0.0588	I
120.	0.0585	I
150.	0.0590	I
180.	0.0600	I
210.	0.0614	I
240.	0.0624	I
270.	0.0622	I
300.	0.0612	I
330.	0.0606	I
360.	0.0602	I
390.	0.0597	I
420.	0.0594	I
450.	0.0592	I
480.	0.0590	I
510.	0.0589	I
540.	0.0588	I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

<*** EXPERIMENTAL CONDITIONS ***>

<* WALL EFFECTS ARE TAKEN INTO ACCOUNT *>

GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED

POLARIZATION IS PERPENDICULAR

RESONANCE WAVE LENGTH OF NA D2 LINE

= 0.589E-04 (CM)

SPPLITTING OF HYPERFINE DOUBLET AT GRAND STATE

= 177.2 (*10MHZ)

DENSITY OF VAPOR = 0.105E 16 (PER CUBIC CM)

VAPOR PRESSURE = 0.067 (MMHG)

TEMPERATURE OF GAS = 619.

HALF OF 1/E POINTS' WIDTH OF INCIDENT LIGHT

= 24.0 (*10MHZ)

1/E POINTS' DOPPLER WIDTH

= 227.2 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 1

= 4.3 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 2

= 2.9 (*10MHZ)

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-270.	1.0166	0.0002	1.0083	0.5E-01	24.7	0.0700
-240.	1.0191	0.0004	1.0095	0.2E-01	24.7	0.0696
-210.	1.0225	0.0010	1.0112	0.9E-02	24.7	0.0691
-180.	1.0265	0.0023	1.0132	0.4E-02	24.6	0.0685
-150.	1.0298	0.0047	1.0148	0.2E-02	24.6	0.0680
-120.	1.0310	0.0081	1.0155	0.1E-02	24.5	0.0678
-90.	1.0288	0.0124	1.0145	0.7E-03	24.6	0.0681
-60.	1.0221	0.0168	1.0113	0.5E-03	24.7	0.0690
-30.	1.0108	0.0196	1.0058	0.4E-03	24.8	0.0707
0.	0.9978	0.0195	0.9994	0.4E-03	25.0	0.0730
30.	0.9885	0.0164	0.9946	0.5E-03	25.1	0.0746
60.	0.9858	0.0125	0.9930	0.7E-03	25.1	0.0748
90.	0.9873	0.0107	0.9938	0.8E-03	25.1	0.0743
120.	0.9881	0.0111	0.9942	0.8E-03	25.1	0.0741
150.	0.9847	0.0117	0.9925	0.7E-03	25.2	0.0747
180.	0.9793	0.0110	0.9897	0.8E-03	25.2	0.0757
210.	0.9734	0.0084	0.9867	0.1E-02	25.3	0.0768
240.	0.9712	0.0058	0.9855	0.1E-02	25.3	0.0770
270.	0.9715	0.0040	0.9857	0.2E-02	25.3	0.0770
300.	0.9731	0.0029	0.9865	0.3E-02	25.3	0.0767
330.	0.9758	0.0022	0.9878	0.4E-02	25.3	0.0762
360.	0.9790	0.0012	0.9894	0.7E-02	25.2	0.0757
390.	0.9818	0.0005	0.9909	0.2E-01	25.2	0.0752
420.	0.9842	0.0002	0.9920	0.4E-01	25.2	0.0749
450.	0.9859	0.0001	0.9929	0.9E-01	25.1	0.0746
480.	0.9873	0.0001	0.9936	0.1E 00	25.1	0.0744
510.	0.9884	0.0001	0.9942	0.1E 00	25.1	0.0742
540.	0.9893	0.0000	0.9946	0.1E 00	25.1	0.0741

** ANGLE OF INCIDENCE =15.0DEGREE **

<FREQ>
(*10MHZ)

<REAL PART OF THE DIELECTRICCONST>

-270.	1.0166	I
-240.	1.0191	I
-210.	1.0225	I
-180.	1.0265	I
-150.	1.0298	I
-120.	1.0310	I
-90.	1.0288	I
-60.	1.0221	I
-30.	1.0108	I
0.	0.9978	I
30.	0.9885	I
60.	0.9858	I
90.	0.9873	I
120.	0.9881	I
150.	0.9847	I
180.	0.9793	I
210.	0.9734	I
240.	0.9712	I
270.	0.9715	I
300.	0.9731	I
330.	0.9758	I
360.	0.9790	I
390.	0.9818	I
420.	0.9842	I
450.	0.9859	I
480.	0.9873	I
510.	0.9884	I
540.	0.9893	I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =15.0DEGREE **

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRICCONST>

-270.	0.0002	I *
-240.	0.0004	I *
-210.	0.0010	I *
-180.	0.0023	I *
-150.	0.0047	I *
-120.	0.0081	I *
-90.	0.0124	I *
-60.	0.0168	I *
-30.	0.0196	I *
0.	0.0195	I *
30.	0.0164	I *
60.	0.0125	I *
90.	0.0107	I *
120.	0.0111	I *
150.	0.0117	I *
180.	0.0110	I *
210.	0.0084	I *
240.	0.0058	I *
270.	0.0040	I *
300.	0.0029	I *
330.	0.0022	I *
360.	0.0012	I *
390.	0.0005	I *
420.	0.0002	I *
450.	0.0001	I *
480.	0.0001	I *
510.	0.0001	I *
540.	0.0000	I *

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =15.0DEGREE **

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-270.	0.0700	I
-240.	0.0696	I
-210.	0.0691	I
-180.	0.0685	I
-150.	0.0680	I
-120.	0.0678	I
-90.	0.0681	I
-60.	0.0690	I
-30.	0.0707	I
0.	0.0730	I
30.	0.0746	I
60.	0.0748	I
90.	0.0743	I
120.	0.0741	I
150.	0.0747	I
180.	0.0757	I
210.	0.0768	I
240.	0.0770	I
270.	0.0770	I
300.	0.0767	I
330.	0.0762	I
360.	0.0757	I
390.	0.0752	I
420.	0.0749	I
450.	0.0746	I
480.	0.0744	I
510.	0.0742	I
540.	0.0741	I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

<*** EXPERIMENTAL CONDITIONS ***>

<* WALL EFFECTS ARE TAKEN INTO ACCOUNT *>

GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED
POLARIZATION IS PERPENDICULAR

RESONANCE WAVE LENGTH OF NA D2 LINE

= 0.589E-04 (CM)

SP/LITTING OF HYPERFINE DOUBLET AT GRAND STATE

= 177.2 (*10MHZ)

DENSITY OF VAPOR = 0.105E 16 (PER CUBIC CM)

VAPOR PRESSURE = 0.067 (MMHG)

TEMPERATURE OF GAS = 619.

HALF OF 1/E POINTS' WIDTH OF INCIDENT LIGHT

= 24.0 (*10MHZ)

1/E POINTS' DOPPLER WIDTH

= 227.2 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 1

= 4.3 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 2

= 2.9 (*10MHZ)

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-270.	1.0169	0.0002	1.0084	0.3E-01	53.9	0.1663
-240.	1.0195	0.0004	1.0097	0.1E-01	53.8	0.1651
-210.	1.0229	0.0010	1.0114	0.5E-02	53.7	0.1635
-180.	1.0265	0.0023	1.0132	0.2E-02	53.6	0.1616
-150.	1.0290	0.0045	1.0144	0.1E-02	53.5	0.1602
-120.	1.0287	0.0078	1.0144	0.7E-03	53.5	0.1602
-90.	1.0250	0.0115	1.0128	0.5E-03	53.6	0.1620
-60.	1.0181	0.0146	1.0096	0.4E-03	53.8	0.1653
-30.	1.0096	0.0160	1.0055	0.3E-03	54.1	0.1697
0.	1.0013	0.0158	1.0014	0.3E-03	54.5	0.1741
30.	0.9950	0.0144	0.9981	0.4E-03	54.7	0.1774
60.	0.9913	0.0126	0.9961	0.4E-03	54.9	0.1793
90.	0.9892	0.0112	0.9950	0.5E-03	55.0	0.1802
120.	0.9877	0.0102	0.9941	0.5E-03	55.1	0.1810
150.	0.9857	0.0095	0.9931	0.6E-03	55.2	0.1820
180.	0.9833	0.0085	0.9918	0.6E-03	55.3	0.1835
210.	0.9811	0.0068	0.9907	0.8E-03	55.4	0.1848
240.	0.9794	0.0049	0.9897	0.1E-02	55.4	0.1857
270.	0.9780	0.0035	0.9890	0.2E-02	55.5	0.1866
300.	0.9770	0.0026	0.9884	0.2E-02	55.5	0.1872
330.	0.9776	0.0016	0.9888	0.3E-02	55.5	0.1870
360.	0.9793	0.0010	0.9896	0.5E-02	55.4	0.1858
390.	0.9819	0.0005	0.9909	0.1E-01	55.3	0.1843
420.	0.9841	0.0002	0.9920	0.3E-01	55.2	0.1830
450.	0.9859	0.0001	0.9929	0.5E-01	55.2	0.1821
480.	0.9872	0.0001	0.9936	0.7E-01	55.1	0.1814
510.	0.9883	0.0001	0.9941	0.1E 00	55.1	0.1808
540.	0.9892	0.0000	0.9946	0.1E 00	55.0	0.1803

** ANGLE OF INCIDENCE =30.0DEGREE **

<FREQ>
(*10MHZ)

<REAL PART OF THE DIELECTRICCONST>

-270.	1.0169	I
-240.	1.0195	I
-210.	1.0229	I
-180.	1.0265	I
-150.	1.0290	I
-120.	1.0287	I
-90.	1.0250	I
-60.	1.0181	I
-30.	1.0096	I
0.	1.0013	I
30.	0.9950	I
60.	0.9913	I
90.	0.9892	I
120.	0.9877	I
150.	0.9857	I
180.	0.9833	I
210.	0.9811	I
240.	0.9794	I
270.	0.9780	I
300.	0.9770	I
330.	0.9776	I
360.	0.9793	I
390.	0.9819	I
420.	0.9841	I
450.	0.9859	I
480.	0.9872	I
510.	0.9883	I
540.	0.9892	I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =30.0DEGREE **

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRICCONST>

-270.	0.0002	I *
-240.	0.0004	I *
-210.	0.0010	I *
-180.	0.0023	I *
-150.	0.0045	I *
-120.	0.0078	I *
-90.	0.0115	I *
-60.	0.0146	I *
-30.	0.0160	I *
0.	0.0158	I *
30.	0.0144	I *
60.	0.0126	I *
90.	0.0112	I *
120.	0.0102	I *
150.	0.0095	I *
180.	0.0085	I *
210.	0.0068	I *
240.	0.0049	I *
270.	0.0035	I *
300.	0.0026	I *
330.	0.0016	I *
360.	0.0010	I *
390.	0.0005	I *
420.	0.0002	I *
450.	0.0001	I *
480.	0.0001	I *
510.	0.0001	I *
540.	0.0000	I *

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =30.0DEGREE **

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-270.	0.1663	I
-240.	0.1651	I
-210.	0.1635	I
-180.	0.1616	I
-150.	0.1602	I*
-120.	0.1602	I*
-90.	0.1620	I
-60.	0.1653	I
-30.	0.1697	I
0.	0.1741	I
30.	0.1774	I
60.	0.1793	I
90.	0.1802	I
120.	0.1810	I
150.	0.1820	I
180.	0.1835	I
210.	0.1848	I
240.	0.1857	I
270.	0.1866	I
300.	0.1872	I
330.	0.1870	I
360.	0.1858	I
390.	0.1843	I
420.	0.1830	I
450.	0.1821	I
480.	0.1814	I
510.	0.1808	I
540.	0.1803	I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

<*** EXPERIMENTAL CONDITIONS ***>

<* WALL EFFECTS ARE TAKEN INTO ACCOUNT *>

GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED

POLARIZATION IS PERPENDICULAR

RESONANCE WAVE LENGTH OF NA D2 LINE

= 0.589E-04 (CM)

SP/LITTING OF HYPERFINE DOUBLET AT GRAND STATE

= 177.2 (*10MHZ)

DENSITY OF VAPOR = 0.105E 16 (PER CUBIC CM)

VAPOR PRESSURE = 0.067 (MMHG)

TEMPERATURE OF GAS = 619.

HALF OF 1/E POINTS' WIDTH OF INCIDENT LIGHT

= 24.0 (*10MHZ)

1/E POINTS' DOPPLER WIDTH

= 227.2 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 1

= 4.3 (*10MHZ)

HALF OF HALF WIDTH OF PRESSURE BROADENED LINE 2

= 2.9 (*10MHZ)

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRACT ANGLE	REFLECTIVITY
-270.	1.0194	0.0002	1.1526	0.2E-04	90.0	0.9991
-240.	1.0229	0.0006	1.1526	0.2E-04	89.9	0.9982
-210.	1.0268	0.0018	1.1526	0.2E-04	89.8	0.9923
-180.	1.0306	0.0039	1.1526	0.2E-04	89.6	0.9802
-150.	1.0332	0.0068	1.1527	0.2E-04	89.4	0.9658
-120.	1.0343	0.0103	1.1527	0.2E-04	89.1	0.9479
-90.	1.0333	0.0144	1.1529	0.2E-04	88.7	0.9277
-60.	1.0299	0.0184	1.1531	0.2E-04	88.3	0.9081
-30.	1.0235	0.0219	1.1533	0.2E-04	88.0	0.8926
0.	1.0149	0.0242	1.1534	0.2E-04	87.9	0.8847
30.	1.0056	0.0251	1.1534	0.2E-04	87.8	0.8837
60.	0.9973	0.0245	1.1534	0.2E-04	87.9	0.8871
90.	0.9904	0.0231	1.1533	0.2E-04	88.0	0.8966
120.	0.9842	0.0214	1.1532	0.2E-04	88.2	0.9050
150.	0.9794	0.0195	1.1531	0.2E-04	88.4	0.9124
180.	0.9749	0.0172	1.1529	0.2E-04	88.6	0.9228
210.	0.9721	0.0142	1.1528	0.2E-04	88.8	0.9377
240.	0.9702	0.0113	1.1527	0.2E-04	89.1	0.9507
270.	0.9703	0.0086	1.1527	0.2E-04	89.3	0.9607
300.	0.9712	0.0062	1.1526	0.2E-04	89.5	0.9733
330.	0.9729	0.0039	1.1526	0.2E-04	89.7	0.9828
360.	0.9752	0.0022	1.1526	0.2E-04	89.8	0.9899
390.	0.9781	0.0010	1.1526	0.2E-04	89.9	0.9966
420.	0.9810	0.0003	1.1526	0.2E-04	90.0	0.9992
450.	0.9835	0.0001	1.1526	0.2E-04	90.0	0.9995
480.	0.9852	0.0001	1.1526	0.2E-04	90.0	0.9996
510.	0.9865	0.0001	1.1526	0.2E-04	90.0	0.9997
540.	0.9875	0.0001	1.1526	0.2E-04	90.0	0.9997

** ANGLE OF INCIDENCE =45.0DEGREE **

<FREQ>
(*10MHZ)

<REAL PART OF THE DIELECTRICCONST>

-270. 1.0194 I
-240. 1.0229 I
-210. 1.0268 I
-180. 1.0306 I
-150. 1.0332 I
-120. 1.0343 I
-90. 1.0333 I
-60. 1.0299 I
-30. 1.0235 I
0. 1.0149 I
30. 1.0056 I
60. 0.9973 I
90. 0.9904 I
120. 0.9842 I
150. 0.9794 I
180. 0.9749 I
210. 0.9721 I
240. 0.9702 I
270. 0.9703 I
300. 0.9712 I
330. 0.9729 I
360. 0.9752 I
390. 0.9781 I
420. 0.9810 I
450. 0.9835 I
480. 0.9852 I
510. 0.9865 I
540. 0.9875 I

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =45.0DEGREE **

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRICCONST>

-270.	0.0002	I*
-240.	0.0006	I *
-210.	0.0018	I *
-180.	0.0039	I *
-150.	0.0068	I *
-120.	0.0103	I *
-90.	0.0144	I *
-60.	0.0184	I *
-30.	0.0219	I *
0.	0.0242	I *
30.	0.0251	I *
60.	0.0245	I *
90.	0.0231	I *
120.	0.0214	I *
150.	0.0195	I *
180.	0.0172	I *
210.	0.0142	I *
240.	0.0113	I *
270.	0.0086	I *
300.	0.0062	I *
330.	0.0039	I *
360.	0.0022	I *
390.	0.0010	I *
420.	0.0003	I *
450.	0.0001	I *
480.	0.0001	I *
510.	0.0001	I *
540.	0.0001	I *

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =45.0DEGREE **

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-270.	0.9991	I
-240.	0.9982	I
-210.	0.9923	I
-180.	0.9802	I
-150.	0.9658	I
-120.	0.9479	I
-90.	0.9277	I
-60.	0.9081	I
-30.	0.8926	I
0.	0.8847	I *
30.	0.8837	I *
60.	0.8871	I *
90.	0.8966	I *
120.	0.9050	I *
150.	0.9124	I *
180.	0.9228	I *
210.	0.9377	I *
240.	0.9507	I *
270.	0.9607	I *
300.	0.9733	I *
330.	0.9828	I *
360.	0.9899	I *
390.	0.9966	I *
420.	0.9992	I *
450.	0.9995	I *
480.	0.9996	I *
510.	0.9997	I *
540.	0.9997	I *

* GAUSSIAN BEAM WITH A NARROW LINE WIDTH IS INCIDENT & SCANNED *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =10.0DEGREE **

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-270.	0.0614	I	*
-240.	0.0612	I	*
-210.	0.0607	I	*
-180.	0.0601	I	*
-150.	0.0597	I	*
-120.	0.0595	I	*
-90.	0.0597	I	*
-60.	0.0604	I	*
-30.	0.0617	I	*
0.	0.0638	I	*
30.	0.0660	I	*
60.	0.0660	I	*
90.	0.0650	I	*
120.	0.0646	I	*
150.	0.0651	I	*
180.	0.0662	I	*
210.	0.0674	I	*
240.	0.0677	I	*
270.	0.0676	I	*
300.	0.0673	I	*
330.	0.0667	I	*
360.	0.0662	I	*
390.	0.0658	I	*
420.	0.0655	I	*
450.	0.0653	I	*
480.	0.0651	I	*
510.	0.0650	I	*
540.	0.0649	I	*

* WHITE LIGHT IS INCIDENT *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

** ANGLE OF INCIDENCE =30.0DEGREE **

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-270.	0.1664	I
-240.	0.1652	I
-210.	0.1635	I
-180.	0.1615	I
-150.	0.1600	I *
-120.	0.1598	I *
-90.	0.1617	I
-60.	0.1651	I
-30.	0.1697	I
0.	0.1743	I
30.	0.1776	I
60.	0.1795	I
90.	0.1802	I
120.	0.1808	I
150.	0.1820	I
180.	0.1834	I
210.	0.1851	I
240.	0.1859	I
270.	0.1866	I
300.	0.1873	I
330.	0.1874	I
360.	0.1858	I
390.	0.1842	I
420.	0.1830	I
450.	0.1820	I
480.	0.1813	I
510.	0.1808	I
540.	0.1803	I

* WHITE LIGHT IS INCIDENT *

* THE CENTRAL FREQ OF ONE OF D2 LINES (F=2) IS AT FREQ=0. WHEREAS THE OTHER (F=1) IS AT 177.2 *

* THE WALL EFFECT IS TAKEN INTO ACCOUNT *

APPENDIX B

```

1  $$JOUT,MONI,T
2  $:IDENT:RPHYA001,SATSPECT
3  $:OPTION:FORTTRAN
4  $:FORTTRAN
5  C  CALCULATION OF A LINE SHAPE OF A WEAK PROBING BEAM REFLECTED
6  C  FROM A SOLID-NA GAS INTERFACE
7  C  THE STRONG BEAM IS TUNED TO ONE OF NA D2 LINES (F=2, F=J+I,
8  C  J=1/2 & I=3/2), WHEN THE WEAK BEAM IS WHITE
9  C
10 C
11 C  SIMPSON'S METHOD IS USED IN NUMERICAL INTEGRATIONS
12 C
13 C  THE STRONG AND THE WEAK BEAM ARE ALSO SWEEPED ACROSS THE
14 C  RESONANCE LINE WHEN THE WEAK BEAM IS ALSO NARROW
15 C
16 C          *CONTROL INTEGERS*
17 C  IHOLE  =0,CALCULATION FOR WITUOT THE STRONG BEAM
18 C  IHOLE  =1,CALCULATION FOR THE HOLE STRACTURE
19 C  ICONTL  SEE SUBROUTINE SUSCEP
20 C  FOR IPRGM =0,THE PROBING BEAM IS NARROW
21 C  FOR IPRGM =1,THE PROBING BEAM IS WHITE
22 C  FOR IPRGM =2,3,4 SEE SUBROUTINE GRAPH
23 C
24 C          *CONSTANTS*
25 C
26 C  REWAVE  = RESONANCE WAVE LENGTH OF NA D2 LINE
27 C  GASDEN  = DENSITY OF GAS VAPOR (PER CUBIC CM)
28 C  MOLWEI  = MOLCULAR WEIGHT OF ATOM
29 C  HWI     = HALF OF 1/E POINTS WIDTH OF INCIDENT LIGHT(*10MHZ)
30 C  HWN1    = HALF OF HALF WIDTH OF RESONANCE LINE(*10MHZ)
31 C  SORIND  = REFRACTIVE INDEX OF GLASS
32 C  OS1     = OSCIRATOR STLENGTH 1
33 C  TEMPER  = GASTEMPERATURE
34 C  XLIGHT  = VELOCITY OF LIGHT
35 C  PLANK   = PLANK CONSTANT
36 C  ELMASS  = ELECTRON MASS
37 C  GASCON  = GAS CONSTANT
38 C  XLOSHM  = LOSHMIDT'S NUMBER
39 C  AANGLE  = ANGLE OF INCIDENCE (DEGREE)
40 C  ANGINC  = ANGLE OF INCIDECE (RADIAN)
41 C  EFDOP   = 1/E POINTS WIDTH OF DOPPLER SHIFTED RESONANCE FLEQ
42 C  PAI     = PAI
43 C  VWGRD3  =A CONSTANT IN THE POTENTIAL ENERGY OF THE FORM,C/Z**3
44 C           IT IS EXPRESSED IN THE DEBYE UNIT "D**2"
45 C           WHERE D**2 IS 1.E-36 ERG*CM**3
46 C  IPOWER  =THE POWER FACTOR OF THE NORMAL DISTANCE FROM
47 C           THE SURFACE IN THE POTENTIAL
48 C  SHIFAC  =A CONSTANT APPEARING IN THE POTENTIAL ENERGY EXPRESSION
49 C           "SHIFAC*C/Z**3" FOR THE EXCITED ATOM
50 C  VDISMI  =A DISTANCE (FROM THE SURFACE) WHERE THE POTENTIAL
51 C           DIP IS MINIMUM
52 C  YFRELO  =FREQ SHIFT OF NATURAL RESONANCE LINE CORRESPONDING TO
53 C           THE MINIMUM OF THE POTENTIAL DIP
54 C  ZHOLE   =LOCATION OF THE HOLE(SATURATED ATOMS)
55 C

```

```

1  DIMENSION  Z(58),REIND(58),DIERE(58),DIECM(58),REF(58),
2  &  ANGREN(58),DEPTHN(58),REFLOG(58)

```

```

3  GASDEN=1.05E+15

```

```

4  HWI=6.

```

```

5  YFREQS=20.0

```

```

6  TEMPER=619.

```

```

7  POLARI=+1

```

```

8  IPROGM=0

```

```

9  ICONTL=1

```

```

10 REWAVE=5.890E-5

```

```

11 XMOLWE=22.9898

```

```

12 SATFAC=0.6

```

```

13 VWGRD3=4.

```

```

14 IPOWER=3

```

```

15 PX=1./FLOAT(IPOWER)

```

```

16 XE24=1.E+8

```

```

17 SORIND=1.63

```

```

18 OS1 =0.42

```

```

19 SHIFAC=2.4

```

```

20 VDISMI=5.

```

```

21 XLIGHT=2.99793E+10

```

```

22 PLANK =1.05459E-27

```

```

23 ELMASS=9.10953E-28

```

```

24 GASCON=8.31441E+7

```

```

25 BOLTZC=1.38042E-16

```

```

26 VWGX=1.E-36

```

```

27 VWGRAX=VWGRD3*VWX

```

```

28 VWFRSH=SHIFAC*VWGRAX/PLANK

```

```

29 BCXX=PLANK/BOLTZC/TEMPER

```

```

30 BCX=BCXX/SHIFAC

```

```

31 XE12HZ=10.E+11

```

```

32 BC=BCX*XE12HZ

```

```

33 XANGST=1.E-24

```

```

34 PAI =3.14159

```

```

35 XLOSHM=2.68675E+19

```

```

36 DOP =2.*GASCON*TEMPER/XMOLWE

```

```

37 EFCDOP=2.*SQRT(DOP)/REWAVE

```

```

38 XMHZ =10.E+6

```

```

39 XEM7=1.E-7

```

```

40 HWN1=0.8142*XEM7*GASDEN/XMHZ/2.

```

```

41 EFDOP =EFCDOP/XMHZ

```

```

42 DISTM3=VWFRSH/XANGST/XMHZ/YFREQS

```

```

43 DISSHI=CBRT(DISTM3)

```

```

44 YFRELO=-VWFRSH/XANGST/XE12HZ/VDISMI**3

```

```

45 VWC=YFREQS*BC

```

```

46 VWCONS=EXP(VWC)

```

```

47 VLOW=2.*HWN1

```

```

48 VHIG=200.

```

```

49 VWID=VHIG-VLOW

```

```

50 FNVV=VWID/10.

```

```

51 NVV=IFIX(FNVV)

```

```

52 FNV=2.*FLOAT(NVV)

```

```

53 NV=IFIX(FNV)

```

```

54 HV=VWID/FNV

```

```

1      VALPX3=VWFRSH*XE24/XMHZ
2
3      C TO ESTIMATE THE ENERGY FLUX(WATT/CM**2) OF THE STRONG BEAM,
4      C THE COLISION RATE IS ASSUMED TO BE 10MHZ
5      ENGFLX=SATFAC*137.036*2.*XLIGHT*ELMASS*2.*HWN1*XMHZ/REWAVE/DS1
6
7      C
8      C CONST=CONSTANT APPERING IN SUSCEPTIBILITY
9      C
10     CONST1=FLANK*GASDEN*REWAVE
11     CONST2=137.036*ELMASS*SQRT(PI)*4.*EFCIDP*PI**2
12     CONST =CONST1/CONST2
13     WRITE(6,899)CONST
14     IPOLA=IFIX(POLARI)
15     899 FORMAT(1H1,///3X,"THIS IS A CONSTANT APPERING IN",
16     &      " SUSCEPTIBILITY",/23X,E12.5)
17
18     C
19     C VAPOR=VAPOR PRESSURE IN UNIT OF MMHG
20     C
21     VAPOR =760.*TEMPER*GASDEN/XLOSHM/273.2
22
23     NA=8
24     FNA=FLOAT(NA)
25     HA=3.2*HWI/FNA
26
27     CALL POWER(AVEFLX,ENGFLX,NA,HA,HWI)
28     AANGLE=50.
29     ANGINC=AANGLE*PI/180.
30     SORSIN=SORIND*SIN(ANGINC)
31     CN=CONST/SORSIN
32     DOPS=EFDOP*SORSIN
33     DLOW=-0.8*DOPS
34     DHIG=0.8*DOPS
35     EFHWI=1.6*HWI
36     FNDD=DHIG/1.
37     NDD=IFIX(FNDD)
38     FND=2.*FLOAT(NDD)
39     ND=IFIX(FND)
40     HD=2.*DHIG/FND
41     FRESHI=-YFREQS
42     ZHOLE=0.
43
44     C A STRONG & A WEAK BEAM WITH A NARROW LINE WIDTH ARE SIMULTANEOUSLY
45     C SCANNED OVER THE F=2 LINE SO THAT THE DETECTED HOLE IS ALWAYS
46     C AT THE CENTRAL FREQ OF THE UNSHIFTED NATURAL RESONANCE LINE
47     C
48     C THE WALL-GAS INTERACTIONS ARE NOT TAKEN INTO ACCOUNT
49
50     IFREQ=30
51     FSC=2.
52     RATIO=FLOAT(IFREQ)/FSC+0.5
53     RATIO2=2.*RATIO
54     IC=IFIX(RATIO2)
55     DO 200 IZ=1,IC
56     IIZ=IZ-1
57     FIFREQ=-FLOAT(IFREQ)
58     Z(IZ)=FIFREQ+FLOAT(IIZ)*FSC
59     SLOW=Z(IZ)-EFHWI

```

```

C      A CONVOLUTION INTEGRATION W.R.T. A NARROW WEAK BEAM STARTS
      SB=0.
      SBG=0.
      SSUSR=0.
      SSUSC=0.
      DO 600 JB=1,NA+1,2
      JBB=JB-1
      HB=HA*FLOAT(JBB)
      XB=SLOW+HB
      SMB=0.
      SMBG=0.
      SMSUSR=0.
      SMSUSC=0.
      DO 650 LLB=1,2
      LB=LLB-1
      FL=FLOAT(LB)
      XA=XB+HA*FL
      CALL GAUSIA(AIGAUS,Z(IZ),XA,HWI,PAI)
      GAUS=AIGAUS
      IHOLE=0
      CALL SUSCER(RESUM,CMSUM,XA,ND,HD,
&      DLOW,CN,OS1,HWN1,DOPS,
&      ICONTL,IHOLE,SATFAC,FRESHI,IPOGM,Z(IZ),HWI)
      RESUMD=RESUM
      CMSUMD=CMSUM
      IHOLE=1
      CALL SUSCER(RESUM,CMSUM,XA,ND,HD,
&      DLOW,CN,OS1,HWN1,DOPS,
&      ICONTL,IHOLE,SATFAC,FRESHI,IPOGM,Z(IZ),HWI)
      RESUMH=RESUM
      CMSUMH=CMSUM
      RESUM1=RESUMD-RESUMH
      CMSUM1=CMSUMD-CMSUMH
      CALL REFINO(REINDX,REFREX,REFCMX,ALFAX,BETAX,RESUM1,CMSUM1,
&      SORIND,ANGINC,PAI)
      REIND1=REINDX
      DIECM1=REFCMX
      DIERE1=REFREX
      ALFAN1=ALFAX
      BETAN1=BETAX
      XANGLE=SORSIN/REIND1
      ANGREF=ARSIN(XANGLE)
      CALL REFLEC(ALREFLFX,REFLFX,ALFAN1,BETAN1,ANGINC,ANGREF,
&      POLARI,SORIND)
      REFLF=REFLFX
      IF(JBB.EQ.0)GO TO 602
      IF(JBB.EQ.NA)GO TO 603
      IF(LB.EQ.0)MLB=2
      IF(LB.EQ.1)MLB=4
      GO TO 604
602  IF(LB.EQ.0)MLB=1
      IF(LB.EQ.1)MLB=4
      GO TO 604
603  IF(LB.EQ.0)MLB=1

```

```

1
2 IF(LB.EQ.1)MLB=0
3 604 FML=FLOAT(MLB)
4 IF(JBB.EQ.NA.AND.LB.EQ.1) GO TO 605
5 SMSUSR=SMSUSR+RESUM1
6 SMSUSC=SMSUSC+CMSUM1
7 605 SMB=SMB+FML*GAUS*REFLF
8 SMBG=SMBG+FML*GAUS
9 650 CONTINUE
10 SB=SB+SMB
11 SBG=SBG+SMBG
12 SSUSR=SSUSR+SMSUSR
13 SSUSC=SSUSC+SMSUSC
14 600 CONTINUE
15 SUMB=HA*SB/3.
16 SUMBG=HA*SBG/3.
17 REF(IZ)=SUMB/SUMBG
18 ARESUM=SSUSR/(FNA+1.)
19 ACMSUM=SSUSC/(FNA+1.)
20 C CALCULATION OF THE AVERAGE REFRACTIVE INDEX AND DIELECTRIC
21 C CONSTANT
22 CALL REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,ARESUM,
23 & ACMSUM,SORIND,ANGINC,PAI)
24 REIND(IZ)=REINDX
25 DIECM(IZ)=REFCMX
26 DIERE(IZ)=REFREX
27 ALFAN=ALFAX
28 BETAN=BETAX
29 DEPTHN(IZ)=REWAVE/BETAN/PAI/2.
30 ANGLE2=SORSIN/REIND(IZ)
31 ANGREN(IZ)=180.*ARSIN(ANGLE2)/PAI
32 200 CONTINUE
33 IF(INTRAC.EQ.0)WRITE(6,903)AANGLE
34 IF(INTRAC.EQ.1)WRITE(6,904)AANGLE
35 WRITE(6,908)VAPOR,TEMPER,HWI,HWN1,ZHOLE,
36 & AVEFLX,SATFAC
37 WRITE(6,906)
38 WRITE(6,909)
39 WRITE(6,910) (Z(IZ),DIERE(IZ),DIECM(IZ),REIND(IZ),DEPTHN(IZ),
40 & ANGREN(IZ),REF(IZ),IZ=1,IC)
41 WRITE(6,955)
42 CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,DIERE)
43 WRITE(6,956)
44 CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,DIECM)
45 WRITE(6,957)
46 CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,REIND)
47 WRITE(6,958)
48 CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,REF)
49 C THE PROBING BEAM IS WHITE, WHEN THE STRONG BEAM CREATES
50 C A HOLE AT VARIOS INCIDENT FREQ
51 C
52 4100 IVAN=3
53 IPOGM=1
54 ZERO=0.
55 DO 4600 IVANDE=1,IVAN

```

```

1      INTRAC=IVANDE-3
2      ZHOLE=0.
3
4      IF (INTRAC.EQ.-2) ZHOLE=-8.
5      IF (INTRAC.EQ.-1) ZHOLE=+16.
6      IFREQ=34
7      FSC=2.
8      FIFREQ=FLOAT(IFREQ)
9      RATIO=FIFREQ/FSC+0.5
10     RATIO2=2.*RATIO
11     IC=IFIX(RATIO2)
12     IFREQ=IFREQ-IFIX(ZHOLE)
13     IF (INTRAC.LE.-1) FIFREQ=FIFREQ-ZHOLE
14     DO 4000 IZ=1,IC
15         IIZ=IZ-1
16         SLOW=ZHOLE-EFHWI
17         SHIG=ZHOLE+EFHWI
18         IF (ICONTL.EQ.0) SLOW=ZHOLE-2.*EFHWI
19         IF (ICONTL.EQ.0) SHIG=ZHOLE+2.*EFHWI
20         Z(IZ)=-FIFREQ+FLOAT(IIZ)*FSC
21         SSL=Z(IZ)-SLOW
22         SSH=Z(IZ)-SHIG
23         IHOLE=0
24         CALL SUSCER(RESUM,CMSUM,Z(IZ),ND,HD,
25 &         DLOW,CN,OS1,HWN1,DOPS,ICONTL,IHOLE,
26 &         SATFAC,ZHOLE,IPOGM,Z(IZ),HWI)
27         RESUMD=RESUM
28         CMSUMD=CMSUM
29         IF (SSL.GE.ZERO.AND.SSH.LE.ZERO) IHOLE=1
30         IF (IHOLE.NE.1) GO TO 4120
31         CALL SUSCER(RESUM,CMSUM,Z(IZ),ND,HD,
32 &         DLOW,CN,OS1,HWN1,DOPS,ICONTL,IHOLE,
33 &         SATFAC,ZHOLE,IPOGM,Z(IZ),HWI)
34         RESUMH=RESUM
35         CMSUMH=CMSUM
36         RESUM1=RESUMD-RESUMH
37         CMSUM1=CMSUMD-CMSUMH
38         GO TO 4130
39
40 4120 RESUM1=RESUMD
41     CMSUM1=CMSUMD
42 4130 CALL REFINDE(REINDX,REFREX,REFCMX,ALFAX,
43 &     BETAX,RESUM1,CMSUM1,SORIND,ANGINC,PAI)
44     REIND(IZ)=REINDX
45     DIERE(IZ)=REFREX
46     DIECM(IZ)=REFCMX
47     ALFAN=ALFAX
48     BETAN=BETAX
49     DEPTHN(IZ)=REWAVE/BETAN/PAI/2.
50     ANGLE2=SORSIN/REIND(IZ)
51     ANGREN(IZ)=180.*ARSIN(ANGLE2)/PAI
52     CALL REFLEC(ALREFLFX,REFLFX,ALFAN,BETAN,ANGINC,ANGREF,
53 &     POLALI,SORIND)
54     REF(IZ)=REFLFX
55 4000 CONTINUE
56     IF (INTRAC.LE.0) WRITE(6,901) ANGLE

```



```

1      IF (INTRAC.EQ.1) WRITE(6,902) AANGLE
3      IF (IPOLA.LT.0) WRITE(6,898)
4      IF (IPOLA.GT.0) WRITE(6,897)
5      IF (IPOLA.EQ.0) WRITE(6,896)
6      WRITE(6,908) VAPOR, TEMPER, HWI, HWN1, ZHOLE,
7      & AVEFLX, SATFAC
8      WRITE(6,909)
9      WRITE(6,910) (Z(IZ), DIERE(IZ), DIECM(IZ), REIND(IZ), DEPTHN(IZ),
10     & ANGREN(IZ), REF(IZ), IZ=1, IC)
11     WRITE(6,955)
12     CALL GRAPH(ZHOLE, IPROGM, IC, IFREQ, FSC, DIERE)
13     WRITE(6,956)
14     CALL GRAPH(ZHOLE, IPROGM, IC, IFREQ, FSC, DIECM)
15     WRITE(6,957)
16     CALL GRAPH(ZHOLE, IPROGM, IC, IFREQ, FSC, REIND)
17     WRITE(6,958)
18     CALL GRAPH(ZHOLE, IPROGM, IC, IFREQ, FSC, REF)

```

```

19 4600 CONTINUE

```

```

20 C THE SOLID-GAS INTERACTIONS ARE TAKEN INTO ACCOUNT

```

```

21 C

```

```

22 C A HOLE IS FIRST CREATED BY THE STRONG BEAM AT "ZHOLE"

```

```

23 C THEN THE INTENSITY DISTRIBUTION OF THE WEAK WHITE

```

```

24 C PROBING LIGHT IS INVESTIGATED

```

```

25 1100 IP=5

```

```

26     ZERO=0.

```

```

27     IPROGM=1

```

```

28     ZHOLE=-40.

```

```

29     DO 1600 IPW=3, IP

```

```

30     IPOW=IPW-1

```

```

31     IF (IPOW.EQ.2) VWGRND=VWGRD3/VDISMI

```

```

32     IF (IPOW.EQ.3) VWGRND=VWGRD3

```

```

33     IF (IPOW.EQ.4) VWGRND=VWGRD3*VDISMI

```

```

34     IF (IPOW.EQ.2) VALPX=VALPX3/VDISMI

```

```

35     IF (IPOW.EQ.3) VALPX=VALPX3

```

```

36     IF (IPOW.EQ.4) VALPX=VALPX3*VDISMI

```

```

37     PX=1./FLOAT(IPOW)

```

```

38     VALFAX=POW(VALPX, PX)

```

```

39     PVX=PX*VALFAX

```

```

40     CALL VWNORM(VWCCP, PX, PVX, NV, HV, VLOW)

```

```

41     IFREQ=68

```

```

42     FSC=4.

```

```

43     FIFREQ=FLOAT(IFREQ)

```

```

44     RATIO=FIFREQ/FSC+0.5

```

```

45     RATIO2=2.*RATIO

```

```

46     IC=IFIX(RATIO2)

```

```

47     IFREQ=IFREQ-IFIX(ZHOLE)

```

```

48     FIFREQ=FIFREQ-ZHOLE

```

```

49     DO 2000 IZ=1, IC

```

```

50     IIZ=IZ-1

```

```

51     Z(IZ)=-FIFREQ+FLOAT(IIZ)*FSC

```

```

52     IHOLE=0

```

```

53     CALL SUSCEV(RESUM, CMSUM, Z(IZ), ND, HD, NV, HV,

```

```

54     & DLOW, VLOW, CN, OS1, HWN1, DOPS, IHOLE,

```

```

55     & SATFAC, ZHOLE, IPROGM, HWI, PX, PVX, VWCCP)

```

```

RESUMD=RESUM
CMSUMD=CMSUM
IHOLE=1
CALL SUSCEV(RESUM,CMSUM,Z(IZ),ND,HD,NV,HV,
& DLOW,VLOW,CN,OS1,HWN1,DOPS,IHOLE,
& SATFAC,ZHOLE,IPOGM,HWI,FX,PVX,VWCOCP)
RESUMH=RESUM
CMSUMH=CMSUM
RESUM1=RESUMD-RESUMH
CMSUM1=CMSUMD-CMSUMH
CALL REIND(REINDX,REFREX,REFCMX,ALFAX,
& BETAX,RESUM1,CMSUM1,SORIND,ANGINC,PAI)
REIND(IZ)=REINDX
DIERE(IZ)=REFREX
DIECM(IZ)=REFCMX
ALFAN=ALFAX
BETAN=BETAX
DEPTHN(IZ)=REWAVE/BETAN/PAI/2.
ANGLE2=SORSIN/REIND(IZ)
ANGREN(IZ)=180.*ARSIN(ANGLE2)/PAI
CALL REFLEC(ALREFLX,REFLFX,ALFAN,BETAN,ANGINC,ANGREF,
& POLALI,SORIND)
REF(IZ)=REFLFX
2000 CONTINUE
WRITE(6,902)AANGLE
IF(IPOLA.LT.0)WRITE(6,898)
IF(IPOLA.GT.0)WRITE(6,897)
IF(IPOLA.EQ.0)WRITE(6,896)
IF(IPOWER.EQ.2)WRITE(6,832)IPOWER,VWGRND
IF(IPOWER.EQ.3)WRITE(6,833)IPOWER,VWGRND
IF(IPOWER.GE.4)WRITE(6,834)IPOWER,VWGRND
WRITE(6,908)VAPOR,TEMPER,HWI,HWN1,ZHOLE,
& AVEFLX,SATFAC
WRITE(6,909)
WRITE(6,910) (Z(IZ),DIERE(IZ),DIECM(IZ),REIND(IZ),DEPTHN(IZ),
& ANGREN(IZ),REF(IZ),IZ=1,IC)
WRITE(6,955)
IPOGM=2
CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,DIERE)
WRITE(6,956)
CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,DIECM)
WRITE(6,957)
CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,REIND)
WRITE(6,958)
CALL GRAPH(ZHOLE,IPOGM,IC,IFREQ,FSC,REF)
IPOGM=1
1600 CONTINUE
C
C THE STRONG & WEAK NARROW BEAMS ARE SIMULTANEOUSLY SCANNED
C
C THUS THE INTENSITY DISTRIBUTION OF THE PEAK(DETECTED BY THE
C WEAK BEAM) IS OBSERVED
C
5100 FNDD=DHIG/0.8

```

```

1      ICONTL=0
2      NDD=IFIX(FNDD)
3      FND=2.*FLOAT(NDD)
4      ND=IFIX(FND)
5      HD=2.*DHIG/FND
6      IPROGM=3
7      FREQ12=-YFRELO
8      ZERO=0.
9      IHOLE=0
10     CALL SUSCER(RESUM,CMSUM,ZERO,ND,HD,
11     &          DLOW,CN,OS1,HWN1,DOPS,ICONTL,IHOLE,
12     &          SATFAC,ZERO,IPROGM,ZERO,HWI)
13     RESUMD=RESUM
14     IHOLE=1
15     CALL SUSCER(RESUM,CMSUM,Z,ND,HD,
16     &          DLOW,CN,OS1,HWN1,DOPS,ICONTL,IHOLE,
17     &          SATFAC,ZERO,IPROGM,ZERO,HWI)
18     RESUMH=RESUM
19     RESUM1=RESUMD-RESUMH
20     FIS=0.8
21     ISHIFT=2
22     DO 3200 IIVW=2,ISHIFT
23     SHIFAC=FIS*FLOAT(IIVW)
24     BCX=BCXX/SHIFAC
25     BC=BCX*XE12HZ
26     CMSUM11=1.-0.5*SATFAC/(SATFAC+1.)
27     CMSUM1=CMSUM11*CN/HWN1
28     FSC=4.
29     IFREQ=IFIX(FREQ12)
30     RATIO=FREQ12/FSC+0.5
31     IC=IFIX(RATIO)
32     DO 3100 IZ=1,IC
33     IIZ=IZ-1
34     Z(IZ)=-FREQ12+FLOAT(IIZ)*FSC
35     FREZIZ=BC*Z(IZ)
36     FUNZIZ=EXP(-FREZIZ)
37     RESUMZ=FUNZIZ*RESUM1
38     CMSUMZ=FUNZIZ*CMSUM1
39     CALL REFINO(REINDX,REFREX,REFCMX,ALFAX,
40     &          BETAX,RESUMZ,CMSUMZ,SORIND,ANGINC,PAI)
41     REIND(IZ)=REINDX
42     DIERE(IZ)=REFREX
43     DIECM(IZ)=REFCMX
44     ALFAN=ALFAX
45     BETAN=BETAX
46     DEPTHN(IZ)=REWAVE/BETAN/PAI/2.
47     ANGLE2=SORSIN/REIND(IZ)
48     ANGREN(IZ)=180.*ARSIN(ANGLE2)/PAI
49     CALL REFLEC(ALREFLFX,REFLFX,ALFAN,BETAN,ANGINC,ANGREF,
50     &          POLALI,SORIND)
51     REF(IZ)=REFLFX
52 3100 CONTINUE
53     WRITE(6,902)ANGLE
54     IF(IPOLA,LT,0)WRITE(6,898)

```

```

1 IF(IPOLA.GT.0)WRITE(6,897)
2 IF(IPOLA.EQ.0)WRITE(6,896)
3 IF(IPOWER.EQ.2)VWGRND=VWGRD3/VDISMI
4 IF(IPOWER.EQ.3)VWGRND=VWGRD3
5 IF(IPOWER.EQ.4)VWGRND=VWGRD3*VDISMI
6 IF(IPOWER.EQ.2)WRITE(6,832)IPOWER,VWGRND
7 IF(IPOWER.EQ.3)WRITE(6,833)IPOWER,VWGRND
8 IF(IPOWER.GE.4)WRITE(6,834)IPOWER,VWGRND
9 WRITE(6,1908)VAFOR,TEMPER,HWI,HWN1,YFRELO,
10 & SHIFAC
11 WRITE(6,907)
12 WRITE(6,1909)
13 WRITE(6,910) (Z(IZ),DIERE(IZ),DIECM(IZ),REIND(IZ),DEPTHN(IZ),
14 & ANGREN(IZ),REF(IZ),IZ=1,IC)
15 WRITE(6,956)
16 CALL GRAPH(ZHOLE,IPROGM,IC,IFREQ,FSC,DIECM)
17 WRITE(6,958)
18 CALL GRAPH(ZHOLE,IPROGM,IC,IFREQ,FSC,REF)
19
20 3200 CONTINUE
21 C THE INTENSITY DISTRIBUTION OF THE PEAK INTENSITY OF THE
22 C REFLECTED NARROW WEAK BEAM
23 C
24 C THE EFFECT OF VARIOUS POWER FACTORS P=2,3,4 ARE INVESTIGATED
25 C FOR THE SAME VALE OF THE POTENTIAL DEAPTH (CORRESPONDING TO
26 C THE MINIMUM OF THE POTENTIAL DIP) AT THE DISTANCE
27 C 5*E-8 CM AWAY FROM THE WALL SURFACE
28 IP=5
29 FREQ7=180.
30 HWI=20.
31 NA=10
32 HA=3.6*HWI/FLOAT(NA)
33 CMSUM11=0.5*SATFAC/(SATFAC+1.)
34 ZERO=0.
35 DO 5600 IPW=3,IP
36 IPOWER=IPW-1
37 IF(IPOWER.EQ.2)VWGRND=VWGRD3/VDISMI
38 IF(IPOWER.EQ.3)VWGRND=VWGRD3
39 IF(IPOWER.EQ.4)VWGRND=VWGRD3*VDISMI
40 IF(IPOWER.EQ.2)VALPX=VALPX3/VDISMI
41 IF(IPOWER.EQ.3)VALPX=VALPX3
42 IF(IPOWER.EQ.4)VALPX=VALPX3*VDISMI
43 PX=1./FLOAT(IPOWER)
44 VALFAX=POW(VALPX,PX)
45 PVX=PX*VALFAX
46 CALL VWNORM(VWCOCF,PX,PVX,NV,HV,VLOW)
47 VWCP=VWCOCF
48 IFREQ=IFIX(FREQ7)
49 FSC=4.
50 RATIO=(FREQ7-2.*HWI)/FSC+0.5
51 IC=IFIX(RATIO)
52 DO 5000 IZ=1,IC
53 IIZ=IZ-1
54 Z(IZ)=-FREQ7+FLOAT(IIZ)*FSC
55 VLOW1=Z(IZ)-1.6*HWI

```

```

1      AVLOW=ABS(VLOW1)
3      CALL VWNORM(VWCOCP,PX,PVX,NA,HA,AVLOW)
4      FUNZIZ=VWCOCP/VWCP
5      CMSUM1=CN*(1.-FUNZIZ*CMSUM11)/HWN1
1130   CALL REFINDEX(REINDX,REFREX,REFCMX,ALFAX,
6      &    BETAX,RESUM1,CMSUM1,SORIND,ANGINC,PAI)
7      REIND(IZ)=REINDX
8      DIERE(IZ)=REFREX
9      DIECM(IZ)=REFCMX
10     ALFAN=ALFAX
11     BETAN=BETAX
12     DEPTHN(IZ)=REWAVE/BETAN/PAI/2.
13     ANGLE2=SORSIN/REIND(IZ)
14     ANGREN(IZ)=180.*ARSIN(ANGLE2)/PAI
15     CALL REFLEC(ALREFLFX,REFLFX,ALFAN,BETAN,ANGINC,ANGREF,
16     &    POLALI,SORIND)
17     REFLOG(IZ)=ALREFLFX
18     REF(IZ)=REFLFX
19     REFLOG(IZ)=ALREFLFX
20
5000   CONTINUE
21
22     WRITE(6,902)AANGLE
23     WRITE(6,960)
24     IF(IPOLA.LT.0)WRITE(6,898)
25     IF(IPOLA.GT.0)WRITE(6,897)
26     IF(IPOLA.EQ.0)WRITE(6,896)
27     IF(IPOWER.EQ.2)WRITE(6,832)IPOWER,VWGRND
28     IF(IPOWER.EQ.3)WRITE(6,833)IPOWER,VWGRND
29     IF(IPOWER.GE.4)WRITE(6,834)IPOWER,VWGRND
30     WRITE(6,908)VAPOR,TEMPER,HWI,HWN1,ZHOLE,
31     &    AVEFLX,SATFAC
32     WRITE(6,909)
33     WRITE(6,910) (Z(IZ),DIERE(IZ),DIECM(IZ),REIND(IZ),DEPTHN(IZ),
34     &    ANGREN(IZ),REF(IZ),IZ=1,IC)
35     WRITE(6,956)
36     IPROGM=4
37     CALL GRAPH(ZHOLE,IPROGM,IC,IFREQ,FSC,DIECM)
38     WRITE(6,958)
39     CALL GRAPH(ZHOLE,IPROGM,IC,IFREQ,FSC,REF)
40     WRITE(6,959)
41     CALL GRAPH(ZHOLE,IPROGM,IC,IFREQ,FSC,REFLOG)
42
5600   CONTINUE
43
833   FORMAT(1H0,5X,"** VAN DER WAAL'S POTENTIAL ENERGY HAS THE",
44   &    " FORM OF THE INVERSE **",/10X,I1,"RD POWER OF THE NORMAL",
45   &    " DISTANCE FROM THE SURFACE",/16X,"WITH A CONSTANT",F5.1,
46   &    " IN UNIT OF E-12*ERG PER A**3",/)
47
832   FORMAT(1H0,5X,"** VAN DER WAAL'S POTENTIAL ENERGY HAS THE",
48   &    " FORM OF THE INVERSE **",/10X,I1,"ND POWER OF THE NORMAL",
49   &    " DISTANCE FROM THE SURFACE",/16X,"WITH A CONSTANT",F5.1,
50   &    " IN UNIT OF E-12*ERG PER A**2",/)
51
834   FORMAT(1H0,5X,"** VAN DER WAAL'S POTENTIAL ENERGY HAS THE",
52   &    " FORM OF THE INVERSE **",/10X,I1,"TH POWER OF THE NORMAL",
53   &    " DISTANCE FROM THE SURFACE",/16X,"WITH A CONSTANT",F5.1,
54   &    " IN UNIT OF E-12*ERG PER A**4",/)
55
898   FORMAT(1H0,23X,"POLARIZATION IS PARALLEL")

```

```

897  FORMAT(1H0,23X,"POLARIZATION IS PERPENDICULAR")
896  FORMAT(1H0,23X,"PLANE POLARIZED LIGHT AT 45 DEGREE")
908  FORMAT(1H0,17X,"*< EXPERIMENTAL CONDITIONS >*", /
&    1H0,2X,"<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH>",
&    " <LOC HOLE> <ENG FLX STG> <SATFAC>",/3X,"(MMHG)",
&    " (KELVIN) <HWI> <HWN1> (*10MHZ)",3X,"(*10MHZ)",
&    3X,"(WATT/CM**2)",/1H0,
&    3X,F5.3,4X,F4.0,4X,F4.1,3X,F4.1,
&    11X,F6.1,6X,E10.3,1X,F6.2,/)
1908 FORMAT(1H0,17X,"*< EXPERIMENTAL CONDITIONS >*", /
&    1H0,2X,"<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH>",
&    " <MIN REFL OCCURED AT FREQ> <SHIFAC>",/3X,"(MMHG) ",
&    " (KELVIN) <HWI> <HWN1> (*10MHZ)",10X,"(*E12HZ)",4X,/1H0,
&    3X,F5.3,4X,F4.0,4X,F4.1,3X,F4.1,18X,F6.2,14X,F3.1,/)
901  FORMAT(1H1,///3X,"*** A WHITE PROBING BEAM INCIDENT ON",
&    " A GLASS-NA GAS INTERFACE ***",/20X,"AT AN ",
&    " INCIDENT ANGLE OF ",F4.1," DEGREE",/)
903  FORMAT(1H1,///3X,"*** A NARROW WEAK PROBING BEAM INCIDENT",
&    " ON A GLASS-NA GAS INTERFACE ***",/20X,
&    " AT AN INCIDENT ANGLE OF ",F4.1," DEGREE",/)
902  FORMAT(1H1,///,"*** THE STRONG & WEAK BEAMS INCIDENT ON A",
&    " THIN METAL LAYER DEPOSITTED ON ***",/10X,
&    " A GLASS-NA GAS INTERFACE AT AN INCIDENT ",
&    " ANGLE OF ",F4.1," DEGREE",/)
904  FORMAT(1H1,///3X,"*** A NARROW WEAK PROBING BEAM INCIDENT",
&    " ON A THIN METAL LAYER DEPOSIT-",/10X,
&    " TED ON A GLASS-NA GAS AT AN INCIDENT ANGLE OF ",F4.1,
&    " DEGREE",/)
906  FORMAT(1H0,13X,"LINE SHAPE OF THE HOLE IS EXAMINED",/)
907  FORMAT(1H0,13X,"LINE SHAPE OF THE HOLE ISN'T EXAMINED",/)
910  FORMAT(1H ,4X,F5.0,4X,F8.6,1X,F8.6,1X,F8.6,1X,E8.2,2X,F4.1,
&    4X,F8.6)
909  FORMAT(1H0,"INCIDENT FREQ",1X,"DIELECTRIC CONST",2X,"REFRACT",
&    4X,"PENET",2X,"REFRCT",2X,"REFLECTIVITY",/2X,"(*10MHZ)",
&    6X,"REAL",4X,"IMAG",5X,"INDEX",6X,"DEPTH",3X,"ANGLE",/)
955  FORMAT(1H1,///"<FREQ>",34X,"<REAL PART OF THE DIELECTRIC",
&    " CONST>",/)
1909 FORMAT(1H0,"INCIDENT FREQ",1X,"DIELECTRIC CONST",2X,"REFRACT",
&    4X,"PENET",2X,"REFRCT",2X,"REFLECTIVITY",/2X,"(*E12HZ)",
&    6X,"REAL",4X,"IMAG",5X,"INDEX",6X,"DEPTH",3X,"ANGLE",/)
956  FORMAT(1H1,///"<FREQ>",34X,"<IMAG PART OF THE DIELECTRIC",
&    " CONST>",/)
957  FORMAT(1H1,///"<FREQ>",34X,"<REFRACTIVE INDEX>",/)
958  FORMAT(1H1,///"<FREQ>",34X,"<REFLECTIVITY>",/)
959  FORMAT(1H1,///"<FREQ>",20X,"< NATURAL LOG OF THE REFLECTIVITY",
&    " >")
960  FORMAT(1H0,"*** THE DISTRIBUTION OF THE PEAK INTENSITY (DUE",
&    " TO THE HOLE) IS EXAMINED ***")
9000 STOP
END

```

C SUBROUTINE TO GRAPH DATA

```

SUBROUTINE GRAPH(ZHOLE,IPROGM,IC,IFREQ,FSC,VALUE)
DIMENSION PLOT(83),VALUE(IC)

```

```
DATA DASH/'-'/,BAR/'I'/,X/'*'/,BLANK/' '/
```

```
DO 5 I=1,IC
```

```
IF(I.EQ.1)YMAX=VALUE(I)
```

```
IF(I.EQ.1)YMIN=VALUE(I)
```

```
IF(I.GE.2) GO TO 2
```

```
GO TO 5
```

```
2 BIG=VALUE(I)-YMAX
```

```
SMALL=VALUE(I)-YMIN
```

```
ZERO=0.
```

```
IF(BIG.GE.ZERO) YMAX=VALUE(I)
```

```
IF(SMALL.LT.ZERO) YMIN=VALUE(I)
```

```
5 CONTINUE
```

```
DIF=YMAX-YMIN
```

```
DO 10 K=1,83
```

```
PLOT(K)=DASH
```

```
10 CONTINUE
```

```
IF(IPROGM.EQ.3)WRITE(6,21)PLOT
```

```
IF(IPROGM.NE.3)WRITE(6,20)PLOT
```

```
20 FORMAT(1X,'(*10MHZ)',/1H0,17X,83A1)
```

```
21 FORMAT(1X,'(*E+13HZ)',/1H0,17X,83A1)
```

```
DO 30 K=1,83
```

```
PLOT(K)=BLANK
```

```
30 CONTINUE
```

```
PLOT(1)=BAR
```

```
DO 60 KZ=1,IC
```

```
KZZ=KZ-1
```

```
Z=-FLOAT(IFREQ)+FLOAT(KZZ)*FSC
```

```
YDIF=VALUE(KZ)-YMIN
```

```
YN=YDIF*80./DIF+2.5
```

```
NY=IFIX(YN)
```

```
PLOT(NY)=X
```

```
WRITE(6,40)Z,VALUE(KZ),PLOT
```

```
40 FORMAT(1X,F5.0,2X,F8.6,1X,83A1)
```

```
PLOT(NY)=BLANK
```

```
60 CONTINUE
```

```
IF(IPROGM.EQ.0)WRITE(6,50)
```

```
IF(IPROGM.EQ.1)WRITE(6,51)ZHOLE,ZHOLE
```

```
IF(IPROGM.EQ.2)WRITE(6,52)ZHOLE
```

```
IF(IPROGM.GE.3)WRITE(6,53)
```

```
50 FORMAT(1H0,'* THE NARROW STRONG & WEAK BEAMS ARE SIMULTANE',  
& 'OUSLY SCANNED OVER NA F=2 LINE(F=J+I,J=1/2&I=3/2) *',/  
& 1X,'* THE SATURATED ATOMS DETECTED BY THE WEAK BEAM ARE',  
& 'NEARLY AT REST */1X,'* THE PEAK FREQ IS THEREFORE THAT',  
& ' OF THE UNSHIFTED LINE *')
```

```
51 FORMAT(1H0,'* THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED',  
& ' TO ',F5.1,' *',/1X,'* THEN THESE SATURATED ATOMS ARE',  
& ' DETECTED BY THE WEAK WHITE LIGHT RESULT IN THE PEAK AT ',  
& F5.1,' *')
```

```
52 FORMAT(1H0,'THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO ',  
& F5.1,' *',/1X,'THEN THESE SATURATED ATOMS UNDER THE',  
& ' WALL INTERACTIONS ARE DETECTED BY WHITE LIGHT *')
```

```
53 FORMAT(1H0,'* THE NARROW STRONG & WEAK BEAMS ARE SIMULTANE',  
& 'OUSLY SCANNED OVER NA F=2 LINE(F=J+I,J=1/2&I=3/2) *',/  
& 1X,'* THE SATURATED ATOMS DETECTED ARE NEARLY AT REST *',
```

```

& /1X,"*THE DISTRIBUTION OF THE PEAKS (DUE TO THE HOLES) IS ",
& "PLOTTED *")

```

```

RETURN
END

```

```

C
C SUBROUTINE TO CALCULATE AVERAGE POWER
C

```

```

SUBROUTINE POWER(AVEFLX,ENGFLX,N,H,HWI)

```

```

SRAA=0.

```

```

DO 10 J=1,N+1,2

```

```

JL=J-1

```

```

D=H*FLOAT(JL)-1.6*HWI

```

```

A1=-D**2/HWI**2

```

```

EA1=EXP(A1)

```

```

SRAA=SRAA+EA1

```

```

10 CONTINUE

```

```

AVEFLX=ENGFLX*SRAA/(FLOAT(N)+1)

```

```

RETURN

```

```

END

```

```

C SUBROUTINE TO CALCULATE SUSCEPTIBILITY FOR THE WEAK PROBING

```

```

C BEAM

```

```

C SOLID-GAS INTERACTIONS ARE ABSENT

```

```

SUBROUTINE SUSCER(RESUM,CMSUM,Z,N,H,

```

```

& XLOWAL,C,OS1,HWN1,DOPS,ICONTL,

```

```

& IHOLE,SATFAC,ZHOLE,IPOGM,XINC,HWI)

```

```

5 SREAB=0.

```

```

SCMAB=0.

```

```

DO 16 LA=1,N+1,2

```

```

LLA=LA-1

```

```

DAL=H*FLOAT(LLA)+XLOWAL

```

```

SRRAB=0.

```

```

SCRAB=0.

```

```

DO 17 II=1,2

```

```

NN=II-1

```

```

FNN=FLOAT(NN)

```

```

AL=DAL+H*FNN

```

```

ALL1=-4.*(AL**2)/DOPS**2

```

```

EAL1=EXP(ALL1)

```

```

C IF ICONTL=0, A FREQ DEPENDENCE ON SATFAC IS NOT TAKEN INTO

```

```

C ACCOUNT

```

```

C IF ICONTL=1, IT IS TAKEN ACCOUNT

```

```

IF(ICONTL.EQ.0)GO TO 100

```

```

IF(IPOGM.EQ.0.AND.IHOLE.EQ.1)ALL2=((Z-XINC)**2)/HWI**2

```

```

IF(IPOGM.EQ.1.AND.IHOLE.EQ.1)ALL2=((Z-ZHOLE)**2)/HWI**2

```

```

IF(ALL2.GT.50)GO TO 80

```

```

EAL2=EXP(-ALL2)

```

```

SATFA1=SATFAC*EAL2

```

```

GO TO 100

```

```

80 SATFA1=0.

```

```

100 ALZ1=AL-Z

```

```

IF(IPOGM.EQ.0.AND.IHOLE.EQ.1)ALZP1=AL+Z

```

```

IF(IPOGM.EQ.1.AND.IHOLE.EQ.1)ALZP1=AL+ZHOLE

```

```

IF(IPOGM.EQ.3)ALZP1=AL

```

```

IF(IHOLE.EQ.1)ALZP2=ALZP1**2

```



```

1      IF(IPROGM.EQ.3)ALZP1=AL
3      ALZ2=ALZ1**2
4      HWN2=HWN1**2
5      ALX1=ALZ2+HWN2
6      IF(ICONTL.EQ.0)SATFA1=SATFAC
7      SATFA2=1.+SATFA1
8      IF(IHOLE.EQ.1)HOLE=SATFA1*HWN2/2./(ALZP2+HWN2*SATFA2)
9      RFAL1=EAL1*OS1*ALZ1/ALX1
10     CFAL1=EAL1*HWN1*OS1/ALX1
11     IF(IHOLE.EQ.1)RFAL1=HOLE*RFAL1
12     IF(IHOLE.EQ.1)CFAL1=HOLE*CFAL1
13     IF(LLA.EQ.0)GO TO 12
14     IF(LLA.EQ.N)GO TO 13
15     IF(NN.EQ.0)NC=2
16     IF(NN.EQ.1)NC=4
17     GO TO 14

```

```

18     12      IF(NN.EQ.0)NC=1
19             IF(NN.EQ.1)NC=4
20             GO TO 14

```

```

21     13      IF(NN.EQ.0)NC=1
22             IF(NN.EQ.1)NC=0

```

```

23     14      FNC=FLOAT(NC)
24             SRRAB=SRRAB+FNC*RFAL1
25             SCRAB=SCRAB+FNC*CFAL1

```

```

26     17      CONTINUE
27             SREAB=SRRAB+SREAB
28             SCMA=SCRAB+SCRAB

```

```

29     16      CONTINUE
30             FUNREY=H*SREAB/3.
31             FUNCMY=H*SCMA/3.

```

```

32     31      RESUM=C*FUNREY
33             CMSUM=C*FUNCMY

```

```

34     32      RETURN
35     END

```

```

36     C      SUBROUTINE FOR THE SUSCEPTIBILITY OF THE WEAK BEAM
37     C      WHERE THE SORID-GAS INTERACTIONS ARE FORM OF 1/Z**P
38     C

```

```

39     SUBROUTINE SUSCEV(RESUM,CMSUM,Z,N,H,NV,HV,
40     & XLOWAL,VLOW,C,OS1,HWN1,DOPS,
41     & IHOLE,SATFAC,ZHOLE,IPROGM,HWI,PX,PVX,VWCOC)

```

```

42     SRR=0.
43     SCR=0.
44     IF(IHOLE.EQ.0)NV=0
45     DO 20 IV=1,NV+1,2
46     IF(IHOLE.EQ.0)GO TO 1
47     IVV=IV-1
48     DV=HV*FLOAT(IVV)+VLOW
49     SRRV=0.
50     SCR=0.
51     LLV=2

```

```

52     1      IF(IHOLE.EQ.0)LLV=1
53             DO 15 LV=1,LLV
54             IF(IHOLE.EQ.0)GO TO 2
55             MM=LV-1

```

```

1      FMM=FLOAT(MM)
2      V1=DV+HV*FMM
3      VFREQ=1./V1
4      PX1=PX+1.
5      VY=POW(VFREQ,PX1)
6
7      2   IF(IHOLE.EQ.0)VY=1.
8      5   SREAB=0.
9      SCMA=0.
10     DO 16 LA=1,N+1,2
11     LLA=LA-1
12     DAL=H*FLOAT(LLA)+XLOWAL
13     SRRAB=0.
14     SCRAB=0.
15     DO 17 II=1,2
16     NN=II-1
17     FNN=FLOAT(NN)
18     AL=DAL+H*FNN
19     ALL1=-4.*(AL**2)/DOPS**2
20     EAL1=EXP(ALL1)
21     IF(IPROGM.EQ.1.AND.IHOLE.EQ.1)ALL2=((Z-ZHOLE)**2)/HWI**2
22     IF(ALL2.GT.50)GO TO 80
23     EAL2=EXP(-ALL2)
24     SATFA1=SATFAC*EAL2
25     GO TO 100
26
27     80   SATFA1=0.
28     100  ALZ1=AL
29     IF(IHOLE.EQ.1)ALZP1=AL+Z+V1
30     IF(IHOLE.EQ.1)ALZP2=ALZP1**2
31     ALZ2=ALZ1**2
32     HWN2=HWN1**2
33     ALX1=ALZ2+HWN2
34     SATFA2=1.+SATFA1
35     IF(IHOLE.EQ.1)HOLE=SATFA1*HWN2/2./ (ALZP2+HWN2*SATFA2)
36     RFAL1=VY*EAL1*OS1*ALZ1/ALX1
37     CFAL1=VY*EAL1*HWN1*OS1/ALX1
38     IF(IHOLE.EQ.1)RFAL1=HOLE*RFAL1
39     IF(IHOLE.EQ.1)CFAL1=HOLE*CFAL1
40     IF(LLA.EQ.0)GO TO 12
41     IF(LLA.EQ.N)GO TO 13
42     IF(NN.EQ.0)NC=2
43     IF(NN.EQ.1)NC=4
44     GO TO 14
45
46     12   IF(NN.EQ.0)NC=1
47         IF(NN.EQ.1)NC=4
48         GO TO 14
49
50     13   IF(NN.EQ.0)NC=1
51         IF(NN.EQ.1)NC=0
52
53     14   FNC=FLOAT(NC)
54     SRRAB=SRRAB+FNC*RFAL1
55     SCRAB=SCRAB+FNC*CFAL1
56
57     17   CONTINUE
58     SREAB=SRRAB+SREAB
59     SCMA=SCRAB+SCRAB
60
61     16   CONTINUE

```

FUNREV=H*SREAB/3.

FUNCMV=H*SCMAB/3.

IF(IHOLE.EQ.0)GO TO 3

IF(IVV.EQ.0)GO TO 22

IF(IVV.EQ.NV)GO TO 23

IF(MM.EQ.0)MC=2

IF(MM.EQ.1)MC=4

GO TO 24

22 IF(MM.EQ.0)MC=1

IF(MM.EQ.1)MC=4

GO TO 24

23 IF(MM.EQ.0)MC=1

IF(MM.EQ.1)MC=0

24 FMC=FLOAT(MC)

SRRV=SRRV+FMC*FUNREV

SCRV=SCRV+FMC*FUNCMV

15 CONTINUE

SRR=SRR+SRRV

SCR=SCR+SCRV

20 CONTINUE

RESUM=C*PVX*HV*SRR/3./VWCOCF

CMSUM=C*PVX*HV*SCR/3./VWCOCF

GO TO 4

3 RESUM=C*FUNREV

CMSUM=C*FUNCMV

4 RETURN

END

SUBROUTINE VWNORM(VWCOCF,PX,FVX,N,H,XALLX)

SRAA=0.

DO 10 J=1,N+1,2

JL=J-1

D=H*FLOAT(JL)+XALLX

SRAAA=0.

DO 5 L=1,2

M=L-1

DM=FLOAT(M)

DV=D+H*DM

VFREQ=1./DV

PX1=PX+1.

VY=POW(VFREQ,PX1)

IF(JL.EQ.0)GO TO 2

IF(JL.EQ.N)GO TO 3

IF(M.EQ.0)ML=2

IF(M.EQ.1)ML=4

GO TO 4

2 IF(M.EQ.0)ML=1

IF(M.EQ.1)ML=4

GO TO 4

3 IF(M.EQ.0)ML=1

IF(M.EQ.1)ML=0

4 DML=FLOAT(ML)

SRAAA=SRAAA+DML*VY

5 CONTINUE

SRAA=SRAA+SRAAA

```

10  CONTINUE
    VWCCP=FVX*H*SRAA/3.

```

```

    RETURN
    END

```

```

C  SUBROUTINE TO CALCULATE REFRACTIVE INDEX

```

```

    SUBROUTINE REFINDEX(REINDEX,REFREX,REFCMX,ALFAX,BETAX,
&  SUSREX,SUSCMX,SOINDEX,ANGINC,PAI)

```

```

    REFREX=1.+4.*PAI*SUSREX
    REFCMX=2.*PAI*SUSCMX
    PART1=(SOINDEX*SIN(ANGINC))**2

```

```

    A=REFREX-PART1
    B=2.*REFCMX
    C=REFREX+PART1

```

```

    D=0.
    AB=ABS(A)
    IF(AB-B)60,61,62

```

```

60  X=A/B
    XRT=1.+X**2
    ALFA=B*(SQRT(XRT)+X)/2.
    ALFAX=SQRT(ALFA)
    BETA=B*(SQRT(XRT)-X)/2.
    E=SQRT(XRT)-X

```

```

    IF(E.EQ.D)BETAX=0.
    BETAX=SQRT(BETA)
    PARTX=(C+B*XRT)/2.
    REINDEX=SQRT(PARTX)
    GO TO 63

```

```

61  ALFA=B*(1.+SQRT(2.))/2.
    ALFAX=SQRT(ALFA)
    BETA=B*(SQRT(2.)-1.)/2.
    BETAX=SQRT(BETA)
    PARTE=(C+B*(1.+SQRT(2.)))/2.
    REINDEX=SQRT(PARTE)
    GO TO 63

```

```

62  Y=B/A
    YRT=1.+Y**2
    ALFA=A*(1.+SQRT(YRT))/2.
    IF(A.LT.D)ALFA=AB*(SQRT(YRT)-1.)/2.
    ALFAX=SQRT(ALFA)
    BETA=A*(SQRT(YRT)-1.)/2.
    IF(A.LT.D)BETA=AB*(SQRT(YRT)+1.)/2.
    F=SQRT(YRT)-1.
    IF(F.EQ.D)BETAX=0.
    BETAX=SQRT(BETA)
    PARTY=(C+A*YRT)/2.
    REINDEX=SQRT(PARTY)

```

```

63  IF(REINDEX**2-PART1)64,65,65
64  XC1=REFCMX**2/(PART1-REINDEX**2)
    XC2=XC1+PART1

```

```

    XC=SQRT(XC2)
    REINDEX=XC

```

```

65  RETURN
    END

```

```

C      SUBROUTINE TO DEFINE A FUNCTIONAL FORM OF REFLECTIVITY
      SUBROUTINE REFLEC(ALREFLFX,REFLFX,ALFAX,BETAX,ANGINC,
&      ANGREF,POLARI,SOINDX)
      AC=COS(ANGINC)
      AS=SIN(ANGINC)
      RNUMAX=(SOINDX*AC-ALFAX)**2+BETAX**2
      RDENO=(SOINDX*AC+ALFAX)**2+BETAX**2
      ALRNUMAX=ALOG(RNUMAX)
      ALRDENO=ALOG(RDENO)
      SA=SOINDX*AS
      XA=(ALFAX**2-BETAX**2+SA**2)*AC
      XAN=XA-SOINDX*ALFAX
      XB=2.*ALFAX*BETAX
      XBN=XB-SOINDX*BETAX
      RNPER=XAN**2+XBN**2
      XAD=XA+SOINDX*ALFAX
      XBD=XB+SOINDX*BETAX
      RDPER=XAD**2+XBD**2
      ALRNPER=ALOG(RNPER)
      ALRDPER=ALOG(RDPER)
      IF(POLARI)701,702,703
701    REFLFX=RNPER/RDPER
      ALREFLFX=ALRNPER-ALRDPER
      GO TO 705
703    REFLFX=RNUMAX/RDENO
      ALREFLFX=ALRNUMAX-ALRDENO
      GO TO 705
702    REFLFX=.5*(RNPER/RDPER+RNUMAX/RDENO)
      ALREFLFX=0.5*(ALRNPER+ALRNUMAX-ALRDPER-ALRDENO)
705    RETURN
      END

```

```

C      SUBROUTINE TO FIND THE SQUARE OF THE INCIDENT LIGHT AMPLITUDE

```

```

      SUBROUTINE GAUSIA(AIGAUS,XINC,AAZ,HWI,PAI)
      AZ=AAZ-XINC
      AIG=(AZ**2)/HWI/HWI
      AIGAUS=EXP(-AIG)
      RETURN
      END
$EXECUTE
$LIMIT:20,37K,,6000
$SYSOUT:06,ORG
$ENDJOB

```

*** A NARROW WEAK PROBING BEAM INCIDENT ON A GLASS-NA GAS INTERFACE ***
AT AN INCIDENT ANGLE OF 50.0 DEGREE

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
(MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

0.067 619. 6.0 4.3 0. 0.381E-02 0.60

LINE SHAPE OF THE HOLE IS EXAMINED

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-30.	1.009233	0.020138	1.248946	0.13E-04	88.8	0.933322
-28.	1.008599	0.020245	1.248949	0.13E-04	88.7	0.933047
-26.	1.007954	0.020344	1.248952	0.13E-04	88.7	0.932802
-24.	1.007297	0.020435	1.248954	0.13E-04	88.7	0.932588
-22.	1.006629	0.020515	1.248956	0.13E-04	88.7	0.932409
-20.	1.005948	0.020585	1.248958	0.13E-04	88.7	0.932270
-18.	1.005253	0.020642	1.248959	0.13E-04	88.7	0.932176
-16.	1.004542	0.020682	1.248960	0.13E-04	88.7	0.932138
-14.	1.003816	0.020702	1.248960	0.13E-04	88.7	0.932174
-12.	1.003082	0.020692	1.248959	0.13E-04	88.7	0.932313
-10.	1.002366	0.020644	1.248957	0.13E-04	88.7	0.932603
-8.	1.001704	0.020560	1.248954	0.13E-04	88.7	0.933094
-6.	1.001134	0.020453	1.248951	0.13E-04	88.7	0.933795
-4.	1.000672	0.020347	1.248948	0.13E-04	88.8	0.934621
-2.	1.000308	0.020269	1.248945	0.13E-04	88.8	0.935271
0.	1.000000	0.020241	1.248944	0.13E-04	88.8	0.935484
2.	0.999692	0.020269	1.248945	0.13E-04	88.8	0.935269
4.	0.999328	0.020347	1.248947	0.13E-04	88.8	0.934644
6.	0.998866	0.020453	1.248950	0.13E-04	88.7	0.933899
8.	0.998296	0.020560	1.248953	0.13E-04	88.7	0.933323
10.	0.997634	0.020644	1.248955	0.13E-04	88.7	0.932980
12.	0.996918	0.020692	1.248956	0.12E-04	88.7	0.932844
14.	0.996184	0.020702	1.248956	0.12E-04	88.7	0.932856
16.	0.995458	0.020682	1.248955	0.12E-04	88.7	0.932968
18.	0.994747	0.020642	1.248953	0.12E-04	88.7	0.933148
20.	0.994052	0.020585	1.248951	0.12E-04	88.7	0.933379
22.	0.993371	0.020515	1.248949	0.12E-04	88.8	0.933651
24.	0.992703	0.020435	1.248946	0.12E-04	88.8	0.933957
26.	0.992046	0.020344	1.248943	0.12E-04	88.8	0.934294
28.	0.991401	0.020245	1.248940	0.12E-04	88.8	0.934659
30.	0.990767	0.020138	1.248937	0.12E-04	88.8	0.935049

<IMAG PART OF THE DIELECTRIC CONST>

A black and white photograph of a starry night sky. The image is overlaid with a grid of horizontal lines, creating a series of bands across the frame. Stars of varying brightness are scattered throughout the sky. Some stars appear in pairs or small groups, while others are isolated. The overall composition suggests a scientific or astronomical observation, possibly related to the study of star distribution or the effects of atmospheric conditions on star visibility.

* THE NARROW STRONG & WEAK BEAMS ARE SIMULTANEOUSLY SCANNED OVER NA $F=2$ LINE ($F=J+1, J=1/2 \& I=3/2$) *

* THE SATURATED ATOMS DETECTED BY THE WEAK BEAM ARE NEARLY AT REST *

* THE PEAK FREQ IS THEREFORE THAT OF THE UNSHIFTED LINE *

*** A WHITE PROBING BEAM INCIDENT ON A GLASS-NA GAS INTERFACE ***
 AT AN INCIDENT ANGLE OF 50.0 DEGREE

POLARIZATION IS PERPENDICULAR

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
 (MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

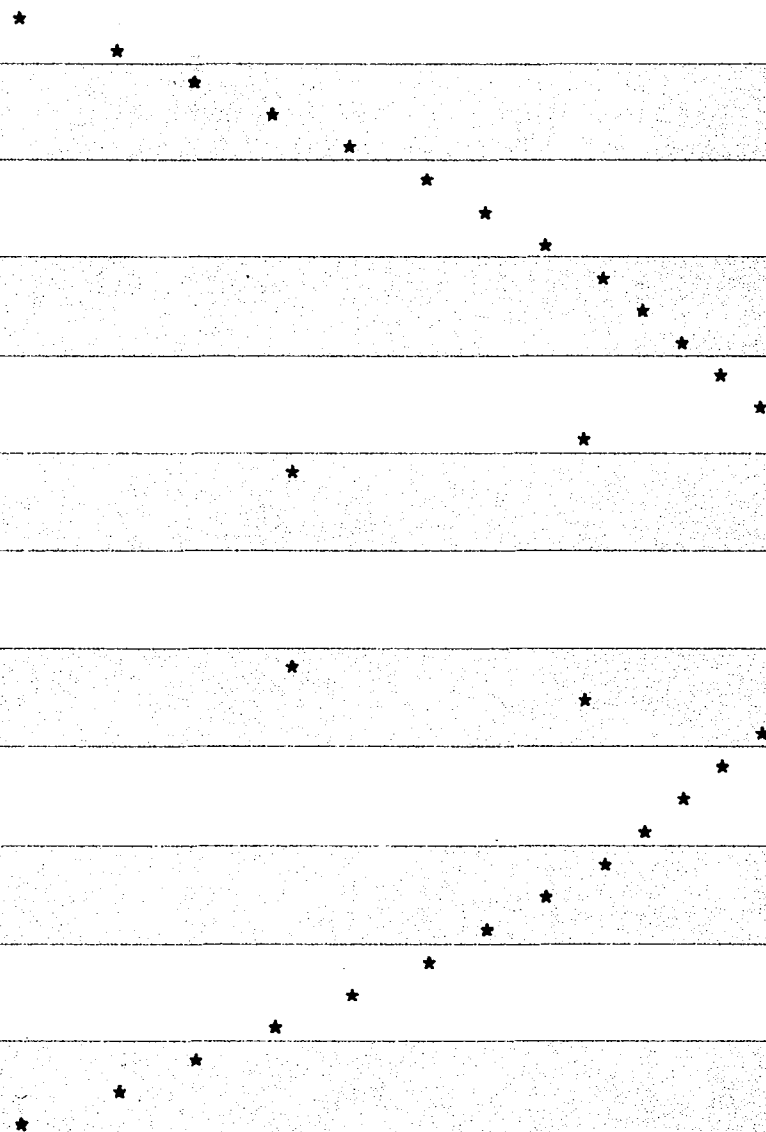
0.067 619. 6.0 4.3 0. 0.381E-02 0.60

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-34.	1.010828	0.019954	1.248942	0.13E-04	88.8	0.891584
-32.	1.010234	0.020081	1.248945	0.13E-04	88.8	0.891015
-30.	1.009632	0.020201	1.248948	0.13E-04	88.8	0.890484
-28.	1.009024	0.020314	1.248951	0.13E-04	88.7	0.889991
-26.	1.008408	0.020420	1.248954	0.13E-04	88.7	0.889535
-24.	1.007786	0.020518	1.248957	0.13E-04	88.7	0.889119
-22.	1.007158	0.020609	1.248959	0.13E-04	88.7	0.888742
-20.	1.006525	0.020692	1.248961	0.13E-04	88.7	0.888404
-18.	1.005887	0.020768	1.248963	0.13E-04	88.7	0.888105
-16.	1.005244	0.020836	1.248965	0.13E-04	88.7	0.887847
-14.	1.004597	0.020896	1.248966	0.13E-04	88.7	0.887629
-12.	1.003947	0.020948	1.248967	0.13E-04	88.7	0.887450
-10.	1.003294	0.020993	1.248968	0.13E-04	88.7	0.887312
-8.	1.002126	0.020749	1.248960	0.13E-04	88.7	0.888677
-6.	1.001024	0.020342	1.248948	0.13E-04	88.8	0.890852
-4.	1.000131	0.019697	1.248929	0.13E-04	88.8	0.894196
-2.	0.999815	0.019071	1.248911	0.13E-04	88.8	0.897382
0.	1.000000	0.018812	1.248905	0.13E-04	88.8	0.898669
2.	1.000185	0.019071	1.248912	0.13E-04	88.8	0.897338
4.	0.999869	0.019697	1.248929	0.13E-04	88.8	0.894227
6.	0.998976	0.020342	1.248947	0.13E-04	88.8	0.891105
8.	0.997874	0.020749	1.248958	0.13E-04	88.7	0.889212
10.	0.996706	0.020993	1.248965	0.12E-04	88.7	0.888150
12.	0.996053	0.020948	1.248963	0.12E-04	88.7	0.888452
14.	0.995403	0.020896	1.248961	0.12E-04	88.7	0.888792
16.	0.994756	0.020836	1.248959	0.12E-04	88.7	0.889171
18.	0.994113	0.020768	1.248956	0.12E-04	88.7	0.889588
20.	0.993475	0.020692	1.248954	0.12E-04	88.7	0.890042
22.	0.992842	0.020609	1.248951	0.12E-04	88.7	0.890533
24.	0.992214	0.020518	1.248948	0.12E-04	88.8	0.891060
26.	0.991592	0.020420	1.248945	0.12E-04	88.8	0.891622
28.	0.990976	0.020314	1.248942	0.12E-04	88.8	0.892220
30.	0.990368	0.020201	1.248938	0.12E-04	88.8	0.892853
32.	0.989766	0.020081	1.248935	0.12E-04	88.8	0.893519
34.	0.989172	0.019954	1.248931	0.12E-04	88.8	0.894218

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRIC CONST>

-34. 0.019954 I
-32. 0.020081 I
-30. 0.020201 I
-28. 0.020314 I
-26. 0.020420 I
-24. 0.020518 I
-22. 0.020609 I
-20. 0.020692 I
-18. 0.020768 I
-16. 0.020836 I
-14. 0.020896 I
-12. 0.020948 I
-10. 0.020993 I
-8. 0.020749 I
-6. 0.020342 I
-4. 0.019697 I
-2. 0.019071 I
0. 0.018812 I*
2. 0.019071 I
4. 0.019697 I
6. 0.020342 I
8. 0.020749 I
10. 0.020993 I
12. 0.020948 I
14. 0.020896 I
16. 0.020836 I
18. 0.020768 I
20. 0.020692 I
22. 0.020609 I
24. 0.020518 I
26. 0.020420 I
28. 0.020314 I
30. 0.020201 I
32. 0.020081 I
34. 0.019954 I

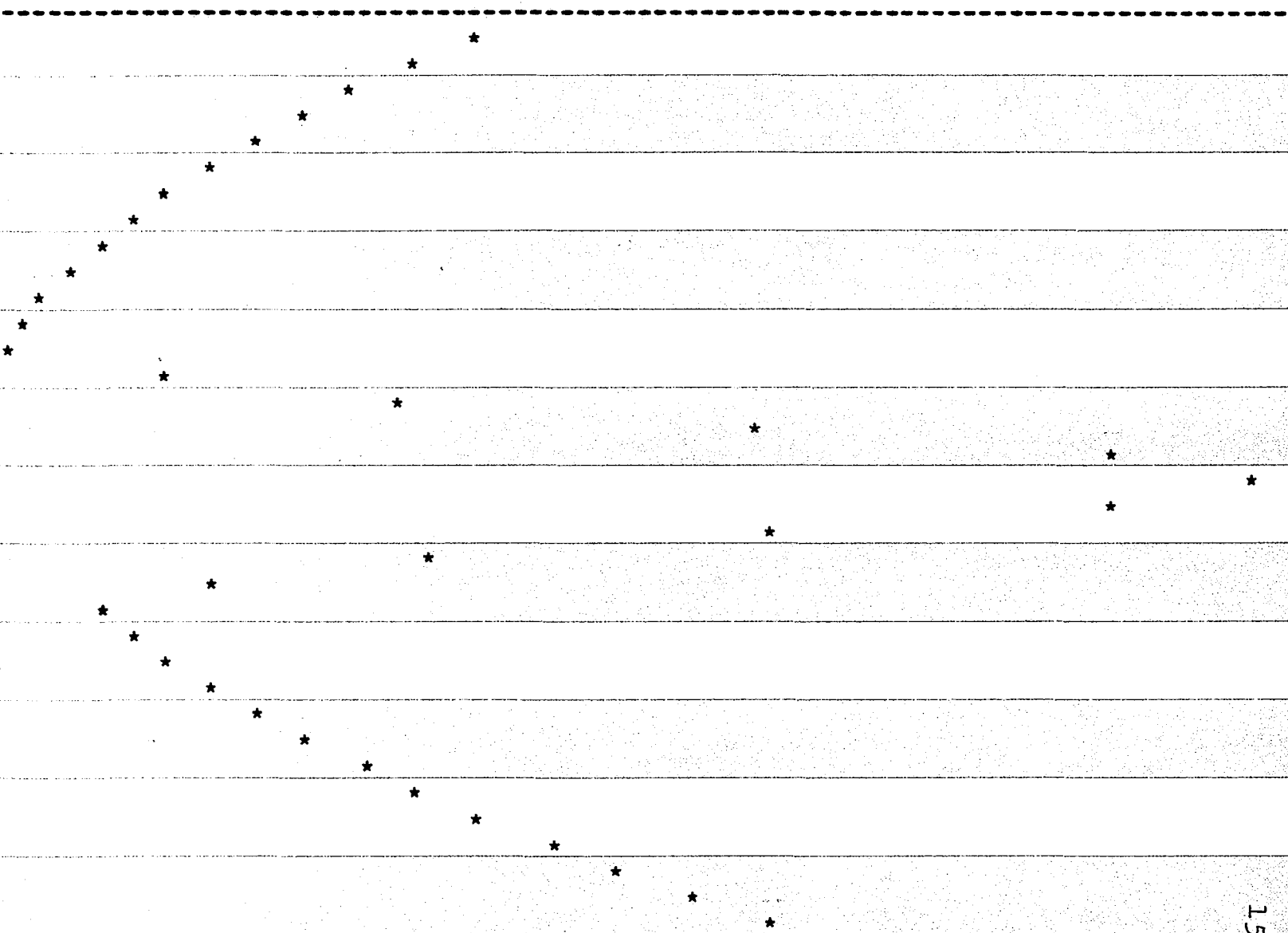


* THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO 0. *
* THEN THESE SATURATED ATOMS ARE DETECTED BY THE WEAK WHITE LIGHT RESULT IN THE PEAK AT 0. *

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-34. 0.891584 I
-32. 0.891015 I
-30. 0.890484 I
-28. 0.889991 I
-26. 0.889535 I
-24. 0.889119 I
-22. 0.888742 I
-20. 0.888404 I
-18. 0.888105 I
-16. 0.887847 I
-14. 0.887629 I
-12. 0.887450 I
-10. 0.887312 I
-8. 0.888677 I
-6. 0.890852 I
-4. 0.894196 I
-2. 0.897382 I
0. 0.898669 I
2. 0.897338 I
4. 0.894227 I
6. 0.891105 I
8. 0.889212 I
10. 0.888150 I
12. 0.888452 I
14. 0.888792 I
16. 0.889171 I
18. 0.889588 I
20. 0.890042 I
22. 0.890533 I
24. 0.891060 I
26. 0.891622 I
28. 0.892220 I
30. 0.892853 I
32. 0.893519 I
34. 0.894218 I



* THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO 0. *
* THEN THESE SATURATED ATOMS ARE DETECTED BY THE WEAK WHITE LIGHT RESULT IN THE PEAK AT 0. *

*** A WHITE PROBING BEAM INCIDENT ON A GLASS-NA GAS INTERFACE ***
 AT AN INCIDENT ANGLE OF 50.0 DEGREE

POLARIZATION IS PERPENDICULAR

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
 (MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

0.067 619. 6.0 4.3 -8.0 0.381E-02 0.60

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-42.	1.013120	0.019380	1.248926	0.13E-04	88.8	0.894220
-40.	1.012560	0.019534	1.248930	0.13E-04	88.8	0.893508
-38.	1.011992	0.019680	1.248934	0.13E-04	88.8	0.892831
-36.	1.011414	0.019821	1.248938	0.13E-04	88.8	0.892189
-34.	1.010828	0.019954	1.248942	0.13E-04	88.8	0.891584
-32.	1.010234	0.020081	1.248945	0.13E-04	88.8	0.891015
-30.	1.009632	0.020201	1.248948	0.13E-04	88.8	0.890484
-28.	1.009024	0.020314	1.248951	0.13E-04	88.7	0.889991
-26.	1.008408	0.020420	1.248954	0.13E-04	88.7	0.889535
-24.	1.007786	0.020518	1.248957	0.13E-04	88.7	0.889119
-22.	1.007158	0.020609	1.248959	0.13E-04	88.7	0.888742
-20.	1.006525	0.020692	1.248961	0.13E-04	88.7	0.888404
-18.	1.005887	0.020768	1.248963	0.13E-04	88.7	0.888105
-16.	1.004914	0.020776	1.248963	0.13E-04	88.7	0.888189
-14.	1.003874	0.020749	1.248961	0.13E-04	88.7	0.888458
-12.	1.002698	0.020659	1.248958	0.13E-04	88.7	0.889058
-10.	1.001551	0.020530	1.248954	0.13E-04	88.7	0.889846
-8.	1.000622	0.020419	1.248950	0.13E-04	88.7	0.890515
-6.	1.000023	0.020388	1.248949	0.13E-04	88.8	0.890748
-4.	0.999742	0.020464	1.248951	0.13E-04	88.7	0.890400
-2.	0.999636	0.020628	1.248955	0.13E-04	88.7	0.889597
0.	0.999491	0.020814	1.248961	0.13E-04	88.7	0.888687
2.	0.999339	0.021090	1.248969	0.13E-04	88.7	0.887334
4.	0.998679	0.021078	1.248968	0.13E-04	88.7	0.887479
6.	0.998020	0.021057	1.248967	0.13E-04	88.7	0.887663
8.	0.997362	0.021029	1.248966	0.12E-04	88.7	0.887887
10.	0.996706	0.020993	1.248965	0.12E-04	88.7	0.888150
12.	0.996053	0.020948	1.248963	0.12E-04	88.7	0.888452
14.	0.995403	0.020896	1.248961	0.12E-04	88.7	0.888792
16.	0.994756	0.020836	1.248959	0.12E-04	88.7	0.889171
18.	0.994113	0.020768	1.248956	0.12E-04	88.7	0.889588
20.	0.993475	0.020692	1.248954	0.12E-04	88.7	0.890042
22.	0.992842	0.020609	1.248951	0.12E-04	88.7	0.890533
24.	0.992214	0.020518	1.248948	0.12E-04	88.8	0.891060
26.	0.991592	0.020420	1.248945	0.12E-04	88.8	0.891622

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRIC CONST>

-42.	0.019380	I *
-40.	0.019534	I
-38.	0.019680	I
-36.	0.019821	I
-34.	0.019954	I
-32.	0.020081	I
-30.	0.020201	I
-28.	0.020314	I
-26.	0.020420	I
-24.	0.020518	I
-22.	0.020609	I
-20.	0.020692	I
-18.	0.020768	I
-16.	0.020776	I
-14.	0.020749	I
-12.	0.020659	I
-10.	0.020530	I
-8.	0.020419	I
-6.	0.020388	I
-4.	0.020464	I
-2.	0.020628	I
0.	0.020814	I
2.	0.021090	I
4.	0.021078	I
6.	0.021057	I
8.	0.021029	I
10.	0.020993	I
12.	0.020948	I
14.	0.020896	I
16.	0.020836	I
18.	0.020768	I
20.	0.020692	I
22.	0.020609	I
24.	0.020518	I
26.	0.020420	I

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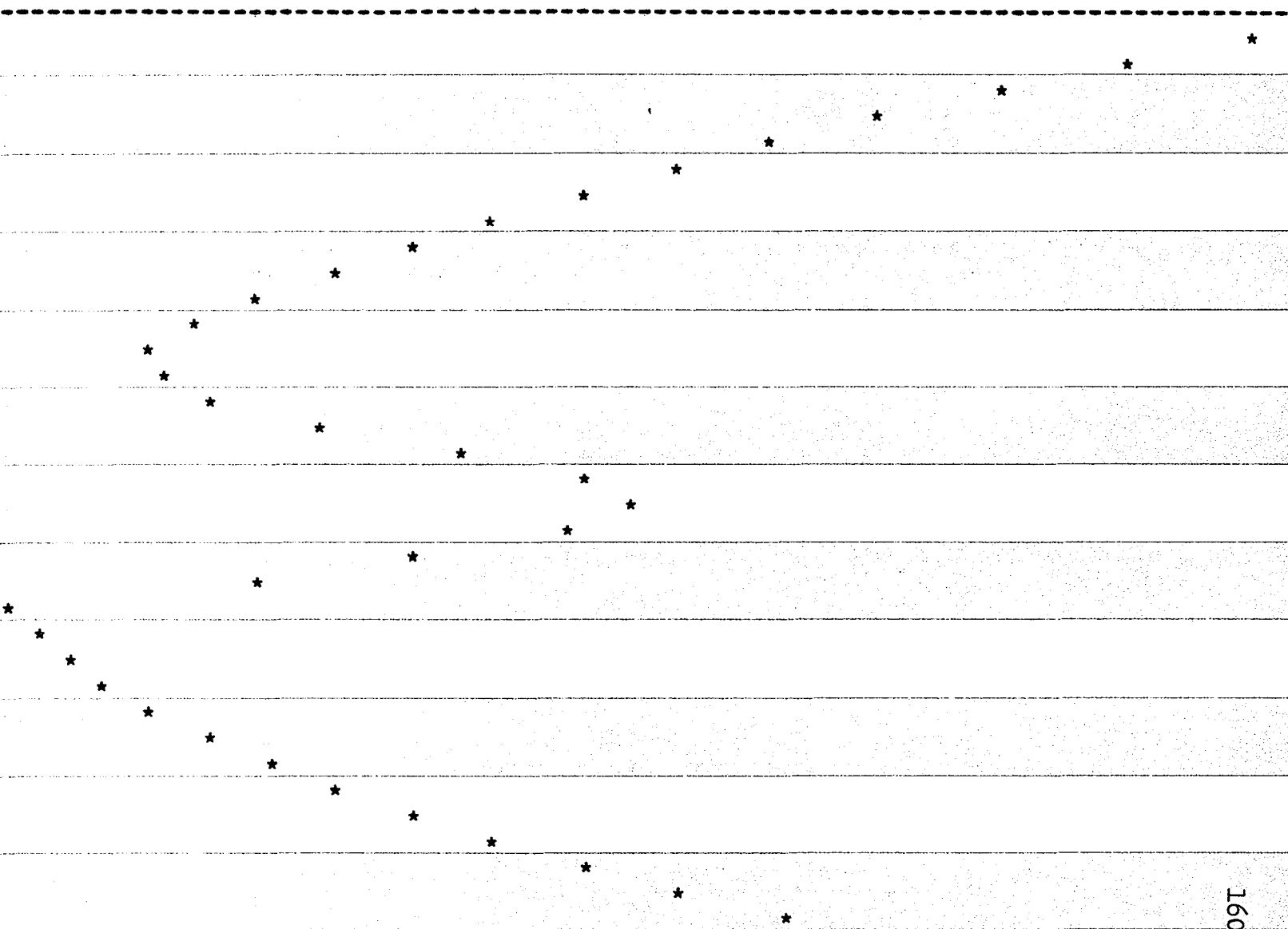
* THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO -8.0 *

* THEN THESE SATURATED ATOMS ARE DETECTED BY THE WEAK WHITE LIGHT RESULT IN THE PEAK AT -8.0*

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-42. 0.894220 I
-40. 0.893508 I
-38. 0.892831 I
-36. 0.892189 I
-34. 0.891584 I
-32. 0.891015 I
-30. 0.890484 I
-28. 0.889991 I
-26. 0.889535 I
-24. 0.889119 I
-22. 0.888742 I
-20. 0.888404 I
-18. 0.888105 I
-16. 0.888189 I
-14. 0.888458 I
-12. 0.889058 I
-10. 0.889846 I
-8. 0.890515 I
-6. 0.890748 I
-4. 0.890400 I
-2. 0.889597 I
0. 0.888687 I
2. 0.887334 I*
4. 0.887479 I*
6. 0.887663 I*
8. 0.887887 I*
10. 0.888150 I*
12. 0.888452 I*
14. 0.888792 I*
16. 0.889171 I*
18. 0.889588 I*
20. 0.890042 I*
22. 0.890533 I*
24. 0.891060 I*
26. 0.891622 I*



* THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO -8.0 *

* THEN THESE SATURATED ATOMS ARE DETECTED BY THE WEAK WHITE LIGHT RESULT IN THE PEAK AT -8.0*

*** A WHITE PROBING BEAM INCIDENT ON A GLASS-NA GAS INTERFACE ***
 AT AN INCIDENT ANGLE OF 50.0 DEGREE

POLARIZATION IS PERPENDICULAR

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
 (MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

0.067 619. 6.0 4.3 16.0 0.381E-02 0.60

INCIDENT FREQ (*10MHZ)	DIELECTRIC REAL	CONST IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-18.	1.005887	0.020768	1.248963	0.13E-04	88.7	0.888105
-16.	1.005244	0.020836	1.248965	0.13E-04	88.7	0.887847
-14.	1.004597	0.020896	1.248966	0.13E-04	88.7	0.887629
-12.	1.003947	0.020948	1.248967	0.13E-04	88.7	0.887450
-10.	1.003294	0.020993	1.248968	0.13E-04	88.7	0.887312
-8.	1.002638	0.021029	1.248969	0.13E-04	88.7	0.887215
-6.	1.001980	0.021057	1.248969	0.13E-04	88.7	0.887158
-4.	1.001321	0.021078	1.248970	0.13E-04	88.7	0.887141
-2.	1.000661	0.021090	1.248970	0.13E-04	88.7	0.887165
0.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
2.	0.999339	0.021090	1.248969	0.13E-04	88.7	0.887334
4.	0.998679	0.021078	1.248968	0.13E-04	88.7	0.887479
6.	0.998020	0.021057	1.248967	0.13E-04	88.7	0.887663
8.	0.997689	0.020969	1.248964	0.13E-04	88.7	0.888143
10.	0.997338	0.020883	1.248962	0.12E-04	88.7	0.888615
12.	0.997021	0.020787	1.248959	0.12E-04	88.7	0.889131
14.	0.996609	0.020703	1.248956	0.12E-04	88.7	0.889598
16.	0.996006	0.020647	1.248954	0.12E-04	88.7	0.889954
18.	0.995200	0.020615	1.248953	0.12E-04	88.7	0.890210
20.	0.994259	0.020592	1.248951	0.12E-04	88.7	0.890445
22.	0.993300	0.020555	1.248950	0.12E-04	88.7	0.890744
24.	0.992426	0.020495	1.248948	0.12E-04	88.8	0.891148
26.	0.991592	0.020420	1.248945	0.12E-04	88.8	0.891622
28.	0.990976	0.020314	1.248942	0.12E-04	88.8	0.892220
30.	0.990368	0.020201	1.248938	0.12E-04	88.8	0.892853
32.	0.989766	0.020081	1.248935	0.12E-04	88.8	0.893519
34.	0.989172	0.019954	1.248931	0.12E-04	88.8	0.894218
36.	0.988586	0.019821	1.248927	0.12E-04	88.8	0.894950
38.	0.988008	0.019680	1.248923	0.12E-04	88.8	0.895714
40.	0.987440	0.019534	1.248918	0.12E-04	88.8	0.896508
42.	0.986880	0.019380	1.248914	0.12E-04	88.8	0.897333
44.	0.986330	0.019221	1.248909	0.12E-04	88.8	0.898187
46.	0.985790	0.019056	1.248905	0.12E-04	88.8	0.899069
48.	0.985259	0.018885	1.248900	0.12E-04	88.9	0.899979
50.	0.984740	0.018708	1.248895	0.12E-04	88.9	0.900916

<FREQ>
(*10MHZ)

<IMAG PART OF THE DIELECTRIC CONST>

-18.	0.020768	I
-16.	0.020836	I
-14.	0.020896	I
-12.	0.020948	I
-10.	0.020993	I
-8.	0.021029	I
-6.	0.021057	I
-4.	0.021078	I
-2.	0.021090	I
0.	0.021094	I
2.	0.021090	I
4.	0.021078	I
6.	0.021057	I
8.	0.020969	I
10.	0.020883	I
12.	0.020787	I
14.	0.020703	I
16.	0.020647	I
18.	0.020615	I
20.	0.020592	I
22.	0.020555	I
24.	0.020495	I
26.	0.020420	I
28.	0.020314	I
30.	0.020201	I
32.	0.020081	I
34.	0.019954	I
36.	0.019821	I
38.	0.019680	I
40.	0.019534	I
42.	0.019380	I
44.	0.019221	I
46.	0.019056	I
48.	0.018885	I
50.	0.018708	I*

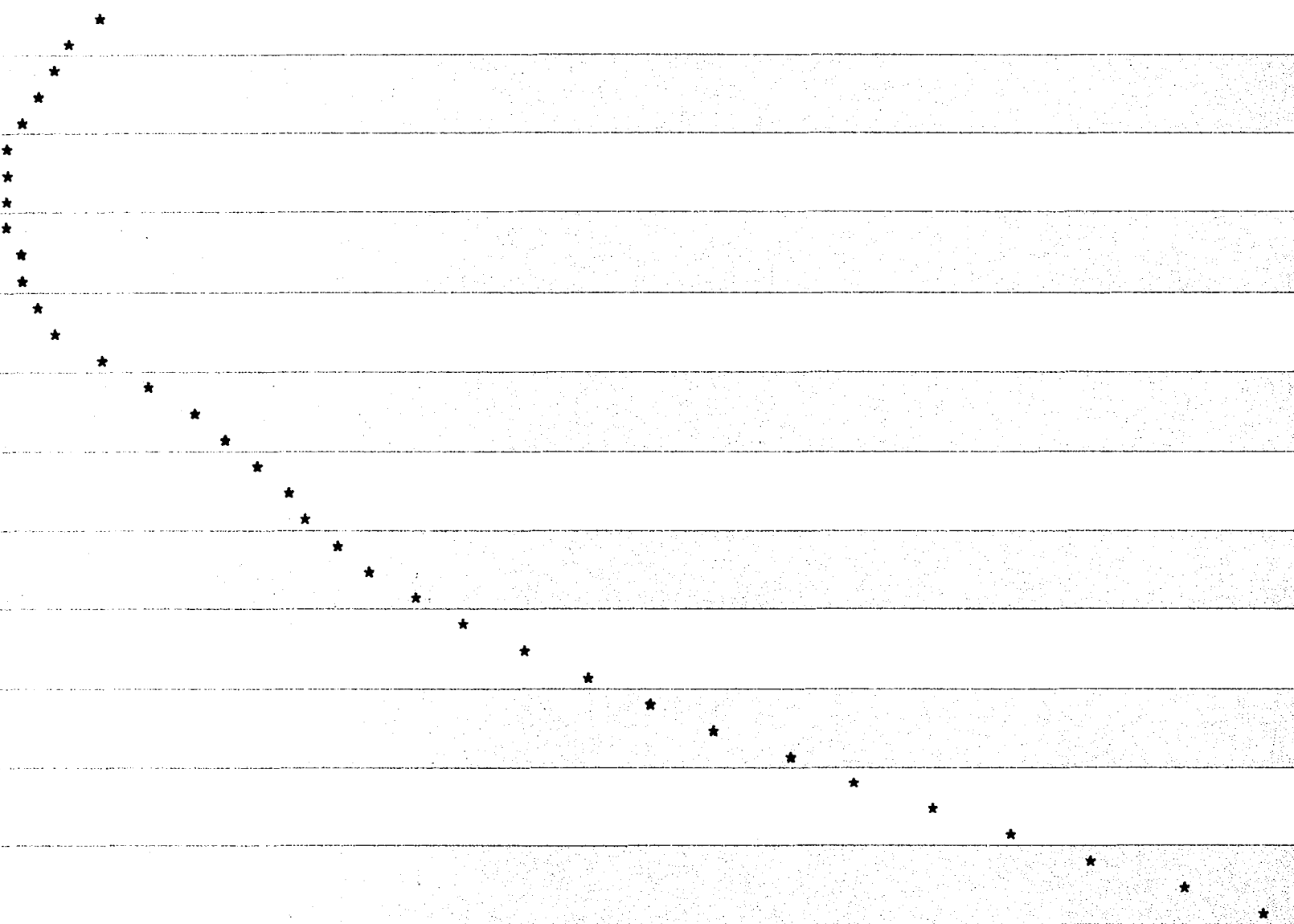
* THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO 16.0 *

* THEN THESE SATURATED ATOMS ARE DETECTED BY THE WEAK WHITE LIGHT RESULT IN THE PEAK AT 16.0*

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-18.	0.888105	I
-16.	0.887847	I
-14.	0.887629	I
-12.	0.887450	I
-10.	0.887312	I
-8.	0.887215	I
-6.	0.887158	I
-4.	0.887141	I
-2.	0.887165	I
0.	0.887230	I
2.	0.887334	I
4.	0.887479	I
6.	0.887663	I
8.	0.888143	I
10.	0.888615	I
12.	0.889131	I
14.	0.889598	I
16.	0.889954	I
18.	0.890210	I
20.	0.890445	I
22.	0.890744	I
24.	0.891148	I
26.	0.891622	I
28.	0.892220	I
30.	0.892853	I
32.	0.893519	I
34.	0.894218	I
36.	0.894950	I
38.	0.895714	I
40.	0.896508	I
42.	0.897333	I
44.	0.898187	I
46.	0.899069	I
48.	0.899979	I
50.	0.900916	I



* THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO 16.0 *
* THEN THESE SATURATED ATOMS ARE DETECTED BY THE WEAK WHITE LIGHT RESULT IN THE PEAK AT 16.0 *

*** THE STRONG & WEAK BEAMS INCIDENT ON A THIN METAL LAYER DEPOSITTED ON ***
A GLASS-NA GAS INTERFACE AT AN INCIDENT ANGLE OF 50.0 DEGREE

POLARIZATION IS PERPENDICULAR

** VAN DER WAAL'S POTENTIAL ENERGY HAS THE FORM OF THE INVERSE **
3RD POWER OF THE NORMAL DISTANCE FROM THE SURFACE
WITH A CONSTANT 4.0 IN UNIT OF E-12*ERG PER A**3

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
(MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

0.067 619. 6.0 4.3 -40.0 0.381E-02 0.60

INCIDENT FREQ (*10MHZ)	DIELECTRIC REAL	CONST IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-108.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-104.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-100.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-96.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-92.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-88.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-84.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-80.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-76.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-72.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-68.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-64.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-60.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-56.	0.999999	0.021094	1.248969	0.13E-04	88.7	0.887230
-52.	0.999977	0.021091	1.248969	0.13E-04	88.7	0.887245
-48.	0.999778	0.021066	1.248968	0.13E-04	88.7	0.887398
-44.	0.999163	0.020968	1.248965	0.13E-04	88.7	0.887964
-40.	0.998640	0.020851	1.248961	0.13E-04	88.7	0.888612
-36.	0.998925	0.020879	1.248962	0.13E-04	88.7	0.888436
-32.	0.999629	0.021010	1.248967	0.13E-04	88.7	0.887695
-28.	0.999951	0.021080	1.248969	0.13E-04	88.7	0.887303
-24.	0.999998	0.021093	1.248969	0.13E-04	88.7	0.887235
-20.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-16.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-12.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-8.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
-4.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
0.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
4.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
8.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
12.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
16.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
20.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
24.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230
28.	1.000000	0.021094	1.248969	0.13E-04	88.7	0.887230

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-108.	0.887230	I*
-104.	0.887230	I*
-100.	0.887230	I*
-96.	0.887230	I*
-92.	0.887230	I*
-88.	0.887230	I*
-84.	0.887230	I*
-80.	0.887230	I*
-76.	0.887230	I*
-72.	0.887230	I*
-68.	0.887230	I*
-64.	0.887230	I*
-60.	0.887230	I*
-56.	0.887230	I*
-52.	0.887245	I *
-48.	0.887398	I *
-44.	0.887964	I *
-40.	0.888612	I *
-36.	0.888436	I *
-32.	0.887695	I *
-28.	0.887303	I *
-24.	0.887235	I*
-20.	0.887230	I*
-16.	0.887230	I*
-12.	0.887230	I*
-8.	0.887230	I*
-4.	0.887230	I*
0.	0.887230	I*
4.	0.887230	I*
8.	0.887230	I*
12.	0.887230	I*
16.	0.887230	I*
20.	0.887230	I*
24.	0.887230	I*
28.	0.887230	I*

THE CENTRAL FREQ OF THE STRONG BEAM IS TUNED TO -40.0*
THEN THESE SATURATED ATOMS UNDER THE WALL INTERACTIONS ARE DETECTED BY WHITE LIGHT *

*** THE STRONG & WEAK BEAMS INCIDENT ON A THIN METAL LAYER DEPOSITTED ON ***
A GLASS-NA GAS INTERFACE AT AN INCIDENT ANGLE OF 50.0 DEGREE

POLARIZATION IS PERPENDICULAR

** VAN DER WAAL'S POTENTIAL ENERGY HAS THE FORM OF THE INVERSE **
4TH POWER OF THE NORMAL DISTANCE FROM THE SURFACE
WITH A CONSTANT 20.0 IN UNIT OF E-12*ERG PER A**4

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <MIN REFL OCCURED AT FREQ> <SHIFA
(MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*E12HZ)

0.067 619. 6.0 4.3 -72.82 1.6

LINE SHAPE OF THE HOLE ISN'T EXAMINED

INCIDENT FREQ (*E12HZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-73.	0.999296	0.005512	1.248673	0.13E-04	89.7	0.968991
-69.	0.999317	0.005345	1.248672	0.13E-04	89.7	0.969916
-65.	0.999338	0.005182	1.248671	0.13E-04	89.7	0.970812
-61.	0.999358	0.005025	1.248670	0.13E-04	89.7	0.971684
-57.	0.999377	0.004872	1.248669	0.13E-04	89.7	0.972530
-53.	0.999396	0.004724	1.248668	0.13E-04	89.7	0.973351
-49.	0.999415	0.004581	1.248667	0.13E-04	89.7	0.974147
-45.	0.999432	0.004442	1.248666	0.13E-04	89.7	0.974922
-41.	0.999450	0.004307	1.248665	0.13E-04	89.7	0.975670
-37.	0.999466	0.004176	1.248664	0.13E-04	89.7	0.976399
-33.	0.999483	0.004049	1.248663	0.13E-04	89.8	0.977106
-29.	0.999498	0.003926	1.248663	0.13E-04	89.8	0.977792
-25.	0.999514	0.003807	1.248662	0.13E-04	89.8	0.978458
-21.	0.999528	0.003691	1.248661	0.13E-04	89.8	0.979105
-17.	0.999543	0.003579	1.248661	0.13E-04	89.8	0.979732
-13.	0.999557	0.003470	1.248660	0.13E-04	89.8	0.980340
-9.	0.999570	0.003365	1.248660	0.13E-04	89.8	0.980931
-5.	0.999583	0.003262	1.248659	0.13E-04	89.8	0.981504

<FREQ>
(*E+13HZ)

<REFLECTIVITY>

-72.	0.968991	I*
-68.	0.969916	I
-64.	0.970812	I
-60.	0.971684	I
-56.	0.972530	I
-52.	0.973351	I
-48.	0.974147	I
-44.	0.974922	I
-40.	0.975670	I
-36.	0.976399	I
-32.	0.977106	I
-28.	0.977792	I
-24.	0.978458	I
-20.	0.979105	I
-16.	0.979732	I
-12.	0.980340	I
-8.	0.980931	I
-4.	0.981504	I

* THE NARROW STRONG & WEAK BEAMS ARE SIMULTANEOUSLY SCANNED OVER NA F=2 LINE(F=J+I,J=1/2&I=3/2) *

* THE SATURATED ATOMS DETECTED ARE NEARLY AT REST *

*THE DISTRIBUTION OF THE PEAKS (DUE TO THE HOLES) IS PLOTTED *

*** THE STRONG & WEAK BEAMS INCIDENT ON A THIN METAL LAYER DEPOSITED ON ***
A GLASS-NA GAS INTERFACE AT AN INCIDENT ANGLE OF 50.0 DEGREE

*** THE DISTRIBUTION OF THE PEAK INTENSITY (DUE TO THE HOLE) IS EXAMINED ***

POLARIZATION IS PERPENDICULAR

** VAN DER WAAL'S POTENTIAL ENERGY HAS THE FORM OF THE INVERSE **
3RD POWER OF THE NORMAL DISTANCE FROM THE SURFACE
WITH A CONSTANT 4.0 IN UNIT OF E-12*ERG PER A**3

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
(MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

0.067 619. 20.0 4.3 0.381E-02 0.60

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-180.	0.999598	0.003757	1.248662	0.13E-04	89.8	0.978735
-176.	0.999598	0.003754	1.248662	0.13E-04	89.8	0.978749
-172.	0.999598	0.003752	1.248662	0.13E-04	89.8	0.978763
-168.	0.999598	0.003749	1.248662	0.13E-04	89.8	0.978778
-164.	0.999598	0.003746	1.248662	0.13E-04	89.8	0.978794
-160.	0.999598	0.003743	1.248662	0.13E-04	89.8	0.978810
-156.	0.999598	0.003740	1.248662	0.13E-04	89.8	0.978827
-152.	0.999598	0.003737	1.248662	0.13E-04	89.8	0.978844
-148.	0.999598	0.003734	1.248662	0.13E-04	89.8	0.978864
-144.	0.999598	0.003730	1.248662	0.13E-04	89.8	0.978883
-140.	0.999598	0.003727	1.248662	0.13E-04	89.8	0.978904
-136.	0.999598	0.003723	1.248662	0.13E-04	89.8	0.978925
-132.	0.999598	0.003719	1.248662	0.13E-04	89.8	0.978948
-128.	0.999598	0.003715	1.248662	0.13E-04	89.8	0.978970
-124.	0.999598	0.003710	1.248662	0.13E-04	89.8	0.978995
-120.	0.999598	0.003706	1.248661	0.13E-04	89.8	0.979021
-116.	0.999598	0.003701	1.248661	0.13E-04	89.8	0.979048
-112.	0.999598	0.003695	1.248661	0.13E-04	89.8	0.979078
-108.	0.999598	0.003690	1.248661	0.13E-04	89.8	0.979108
-104.	0.999598	0.003684	1.248661	0.13E-04	89.8	0.979140
-100.	0.999598	0.003678	1.248661	0.13E-04	89.8	0.979173
-96.	0.999598	0.003672	1.248661	0.13E-04	89.8	0.979211
-92.	0.999598	0.003665	1.248661	0.13E-04	89.8	0.979248
-88.	0.999598	0.003658	1.248661	0.13E-04	89.8	0.979289
-84.	0.999598	0.003650	1.248661	0.13E-04	89.8	0.979332
-80.	0.999598	0.003642	1.248661	0.13E-04	89.8	0.979378
-76.	0.999598	0.003633	1.248661	0.13E-04	89.8	0.979429
-72.	0.999598	0.003623	1.248661	0.13E-04	89.8	0.979483
-68.	0.999598	0.003613	1.248661	0.13E-04	89.8	0.979538
-64.	0.999598	0.003602	1.248661	0.13E-04	89.8	0.979600
-60.	0.999598	0.003590	1.248661	0.13E-04	89.8	0.979667
-56.	0.999598	0.003577	1.248661	0.13E-04	89.8	0.979740
-52.	0.999598	0.003563	1.248661	0.13E-04	89.8	0.979819
-48.	0.999598	0.003548	1.248661	0.13E-04	89.8	0.979903
-44.	0.999598	0.003531	1.248661	0.13E-04	89.8	0.979997

<FREQ>
(*10MHZ)

<REFLECTIVITY>

-180.	0.978735	I*
-176.	0.978749	I *
-172.	0.978763	I *
-168.	0.978778	I *
-164.	0.978794	I *
-160.	0.978810	I *
-156.	0.978827	I *
-152.	0.978844	I *
-148.	0.978864	I *
-144.	0.978883	I *
-140.	0.978904	I *
-136.	0.978925	I *
-132.	0.978948	I *
-128.	0.978970	I *
-124.	0.978995	I *
-120.	0.979021	I *
-116.	0.979048	I *
-112.	0.979078	I *
-108.	0.979108	I *
-104.	0.979140	I *
-100.	0.979173	I *
-96.	0.979211	I *
-92.	0.979248	I *
-88.	0.979289	I *
-84.	0.979332	I *
-80.	0.979378	I *
-76.	0.979429	I *
-72.	0.979483	I *
-68.	0.979538	I *
-64.	0.979600	I *
-60.	0.979667	I *
-56.	0.979740	I *
-52.	0.979819	I *
-48.	0.979903	I *
-44.	0.979997	I *

* THE NARROW STRONG & WEAK BEAMS ARE SIMULTANEOUSLY SCANNED OVER NA F=2 LINE($F=J+I, J=1/2 \& I=3/2$) *

* THE SATURATED ATOMS DETECTED ARE NEARLY AT REST *

*THE DISTRIBUTION OF THE PEAKS (DUE TO THE HOLES) IS PLOTTED *

*** THE STRONG & WEAK BEAMS INCIDENT ON A THIN METAL LAYER DEPOSITTED ON ***
A GLASS-NA GAS INTERFACE AT AN INCIDENT ANGLE OF 50.0 DEGREE

*** THE DISTRIBUTION OF THE PEAK INTENSITY (DUE TO THE HOLE) IS EXAMINED ***

POLARIZATION IS PERPENDICULAR

** VAN DER WAAL'S POTENTIAL ENERGY HAS THE FORM OF THE INVERSE **
2ND POWER OF THE NORMAL DISTANCE FROM THE SURFACE
WITH A CONSTANT 0.8 IN UNIT OF E-12*ERG PER A**2

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
(MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

0.067 619. 20.0 4.3 0.381E-02 0.60

INCIDENT FREQ (*10MHZ)	DIELECTRIC REAL	CONST IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-180.	0.999598	0.003801	1.248662	0.13E-04	89.8	0.978486
-176.	0.999598	0.003800	1.248662	0.13E-04	89.8	0.978495
-172.	0.999598	0.003798	1.248662	0.13E-04	89.8	0.978505
-168.	0.999598	0.003796	1.248662	0.13E-04	89.8	0.978516
-164.	0.999598	0.003794	1.248662	0.13E-04	89.8	0.978526
-160.	0.999598	0.003792	1.248662	0.13E-04	89.8	0.978538
-156.	0.999598	0.003790	1.248662	0.13E-04	89.8	0.978550
-152.	0.999598	0.003788	1.248662	0.13E-04	89.8	0.978562
-148.	0.999598	0.003785	1.248662	0.13E-04	89.8	0.978574
-144.	0.999598	0.003783	1.248662	0.13E-04	89.8	0.978588
-140.	0.999598	0.003780	1.248662	0.13E-04	89.8	0.978602
-136.	0.999598	0.003778	1.248662	0.13E-04	89.8	0.978617
-132.	0.999598	0.003775	1.248662	0.13E-04	89.8	0.978633
-128.	0.999598	0.003772	1.248662	0.13E-04	89.8	0.978650
-124.	0.999598	0.003769	1.248662	0.13E-04	89.8	0.978667
-120.	0.999598	0.003766	1.248662	0.13E-04	89.8	0.978686
-116.	0.999598	0.003762	1.248662	0.13E-04	89.8	0.978705
-112.	0.999598	0.003758	1.248662	0.13E-04	89.8	0.978726
-108.	0.999598	0.003754	1.248662	0.13E-04	89.8	0.978747
-104.	0.999598	0.003750	1.248662	0.13E-04	89.8	0.978771
-100.	0.999598	0.003746	1.248662	0.13E-04	89.8	0.978796
-96.	0.999598	0.003741	1.248662	0.13E-04	89.8	0.978822
-92.	0.999598	0.003736	1.248662	0.13E-04	89.8	0.978850
-88.	0.999598	0.003731	1.248662	0.13E-04	89.8	0.978879
-84.	0.999598	0.003725	1.248662	0.13E-04	89.8	0.978913
-80.	0.999598	0.003719	1.248662	0.13E-04	89.8	0.978946
-76.	0.999598	0.003712	1.248662	0.13E-04	89.8	0.978983
-72.	0.999598	0.003705	1.248661	0.13E-04	89.8	0.979023
-68.	0.999598	0.003697	1.248661	0.13E-04	89.8	0.979067
-64.	0.999598	0.003689	1.248661	0.13E-04	89.8	0.979113
-60.	0.999598	0.003680	1.248661	0.13E-04	89.8	0.979164
-56.	0.999598	0.003670	1.248661	0.13E-04	89.8	0.979219
-52.	0.999598	0.003659	1.248661	0.13E-04	89.8	0.979280
-48.	0.999598	0.003647	1.248661	0.13E-04	89.8	0.979346
-44.	0.999598	0.003634	1.248661	0.13E-04	89.8	0.979420

*** THE STRONG & WEAK BEAMS INCIDENT ON A THIN METAL LAYER DEPOSITED ON ***
A GLASS-NA GAS INTERFACE AT AN INCIDENT ANGLE OF 50.0 DEGREE

*** THE DISTRIBUTION OF THE PEAK INTENSITY (DUE TO THE HOLE) IS EXAMINED ***

POLARIZATION IS PERPENDICULAR

** VAN DER WAAL'S POTENTIAL ENERGY HAS THE FORM OF THE INVERSE **
4TH POWER OF THE NORMAL DISTANCE FROM THE SURFACE
WITH A CONSTANT 20.0 IN UNIT OF E-12*ERG PER A**4

* < EXPERIMENTAL CONDITIONS > *

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
(MMHG) (KELVIN) <HWI> <HWN1> (*10MHZ) (*10MHZ) (WATT/CM**2)

0.067 619. 20.0 4.3 0.381E-02 0.60

INCIDENT FREQ (*10MHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-180.	0.999598	0.003725	1.248662	0.13E-04	89.8	0.978914
-176.	0.999598	0.003722	1.248662	0.13E-04	89.8	0.978930
-172.	0.999598	0.003718	1.248662	0.13E-04	89.8	0.978948
-168.	0.999598	0.003715	1.248662	0.13E-04	89.8	0.978967
-164.	0.999598	0.003712	1.248662	0.13E-04	89.8	0.978986
-160.	0.999598	0.003708	1.248662	0.13E-04	89.8	0.979006
-156.	0.999598	0.003705	1.248661	0.13E-04	89.8	0.979025
-152.	0.999598	0.003701	1.248661	0.13E-04	89.8	0.979046
-148.	0.999598	0.003697	1.248661	0.13E-04	89.8	0.979069
-144.	0.999598	0.003693	1.248661	0.13E-04	89.8	0.979092
-140.	0.999598	0.003688	1.248661	0.13E-04	89.8	0.979117
-136.	0.999598	0.003684	1.248661	0.13E-04	89.8	0.979141
-132.	0.999598	0.003679	1.248661	0.13E-04	89.8	0.979168
-128.	0.999598	0.003674	1.248661	0.13E-04	89.8	0.979196
-124.	0.999598	0.003669	1.248661	0.13E-04	89.8	0.979225
-120.	0.999598	0.003664	1.248661	0.13E-04	89.8	0.979255
-116.	0.999598	0.003658	1.248661	0.13E-04	89.8	0.979287
-112.	0.999598	0.003652	1.248661	0.13E-04	89.8	0.979321
-108.	0.999598	0.003645	1.248661	0.13E-04	89.8	0.979357
-104.	0.999598	0.003639	1.248661	0.13E-04	89.8	0.979394
-100.	0.999598	0.003632	1.248661	0.13E-04	89.8	0.979434
-96.	0.999598	0.003624	1.248661	0.13E-04	89.8	0.979477
-92.	0.999598	0.003616	1.248661	0.13E-04	89.8	0.979522
-88.	0.999598	0.003608	1.248661	0.13E-04	89.8	0.979569
-84.	0.999598	0.003599	1.248661	0.13E-04	89.8	0.979620
-80.	0.999598	0.003589	1.248661	0.13E-04	89.8	0.979673
-76.	0.999598	0.003579	1.248661	0.13E-04	89.8	0.979729
-72.	0.999598	0.003568	1.248661	0.13E-04	89.8	0.979791
-68.	0.999598	0.003556	1.248661	0.13E-04	89.8	0.979857
-64.	0.999598	0.003544	1.248661	0.13E-04	89.8	0.979927
-60.	0.999598	0.003530	1.248661	0.13E-04	89.8	0.980003
-56.	0.999598	0.003515	1.248661	0.13E-04	89.8	0.980084
-52.	0.999598	0.003500	1.248660	0.13E-04	89.8	0.980174
-48.	0.999598	0.003482	1.248660	0.13E-04	89.8	0.980269
-44.	0.999598	0.003464	1.248660	0.13E-04	89.8	0.980374

APPENDIX C

1
2 \$\$JOUT,MONI,T

3 \$:IDENT:RPHYA001,GAS FLOW

4 \$:OPTION:FORTTRAN

5 \$:FORTTRAN

6 C THE INTENSITY DISTRIBUTION OF THE WEAK REFLECTED BEAM FROM
7 C A GLASS-GAS FLOW INTERFACE

8 C
9 C THE GAS FLOW VELOCITY IS ASSUMED TO BE PARALLEL TO THE WALL

10 C
11 C THE INSTANTANEOUS VELOCITY IS ASSUMED TO HAVE THE MEAN PLUS
12 C THE FLUCTUATING PART

13 C
14 C A GAUSSIAN FLUCTUATION IS ASSUMED

15 C
16 C SIMPSON'S METHOD IS USED IN NUMERICAL INTEGRATIONS

17 C
18 C *CONSTANTS*

19 C
20 C REWAVE = RESONANCE WAVE LENGTH OF NA D2 LINE

21 C GASDEN = DENSITY OF GAS VAPOR (PER CUBIC CM)

22 C MOLWEI = MOLECULAR WEIGHT OF ATOM

23 C HWI = HALF OF 1/E POINTS WIDTH OF INCIDENT BEAM

24 C HWN1 = HALF OF HALF-VALUE WIDTH OF RESONANCE LINE

25 C SORIND = REFRACTIVE INDEX OF GLASS

26 C OS1 = OSCILLATOR STLENGTH 1

27 C TEMPER = GAS TEMPERATURE

28 C XLIGHT = VELOCITY OF LIGHT

29 C PLANK = PLANK CONSTANT

30 C ELMASS = ELECTRON MASS

31 C GASCON = GAS CONSTANT

32 C XLOSHM = LOSHMIDT'S NUMBER

33 C AANGLE = ANGLE OF INCIDENCE (DEGREE)

34 C ANGINC = ANGLE OF INCIDENCE (RADIAN)

35 C EFDOP = 1/E POINTS' WIDTH OF DOPPLER BROADENED RESONANCE LINE

36 C PAI = PAI

37 DIMENSION Z(58),REIND(58),DIERE(58),DIECM(58),REF(58),

38 & ANGREN(58),DEPTHN(58),REFLOG(58)

39 GASDEN=1.05E+15

40 HWI=2.

41 HWN1=200.

42 TF1=10.

43 TEMPER=300.

44 POLARI=+1

45 IPROGM=1

46 REWAVE=5.89E-5

47 XMOLWE=22.9898

48 SATFAC=0.8

49 SORIND=1.63

50 OS1 =0.42

51 XLIGHT=2.99793E+10

52 PLANK =1.05459E-27

53 ELMASS=9.10953E-28

54 GASCON=8.31441E+7

55 BOLTZC=1.38042E-16

PAI =3.14159

XLOSHM=2.68675E+19

DOP =2.*GASCON*TEMPER/XMOLWE

EFCDOP=2.*SQRT(DOP)/REWAVE

X10KHZ =1.E+4

X10MHZ=1.E+7

EFDOP =EFCDOP/X10KHZ

C TO ESTIMATE THE ENERGY FLUX(WATT/CM**2) OF THE STRONG BEAM,

C THE COLISION RATE IS ASSUMED TO BE 10MHZ

ENGFLX=SATFAC*137.036*2.*XLIGHT*ELMASS*2.*HWN1*X10MHZ/REWAVE/OS1

C

C CONST=CONSTANT APPERING IN SUSCEPTIBILITY

C

CONST1=FLANK*GASDEN*REWAVE

CONST2=137.036*ELMASS*SQRT(PAI)*4.*EFCDOP*PAI**2

CONST =CONST1/CONST2

WRITE(6,899)CONST

IPOLA=IFIX(POLARI)

899 FORMAT(1H1,///,3X,"THIS IS A CONSTANT APPERING IN",
& " SUSCEPTIBILITY",/23X,E12.5)

C

C

VAPOR=VAPOR PRESSURE IN UNIT OF MMHG

C

VAPOR =760.*TEMPER*GASDEN/XLOSHM/273.2

NA=8

FNA=FLOAT(NA)

HA=3.2*HWI/FNA

CALL POWER(AVEFLX,ENGFLX,NA,HA,HWI)

AANGLE=40.

ANGINC=AANGLE*PAI/180.

SORSIN=SORIND*SIN(ANGINC)

CN=CONST*2./SORSIN/TF1/SQRT(PAI)

DLOW=-0.8*TF1

DHIG=0.8*TF1

FNDD=0.8*DHIG/0.1

NDD=IFIX(FNDD)

FND=2.*FLOAT(NDD)

ND=IFIX(FND)

HD=2.*DHIG/FND

TVL=X10KHZ*REWAVE*TF1/SORSIN

TKE=TVL**2

IVAN=3

ZERO=0.

DO 4600 IVANDE=1,IVAN

INTRAC=IVANDE-3

FVM=0.

IF(INTRAC.EQ.-2)FVM=+3.

IF(INTRAC.EQ.-1)FVM=+12.

VM=X10KHZ*REWAVE*FVM/SORSIN

IFREQ=18

FSC=2.

FIFREQ=FLOAT(IFREQ)

```

1      RATIO=FIFREQ/FSC+0.5
2      RATIO2=2.*RATIO
3      IC=IFIX(RATIO2)
4      IFREQ=IFREQ-IFIX(FVM)
5      IF(INTRAC.LE.-1)FIFREQ=FIFREQ-FVM
6      DO 4000 IZ=1,IC
7      IIZ=IZ-1
8      Z(IZ)=-FIFREQ+FLOAT(IIZ)*FSC
9      IHOLE=0
10     CALL SUSCER(RESUM,CMSUM,Z(IZ),ND,HD,
11     & DLOW,CN,OS1,HWN1,IHOLE,
12     & SATFAC,FVM,TF1,HWI)
13     RESUMD=RESUM
14     CMSUMD=CMSUM
15     IHOLE=1
16     CALL SUSCER(RESUM,CMSUM,Z(IZ),ND,HD,
17     & DLOW,CN,OS1,HWN1,IHOLE,
18     & SATFAC,FVM,TF1,HWI)
19     RESUMH=RESUM
20     CMSUMH=CMSUM
21     RESUM1=RESUMD-RESUMH
22     CMSUM1=CMSUMD-CMSUMH
23
24 4130 CALL REIND(REINDX,REFREX,REFCMX,ALFAX,
25     & BETAX,RESUM1,CMSUM1,SORIND,ANGINC,PAI)
26     REIND(IZ)=REINDX
27     DIERE(IZ)=REFREX
28     DIECM(IZ)=REFCMX
29     ALFAN=ALFAX
30     BETAN=BETAX
31     DEPTHN(IZ)=REWAVE/BETAN/PAI/2.
32     ANGLE2=SORSIN/REIND(IZ)
33     ANGREN(IZ)=180.*ARSIN(ANGLE2)/PAI
34     CALL REFLEC(ALREFLFX,REFLFX,ALFAN,BETAN,ANGINC,ANGREF,
35     & POLALI,SORIND)
36     REF(IZ)=REFLFX
37     REFLOG(IZ)=ALREFLFX
38
39 4000 CONTINUE
40     WRITE(6,901)AANGLE
41     IF(IPOLA.LT.0)WRITE(6,898)
42     IF(IPOLA.GT.0)WRITE(6,897)
43     IF(IPOLA.EQ.0)WRITE(6,896)
44     WRITE(6,840)
45     WRITE(6,850)VM,TKE,TVL
46     WRITE(6,908)VAPOR,TEMPER,HWI,HWN1,FVM,
47     & AVEFLX,SATFAC
48     WRITE(6,909)
49     WRITE(6,910) (Z(IZ),DIERE(IZ),DIECM(IZ),REIND(IZ),DEPTHN(IZ),
50     & ANGREN(IZ),REF(IZ),IZ=1,IC)
51     WRITE(6,955)
52     CALL GRAPH(FVM,VM,TF1,TVL,IC,IFREQ,FSC,DIERE)
53     WRITE(6,956)
54     CALL GRAPH(FVM,VM,TF1,TVL,IC,IFREQ,FSC,DIECM)
55     WRITE(6,958)
56     CALL GRAPH(FVM,VM,TF1,TVL,IC,IFREQ,FSC,REF)

```

```

1      WRITE(6,960)
2      CALL GRAPH(FVM,VM,TF1,TVL,IC,IFREQ,FSC,REFLOG)

```

```

3      4600 CONTINUE
4      898 FORMAT(1H0,23X,"POLARIZATION IS PARALLEL")
5      897 FORMAT(1H0,23X,"POLARIZATION IS PERPENDICULAR")
6      896 FORMAT(1H0,23X,"PLANE POLARIZED LIGHT AT 45 DEGREE")
7      908 FORMAT(1H0,17X,"*< EXPERIMENTAL CONDITIONS >*",
8      & 1H0,2X,"<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH>",
9      & " <LOC HOLE> <ENG FLX STG> <SATFAC>",/3X,"(MMHG) ",
10     & "(KELVIN) <HWI> <HWN1> (*10KHZ)",3X,"(*10KHZ)",
11     & 3X,"(WATT/CM**2)",/1H0,
12     & 3X,F5.3,4X,F4.0,4X,F4.1,2X,F6.1,10X,
13     & 3X,F6.1,6X,E10.3,5X,F3.1,/)
14     901 FORMAT(1H1,///,3X,"*** A NARROW PROBING BEAM INCIDENT ON",
15     & " A GLASS-NA GAS FLOW INTERFACE ***",/20X,"AT AN ",
16     & " INCIDENT ANGLE OF ",F4.1," DEGREE",/)
17     906 FORMAT(1H0,13X,"LINE SHAPE OF THE HOLE IS EXAMINED",/)
18     910 FORMAT(1H ,4X,F5.0,4X,F8.6,1X,F8.6,1X,F8.6,1X,E8.2,2X,F4.1,
19     & 4X,F8.6)
20     909 FORMAT(1H0,"INCIDENT FREQ",1X,"DIELECTRIC CONST",2X,"REFRACT",
21     & 4X,"PENET",2X,"REFRCT",2X,"REFLECTIVITY",/2X,"(*10KHZ)",
22     & 6X,"REAL",4X,"IMAG",5X,"INDEX",6X,"DEPTH",3X,"ANGLE",/)
23     955 FORMAT(1H1,///,"<FREQ>",34X,"<REAL PART OF THE DIELECTRIC",
24     & " CONST>",/)
25     956 FORMAT(1H1,///,"<FREQ>",34X,"<IMAG PART OF THE DIELECTRIC",
26     & " CONST>",/)
27     957 FORMAT(1H1,///,"<FREQ>",34X,"<REFRACTIVE INDEX>",/)
28     958 FORMAT(1H1,///,"<FREQ>",34X,"<REFLECTIVITY>",/)
29     840 FORMAT(1H0,25X,"***GAS FLOW CONDITONS ***")
30     960 FORMAT(1H1,///,"<FREQ>",20X,"< NATURAL LOG OF THE REFLECTIVTY>")
31     850 FORMAT(1H0,19X,"< MEAN FLOW VELOCITY=",F7.2,"(CM/SEC) >",/
32     & 20X,"< TURBULENCE KINETIC ENGY=",F7.2,"(CM/SEC)**2 >",/
33     & 20X,"< INTENCITY OF TURBULENCE=",F7.2,"(CM/SEC) >")
34     9000 STOP
35     END

```

```

36     C SUBROUTION TO GRAPH DATA

```

```

37     SUBROUTINE GRAPH(FVM,VM,TF1,TVL,IC,IFREQ,FSC,VALUE)
38     DIMENSION PLOT(83),VALUE(IC)
39     DATA DASH/'-'/,BAR/'I'/,X/'*'/,BLANK/' '/
40     DO 5 I=1,IC
41     IF(I.EQ.1)YMAX=VALUE(I)
42     IF(I.EQ.1)YMIN=VALUE(I)
43     IF(I.GE.2) GO TO 2
44     GO TO 5
45     2    BIG=VALUE(I)-YMAX
46     SMALL=VALUE(I)-YMIN
47     ZERO=0.
48     IF(BIG.GE.ZERO) YMAX=VALUE(I)
49     IF(SMALL.LT.ZERO) YMIN=VALUE(I)
50     5    CONTINUE
51     DIF=YMAX-YMIN
52     DO 10 K=1,83
53     PLOT(K)=DASH

```

```

10  CONTINUE
    WRITE(6,20)PLOT
20  FORMAT(1X,"(*10KHZ)",/1H0,17X,83A1)
    DO 30 K=1,83
        PLOT(K)=BLANK
30  CONTINUE
    PLOT(1)=BAR
    DO 60 KZ=1,IC
        KZZ=KZ-1
        Z=-FLOAT(IFREQ)+FLOAT(KZZ)*FSC
        YDIF=VALUE(KZ)-YMIN
        YN=YDIF*80./DIF+2.5
        NY=IFIX(YN)
        PLOT(NY)=X
        WRITE(6,40)Z,VALUE(KZ),PLOT
40  FORMAT(1X,F5.0,2X,F8.6,1X,83A1)
        PLOT(NY)=BLANK
60  CONTINUE
    WRITE(6,61)
    WRITE(6,62)FVM,VM,TF1,TVL
62  FORMAT(1H0,"* THE FREQ SHIFT",F7.2," (*10KHZ) CORESPONDS TO THE ",
& "MEAN FLOW VELOCITY",F7.2," (CM/SEC) *",/1X,"* THE 1/E ",
& "POINTS' WIDTH",F7.2," (*10KHZ) CORESPONDS TO THE FLUCTUATIO",
& "N OF",F7.2," (CM/SEC) *")
61  FORMAT(1H0,20X,"**** THE CENTRAL FREQ OF THE UNSHIFTED ",
& "NATURAL LINE IS LOCATED AT FREQ=0. ****")
    RETURN
    END
    SUBROUTINE POWER(AVEFLX,ENGFLX,N,H,HWI)
    SRAA=0.
    DO 10 J=1,N+1,2
        JL=J-1
        D=H*FLOAT(JL)-1.6*HWI
        A1=-D**2/HWI**2
        EA1=EXP(A1)
        SRAA=SRAA+EA1
10  CONTINUE
    AVEFLX=ENGFLX*SRAA/(FLOAT(N)+1)
    RETURN
    END
    SUBROUTINE SUSCER(RESUM,CMSUM,Z,N,H,
& XLOWAL,C,OS1,HWN1,
& IHOLE,SATFAC,FVM,TF1,HWI)
    Z=Z-FVM
5  SREAB=0.
    SCMAB=0.
    DO 16 LA=1,N+1,2
        LLA=LA-1
        DAL=H*FLOAT(LLA)+XLOWAL
        SRRAB=0.
        SCRAB=0.
        DO 17 II=1,2
            NN=II-1
            FNN=FLOAT(NN)

```

```

1      V1=DAL+H*FNN
3      FVN=-4.*V1**2/TF1**2
4      EAL1=EXP(FVN)
5      ZHOLE=V1
6      ALL2=(Z-ZHOLE)**2/HWI**2
7      IF(ALL2.GT.60)GO TO 90
8      EAL2=EXP(-ALL2)
9      SATFA1=SATFAC*EAL2
10     GO TO 100
11     90      SATFA1=0.
12     100     ALZ1=-Z-V1
13           ALZ2=ALZ1**2
14           IF(IHOLE.EQ.1)ALZP1=0.
15           IF(IHOLE.EQ.1)ALZP2=ALZP1**2
16           ALZ2=ALZ1**2
17           HWN2=HWN1**2
18           ALX1=ALZ2+HWN2
19           SATFA2=1.+SATFA1
20           IF(IHOLE.EQ.1)HOLE=SATFA1*HWN2/2./((ALZP2+HWN2*SATFA2)
21           RFAL1=EAL1*OS1*ALZ1/ALX1
22           CFAL1=EAL1*HWN1*OS1/ALX1
23           IF(IHOLE.EQ.1)RFAL1=HOLE*RFAL1
24           IF(IHOLE.EQ.1)CFAL1=HOLE*CFAL1
25           IF(LLA.EQ.0)GO TO 12
26           IF(LLA.EQ.N)GO TO 13
27           IF(NN.EQ.0)NC=2
28           IF(NN.EQ.1)NC=4
29           GO TO 14
30     12      IF(NN.EQ.0)NC=1
31           IF(NN.EQ.1)NC=4
32           GO TO 14
33     13      IF(NN.EQ.0)NC=1
34           IF(NN.EQ.1)NC=0
35     14      FNC=FLOAT(NC)
36           SRRAB=SRRAB+FNC*RFAL1
37           SCRAB=SCRAB+FNC*CFAL1
38     17      CONTINUE
39           SREAB=SRRAB+SREAB
40           SCMAE=SCRAB+SCRAB
41     16      CONTINUE
42           FUNREY=H*SREAB/3.
43           FUNCMY=H*SCMAE/3.
44     31      RESUM=C*FUNREY
45           CMSUM=C*FUNCMY
46           Z=Z+FVM
47     32      RETURN
48           END
49     C      SUBROUTINE TO CALCULATE REFRACTIVE INDEX
50
51           SUBROUTINE REFIND(REINDX,REFREX,REFCMX,ALFAX,BETAX,
52           &      SUSREX,SUSCMX,SOINDX,ANGINC,PAI)
53           REFREX=1.+4.*PAI*SUSREX
54           REFCMX=2.*PAI*SUSCMX
55           PART1=(SOINDX*SIN(ANGINC))**2

```

```

1      A=REFREX-PART1
2      B=2.*REFCMX
3      C=REFREX+PART1
4      D=0.
5      AB=ABS(A)
6      IF(AB-B)60,61,62
7
8 60     X=A/B
9      XRT=1.+X**2
10     ALFA=B*(SQRT(XRT)+X)/2.
11     ALFAX=SQRT(ALFA)
12     BETA=B*(SQRT(XRT)-X)/2.
13     E=SQRT(XRT)-X
14     IF(E.EQ.D)BETAX=0.
15     BETAX=SQRT(BETA)
16     PARTX=(C+B*XRT)/2.
17     REINDX=SQRT(PARTX)
18     GO TO 63
19
20 61     ALFA=B*(1.+SQRT(2.))/2.
21     ALFAX=SQRT(ALFA)
22     BETA=B*(SQRT(2.)-1.)/2.
23     BETAX=SQRT(BETA)
24     PARTE=(C+B*(1.+SQRT(2.)))/2.
25     REINDX=SQRT(PARTE)
26     GO TO 63
27
28 62     Y=B/A
29     YRT=1.+Y**2
30     ALFA=A*(1.+SQRT(YRT))/2.
31     IF(A.LT.D)ALFA=AB*(SQRT(YRT)-1.)/2.
32     ALFAX=SQRT(ALFA)
33     BETA=A*(SQRT(YRT)-1.)/2.
34     IF(A.LT.D)BETA=AB*(SQRT(YRT)+1.)/2.
35     F=SQRT(YRT)-1.
36     IF(F.EQ.D)BETAX=0.
37     BETAX=SQRT(BETA)
38     PARTY=(C+A*YRT)/2.
39     REINDX=SQRT(PARTY)
40
41 63     IF(REINDX**2-PART1)64,65,65
42 64     XC1=REFCMX**2/(PART1-REINDX**2)
43     XC2=XC1+PART1
44     XC=SQRT(XC2)
45     REINDX=XC
46
47 65     RETURN
48     END

```

C SUBROUTINE TO DEFINE A FUNCTIONAL FORM OF REFLECTIVITY

```

18      SUBROUTINE REFLEC(ALREFLFX,REFLFX,ALFAX,BETAX,ANGINC,
19      &  ANGREF,POLARI,SOINDX)
20      AC=COS(ANGINC)
21      AS=SIN(ANGINC)
22      RNUMAX=(SOINDX*AC-ALFAX)**2+BETAX**2
23      RDENO=(SOINDX*AC+ALFAX)**2+BETAX**2
24      ALRNUMAX=ALOG(RNUMAX)
25      ALRDENO=ALOG(RDENO)

```

```
SA=SOINDX*AS
XA=(ALFAX**2-BETAX**2+SA**2)*AC
XAN=XA-SOINDX*ALFAX
XB=2.*ALFAX*BETAX
XBN=XB-SOINDX*BETAX
RNPER=XAN**2+XBN**2
XAD=XA+SOINDX*ALFAX
XBD=XB+SOINDX*BETAX
RDPER=XAD**2+XBD**2
ALRNPER=ALOG(RNPER)
ALRDPER=ALOG(RDPER)
IF (POLARI) 701,702,703
701  REFLFX=RNPER/RDPER
    ALREFLFX=ALRNPER-ALRDPER
    GO TO 705
703  REFLFX=RNUMAX/RDENO
    ALREFLFX=ALRNUMAX-ALRDENO
    GO TO 705
702  REFLFX=.5*(RNPER/RDPER+RNUMAX/RDENO)
    ALREFLFX=0.5*(ALRNPER+ALRNUMAX-ALRDPER-ALRDENO)
705  RETURN
    END
```

```
$:EXECUTE
$:LIMITS:10,27K,,5000
$:SYSOUT:06,ORG
$:ENDJOB
```


*** A NARROW PROBING BEAM INCIDENT ON A GLASS-NA GAS FLOW INTERFACE ***
AT AN INCIDENT ANGLE OF 40.0 DEGREE

POLARIZATION IS PERPENDICULAR

***GAS FLOW CONDITONS ***

< MEAN FLOW VELOCITY= 0. (CM/SEC) >
< TURBULENCE KINETIC ENGY= 31.60(CM/SEC)**2 >
< INTENCITY OF TURBULENCE= 5.62(CM/SEC) >

< EXPERIMENTAL CONDITIONS >

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
(MMHG) (KELVIN) <HWI> <HWN1> (*10KHZ) (*10KHZ) (WATT/CM**2)

0.033 300. 2.0 200.0 0. 0.238E 00 0.8

INCIDENT FREQ (*10KHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-18.	1.000010	0.000058	1.047743	0.30E-04	90.0	0.999037
-16.	1.000009	0.000058	1.047743	0.30E-04	90.0	0.999037
-14.	1.000008	0.000058	1.047743	0.30E-04	90.0	0.999037
-12.	1.000007	0.000058	1.047743	0.30E-04	90.0	0.999037
-10.	1.000006	0.000058	1.047743	0.30E-04	90.0	0.999037
-8.	1.000005	0.000057	1.047743	0.30E-04	90.0	0.999047
-6.	1.000003	0.000056	1.047743	0.30E-04	90.0	0.999057
-4.	1.000002	0.000055	1.047743	0.30E-04	90.0	0.999089
-2.	1.000001	0.000053	1.047743	0.30E-04	90.0	0.999111
0.	1.000000	0.000052	1.047743	0.30E-04	90.0	0.999122
2.	0.999999	0.000053	1.047743	0.30E-04	90.0	0.999111
4.	0.999998	0.000055	1.047743	0.30E-04	90.0	0.999089
6.	0.999997	0.000056	1.047743	0.30E-04	90.0	0.999057
8.	0.999995	0.000057	1.047743	0.30E-04	90.0	0.999047
10.	0.999994	0.000058	1.047743	0.30E-04	90.0	0.999037
12.	0.999993	0.000058	1.047743	0.30E-04	90.0	0.999037
14.	0.999992	0.000058	1.047743	0.30E-04	90.0	0.999037
16.	0.999991	0.000058	1.047743	0.30E-04	90.0	0.999037
18.	0.999990	0.000058	1.047743	0.30E-04	90.0	0.999036

<FREQ>
(*10KHZ)

<REFLECTIVITY>

-18. 0.999037 I*
-16. 0.999037 I*
-14. 0.999037 I*
-12. 0.999037 I*
-10. 0.999037 I*
-8. 0.999047 I
-6. 0.999057 I
-4. 0.999089 I
-2. 0.999111 I
0. 0.999122 I
2. 0.999111 I
4. 0.999089 I
6. 0.999057 I
8. 0.999047 I
10. 0.999037 I*
12. 0.999037 I*
14. 0.999037 I*
16. 0.999037 I*
18. 0.999036 I*

*

*

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**** THE CENTRAL FREQ OF THE UNSHIFTED NATURAL LINE IS LOCATED AT FREQ=0. ****

* THE FREQ SHIFT 0. (*10KHZ) CORESPONDS TO THE MEAN FLOW VELOCITY 0. (CM/SEC) *

* THE 1/E POINTS' WIDTH 10.00 (*10KHZ) CORESPONDS TO THE FLUCTUATION OF 5.62 (CM/SEC) *

*** A NARROW PROBING BEAM INCIDENT ON A GLASS-NA GAS FLOW INTERFACE ***
 AT AN INCIDENT ANGLE OF 40.0 DEGREE

POLARIZATION IS PERPENDICULAR

***GAS FLOW CONDITONS ***

< MEAN FLOW VELOCITY= 1.69(CM/SEC) >
 < TURBULENCE KINETIC ENGY= 31.60(CM/SEC)**2 >
 < INTENCITY OF TURBULENCE= 5.62(CM/SEC) >

< EXPERIMENTAL CONDITIONS >

<VAPOR> <TEMP> <1/2 OF HALF-VALUE WIDTH> <LOC HOLE> <ENG FLX STG> <SATFAC>
 (MMHG) (KELVIN) <HWI> <HWN1> (*10KHZ) (*10KHZ) (WATT/CM**2)

0.033 300. 2.0 200.0 3.0 0.238E 00 0.8

INCIDENT FREQ (*10KHZ)	DIELECTRIC CONST REAL	IMAG	REFRACT INDEX	PENET DEPTH	REFRCT ANGLE	REFLECTIVITY
-15.	1.000010	0.000058	1.047743	0.30E-04	90.0	0.999037
-13.	1.000009	0.000058	1.047743	0.30E-04	90.0	0.999037
-11.	1.000008	0.000058	1.047743	0.30E-04	90.0	0.999037
-9.	1.000007	0.000058	1.047743	0.30E-04	90.0	0.999037
-7.	1.000006	0.000058	1.047743	0.30E-04	90.0	0.999037
-5.	1.000005	0.000057	1.047743	0.30E-04	90.0	0.999047
-3.	1.000003	0.000056	1.047743	0.30E-04	90.0	0.999057
-1.	1.000002	0.000055	1.047743	0.30E-04	90.0	0.999089
1.	1.000001	0.000053	1.047743	0.30E-04	90.0	0.999111
3.	1.000000	0.000052	1.047743	0.30E-04	90.0	0.999122
5.	0.999999	0.000053	1.047743	0.30E-04	90.0	0.999111
7.	0.999998	0.000055	1.047743	0.30E-04	90.0	0.999089
9.	0.999997	0.000056	1.047743	0.30E-04	90.0	0.999057
11.	0.999995	0.000057	1.047743	0.30E-04	90.0	0.999047
13.	0.999994	0.000058	1.047743	0.30E-04	90.0	0.999037
15.	0.999993	0.000058	1.047743	0.30E-04	90.0	0.999037
17.	0.999992	0.000058	1.047743	0.30E-04	90.0	0.999037
19.	0.999991	0.000058	1.047743	0.30E-04	90.0	0.999037
21.	0.999990	0.000058	1.047743	0.30E-04	90.0	0.999036

<FREQ>
(*10KHZ)

<REFLECTIVITY>

-15. 0.999037 I*
-13. 0.999037 I*
-11. 0.999037 I*
-9. 0.999037 I*
-7. 0.999037 I*
-5. 0.999047 I
-3. 0.999057 I
-1. 0.999089 I
1. 0.999111 I
3. 0.999122 I
5. 0.999111 I
7. 0.999089 I
9. 0.999057 I
11. 0.999047 I
13. 0.999037 I*
15. 0.999037 I*
17. 0.999037 I*
19. 0.999037 I*
21. 0.999036 I*

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*

**** THE CENTRAL FREQ OF THE UNSHIFTED NATURAL LINE IS LOCATED AT FREQ=0. ****

* THE FREQ SHIFT 3.00 (*10KHZ) CORRESPONDS TO THE MEAN FLOW VELOCITY 1.69 (CM/SEC) *

* THE 1/E POINTS' WIDTH 10.00 (*10KHZ) CORRESPONDS TO THE FLUCTUATION OF 5.62 (CM/SEC) *