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Impact of Teachers' Planned Questions on Opportunities for Students to Reason Mathematically in Whole-class Discussions Around Mathematical Problem-solving Tasks

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Impact of Teachers' Planned Questions on Opportunities for Students to Reason Mathematically in Whole-class Discussions Around Mathematical Problem-solving Tasks

by

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Abstract

While professional developers have been encouraging teachers to plan for discourse around problem solving tasks as a way to orchestrate mathematically productive discourse (Stein, Engle, Smith, & Hughes, 2008; Stein, Smith, Henningsen, & Silver, 2009) no research has been conducted explicitly examining the relationship between the plans that teachers make for orchestrating discourse around problem solving tasks and the outcomes of implementation of those plans. This research study is intended to open the door to research on planning for discourse around problem solving tasks.

This research study analyzes how 12 middle school mathematics teachers participating in the Mathematics Problem Solving Model professional development research program implemented lesson plans that they wrote in preparation for whole-class discussions around cognitively demanding problem solving tasks. The lesson plans consisted of the selection and sequencing of student solutions to be presented to the class along with identification of the mathematical ideas to be highlighted in the student solutions and questions that would help to make the mathematics salient. The data used for this study were teachers' lesson plans and the audio-recordings of the whole-class discussions implemented by the teachers.

My research question for this study was: How do teachers' written plans for orchestrating mathematical discourse around problem solving tasks influence the opportunities teachers create for students to reason mathematically? To address this research question, I analyzed the data in three different ways. First, I measured fidelity to the literal lesson by comparing what was planned in the ISAs to what was actually took

place in the implemented debriefs. That is, I analyzed the extent to which the teachers were implementing the basic steps in their lesson (i.e. sharing the student work they identified, addressing the ideas to highlight and the planned questions). Second, I analyzed the teachers' fidelity to the intended lesson by comparing the number of high-press questions in the lesson plans (that is, questions that create opportunities for the students to reason mathematically) to the number of high-press questions in the implemented discussion. I compared these two sets of data using a linear regression analysis and t-tests. Finally, I conducted a qualitative analysis, using grounded theory, of a subset of four teachers from the study. I examined the improvisational moves of the teachers as they addressed the questions they had planned, building a theory of how the different ways that teachers implemented their planned questions affected the opportunities for their students to reason mathematically around those planned questions.

My findings showed that it was typical for the teachers to implement most of the steps of their lesson plans faithfully, but that there was not a statistically significant correlation between the number of high-press questions they planned and the number of high-press questions they asked during the whole-class discussions, indicating that there were other factors that were influencing the frequency with which the teachers were asked these questions that prompted their students to reason mathematically. I hypothesize that these factors include, but are not limited to, the norms in the classrooms, teachers' knowledge about teaching mathematics, and teachers' beliefs about mathematics. Nevertheless, my findings did show that in the portions of the whole-class discussions where the teachers had planned at least one high-press question, they, on average, asked more high-press questions than when they did not plan to ask any.

Finally, I identified four different ways that teachers address their planned questions which impacted the opportunities for students to reason mathematically. Teachers addressed their questions as drop-in (they asked the question and then moved on as soon as a response was elicited), embedded (the ideas in the question were addressed by a student without being prompted), telling (the teacher told the students the ‘response’ to the question without providing an opportunity for the students to attempt to answer the question themselves) and sustained focus (the teacher sustained the focus on the question by asking the students follow-up questions).

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"I can do all things through Christ who strengthens me."

Philippians 4:13

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Glossary of Terms

Debrief, problem-solving – A whole-class discussion following the implementation of a problem-solving task in which students share their solution strategies and the teacher orchestrates the discussion to help students focus on the important mathematical ideas in the task.

Design a Dartboard – One of the three problem-solving tasks the teachers in this study were required to implement as part of their Mathematics Problem Solving Model professional development experience. The teachers implemented this task with their students, planning and orchestrating a whole-class discussion around the ideas in this task. Please refer to Chapter 4, Task Analysis, for a detailed description of this task.

Episode – Used in the analysis of a problem-solving debrief. A portion of a problem-solving debrief in which a planned question is being addressed. A standard episode would occur when the teacher asks the planned question; a student (or multiple students) answers the question; and continues as long as the ideas generated by the question are being discussed. An episode also can occur when the ideas in a planned question are being addressed, either by the teacher or a student, without the question first being asked by the teacher.

High-press questions – Part of the question coding scheme developed for this research study that differentiated between high-press questions and low-press questions. There are questions planned and/or asked by the teacher that are intended to encourage students to reason mathematically about the problem-solving task and/or the mathematical ideas present within the problem-solving task. High-press questions were identified both in the Instructional Sequence Analyses as well as in the implemented debriefs. See Chapter 7, Research Sub-Question 2, for a description of this coding scheme.

Implementation Fidelity Analysis (IFA) – Data analysis tool that was developed and used for this research study to gauge how closely teachers were implementing the Instructional Sequence Analyses (ISAs). This data analysis tool focused specifically on (1) whether or not the student work identified in the ISA was presented, (2) whether or not the teacher addressed the planned questions within a segment, and (3) whether or not the mathematical ideas to highlight were addressed. The Implementation Fidelity Analysis analyzes each segment in a problem-solving debrief separately, assigning a level of fidelity. A faithful implementation of a segment is when the identified student work was presented, and both the planned questions and ideas to highlight were addressed. A partially faithful implementation is when the student work was presented and either the idea to highlight was addressed or the identified questions were planned, but not both. A non-faithful implementation of a segment is when either the student work was never presented or the student work was presented but neither the idea to highlight nor the planned questions were addressed.

Instructional Sequence Analysis (ISA) – The lesson planning protocol that the teachers participating in the Mathematics Problem Solving Model professional development used to plan for the problem-solving debriefs as part of the professional development. The key features of the ISA were the selection and sequencing of student work to be presented during the problem-solving debrief, the identification of mathematical ideas to highlight for each piece of student work and questions to ask to help make the mathematics salient to the students during the debrief. See Appendix A for an example of a blank ISA.

Inquiry Oriented Teacher Analysis (IOTA) – A coding scheme to code teacher utterances. In contrast to the question codes (see below), the IOTA coding scheme is intended to code most teacher utterances in the context of students discussing their mathematical thinking. These codes are grouped into the subcategories of revoicing, telling, questioning, and managing. The codes were used in research sub-question 3 to identify varying patterns in teacher discourse moves as the planned questions from the ISA were being implemented. See Appendix E for the complete coding scheme.

Intended Lesson - Part of my theoretical framework that differentiates between a literal lesson and an intended lesson. The intended lesson is the opportunities for students to learn as planned within a lesson plan. In my research study, the intended lesson is identified as the opportunities for students to reason mathematically based upon the nature of questions planned in the Instructional Sequence Analysis.

Literal Lesson – Part of my theoretical framework that differentiates between a literal lesson and an intended lesson. The literal lesson represents the specific steps identified in a lesson plan. In this research study, the literal lesson plan within the Instructional Sequence Analysis is identified as (1) present the identified piece of student work, (2) ask the planned questions, and (3) orchestrate the discourse to ensure that the planned ideas to highlight are being addressed. These steps are repeated for each identified piece of student work. This construct of a literal lesson plan was used to develop the Implementation Fidelity Analysis, which was used to answer research sub-question 1.

Low-press questions – Part of the question coding scheme developed for this research study that differentiated between high-press questions and low-press questions. These are questions planned by the teacher that do not prompt students to reason mathematically. They are primarily characterized as questions that prompt students to share their thinking without necessarily attending to the mathematics or short-response questions with a single correct response. Low-press questions were identified in the Instructional Sequence Analyses, but not in the implemented debriefs. See Chapter 7, Research Sub-Question 2, for a description of this coding scheme.

Mathematical Problem Solving Model (MPSM) – The model for the professional development program that the teachers participated in as part of the larger research study that this study has drawn from. The MPSM focuses on the use problem-solving tasks as a way to teach mathematics and the use of formative assessment to plan subsequent instruction. See Chapter 3, Professional Development Description, for a complete overview of the program.

Question codes – A coding scheme developed for this research study, used to identify the nature of questions, both planned in the ISA and asked by the teachers in the problem-solving debriefs. The codes identify the type of discourse the teachers are prompting their students to engage in during the problem-solving debrief (e.g. sharing their strategies, reasoning about errors in their strategies, making connections between strategies). The codes fall into two separate categories: high-press questions and low-press questions. Only the high-press question codes were used to analyze teacher questions and, consequently, not all teacher questions in the problem-solving debriefs were coded using the question codes. See Chapter 7, Research Sub-Question 2, for a description of this coding scheme and Appendix C for a description of all codes.

Segment – The portion of a problem-solving debrief centered on an identified piece of student work. A segment may be in reference to either the written portion of the ISA that is focused on a particular piece of student work or it may refer to the implemented portion of the problem-solving debrief in which that piece of student work is being discussed by the class. In the ISA, a segment refers to an identified piece of student work along with the ideas to highlight and the questions to make the mathematics salient that the teacher recorded in conjunction with that piece of student work. In the implemented problem-solving debrief, a segment begins when the teacher brings up the student work to be discussed (either by inviting the student to share or showing the work to the class without explicitly attributing the work to a particular student). The segment continues as long as the student work is under discussion. A segment ends either when the teacher tells the student they can sit down, or the teacher brings up a new piece of student work to be discussed.

Snack Shack – One of the three problem-solving tasks the teachers in this study were required to implement as part of their Mathematics Problem Solving Model professional development experience. The teachers implemented this task with their students, planning and orchestrating a whole-class discussion around the ideas in this task. Please refer to Chapter 4, Task Analysis, for a detailed description of this task.

Spinner Elimination – One of the three problem-solving tasks the teachers in this study were required to implement as part of their Mathematics Problem Solving Model professional development experience. The teachers implemented this task with their students, planning and orchestrating a whole-class discussion around the ideas in this task. Please refer to Chapter 4, Task Analysis, for a detailed description of this task.

Chapter 1. Introduction and Theoretical Framework

This research study is part of a larger NSF-funded research project[1] validating the Mathematics Problem Solving Model (MPSM), which is a professional development program for middle school mathematics teachers designed by Education Northwest, a research laboratory in Portland, Oregon. The MPSM promotes the use of cognitively demanding problem-solving tasks and whole-class discussions around problem-solving tasks (referred to as problem-solving debriefs) as a way for students to deepen their mathematical understanding. Before introducing my research questions, I will provide some background on the use of discourse around problem-solving tasks to promote mathematical learning. I also describe a protocol for discourse planning that was used by the teachers in the program and is central to my research study. This discussion is followed by an introduction to my primary research question and my three sub-questions. After introducing my theoretical framework, I will discuss the three sub-questions in greater detail followed by an explanation of how these three questions are intended to address my primary research question.

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[1] NSF DRL 0437612 The opinions expressed in this research project are those of the authors and do not necessarily represent the views of the National Science Foundation.

Background and Research Questions

I open this chapter with some background information about whole-class discussions around problem-solving tasks as this forms the basis for my research study. This section is concluded with an introduction to my research questions.

Discourse around Problem Solving Tasks as a Means to Promote Mathematical Learning. From Schoenfeld's research on the teaching of mathematical problem solving (1979) to Cognitively Guided Instruction (Carpenter, Fennema, Franke, Levi, & Empson, 2000) the potential role of problem solving in the mathematics classroom has developed from a skill to be learned on its own to a vehicle for mathematical learning to take place. One way for students to learn mathematics through problem-solving tasks is for students to share their problem-solving strategies during a whole-class discussion while the teacher scaffolds the discourse, creating opportunities for students to engage in thinking about mathematics and mathematical problem solving. Discourse around problem-solving tasks that promotes learning is characterized by students making connections between strategies, extending and generalizing solutions, making conjectures, verifying and modifying claims on the basis of mathematical evidence, and making sense of mathematical ideas (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997).

One of the many roles for teachers in orchestrating discourse is the use of questions to promote mathematical reasoning. Such questions include asking students to provide justification for the strategies they used (Hiebert et al., 1997; Kazemi & Stipek, 2001), questions that lead students to make sense of the mathematical ideas used to solve a task (Boaler & Humphreys, 2005; Sherin, 2002), questions that prompt students to make connections between strategies (Hiebert & Wearne, 1993; Kazemi & Stipek, 2001), and questions that encourage students to formulate and prove conjectures and generalizations around the mathematics in the task (Fraivillig, Murphy, & Fuson., 1999; Hiebert, & Wearne, 2003; Yackel & Hanna, 2003). Research has shown that students demonstrate positive learning gains in classrooms where the teacher regularly requires students to reason mathematically (Cobb, Wood, Yackel, Nichols, Wheatly, Trigatti, & Perlwitz, 1991; Silver & Stein 1996; Carpenter, Fennema , & Franke, 1997; Hiebert 2003).

Students sharing their problem-solving strategies in the context of whole-class discussions is one way that teachers can promote new mathematical thinking around a problem-solving task (Hiebert, 2003), creating opportunities for students to engage in rich mathematical discourse as described above. However, students sharing solution strategies as a way to generate worthwhile discourse comes with the caveat that students randomly volunteering to share how they solved a task may lead to limited opportunities for students to engage in mathematical thinking around a problem-solving task because the students, on their own, do not necessarily recognize the mathematical nature of the tasks they are engaging with (Leinhardt, 2001; Nathan & Knuth, 2003; Stein, Engle, Smith, & Hughes, 2008; Williams & Baxter, 1996). The teacher's role in orchestrating discourse around a mathematical problem-solving task is a critical one as it is the teacher's responsibility to create opportunities for students to reason mathematically about the problem-solving task (Chazan & Ball, 2001).

Planning for Discourse around Problem-Solving Tasks. Professional developers have proposed that teachers may create a platform upon which worthwhile mathematical discourse is more likely to emerge by deliberately planning for the whole-class discussion following the implementation of a problem-solving task (Stein, Engle, Smith, & Hughes, 2008; Stein, Smith, Henningsen, & Silver, 2009). This may be done by identifying which solution strategies will be discussed (selecting), determining the order in which those solution strategies will be presented so as to build upon students' understanding and move forward a mathematical agenda (sequencing), and planning appropriate questions to help students make connections amongst each other's' strategies and to the underlying mathematical ideas in the task. My research study analyzed the questions that teachers planned for whole-class discussions. Additional information about the process of planning for discourse around problem-solving tasks may be found in Chapter 3, Professional Development Description.

Once a sequencing of student work is identified, the teacher plans questions intended to move forward the mathematical discourse. Planning for mathematical discourse has the potential to yield positive results since "rather than having mathematical discussions consist of separate presentations of different ways to solve a particular problem, the goal is to have student presentations build on each other to develop powerful mathematical ideas" (Stein et al., 2008, p. 330). Planning questions to ask during the whole-class discussion is a way for the teacher to be deliberate about orchestrating the discourse, focus the discourse on the aspects of the students' problem-solving strategies that is relevant to the mathematical ideas the teacher hopes will emerge.

For the remainder of my dissertation, I will be referring to these whole-class discussions around problem-solving tasks as problem-solving debriefs. While this language is not widely used in mathematics education communities, it was the language that was used in the professional development program from which I have drawn my data. The term debrief is appropriate to describe this type of discourse in which students are coming together as a class to meaningfully discuss their thinking and reasoning around a problem-solving task because it refers to students reflecting upon their experiences with a problem-solving task in meaningful ways.

The Research Questions

While it has been proposed that preplanning questions will lead teachers to orchestrate more effective discussions than if they were to lead a whole-class discussion without planning ahead of time what questions they will ask (Stein et al., 2008), no research has been done to investigate the impact that preplanning questions can have on problem-solving debriefs. This research study addresses this gap in the literature by focusing on the relationship between the questions teachers preplan for discourse around mathematical problem-solving tasks and what actually takes place during the implemented problem-solving debrief. I did this by analyzing the enactment of teacher-written plans for whole-class discussions around a problem-solving task, examining the impact that preplanned questions had on the opportunities that teachers created for their students to engage in meaningful mathematical discourse. The research question for this study is:

How do teachers' written plans for orchestrating mathematical discourse around problem-solving tasks influence the opportunities teachers create for students to reason mathematically?

This question will be addressed with three sub-questions:

1. Do teachers enact their written plans for problem-solving debriefs in the classroom as they had planned prior to implementation?
2. Is there a correlation between the number of questions teachers plan that promote mathematical reasoning around problem-solving tasks and those that they actually ask during whole-class discussions?
3. How do teachers' improvisational moves during whole-class discussions influence the enactment of the questions that were planned by the teacher prior to implementation?

This dissertation analyzed how the teachers participating in the MPSM program implemented written plans that they created for mathematical problem-solving tasks, with a particular focus on the questions the teachers planned. Below, I will discuss each of the research sub-questions and explain how each of these questions contributes to my primary research question. But first, I will briefly describe some highlights of the MPSM professional development program that are relevant to my research to provide some background (for a complete description of the MPSM professional development program, please refer to Chapter 3, Professional Development Background).

Implementation of Problem-Solving Debriefs. As part of the MPSM professional development research experience, the participating middle-school mathematics teachers were provided with a set of problem-solving tasks that were considered to be cognitively demanding for students in middle school mathematics courses. The teachers were required to implement these tasks in their classrooms (see Chapter 4, Task Analysis, to see the tasks implemented by the teachers used in this study). After the students had time to work on the tasks in class, the teachers collected their students' written work and used their solutions to plan a problem-solving debrief. The teachers analyzed their students' work; selected pieces of student work to be shared with the whole class that would bring out the important mathematical ideas in the task; planned a specific order in which the student work would be presented; identified the important ideas to be highlighted for each piece of student work; and planned questions to be asked to help make the mathematics salient. The teachers documented these plans in a planning form called an Instructional Sequence Analysis (ISA). For an example of an ISA, see Appendix A. My research study examined the MPSM teachers' uses of their ISAs during the MPSM problem-solving debriefs, with a particular focus on their planned questions. In the next section, I discuss my theoretical framework. This is followed by a discussion of my three sub-questions, briefly highlighting the theoretical framework and data analysis methods used to answer each question. For a complete discussion of the methods used to address the research questions, see Chapter 5, Method.

Theoretical Framework

My primary research question is: How do teachers' written plans for orchestrating mathematical discourse around problem-solving tasks influence the opportunities teachers create for students to reason mathematically? This question focuses on what the teacher planned in the ISA and how that plan was implemented in the problem-solving debrief. To guide my research, I have adapted two frameworks from research on intended and enacted curriculum to fit the perspective of teachers' self-written lesson-plans. While research on intended and enacted curriculum frequently refers to curriculum as national, state, district, or school-level standards (Porter, 2004; Tarr, Reys, Reys, Chavez, Shih, & Osterlind 2008), I am not focusing on any of these definitions of curriculum enactment to guide my research because, for my research study, I examined how teachers enacted a single lesson (as opposed to, for example, an entire unit in a textbook). To support my research on teacher planning and implementation, I adapted two frameworks for intended and enacted curriculum that focused on the implementation of individual lessons. Also, I focused on theoretical frameworks that perceived the teacher as the primary decision-maker concerning what would take place in the classroom, both in the planning phase and the implementation phase. Below, I discuss these two frameworks and then describe my own 'hybrid' framework that blended these two frameworks together for the purpose of making sense of teachers' planning for and implementation of a problem-solving debrief.

The Temporal Phases of Curriculum Implementation. Stein, Remillard, and Smith (2007) suggested a framework in which intended curriculum refers to the teacher's personal plans for instruction. In their framework, the written curriculum (i.e. textbooks and teaching materials) influences the teacher's intended curriculum. As teachers use curricular materials to plan for what takes place in the classroom, the teachers must make important decisions about what to cover within a text, how to interpret curricular materials, and what aspects of the curriculum to emphasize. In turn, when teachers implement their intended curriculum in the classroom, how that curriculum plays out in the classroom is going to look different from what had been originally intended. What students experience with respect to the enacted curriculum is subsequently going to influence what the students learn. Finally, how the enacted curriculum plays out in the classroom and what students come to learn as a consequence of the enacted curriculum is going to, in turn, influence the plans the teacher makes in subsequent lessons. The boxes in figure 1 illustrate these temporal phases. This framework demonstrates the teacher as central to the implementation process by bridging the gap between the written curriculum and the enacted curriculum with the intended curriculum.

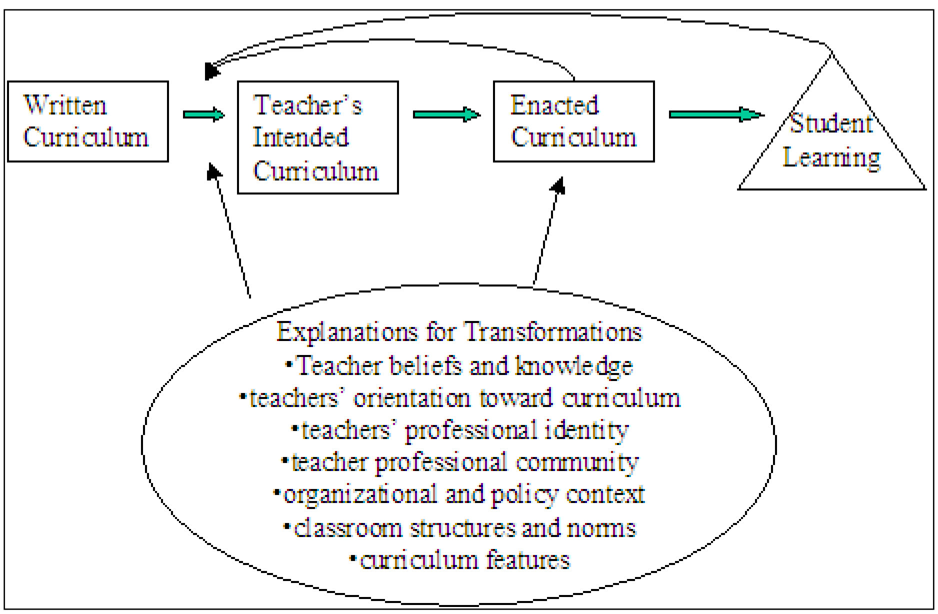


Figure 1. The temporal phases of curriculum implementation (Stein et al. 2007, 322)

Research has shown that there are several influencing factors that may contribute to the variations that exist between a written, intended, and enacted curriculum (Stein et al., 2007). What happens in these phases of implementation is influenced by teachers' beliefs, knowledge, and professional identity, among other contributing factors (see, for example, Stein, Baxter, & Leinhardt, 1990; Cohen, 1990; Spillane, 1999; 1997; Remillard & Bryans, 2004). This framework supports how I think about teachers' planning and implementation of a problem-solving debrief because it takes into consideration not only the influences that teachers have on the planning of a lesson, but also its implementation, recognizing that both the intended curriculum and enacted curriculum are influenced by teacher decisions. While this research study recognizes that teachers' beliefs, knowledge, and professional identities influence how teachers enact a curriculum in their classroom, this study does not explicitly study these factors.

Literal versus Intended Lesson. The other framework that was useful for developing my own theoretical framework focused on the lesson plan as central to their analysis of the enacted curriculum. In their research on teachers' fidelity to a reform-oriented curriculum, Brown, Pitvorec, Ditto, and Kelso (2009) interpreted a textbook lesson as having both a literal lesson and an intended lesson. The literal lesson plan represents the steps laid out by the textbook authors in a written lesson plan for the teacher to complete during the enactment of the lesson. A lesson step might be an instruction for students to work on a particular problem in a small group, or it may be a prompt for the teacher to pose a particular question for the class to discuss as a whole. The intended lesson plan represents the learning opportunities that the textbook author hopes the lesson will bring forth.

This perspective of a literal lesson versus an intended lesson has been used in various studies on fidelity of implementation, although the interpretation of what counted as a literal lesson or an intended lesson varied between studies. For example, Porter (2005) viewed academic content as both the topics to be covered and the cognitive demand linked to those topics. Tarr et al. (2008) measured both how much the teachers were using standards-based textbook materials and the extent to which the teachers were establishing a learning environment consistent with the tenets of the NCTM Principles and Standards for School Mathematics (2000). Brown et al. (2009) perceived authors of a textbook lesson as making decisions both about what aspects of the lesson content to cover, and also the opportunities to learn that content (opportunities to reason about mathematics and opportunities to communicate about mathematics). I used this idea of a literal lesson and an intended lesson to allow me to examine how teachers implemented their ISAs from contrasting perspectives. I will describe this more, following the description of the theoretical framework that I developed for my research study using these Stein et al.'s temporal phases for curriculum implementation (2007) and Brown et al.'s framework for literal versus planned lessons.

A Hybrid Framework. For the theoretical framework that guided this study, I blended together Stein et al.'s (2007) framework of the temporal phases of curriculum enactment with Brown et al.'s (2009) framework for literal versus intended lesson. I replaced the Teacher's Intended Curriculum in Stein et al.'s framework with the Teacher's Literal Lesson and Teacher's Intended Lesson (see figure 2 below). Another key difference between Stein et al.'s framework for the temporal phases of curriculum implementation and my own framework for lesson implementation is that their framework focused on teachers implementing a written curriculum by adapting it to fit their own plans for classroom instruction, while my research study is focused on teachers implementing plans that they wrote themselves. A written curriculum may refer to standards that the teachers are expected to adhere to or textbook lessons that they are attempting to implement. While such curriculum usually would be directly influencing the intended lesson, the lessons developed in my research study are not being directly influenced by such outside sources. As a result, the lines connecting the Written Curriculum to the Teacher's Intended Lesson and Teacher's Literal Lesson are dashed; indicating that the influences that the teachers' curricula that they are using in their own classrooms may have on these lessons is going to be indirect. I also changed the language from curriculum to lesson to indicate that I was specifically thinking about these temporal phases at the lesson level and, in particular, the enactment of teachers' self-written lesson plans. Note also, that, although it is not included in figure 2, I still consider Stein et al.'s explanations for transformations of the lesson as an important element of my framework.

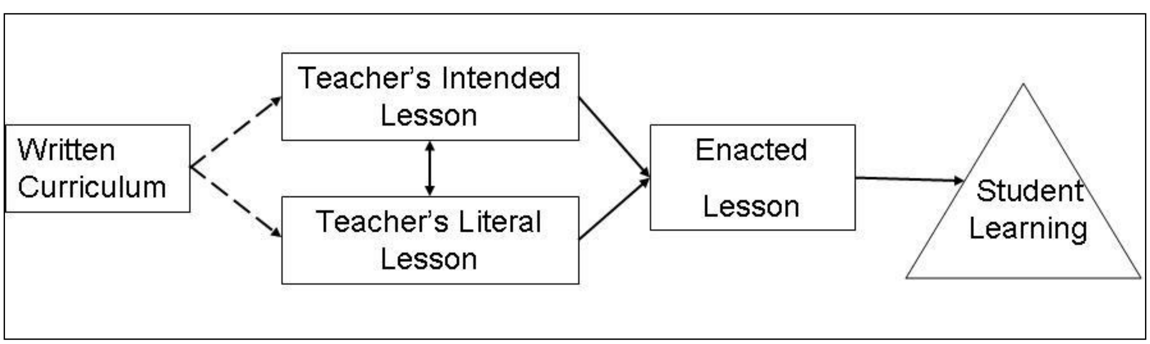


Figure 2. My framework for teacher-written lesson plans for discourse around problem-solving tasks

For my own research, I was interested in the fidelity with which the teachers were implementing their own ISAs. To do this, for research sub-question 1, I examined the extent to which the teachers followed the basic components of their lessons (for example, sharing the pieces of student work that they said they would and asking the questions they had planned to ask). This part of my analysis explicated the teachers' fidelity to the literal lesson. For research sub-question 2, I analyzed the teachers' planned questions with respect to the opportunities the teachers had intended to create for students to engage in mathematical reasoning and compared those to the questions teachers asked during the implemented problem-solving debrief. This looked at fidelity to the teachers' intended lesson. Finally, in research sub-question 3, I looked at how the teachers implemented their planned questions to understand how the teachers' utterances transformed the way that the students interacted with the planned questions. In Chapter 5, Method, I describe in greater detail how the analysis I conducted for these sub-questions each addressed a different element of my theoretical framework.

Discussion of the Research Questions

Research Sub-Question 1: Do teachers enact their written plans for problem-solving debriefs in the classroom as they had planned prior to implementation? This sub-question is intended to address the teachers' fidelity to the literal lesson. Although the teachers were required to follow up the implementation of an MPSM problem-solving task by creating an ISA and orchestrating a problem-solving debrief using the ISA, there is no guarantee that the enactment of the ISA will completely resemble what was laid out in the document. For example, a teacher may not share every piece of student work that was identified in the ISA. Reasons why this may occur include insufficient time, absence of a presenting student, or because the discourse took a different direction than originally intended and the teacher chose not to implement the remainder of the ISA as planned. Also, a teacher may have a student share their work as planned in the ISA, but not address the questions as planned.

Using Brown et al.'s (2009) concept of fidelity to the literal lesson, research sub-question 1 is intended to address the extent to which the teachers followed the basic steps of their ISAs, or literal lesson. The analysis for research sub-question 1 addressed whether or not the teachers actually presented the student work, asked the planned questions, and addressed the identified mathematical ideas to highlight as described in the ISA. This analysis was useful for identifying whether or not the teachers showed evidence of attempting to use the ISAs as planned. This was important for this research study because, if the teachers did not follow the ISA (or parts of the ISA), then it may be assumed that the plans in the ISA did not influence what took place during the problem-solving debriefs. The analysis for research sub-question 1 was used to inform the analyses for research sub-questions 2 and 3 by showing which parts of their ISAs the teachers did not use (for example, a piece of student work that was never shared). A detailed description of the analysis and results for research sub-question 1 may be found in Chapter 6.

Research Sub-Question 2: Is there a correlation between the number of questions teachers plan that promote mathematical reasoning around the problem-solving task and those that they actually ask during the whole-class discussion? This sub-question is intended to address the teachers' enactment of their intended lesson, or the opportunities the teachers included in their ISA for students to reason mathematically. Research shows that, in order for students to engage in discourse in which they are reasoning mathematically, teachers need to promote that level of engagement by asking pressing questions, encouraging students to provide mathematical justification for their problem-solving strategies, address errors and misconceptions in their work, and make connections between problem-solving strategies and the mathematics within the task (Boaler & Humphreys, 2005; Cobb et. al., 1997; Fraivillig and Murphy, 1999; Hiebert & Wearne, 1993; Kazemi & Stipek, 2001). In this research study, I used Kazemi and Stipek's conceptualization of high-press and low-press classrooms to characterize questions that promote mathematical reasoning (2001). In my study, I made the assumption that if teachers ask questions intended to elicit mathematical reasoning from the students in their classrooms, then the teachers are creating opportunities for their students to reason mathematically. In this part of my analysis, I focused on the teachers' intended lesson, that is, the opportunities for students to reason mathematically that were present in the planned questions in the ISA.

Based upon this assumption that, if teachers want their students to be reasoning mathematically, teachers should be asking high-press questions, I posited that the more high-press questions a teacher plans for in the ISA, the more high-press questions that teacher will ask during the problem-solving debrief, therefore creating more opportunities for students to reason mathematically. In research sub-question 2, I test this hypothesis, comparing the number of high-press questions planned in the ISA to the number of high-press questions asked in the implemented problem-solving debrief. To do this, I created a coding scheme to code the questions teachers planned in their ISAs, differentiating between high-press questions and low-press questions. I also use this coding scheme to identify the high-press questions the teachers asked in their implemented problem-solving debriefs. Descriptions of the analysis and results for research sub-question 2, including a discussion of the framework used to distinguish between high-press and low-press questions and a description of the coding scheme may be found in chapter 7.

Research Sub-Question 3: How do teachers' improvisational moves during whole-class discussions influence the enactment of the questions that were planned by the teacher prior to implementation? It is widely recognized that what takes place in the classroom is going differ from what was initially planned, whether it is based upon the implementation of published curricular materials, or teachers' self-written lesson plans (Lee & Takahashi, 2011; Remillard, 1999; Schneider, Krajcick, & Blumenfeld, 2005; Superfine, 2009). The discrepancies that inevitably occur are influenced by "teachers' beliefs and knowledge, teachers' orientation towards a curriculum, teachers' professional identity, teacher professional community, organizational and policy context, classroom structures and norms, [and] curriculum features" (Stein, Remillard, & Smith, 2007, p. 322). As a result of these and other influences, it is logical to assume that the teacher will make changes and alterations to what was planned in the ISA. My final research sub-question is intended to complement research questions 1 and 2 by providing a deeper look into the enacted lesson, or what is actually taking place as the teachers are implementing their ISAs.

Keeping in line with the thrust of research sub-question 2, this final data analysis is focused specifically on how the teachers enacted the questions they planned in their ISAs. I used grounded theory (Strauss & Corbin, 2007) to examine how the planned questions were implemented by the teachers. I developed and tested some theories to help explain how these variations in implementation might affect the ways in which the students experienced the planned questions and how this might impact (positively or negatively) the opportunities the students had to reason mathematically. To help me with this analysis, I used a second coding scheme to code teacher utterances. I used the Inquiry-Oriented Teaching Analysis (IOTA) codes developed by Rasmussen, Kwon, and Marrongelle (2009). This coding scheme was useful for this stage of the analysis because it includes codes for all types of teacher utterances (e.g. questions, telling, revoicing, managing). Also, it was developed to be used in the context of students attempting to share their thinking and reasoning about mathematical tasks and ideas, which is the type of mathematical activity we expected students to be engaged in during a problem-solving debrief. Chapter 8 provides a detailed description of the analysis and results for research sub-question 3. The complete IOTA coding scheme may be found in Appendix E.

I would like to make a note here that the intent of this research study is not to analyze the relationship between what the teachers planned and what their students did. Rather, the intent is to look at what the teachers did during the problem-solving debrief in response to what they planned in the ISA. That is, I examined the questions the teachers asked and the moves the teacher made as they implemented their ISAs, not how students responded to the implementation of the ISAs. My reason for this focus in my analysis is both theoretical and practical. First, I chose to focus on teacher moves because, in reference to the temporal phases of curriculum enactment, there is a better defined relationship between the moves teachers make in response to a self-written lesson plan than what students do or learn in response to the enactment of a lesson plan. While students' participation in mathematical discourse may enable mathematical development, it does not determine it (Cobb & Boufi, 1997), which is why teachers' questions focused on mathematical reasoning are referred to as 'opportunities' for students to reason mathematically. My choice to focus on teacher moves was also practical because, in the context of whole-class discussions, it is very difficult to make claims about how students are reasoning and what students are learning because it is not possible for all students to be engaged in a whole-class discussion all of the time. Below, I describe in greater detail the three sub-questions for this study and a discussion of how these are intended to address the primary research question.

A Road Map to the Remainder of the Dissertation

Before discussing the analysis and results for my research questions, I will be laying the groundwork needed to fully understand this study. In Chapter 2, Literature Review, I provide a review of research on discourse and planning, situating my own research within this literature. In Chapter 3, Professional Development Description, I give a complete overview of the MPSM professional development program including the critical features of the program, assumptions about student learning, some of the key professional development activities, a detailed description of the MPSM model itself, and a discussion of adult learning framed within the experiences of the MPSM professional development experience. In Chapter 4, Task Analysis, I share the three MPSM problem-solving tasks that the teachers implemented as part of their professional development experience and from which the data for this research study is drawn. This chapter includes a discussion of curriculum placement as well as some of the key strategies that students used to solve the tasks.

In Chapter 5, Method, I provide information about the teachers participating in the MPSM professional development and in my study. I also describe data collection methods and give an overview of the data analysis methods used to address the three research sub-questions. Chapters 6, 7, and 8 are my data analysis chapters in which I address research sub-questions 1-3, respectively. These chapters include more detailed descriptions of the data analysis methods used, including inter-rater reliability analyses (where applicable), and a discussion of the results, including how the results served to address my primary research question. Chapter 9 concludes this dissertation with a summary of my findings, possible implications for both researchers and practitioners, and a discussion of the limitations of this study as well as suggestions for future research.

Chapter 2. Literature Review

Introduction

My study focuses on the enactment of teacher-written lesson plans for discourse around mathematical problem-solving tasks to examine the interplay between the teachers' use of planned questions and their improvisational moves as they implemented those planned questions. My analysis examines two areas of research on teaching. These are: research on teachers' orchestration of classroom discourse and research on teacher planning. As a result, my literature review is split into two sections. First, I will discuss the role of discourse as an opportunity for students to reason mathematically around problem-solving tasks. Second, I will discuss research on teacher planning.

In the section on discourse around mathematical problem solving, I will open with a discussion of why the mathematics education community is increasingly focused on discourse around problem-solving tasks as a way for students to learn mathematics, followed by a discussion of some of the issues that have emerged when teachers attempt to engage their students in discussions of their problem-solving strategies. I will then describe some frameworks that help to differentiate between students simply sharing their problem-solving strategies and students engaging in meaningful mathematical discourse.

Finally, I will share some research studies that highlight the benefits of discursive practices that have been recognized as opportunities for students to reason mathematically.

In the section on planning, I open with a discussion of the challenges that come with reflecting-in-action and compare this to the benefits of reflective planning. I then go on to discuss the need for reflecting-in-action in the context of, first, the enactment of published curricular materials, and, second, in the context of teacher's own lesson plans. Finally, I describe two research studies that provide some insight into the potential benefits of planning for discourse. I will conclude my literature review with a discussion of how my research questions are intended to move forward both the literature on discourse around mathematical problem solving and the literature on planning and, more specifically, the enactment of teachers' lesson plans.

Discourse around Mathematical Problem Solving

Discourse around students sharing solutions to problem-solving tasks. The NCTM Principles and Standards for School Mathematics (2000) advocates for students to be able to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" as well as emphasizing that students need to "analyze and evaluate the mathematical thinking and strategies of others" (p. 268). Also, the Common Core Standards for Mathematics Practices includes "make sense of problems and persevere in solving them" and "construct viable arguments and critique the reasoning of others" (2010). These standards encourage teachers to move away from 'traditional' teacher-centered instruction and towards reform instruction in which problem solving and the students' voices play a more central role in the classroom.

The implementation of mathematical problem-solving tasks to generate discourse around important mathematical ideas is a method of instruction that has been supported by many research studies and professional development programs as a potentially effective way to generate worthwhile learning opportunities as students both reason about the mathematics in the task and learn to communicate their mathematical thinking (Boaler, 1998; Hiebert & Wearne, 1993; Larsen & Bartlo, 2009; Silver & Stein, 1996). This instructional approach involves the careful selection of a problem-solving task designed to bring out certain mathematical ideas. The students work on the task in groups, the teacher then orchestrates a sharing and discussion time in which students report out on their solution methods with the whole class. This whole-class discussion is intended to create a venue in which students are encouraged to consider multiple solution methods and where mathematical thinking and argumentation is valued.

Dilemmas of engaging students in discourse around problem-solving tasks. Research has shown that teachers often struggle with successfully orchestrating discussions around problem-solving tasks in a way that ensures their students are reasoning mathematically, even when the tasks were originally designed to promote mathematical thinking and reasoning (Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996). Unfortunately, students who are regularly communicating in their classrooms with their peers are not necessarily engaging in mathematical reasoning (Nathan & Knuth, 2003). Classroom environments in which students share their solutions to mathematical tasks have occasionally been referred to as “show and tell” (Ball, Lubienski, & Mewborn, 2001) because the teacher is focused on ensuring that students’ thinking is valued and listened to, but fails to focus on the mathematical nature of the discourse. In this context, teachers’ questions were focused on encouraging students to explain their thinking, but they failed to hold their students accountable to the mathematical nature of their argumentation (Cobb, Wood, & Yackel, 1993; Hiebert & Wearne, 1993). Research shows that teachers sometimes deliberately avoid scaffolding students’ discourse in order to keep the discourse student-centered (Nathan & Knuth, 2000, Heaton, 2000) to the detriment of the mathematical quality of the discourse. Such teachers were often observed teaching with the expectation that important mathematical ideas would emerge and develop as a result of student-to-student discussions, but this often turned out to not be the case (Smith 1996; Williams & Baxter 1996; Chazan & Ball 1999; Lobato, Clark, & Ellis, 2005; Baxter & William 2010).

When the goal for classroom discourse is simply that the students do more of the talking and that the teacher is not positioned as the mathematical authority in the classroom, the quality of the discourse may be weakened with respect to the mathematical nature of the discourse. Nathan & Knuth (2003) worked with a middle school mathematics teacher as she undertook to support her students to engage in more dialogue between each other without first filtering their ideas through her. The teacher did this by encouraging multiple students to share their thinking, emphasizing that there is more than one way to solve a math problem, and encouraging students to listen carefully to others. Closer analysis of the classroom discourse, though, revealed that there was a lack of clear mathematical argumentation. That is, without the teacher participating as a mathematical authority in the classroom, the students were not always able to construct or verify meaningful mathematical ideas.

It can be difficult to distinguish between communication as a goal for instruction and communication as a means to understanding mathematics (Lampert & Cobb, 2003) because students who attempt to communicate their reasoning are well-situated to develop their mathematical understanding (Hiebert & Wearne, 1997). Williams and Baxter (1996) documented a case study in which a middle-school teacher implementing a reform-oriented curriculum (QUASAR) successfully built up social norms so that her students were using communication as a way to develop each other's mathematical understanding. However, her students sometimes treated communication as a goal in itself, rather than a means for building mathematical understanding. For example, when the teacher asked a group of students how they found a particular solution, a student responded that they had worked as a group. The student assumed that because the teacher put so much emphasis on communication, working together was the most important part of how they arrived at a solution. Also, when students shared their solutions with the class, students rarely asked questions and, when they did, they were typically questions in which the answer was already known, but they asked it anyway because they knew they were supposed to ask questions.

Taking discourse to the next level. The above-mentioned research studies demonstrated how simply shifting the dialogue away from the teacher and toward the students is an insufficient method for ensuring that the students are engaging in meaningful mathematical discourse. In a later study by Baxter and Williams (2010), two teachers who showed evidence of successfully implementing a reform-oriented curriculum, including high achievement gains on the part of their students, were documented as using both social scaffolding *and* analytic scaffolding to achieve the desired learning goals of discourse-oriented instruction. The authors defined analytic scaffolding as teachers providing students with additional explanations and alternative solutions strategies as well as asking students to make generalizations, explore alternative solutions, and analyze incorrect solutions. In this section of my literature review, I discuss research studies that distinguish between discourse in which sharing is central, from discourse that promotes mathematical reasoning and sense-making. I refer to this latter type of discourse as high-level in my research study and consider it to be worthwhile mathematical discourse.

Cobb, Boufi, McClain, and Whitenack (1997) documented a potential shift in classroom discussions away from students sharing their solutions and towards students discussing their mathematical activity as an object in and of itself. They referred to this type of discourse as collective reflection. One example they gave of this shift was in a first-grade classroom in which the students were trying to find all possible combinations of numbers that add up to five (e.g. 0+5, 5+0, 1+4, etc.). Initially, the students suggested random combinations. When the teacher asked the class how they *knew* that they had found all possible combinations, the discourse shifted to the students reasoning about the solutions already presented and attempting argumentations that would allow them to know for certain that they had found all possible solutions. The authors argued that this shift in the discourse generated opportunities for the students to reorganize their prior activity and supported students' mathematical development.

Brendefur and Frykholm (2000) defined four levels of mathematical communication that are helpful for seeing beyond categorizing discourse as either teacher-focused or student-focused. These levels are uni-directional, contributive, reflective, and instructive communication. Uni-directional communication is when the teacher dominates the discourse, typically with lecturing, asking closed questions (questions in which there is an expected response) and rarely providing opportunities for students to share their strategies or thinking. Contributive communication involves communication between the teacher and students or between students and is focused upon the development and sharing of strategies. This level of communication tends to be informal with a higher level of mathematical activity, but minimal focus on mathematical reasoning. Sharing at this level tends to be centered on what students did to complete the mathematical tasks rather than why and is similar to the discourse described in the previous section in which student communication is high but opportunities for reasoning and argumentation are limited.

Reflective communication tends to emerge from contributive communication, but the focus of the discourse shifts such that what the class was doing in action becomes an object of discussion in itself. This shift occurs, for example, when students begin to justify or challenge conjectures posed by their peers. Reflective communication is meant to be congruent to collective reflection in which mathematical activity becomes the object of discussion as students reflect upon their work (Brendefur & Frykholm, 2000; Cobb et al., 1997). Finally, instructive communication pertains to the teacher using classroom discourse to shape subsequent instruction, which may be viewed as a type of formative assessment. These four levels of communication are meant to be viewed as a hierarchy, but they are also viewed as embedded within one another. For example, a teacher that engages her class in reflective communication is going to also, on occasion, engage in uni-directional communication when, for example, it is necessary to summarize some important mathematical ideas. Also, reflective communication is not likely to occur without contributive communication taking place first.

In Brendefur and Frykholm's research study (2000), they described the changes that occurred in a pre-service teacher's methods of instruction with particular attention to how she grew and developed as a teacher by deliberately creating more opportunities for her students to engage, not only in contributive communication, but also in reflective communication. In their case study, a key factor in her methods of instruction that effectively allowed this shift to occur was the types of questions she was asking. Her students were more deeply engaged in mathematical discussions when she asked open-ended questions that were focused on the mathematics.

Kazemi and Stipek (2001) provide further clarification of the differences between contributive communication and reflective communication in their conceptualization of low-press teacher-student interactions versus high-press ones. In their analysis of four upper-elementary classrooms characterized as having positive social norms, they found that, in two of the classrooms, students were engaged in richer mathematical discussions than in the other two classrooms. In order to explain the differences between these two sets of classes, the authors identified four sociomathematical norms that differentiated the high-press classes from the low-press classes:

> (a) an explanation consists of a mathematical argument, not simply a procedural description or summary;
>
> (b) mathematical thinking involves understanding relations among multiple strategies;
>
> (c) errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies; and
>
> (d) collaborative work involves individual accountability and reaching consensus through mathematical argumentation. (Kazemi & Stipek, 2001, 64).

Using transcripts from these four classes, this study demonstrated how providing opportunities for students to share their problem-solving strategies did not guarantee student engagement in worthwhile mathematical discourse because, when students were asked to share their work without the requirement of clear mathematical explanations, they were simply verbalizing what they already knew, without opportunity to develop or refine their thinking (Lampert & Cobb, 2003). These research studies are demonstrative of the differences that can occur between high-level discourse and low-level discourse. In the next section, I will describe and discuss some research studies that provide some additional details of the characteristics of high-level discourse and how they create opportunities for students to learn.

Opportunities to learn through high-level discourse. Current trends in mathematics education highlight the importance of students' high-level engagement in mathematical discourse because it creates opportunities for students to deepen their mathematical understanding. In particular, mathematical activities such as reasoning and justification; making connections to other students' strategies or to the underlying mathematics of a problem-solving task; and addressing errors are all ways that students can build mathematical understanding as they engage in discourse around problem-solving tasks. In this section, I highlight some research studies that point to the benefits of discourse around mathematical problem-solving tasks as a way for students to learn.

In order for students to be engaged in meaningful mathematical discourse, it is necessary for teachers to press their students for clear and accurate explanations of their problem-solving strategies, accompanied by reasoning and justification for their strategies. When students make sense of the mathematics they are doing, providing reasoning and justification for their mathematical activity, they are developing mathematical proficiency in addition to gaining an understanding of what it means to reason and argue mathematically (Hiebert et al., 1997). Franke et al. (2009) found that the ways in which teachers followed up on students' initial explanations of their thinking impacted the opportunities that the students had to connect their thinking to the mathematics being learned as well as to the thinking of others in the class. In a case study by Ball and Bass (2003), the questions that the teacher in a 3rd grade classroom asked prompted students to clarify how they knew that their solutions to a problem-solving task were correct and how they knew they had found all possible solutions.

These questions created opportunities for the students to make sense of the mathematics and to reason mathematically. Without these promptings by the teacher, those opportunities would not have emerged (Franke et al., 2009).

Discourse around problem-solving tasks also creates opportunities for students to learn as they make connections to the underlying mathematics as well as to each others' strategies. Hiebert and Wearne (1993) provided evidence of the benefit of creating opportunities to compare and connect strategies in an analysis of six second-grade classrooms' instructional practices as they covered multi-digit addition and subtraction. Two of the classes were deliberately designed for students to solve fewer tasks overall, but to spend more time sharing multiple strategies and analyzing the connections between the solutions shown. In the comparison, they found that, in the classes in which discourse centered on comparing strategies, the students showed higher gains from the pre- to post-tests on all types of questions related to addition and subtract of multi-digit numbers, but particularly story problems and place value problems.

Discourse around problem-solving tasks can create a venue for students to address their errors in ways that deepen their understanding of the mathematics they are doing. Teachers may be tempted to ignore student errors for the sake of not causing embarrassment for the student with the error, but it can be beneficial for teachers to provide opportunities for students to discuss errors in students' strategies. Instead, errors should be treated as opportunities to improve understanding, both for the student that made the error as well as for other students who engage in making sense of the error (Hiebert et al., 1997; Silver & Stein, 1996). The low-press teachers in Kazemi and

Stipek's study (2001) were observed handling student mistakes by correcting the mistake for the students and then moving on, failing to provide an opportunity for students to reason through the error on their own. In another research study, teachers created opportunities for students to change an answer given, but did not provide opportunity for the students to reason about why their mistakes were incorrect (Fraivillig, Murphy, & Fuson, 1999). These approaches to addressing errors are problematic as they fail to provide learning opportunities for students. In contrast, high-press teachers who create opportunities for students to reason about their errors and give them time to identify their own mistakes, create opportunities for the students to deepen their understanding of the mathematics (Fraivillig, Murphy, & Fuson, 1999; Kazemi & Stipek, 2001).

Effective classroom discourse is characterized by students making connections between strategies, extending and generalizing solutions, making conjectures, verifying and modifying claims on the basis of mathematical evidence, and making sense of mathematical ideas. Questions a teacher might ask to promote mathematical reasoning include requests for students to provide justification for the strategies they used (Hiebert et al., 1997; Kazemi & Stipek, 2001), questions that lead students to make sense of the mathematical ideas used to solve the task (Boaler & Humphreys, 2005; Sherin, 2002), questions that prompt students to make connections between strategies (Hiebert & Wearne, 1993; Kazemi & Stipek, 2001), and questions that encourage students to formulate and prove conjectures and generalizations around the mathematics in the task (Fraivillig, Murphy, & Fuson., 1999; Hiebert, & Wearne, 2003; Yackel & Hanna, 2003). Research has shown that classrooms that support student discourse around these types of activities demonstrate positive learning gains (Cobb, Wood, Yackel, Nichols, Wheatly, Trigatti, & Perlwitz, 1991; Silver & Stein 1996; Carpenter, Fennema , & Franke, 1997; Hiebert 2003).

Students should have the opportunity to engage in this type of discourse, both with the teacher and their peers (NCTM, 1991). Students sharing their problem-solving strategies in the context of whole-class discussions is a useful way for students to build new mathematical thinking (Hiebert, 2003), creating opportunity for students to engage in rich mathematical discourse as described above. However, students sharing solution strategies as a way to generate worthwhile discourse comes with the caveat that students randomly volunteering to share how they solved a task may lead to limited opportunities for students to engage in mathematical thinking around a problem-solving task (Leinhardt, 2001; Nathan & Knuth, 2003; Stein, Engle, Smith, & Hughes, 2008; Williams & Baxter, 1996). Consequently, the teacher's role in orchestrating discourse around a mathematical problem-solving task is a critical one as it is the teacher's responsibility to create opportunities for students to reason mathematically about the problem-solving task (Chazan & Ball, 2001). In response to the demands of orchestrating a productive mathematical discussion, it has been recommended that teachers reflect upon students' solutions and deliberately plan for discourse around mathematical problem-solving tasks (Stein, Engle, Smith & Hughes, 2008). In the second half of this literature review, I will discuss the literature on teachers' planning and the potential impact that the enactment of lesson plans can have on classroom discourse.

Teacher Planning and Lesson Implementation

My research study is intended to assess the impact of reflective planning on classroom discourse that is focused on students sharing their problem-solving strategies. Such planning gives teachers the opportunity to reflect upon how student work connects to the learning needs of the students, and then identify questions to ask that will support students to engage in discourse around the selected ideas. In this section of my literature review, I will discuss the tension that exists between the practice of teachers reflecting and planning prior to implementation of a whole-class discussion and the practice of making on-the-spot decisions during the orchestration of a whole-class discussion. I argue that, while reflecting beforehand does appear to have some benefits for creating worthwhile mathematical discourse, the need to "reflect-in-action" is an ever-present reality and, particularly in the case of mathematical discourse, must be taken into account when analyzing the discursive practices of the teacher. Following a discussion of the existing research on planning and reflecting-in-action, I will then explain how my research is attempting to address and better understand this tension.

Reflecting-in-action. The role the teacher plays in orchestrating mathematical discourse is perceived as a decision-making process that takes place as teachers respond to what their students say in the classroom (Heaton, 2000; Lampert, 2001; Leinhardt & Steele, 2005). Effective instruction requires "reflecting-in-action" that leads to improvisational moves. That is, teachers must make instantaneous decisions as they reflect upon their students' thinking and then attempt to translate that information into opportunities to advance students' mathematical thinking and connect their thinking to conventional mathematics (Chazan & Ball, 1999; Heaton, 2000; Krussel, Edwards & Springer, 2004; Schifter, 2001; Sherin, 2002). Research studies have shown that, by being responsive to students' thinking, it is possible to move discourse in directions that both attend to students' thinking and also progress in mathematically productive ways (Heaton, 2000; Leinhardt & Steele, 2005; Schifter, 2001).

Research shows that, as teachers learn to orchestrate discussions around students' thinking, they struggle to attend to students' thinking and how teachers respond to students' unique ways of thinking can vary. For example, Sherin (2002) conducted a study on two experienced teachers who agreed to implement a reform-based unit on linear functions that was distinctly different from their previous style of teaching. The purpose of the study was to investigate the ways that the teachers built new pedagogical content knowledge as they encountered novel thinking from their students as they worked on problem-solving tasks. Sherin distinguished between three ways that teachers responded to novel aspects of a lesson. (1) Transform is when teachers adapt the lesson to fit their preexisting knowledge about the mathematical topic affording no opportunities to build new understanding. (2) Adapt is when the teacher accepts a novel way of thinking contributed by a student, but does nothing to modify the lesson as a result. (3) Negotiate is when the teacher makes changes to the lesson as it is being implemented in response to the students' novel ideas (negotiate). This last approach would be considered successful "reflecting-in-action" because it represents the teacher attending to students' thinking during classroom discourse. Out of the 17 lessons implemented by these two teachers (34 total), it was found that adapt was the most common response, having occurred in 82% of the lessons while transform was evident in 53% of the lessons. In contrast, negotiate occurred only 29% of the time. While it is significant that this "reflection-in-action" was taking place, there were still many instances in which these teachers were missing opportunities to respond in the moment to students' novel ways of thinking about the tasks.

Reflective planning. Research has shown that for worthwhile mathematical discourse to take place, teachers need to be responsive to students' ideas and ways of thinking (Chazan and Ball, 1999; Fraivillig, Murphy, and Fuson, 1999; Heaton, 2000; Schifter, 2001; Sherin, 2002). The study described in the previous section showed that this is not easy for teachers to do and that teachers often fail to respond to students' thinking in the moment of implementation. One response to this challenge of attending to students' thinking is to provide teachers with the opportunity to slow down the reflection process. Reflection upon student work and students' engagement in the classroom has been shown to be beneficial for developing teaching practices. Kazemi and Franke (2004) found that teachers who collectively analyzed student work were able to shift their thinking from simply identifying the different strategies used by students towards developing possible learning trajectories. Davies and Walker (2005) studied teachers who observed videotapes of themselves teaching with particular attention to student talk. This helped the teachers to be more effective in how they responded to their students' talk.

Reflection upon curricular lesson plans has also shown to benefit students opportunities to learn. In a study of four elementary school teachers implementing problem-based curricula (Rigelman, 2009), the teachers collaborated with the researcher, analyzing the curriculum for ways in which the lesson supported mathematical discourse. By deliberately planning strategies to increase opportunities for high-level discourse, the teachers were able to elicit higher levels of discourse than what the curriculum had originally intended. These studies demonstrate the value of focused reflection for the purpose of changing teachers' instructional practices. However, these examples of reflective practice are within the context of supported professional development and are not examples of daily teaching practices.

Enactment of curricular materials. When it comes to the enactment of reform curriculum, it is necessary for teachers to be responsive to students' thinking. This requires that teachers must make on-the-fly decisions about how to respond to student thinking. When teachers are using new curricular materials in which the concepts, skills and tasks presented in the curriculum are unfamiliar to the teacher, the need to make on-the-fly decisions increases because it is harder for them to anticipate student responses. Research shows that it is typical for teachers to digress from the original lesson plan when they are faced with challenges of implementation. In an analysis of teachers' implementation of a reform curriculum in a science classroom, even when teachers were faithfully enacting the curriculum, they tended to be less faithful to the intended curriculum when the topics were more challenging for the students, when they were required to respond to students' ideas, when they had to support their students in performing investigations, and when they led their students in small-group discussions (Schneider, Krajcick, & Blumenfeld, 2005).

When challenges arise, teachers often fall back into old routines. Superfine (2009) found that teachers' uses of the teacher guides in the Connected Math Project (CMP) curriculum was heavily influenced by their prior teaching practices and experiences. For example, when the teachers in his study were faced with the challenge of supporting students who were struggling with the content, they were more likely to rely on their prior conceptualizations of teaching rather than utilizing the recommendations provided in the teacher's guide. In a case study of two elementary teacher's use of curriculum (Remillard, 1999), the teachers made improvisational moves in their classrooms for differing reasons. One teacher was observed constructing new questions for her students when she saw that they needed to refine their thinking on a problem-solving task while the other teacher was observed making improvisational moves such as inserting direct instruction into the lesson when she observed her students struggling with a task. These studies suggest that a teacher's ability to successfully reflect-in-action is going to impact how they implement a lesson. In the case of the teacher who resorted to direct instruction when she observed her students struggling, she was unable to scaffold the discourse to continue to promote students' mathematical reasoning. In contrast, the teacher who constructed new questions was able to make on-the-fly decisions about how to continue to move students' thinking forward. These studies are similar to Henningsen and Stein's study (1997) that demonstrates how the cognitive demand of a task can either be sustained or degenerate depending upon the types of support provided by the teachers.

Enactment of teacher-written lesson plans. Implementation of curricular materials bears some semblance to the notion of implementing one's own written lesson-plan. However, these experiences are not identical. For one thing, there is an attitude that when implementing curricular materials, the teacher is making decisions about what he or she wishes to use, using some parts of the curriculum and rejecting others (Remillard, 1999). In contrast, the implementation of a self-written lesson plan is more personal because it is the documentation of the teacher's ideas about what she intends to do with her students. Research on teachers' lesson plans is often limited to analyzing what the teacher plans and how (e.g. Leinhardt, 1989). Little research has been done, though, on the enactment of teacher-written lesson plans. While it may be easy to assume that a teacher-written lesson plan will be more closely followed than a textbook lesson, making on-the-spot modifications to a teacher-written lesson is still essential as the teacher must respond to student contributions which cannot be anticipated beforehand. In a study of a group of teachers who shared and observed each other's lessons and then reflected in groups upon the enactment of their lesson plans as part of a professional development, it was observed that when a lesson plan had to be altered during its implementation, the teachers did not treat it as failure to successfully implement the lesson plan, but simply regarded it as a design issue that required immediate revision (Lee & Takahashi, 2011). Leaving room for reflection-in-action to take place may be important because over-planning (e.g. writing a script) runs the risk of the teacher not providing enough space for students to openly engage in discourse in the classroom, as was found to be the case in a study in which teachers were either asked on the spot to teach a lesson or asked to prepare a lesson plan in advance (Zahorik, 1970). Teachers with extensive lesson plans have been documented providing too much feedback, not providing enough pause time for students to reflect, and "talking over" their students (Walshaw & Anthony, 2008).

Planning for Discourse

While it may appear that writing a lesson plan is a way to eliminate the need for reflection-in-action, I argue that it remains necessary for teachers to do so as they attempt to create dialogue in their classroom. Preplanned questions do not eliminate the need for teachers to follow-up on student responses, encouraging them to provide clear mathematical responses (Kazemi & Stipek, 2001), nor does it help the teachers to be prepared for unexpected responses and reactions from their students (Chazan & Ball, 1999). An extreme example of the potential interplay between planning and improvisation is seen in a class taught by Magdalene Lampert (Leinhardt & Steele, 2005). While creation of a lesson plan is typically perceived as a series of linear steps, Lampert's discourse-oriented lesson on functions was developed in a more dynamic way, in which she perceived the relationship between the concepts of a function as a web in which she guided her lessons through the web, allowing the direction of her lessons to be guided by the thinking of her students, rather than a predetermined series of tasks. Lampert planned her lessons so that they were never intended to be followed rigidly. Instead, by continuously reflecting upon the thinking of her students throughout implementation, she carefully navigated the lessons to help address all the important learning goals. Her method of discourse-oriented instruction is an example of instructive communication as described earlier (Brendefur & Frykholm, 2000).

Connecting research on planning back to the earlier discussion of discourse creating opportunities for students to reason mathematically, careful attention to the opportunities for students to reason mathematically when planning a lesson may lead to more opportunities for students to engage in high-level discourse. For example, in a study in which teachers collaborated with the researcher to modify lesson plans, with a particular eye to creating opportunities for students to engage in high-level discourse, an analysis of the enacted lessons revealed that students did engage in more mathematical reasoning as a result of the additional planning (Rigelman, 2009). While this study supports the notion that it is possible for teachers to create more opportunities for students to engage in worthwhile mathematical discourse through the act of deliberate planning, more research is needed to understand how teachers' plans for discourse around mathematical problem-solving tasks can influence the opportunities that teachers create for students to reason mathematically.

Connecting the Literature to My Research

It is almost an oxymoron to propose that teachers plan discourse given that teachers have little control over how the students will respond to planned questions. Research on teacher implementation of lesson plans (both curricular and self-written) suggests that making on-they-fly decisions is not a question of whether or not the plans are modified when teachers implement them, but a question of how the plans are modified upon implementation. Teachers participating in the MPSM professional development program were not expected to create a rigorous lesson plan. Rather, the Instructional Sequence Analysis was a venue for the teachers to make tentative plans about what questions they would like to ask their students as a way to generate discourse around their problem-solving strategies. The ISA still leaves room for teachers to respond spontaneously to student talk once the questions have been posed. By limiting the number of questions planned, teachers are opening the discourse for new student thinking without creating an agenda that is so rigid that students are not able to engage in discourse because they are simply answering a series of questions. My research study will be examining this balance between reflective planning and reflection-in-action by focusing on how teachers implement their planned questions and how the improvisational moves of the teacher as they implement those questions impacts the opportunities they create for their students to reason mathematically.

The lesson planning process affords teachers time to reflect on their students' work and then make decisions on how they wish to organize the discourse, choosing questions to ask that will help move their mathematical agenda forward and, hopefully, support their students to reason mathematically. I intend to investigate the impact that a teacher-written lesson plan can have on mathematical discourse around problem-solving tasks given that the outcomes of the enactment of the lesson plan cannot be anticipated simply based upon the contents of the lesson plan. Research sub-question 1 (Do teachers enact their written plans for problem-solving debriefs in the classroom as they had planned prior to implementation?) addresses that fact that enactment of lesson plans is frequently different from what was originally planned and is an analysis of how closely the teachers followed the basic contents of their ISAs.

Since opportunities for students to deepen their mathematical understanding emerge when students engage in reasoning and justification, making connections between students' strategies and to the mathematics, and reasoning through errors, my research study includes a particular focus on the questions teachers planned that were intended to prompt students to engage in these mathematical practices. In research sub-question 2 (Is there a correlation between the number of questions teachers plan that promote mathematical reasoning around problem-solving tasks and those that they actually ask during whole-class discussions?), I analyze the relationship between the number of questions teachers plan in their ISA intended to prompt students to reason mathematically to the frequency with which the teachers actually ask such questions during implementation of the ISA. My hypothesis is that the more of these questions a teacher plans, the more they will subsequently ask during the problem-solving debrief.

While the ISA is meant to create some structure as the teacher guides the class in discourse around mathematical problem solving, teachers must still engage in reflection-in-action as they are not able to fully anticipate student responses and may still need to make some in-the-moment decisions about how to guide the discourse. My research is intended to examine the relationship between what the teachers plan in the ISA (in particular, the planned questions) and what actually takes place during the lesson. In particular, research sub-question 3 (How do teachers' improvisational moves during whole-class discussions influence the enactment of the questions that were planned by the teacher prior to implementation?) is intended to examine the improvisational moves of the teachers as they address the questions that they planned in the ISA. My analysis looks at how the teachers' improvisational moves, prompted by in-the-moment decision-making, influenced (either positively or negatively) the opportunities for students to engage in reasoning around mathematical problem-solving tasks.

These three sub-questions are intended to support the overarching research question for this study (How do teachers' written plans for orchestrating mathematical discourse around problem-solving tasks influence the opportunities teachers create for students to reason mathematically?). This research study is intended to bring together research on planning, curriculum enactment, and discourse analysis by analyzing teachers' enactment of self-written lesson plans with a particular eye on the opportunities that the plans afford for students to reason mathematically, recognizing that teachers' improvisational moves will also, inevitably, impact the outcomes of the discourse. Since mathematics educators are advocating for teachers to make plans concerning how they will orchestrate discourse around mathematical problem-solving tasks (Stein, 2008; Stein, Smith, & Silver, 2009), research is needed to allow us to better understand how such planning practices can impact the opportunities that teachers create for their students to reason mathematically.

In the next chapter, I discuss the MPSM professional development program including a discussion of assumptions about student learning, the major learning activities experienced by the teachers, a discussion of the MPSM models and the elements within the model, and a discussion of assumptions about adult learning that motivated the activities in the professional development experience. Throughout the chapter I include references to literature and research that promote the implementation of problem-solving tasks as supported by the MPSM model.

Chapter 3. Professional Development Description

In this chapter, I describe the Mathematics Problem Solving Model professional development research program. This chapter is intended to give the reader a complete picture of the professional development program that was experienced by the teachers. I begin with a description of the assumptions about student learning that motivated the structure of the program and the critical issues of the professional development experience, including how much time the teachers spent doing the professional development and compensation they received for participating in the program. Before going into a discussion of the model itself, I include a section describing some of the major aspects of the professional development experience that the teachers received to address the components of the framework. I then give a complete overview of the Mathematics Problem Solving Model (MPSM), which is the framework for implementing problem-solving tasks that was used for this program. I conclude this chapter with a discussion of the assumptions about adult learning that motivated the professional development activities the teachers experienced.

While this chapter paints a complete picture of the professional development experienced by the teachers that participated in my study, my research study is focused on only a small part of this professional development experience. That is, this study analyzes the teachers' implementation of the plans they wrote for orchestrating discourse around problem-solving tasks. There are many other features of this professional development experience that are not addressed in my study. This includes, but is not limited to, identification of learning goals, task selection, and formative feedback. I include these elements in my description of the MPSM professional development program so that the reader may have a complete understanding of the professional development that the teachers participating in my study experienced.

Overview and Program Goals

The Mathematics Problem Solving Model (MPSM) is a professional development program designed by Education Northwest, a non-profit organization in Portland, Oregon that conducts educational research and provides professional development and educational materials to K-12 teachers in the northwest. The MPSM is a research-based framework for teaching mathematics through problem solving with the use of formative assessment to enhance instruction. Formative assessment is when teachers use their understanding of student thinking to make decisions about subsequent instruction. The goal of the professional development program is to increase teachers' understanding of mathematics and their ability to implement cognitively demanding problem-solving tasks in their mathematics classrooms, including the use of students' performance on mathematics problem-solving tasks as a resource for formative assessment to both modify instruction and to provide students with opportunities to improve their proficiency in mathematical thinking and mathematical problem solving. Through an

NSF-funded research project[2], the MPSM professional development program has been implemented at Education Northwest with four cohorts of teachers beginning in the 2006-2007 school year and concluding in the 2009-2010 school year. This has been part of an NSF-funded research project, validating the professional development program.

Critical Issues and Structural Description

Participating teachers were involved with the Mathematics Problem Solving Model professional development program for a complete school year with professional development activities taking place both during the summer and integrated throughout the following school year. Teachers spent eight full days engaged in professional development during the summer (typically, five days in July, followed by three follow-up days in August). During the school year following their summer professional development experiences, the teachers met together for three additional professional development days, meeting for one Saturday in fall, winter, and spring. The total number of contact hours came to about 78 hours. During the following school year, the teachers also participated in an online learning management system (MOODLE) and received five one-on-one coaching sessions with a professional developer to support implementation of the model in their everyday practices. The professional development seminars took place in downtown Portland at the Education Northwest facilities. As part of the research project's data collection requirements, the teachers implemented five problem-solving tasks in their classrooms.

[2] NSF DRL 0437612 The opinions expressed in this research project are those of the authors and do not necessarily represent the views of the National Science Foundation.

The MPSM professional development program was implemented with four different cohorts of teachers over the course of four years. The cohorts ranged in size from approximately 10 participants to 18. The participants of the MPSM professional development program were middle school mathematics teachers who joined the program on a voluntary basis. Teachers were recruited through letters sent to the schools and an advertisement in a local practitioners' journal. Teachers also had the option of receiving six credit hours for their participation in the professional development program. The credit was professional development credit only, which means that it was useful for moving teachers up the pay scale, but could not be used towards completion of a degree. Teachers were required to pay a small fee to receive the credit and were required to submit an additional paper discussing some aspect of the professional development that impacted their teaching practice. For a description of the teachers participating in the professional development research program, including years of experience and school demographics, see Chapter 5, Method.

The professional development was advertised and implemented as part of a large research project in which the researchers were validating the professional development program. The teachers signed consent forms agreeing to participate in the program and the teachers also collected signed consent forms from the parents of their students. Data collection occurred continually throughout the program and drove many of the teachers' classroom tasks. During the summer, this included completion of tests and surveys by the teachers, collection of reflective writing pieces, and continual video recording of the professional development sessions. During the school year, the teachers were required to submit their students' standardized test results. When the teachers implemented the problem-solving tasks in their classrooms, they were required to submit their lesson plans and all student work. During the implementation of the task, the teachers wore audio-recorders. As incentive and compensation for their participation in the project, the teachers received a stipend of $1,500. They received half of their stipend after completing the summer professional development and the other half was given to them after all of their data was delivered at the end of the school year. There were also control groups in years one, two, and three who did not participate in the professional development, but data was collected from their classrooms. These teachers received a smaller stipend for their participation and were invited to participate as treatment teachers the following year.

Assumptions about Student Learning

The Mathematics Problem Solving Model and the professional development program were designed to support the following two assumptions about how students learn mathematics:

1) Two important influences on how children learn mathematics are the tasks and problems they engage in and the interactions they have about them.
2) Teachers' ability to understand students' mathematical development is enhanced by their ability to notice and describe what students say and do.

These assumptions were articulated to the teachers participating in the professional development at the beginning of the program and were regularly referred to throughout the professional development in order to make explicit to the teachers how the professional development they were receiving supported these assumptions. A description of these assumptions is given below.

Two important influences on how children learn mathematics are the tasks and problems they engage in and the interactions they have about them. This assumption about student learning is reflective of the increasingly common perspective that effective instructional practice revolves around the use of problem-solving tasks and student-centered discussions about these tasks (See, for example, Boaler & Humphreys, 2005; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier & Human, 1997; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; Wood, Nelson, & Scott, 2001). In an analysis of the types of tasks that teachers implemented in their classroom, it was found that higher learning gains were achieved when teachers implemented tasks that were cognitively demanding. Moreover, it was found that the greatest learning gains were evident when multiple solutions were identified, multiple representations were used, and student explanations were expected. Learning gains were considerably less when a single solution was accepted, single representations were used and communication was not required of students (Stein & Lane, 1996). Students that engage in cognitively demanding tasks have opportunities to learn problem solving, reasoning skills, and higher order thinking. In contrast, students who engage in tasks that are of low cognitive demand only have opportunities to learn facts and procedural skills (Wood & Turner-Vorbeck, 2001).

As mentioned earlier, the learning opportunities within a task are dependent upon how the teacher leads students to engage in the task. This includes how the teacher introduces the tasks to the students as well as what types of discussions the students engage in following implementation of the task. While reporting out solutions is a common way to follow up the implementation of a problem-solving task, it is also necessary to create opportunities for students to clarify their solutions, provide their reasoning and justification for the approaches they take and connect it to relevant mathematical content (Hiebert et al., 1997; Kazemi & Stipek, 2001). Also, students should have opportunities to look for patterns and make generalizations around problem-solving tasks, attempting to defend and justify their conjectures, with reasoning playing a central role in the process (Yackel & Hanna, 2003).

Teachers' ability to understand students' mathematical development is enhanced by their ability to notice and describe what students say and do. Analysis of student work is a valuable tool for creating effective mathematical instruction. An awareness of students' understanding and thinking around mathematical concepts and tasks makes it possible to appropriately choose tasks that are of high cognitive demand, but still within the students' reach (Stein, Grover, et al., 1996). Knowledge of student work also supports appropriate teacher questions useful for building upon student thinking to develop and make connections between mathematical ideas (Grouws, 2003). Professional development programs such as Cognitively Guided Instruction (CGI) and Integrating Mathematics Assessment (IMA) are examples of programs in which teachers are encouraged to attend closely to what their students are saying and doing as they engage in mathematics tasks and then use this information to shape subsequent instruction. Research from both of these programs showed that students of CGI and IMA teachers demonstrated greater learning gains compared to control groups of teachers who were not trained to attend to students' thinking, particularly on assessment of conceptual understanding and problem solving (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Saxe et al., 2001). The CGI research also showed that teachers who attended closely to their students' thinking and reasoning were better prepared to project students' learning trajectories and build subsequent instruction based upon that information than teachers who never asked their students for explanations beyond how they obtained a solution (Franke, Carpenter, et al. 2001). Similarly, Yackel, Cobb, and Wood (1999) documented a teacher who was able to meaningfully formulate subsequent instructional tasks building towards his mathematical learning goals based upon information he obtained about his students' thinking as they engaged in discourse around mathematical problem-solving tasks.

Teaching mathematics through problem solving is an overwhelming task for a teacher, particularly when they are doing it for the first time. The challenges that come with selecting and implementing cognitively demanding mathematical tasks that support mathematical learning can be eased when teachers know how their students think about the mathematics in the task and use that information to move forward with instruction. Teacher's knowledge about students' thinking can be improved by posing problems specifically designed to access students' thinking (Lesh, Hoover, Hole, Kelly, & Post, 2000) and asking good questions during instruction that access students' thinking (Wiliam, 2007). Teachers may also gain a deeper understanding of their students' understanding by listening attentively to their students' thinking (Schifter, 1998) and examining student representations to make a connection between their idiosyncratic ways of thinking and more conventional mathematics (Mewborn, 2003). This knowledge can then be used to further support students' learning and to effectively build subsequent instruction. The Mathematics Problem Solving Model is a framework that demonstrates the steps in the teacher's process of implementing a mathematics problem-solving task, with a focus on the use of student work as a formative assessment tool for planning subsequent instruction (see figure 3). Below, I discuss the components of the MPSM model which include mathematical learning objectives, choosing and implementing a problem-solving task, analyzing student work, and using that analysis to either select and sequence student work, plan a whole-class discussion, or provide feedback and use that to shape a follow-up instructional activity.

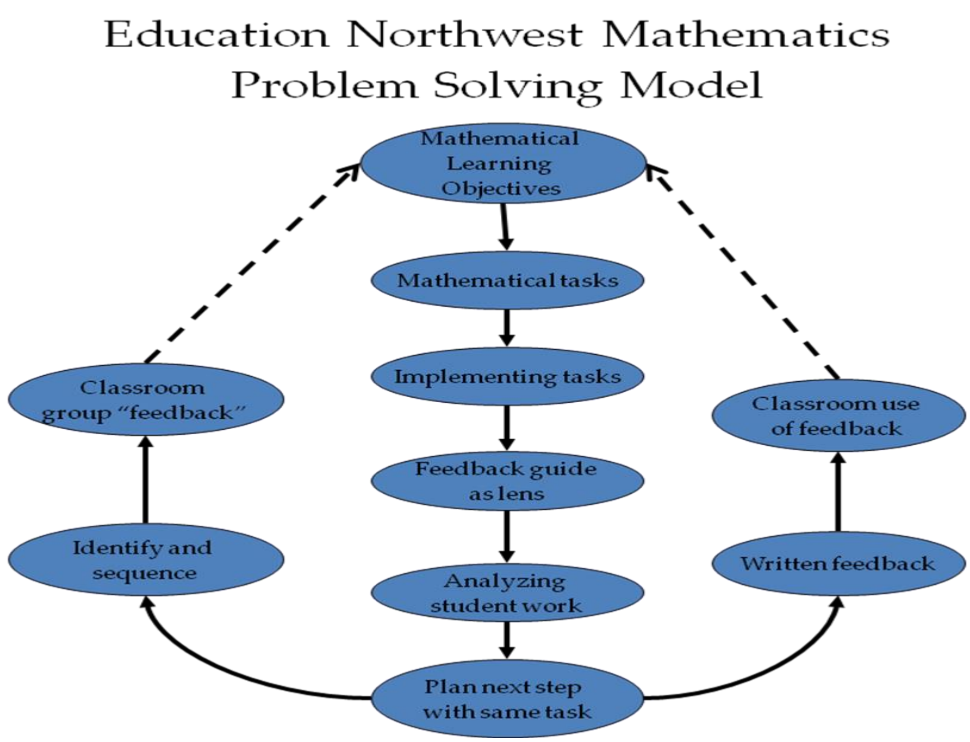


Figure 3. The Education Northwest Mathematics Problem Solving Model (Gummer, Gates, & Strowbridge, 2009)

The Education Northwest Mathematics Problem Solving Model

The Education Northwest Mathematics Problem Solving Model (MPSM) is a research-based model designed to conceptualize critical elements of the formative assessment process of effectively teaching mathematics through problem solving. It is focused on the selection and implementation of mathematics problem-solving tasks, analysis of student work through the lens of a formative feedback guide, orchestration of classroom discourse around the problem-solving task, and the subsequent use of written feedback in the classroom. Figure 3 shows the elements of the model sequenced together in a cyclic fashion. Each part of the model represents a step that the teacher takes in the process of planning, implementing, and following up on a problem-solving task in their mathematics classroom. The professional development program described in this paper was designed to support participating teachers in gaining an understanding of the model and to develop their capacity to incorporate it into their beliefs about teaching mathematics and their everyday mathematical practices.

Within the model, the teacher begins the implementation of a problem-solving task by first identifying a mathematical learning objective that she[3] would like her students to accomplish. With this learning objective in mind, the teacher then identifies a suitable problem-solving task that has the potential to be cognitively demanding for her students (a definition of cognitive demand will be given below). As the teacher implements the problem-solving task with her students, she must be careful to maintain

[3] For sake of simplicity, throughout this chapter I use female pronouns to refer to the teacher and male pronouns to refer to a single student. There were male teachers participating in the program as well as female.

the cognitive demand of the task. This may be done, for example, by asking the students questions that will move their thinking forward without over-simplifying the task for them. After the task has been [mostly] completed by the students, the teacher then collects the students' written work and analyzes the students' work with the use of a feedback guide (which will be described in detail below). The analyzed student work may then be used to move instruction forward in two distinct ways. The teacher may choose to make one or both of these instructional moves. The teacher may provide written feedback to her students and plan for further instruction using the feedback with the express purpose of helping students to improve their problem-solving skills. Alternatively, the teacher may select a sampling of student work useful for demonstrating the important features in the task. These features may include the key mathematical ideas in the task. It also may be a focus on a problem-solving process such as clearly communicating a problem-solving strategy or different methods of representing the problem situation. The teacher uses the selected set of student work to plan for a whole-class discussion that is focused on the important features of the problem-solving task by first carefully sequencing the students' work in such a way to bring to the forefront those key features in the task and preplanning questions to ask her students that will help focus the discussion on those ideas.

Once the task is fully completed, the teacher then uses the new information gained about her students' understanding to plan and implement a new task that will continue to move forward her students' thinking about the mathematics within the task. The more a teacher completes a full cycle of the MPSM, the more knowledge she will have about her students, allowing her to make increasingly informed decisions about which task is appropriate for moving forward the mathematical thinking of the class as a whole. Below is a more thorough description of the components of the Mathematics Problem Solving Model, including a discussion of what current educational research has to say about these elements of the model, and how these elements of the MPSM were incorporated into the professional development. Before I go into a more detailed discussion of the MPSM, I will first discuss some of the professional development activities experienced by the teachers that were facilitative in deepening their understanding of the elements of the model.

Professional Development Activities that Attended to the Model

The MPSM professional development program was designed around this model, with teachers being made explicitly aware of the model through the course of the professional development. There is not enough space here to provide a complete description of everything that happened in the professional development, including all of the refinements that were made over the four years of its implementation. However, in order to give a sense of what did happen during the professional development, I briefly describe below four professional development activities that were central to the program. These four major activities took place consistently across the four years and all served to address key aspects of the model and the professional development experience as envisioned by the professional developers. I will frequently refer to these four activities throughout my discussion of the Mathematics Problem Solving Model, providing more detail about each of these activities as they are revisited in the remainder of this paper.

Implementation of MPSM tasks. For the purpose of facilitating instruction around the MPSM, a collection of problem-solving tasks considered appropriate for implementation in a middle school mathematics classroom was pre-assembled by the professional developers. While the number of MPSM tasks used from year-to-year varied, three tasks were used all four years. These were Snack Shack, Design a Dartboard, and Spinner Elimination (see chapter 4 for a complete description of these tasks). These problem-solving tasks played a central role in the professional development experience. In preparation for these tasks being used by the teachers, considerable time during the professional development was devoted to the teachers solving these tasks themselves, playing the role of the student. The members of the professional development team took turns playing the role of the teacher, implementing the tasks as intended in the MPSM. That is, the professional developer would enact certain elements of the MPSM model such as deliberately selecting and sequencing solutions and then leading a discussion based upon those solutions. Following this whole-group discussion, the professional developer would often explain the choices made in leading the whole-group discussion. This allowed the teachers to gain an understanding of the mathematics involved in the task and to observe how the tasks are intended to be implemented, according to the MPSM. The teachers also analyzed pre-existing student work from these tasks as a way to experience the formative assessment practices of the MPSM (i.e. using student work to plan subsequent instruction).

Following the summer professional development sessions, the teachers took these MPSM tasks, and other tasks developed during the sessions, into their own classrooms and implemented them according to the MPSM. The teachers began the initial planning process in the summer by identifying their mathematical learning objectives and considering where they intended to place the tasks in their curricula.

Identifying the cognitive demand of a task. The effectiveness of a mathematics problem-solving task will vary depending upon the level of thinking required to successfully complete the task. To support the teachers in suitable tasks, we used the concept of cognitive demand as drawn from Stein, Smith et al.'s (2009) Implementing standards-based mathematics: A casebook for professional development in which they identify four levels of cognitive demand (memorization, procedures without connections, procedures with connections, and doing mathematics). It is the intent of the professional development program that the teachers are able to identify and select tasks for implementation in their classrooms that are of high cognitive demand (that is, procedures with connections and doing mathematics). The tasks developed by the MPSM research team are considered to be at the level of doing mathematics and these were provided to the teachers during the summer professional development as a resource for the teachers to become familiar with the characteristics of a task that is doing mathematics. In order to further build the teachers' understanding of cognitively demanding tasks, the teachers were introduced to Stein et al.'s (2009) definitions of the four levels of cognitive demand and given a collection of middle school mathematics tasks that were of varying levels of cognitive demand which they were asked to categorize according to their potential level of cognitive demand. The purpose of this task was to familiarize the teachers with the four levels of cognitive demand and to increase their facility with recognizing the cognitive demand of a problem-solving task they may be considering using in their classrooms. Later in the professional development, the teachers were led through another activity in which they adapted tasks from their own textbooks to raise the cognitive demand of those tasks.

Examples of discourse in the classroom. Since many teachers are only familiar with traditionally taught classrooms in which discourse around problem solving is not a common occurrence, the professional development program showed videos to the teachers for the purpose of increasing their understanding of what it looks like for a teacher to successfully engage his or her students in discourse around a problem-solving task. The goal of these activities was to engage teachers in reflective discussions around samples of classroom instruction that support the MPSM, creating opportunities for teachers to "not only to see alternative conceptions of teaching but also to build their own understandings as they interact with these cases and their colleagues" (Merseth, 1996, p. 734).

Adapted from Boaler and Humphreys' book, *Connecting mathematics ideas, Middle school video cases to support teaching and learning* (2005), the teachers were shown a video of Cathy Humphreys implementing The Border Problem, a problem-solving task designed to engage students in thinking about algebraic representations. The teachers were given a handout from the Boaler and Humphreys book of nine types of questions teachers might ask with a description of each question type. They were then asked to review the transcript of the video they just watched to identify the different question types. The teachers were led in further discussion as they compared these question types to *Examples of Effective Questions,* adapted from NCTM, and discussed which question types they felt would be most useful in moving students' thinking forward.

Another classroom example of effective classroom discourse was drawn from the Mathematics in the City Research Project (Fosnot et al., 2006). This video was shown to give the teachers an opportunity to follow another teacher's thinking in planning, implementing, and facilitating classroom discourse around a problem-solving task. The teachers watched videos of Joel, a middle school teacher, implementing a problem-solving task with his sixth grade class, including Joel posing the problem, the students working on the problem as Joel gives feedback, students discussing the task in small groups, Joel engaging the students in a whole-class discussion of the task, and, finally, an interview of Joel sharing the goals and decisions he made as he was implementing the problem. The teachers interacted with the videos by solving the task themselves and discussing possible solutions, discussing the questions asked by Joel as the students worked on the task, reviewing student solutions and predicting in what sequence Joel might choose to discuss those solutions with the class, and reflecting upon how Joel's classroom moves connect to the Mathematics Problem Solving Model.

Reading and Reflection on Practitioner Articles. The participating teachers were assigned, as homework, a variety of practitioner articles related to problem solving, classroom discourse, and formative assessment. Examples of these articles include a review of research on formative feedback in the classroom (Wiliam, 2007) and an article about classroom discourse called *Never Say Anything a Kid Could Say*, from Mathematics Teaching in the Middle School (Reinhart, 2000). The articles read by the teachers changed some over the four years as new and more appropriate articles became available. The purpose of reviewing these articles with the teachers was to raise their awareness of what research says about classroom practices that align with the MPSM and to provide an opportunity for the teachers to reflect upon their own teaching practices. In addition to the articles that the teachers read during the summer, the teachers also read through the book *Implementing standards-based mathematics: A casebook for professional development* (Stein, et al., 2009). During the school year, the teachers discussed the book via MOODLE, an online learning management system, sharing their general impressions of the case studies as well as discussing how they felt the reading connected to their own teaching practices.

The Elements of the MPSM Model

We now shift back to a discussion of the MPSM Model itself and the role it played in the professional development. As a reminder and for easy reference, I have included the model again in figure 4. Below is a description if each of the elements of the model including a discussion of the research literature that supports the views expressed in the model and descriptions of how the professional development experiences described above supported these elements. Also there are some references to responses that were received from the teachers participating in the professional development. These responses are based upon informal feedback forms that the teachers filled out at the end of each professional development day.

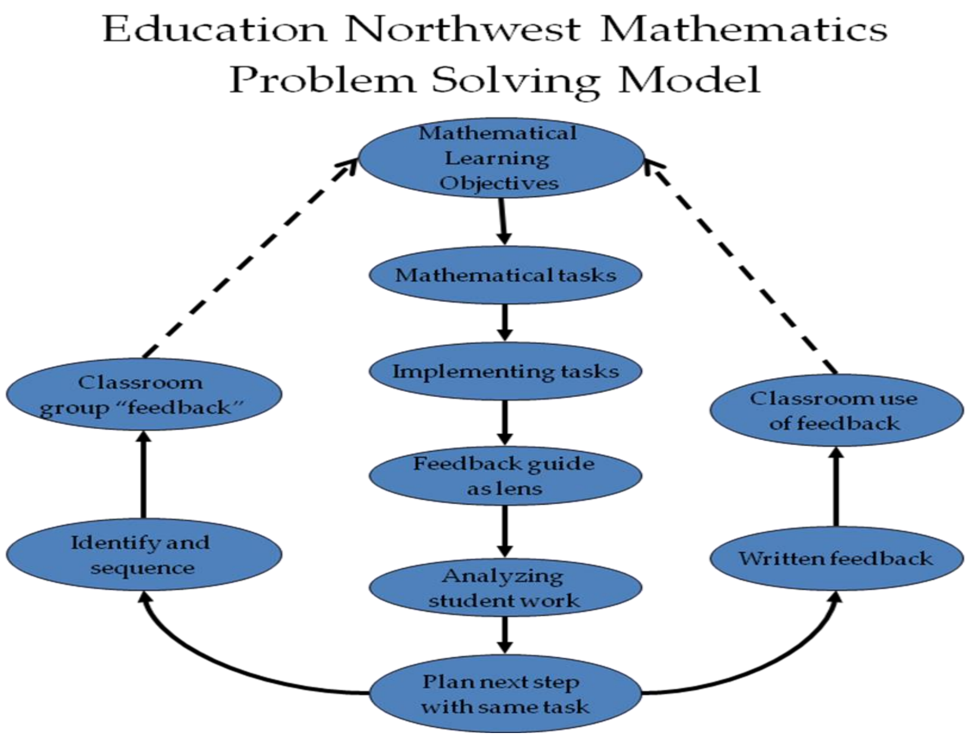


Figure 4. The Education Northwest MPSM, Revisited (Gummer, Gates, & Strowbridge, 2009)

Mathematical Learning Objectives. Before a problem-solving task is selected to be implemented in a mathematics classroom, a mathematical learning objective must first be chosen to ensure that the task is useful for moving forward the mathematical agenda in the classroom. Teachers' instructional moves that lead towards building conceptual understanding should begin with, and be subsequently guided by, the identification of an instructional learning goal (Sherin, 2002; Stein, Smith, Henningsen, & Silver, 2009). The MPSM program emphasizes that, in order to successfully teach mathematics through the use of problem-solving tasks, the implementation of a problem-solving task should take place concurrently with the mathematical concepts and procedures in the classroom's curriculum (NCTM 2000; Lesh & Zawojewski, 2007). A clearly identified learning objective is central to instructional planning, but also to formative assessment:

> Learning goals are the starting, ending, and recycling points in the selection and implementation of quality assessment tools, in interpretation and analysis of student work, and in the use of results to provide informative feedback and take action that will further students' progress. (Herman et al., 2006, p. 5)

The 'Examples of Discourse in the Classroom' activity served to demonstrate to teachers the potential value of articulating a learning goal when implementing a problem-solving task. In the follow-up interview, Joel was able to articulate exactly what his learning goal was for the problem-solving task he had selected. From the interview and classroom video, it was clear that Joel was successful in selecting a task and then orchestrating discourse in his classroom to accomplish that learning goal. Participating teachers expressed that the effectiveness with which Joel utilized his learning goal was eye-opening for them to understand how important having a learning goal can be. As teachers planned to implement their five problem-solving tasks in their classroom, teachers were first asked to examine the mathematical content and skills required for successful implementation of the tasks and then looked at their own curriculum to see where these tasks would best fit to support a mathematical learning goal appropriate for their class. By emphasizing the importance of a clearly articulated mathematical learning goal, the MPSM program discourages the concept of "problem solving Friday" in which a problem-solving task is essentially "dropped" into the curriculum without regard for how the task fits into the content being covered in the curriculum.

Research unfortunately shows that when teachers plan for instruction, they typically focus on selecting an activity without identification of a learning goal (Brown, 1988; Young, Reiser, & Dick, 1998). This tended to be true of the teachers participating in the MPSM professional development program as well, as observed by one of the professional developers when visiting the teachers' classrooms. While the professional development emphasized the importance of having a mathematical learning goal when implementing a problem-solving task, the design of the research study did not allow the participating teachers to identifying a learning goal *before* choosing a task because the teachers were required to implement the three MPSM tasks, regardless of whether it fit with a mathematical learning goal from their curriculum. That is, as part of data collection for the research study, the teachers were given three tasks to implement in their classroom during the following school year and they had to identify a learning goal based upon the task that was given to them, which is backwards from how the model was intended. The data collected from the teachers also showed that when choosing their own tasks, they were more likely to choose a task and then select a mathematical learning objective for the task.

Mathematical Tasks. Once a mathematical learning objective has been identified, a problem-solving task that supports that learning objective is then selected. One major assumption about how students learn mathematics is that students build meaning and understanding through engagement with mathematical tasks (Hiebert et al., 1997). Research has shown that students demonstrate greater learning gains and more flexible reasoning skills in classrooms in which students spend more time on tasks that engage students in sense-making and building conceptual understanding as opposed to classrooms in which students are demonstrated a skill and then practice that skill by solving several related problems (Boaler, 1998; Hiebert & Wearne, 1993; Silver & Stein, 1996). In order for this to happen, the appropriate task must be thoughtfully selected as "the mathematical tasks with which students become engaged determine not only what substance they learn but also how they come to think about, develop, use, and make sense of mathematics" (Stein, Grover, & Henningsen, 1996, p. 459). Therefore, in order to successfully select a mathematics problem-solving task, the teacher must take into consideration, not only the mathematical content of the task, but also the level of thinking in which the task will afford opportunity for the students to be engaged. In order to clarify what is meant by students building understanding through mathematical tasks, this section will start with a discussion of how problem solving is defined in the MPSM and then further discuss the concept of students engaging with mathematical tasks within the framework of cognitively demanding tasks (Stein et al., 2009).

Defining problem solving as utilized in the MPSM. While there are many different definitions for problem solving (Schoenfeld, 1992), the MPSM focuses in particular on the use of open-ended problem-solving tasks as a means for building new mathematical knowledge. An open-ended problem-solving task is one that has multiple solution methods that may lead to multiple possible solutions. When solving and discussing open-ended problem-solving tasks, the focus is not on finding the answer, but on the processes that students use to arrive at their solutions. Genuine problem solving means that students are responsible for choosing their method or procedure that they will use to solve the task and then reflecting on the problem-solving experience, affording opportunities to deepen understanding of the problem-solving process and the mathematical concepts underlying the problem-solving process (Hiebert et al., 1997). Such problem-solving tasks should be non-routine and challenging, but not inaccessible (Becker & Shimada, 1997).

The Mathematics Problem Solving Model strongly encourages the placement of problem-solving tasks into the curriculum in such a way that they create opportunities for students to build new mathematical knowledge and deepen conceptual understanding (Boaler, 1998; Hiebert & Wearne, 1993; Silver & Stein, 1996). Stanic and Kilpatrick (1989) identified the potential for problem solving to be used as a tool for teaching new concepts and skills. In addition to this definition of problem solving, a well-chosen problem-solving task also has the potential to support students in making connections between mathematical concepts, representations, and operations, deepening conceptual understanding, and creating richer conceptual structures (Donovan & Bransford, 1999; Kahan & Wyberg, 2003). Problem solving is also defined as model-eliciting in which students mathematize real-world problems by mathematically interpreting a contextual problem (Bonotto, 2002; Lesh, Hoover, Hole, Kelly, & Post, 2000). All of these conceptualizations of problem solving support how problem solving is perceived within the MPSM. Below is a discussion of the types of tasks that best support this type of learning in the mathematics classroom.

Cognitively demanding tasks. The appropriate selection of problem-solving tasks is critical for the success of the Mathematics Problem Solving Model. Selection of a problem-solving task goes beyond finding a task that will keep students busy throughout a class period to selecting a task that will lead students to "understanding fundamental mathematical concepts and principles and to acquiring skill in the use of basic mathematics techniques" (Marcus & Fey, 2003, p. 55). The MPSM professional development program used Stein et al.'s conceptualization of cognitively demanding tasks as a framework to be used for the selection of tasks. There are four levels of cognitive demand that a task may elicit: memorization, procedures without connections, procedures with connections, and doing mathematics (Stein et al., 2009). *Memorization* tasks are straight-forward tasks that require students to recall previously learned information to solve and do not afford opportunities to use procedures either because the task does not require the execution of a procedure or not enough time is allotted to allow for the use of a procedure. *Procedures without connections* tasks are algorithmic in nature, requiring the use of previously learned procedures. Memorization tasks and procedures without connections tasks are considered low-level tasks that require little to no cognitive demand to complete, are focused on producing correct answers, having no connections to related concepts, and require little explanations. Research shows that students who only ever engage in tasks of low cognitive demand have a very difficult time connecting the mathematics they learn to problems outside of a textbook or where the procedure to be applied is not made explicit (Boaler, 1998). While there is an appropriate time and place for students to perform tasks of low cognitive demand, the MPSM is not intended to be used with these types of tasks.

Tasks considered to be of high cognitive demand are procedures with connections tasks and doing mathematics tasks. *Procedures with connections* tasks focus students on the use of broad procedural pathways for the purpose of deepening understanding of the concepts underlying the procedures, requiring students to engage with conceptual ideas in order to successfully complete the task. This is often done through the use of multiple representations. *Doing mathematics* tasks require complex, non-algorithmic thinking, and require considerable cognitive effort. These tasks require students to explore mathematical relationships, processes, and concepts, demand self-monitoring of one's thinking. In addition, doing mathematics tasks require students to access relevant knowledge from past experiences and students must be able to examine these tasks in order to recognize constraints that may limit possible strategies and solutions (Stein et al., 2009). Research has shown that a major contributing factor to the success of students in high achieving schools is that their teachers engage them in tasks of high cognitive demand (Boaler & Staples, 2008). This is likely due to the fact that tasks of high cognitive demand are designed to build conceptual understanding (Boston & Smith, 2009) and promote reasoning skills (Yackel & Hanna, 2003).

The MPSM professional development supported teachers in the identification and use of cognitively demanding tasks in several ways. In the activity, *identifying cognitive demand of a task*, the teachers analyzed problem-solving tasks for the purpose of categorizing tasks by their level of cognitive demand and discussing the features of the tasks that placed them into their respective categories. This task is meant to help teachers to recognize the features of a task that make them high-level or low-level. The teachers often disagreed on the level of cognitive demand of some of the tasks and the teachers learned that some task features can be misleading in identification of the cognitive demand. For example, teachers participating in the professional development recognized that just because a task was a story problem or required the use of manipulatives did not guarantee that the task had a high cognitive demand. Teachers are often distracted by certain features when selecting a problem-solving task and this activity was helpful for teachers to build acuity for recognizing when a task is of high cognitive demand.

It was also the intent of the professional development that teachers should learn how to find or develop cognitively demanding tasks of their own. If cognitively demanding tasks are not readily available to teachers, then they must be able to adapt the tasks available to them in order to raise the cognitive demand of the task. As an extension of the *identifying cognitive demand* task, the teachers selected tasks from their own textbooks and made changes to the tasks based upon some guidelines given by the professional developers to raise the cognitive demand of the tasks. For example, one recommendation for raising the cognitive demand of a task was to remove excess scaffolding from the task (i.e. step-by-step directions that lead the students to the answer). The teachers later implemented some of these tasks in their own classroom.

Implementation of MPSM tasks with the teachers was another strategy used in the professional development program to raise teachers' awareness of what a 'doing mathematics' task looks like. 'Doing mathematics' tasks are rarely implemented in a typical middle school classroom, making it important for the teachers to experience first-hand what such a task looks and feels like as it is being implemented. The teachers used some of these tasks in their classrooms as part of the professional development experience in order to guarantee that the teachers did have the opportunity to implement tasks that are ‘doing mathematics’.

Implementing Tasks. It is necessary to select a task that is of high cognitive demand in order to give students opportunities to think and reason mathematically. However, even when tasks are set up as high cognitive demand, the level at which the students engage with the tasks may be lower. How the teacher supports students’ thinking and reasoning during implementation of the task is a major factor in how the students ultimately engage with the task (Henningsen & Stein, 1997; Stein et al., 1996). Stein et al. (2007) used the Mathematical Tasks Framework as a lens to understand how a task can evolve during the implementation process (see figure 5). The basic premise of the framework is that the nature of the task can change as it is set up by the teacher and then implemented by the students. Consequently, what the students learn when completing the task will also be impacted. Note that this framework is similar to the temporal phases of curriculum enactment that I used to develop my theoretical framework (see figure 1 in Chapter 1). Because the ways in which a task is implemented can greatly impact the learning opportunities of the students, the set-up of the task was referred to as the launch and teachers were encouraged to very seriously consider how they introduced a task to their students as it has the potentially to alter how the students experience the task.

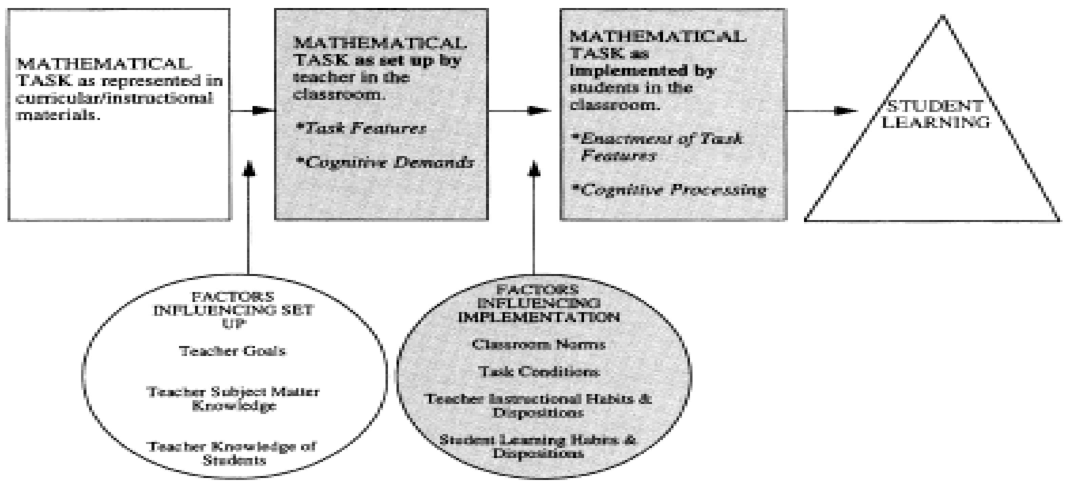


Figure 1. **Relationship among various task-related variables and student learning. Shaded portions represent areas under investigation**

Figure 5. Mathematical Tasks Framework

Maintaining cognitive demand of a task. Just as important as the selection of a cognitively demanding task that supports a mathematical learning goal, the MPSM emphasizes the importance of implementing classroom instruction in such a way to maintain the cognitive demand of a task. A research study focused on the implementation of mathematics tasks found that, of the tasks set up as high cognitive demand, less than half of the tasks maintained a high-level of cognitive demand (Stein, Grover, & Henningsen, 1996). The researchers identified several factors that contributed to the decline of cognitive demand: (1) tasks became non-problematic as the teacher either reduced the requirements of the task or completed the challenging steps for the students; (2) the task was inappropriate for the students, indicating a lack of knowledge about students on the part of the teacher; (3) the focus shifted to finding the correct answer; (4) too much or too little time was allotted to complete the task; (5) a lack of accountability on the part of the students; and (6) classroom management issues. Factors that contributed to teachers maintaining the cognitive demand of the task included (1) the task built on students' prior knowledge; (2) an appropriate amount of time was allotted for students to work on the task; (3) a high level of performance was modeled either by the teacher or more capable peers; (4) there was sustained pressure for explanation and meaning; (5) scaffolding was provided by the teacher or a peer without taking away from the complexity of the task; (6) students were encouraged to self-monitor their work; and (7) the teacher helped the students to draw conceptual connections (Henningsen & Stein, 1997; Stein, et al., 1996).

Maintaining the cognitive demand of a problem-solving task is particularly challenging for teachers when they see their students are struggling with a task and they want to relieve that anxiety for their students (Henningsen & Stein, 1997). In this sort of situation, it is necessary for teachers to provide scaffolding for their students that allow them to move forward with the task without removing the demands of the task for them. The use of questions is central to how teachers may support their students, as they work on a task, to maintain the cognitive demand. Questions that support students in moving their thinking forward should encourage students to provide mathematical argumentation and make mathematical connections (Kazemi & Stipek, 2001). The MPSM provided teachers with multiple opportunities to discuss how the cognitive demand of tasks, such as the MPSM tasks, might be maintained and how the cognitive demand of the MPSM tasks might be lowered and what to do to avoid this. These discussions arose as the teachers discussed Cathy Humphreys' classroom (Discourse and Feedback in the

Classroom activity) and Joel's classrooms ("Visiting a Classroom" activity) as these classroom episodes provided examples of teachers that were effectively able to maintain the cognitive demand of the tasks they were implementing in their classrooms.

To make sense of the types of questions that can be asked in the classroom as a task is implemented, Boaler and Humphrey's framework of teacher questions (2005) was shared with the participating MPSM teachers (see table 1) and they were asked to identify these different types of questions in a classroom video of Cathy Humphreys implementing the Border Problem with middle school students. The purpose of this activity was for teachers to recognize the importance of the difference types of questions that are asked in the classroom and to become more aware of the types of questions that they are asking in their own classrooms. The teachers revisited these question types when they watched the video of Joel engaging his class in discourse. The teachers were asked to discuss the questioning practices that led Joel to successfully maintain the cognitive demand of the problem-solving task he implemented in his classroom.

Question Type	**Description**
1. Gathering information, checking for a method, leading students through a method	Wants direct answer, usually wrong or right; Rehearses known facts or procedures; Enables students to state facts or procedures
2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them
3. Exploring mathematical meanings and relationships	Points to underlying mathematical relationships and meanings; Makes links between mathematical ideas
4. Probing; getting students to explain their thinking	Clarifies student thinking; Enables students to elaborate their thinking for their own benefit and for the Class

5. Generating discussion	Enables other members of class to contribute and comment on ideas under discussion.
6. Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study and life.
7. Extending thinking	Extends the situation under discussion, where similar ideas may be used
8. Orienting and focusing	Helps students focus on key elements or aspects for the situation in order to enable problem solving
9. Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics at a later point??

Table 1. Teacher questions from Boaler and Humphrey's *Connecting Mathematical Ideas* (2005)

The reading and reflection upon the piece *Never Say Anything a Kid Can Say* (Reinhart, 2000) was another tool used to develop the teachers' thinking about how a problem-solving task is implemented. The article discussed a teacher's experience with learning how to encourage all of his students to be actively engaged in thinking about a problem-solving task. In that article, he recommended that the types of questions asked in a classroom force the students to do the thinking (not just the teacher) and that the manner in which the questions are asked presses all students to be prepared to respond to the questions (for example, directing a question to the whole group and not just an individual). Participating teachers were asked to reflect upon how this article reminded them of their own teaching practices.

Analyzing Student Work. In a typical mathematics classroom, once the students have finished a problem-solving task, the work is handed in and then the teacher assigns a grade. The work is [sometimes] returned to the student, stuffed into their binder, and then never thought about again. One of the assumptions about student learning in the MPSM professional development is that: *Teachers' ability to understand students'* *mathematical development is enhanced by their ability to notice and describe what students say and do* (Franke, Carpenter, Levi, & Fennema, 2001; Sowder, 2007; Franke, Kazemi, & Battey, 2007). As a result, the perspective of the MPSM is that once the students hand in their work, the teacher now has valuable new information about her students and has the opportunity to use that information to gain further understanding of her students' thinking and to move forward more effectively in the learning process. The analysis of student work allows for teachers to gain insight into how their students think about and understand mathematics. This insight into how students are thinking about the mathematics is valuable as teachers implement problem-solving tasks in their classrooms and allows them to better understand how they can support their students in moving forward their thinking, asking appropriate questions to assist students in making connections, and shaping subsequent instruction (Sowder, 2007; Franke et al., 2007).

Several professional development programs have been developed that place student work at the center of teacher learning. Cognitively Guided Instruction (CGI) was a professional development program in which teachers studied children's developmental thinking about addition and subtraction; Integrating Mathematics Assessment (IMA) focused participating teachers' attention on student thinking from video tapes of students working on problem-solving tasks and using that information to inform lessons. The students of teachers in the CGI and IMA programs outperformed students of teachers in control groups, including on computational tests (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Saxe, Gearhart, & Nasir, 2001).

Analysis of student work was a central part of the MPSM. Participating teachers frequently engaged in analysis of student work from the MPSM problem-solving tasks and samples of student work from Joel's classroom. Teachers were encouraged to look beyond the surface features of the students' work (such as whether or not they provided a correct solution) to think about the mathematical ideas that are evident within the student work and how they can provide feedback and modify subsequent instruction to support students in their mathematical development, leading them towards the teacher's identified mathematical learning goals.

Feedback Guide as Lens. The MPSM utilized a formative feedback guide around mathematical problem solving to assist the teachers in thinking about how to respond to student work. This framework consists of five components of problem solving: Conceptual Understanding, Strategies & Reasoning, Communication, Computation & Execution, and Insights. The basic premise behind these components is that as students engage in mathematical problem solving, these five components characterize the skills and thinking that will lead students to be successful, both in completing the task and in developing mathematical understanding through problem solving. In order to support students in their problem-solving experience, the teachers were encouraged to address these components as they planned subsequent instruction around the task. These five problem solving components are grounded in research literature and are similar to the National Research Council's five intertwined strands of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001) which are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

***Conceptual Understanding*.** In the formative feedback guide, conceptual understanding pertains to the student's ability to make sense of the mathematics in a task. Comprehending the mathematics in a problem-solving task includes the appropriate use of mathematical language and mathematical representations in generating a solution to the task and recognizing the mathematical ideas used to solve the task. Students with a good conceptual understanding of a task are able to recognize how a mathematical idea or procedure is useful for solving a task. Also, a student should be able to connect a problem-solving task to old and new mathematical ideas. Research shows that learning with understanding involves an extensive factual knowledge base organized into a conceptual framework (Donovan & Bransford, 2001). That is, the mathematical ideas, terminology, and representations useful for solving a task best serve the learner when they are all interconnected. When working on a problem-solving task, students have opportunities to build conceptual understanding as they make decisions about which representations are useful for solving the task and why (Kilpatrick et al., 2001). The appropriate use of mathematical terminology is another avenue for building conceptual understanding, particularly when it is built up from students' everyday use of mathematical words (NCTM, 2000).

***Strategies & Reasoning*.** *S*uccessfully solving a task involves identifying a strategy and supplying a reason for why that strategy is useful for solving the task. Students should be able to recognize strategies available to them for solving a task and know when a strategy is appropriate to solve a task. Strategies as recommended by Polya (1945) include trying special values or cases, guessing and checking, using diagrams, looking for patterns, listing all possibilities, working backward, creating an equivalent problem, and creating a simpler problem. Students need to be able to not only recognize a strategy useful for solving a task, but flexibly recognize multiple strategies and select the best one (Kilpatrick et al., 2001). Engagement in a mathematical problem-solving task should entail students developing their own, informal strategies, providing explanation and justification for the strategies used, and making connections between multiple strategies. This process guides students' thinking towards a more advanced understanding of problem-solving strategies (Hiebert et al., 1997).

Communication. Students need to be able to communicate their thinking, both verbally to their peers and teacher as they are discussing a problem-solving task and in writing in their final written product of a task. Students should develop their ability to communicate clearly in mathematics in much the same way that students learn to communicate clearly in their language classes (NCTM, 2000). The ability to communicate one's thinking around a mathematical problem-solving task inevitably leads to the use of more concise language and better mathematical representations. Clear communication is a valuable skill in mathematics as it can function as a stepping stone to build mathematical ideas and deepen conceptual understanding, positioning students to build useful mathematical connections (Hiebert et al, 1997). Verbal communication in the classroom creates a forum in which students' thinking about mathematics becomes visible (Fuson, Kalchman, & Bransford, 2001). Within the MPSM problem solving framework, communication refers to writing and talking about a problem-solving task in such a way that the students make it clear to the instructor what they and why they did it. Note that communication, as it pertains to appropriate mathematical terminology and representations, is captured within the conceptual understanding component.

***Computation & Execution*.** It is not only necessary to successfully identify a strategy, but the computation and execution of the strategy must be done accurately in order to arrive at a correct solution. Completing a problem-solving task should afford opportunities for students to use the mathematical skills and procedures they have been learning in the classroom (Hiebert et al., 1997). It is important that students can compute a number accurately and efficiently, but it is also important that students can estimate a solution and recognize when a solution is appropriate. Part of this process involves recognizing when a procedure will lead to a correct solution. While accurate computation and execution may seem contrary to conceptual understanding, mathematical procedures should, in fact, be incorporated into a student's conceptual framework, organized in such a way for facile retrieval and application (Bransford, Brown, & Cocking, 2000). Computations that are founded in conceptual understanding will be more efficient and less prone to error (Kilpatrick et al., 2001).

***Insights*.** Student engagement with a cognitively demanding task should afford opportunities for students to make connections to other mathematics and to real-world contexts. NCTM (2000) advocates for students making connections between the mathematics they are using in their classroom and to real world contexts outside the classroom. Research shows that learners' conceptual frameworks tend to remain deeply rooted in their understanding of the outside world, despite formal instruction, making it vital that students' conceptualizations of the outside world be directly addressed in mathematics instruction (Bransford, Brown, & Cocking, 2000). Making connections to real-world contexts and extending mathematical thinking around a problem-solving task are useful for deepening mathematical understanding (Hiebert et al., 1997). Exploring mathematics problem-solving tasks beyond simply finding a solution can lead to students making, and eventually justifying, conjectures and generalizations related to the task (Kalathil, 2006).

The Feedback Guide. One of the features of the MPSM professional development is a feedback guide based upon the five problem-solving traits. The original version of the feedback guide resembled a rubric and the professional development team felt that the suggested feedback in the guide was summative in nature and not useful for moving students forward in their thinking about the task. By year three of the professional development program, one of the researchers in the project had developed a revised version of the feedback guide, which was used for years three and four of the professional development (Cohen, 2008). The new feedback guide was designed to help teachers give constructive feedback to their students that would help them move forward with the task. Another major difference in the revised formative feedback guide is that the feedback comments were organized not only according to the five problem-solving traits, but also according to the five NCTM process strands: Problem Solving, Reasoning and Proof, Communication, Connections, and Representations (2000).

Plan Next Step with Same Task. Once the students have worked the task and the teacher has had a chance to analyze the student work, the teacher must choose how he or she will then use that information in subsequent classroom instruction. The MPSM model proposes two avenues for subsequent instruction. One option is for the teacher to plan and orchestrate a whole class discussion around samples of student work. Whole classroom discourse around a mathematics problem-solving task creates an opportunity for students to deepen their understanding of the mathematics used to solve the task and also creates an opportunity for students to reason mathematically, justifying their reasoning and making connections between student work (Hiebert et al., 1997). To support the orchestration of this type of discourse, teachers are expected to analyze student work, select samples of student work to help make the mathematics salient to the class as a whole and then select the order in which student work will be presented to best support the development of mathematical ideas. The other instructional process supported by the MPSM model involves the teacher providing her students with written feedback on their work and then using that feedback in the classroom to create opportunities for students to move forward in their thinking about the task.

While this element of the model makes the assumption that the teacher must make a decision about which direction to take when planning for follow-up instruction, the professional development program was not designed in such a way to give teachers the opportunity to make this decision. Originally, in the professional development, teachers were required to both orchestrate classroom discussions around the task and provide written feedback to students for each of the five problem-solving tasks they implemented during the follow-up school year. In later years, in an effort to decrease the load for the teachers, this requirement was changed so that they orchestrated discourse for three of the five tasks and then provided written feedback with follow-up instruction for the other two tasks. This structure of the professional development limited the teachers' opportunities to make the decision about whether or not they wanted to follow-up the implementation of a problem-solving task with a whole-class discussion or with instructional activities using feedback.

Select and Sequence Student Work for Whole-Class Discourse. All students should have the opportunity to engage in discourse both with the teacher and their peers (NCTM, 1991). Students sharing their problem-solving strategies in the context of whole-class discussions is a valuable way for students to build new mathematical thinking (Hiebert, 2003), but this must come with a caveat that students randomly volunteering to share how they solved a task can lead to limited opportunities for students to engage in mathematical thinking around a problem-solving task (Leinhardt, 2001; Nathan & Knuth, 2003; Stein, Engle, Smith, & Hughes, 2008; Williams & Baxter, 1996). The teacher's role in orchestrating discourse around a mathematical problem-solving task is a critical one as it is the teacher's responsibility to create opportunities for students to reason mathematically about the problem-solving task (Chazan & Ball, 2001). The MPSM proposes planning for discourse around a mathematical problem-solving task by identifying which solution strategies will be discussed, determining the order in which those solution strategies will be presented, and planning appropriate questions to make the mathematics salient (Stein et al, 2008; Stein et al, 2009). These three planning activities are described in further detail below.

***Selecting, Sequencing and Planning for Discourse*.** The practice of selecting which pieces of student work will be presented during a whole-class discussion commonly takes place as the teacher navigates the classroom, observing the types of solutions students are generating (Groves, 2004). This approach is practical when a whole-class discussion takes place immediately following students working on the task. However, in the context of the MPSM professional development, teachers are expected to collect student work and take time outside of class to plan for the discourse. Research has shown that reflection of student work outside of the classroom is effective for enabling teachers to recognize their students' mathematical thinking and to make decisions about how to move that thinking forward (Davies & Walker, 2005; Kazemi & Franke, 2004; Nelson, 2001). The purpose of selecting student work is to make sure that particular mathematical ideas become open for discussion during the discourse (Lampert, 2001). Selection of student work may be based upon the ideas present in the solution, representations used, or misconceptions that are evident in the work. Selection of student work is also based less upon mathematical factors such as sharing a particularly well-written solution, or making sure that a student who hasn't presented in a while gets an opportunity to share.

The next step in planning for classroom discourse around mathematical problem-solving tasks is to determine the order in which the selected pieces of student work will be presented during the whole-class discussion. This is referred to in the MPSM as the sequencing of student work.

> "Rather than being at the mercy of when students happen to contribute an idea to a discussion, teachers can select students to present in a particular sequence to make a discussion more mathematically coherent and predictable" (Stein, Engle, Smith, & Hughes, 2008, p. 330).

Ideally, solutions are ordered in such a way that students will have opportunities to further develop the mathematical ideas that are the focus of the lesson. A common and effective way to sequence student solutions for discourse around a mathematical problem-solving task is to start with the least sophisticated solutions to allow the classroom community to progressively build a sophisticated solution to the task (Groves & Doig, 2004). Other ways to sequence include, but are not limited to, sharing incorrect solutions up front to eliminate misconceptions before proceeding with the discussion, sharing two conceptually similar solutions together to create opportunities for students to make connections between the strategies, and sequencing in such a way to allow a mathematical lesson to emerge (Stein et al., 2008).

Once a sequencing of student work is identified, the teacher plans questions intended to move forward the mathematical discourse. Planning for mathematical discourse has the potential to yield positive results since "rather than having mathematical discussions consist of separate presentations of different ways to solve a particular problem, the goal is to have student presentations build on each other to develop powerful mathematical ideas" (Stein et al., 2008, p. 330). Effective classroom discourse is characterized by students making connections between strategies, extending and generalizing solutions, making conjectures, verifying and modifying claims on the basis of mathematical evidence, and making sense of mathematical ideas. Questions a teacher might ask include requests for students to provide justification for the strategies they used (Hiebert et al., 1997; Kazemi & Stipek, 2001), questions that lead students to make sense of the mathematical ideas used to solve the task (Boaler & Humphreys, 2005; Sherin, 2002), questions that prompt students to make connections between strategies (Hiebert & Wearne, 1993; Kazemi & Stipek, 2001), and questions that encourage students to formulate and prove conjectures and generalizations around the mathematics in the task (Fraivillig, Murphy, & Fuson., 1999; Hiebert, & Wearne, 2003; Yackel & Hanna, 2003). Research has shown that classrooms that support student discourse around these types of activities demonstrate positive learning gains (Cobb, Wood, Yackel, Nichols, Wheatly, Trigatti, & Perlwitz, 1991; Silver & Stein 1996; Carpenter, Fennema , & Franke, 1997; Hiebert 2003).

Professional Development around Selecting and Sequencing. Teachers participating in the MPSM professional development program received support in engaging in this method of planning for classroom discourse. First, teachers were able to experience first-hand the implementation of a follow-up discussion of a problem-solving task as the professional developers implemented the MPSM tasks with the teachers. As one of the experiences planned by the professional development team, the professional developer implementing the task navigated around the room, observing the solutions the participating teachers were developing in small groups. After the teachers had completed the task, the professional developer asked specific groups to share their solutions and asked pressing questions that led teachers to recognize important mathematical connections between the solutions. After the discussion, the professional developer shared with the teachers her strategies for selecting and sequencing the solutions they discussed. The participating teachers expressed surprise and pleasure at how effectively this approach allowed for interesting mathematical ideas to be exposed that they themselves were not even aware of.

Multiple times during the professional development, teachers were given samples of actual student work from various problem-solving tasks and were asked to identify the important mathematical ideas evident in the student work, sequence the student work in such a way to develop further thinking about important mathematical ideas in the task, and think about what questions they might ask to make the mathematics salient. Teachers engaged in this activity first with a very simple problem-solving task, then an MPSM problem-solving task, and also with Joel's problem-solving task (before watching the video of his follow-up discourse). Practicing with samples of student work from actual teachers' classrooms made it possible to share with the teachers the actual plans that were developed by the teachers and to compare and contrast their own plans to what actually took place. In some cases, the teachers were pleased to see that the plans they developed were very similar to what the teachers had actually planned, while, in other instances, the teachers were amazed at how different the plans could be, even though they were selecting from the same set of student work.

In order to facilitate planning for whole-class discussions around a problem-solving task, a planning form called the Instructional Sequence Analysis (or ISA) was developed for the teachers to complete (see Appendix A). The ISA included a table with three column and several rows. The first column is for the teacher to put the names of the students whose work is to be presented in the order in which they plan to present their work. In the second column, the teacher identifies the idea to be highlighted in that student's work. For example, the teacher may have picked that student's work because they used a particular representation. In the third column, the teacher identifies the questions they plan to ask in order to make the mathematics salient to the class. There is one row for each piece of student work to be presented. Following the table, space is provided for the teacher to provide any rationale they had for the sequencing of student work they chose and to provide information about any additional follow-up they plan to do after the whole-class discussion. The Instructional Sequence Analysis was used on more than one occasion as the teachers practiced selecting and sequencing student work during the summer professional development. Also, the teachers were shown samples of the ISA completed by teachers in previous cohorts. During the following school year, the teachers used the ISA when planning the whole-class discussions they implemented in their classrooms. The ISA was used both to support the teachers in practicing implementing whole-class discussions around problem-solving tasks in their classrooms and also were used for data collection purposes in the research project.

Implementing Whole-Class Discourse. Once the teacher has planned for discourse by completing the Instructional Sequence Analysis, the obvious next step is that they will implement their plans in the classroom. At this stage, there are some additional decisions that must be made by the teacher. The teachers must choose whether or not they will ask the students whose work is being presented to come up and share their work with the class or if they will present the student work anonymously. They must make decisions such as whether or not they will direct the planned questions at the presenting students or to the class in general. Also, follow-up questions may need to be asked based upon how the students respond to the preplanned questions. The teachers had several opportunities to witness this type of whole-class discussion during the summer professional development. The teachers experienced these whole-class discussions as a student participant when the MPSM tasks were implemented by the professional developers. Also, the video of Joel's whole-class discussion represented an example of a whole-class discussion that promoted mathematical reasoning as he preplanned which pieces of student work would be presented. Also, in the later cohorts, excerpts of whole-class discussions from previous cohorts were shared with the participants.

While the teachers were able to witness examples of the implementation of a whole-class discussion around a problem-solving task, the teachers did not have a chance to practice implementing a whole-class discussion themselves until the following school year. As part of the larger research study, the teachers audio-recorded their whole-class discussions. The purpose of my dissertation is to gain a better understanding of what happens as the teachers implement their ISAs and how their choice of questions in the ISA affects the opportunities they create for students to reason mathematically.

Written Feedback and Classroom Use of Feedback. The other option in the MPSM for following up a mathematical problem-solving task is to provide the students with written feedback that they can use to improve their problem-solving skills. The primary purpose of formative feedback is to increase the knowledge, skills, or understanding of a student within a certain content area (Shute, 2008). Formative feedback involves establishing where the student is in the learning process, determining where they are going, and identifying what that student needs to do in order to get there. An important characteristic of good feedback is that the nature of feedback provided should be guided by the teacher's instructional goals and the teacher's knowledge of the student (or students) in question (Narciss & Huth, 2004).

There are many methods employed to impart formative feedback, but this section is focused specifically on written feedback. Nyquist (2003) identified five different forms of feedback. These are:

- *Weaker feedback*: Feedback which only gives the student knowledge of a score or a grade
- *Feedback*: Feedback which gives the student knowledge of a score or grade and also provides clear goals to work for or knowledge of correct results, often in the form of correct answers to questions the student attempted.
- *Weak formative assessment*: The student is given information about the correct results along with some explanation
- *Moderate formative assessment*: The student is given information about the correct results, some explanation, and some suggestions for improvement

- *Strong formative assessment*: The student is given information about correct results, some explanation, and suggestions for specific activities to undertake in order to improve.

In his meta-analysis of 185 studies, Nyquist found that, while there were positive learning gains for all five of these forms of feedback, strong formative assessment demonstrated the greatest learning gains (Nyquist, 2003). In light of this research, the MPSM formative feedback guide and the professional development experiences encouraged the teachers to not only identify the relationship between the students' performance and what the teachers considers to be ideal performance, but to also include some recommended action that the students can take to decrease the gap between student performance and ideal performance. This action may be implicitly or explicitly stated. In addition, the action may be directive, recommending specific action by students, or facilitative, providing hints or clues to what a student might do next.

Written feedback and the MPSM formative feedback guide. A MPSM feedback guide was developed for the MPSM professional development program that was meant to support teachers as they provided feedback to their students. An analysis of the original MPSM feedback guide that was used with the teachers in cohorts 1 and 2 revealed that the feedback guide was useful for weaker feedback only, providing information to students about the deficiencies in their work, but not giving recommendations for how they could move forward with the task and eliminate those deficiencies (Nyquist, 2003). Research shows that feedback is more effective when the feedback does not simply indicate whether the student was right or wrong, but also provides information pertaining to actions the student may take to increase proficiency (Bangert-Drowns, Kulik, Kulik, & Morgan, 1991; Pridemore & Klein, 1995). In order to increase the efficacy of the feedback guide, the revised feedback guide, used with cohorts 3 and 4 teachers, identifies the deficiencies in student performance, with some explanation, and gives suggestions for specific activities the student may undertake in order to address the deficiency. The new version of the MPSM formative feedback guide was designed to better encapsulate the feedback guide's purpose as a formative assessment tool by prompting the teacher to give more detailed information when providing written feedback and also suggesting possible avenues for improving the issues evident in the student's work (Cohen, 2006). This new feedback guide was designed to be useful as either moderate formative assessment or strong formative assessment.

Teachers practiced giving feedback and using the formative feedback guide during the summer professional development. The teachers began by giving feedback as they would in their own everyday practices and eventually worked towards practicing giving written feedback using the formative feedback guide. Teachers were given samples of student work and asked to pretend to provide feedback to these students as they would in their own classrooms. Teachers then discussed the type of feedback they were giving the students. Later, teachers were given a similar task, but were asked to use the formative feedback guide to help support their thinking as they thought about the types of comments they wanted to give the students. Throughout this process, the teachers reflected on the usefulness of the feedback they were giving, comparing and contrasting to feedback of their peers.

Classroom Use of Feedback. The extent to which the MPSM formative feedback guide is used for formative assessment is dependent upon how the teacher chooses to use the feedback in subsequent instructional activities. "[Formative assessment] is to be interpreted as encompassing all those activities undertaken by teachers, and/or by their students, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged." (Black & Wiliam, 1998). This quote implies that effective feedback should be used to move students' learning forward. It is, unfortunately, common practice for students to receive feedback comments from their teacher, possibly glance at the comments briefly, and then shove the paper into the back of their binder never to be looked at again. In order to ensure that the written feedback generated by the teacher is put to good use and that students actually use the feedback to engage in activities that will improve their performance on problem-solving tasks, the MPSM recommends formal activities in the classroom centered on the feedback. The most common use of feedback is to require that students use the feedback to revise their task solution and resubmit it for additional regard by the teacher or other students. This type of activity is more effective when feedback includes information about how a student may move forward with a task (Day & Cordon, 1993; Nyquist, 2003). Other common uses of feedback include peer feedback and student reflection on written feedback, providing information about how that student might improve. These follow-up activities, in which students reflect upon their work and take action to make improvements, have the potential to be beneficial to students as they encourage learners' engagement in active cognitive and metacognitive processing, thus engendering a sense of autonomy (and perhaps improved self-efficacy)" (Shute, 2008, p. 166).

Adult Learning: Building Understanding of the MPSM through Cognitive Apprenticeship

While the description of the elements of the MPSM given above provided some details about the four professional development activities described at the beginning of this chapter, this section is devoted specifically to describing how the professional development activities referred to as 'implementing MPSM tasks' were used to support teachers in developing their understanding of the MPSM. A common perspective of adult learning is that it needs to be experience-based (Baird, Schneier, & Laird, 1983; Donovan, Bransford, & Pellegrino, 1999). In keeping with this perspective, the MPSM professional development program engaged teachers in the 'implementing MPSM tasks' activities for the purpose of giving them increasingly independent experiences with the model. In this way, the 'implementing MPSM tasks' activities were in alignment with cognitive apprenticeship, a teaching model for adult learners (Merriam, 2007).

Brandt, Farmer, and Buckmaster (1993) defined five phases of cognitive apprenticeship: Modeling, Approximating, Fading, Self-directed Learning, and Generalizing. Modeling is when the professional developers model the activity they wish the learner to perform. Approximating is when the adult learner engages in activities that simulate the desired activity, with support from the professional developers as needed. Approximating activities progress into fading activities in which the learners engage in activities that more closely resemble what they are expected to do. Self-directed learning takes place when the learner independently engages in the newly learned activity. Finally, generalizing takes place as the learner reflects upon the learning that took place and makes personal choices about how that new learning will be applied to his professional practices. The activities developed for the MPSM professional development program reflected these phases of professional learning. In particular, 'implementing MPSM tasks' was an ongoing activity that developed in this manner. Further detail of this professional development activity is described below with respect to the concept of cognitive apprenticeship.

As an introduction to the Mathematics Problem Solving Model, the participating teachers were led through the MPSM problem-solving tasks by the professional development team. The participating teachers played the role of the student as they solved the tasks while the professional developers modeled the elements of the MPSM by playing the role of the teacher. As the professional developers modeled the MPSM, they frequently made the teachers aware of the different elements of the MPSM as they implemented the problem-solving tasks. For example, after implementing a task, the professional developer might talk about how they introduced the task and explain how the way that a task is introduced is going to impact the cognitive demand of the task as the students engage in it. The professional developer would then explain that how the task is introduced is part of the 'implementing tasks' element of the framework. This part of the professional development experience was important because it allowed the teachers to see how a task should be implemented according to the MPSM.

The activities of fading and approximating took place throughout the summer professional development. As the teachers learned about the different elements of the MPSM, they were given activities that gave them experiences with the model. For example, the teachers engaged with the mathematical tasks element of the model by first looking at sample problem-solving tasks and identifying the cognitive demand of the tasks. Later, they actually took tasks from their own textbooks and altered them in order to raise the cognitive demand of the tasks. This latter activity more closely simulated what we expected the teachers to do in their own classrooms as they look for problem-solving tasks to implement with their students.

Another important tool that was used to approximate implementing the MPSM tasks was the use of samples of student work. This made it possible for the teachers to analyze real student work under the guidance of the professional development team. This included providing written feedback as the teachers practiced giving useful feedback comments on samples of students' written work. Also, the teachers used samples of student work to practice selecting and sequencing student work in preparation for a hypothetical whole-class discussion.

Self-directed learning took place in the MPSM professional development program during the following school year when the teachers were asked to implement five problem-solving tasks in a manner consistent with the MPSM. With some variation across the four years, the teachers were asked to implement at least three MPSM tasks (tasks developed for the program that were recognized as doing mathematics) and two tasks of their own choosing. With at least three of the tasks, the teachers were expected to select and sequence student work and plan a whole-class discussion (the left side of the model). With at least two of the tasks the teachers were asked to give written feedback and plan follow-up instructional activities based upon the feedback (the right side of the model). During this phase of the learning process, the teachers were also required to fill out and submit planning forms and provide copies of their students' work with comments as part of data collection for the research project. This gave some accountability for the work they did, pressing them to follow the MPSM as expected. However, at this time, they worked independently and the manner in which they applied the model was under their control.

The final phase of cognitive apprenticeship is generalizing. The teachers received approximately 6 coaching visits during the school year to support their implementation of the MPSM. Since it was the goal of the professional development that the teachers would change their classroom practices to include problem solving as a common approach to teaching mathematics, these coaching visits were intended to support the teachers in their use of cognitively demanding tasks in their everyday classrooms. For the coaching visits, prior to the in-class visit, the teachers were asked to send the coach the goals of the lesson, the lesson itself, and any planned questions and activities. The coach would visit with the teacher before the lesson, encouraging the teacher to refine her thinking about the lesson in order to bring it more in alignment with the philosophy of the MPSM. The coach would then observe the lesson and debrief with the teacher. Following the observation, the coach would send written feedback to the teacher about key observations.

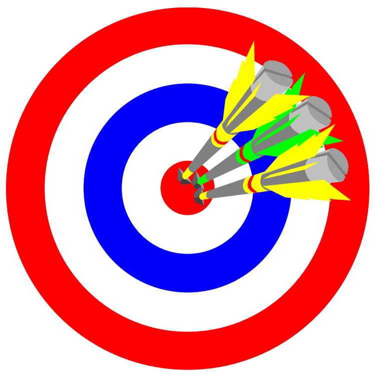
In the next chapter, I share and discuss the three MPSM tasks that were used by the teachers in the MPSM professional development program. These tasks were Spinner Elimination, Design a Dartboard, and Snack Shack. These tasks were implemented by the teachers in their classrooms. I will be discussing some of the common strategies that students provided

Chapter 4. Task Analysis

During the Mathematics Problem Solving Model professional development, the teachers were presented with several tasks, developed for the professional development, that were considered to be ‘doing mathematics’ (for more information, see Chapter 3, Professional Development Description). Three tasks, in particular, were shared with all cohorts and were used as part of their professional development during the school year. That is, all teachers in all cohorts were asked to implement three problem-solving tasks with their students during the school year and either provide the students with formative feedback or orchestrate a problem-solving debrief as follow-up to implementation of the tasks. While there were other problem-solving tasks that the teachers implemented with their students during the school year, these three tasks (Design a Dartboard, Snack Shack, and Spinner Elimination) were required for all teachers in all four cohorts. In preparation for implementing these tasks, the teachers completed these tasks during the summer professional development, debriefed them with a professional developer, and analyzed student work from these tasks.

When the teachers implemented debriefs for these tasks, they collected student work and selected and sequenced which samples of student work they wanted to share with the rest of the class. When they did this, they focused on common misconceptions and common strategies as well as strategies that revealed students' mathematical thinking about the task. When planning for the debriefs, they focused the discussion on sharing the common strategies, discussing the common misconceptions, and asking questions that helped students to focus on the mathematics in the tasks. In this section, I will discuss each of these tasks, beginning with a discussion of where these tasks fit into a middle school curriculum. In my discussion of student solutions, I highlight some of the most common strategies used by the middle school students, share some common misconceptions that students had with the tasks, and discuss some of the important mathematical ideas in the tasks. I encourage the reader to attempt these tasks on their own as a way to provide familiarity with these tasks as they will be referenced frequently throughout this study.

Design a Dartboard



As some of you may know, the traditional dartboard is made up of concentric circles or squares. As a new twist on the traditional dartboard your company wants to make a dart board of some other shape. You are in charge of designing the board. Be sure to use a shape other than a square or circle. The board should have three major sections. The area of the board should be divided so the area has:

15% for the inner section
25% for the middle section
60% for the outer section

- Draw a design for your dartboard
- Show **all** your work using numbers, drawings, and words
- Explain the strategy you used to get your answer

Placement in the curriculum. The Design a Dartboard task focuses on two areas in mathematics: the geometry of shapes and percentages. With respect to geometry, this task has the potential to address finding the areas of shapes, whether it is by counting squares or using area formulas. The task also allows students to compute percentages of area in various ways. While some teachers used this activity immediately following an introduction to percents, others used it as a culminating activity. For example, one teacher used it as a review of percentages before the State tests. Teachers also used the task during their geometry units.

A Common Misconception. A common misconception for the Design a Dartboard task was to make the middle shape 25% of the largest shape and then place the inner-most shape inside of the middle shape. This would result in the middle *section* only taking up 10% of the dartboard and leaving 75% for the outer-most shape instead of the required 60%. There are two ways to avoid this error. One way was to not place the inner-most shape inside of the middle shape. While some teachers accepted this as a valid solution, others made the assumption that the inner shape needed to lie inside of the middle shape because this is how a dartboard is typically constructed. In this case, the middle shape has to take up 40% of the dartboard because the 25% required for the middle section is supposed to surround the inner-most section. It can be verified that this is logical because if the middle shape takes of 40% of the dartboard, this leaves the required 60% for the outer-most section. Many teachers asked their students to explain why they chose to make the middle shape 40% of the dartboard.

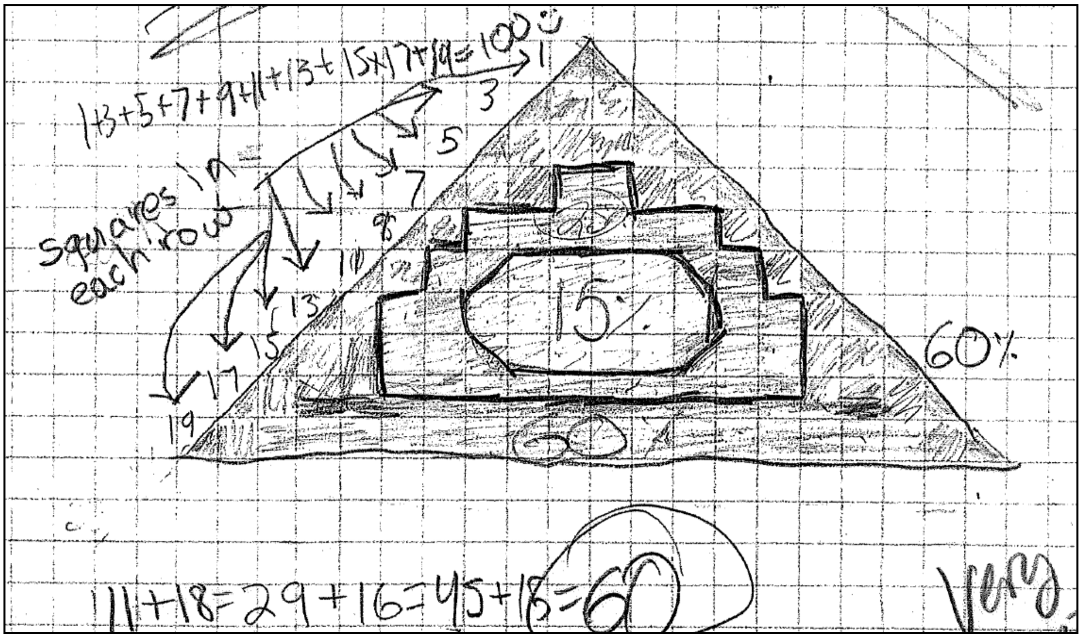


Figure 6. Example of using 100 squares to represent 100% because it made the percentages easier and counting squares to determine area.

A simple solution. A straight-forward entry-point into these tasks was to create shapes using graph paper. Students understand that each square on a piece of graph paper could represent one square unit. Students then let the largest shape be equal to 100 square units, the next shape would be equal to 40 square units, and the smallest shape would be equal to 15 square units. Students that used this strategy were not concerned with the fact that their resulting shapes were irregular. Nevertheless, this allowed students to reason through the task who had a very basic understanding of area as well as percentages. Notice in figure 6, the student used graph paper to create a shape with 100 square units total. While she did use the counting squares method to find the area, she was also able to create a shape that was symmetrical and she counted half squares to determine the total area. Many students made shapes that were more irregular than this one such as the example in figure 7.

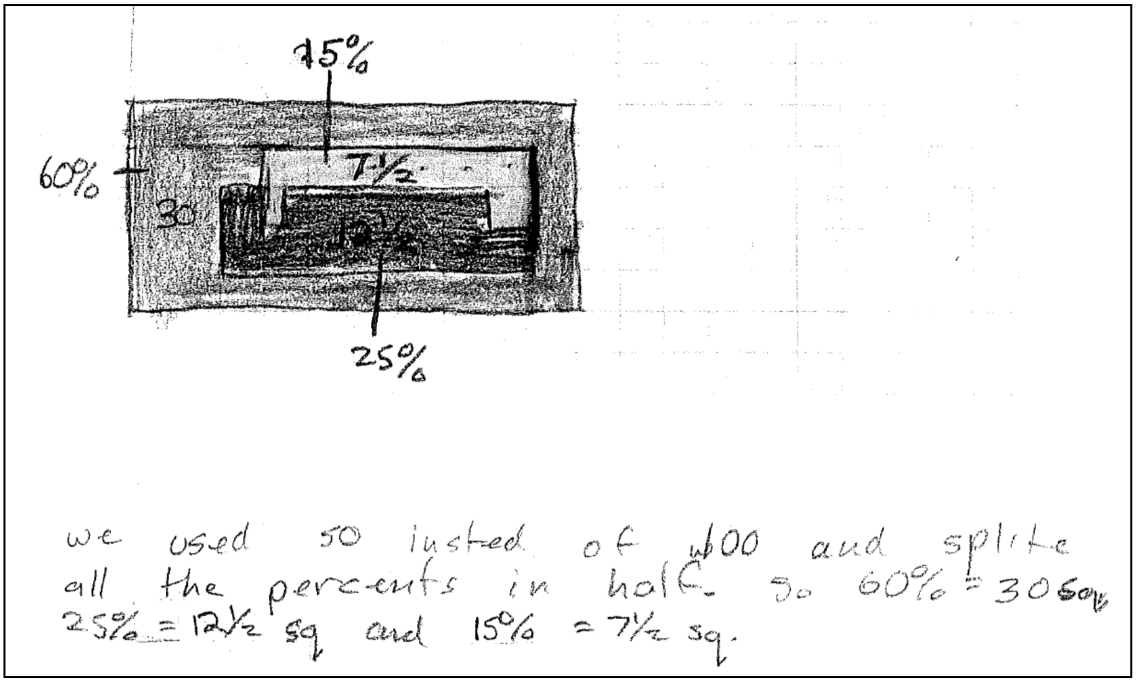


Figure 7. Example in which student divided the given percentages by 2 to determine workable areas and the student configured the shapes on graph paper.

Using Areas other than 100. When students wanted to construct shapes that were larger than 100 square units, they used their understanding of percentages to determine what size each shape had to be. Some students deliberately chose an area of the largest shape that would make it easier for them to know what the other percentages had to be. For example, they might let the largest shape be a multiple of 100 such as 300 square units. In this way, they easily knew that each shape would have an area that was three times the size of their percentages. The inner-most shape would be 45 square units (15x3), and the middle shape would be 120 square units (40x3). Similarly, it was possible to make the area smaller by dividing each percentage by the same number, such as 5, which would reduce the areas of the inner, middle, and outer shapes to 3, 8, and 20. As seen in the illustration below where they divided each percentage by two to determine the areas they would use, the students did not always try to reduce to whole numbers. When students chose the areas of the shapes to be factors or multiples of the given percentages, they often built their shapes starting with the inner shape, and then built the other shapes around it.

Another strategy that involved using areas other than 100 was to draw the outermost shape, determine the area of that shape (either by counting squares or using formulas for the areas of triangles), and then compute the corresponding areas for the middle and inner shapes. Students that used this strategy applied different methods they had used in class to determine the areas of the corresponding shapes. Teachers often chose to ask these students to share their methods as a way to remind the class of how percentages are computed. The most common methods were multiplying by the decimal of the percent and setting up common ratios. For example, if the biggest shape were 250, then the middle shape would be determined by multiplying 250 by .40 and the inner shape would be determined by 250 x .15. Alternatively, students might set of the ratios $\frac{250}{100} = \frac{M}{40}$ and $\frac{250}{100} = \frac{I}{15}$. They would then use these equations to determine the areas for the middle and inner sections.

Determining Dimensions of Regular Shapes. It would appear that necessary prerequisite knowledge for this task would be knowledge of formulas for finding the area of various shapes and algebraic reasoning. However, for many students in middle school, this was not information that they were not knowledgeable about. In some of the lower grades, students had not yet learned formulas for the areas of shapes and, in most classes they did not have knowledge of how to solve equations algebraically to determine the dimensions of a regular shape with a given area. When students wanted to use regular shapes, their most common choices were triangles and rectangles because they were most familiar with their formulas. No student attempted to make the three shapes similar. Instead, they would focus on creating three figures that were the same type of shape, but did not focus on making their dimensions proportional. One exception to this that did not result in a working solution was to make the dimensions proportional to the required percentages. For example, if a student had a 100cm by 200cm rectangle for the outer section, they might make the middle shape a 40cm by 80cm rectangle and the inner shape 15cm by 30cm. This method would result in rectangles that were similar, but the areas of the rectangles (20,000cm^2, 3,200cm^2, and 450cm^2 respectively) do not result in the needed percentages.

Determining dimensions of similar shapes would require algebraic reasoning that these students were not familiar with. Instead of using algebra to construct similar shapes, the students focused on their knowledge of factors to determine the dimensions of rectangles and triangles. For example, in the example with the dimensions that corresponded directly with the given percentages, they might make a rectangle of areas 10x10=100, 5x8=40, and

3x5=15. To determine the areas of triangles, they could use their understanding of triangles as half of a rectangle, including the visualization of cutting a rectangle in half and flipping one of the resulting triangles to the other side. In this way, they would use the factors given above to create [usually isosceles] triangles with the dimensions 20x30, 10x8, and 6x5 (see figure 8).

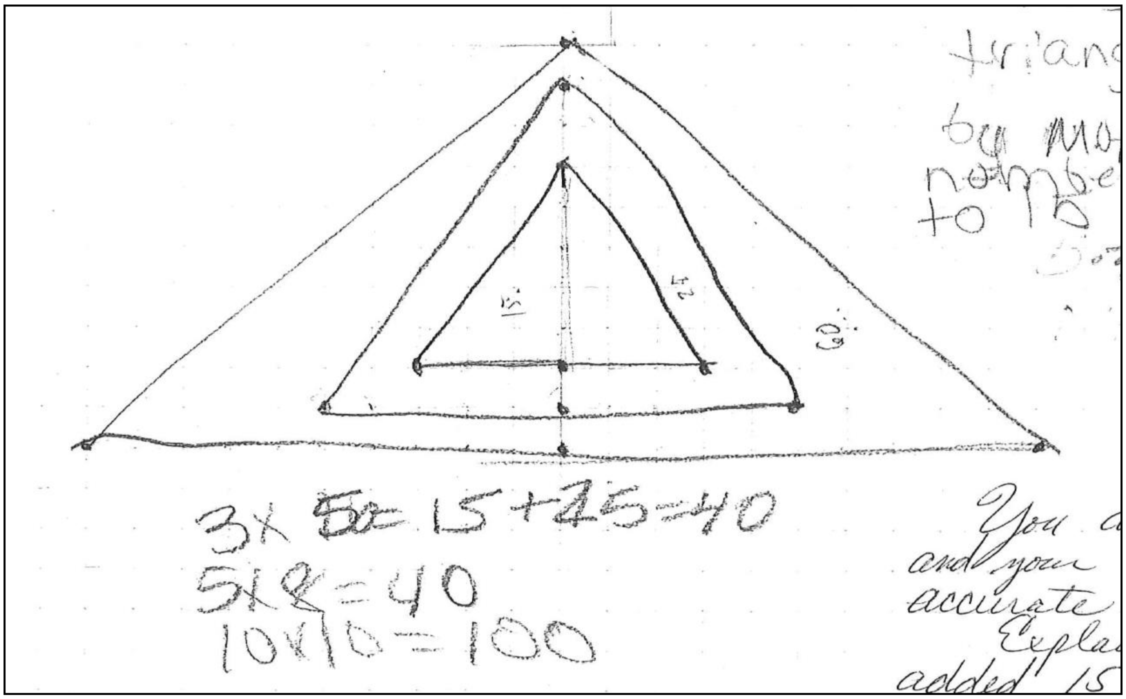
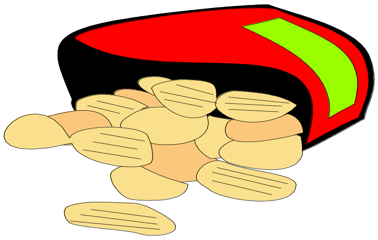
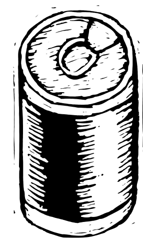


Figure 8. Student uses knowledge of rectangles and triangles to create isosceles triangles.

In the next section, I will discuss the Snack Shack task.

Snack Shack

Here's something to think about…

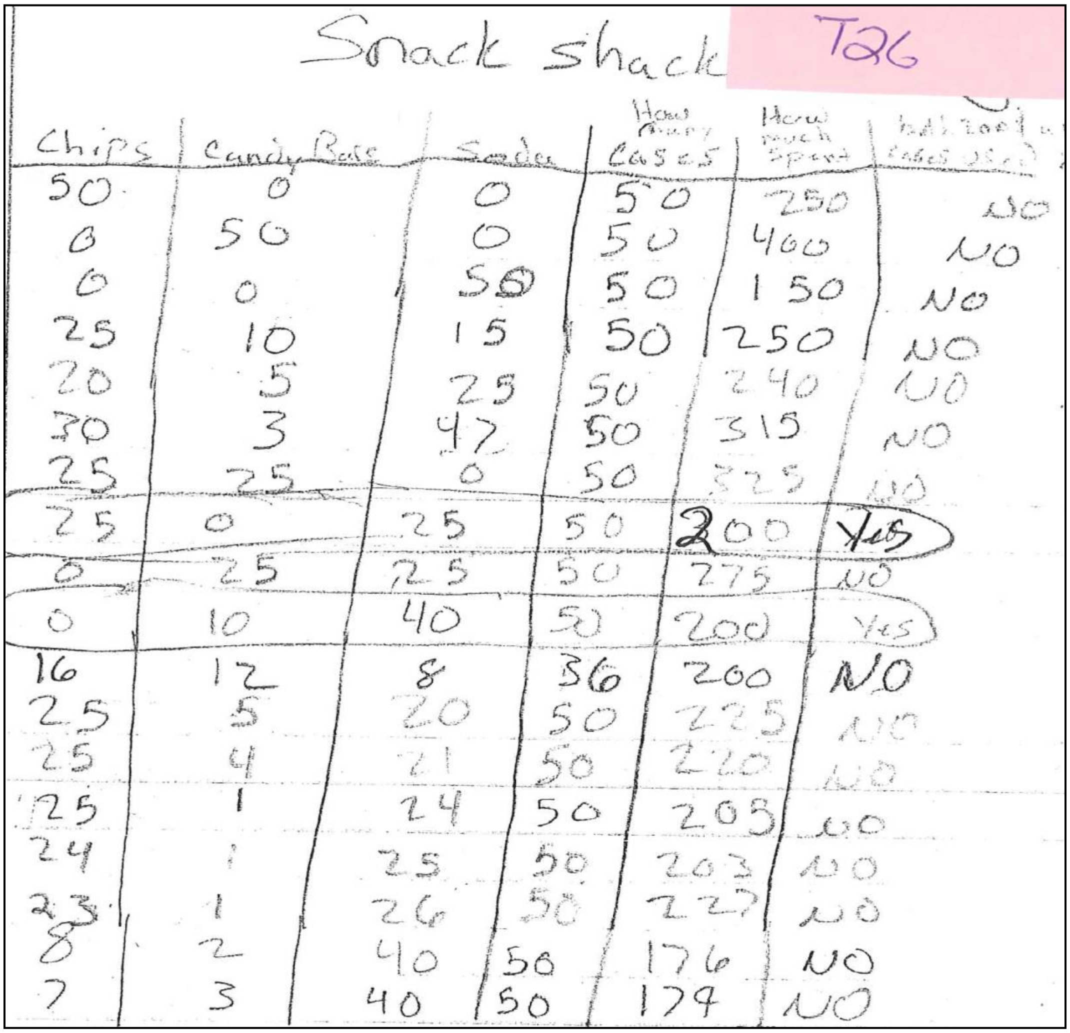
If you were starting a snack stand for students at your school and you had $200 to spend and room to store 50 cases, how would you decide how much of each item to buy if prices were as follows:

Chips	**$5.00/case**
Candy Bars	**$8.00/case**
Soda	**$3.00/case**

- Find as many possible solutions (using exactly $200 and 50 cases) as you can to this problem.
- Show all work using diagrams, charts, numbers, and words.
- Explain the strategies and describe any patterns you used to get your answers.

Placement in the curriculum. The Snack Shack task can be implemented at multiple levels. This task can be used as an entry level task for learning algebraic reasoning, but it can also be used as a culminating activity for students learning systems of linear equations. This task was implemented in middle school classrooms ranging from 6^{th} grade general math up through pre-algebra for advanced 8^{th} grade students. Solutions developed for this task varied significantly, depending upon the class in which it was implemented. One teacher placed this task towards the beginning of the school year as a focus on using tables to organize one's work, while another teacher placed it after a unit on systems of linear equations. There were also teachers who perceived this task as very challenging and, because they did not know where to place it in their curriculum, placed the task at the very end of the school year, or randomly placed it in their curriculum as a general practice in problem solving.

Using Tables to Find Solutions. Because this problem has two constraints, finding even one solution was challenging for most students. For many students, it was difficult to find a strategy to make just one of the constraints work. A common initial strategy was guess and check. Students would often choose three numbers randomly to represent the number of chips, candy, and soda and then add the numbers up on their calculator to check if they added up to a total of 50 cases, or if the cost added up to $200, but not necessarily both. When students plugged random combinations into their calculators in this way, they were not keeping track of what they had tried. As a result, they did not know which combinations they had already tried nor did they have a clear idea of what combinations to try next. For students in lower level math classes, keeping track of their guesses in an organized manner became an important focus in their mathematical development of the task.



Snack shack T26

Chips	Candy Bars	Soda	How many cases	How much spent	[illegible]
50	0	0	50	250	NO
0	50	0	50	400	NO
0	0	50	50	150	NO
25	10	15	50	250	NO
20	5	25	50	240	NO
30	3	47	50	315	NO
25	25	0	50	325	NO
25	0	25	50	200	Yes
0	25	25	50	275	NO
0	10	40	50	200	Yes
16	12	8	36	200	NO
25	5	20	50	225	NO
25	4	21	50	220	NO
25	1	24	50	205	NO
24	1	25	50	203	NO
23	1	26	50	227	NO
8	2	40	50	176	NO
7	3	40	50	179	NO

Figure 9. A table created by a student to keep track of her attempted combinations.

Understanding of organizational strategies developed, starting with numbers randomly plugged into calculators, randomly jotting down their calculations on paper, creating tables to display possible solutions in a more organized way, and developing strategies for which numbers to try next in a series of guesses. As students begin to organize their work, they have to make decisions about what information to display in their tables or charts, how to label their solutions so that other people will understand the data that is being shown, and what number combinations should be tried as they build up their tables. Notice how, in figure 9, a student kept track of the combinations she had tried in an organized table. The first three columns identified how many of each case she was trying, the fourth column was the sum of the cases, the fifth column was the total cost, and the final column noted whether or not the combination worked.

Because there are so many possible combinations to try, lists of combinations can become very cumbersome, sometimes taking up several pages of work. While in the illustration provide, she attempted some patterns to help her look for solutions in a consistent manner, such as trying 50 of each case, much of her guesses were fairly random. If she had continued trying to find all six solutions in this manner, she would have had a much longer list. As a result, students developed strategies for limiting the combinations that they tried. Something that some students noticed was that you could only have an even number of cases for candy bars. The reason for this has to do with the fact that candy bars were the only item with an even numbered cost ($8). If there were an odd number of candy bar cases, then the combination of chips and soda would necessarily be odd too, resulting in an odd total cost.

A useful strategy for finding at least one solution that worked well for students who were using guess and check strategies was to reduce the number of one type of case to zero. By reducing the types of cases used, the problem of finding a solution that worked became simpler. Students that tried this strategy quickly learned that it was not possible to create a working combination with only one type of case because they ended up with either more or less than 50 cases. For example, there could be no more than 25 boxes of candy bars because 25 boxes of candy bars at $8.00 a box came to $200, so more than 25 candy bar boxes would come to too much money. Similarly, there could be no more than 40 cases of chips because 40 cases of chips at $5/case came to $200. Also, students realized that it was not possible to have all soda cases at $3.00/case because they only came to $150. It was possible, however, to reduce the types of cases to just two. Two solutions emerge from this strategy. One is 25 cases of chips and 25 cases of candy bars. The other is 40 cases of soda and 10 cases of candy bars.

Finding Solutions by Making Even Trades. While it was very difficult to find a working solution using guess and check, it was much easier for students to find a solution that satisfied one constraint, but not another. It was then possible for students to use those combinations to strategically adjust the number of cases to get closer to the missing constraint while still satisfying the original constraint. For example, in Teacher 27's class, one student found a combination that added up to a total of 50 cases. She got 30 cases of chips which would be $150, 8 cases of soda which comes to $24, and 12 cases of candy which costs $96. While the total number of cases added up to the necessary 50 cases, the total cost was $270 which was too much money. In this example where the student had found the right number of cases but it cost too much money, it is possible to make even trades that would lower the overall cost.

By swapping out $8.00 cases of candy bars for $3.00 cases of soda, each swap would reduce the cost by $5.00 (see table 2 for a summary of these trades). Alternatively, swapping out candy bars for chips reduces the cost by $3.00, swapping out chips for soda reduced the cost by $2.00. Using the example given above of 30 chips, 8 sodas, and 12 candy bars, the goal is to reduce the overall cost from $270 down to $200. This, feasibly, could be down by swapping out 14 cases of candy bars with 14 cases of soda because each case swapped would reduce the cost by $5.00. However, there are only 12 candy bars. Alternatively, suppose only 8 candy bars were swapped out for 8 sodas (note that I could have swapped out 12 candy bars for 12 sodas, leaving 0 cases of candy bars, but we already have seen an example with 0 cases of candy bars). This trade results in 30 chips, 16 sodas, and 4 candy bars. This still adds up to 50 cases total, but the total is now reduced down to $230. This process of making even trades can now be repeated with another even trade. Since the total cost must be reduced by $30, we could either trade 15 chips for 15 soda or 10 candy bars for 10 chips. Since we only have 4 candy bar bases left, it only makes sense to trade chips for soda. This now gives the combination of 15 chips, 31 sodas, and 4 candy bars which does add up to $200 ($5 x 15 chips + $3 x 31 sodas + $8 x 4 candy bars = $200 total). These trades leading to a correct solution are shown in table 2.

Table 2. demonstration of even trades to reduce cost by $70 while keeping 50 cases

	Chips $5/case	Soda $3/case	Candy Bars $8/case	Total
Right number of cases, too much money.	30 $150	8 $24	12 $96	50 cases $270
Swapped 8 candy bars for 8 sodas.	30 $150	16 $48	4 $32	50 cases $230
Swapped 15 chips for 15 sodas,	15 $75	31 $93	4 $32	50 cases $200

A similar strategy can be used when a combination has been found such that the cost is right but the number of cases is not 50. In Teacher 9's class, two of his students found a combination that added up to $200. They had found 12 cases of chips, 10 cases of candy bars and 20 cases of soda. This combination cost a total of $200, but only added up to 42 cases. There is only one possible combination that will increase the number of cases while maintaining a cost of $200, which is trading one case of candy bars for both a case of chips and a case of soda. This works because the cost of chips and soda together ($5+$3) is equivalent to a case of candy bars ($8) which means that every time this trade is made, the cost is the same but the number of cases increases by one. In order to make this trade and increase the total number of cases by eight (from 42 cases up to 50 cases), it is necessary to trade 8 cases of candy bars for 8 cases, each, of soda and chips. This new combination is 20 cases of chips, 2 candy bars, and 28 cases of soda.

Using Patterns to Find the Remaining Solutions. Once multiple solutions were identified, these solutions were then typically used to find additional solutions. This was most commonly done by putting the found solutions into a table and looking for patterns. To illustrate how this was done, observe table 3 containing the four solutions that have been identified thus far:

Table 3. subset of solutions obtained thus far

Chips	Soda	Candy Bars
0	40	10
15	31	4
20	28	2
25	25	0

They have been organized so that chips are in ascending order while soda and candy bars are in descending order. There is a linear relationship between the solutions. When given a subset of the solutions in a table as shown in table 3, students quickly observe that, as you go from one solution to the next, the chips increase by five while the soda decreases by three and the candy bars decreases by two. Once this pattern is observed, students can quickly identify two additional solutions: 1) 5 chips, 37 sodas, and 8 candy bars; and 2) 10 chips, 34 sodas, and 6 candy bars.

When asked to justify why this taking away five chips and adding back in three cases of soda and 2 cases of candy bars gives another solution, students can explain, first, that the number of cases is preserved because five cases are taken away (the five cases of chips) while five cases are added back in (the three cases of soda and the two cases of candy bars). Also, the cost is preserved because five cases of chips cost $25 while three bases of soda cost $9 and two cases of candy bars cost $16. As a result, $25 is taken out and $9 + $16 = $25 is added back in.

The next question often asked is: “How do we know that we have found all possible solutions?” Two explanations are commonly given to address this question. First, as explained earlier, it is only possible to have an even number of candy bars. As a result, we know that there is, for example, no solution with just one candy bar. Second, because the number of chips on the list of solutions begins at zero and works its way up, we know that there can be no solutions with a negative number of chips. The same thing is true for the number of candy bars. The list has to stop at zero cases of candy bars.

Using algebra to find a solution. While the solution strategies described above were the most commonly used, it is also possible to find solutions using algebraic methods. Algebraic methods were not commonly used because middle school students have had very little, if any, exposure to linear equations. The use of algebraic expressions were used by Teacher 31's students to reason through the task and, with Teacher 31's assistance, they were shown a way to find solutions by solving the expressions using substitution. A challenge for many students was to create algebraic expressions to show both the number of cases and the total cost of the cases. One misconception that emerged was to use variables A, B, and C to represent *both* $A+B+C=50$ and $A+B+C=200$. In this case, students were letting A, B, and C represent, in one equation, the number of cases while in the other equation the variables represented the cost of the cases. It was necessary for students to recognize that A, B, and C should always represent the same thing and that, if A, B, and C represented cases of chips, soda, and candy bars, respectively then the second equation should be $5A+3B+8C=200$.

Even when students were able to successfully derive these two equations, they typically did not know how to use them. In Teacher 31's class, there was one student that made use of the equations. In the class, they had already begun to learn about linear equations and solving systems of linear equations. However, they had only seen systems of equations with two variables. In order for this student to be able to apply systems of linear equations to the problem, he let the number of cases of candy bars be equal to 0. In this way, he reduced the two equations to equations with two variables and was able to find the other variables using the methods he had already learned.

During the debrief, Teacher 31 acknowledged this strategy as 'legitimate', describing it as 'making the problem simpler'. He then led the class through a strategy that was similar to the student's, but used all three variables. He solved the equation

$A + B + C = 50$ for soda: $B = 50 - A - C$ and substituted this into the second equation: $5A + 3(50 - A - C) + 8C = 200$ and simplified the equation to $2A + 5C = 50$. Teacher 31 then explained to his students how, by substituting a value for one of the variables, it is possible to find out what the other variable has to be. For example, if A were equal to 0, then C would have to be equal to 10 and these values can then be inserted into the equation $B = 50 - A - C = 0 - 10 = 40$. He explained to the class that no solution can be a fraction and that they would have to use guess and check to find other solutions. He allowed the class some time to try different numbers and they were able to find additional solutions. This particular solution strategy was not one that Teacher 31's students were able to come up with on their own, and their understanding of how to use it to find solutions was rudimentary at best. However, it demonstrates a way that middle school students might be able to use algebra to solve this task if they had sufficient pre-requisite knowledge (specifically, some experience with solving systems of linear equations).

In the next section, I will discuss the Spinner Elimination task.

Spinner Elimination

Here's something to think about…

Design a spinner that you think will help you to cross out more squares than your opponent on a 50's chart like the one shown in table 4. (See rules for the Spinner Elimination Game.) After you have played a game with your spinner, decide if you want to change the numbers on your spinner.

After you have played several games, answer the following questions:

- Do you think that you have created the best spinner possible or, if you were to play the game again, would you change the numbers on your spinner? Why or why not?
- What advice would you give to someone who wants to cross out the most squares on how to choose numbers for their spinner?

Rules for the Spinner Elimination Game

1. Divide your spinner into eight equal sections.
2. You may choose up to 8 numbers (from zero to nine) to put on your spinner.
3. You may put them on any space you choose on the spinner and you may use the same number as many times as you like.
4. You eliminate squares on the 50's chart by spinning your spinner as many times as you choose and multiplying the product of the spins (E.g. If you spin three times and you spin a 4, then a 3 and then a 4 you would get $4 \times 3 = 12$, $12 \times 4 = 48$. You would eliminate 48 from the 50's chart. If choose to spin only one time and get a 4, then you would eliminate the 4.). Each time you eliminate a square counts as one turn.
5. If your spin creates a product greater than 50 you lose that turn and the next player spins.
6. You can only cross off one number per turn.
7. After 20 turns, the player with the most squares eliminated on their 50's chart wins the game.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**

Table 4. 50's chart for Spinner Elimination Game

Placement in the curriculum. The Spinner Elimination task asks students to create a spinner with some combination of 8 numbers from 0-9. The spinner is used for a game in which the students spin the spinner one or more times and multiply the numbers together. They then cross out a number on a chart numbered from 1-50. They get 20 turns, they can't cross out a number more than once, and if their product is more than 50 they lose their turn. The goal of the task is to create a spinner that will give them the best chance of winning the game. This task focuses largely on multiples and factors, creating opportunities to reason about primes and composites. It also has some opportunities to reason about probability. Because multiples, factors, primes, and composites are concepts that students are expected to already have learned before entry into middle school, this task was often used at the beginning of the school year as a way to introduce students to reasoning and problem solving as well as to provide students with a review of number concepts.

Zero is problematic. Most students quickly recognized that you would never want to put zero on a spinner because any number multiplied by zero is zero. As a result, once zero is spun, every additional spinner during that turn is going to result in another zero. Many students, though, did not realize this would happen. These students made the initial assumption that having a zero on your spinner would allow you to start over because they saw it as causing them to go back down to zero. What they did not realize was that in order to 'start over' they would have to some have 'get back down' to 1, which is not possible.

Should the number one go on the spinner? While students were all able to agree that you would never want a zero on your spinner, the number one was more debatable. When the number one was spun, the result would be no change on the value. For example, if the player had spun a five and then a one, they would still be at five. Because of this, many students saw this as a wasted spin and preferred not to have it on their spinner, while others perceived a one as a free spin in which they did not have to worry about going over 50. While everyone generally agreed that a one on the spinner had no impact on the outcomes of the game, there was one exception to this, which was if the one was spun during the first spin. In this case, since it was not a requirement to spin the spinner more than once, this would make it possible to cross of a one on the 50's chart. Students had mixed feelings about whether or not this made having a one on the spinner worth it. While many students argued that it wasn't worth it, other students recognized that since it counted as one more number that could be crossed of, it would be worth having on your spinner.

Using a wide variety of numbers and avoiding high numbers. After recognizing the role that one and zero play on the spinner, the two most common strategies that students used were to use a wide variety of number and to avoid high numbers. Students often tried all the numbers from one to eight or two to nine on their first spinners. This tended to be a logical first step for this game because it allowed students to see what would happen when they used all of the possible numbers on their spinner. Usually, after trying this initial strategy students would then choose to eliminate the high numbers on their spinner. This was because having large numbers like seven, eight, and nine on a spinner made it more likely to get more than 50 on a turn. In particular, eight and nine were recognized as the most problematic because if you spun, for example, an eight twice in a row, then the resulting number would be 64, which is more than 50, resulting in a lost turn. Whether or not seven should be counted as a 'high' number was debated. If seven were spun twice in a row, it only results in 49, which is still less than 50. However, there were some students that felt that seven was too high to have on the spinner. Others, though, recognized that seven could be a worthwhile number to have on the spinner because it creates more multiples that could be crossed off since it's a prime. This will be discussed in further detail later.

Sticking with very low numbers: A common misconception. When students saw the risks of going over 50 with the larger numbers, some of them attempted to remedy this dilemma by using only very low numbers. For example some students tried making spinners with either just the number two or only the numbers two and three on it. These students reasoned that if they were less likely to go over 50, they would have more opportunities to cross of multiples. More specifically, a misconception that occurred here was that students believed they would be able to get all even numbers if they had just a two on their spinner. What these students discovered was that by limiting the variety of numbers, they were limiting the possible combinations they could make. Students who tried only two on the spinner quickly realized this problem because they could only cross off the exponents of two (2, 4, 8, 16, and 32). While the students that tried only two on their spinner recognized that they could not make enough combinations, the students that tried just two and three were not able to recognize this limitation as easily because there were 15 possible combinations. In Teacher 27's debrief, he led them through a discussion that allowed them to see this problem. He asked them to list out all of the possible combinations that could be made with the numbers two and three and then count how many combinations they got. This allowed them to recognize that they would only ever be able to cross off 15 numbers on the 50's chart.

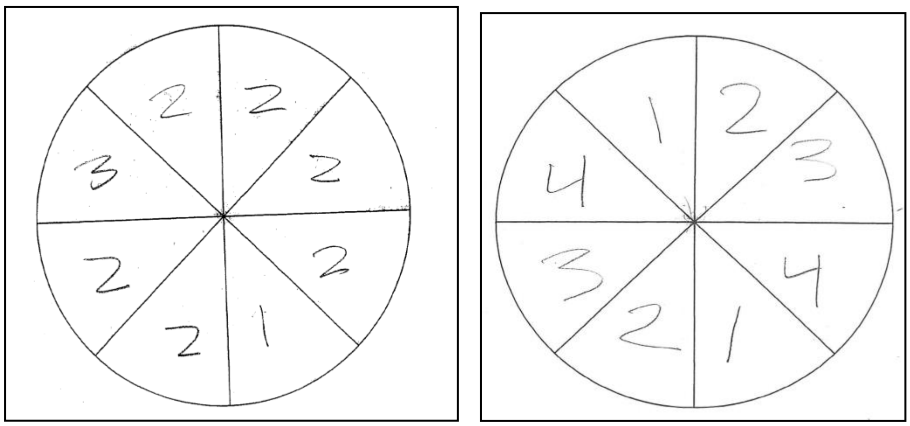


Figure 10. Spinners designed by students with just 1, 2, 3 and 1, 2, 3, 4, respectively.

Reasoning with primes and composites. When students saw that putting only very low numbers on the spinner limited the number of combinations they could spin, they would still want to avoid the higher numbers and, as a result, they would try to compromise by adding another number that is also low. In Teacher 27's class, a student tried a spinner with the numbers one, two, three, and four (see figure 10). This spinner actually has the exact same problem as the spinner with just two's and three's because four is created by multiplying two by itself twice. Consequently, any combinations that are possible with a two, three, and four are also possible with just a two and three. In Teacher 27's class, he exposed this relationship between these two spinners by asking his students to once again list out all of the possible combinations, but this time using the numbers two, three, and four. When the students saw that they were making all of the same combinations, they were able to reason that, because four was a composite of two, they would not have any more combinations with the second spinner. While four does not make any additional combinations because it is a composite of two, adding a five or a seven to the spinner is going to create new combinations. This is because they are prime numbers.

Reasoning through the task in this way allowed students to see an important distinction between prime number and composite numbers. A central idea that students explored was that they could not cross certain numbers off of the 50's chart unless their prime factors were on the spinner. For example, you cannot cross off a 25 unless you have a 5 on the spinner. A similar realization was that, it is not always necessary to have composite numbers on the spinner because the prime numbers may be used instead. Reasoning for this idea connects to the concept of prime factorization. For example, a four is not necessary on the spinner because the two may be used in place of the four by spinning a two twice. Although 16 = 4x4 means that 16 can be crossed off by spinning a four twice, it is also true that 16 = 2x2x2x2, so the 16 may also be crossed off by spinning two four times. Similarly, 24 = 4x6 = 2x2x2x3.

Students further recognized that there were some numbers on the 50's chart that could never be crossed off, even if you have all numbers from two to nine on the spinner. This was because they were either prime numbers greater than nine (like 17) or numbers with prime factors greater than nine (like 22 = 2x11). While students were capable of understanding these concepts related to primes and composites with respect to the task, many students did not make these connections on their own. It was typically necessary for the teachers to draw their students' attention to these ideas.

Spinning strategies and probabilities on the spinner. While the purpose of this activity was to select a spinner that would help you win, most students recognized that there were also appropriate strategies for playing the game that were important for increasing your chances of winning the game. Specifically, students had to make decisions about when to stop spinning the spinner. In some cases, students would continue to spin the spinning even if they had opportunities to cross off a square on the 50's chart. For example, when a student spins once, and spin, for example, a 5, they could choose to cross that number off of the board, or they could choose to spin again. Some students would choose to spin again, wanting to cross off some of the bigger number first. Most students, though, agreed that it was best to cross off a number as soon as one became available. As a result, if a student spun a 3 in their first turn they would immediately cross off the 3 on the 50's chart. During their next turn, they might spin a 3 again and they would not be able to cross off the 3, but they would just spin again. They then might spin a 5, giving them 3x5 = 15 as an option to cross off on the board. If this spot were available, most students would choose to cross it off. Other students, though, would consider choosing to spin again. At this point, depending upon what is on their spinner, they would have a varying probability of going over 50.

In some classes, students reasoned about the probability of going over 50 after spinning a certain combination. For example, if a student spins a 5 first, they know it is safe to spin again because they have a 0% probability of going over 50. This is because the largest number they could spin was a 9 which would result in 45. In contrast, if a student had already spun up to a number greater than 25, like 27, they have a 100% probability of going over 50 because on their next spin because the smaller number they could spin was a 2 which would give them 54. Students could reason further about the probabilities of going over 50 if, for example, they have a spinner from 2 through 9 with one of each number on the spinner. Then if they had spun a 15, the numbers 2 and 3 would not cause them to go over 50 (giving them the products 30 and 45 respectively) while landing on any of the remaining numbers on the spinner would cause them to go over 50. As a result, they can determine that they have a 2/8 chance of staying under 50, or a 25% chance and a 6/8 chance of going over 50, or a 75% chance. Teacher 27 discussed these types of probabilities with his students because some of his students chose to keep spinning even if they had blank spots on their 50's chart that they could have cross out. They reasoned about how the decision of when to stop spinning would depend upon the individual and that some students would want to risk it, while others would prefer to be safe.

In the next chapter, I describe the methods used in my study including a description of the participating teachers, data collection methods, and a description of the research designs for my three sub-questions.

Chapter 5. Method

Participants and Context

The participants in my research study were drawn from the middle school mathematics teachers that were involved in the Mathematics Problem Solving Model professional development program. The professional development program ran for a total of four years with teachers participating in the program for one year. The total number of teachers who participated in the Mathematics Problem Solving Model professional development program over the four years was about 50. For this research study I used data collected from 12 teachers from cohorts 2 and 3. Nine of the participants were from cohort 2 and three of the participants were from cohort 3. I did not use data collected from cohort 1 because the data collected from cohort 1 was different from the remaining years. The teachers that participated in this study from cohorts 2-4 were selected based upon the availability of complete data sets. For a complete description of the professional development program and information pertaining to the demographics of the teachers, see Chapter 3, Professional Development Description. Below, I provide a description of the teachers who participated in the professional development program. The information given includes years of experience, both as a licensed teacher and as a middle school mathematics instructor; grade levels taught; school settings, percent free and reduced lunch, percentage of schools that met adequate yearly progress in the year prior to the professional development; and availability of reform curricular materials. This information is provided for each cohort and then I provide a summary for the teachers whose data was used in my study.

Summary of cohort 1 treatment group. Ten middle school teachers participated in the professional development in the 2006-2007 school year (cohort 1). Years of experience as licensed teachers ranged from a minimum of five years teaching experience to a maximum of 29 years. The average number of years of experience as a licensed teacher was approximately 12 years. Within the area of teaching math in the middle schools, these teachers averaged a total of 10 years of experience, ranging from two to 20. Two teachers taught 6^{th} grade math, two taught 7^{th} grade math, two taught 8^{th} grade math, and four taught $7^{th}/8^{th}$ grade math.

Five of these teachers taught in an urban setting, four taught in towns with populations less than 50,000, and one taught in a rural setting. Percent of students receiving free and reduced lunch ranged from 20.2% to 64.3%, with an average of 41.35%. Seventy percent of the teachers participating in the professional development program came from schools that did not meet criteria for adequate yearly progress in the previous school year. Fifty percent of the treatment teachers used at least some reform-based materials in their classrooms.

Summary of Cohort 2 Treatment Group. Eleven middle school teachers participated in the professional development in the 2007-2008 school year (cohort 2).

Years of experience as licensed teachers ranged from a minimum of one year of teaching experience to a maximum of 28 years. The average numbers of years of experience as a licensed teacher was approximately nine years. Within the area of teaching math in the middle schools, these teachers averaged a total of approximately five years. Years of experience within the field of math in the middle schools ranged from one to 10 years. Three teachers taught 6^{th} grade math, six taught 7^{th} grade math, and two taught 8^{th} grade math.

Five of these teachers taught in an urban setting, one taught in a town with a population less than 50,000, and four taught in a rural setting. One of the schools was a private prep school. Percent of students receiving free and reduced lunch ranged from 0% to 70.2%, with an average of 33%. It is worth noting that four of the teachers taught at schools with over 59% percent of students receiving free and reduced lunch while the remaining seven teachers taught at schools with less than 22% of students receiving free and reduced lunch. 55% of the teachers participating in the professional development program came from schools that did not meet criteria for adequate yearly progress in the previous school year. 20% of the treatment teachers used at least some reform-based materials in their classrooms (one teacher was not included in this number as the type of curriculum used by the teacher was not made available to us).

Summary of cohort 3 treatment group. Seventeen middle school teachers participated in the professional development in the 2008-2009 school year (cohort 3). Years of experience as licensed teachers ranged from a minimum of three years teaching experience to a maximum of 33 years. The average numbers of years of experience as a licensed teacher was approximately 13 years. Within the area of teaching math in the middle schools, these teachers averaged eight years. Years of experience within the field of math in the middle schools ranged from one to 20. Two teachers taught 6^{th} grade math, eight taught 7^{th} grade math, six taught 8^{th} grade math, and one teacher taught 8^{th} grade content to 6^{th} grade students.

Nine of these teachers taught in an urban setting, four taught in towns with populations less than 50,000, and four taught in a rural setting. Percent of students receiving free and reduced lunch ranged from 25.5% to 65.9%, with an average of 39.28%. 88% of the teachers participating in the professional development program came from schools that did not meet criteria for adequate yearly progress in the previous school year. 71% of the treatment teachers used at least some reform-based materials in their classrooms (three teachers were not included in this number as the type of curriculum they used was not made available to us).

Summary of cohort 4 treatment group. Twelve middle school teachers participated in the professional development in the 2009-2010 school year (cohort 4). Of those 12 teachers, five completed the program, providing us with data. Of those teachers that completed the program, years of experience as licensed teachers ranged from a minimum of two years teaching experience to a maximum of 25 years. The average numbers of years of experience as a licensed teacher was approximately 10.4 years. Within the area of teaching math in the middle schools, these teachers averaged three years. Years of experience within the field of math in the middle schools ranged from two to four. Two teachers taught 6th grade math, two taught 7th grade math, and one taught 8th grade math.

One of these teachers taught in an urban setting, one taught in a suburban setting, two taught in towns with populations less than 50,000, and one taught in a rural setting. Percent of students receiving free and reduced lunch ranged from 14.2% to 61.7%, with an average of 40.34%. 50% of the teachers participating in the professional development program came from schools that did not meet criteria for adequate yearly progress in the previous school year. 50% of the treatment teachers used at least some reform-based materials in their classrooms (This data was not available from one of these teachers).

Summary of teachers used in this study. The data collected from 12 of the middle school teachers who had participated in the professional development were used in my study. Nine of these teachers completed the professional development in the 2007-2008 school year (cohort 2). Three of these teachers participated in the 2008-2009 school year. Years of experience as licensed teachers ranged from a minimum of one year of teaching experience to a maximum of 33 years. The average numbers of years of experience as a licensed teacher was approximately 11.25 years. Within the area of teaching math in the middle schools, these teachers ranged from 1-12 years of experience and averaged a total of approximately 5.5 years. Four teachers taught 6th grade math, five taught 7th grade math, and three taught 8th grade math.

The teachers in my study taught in 8 schools, with four pairs of teachers working the same school. Two teachers taught in an urban setting, four taught in a suburban setting, and six taught in a rural setting. One of the schools was a private prep school while the remaining schools were public schools. Percent of students receiving free and reduced lunch ranged from 0% to 70.2%, with an average of 29.3%. Three of the teachers taught at schools with over 59% percent of students receiving free and reduced lunch while the remaining teachers taught at schools with less than 30% of students receiving free and reduced lunch. 50% of the teachers came from schools that did not meet criteria for adequate yearly progress in the previous school year. 33% of the teachers reported using at least some reform-based materials in their classrooms.

Data Collection

As part of their participation in the MPSM professional development program, teachers were required to implement 5 cognitively demanding problem-solving tasks during the school year, orchestrating a problem-solving debrief with the students, and/or providing written feedback after students had an opportunity to work on the tasks. During the four years that the program was implemented, the requirements changed as to which tasks the teachers were required to implement and whether they were to give written feedback on students work or orchestrate a problem-solving debrief as their follow-up instruction. In year 1, all teachers were required to implement the same five MPSM tasks, chosen by the researchers, and to both orchestrate a problem-solving debrief with the students and provide written feedback for all five tasks[4]. In year 2, the teachers were required to implement three MPSM tasks chosen by the researchers and two tasks of their own choosing. They were also required to provide written feedback

[4] Because year 1 was a pilot year for the program and because of how many changes were made to the professional development experience from year 1 to year 2, I chose not to use the data collected in year 1 for my study.

and orchestrate a problem-solving debrief for all five tasks. In year 3, the teachers were required to implement three MPSM tasks chosen by the researchers and two of their own choosing; they were required to implement a problem-solving debrief for three of the tasks (two MPSM and one of their own choosing) and to provide written feedback on the other two tasks. In year 4, the teachers were required to implement three MPSM tasks chosen by the researchers, one task of their own choosing, and one task of their own choosing from a collection of tasks designed by Lesh and his colleagues (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). In year 4, teachers were required to orchestrate problem-solving debriefs for two MPSM tasks and the Lesh et al. task. For the remaining two tasks, they were required to provide written feedback. Teachers were expected to implement the tasks at a time during the school year when the task would fit appropriately with the content that was being covered in their curriculum at the time of implementation.

For the tasks in which the teachers orchestrated a problem-solving debrief, once the task had been at least partially completed by the students, the teachers collected the student work and used it to complete a debrief planning form, called an Instructional Sequence Analysis (ISA). To see an example of a blank Instructional Sequence Analysis, see Appendix A. The teachers chose two to six pieces of student work to be shared during the problem-solving debrief which was implemented on a later class day. Teachers implemented their ISAs as early as the next class day, but teachers sometimes delayed the problem-solving debrief as much as two weeks later. In the ISA, the teacher wrote down (1) the order of the students whose work was to be shared with the class, (2) the ideas the teacher wished to highlight for each piece of student work, and (3) questions planned by the teacher to help make the mathematics salient to the students. The teacher was also required to give a rationale for the selection and sequencing made in the Instructional Sequence Analysis.

As part of the research project, the teachers submitted student solutions and the ISA they had completed for the task (when they orchestrated a debrief). The problem-solving debriefs were audio-recorded. Whenever possible, two audio-recorders were used. One audio-recorder was placed to pick up the teacher's voice (often wore around the teacher's neck or attached to her lapel) and the other was placed somewhere else in the classroom to pick up students' voices. In years 1 and 2, a researcher was always present for the problem-solving debrief to assist with the audio-recorder and to collect data. In years three and four, the teachers were given the audio recorders to run during the debriefs and asked to submit their data online through a secure website.

For my research study, I analyzed the debriefs that were implemented for the three MPSM tasks. I used two data sources for this research study. These were (1) the Instructional Sequence Analysis, and (2) audio recordings of the implementation of the ISAs (which were transcribed with time stamps on all teacher utterances). The students' work was occasionally used as a resource for making sense of the data, but was not explicitly used for the study.

Not all data sets of the problem-solving debriefs for the MPSM tasks were useable for my analysis due to issues with the data collected. These issues primarily included the teacher forgetting to turn on the audio-recorder, the audio recorder malfunctioning, or the audio-recording being inaudible. Sometimes a teacher would misplace their ISA, so it would not get turned in. There was also evidence that a couple ISAs were written following the implemented problem-solving debrief and these data sets were consequently not used in my analysis since my study is looking at how teachers used the ISAs to implement their debriefs and if they completed the ISA after the debrief, then it may have been written as a report of what happened, not what was planned. Once I eliminated incomplete data sets for this study, I had a total of 25 problem-solving debriefs to analyze. These came from 12 teachers: nine from cohort 2 and three from cohort 3. There was no useable data from cohort 4. Three teachers had one complete data set (these were all from the three teachers in cohort 3), five teachers had two complete data sets, and four teachers had data sets for all three of the MPSM tasks.

There is a noteworthy discrepancy between how much data was useable from year 2 in comparison to years three and four. One reason why so little of the data was available from cohorts 3 and 4 was because the teachers were only required to implement two problem-solving debriefs for the MPSM tasks, making less data available to begin with. Also, in year 2, somebody from the research team was present for every problem-solving debrief, assisting the teachers with the audio-recorders and collecting the data from the teachers at the end of the debriefs, while in years 3 and 4 the teachers were responsible for recording their own classes and submitting the data themselves. Because the teachers were required to run the audio-recorders themselves, they were only given one (as opposed to each researcher always having two), which meant that if they had trouble with the recorder there was no backup. Also, it was easy for the teachers to forget to turn on the recorders in the midst of teaching and they sometimes ran into difficulties with downloading the audio recordings online. The research team has also speculated that the most motivated middle school teachers joined the MPSM program in the first couple years that it was offered and that, by the final year, the teachers participating in the program took the requirements of the professional development and research program less seriously and, consequently, did not implement all of their tasks as required.

Research Design

In this section, I will briefly describe the data analysis I did for each research sub-question with a particular focus on how, for each sub-question, I made use of the data to address a particular element of my theoretical framework. I conclude this section with a discussion of how these sub-questions worked together to address my primary research question. A more detailed description of the data analysis tools I used to answer each sub-question is given within their respective chapters.

Research Sub-question 1: Fidelity to the Literal Lesson. My first sub-question (Do teachers enact their written plans for problem-solving debriefs in the classroom as they had planned prior to implementation?) is intended to address the extent to which teachers are following the literal lesson. In the ISA, the teachers planned for certain pieces of student work to be shared in a particular order, planned to highlight specific ideas in the students' solutions, and wrote down specific questions to ask to help make the mathematics salient to the students[5]. For my study, I considered these three pieces of

[5] For an example of an ISA, see Appendix A

the ISA to represent the literal lesson because they represented a specific agenda that the teachers were supposed to follow as they implemented their problem-solving debriefs. In my analysis of the literal lesson in research sub-question 1, I further specify the steps of the literal lesson in an ISA as follows:

1. The teacher will provide an opportunity for a piece of student work to be shared with the class either by allowing the student to present his or her work, or the teacher, herself, showing the work to the class.
2. The teacher will ask the questions, or a close approximation of the questions, identified in the ISA. A question might alternatively be addressed if a student provides a "response" to the question without having to be prompted by the teacher.
3. The teacher scaffolds the sharing of the piece of student work to ensure that the mathematical ideas proposed by the teacher in the ISA surrounding the piece of student work emerge during the segment.
4. Repeat steps 1-3 for the next piece of student work.

The framework used for this study, as depicted in figure 2 in Chapter 1, is useful for making sense of the relationship between what teachers plan and what actually takes place during implementation of the lesson. When a teacher selects pieces of student work to be shared and plans questions to ask during that student's presentation (the literal lesson), the teacher has made some pedagogical decisions about what content to focus on and how to engage the students in thinking about the content (the intended lesson). Since my research question is focusing on the effect that the planned questions have on the enacted lesson, it is necessary to assess whether the teachers are simply asking those questions. That is, whether the teacher's literal lesson is enacted as stated in the ISA. If the answer to this question is yes, then it may be assumed that what was recorded in the literal lesson impacted what took place in the enacted lesson. If the teacher did not follow the literal lesson, then it cannot be assumed that the ISA impacted what took place during the problem-solving debrief.

Research Sub-Question 2: Fidelity to the Intended Lesson. I considered the types of questions the teachers planned to ask in their ISAs to be an indication of the level of mathematical reasoning they would be encouraging their students to engage in. A primary assumption underlying this study is that the types of questions teachers ask during discourse centered on mathematical problem solving will influence the opportunities students have to engage in discourse around mathematical problem solving. I used Kazemi and Stipek's conceptualization of high-press versus low-press (2001) to guide the development of a coding scheme to differentiate between questions that prompt students to engage in mathematical reasoning and those that do not. The discussion of the coding scheme may be found in the Chapter 6 and a complete description of the coding scheme may be found in Appendix C.

Sub-question 2 asks: "Is there a correlation between the number of questions teachers plan that promote mathematical reasoning around the problem-solving task and those that they actually ask during the whole-class discussion?" To answer this question, I analyzed the questions planned by the teacher, differentiating between high-press questions and low-press questions to identify the opportunities the teachers had planned for students to engage in mathematical reasoning and subsequently assess whether the teachers utilized these opportunities to engage their students in mathematical discourse during the enacted lesson. Further details on how I conducted this analysis and the results will be given in Chapter 6.

Research Sub-Question 3: Transformation of the Enacted Lesson. My third research sub-question, "How do teachers' improvisational moves during whole-class discussions influence the enactment of the questions that were planned by the teacher prior to implementation?" is intended to take a closer look at the teachers' discourse moves during their problem-solving debriefs; that is, the enacted lesson. While research sub-questions 1 and 2 both analyzed the enacted lesson through the lens of how teachers were implementing their literal lessons (the steps laid out in the ISAs) and the intended lessons (the level of discourse planned in the ISA), research sub-question 3 is included in this research study to take into account the many factors that can transform an enacted lesson such as teachers' beliefs and knowledge about mathematics, teachers' professional identity, and classroom structure and norms, among other contributing factors (Stein et al, 2007).

While I did not analyze these contributing factors, since that was outside of the scope of this research study, I analyzed the audio-recording and transcripts of the implemented problem-solving debriefs using grounded theory (Strauss & Corbin, 2007) with the understanding that what took place during the enacted lesson (the implemented problem-solving debrief) was only partially influenced by the literal and intended lessons (the ISAs) and that there are other, invisible factors, modifying what took place during the debriefs. My intention in this final stage of my analysis was to gain an understanding of how the teachers might be implementing the questions from their ISAs in ways that might modify the opportunities the teachers are creating for the students to reason about the mathematics in the tasks. I argue that these transformations of the questions have the potential to either diminish the opportunities for students to reason mathematically or afford the students more opportunities to reason mathematically.

The Primary Research Question: The Big Picture. The analyses for these three sub-questions is intended to support the analysis for my primary research question which is "How do teachers' written plans for orchestrating mathematical discourse around problem-solving tasks influence the opportunities teachers create for students to reason mathematically?" My research is focused on the relationship between what the teachers planned in their ISAs and how they implemented those plans, with a particular eye on teacher moves. By breaking this analysis into three sub-questions, I focused on different aspects of teacher implementation. That is, using my framework as a guideline, I focused on teachers' implementation of the literal lesson (the steps in the ISA) and the intended lesson (the level of mathematical reasoning the teacher planned for their students to be engaged in) as well as how those plans played out during the implemented lesson (taking into account that outside factors can transform what actually takes place during lesson implementation).

These three viewpoints on lesson implementation provided a picture of what happened as the teachers are implementing their ISAs. Research sub-questions 1 and 2 allow us to see whether or not faithful enactment of the literal lesson (i.e. the steps in the

ISA) ensured that the opportunities the teachers designed for students to reason mathematically through the planning of high-press questions led to an increase in the opportunities for students to reason mathematically in the enacted lesson. Alternatively, the analysis in research sub-question 3 investigates how on-the-fly decisions about implementation of an ISA can also impact the opportunities the teachers created for students to engage in mathematical discourse (either positively or negatively). By conducting both a quantitative analysis of the types of questions the teachers asked in relation to the types of questions they planned (research sub-question 2) and a qualitative analysis of how the teachers were implementing the questions they planned (research sub-question 3), this analysis offers a more robust picture of how teachers implemented their ISAs.

At the conclusion of my analysis of research sub-question 3, I include a discussion of the four teachers, focusing on how they implemented their ISAs overall. The intention of this discussion is to provide a synthesis of my research, focusing on how each aspect of my research contributes to an understanding of how these four teachers implemented their ISAs and how these variations in implementation impacted the opportunities these teachers created for their students to reason mathematically.

The next three chapters are a discussion of my analysis and results, starting with research sub-question 1.

Chapter 6. Research Sub-Question 1

The purpose of this chapter is to address research sub-question 1: Do teachers enact their written plans in the classroom as they had planned prior to implementation? This part of my data analysis is focused on the teachers' implementation of the literal lesson, as described in my theoretical framework in Chapter 1. To answer this question, I developed a data analysis tool called the Implementation Fidelity Analysis Tool (IFA). In this chapter, I will discuss the IFA by explaining how it is useful for addressing the teachers' fidelity to the literal lesson in their ISAs, describing the data analysis tool itself, including a description of the different levels of fidelity to the literal lesson, the criteria for which these levels were assigned, and the specific processes of data analysis that were used to assign these levels. Next, I will provide examples from the data to describe how the data analysis tool is used. This is followed by a discussion of the inter-rater reliability of the tool. Finally, I share the results of my analysis along with a discussion of the findings for my study and how these contribute to my main research question.

Method: Implementation Fidelity Analysis Tool

The Implementation Fidelity Analysis tool measures the extent to which the teachers followed their Instructional Sequence Analysis (ISA) during implementation of their problem-solving debriefs. The key elements of the ISA is the selection and sequencing of student work to be presented, the identification of the main ideas the teacher wishes to highlight within each chosen piece of student work, and questions planned to help make the mathematics salient within the debrief. See figure 11 for an example of a blank ISA and table 7 for an example of an ISA completed by one of the participating teachers (a larger example of a blank ISA is shown in Appendix A and additional examples of completed ISAs are given in Appendix F).

Instructional Sequence Analysis

Teacher Name ______________________ **Task Name**__________________________________ **Date** ____________

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so. *Be sure to indicate whether student work to be shared has misconceptions or mistakes that were common or important and need to be addressed to achieve your mathematical goals. Identify those pieces of student work that incorporate important mathematical concepts, ideas, or procedures that can help you meet your mathematical goals.* Identify the questions you will ask the student or the class during the discussion to expose and/or address any misconceptions or to illuminate the important ideas of the lesson.

Mathematical Learning Goal:____

Order of Sharing	Student Name	Ideas to Highlight	Questions to Make the Math Salient
1			
2			
3			
4			

Please add additional cells to the table above for additional samples of student work you will share in the lesson debrief.

Rationale: For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares.

____
____
____
____
____

Figure 11. Blank ISA

Fidelity to the Literal Lesson. With respect to my theoretical framework, this data analysis tool is intended to measure fidelity to the literal lesson, or the extent to which the enacted lesson resembles the literal lesson, where the literal lesson is the steps laid out in the written lesson plan (Brown, 2009). While every ISA completed by a participating teacher is going to be different with respect to what ideas will be highlighted and what questions the teacher plans to ask, the general format of the ISA assumes the teachers will follow certain steps as they implement each the ISA. The steps of the literal lesson are:

1. The teacher will provide an opportunity for the piece of student work to be shared with the class, either by allowing the student to present his or her work, or the teacher, herself, showing the work to the class.
2. The teacher will ask the questions (or an approximate variation of the questions) that they identified in the ISA. A question might alternatively be considered addressed if a student provides a "response" to the question before the teacher asks it.
3. The teacher scaffolds the sharing of the piece of student work to ensure that the mathematical ideas proposed by the teacher in the ISA surrounding the piece of student work emerge during the segment.
4. Repeat steps 1-3 for the next piece of student work.

The Implementation Fidelity Analysis identifies the extent to which the teachers followed these steps as they implemented their ISAs.

Fidelity to the literal lesson is measured for each piece of student work identified by the teacher in the ISA. According to the format of the ISAs, the planned debrief is broken up by segments in which one segment is characterized by a selected piece of student work to be demonstrated accompanied by an identified Idea to Highlight and some questions the teacher plans to ask. Within the enacted debrief, a segment refers to the time spent during the debrief focused on the selected piece of student work previously identified in the ISA. A segment begins when a piece of student work is brought forth to be shared (either by the student or the teacher) and a segment ends when another piece of student work is brought forward to be shared or the focus of the discussion is no longer related to that student's work. These segments are delineated by time stamps from the audio-recording and breaking points from one segment to the next are also labeled in the transcripts. Breaking down the whole-class discussions by these segments is reasonable as the teachers were requested to structure their problem-solving debriefs around the sequential demonstration of student work (Stein, Smith, et al., 2009).

Levels of fidelity. For each segment, one of three levels of fidelity to the literal lesson is assigned, with two of the three levels broken down into two sub-levels. Fidelity level 1 is considered to be faithful implementation to the literal lesson. During a faithful implementation of a segment of a debrief, the teacher clearly followed the ISA, sharing the selected pieces of student work, addressing the questions identified in the ISA, and addressing the mathematical ideas proposed by the teacher in the ISA. Levels 2a and 2b are considered to be a partially faithful implementation of the literal lesson. If a segment of a debrief is partially faithful then there is evidence that the teacher was attempting to follow the ISA, but either the questions were not addressed but the Idea to Highlight was (level 2a), or the proposed Idea to Highlight did not emerge even though the questions were asked (level 2b). Finally, levels 3a and 3b are considered to be non-faithful implementation of the literal lesson. Either the student work was never presented (level 3b) or the student did present, but the planned questions were never addressed and the Idea to Highlight did not emerge (level 3a). While these three levels of fidelity point to a hierarchy of fidelity to the literal lesson in which the levels range from more to less faithful implementation, level 2a is not necessarily intended to indicate a more faithful implementation of the ISA than level 2b. In both cases, a certain element of the ISA was not being addressed during the segment of the problem-solving debrief.

In addition to these levels of fidelity, a participation code of + or – will be assigned to the segments indicating evidence of participation from the class. A code of + is given to a segment in which students other than the presenting student spoke up during the segment. A code of – is given to segments in which only the teacher and/or the presenting student spoke. This code serves the purpose of distinguishing between discussions that are limited to the teacher and presenting student talking about the student's work and those in which there is evidence of participation from other members of the classroom. In the case where a segment is assigned a level 3b, a participation code is not assigned because the segment is essentially missing from the debrief.

Assigning a Level of Fidelity. In order to identify the level of fidelity, the audio recordings of the debriefs were transcribed and broken up into segments as described above. A level of fidelity was assigned to each segment by answering a series of yes or no criteria questions with respect to the proposed mathematical ideas and the planned questions as identified by the teacher in the ISA. The questions are:

1. Was the identified student's work shown?
2. Were the planned questions addressed by a student (either as a result of the teacher asking the question, or unprompted)?
3. Were the important ideas surrounding this piece of student work, as proposed by the teacher in the ISA, evident in the segment?
4. Did at least one other student, other than the presenting student, contribute to the discussion?

These questions are used to assign one of the levels of fidelity to each segment. See table 5 for a summarized description of these levels and the criteria used to assign levels of fidelity.

Table 5. Implementation Fidelity Analysis Tool: Four Levels of Fidelity

Level	Description	Criteria
1	The student work was shared. The questions were asked and the proposed mathematical ideas were addressed during the debrief.	Questions 1-3 are answered yes
2a	The student work was shared. The questions were not addressed, but the proposed mathematical ideas emerged as planned.	Questions 1 and 3 are yes, but Question 2 is no
2b	The student work was shared. The questions were addressed by the students, but the proposed mathematical ideas were either not addressed or inadequately addressed.	Questions 1 and 2 are yes, but Question 3 is no.
3a	The student work was shared, but the proposed mathematical ideas were not addressed and the questions planned by the teacher were never asked.	Question 1 is yes, but Questions 2 and 3 are both no.
3b	The student's work was never shared	Question 1 is no.
Participation Code: In addition to the assignment of one of the four levels described above, each segment will be assigned a participation code.		
+	At least one other student other than the presenting student made a verbal contribution to the discussion.	Question 4 is answered yes.
-	Only the student whose work was being presented spoke during the segment	Question 4 is answered no.

Data analysis procedures. To assign the levels of fidelity to each segment within a debrief, I created a Microsoft Word document for every debrief. In that document, a template was created for each segment in which I kept track of my responses to the four criteria questions along with an explanation for my responses and relevant time stamps. (See table 6 for a blank version of the implementation fidelity template and see table 10 for a completed example). I completed these tables by listening to the audio recording and simultaneously reading the completed transcripts. As I listened, I was looking for evidence of the planned questions and ideas to highlight being addressed. In the template, I recorded the necessary yes/no responded to the criteria questions, the time stamps from the transcript for the times in which these episodes took place, and a brief description of how these events were taking place in the debrief. (Note that, while the time stamps are not necessary for the assignment of a level of fidelity, they are recorded for future reference when answering research sub-question 3). In the case of criteria question 2 (Were any of the planned questions addressed?), I answered the question (yes/no) for each planned question and, if at least half of the planned questions were addressed during the segment, then an overall response of yes was assigned for that segment. Once all of the criteria questions were answered for a segment, a level of fidelity was assigned to that segment based upon the responses to the criteria questions (see table 5).

Table 6. Template Used for Analysis of Research Sub-Question 1

Task: ____________________________	Segment #:	
Teacher's plan for the instructional segment as described in the ISA including questions.		
Idea to Highlight:		
Planned question(s)		
	Answer and explanation	**Times**
1. Was the identified student's work shown?		
2. Were any of the planned questions addressed?		
3. Did the mathematical idea as proposed by the teacher become evident in the discussion? At some point was it the focus of the discussion?		
4. Did at least one other student, other than the presenting student contribute to the discussion?		
Level of Fidelity		

Once a level of fidelity was assigned to all segments in all debriefs, the data were consolidated into an excel spreadsheet with a column created for each task implemented by each teacher (grouped first by teacher, then by task). In that column, the levels of fidelity were listed for every segment in the order in which they had been planned. As a result, each implemented task was assigned from 2 to 6 levels of fidelity assigned, depending upon how many segments had been planned. See Appendix B for the complete results of my analysis. In the next section, I will provide two examples of how I used the IFA to assign levels of fidelity to the segments implemented from an ISA.

Table 7. Teacher 3's ISA for Spinner Elimination

Student	Ideas to Highlight	Questions to make the math salient
1. Jake	Change of numbers on spinner used bigger values.	"Your advice to choose new numbers was to "spread them out". Does your work show this? Explain your thinking."

2. Jeff	**-Uncover the issue with using numbers like 7 and 9.**	**What did you discover in your experiment with using spinner number 2 numbers?**
3. Tara	-Only 2s lead to even number choices	What did you discover happens when using all similar numbers?
4. Jenny	-Using numbers higher than 4 more likely to lose a turn.	Relay your conclusion about using numbers less than 4. What did two ones do to your products?
5. Brian	**-Students should recognize that spinner numbers are low prime numbers.**	**Can you think of a number to replace the 4 that would work just as well?**

Rationale: My choices show a slight development in thinking about using small prime numbers for the spinner. I used some samples (initially) that would not prove overly successful 7s, 9s, only 2s. I ended with 2 sample papers that came the closest to using all prime factors.
Mathematical Learning Goal: "Students will recognize prime numbers (or their multiples) connect to all numbers."

Examples of the Implementation Fidelity Analysis

In this section, I discuss two examples of the Implementation Fidelity Analysis applied to two segments from the same debrief[6]. Table 7, above, shows the ISA that Teacher 3 completed following implementation of the Spinner Elimination task and prior to implementation of the Spinner Elimination debrief. The segments that I will be discussing in further detail are in bold print. For a complete description of the Spinner Elimination task, please refer to Chapter 4, Task Analysis. Below is a discussion of the IFA for segments 2 (student Jeff) and 5 (student Brian). Along with an explanation for how the levels of fidelity were assigned to these two segments, I also include the transcripts of these segments (tables 9 and 12) and the document used to assign the appropriate level of fidelity to these segments (tables 10 and 13).

Discussion of Segments 2 and 3. In segment 2 (Jeff's work), Teacher 3 chose to focus on the difficulties that can arise from selecting large numbers to go on the spinner.

[6] The names of the students have been changed. The teachers have been given numbers as identifiers; this allowed me to either identify a teacher as, for example, Teacher 3 when referencing the teacher in text and as T3 when identifying a teacher in a table or a graph where space was limited.

In particular, if a large number is spun more than once in a single turn, then the player will get a product that is greater than 50, causing him or her to lose a turn. For example, if the 9 is spun twice in a row, the product is 81 and the player cannot cross the number off of the 50s chart and he or she will lose a turn. In segment 5 (Brian's work), the teacher planned to lead students to recognize that Brian was using not just low numbers, but low prime numbers. Teacher 3 planned on having the class recognize that if they had at least one 2 on their spinner, then it was not absolutely necessary to have a 4 on the spinner because multiples of 4 will come up when a 2 is spun twice in one turn.

Example 1: Segment 2. In order to demonstrate how the IFA tool is used to assign a level of fidelity, I show how a level of fidelity was assigned to segment 2 of Teacher 3's Spinner Elimination debrief. Table 8 shows an excerpt from Teacher 3's Spinner Elimination ISA, identifying what she had planned for this portion of her debrief, including an excerpt from her rationale that was relevant to segment 2. Immediately following that, in table 9, I show the transcript of the portion of the implemented debrief in which segment 2 was being addressed. In the following paragraph, I discuss my rationale for how I assigned a level of fidelity to this segment.

Table 8. Excerpt 1 from Teacher 3's Spinner Elimination ISA

Order of sharing	Student name	Ideas to Highlight	Questions to make the math salient
2	Jeff	Uncover the issue with #s like 7 and 9.	"What did you discover in your experiment with using spinner #2 #s?"

Rationale: I used some samples (Initially) that would not prove overly successful 7s, 9s.

Table 9. Teacher 3 Spinner Elimination Segment 2 Transcript

Teacher3	All right. I would like Jeff. Would you bring yours up, please?
Jeff	For the first one, I did just as an experiment 1, 2, 3, 4, 5, 6, 7, 8.
Teacher3	And how did your experiment go?
Jeff	That was okay, but I think I started out and once I got a low number like 8 or

	something I spun again and went over, so that was my problem, not the spinner's problem. And, yeah, that one worked out pretty well, but then for the second one I tried to switch it up as an experiment and I did 1, 1, 2, 2, 7, 7, 9 and 9 and I spread them out more on the spinner, so there's like 1 and 1, 9 and 9, 2 and 2, and 7 and 7.
Teacher3	How did you choose doubles, you know, 1, 1, 2, 2. How did you pick 7 and 9? How did you pick those numbers?
Jeff	Um, I wanted to do the extreme high and the extreme low and then… 2, I like 2 for some reason so I just did that and then 7 was 9 minus 2, I guess, and it was kind of in the middle between 2 and 9, so I picked that and that one didn't work so well because I just went over every time like Jake said, so I said have at least 6 different numbers and keep those numbers below 8.
Teacher3	So that was your final conclusion there, have at least 6 different numbers. By that you mean 1, 2, 3, 4, 5, 6. Are those different?
Jeff	Yeah. Those are all different numbers. So don't have only 4 different numbers.
Teacher3	And keep them below 8. So you would still have 7, and 6, and…
Jeff	Yeah.
Teacher3	Okay. Any questions for Jeff?
Teacher3	Thank you.

In segment 2 of the ISA, the teacher planned to ask "What did you discover in your experiment with using spinner #2 numbers?" In the implemented debrief, the teacher does not directly ask this question (she did ask a similar question with respect to the student's first spinner when she asks him "And how did your experiment go?" but not the student's second spinner as originally planned). However, in this debrief, the planned question is considered to be addressed because the student stated, with respect to his second spinner, "that one didn't work so well because I just went over every time like Jake said", essentially reporting that he discovered that the larger numbers on his second spinner were causing him to go over 50. This statement about his spinner also served to address the teacher's proposed mathematical idea that the larger numbers on a spinner were problematic. The student went on to say that, in response to this dilemma, he concluded that his next spinner would only have numbers less than 8. Since the question was address and the proposed Idea to Highlight came up during the discussion, this segment was assigned a level 1. Also, because Jeff was the only student speaking during this entire segment, a – code was assigned for student participation. Below, in table 10, is the completed Implementation Fidelity Analysis for segment 2.

Table 10. Implementation Fidelity Analysis for Segment 2 of Teacher 3's Spinner Elimination debrief

Teacher's plan for the instructional segment as described in the ISA including questions.	Jeff – Uncover the issue with using #'s like 7 & 9. "What did you discover in your 'experiment with using spinner #2 #'s'?"	
Idea to Highlight:	Uncover the issue with using #'s like 7 and 9	
Planned question(s)	What did you discover in your experiment with using spinner number 2's numbers?	
	Answer and explanation	**Times**
1. Was the identified student's work shown?	yes	6:15-8:26
2. Were any of the planned questions addressed? Identify the times in which the questioning occurred.	Yes Jeff said "that didn't work so well b/c I just went over every time like Jake said."	7:19
3. Did the mathematical idea as proposed by the teacher become evident in the discussion? At some point was it the focus of the discussion?	Yes Jeff identified that the large numbers were causing him to go over 50. His conclusion was to keep the numbers below 8.	7:19
4. Did at least one other student, other than the presenting student contribute to the discussion?	No	
Level of Fidelity	1-	

Example 2: Segment 3. My second example is the fifth and last segment of Teacher 3's debrief for Spinner Elimination. I selected this segment of this debrief as an example because it is in this segment that the teacher finally addresses the goal of her lesson which was: "Students will recognize prime numbers (or their multiples) connect to all numbers", as stated in her Ideas to Highlight section of segment 5 (see Table 11). She planned to ask the question of "Can you think of a number to replace the 4 that would

work just as well?" This question could potentially allow her students to recognize that having 2's on a spinner would potentially allow the player to cross off all multiples of 4 on the 50's chart by spinning the 2 at least twice in one turn. See table 11 for the excerpt of segment 5 from Teacher 3's Spinner Elimination ISA and see table 12 for the transcript of the implementation of segment 5. Below I discuss the assignment of fidelity for this segment.

Table 11. Excerpt 2 from Teacher 3's Spinner Elimination ISA (Segment 5)

Order of sharing	Student name	Ideas to Highlight	Questions to make the math salient
5	Brian	-Students should recognize that spinner numbers are <u>low</u> prime numbers.	Can you think of a number to replace the 4 that would work just as well?

Rationale: <u>I ended with 2 sample papers that came the closest to using all prime factors.</u>

Table 12. Teacher 3 Spinner Elimination Segment 5 Transcript

Teacher 3:	And Brian. Would you mind? Thank you. Talking about you and your partner's thinking.
Brian:	Me and my partner, Jordan, did different things. But, um, I mostly chose on the lower side of numbers. 1, 5, 2, 2, 3, 5, 4, 3.
Teacher 3:	Why?
Brian:	The one was like a safety net in case I spun again because I wanted to. It's like a safety net. I thought it might help me. Um, 5s I figured that if I get anything times 5 I've got a pretty good range. I've got 20 numbers I could have with 5s. [*inaudible*]. And 2, is just a good number. It's got 25 multiples in 50 and then, um, 3 and 4 are just lower numbers and they've got a lot of multiples and so that worked.
Teacher 3:	I noticed that with your first guess you almost followed Jenny's recommendation of picking all the numbers less than six with two repeats.
Brian:	I did pretty good on that turn. I got 8 only. That's pretty good. And possibly, I had a pretty good score but I think I would switch a 5 with a 4 and it might make my score a bit better because a 5 I ended up busting. That's the only time I busted with the 5, so I went 15223243 instead of, I didn't do that.
Teacher 3:	So you changed your five to a two. Looks like. Is that the only change you made? Any just random low numbers? Could you leave your paper up there for a second. Class, if you were looking at Brian's spinner 2 selections, most of those numbers on spinner 2 are what type of numbers?
Stud.	prime

Teacher 3:	Why?…. Why, Jordan?
Jordan:	Because if you have like a lot of prime numbers it won't usually like go over. Because if you multiply a lot with something like 6 or 7 it could go over. But if you use a lot of prime numbers you could get a different variety on the chart.
Teacher 3:	Of what type of number?
Jordan:	Prime.
Teacher 3:	But get a different variety on the chart of what type of number?
Jason	Composite.
Teacher 3:	Yeah. Because composite numbers are all products of prime numbers. And possibly, Brian, if you could change one number on spinner number two for your third assessment what would they be?
Brian:	I'd probably change a 1 to a 2.
Teacher 3:	Why?
Brian	I'd change it because the 1 really didn't help me at all. It didn't give me any points. I got maybe 1 point off of it. It's not really worth anything.
Teacher 3:	Right. Because you could just stop and get the same answer as if you spun a one.

In this second example, the teacher had planned to ask "Can you think of a number to replace the 4 that would work just as well?" but nowhere in this segment did anyone discuss the possibility of replacing a 4 with a different number. An appropriate response to this question would have been to replace the 4 with a 2 because, if the spinner landed on the 2 twice, it would be the same as spinning a 4 once. This idea was never brought up during the segment. However, the proposed mathematical idea, that she wanted her students to recognize that the presenting student's spinner contained low prime numbers, did come up during the debrief. The teacher directed the class's attention to the fact that his spinner contained mostly prime numbers when she asked "most of those numbers on spinner two are what type of numbers?" A student in the class responded that they were mostly prime numbers and, with the teacher's prompting, explained that having mostly primes was beneficial because it could potentially give you a wider variety of composite numbers on the 50's chart. The teacher closed this discussion by stating that “composite numbers are all products of prime numbers.” Analysis of this segment, as shown in table 13, resulted in the assignment of level 2a+ to the segment because, although the planned question was never addressed, the proposed mathematical idea did emerge. The + was assigned because an additional student shared during the discussion. Below is the Implementation Fidelity Analysis completed for this segment.

Table 13. Implementation Fidelity Analysis for Segment 5 of Teacher 3’s Spinner Elimination debrief

Teacher’s plan for the instructional segment as described in the ISA including questions.	Brian – Students should recognize that spinner numbers are low prime #’s. “Can you think of a # to replace the 4 that would work just as well?”	
Idea to Highlight:	Students should recognize that spinner numbers are low prime #’s.	
Planned Question:	Can you think of a number to replace the 4 that would work just as well?	
	Answer and explanation	**Times**
1. Was the identified student’s work shown?	Yes	13:55-18:08
2. Were any of the planned questions addressed?	No. Nowhere in the segment did anyone discuss replacing a 4 with another number.	
3. Did the mathematical idea as proposed by the teacher become evident in the discussion? At some point was it the focus of the discussion?	Yes. Teacher 3 focused the class’s attention on the fact that most of Brian’s numbers on his second spinner were low prime numbers. With a little prompting, a student in the class stated that this would lead to a variety of composite numbers on the 50’s Chart.	16:00-16:48
4. Did at least one other student, other than the presenting student contribute to the discussion?	Yes	16:14-16:48
Level of convergence	2a+	

Discussion of the Two Examples. It is important to be clear that the purpose of the Implementation Fidelity Analysis is not to analyze the mathematical nature of the problem-solving debrief. Rather, the tool is simply intended to determine the extent to which the discussion was implemented as intended by the teacher. What we learn from this tool is that, in the first example, the segment was well aligned with the ISA as the planned question was addressed by the student and the proposed idea to highlight was evident. In the second example, although the teacher was successful in focusing the discussion on the proposed idea to highlight, this was not done with the help of the planned question. It is clear here that the teacher stuck to her plan of focusing the class's attention on the fact that the student was using low prime numbers on his spinner, but this was not done with the help of the question the teacher planned.

What we may observe from these two examples is that the teacher is not consistently using the planned questions to focus the discussion on the planned ideas to highlight. Particularly in the second example, Teacher 3 disregarded her planned question and chose a different approach for bringing out the planned idea to highlight. This raises questions about the relationship between the nature of the teacher's plans and the outcomes of the problem-solving debrief. For example, are segments more likely to be faithfully implemented if there is a clear relationship between the planned questions and the planned ideas to highlight? Also, does the way that a question from the ISA is addressed affect the opportunities for the students to think and reason about the question?

Notice that, although a lower level of fidelity was assigned in the second example, this does not necessarily reflect the strength or weakness of the segment with respect to the mathematical content of the discussion within the segments. The first segment may be considered weak with respect to its mathematical content because it was only useful for drawing attention to the idea that larger numbers on the spinner are more likely to cause the player to go over fifty. In contrast, mathematical concepts were more evident in the second example because, in the second segment, the class's attention was drawn to the fact that the student's spinner contained mostly prime numbers and that the prime numbers were useful for creating a wide variety of composite numbers on the chart. While this tool is helpful for demonstrating the relationship between the ISA and the implemented debrief, more analysis is required to understand the quality and nature of the discourse. This further analysis is reserved for research sub-questions 2 and 3.

Inter-Rater Reliability of the Implementation Fidelity Analysis Tool

After I developed the Implementation Fidelity Analysis tool, I collaborated with another member of the MPSM research team to assess the inter-rater reliability of the tool. After collaborating together on the analysis of a couple of debriefs, allowing me to share the Implementation Fidelity Analysis tool and to make sure that the other coder understood how to respond to the questions designed to lead to the designation of a level of fidelity, we each completed an analysis of three debriefs from three different teachers from cohort 2. Between the three teachers, there was a total of 14 segments, giving us a segment population of n=14. Because we both agreed that one of the 14 segments was never implemented (giving that segment a fidelity level of 3b, the majority of the results shown below were for a population of n=13). Below, I share the percentage of agreement for each of the four questions from the Implementation Fidelity Analysis tool and then discuss the overall outcome that our responses to these questions had on the assignment of the levels of fidelity.

Question 1: Was the identified student's work shown? Of the 14 segments identified in the ISAs by the three teachers, we demonstrated 100% agreement. Of the 14 segments identified in the teachers' lesson plans, all but one of the students' work were shared. Because no discourse took place around that student's work, the remaining questions will be analyzed with a population of 13 segments. See table 14 for a summary of our analysis for question 1.

Table 14. Inter-rater reliability comparison for question 1 of IFA

(A1 = my responses, B1 = colleague's responses, 1 = yes, 0 = no)				
TeacherID	Segment	A1	B1	Agreement?
23	1	1	1	1
23	2	0	0	1
23	4[7]	1	1	1
31	1	1	1	1
31	2	1	1	1
31	3	1	1	1
31	4	1	1	1
31	5	1	1	1
31	6	1	1	1
3	1	1	1	1
3	2	1	1	1
3	3	1	1	1
3	4	1	1	1
3	5	1	1	1
				%agreement:100%

Question 2: Were any of the planned questions addressed by a student (either as a result of the teacher asking the question, or unprompted)? We agreed on 11 out of 13 of the segments, yielding 84.6% consistency (see table 15). In both cases, the other researcher thought that the planned questions were addressed by the student, while I did not.

[7] Data skips from segment 2 to segment 4 because the data for segment 3 of Teacher 23's debrief is missing.

Table 15. Inter-rater reliability comparison for question 2 of IFA

(A2 = my responses, B2 = colleague's responses, 1 = yes, 0 = no)				
TeacherID	Segment	A2	B2	Agreement?
23	1	1	1	1
23	2	NA	NA	
23	4	0	1	0
31	1	1	1	1
31	2	1	1	1
31	3	1	1	1
31	4	1	1	1
31	5	1	1	1
31	6	1	1	1
3	1	1	1	1
3	2	1	1	1
3	3	1	1	1
3	4	1	1	1
3	5	0	1	0
				%Agreement:84.6%

Question 3: Was the mathematical idea as proposed by the teacher evident in the segment? We agreed upon 12 out of 13 of the segments, yielding 92.3% reliability. In the instance that we did not agree, the other researcher thought that the mathematical idea as proposed by the teacher was not evident, while I did (see table 16).

Table 16. Inter-rater reliability comparison for question 3 of IFA

(A3 = my responses, B3 = colleague's responses, 1 = yes, 0 = no)				
TeacherID	Segment	A3	B3	Agreement?
23	1	1	1	1
23	2	NA	NA	
23	4	1	1	1
31	1	1	1	1
31	2	0	0	1
31	3	1	1	1
31	4	1	1	1
31	5	1	1	1
31	6	1	1	1
3	1	1	1	1
3	2	1	1	1
3	3	1	1	1
3	4	1	0	0
3	5	1	1	1
				%Agreement:92.3%

Levels of fidelity. The levels of fidelity that are assigned to the segments in a debrief are completely dependent upon the responses given to the three questions (see Table 5 above for the criteria used to assign these questions). Recall that the levels of fidelity are ranked as faithful implementation (1), partially faithful implementation (2a/2b), and non-faithful implementation (3a/3b). Within these parameters and based objectively on the responses to criteria questions 1-4, we agreed on 11 out of the 14 segments, showing a 78.6 % rate of consistency (see table 17). Because our inconsistent results from criteria questions 2 and 3 were never from the same segment, the overall fidelity is less consistent than the results for each individual criteria question.

Table 17. Inter-rater reliability comparison for assignment of fidelity

(AL = level of fidelity assigned based upon me, BL = level of fidelity assigned by colleague)										
TeacherID	Segment	A1	B1	A2	BT2	A3	B3	AL*	BL*	Agreement?
23	1	1	1	1	1	1	1	1	1	1
23	2	0	0	NA	NA	NA	NA	3b	3b	1
23	4	1	1	0	1	1	1	2a	1	0
31	1	1	1	1	1	1	1	1	1	1
31	2	1	1	1	1	0	0	2b	2b	1
31	3	1	1	1	1	1	1	1	1	1
31	4	1	1	1	1	1	1	1	1	1
31	5	1	1	1	1	1	1	1	1	1
31	6	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
3	2	1	1	1	1	1	1	1	1	1
3	3	1	1	1	1	1	1	1	1	1
3	4	1	1	1	1	1	0	1	2b	0
3	5	1	1	0	1	1	1	2b	1	0
										%agreement:78.6%

Question 4: Did at least one other student, other than the presenting student contribute to the discussion? The last question in the Implementation Fidelity Analysis does not affect the levels of fidelity shown above. Rather, this is a separate analysis that

simply rates the level of participation of the class during the segment. Of the 13 segments, we agreed upon 11 out of 13 of the segments, yielding 84.6% consistency (see table 18). In both cases of disagreement, I said that there were contributions from other students and the other researcher did not. Part of the reason for the inconsistencies here is because I originally tried to require that the contributions from the students were of some value to the discussion. This meant that when students made minor contributions to the discussion, such as answering a short-response question, this was not necessarily counted. Because this was producing inconsistent results, I later changed this criteria to simply require that students were contributing to the discussions, whether the contributions were significant or not.

Table 18. Inter-rater reliability comparison for question 4 of IFA

(S4 = my responses, T4 = Colleague's responses, 1 = yes, 0 = no)				
TeacherID	Student	S4	T4	Agreement?
23	1	1	0	0
23	2	NA	NA	NA
23	4	1	0	0
31	1	1	1	1
31	2	0	0	1
31	3	1	1	1
31	4	1	1	1
31	5	0	0	1
31	6	1	1	1
3	1	0	0	1
3	2	0	0	1
3	3	0	0	1
3	4	0	0	1
3	5	1	1	1
				%Agreement:84.6%

Based upon the results from the inter-rater reliability analysis, it may be concluded that the Implementation Fidelity Analysis yields reasonably consistent results.

Results

Twenty-five debriefs were planned and implemented by twelve teachers for this analysis. Within these twenty-five debriefs, a total of 107 segments were planned. Grouping all 107 segments together, disregarding which teacher or task the segments belonged to, 67 of the 107 (about 63%) were implemented faithfully (level 1), in which the student's work was presented, the planned questions were addressed, and the identified ideas to highlight was discussed. 15 of the 107 segments (or about 14%) were not implemented faithfully either because the student's work was not presented (level 3a) or because the teacher presented the student's work without addressing either the ideas to highlight or the planned questions (level 3b). This left 25 of the 107 segments (about 23%) that were implemented partially faithfully, with the student's work being addressed and either the ideas to highlight being addressed or the planned questions being addressed, but not both. See table 19 and figure 12 for a summary of this data. The complete set of results may be found in Appendix B.

Table 19. Summary of Results from Implementation Fidelity Analysis Tool

Level of Fidelity	#segments (%)
Level 1	67 (62.62%)
Level 2	25 (23.36%)
Level 2a	23 (21.495%)
Level 2b	2 (1.869%)
Level 3	15 (14.02%)
Level 3a	7 (6.54%)
Level 3b	8 (7.48%)
Total	107

Interestingly, 23 of the 25 segments that were implemented partially faithfully were because the teacher never addressed the planned questions but still addressed the ideas to highlight. This may be because the teachers tended to plan their questions such that if the question was addressed, then the Ideas to Highlight would consequentially be highlighted, which would result in level 1 fidelity.

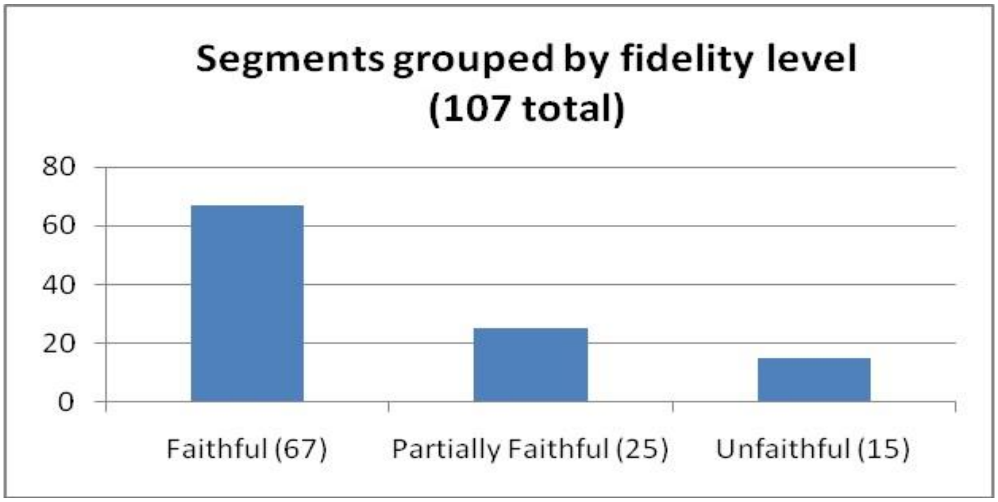


Figure 12. Distribution of segments by levels of fidelity

It is important for this overall research study to know that the teachers were using their ISAs in their classrooms. Since partially faithful implementation shows that the teacher used at least part of the ISA in their debrief (either addressing the ideas to highlight or asking the planned questions), which allows us to consider both faithful and partially faithful segments as being implemented by the teacher with some attention to the ISA. We see that 92 of the 107 segments (or about 86%) were implemented as described in the ISA. Also, 90 of the 107 segments (about 84%) were implemented faithfully with respect to the mathematics identified in the Ideas to Highlight. This suggests that, overall, the teachers were following the ISAs as they were implementing their debriefs. One possible explanation for these high levels of consistency is that the teachers knew that they were expected by the research team to follow the ISA during the debriefs, as both the ISA and the audio-recording of the debrief would be collected. It would also be hoped, though, that the act of writing up an ISA would be helpful for developing in the teacher's mind what he or she wishes to address with the students during the debrief and that they would, in turn, find it useful and natural to follow along with ISA to keeps those important ideas in focus.

It is also possible that so many of the partially faithful segments were due to the teachers addressing their identified ideas to highlight without asking the planned questions (23 out of 25) was because it was easier for the teachers to remember what the big ideas were in the ISAs, but may have had a harder time remembering what the questions were that they were going to ask. If they were running their debriefs without holding the ISA directly in front of them, they may have failed to follow the ISA exactly as planned simply because they couldn't remember all of the details.

When looking at the data grouped by debrief, we see further evidence that the teachers were committed to using the ISAs. Findings showed that all of the teachers were using the ISA to some extent during their problem-solving debriefs. Specifically, of the 25 debriefs implemented by the 12 teachers, only eight of those 25 debriefs had been identified as having any non-faithfully implemented segments in them (levels 3a or 3b). Looking at the same data by teacher, these eight debriefs containing non-faithful segments were implemented by the same four teachers. Also, of the 15 segments that were implemented non-faithfully, 11 were implemented by the same two teachers. While these two teachers had a strongly tendency towards non-faithfully implementing their ISAs as planned (implementing two to three segments non-faithfully for each ISA), the other two teachers only ever had one non-faithfully implemented segment per debrief. Interestingly, the teachers with a high tendency towards non-faithfully implementing a debrief worked at the same school, although they both reported never having time to work together. It is possible, though, that norms or policies within the school encouraged teachers to adapt instruction.

Regardless of the inclusion of segments that were not faithfully implemented, even those debriefs showed some evidence of faithful implementation as all eight of the debriefs with at least one non-faithful segment also had at least one other segment that was identified as faithfully implemented. This suggests that, while these teachers were neglecting some parts of their ISAs, they were attentive to other parts of the tool. Generally speaking, 24 of the 25 total debriefs had at least one level 1 segment. The one debrief that was lacking a level 1 segment contained only two segments, both of which were assigned level 2a, suggesting that the teacher was following the big ideas of the ISA, but was neglecting the planned questions.

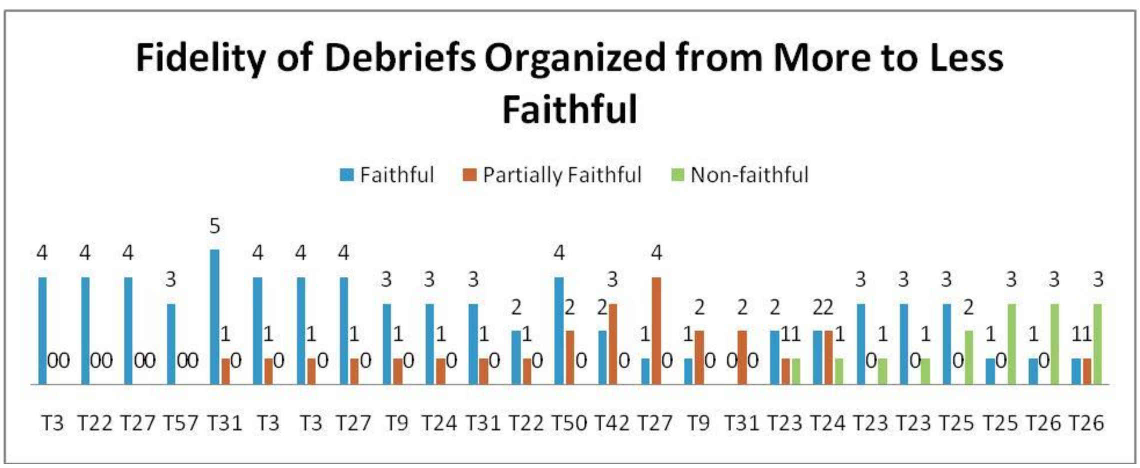


Figure 13. Distribution of fidelity of segments by debrief (organized from more to less faithfully implemented)

Figure 13 shows the distribution of faithful, partially faithful, and non-faithful segments within each debrief. The debriefs in this table are roughly organized from more to less faithful to demonstrate the overall trend for teachers to follow the ISA they created for their debriefs. Note that four of the debriefs were implemented completely faithfully, eight of the debriefs were implemented faithfully, with the exception of one partially faithful segment, and two debriefs were implemented faithfully with the exception of one non-faithful segment (both Teacher 23). This shows 15 of the 25 debriefs implemented with a strong tendency towards faithful implementation.

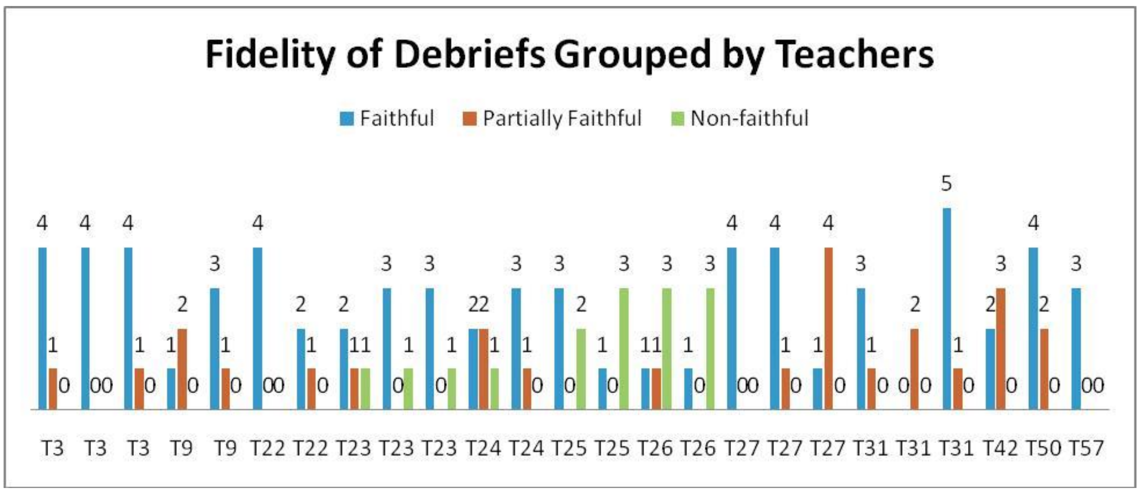


Figure 14. Distribution of fidelity of segments by debrief (grouped by teacher)

Figure 14 also shows the distribution of faithful, partially faithful and non-faithful segments within each debrief, but the segments have been grouped by teachers. This table allows us to see the individual tendencies of the teachers to be more or less faithful when implementing their problem-solving debriefs. Notice, in particular, teacher 3's tendency to faithfully implement her debriefs with two of her debriefs containing four out of five segments faithfully implemented (80%) and a third debrief faithfully implemented with all four segments (100%). In contrast, teachers 25 and 26 have an unusually strong tendency to not faithfully implement their debriefs. Out of teacher 25's two implemented debriefs, she implemented two out of five (40%) and three out of four (75%) of her planned segments *non*-faithfully. Similarly, teacher 26 implemented three out of six (50%) and three out of five (60%) of his debriefs non-faithfully. These findings for teachers 25 and 26 particularly stand out because they were the only teachers that implemented more than one non-faithful segment in a single debrief and they did it for all of their debriefs. It is interesting to note that teachers 25 and 26 are both from the same school, although during the time that this data was collected, they had both mentioned that they rarely had time to collaborate.

While teachers 3, 25, and 26 all showed consistencies with the fidelity or lack of fidelity with which they implemented their debriefs, some teachers showed some surprising difference in how they faithfully implemented their debriefs. Teachers 27 and 31 each implemented two debriefs that were, for the most part, faithfully implemented (75%-100% of their segments were faithfully implemented), but they each had a third debrief that was not implemented faithfully (20% and 0% faithful, respectively). This raises the question of what made those debriefs that were less faithfully implemented different from the faithfully implemented ones. In particular, did the nature of the plans they wrote in their ISAs have an impact on the level of fidelity with which they implemented those debriefs? I look more closely at teacher 27's debrief and discuss further the discrepancies between his ISA and the enacted debrief when I address research sub-question 3.

Participation codes. Along with the levels of fidelity, each segment was also assigned a participation code. Each segment was assigned a code of '+' or '–' based upon whether or not there was evidence of participation from the rest of the class (that is, other than the presenting student). A code of '+' (referred to below as a participation code) was assigned if a student other than the presenting student made a contribution to the segment. A code of '–' (referred to below as a non-participation code) was assigned if no student other than the presenting student made a contribution to the discussion. This code was not assigned to segments that were coded as 3b because, if the student work was never presented, then there was no discussion around which students might participate. As a result, the participation/non-participation codes were assigned to a total of 99 segments. See a breakdown of these participation codes in Table 20.

Table 20. Breakdown of Participation Codes by Level of Fidelity

	Faithful	Partially Faithful	Non-Faithful	Total
Participation	45 (67%)	14 (56%)	4 (57%)	63(63.6%)
Non-Participation	22 (33%)	11 (44%)	3 (43%)	36 (36.4%)
Total	67	25	7	99

Out of a total of 99 segments, 63 segments were given a participation code (63.6%) and 36 segments were given a non-participation code (36.4%). The percentage distribution of these codes within the partially faithful and non-faithful segments was very similar with 56% of the partially faithful segments and 57% of the non-faithful segments being assigned the participation code. In contrast, the faithful segments had a slightly higher level of participation with 67% of these segments being assigned the participation code. This suggests that either the teachers have an easier time engaging the class when they use their plans from the ISA, or they have an easier time following the ISA when their students are willing to participate.

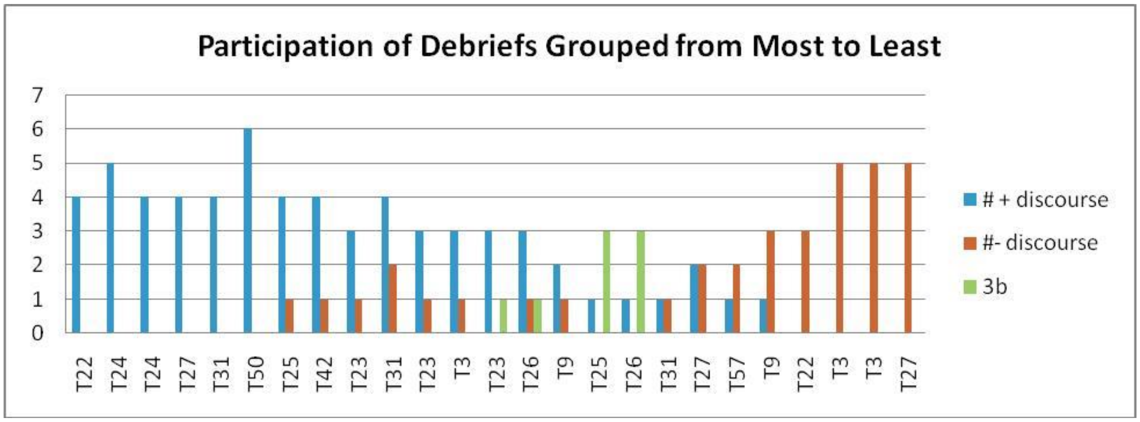


Figure 15. Evidence of participation per segment, grouped by debrief from most to least participation.

Figure 15 shows the distribution of participation levels within each debrief. The chart is roughly organized according to the number of segments assigned a participation code and the number of segments assigned a non-participation code (the third bar represents segments that were assigned a code of 3b and, thus, could not be given a participation code). The table demonstrates teachers' tendencies to either run their class so that students are regularly contributing or to run their class so that students are almost never participating. Notice how there are six debriefs in which students were participating for all segments. Conversely, there are four segments in which students were never found to be participating.

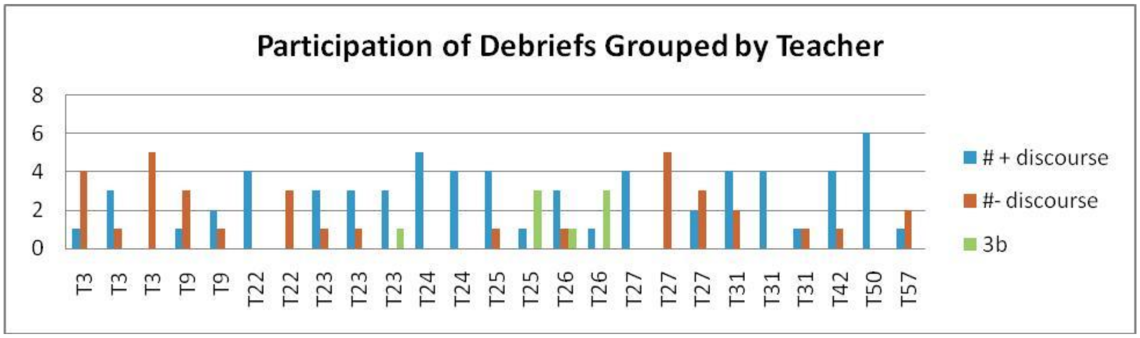


Figure 16. student participation in debriefs, grouped by teachers

Figure 16 shows the same data grouped by teachers. Note that teachers 22 and 27 each have a debrief in which someone is participating in every segment and a debrief in which no students were observed participating other than, possibly, the presenting student. (Teacher 27 also had a debrief in which the students were contributing half the time and the other half of the time they were not.). This would suggest that something was different about their debriefs that affected the level of participation of the students. On the other hand, there was also a teacher (teacher 24) who had two debriefs in which students were regularly participating and a teacher (teacher 3) who had two debriefs in which no students ever participated. This suggests that these teachers may be following a pattern of instruction that either gives opportunities for students to share or fails to give opportunities to allow students to share. These patterns are worth investigating to see whether or not there is a relationship between the teachers' plans that is affecting the outcomes of their debriefs, or if it is the improvisational moves of the teachers that are causing more or less participation from the class. These are questions that will be investigated further in research sub-question 3.

Conclusions

The Implementation Fidelity Analysis tool was designed to answer research sub-question 1 (Do teachers enact their written plans in the classroom as they had planned prior to implementation?). The quick answer to this question is that, yes, teachers did show a tendency to implement at least part of their ISAs as planned. Nearly two thirds of all the segments planned by the teachers were faithfully implemented and 86% of all the segments were implemented with at least partial fidelity. An examination of the segments grouped within their respective debriefs also supported the conclusion that most of the teachers were implementing their problem-solving debriefs with at least some fidelity to the ISAs. In particular, more than half of the problem-solving debriefs were implemented in such a way that no more than one segment was implemented unfaithfully. Also, twenty-four of the twenty-five debriefs in the study were implemented with at least one of the segments implemented faithfully. This suggests that, even if there wasn't perfect fidelity to the ISAs, all of the teachers at least attempted to implement their debriefs according to their written plans.

There was, however, some exceptions to this trend to implement the ISAs faithfully. Two teachers demonstrated a very strong tendency to not faithfully follow the plans they documented in their ISAs. Both of these teachers failed to faithfully implement either two or three of their segments for each debrief they implemented. These were the only teachers to show non-faithful implementation on more than one segment within a debrief. Also, there were two teachers that showed a strong tendency to faithfully implement their ISAs for two of the three debriefs they implemented, but they then demonstrated an, overall, less faithful implementation on a third debrief. These examples suggest that teachers' improvisational moves may continue to heavily impact the outcomes of a lesson despite the use of a lesson plan.

In addition to analyzing the extent to which the teachers were addressing their planned questions and ideas to highlight, the IFA tool also examined whether or not students other than the presenting student were contributing to the discussion. The results showed some tendency for the students to participate in the debriefs (63.6% of the segments were assigned the participation code). Interestingly, students were more likely to participate when the ISA was being implemented faithfully (67% of the segments identified as faithfully implemented were assigned a participation code versus 56% and 57% for the partially faithful and non-faithful segments, respectively). It is difficult to say, however, why this trend towards greater participation in the faithful segments exists. It might be that when teachers are following the ISA they have an easier time engaging their students in the discussion, or it might be that the teachers have an easier time following the ISA when their students are willing to participate.

Application of the Implementation Fidelity Analysis to the Research Questions. The levels of the Implementation Fidelity Analysis tool were intended to capture the extent to which the teachers were implementing the questions and ideas they had identified in the ISA. It does not address how they engaged students in mathematical discourse. The Implementation Fidelity Analysis only captures fidelity to the literal lesson. That is, it measures the extent to which the teachers followed the steps in their ISAs, regardless of the mathematical content that was developed in the plan. This data analysis tool was used to answer research sub-question 1 (Do teachers enact their written plans in the classroom as they had planned prior to implementation?). This is an essential question to ask when studying the impact that a written lesson plan has on the enacted lesson because, if a teacher fails to adhere to the basic steps in the lesson plan, then it is difficult to attribute the outcomes of the debrief to the written lesson plan.

In order to accurately answer research sub-question 2 (Is there a correlation between the number of questions teachers plan that promote mathematical reasoning and argumentation and those that they actually ask during the whole-class discussion?), it is necessary to know the level at which the teacher adhered to the lesson plan during implementation. Research sub-question 2 addresses the influence that the types of questions the teachers planned to ask had on the nature of the discourse. If a teacher was not using the questions from the ISA in his or her debrief, then we cannot assume any connections exist between the types of questions the teacher planned to ask and the nature of the discourse that subsequently took place.

A faithful implementation of the ISA does not necessarily imply that what took place during the problem-solving debrief was a perfect reflection of what the teacher had intended in the write-up of an ISA. For research sub-question 3 ("How do teachers' improvisational moves during whole-class discussions influence the enactment of the questions that were planned by the teacher prior to implementation?") I analyzed the episodes in the debriefs in which the planned questions were being addressed. The purpose of this analysis was to gain a better understanding of how a question that was planned in the ISA can change as it is being implemented by the teacher. In particular, I examined the teachers' moves that impacted how a planned question was addressed. I continue this discussion of my research analysis and results in the following chapter in which I discuss the analysis and results for research sub-question 2.

Chapter 7. Research Sub-Question 2

This chapter is devoted to the analysis of research sub-question 2, the part of the analysis focused on the intended lesson, or the opportunities to reason mathematically, that were identified in the lesson plan. This portion of the analysis addresses the nature of questions that the teachers planned with a particular focus on whether or not the planned questions were intended to create opportunities for students to reason mathematically. In this chapter, I will describe the coding scheme that I developed to differentiate between questions that promote mathematical reasoning (high-press) and those that do not (low-press). My description of the coding scheme includes a discussion of Kazemi and Stipek's (2001) conceptualization of a high-press mathematics classroom and a low-press mathematics classroom and how this framework was used to develop my codes. I include a discussion of my coding scheme, including a discussion of inter-rater reliability. The analysis section of this chapter includes a description of how I used this coding scheme to analyze my data, the results from this analysis, and I conclude with a discussion of how these findings contribute to the larger study.

Development of the Question Codes

To answer research sub-question 2 (Is there a correlation between the number of questions teachers plan that promote mathematical reasoning and argumentation and those that they actually ask during the whole-class discussion?), I developed a coding scheme to analyze teacher questions from the Instructional Sequence Analysis (ISA) and their implemented debriefs (see Appendix A to see an example of the ISA). My primary goal for developing this coding scheme was that the codes differentiated the questions as either high-press (promoting thinking and reasoning about the mathematics in the task) or low-press (promoting communication, but not necessarily mathematical reasoning). To define what I mean by high-press and low-press, I used Kazemi and Stipek's concept of high-press and low-press teacher-student interactions as a basis for my coding scheme (2001). In their analysis of four upper-elementary classrooms that were characterized as having positive social norms, that is, characterized by students working together and sharing their problem-solving strategies with one another, they found that, in two of the classrooms, students were engaged in richer mathematical discussions than in the other classrooms. They identified four sociomathematical norms that differentiated the high-press classes from the low-press classes:

(a) an explanation consists of a mathematical argument, not simply a procedural description or summary;

(b) mathematical thinking involves understanding relations among multiple strategies;

(c) errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies; and

(d) collaborative work involves individual accountability and reaching consensus through mathematical argumentation. (Kazemi & Stipek, 2001, 64).

This conceptualization of high-press and low-press discourse was helpful for developing a coding scheme that clearly distinguished between high-level and low-level mathematical discourse because their characterizations of low-press versus high-press were primarily developed in the context of students sharing out their problem-solving strategies in a manner similar to the debriefs I am analyzing. Below is an overview of the final coding scheme developed for my analysis. See Appendix C for a complete description of the codes and examples of coded questions from the data.

Question Codes

High Press-

H1. Reasoning and Justification
H2. Addressing Errors and Misconceptions, Verifying a Solution
H3. Generalizations, Conjectures, New Strategies
H4. Making Connections
H5. Clarifying other Students' Thinking

Low-Press-

L1. Sharing and Explaining
L2. Short-Answer Questions, Recall Facts, Procedural Answer
L3. Non-Mathematical Questions

While my coding scheme was inspired by Kazemi and Stipek's high-press versus low-press teacher-student interactions, my codes were not meant to be a perfect match between these norms. Table 21 shows the relationship between my codes and the characterizations of high-press and low-press identified in Kazemi and Stipek's study. In this table, the first column contains my question codes with a brief description of the code, the second column gives both references to the sociomathematical norms given in the quotation above as well as additional quotes from Kazemi and Stipek's article describing characteristics of the high-press and low-press classrooms. I also include a third column that includes additional references to educational research that further support these categories of questions as promoting mathematical reasoning (high-press) or not (low-press).

Table 21 Relating Question Codes to Kazemi and Stipek's high-press/low-press framework (2001)

The Question Codes	**Sociomathematical Norm**	**Other Supporting References**
H1 Reasoning and Justification. Questions that create an opportunity for students to provide rationale to support their reasoning and/or solution	*An explanation consists of a mathematical argument* "**In high-press interactions, students learned that they could justify their actions by triangulating verbal, graphical, and numerical strategies" (67)**	Ball & Bass (2003) Yackel & Cobb (1996) Wood & Turner-Vorbeck (2001)
H2 Addressing Errors and Misconceptions, Verifying a Solution. Prompts students to verify whether or not a solution is correct *or* discussing what makes a solution, or part of a solution, incorrect or problematic	*Errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies.* " **Verification was an integral part of group activities during the lesson" (66)**	Baxter & Williams (2010) Fraivillig and Murphy (1999) Hiebert et al (1997)
H3 Generalizations, Conjectures, and New Strategies. Prompt students to move forward with their thinking on the task by coming up with new ideas on how to solve the task or exploring the mathematical ideas surrounding the task	**"the teacher pressed them to think how else they could conceptualize 7/6" (68)**	Fraivillig & Murphy (1999) Hiebert et al (1997) Jansen (2009) Rasmussen & Marrongelle (2006)
H4 Making Connections. Prompts students to make connections between strategies, representations, other mathematics (besides that which is central to solving the task), or real-world context.	*Mathematical thinking involves understanding relations among multiple strategies* "**The teacher initiated a discussion that required students to 'focus on the mathematical concept of equivalence and its relation to the process of adding fractional parts." (70)**	Hiebert & Wearne (1993) Fraivillig & Murphy (1999) Carpenter, Ansell, & Levi (2001)
H5 Clarifying other Students'	*Collaborative work involves individual*	O'Connor (1998)

Thinking. Prompt students to make sense of what other students are saying or thinking.	*accountability* *"***She invited everyone, not just the students at the board, to think about how the students had solved the problem."** **"both [high-press teachers] invited all members of the group to contribute to the explanation of their group's work." (77)**	Hiebert et al. (1997)
L1 Sharing and Explaining a request for the presenting student to describe the work that they did, or explain their thinking on the task.	*An explanation … [is] not simply a procedural description or summary* *"***In low-press exchanges …Students described solutions primarily by summarizing the steps they took to solve a problem" (68)**	Ball, Lubienski, & Mewborn (2001) Cobb, Wood, & Yackel (1993)
L2 Short-Answer Questions, Recall Facts, Procedural Answer there is only one appropriate answer and/or these questions would be evaluated as right or wrong based upon text-book knowledge	"**[in the low-press classes] there was no evidence that the teachers were looking for detailed responses" (69)** **"Ms. Reed called on one student after another until she called on a student who provided the correct solution"**	Cazden (2001)
L3 Non-Mathematical Questions questions that fail to focus on the mathematical content.	"**In low-press exchanges, connections were limited to nonmathematical aspects of students' strategies."** **"both [low-press teachers] were primarily concerned with managerial and procedural instructions"**	

The codes L2 (Short-Answer Questions, Recall Facts, Procedural Answer) and H3 (Generalizations, Conjectures, and New Strategies) do not have a clear connection to the Kazemi and Stipek framework. L2 questions (Short-Answer Questions, Recall Facts, Procedural Answer) were not something that was explicitly addressed in Kazemi and Stipek's framework, although they did characterize the teachers in low-press questions as focusing on correct solutions over detailed solutions (see quotes in table 21 for L2). These types of questions occurred frequently in the teacher's ISA's. Because these types of questions are closed-ended, they are not useful for promoting mathematical reasoning and/or argumentation which is why they are labeled as low-press questions. I included H3 (Generalizations, Conjectures, and New Strategies) in the high-press category because I consider these types of questions important for generating discourse focused on reasoning and argumentation because generating new ideas is an important step towards creating opportunities for students to reason mathematically (Jansen, 2009; Rasmussen & Marrongelle, 2006). In a research study that analyzed the practices of a successful elementary school teacher, they found that she frequently supported students in thinking about the mathematics in their problem-solving tasks by encouraging them to make generalizations and conjectures as well as try alternate solutions methods and look for more efficient solution methods (Fraivillig & Murphy, 1999). Also, a primary goal in discourse around student-generated problem-solving solutions is to make the methods more powerful and efficient (Hiebert et al, 1997). In order to do this, students should be thinking about new strategies beyond their initial attempts to solve the task. I found in the data for this research study that it was common for teachers to plan questions that prompted their students to think about the task in a new way. That is, asking them to develop new strategies or look for additional solutions. The complete question coding which I used for my data analysis may be found in Appendix C.

Inter-rater Reliability

To assess the inter-rater reliability of the question codes, I brought in a secondary coder who was also a mathematics education PhD student. To train the secondary coder, I shared the coding scheme, going over the codes with her. Then, we reviewed several ISA's, individually coding each question and then sharing our codes, discussing why we coded them as we had. In this process, we were working to build a mutual understanding of what each code represented. Following this training session, the secondary coder and I individually coded four other Instructional Sequence Analyses, different from the ones that we had trained with. There were a total of 26 questions that we coded. Overall, our codes were 77% consistent. Because I will only be utilizing this coding scheme to differentiate between high-press questions and low-press question, I also compared our codes by whether they were high-press or low-press. In other words, I reanalyzed the consistency of our codes, counting a code as consistent between coders if we both coded a question as high-press or both coded a question as low-press. For example, I counted a question as consistent because we both coded it as high-press, even though I coded it as H2 (reasoning through errors) and the secondary coder coded it as H1 (providing justification). A comparison of our codes for high-press versus low press were 85% consistent.

Analysis and Results

The data used for the analysis of research sub-question 2 included the ISAs of all the teachers and the audio-recordings of their problem-solving debriefs for the three MPSM problem-solving tasks. There were a total of 12 teachers whose data were used and a total of 25 problem-solving debriefs that were analyzed. Four teachers turned in complete data sets for all three of the tasks, five teachers turned in complete data sets for two of the tasks, and three teachers turned in a complete data set for only one of the tasks. The three teachers who submitted complete data sets for just one of the problem-solving tasks were all from cohort 3 while the remaining teachers were all from cohort 2. Of the 25 implemented problem-solving debriefs, seven were Snack Shack tasks, ten were Design a Dartboard tasks, and eight were Spinner Elimination tasks. For a full description and analysis of these tasks, refer to Chapter 4, Task Analysis. For a full description of the participating teachers and demographic information about their schools, see Chapter 5, Method.

Coding the questions. The question codes, as described above, were used to analyze both the questions planned in the ISA and the questions asked by the teacher in the implemented debrief. Identification of the questions to be coded in the ISAs was straightforward, because the questions were literally listed out for me in the column for questions to make the mathematics salient (see Appendix A for a blank version of the ISA and see Appendix F for samples of completed ISAs). The criteria for what constitutes a question in the implemented debrief and which questions I would be coding needed to be made explicit. A question in the implemented debrief was defined as an utterance made by the teacher that prompted some sort of response from a student. As a result, both direct questions ("Why did you choose that strategy?") and indirect questions ("please tell the class why you chose that strategy.") were considered in my analysis. Within the problem-solving debriefs, teachers asked a large number of a wide variety of questions. For example, teachers check in on student understanding ("Does that make sense to everyone?"), they call on students ("Sierra, did you have something to share?"), they direct students to move around the classroom ("Anthony, please bring your paper up here."), they give behavioral directions ("be quiet, please"), etc. As a result, the number of questions a teacher asks during a debrief can be copious. Rather than coding all of these questions, my analysis was focused on coding only the high-press questions. As a result, the number of questions I coded was considerably less than what was actually asked by the teacher. For example, Teacher 23 made twenty-three utterances that would be considered questions in the first 10 minutes of her debrief of the *Snack Shack* task. In contrast, in that same block of time, I coded just two of those questions as high-press questions.

I chose to only code the high-press questions in the implemented debriefs because research sub-question 2 is specifically focused on the teachers' use of high-press questions in their problem-solving debriefs. The assumption I am making is that a teacher who maintains press for high cognitive demand in his or her classroom is going to do so with the use of high-press questions. As a result, my research is focused on the use of the high-press questions in the classroom and research sub-question 2 is intended to analyze the relationship between the high-press questions that were planned in the ISA and the high-press questions that were asked in the implemented debrief.

In order to ensure consistency in my analysis for all teachers and debriefs, I followed certain rules when assigning codes. (1) A code was assigned to a high-press question only if students were given an opportunity to answer the question. This applies even if no student was able to answer it. However, if a teacher asked a high-press question and then immediately provided an answer to the question without waiting for a response, then the question was not counted. (2) A question that was repeated several times was only counted once. It is often the case that a teacher will ask a question several times or reword a question in a couple different ways before students give a response. In this case of repetition, the question was counted once. (3) If several students gave a response to the same question, it was counted only once. However, if the question was directed to one group of students and, after they responded to it, redirected to another group, it was counted twice. An example of this would be if the teacher asks the presenter a question and, after that student responds, redirects the same question to the class in order to see if someone in the group could answer the question better or differently. (4) If a teacher asked the class to respond to a question by thinking quietly to oneself or discussing it in small groups, this question was only counted if the students were then asked to share out their responses.

Two data analysis approaches. The following sections address the processes that were used to analyze the data. The data was analyzed in two ways. First, the number of high-press questions asked in the ISAs was compared to the number of high-press questions asked during the implemented debrief. In this manner, the planned and implemented debriefs were compared overall. That is, the frequency of high-press questions planned in the entire ISA was compared to the frequency of high-press questions asked in the entire debrief. The second way in which the data was analyzed was by segment (a segment is the portion of the ISA/debrief that is focused on a single, selected piece of student work). That is, I compared the number of high-press questions planned around the presentation of a particular piece of student work to the number of high-press questions asked during the portion of the implemented debrief in which that student's work was being discussed. The benefit of this latter approach was that it allowed me to disregard segments that were not faithfully implemented (as determined in the analysis for research sub-question 1) as well as portions of the debrief that were attending to things other than what was planned in the ISA (such as reviewing what the problem was asking). While these may be interesting instructional moves, they were not answering my research questions.

Analysis of the overall debriefs. In order to consistently compare the number of high-press questions planned for and asked in a problem-solving debrief, I chose to look at the rate at which the planned and asked questions were being addressed with respect to the length of the debrief. If a teacher asked five high-press questions during a thirty minute debrief, this would be different from if another teacher had asked five high-press questions during a ten minute debrief. Similarly, if a teacher had planned one high-press question for a debrief that was only ten minutes long, this would be different from a teacher who planned one high-press question for a debrief that was thirty minutes long. As a result, I chose to consider the average number of high-press questions asked by the teacher per 10 minutes of debrief time as well as the number of high-press questions planned per 10 minutes of the debrief. This latter value is meant to represent the frequency (per 10 minutes) which we would expect to see the planned questions appear, given the length of the debrief, if the teacher were to ask all of the planned high-press questions during the debrief. For example, if a teacher were to plan six high-press questions and her debrief was 30 minutes long, then we would say that this teacher planned for two high-press questions to be asked every ten minutes of her debrief. On the other hand, if another teacher planned four high-press questions, but his debrief was 16 minutes long, then he planned for 2 ½ high-press questions to be asked per 10 minutes. This allowed me to compare the frequency of high-press questions asked in the debrief to the number of high-press questions planned for the debrief given the varying lengths of the teachers' implemented debriefs.

Organization of the Data. Once the questions from the ISAs and the high-press questions asked by the teachers in their debriefs were coded, a table was constructed in which each task implemented by a teacher was assigned a column (see Appendix D for the complete table). The first rows of the table contained the following data: (1) the total number of high-press questions planned by the teachers in the ISA, (2) the total number of high-press questions asked by the teacher in the implemented debrief, and (3) the total length of the implemented debrief. The value of row three was determined by measuring the time on the audio-recording from when the teacher brought the class together to begin discussing student work on the task and ending with either the end of the audio recording or when the class was no longer discussing the task. Using the data in rows one through three, the following additional rows were added: (4) The average number of high-press questions planned per 10 minutes of the debrief, which was calculated as: ((number of high-press questions planned)/(number of minutes in a debrief))*10 and (5) the average number of high-press questions asked during a debrief per 10 minutes, which was calculated as: ((number of high-press questions asked)/(number of minutes in a debrief))*10.

Results. Using a correlation analysis, the number of questions planned per 10 minutes of the debrief was compared to the number of questions asked per 10 minutes of the debrief. This data showed that there was a negative correlation between these data sets with a very weak R^2 of .0043 (see figure 17). A review of the corresponding scatter plot, though, made it clear that there was an outlier within the data set that was skewing the data. This outlying data value had an unusually high number of high-press questions planned per 10 minutes (6.25 high-press questions planned per 10 minutes, which is more than twice as many planned high-press questions than any other debrief). This was coupled with an unusually low number of high-press questions asked during the debrief (1.25 high-press questions asked per 10 minutes of the debrief). By removing this outlier, the data did reveal a positive correlation with a slightly significant, but still very weak R^2 of 0.1004 (see figure 18).

Figure 17. # high-press questions planned versus # high-press questions asked (including outlier)

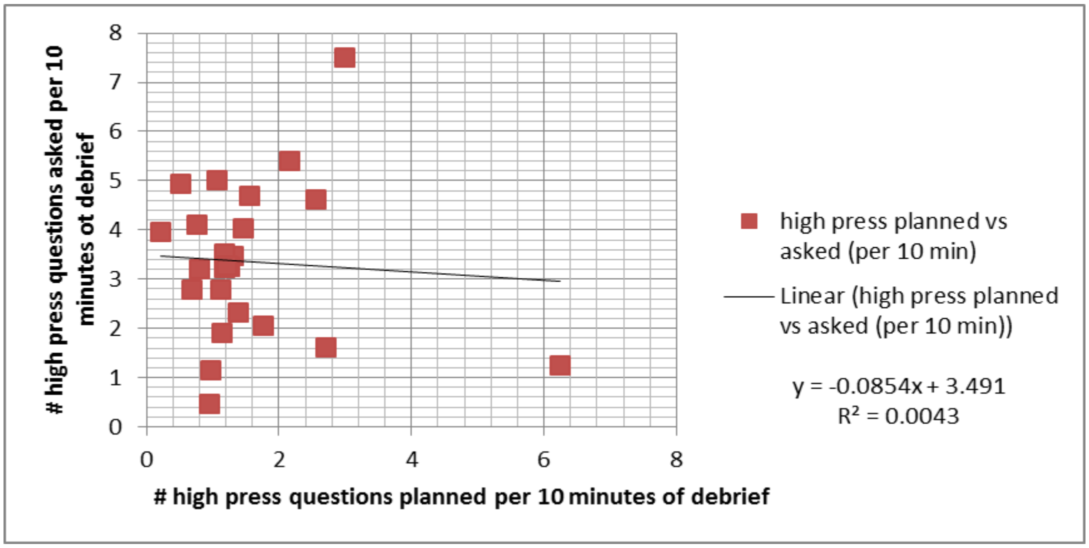
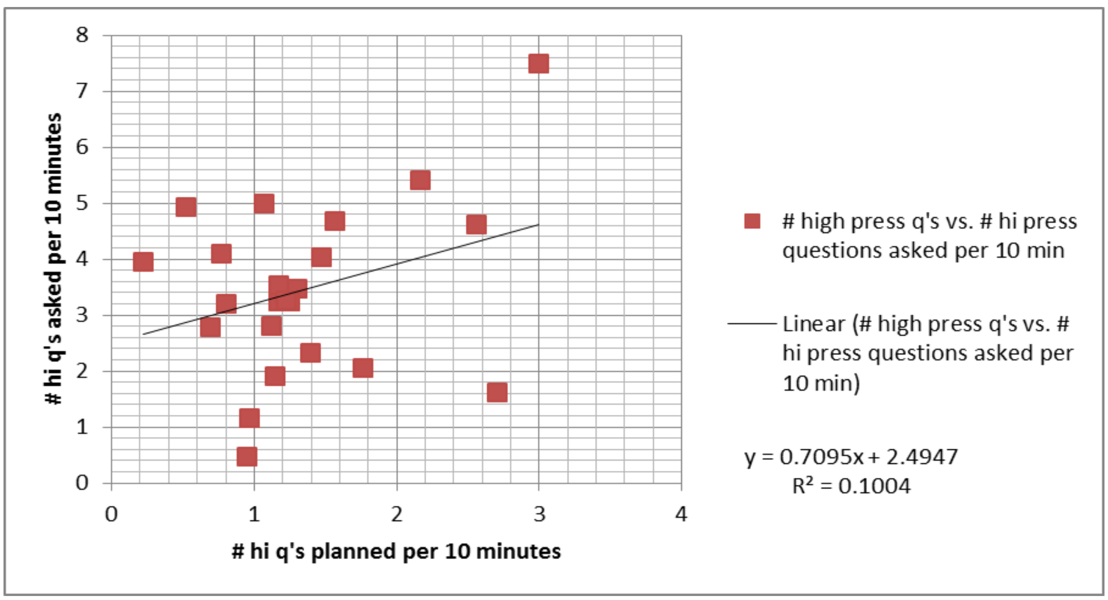


Figure 18. # high-press questions planned vs. # high-press questions asked (excluding outlier)



Overall, the data suggests that the relationship between the high-press questions the teachers planned in their ISAs and those they actually asked during a debrief were only slightly related, suggesting that there were other factors that influenced the frequency with which the teachers asked high-press questions during the implemented problem-solving debriefs. One of the possible reasons why this analysis showed only a slight correlation is because this analysis compared all debriefs in their entirety, regardless of how closely the teachers were following the ISAs, whether or not they were asking the planned questions, and how much time was additionally spent discussing things other than what was planned in the segments. In many cases, the teachers would engage their classes in lengthy discussions about the task prior to discussing the student work as planned in the ISA. Also, the teachers occasionally led follow-up discussion after presenting the student work. These episodes before and after the discussion of student work represented a part of the debrief, but did not necessarily reflect what was planned in the debrief. In order to more clearly connect the planned and asked high-press questions of the teachers in their implemented debriefs, I conducted a second analysis of the data, this time focusing on the segments in which the teachers faithfully implemented the questions they planned in their ISA's. This analysis is discussed in the next section.

Analysis by Segment. Since this analysis is intended to look specifically at the relationship between the number of high-press questions planned for each segment in the ISA and the number of high-press questions asked in the implemented debrief, an analysis in which the planned questions more clearly corresponded with what was taking place in the implemented debriefs was appropriate. As a result, I narrowed the scope of what I was analyzing by comparing the number of high-press questions planned within each segment of the ISA to the number of high-press questions asked in the corresponding segment within the implemented debrief. As a result, my unit of analysis became each individual segment of a debrief. In addition, I only used the segments of the debriefs that were faithfully implemented with respect to the planned questions. That is, I only included in this analysis the segments that were assigned either level 1 (faithfully implemented) or level 2b (partially faithfully implemented in which the questions were addressed, but the Ideas to Highlight were not; recall that only two segments fell into this category). By narrowing down the scope of my analysis, I focused on the portions of the debriefs in which (1) the teacher addressed the presentation of student work as described in their ISA and (2) attended to the questions in the ISA, according to the Implementation Fidelity Analysis.

For each segment that was identified as either level 1 (faithful) or level 2b (partially faithful with respect to the planned questions), I counted the number of high-press questions that were planned for that segment. The number of high-press questions planned for a segment ranged from no (0) high-press questions planned to three high-press questions planned. Nineteen segments were planned with no high-press questions (This implied that the questions that were planned were all coded as low-press); 36 segments were planned with one high-press question; nine were planned with two high-press questions and two were planned with three high-press questions. Three was the highest number of high-press questions planned for a single segment. Using the time stamps identified in the IFA, I counted the number of times that the teacher asked a high-press question during each segment. I used the same criteria as described earlier to determine when to count a high-press question.

Results. Table 22 is a summary of the findings from this analysis. The table shows the number of segments that were implemented such that the given number of planned high-press questions resulted in a corresponding number of high-press questions being asked by the teacher during that segment. For example, there were seven segments in which no high-press questions were planned and, correspondingly, no high-press questions were asked. Also, There was just one segment in which two high-press questions were planned and nine high-press questions were then asked during the corresponding segment in the debrief.

Table 22. Frequency table of high-press questions asked given planned questions

		Number of High Press Questions Planned			
		0	1	2	3
Number of High-Press Questions Asked During the Segment	0	7*	5	0	0
	1	5	6	4	0
	2	2	11	1	0
	3	4	5	0	1
	4	1	5	1	0
	5	0	1	1	1
	6	0	3	0	0
	7	0	0	0	0
	8	0	0	1	0
	9	0	0	1	0

*This number means that 7 segments were implemented such that 0 high-press questions were planned and 0 high-press questions were asked.

A correlation analysis comparing the number of high-press questions planned within each segment of the ISA's to the high-press questions asked in the implemented debriefs reveals a positive correlation between the number of questions planned and the number of questions asked in the implemented debrief (see figure 19). The correlation coefficient of 0.1489, while stronger than what was found in the previous analysis, is still quite weak. This finding provides further evidence of the simple fact that all teachers are unique in how they teach and, as a result, the number of high-press questions they ask per segment is going to vary significantly.

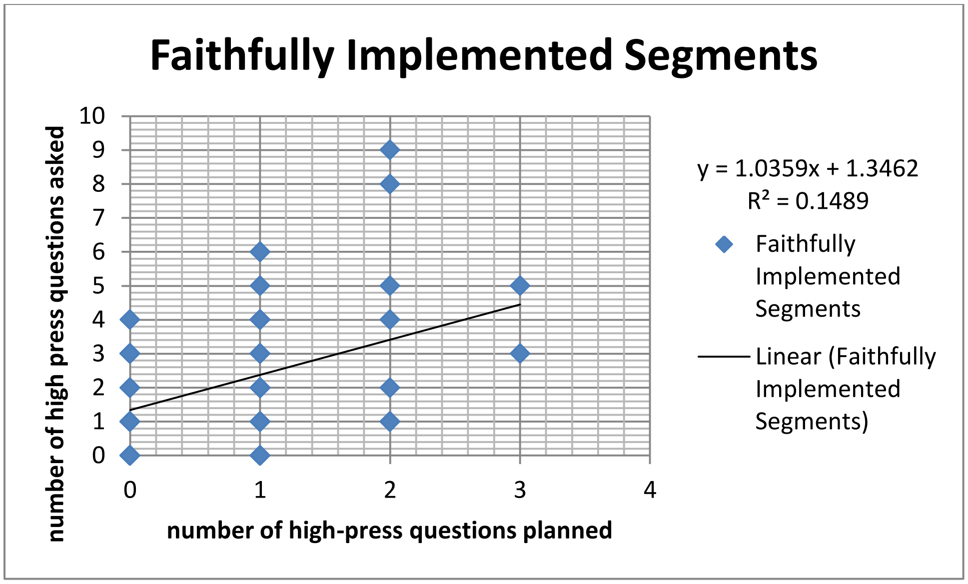


Figure 19. Linear regression of high-press questions planned and asked per segment

Although the data continues to show much variation, I conducted a t-test which showed that the average number of high-press questions the teachers asked did increase when the teachers planned to ask at least one high-press question in comparison to when they did not plan any. The segments in which no high-press questions were planned recorded a mean score of 1.32 high-press questions asked during the implemented segment, with a standard deviation of 1.32, while the segments in which exactly one high-press question was planned recorded a mean score of 2.39 high-press questions asked during the implemented segment, with a standard deviation of 1.71. Hypothesis testing shows that there is a significant difference at 5% significance level (t-value = -2.56 and p-value < .05) in the mean score. The mean difference between these two groups is -1.073, with a t-value of -2.56 and a p-value of 0.007 (<.05), which indicates that the mean number of high-press questions asked within a segment is significantly higher when one high-press question is planned for a segment compared to when no high-press question is planned.

However, the difference between the number of high-press questions asked in a segment when the teacher planned more than one high-press question in the ISA was not significantly different compared to when the teachers planned for just one high-press question. When the teachers had planned either two or three high-press questions, the mean number of high-press questions asked within a segment was 3.64 with a standard deviation of 2.87. When compared to the segments in which one high-press question was planned (mean of 2.39, standard deviation of 1.71), the mean difference between these two groups is -1.247, with a t-value of -1.37 and a p-value of .098 (>.05), which indicates that there is not a significant difference in the number of high-press questions asked in these two groups. The failure of the t-test is likely due in part to the small sample size of segments in which more than one high-press question was planned (there were 11 segments in which more than one high-press question was planned versus 36 segments in which exactly one high-press questions was planned).

Conclusions and Implications

The lack of correlation between the number of high-press questions planned and the number of high-press questions asked, whether the data is grouped by overall debrief or by segments, leads me to surmise that the number of high-press questions planned for a debrief plays only a minor contributing role with respect to the types of questions the teachers are asking when implementing their debriefs. The teachers are making on-the-fly decisions to ask either more or less high-press questions during the problem-solving debriefs and these decisions are not completely dependent upon the nature of the questions that the teachers planned prior to implementation.

While this analysis failed to reveal a correlation between the number of high-press questions the teachers planned for their problem-solving debriefs and the number of high-press questions they actually asked during the implemented debrief, my analysis of the data did show that teachers who pre-planned at least one high-press question for a segment did, on average, ask more high-press questions during that segment compared to when they did not plan any high-press questions for a segment. That is, planning at least one high-press question for a segment did, in fact, make a difference with respect to the number of high-press questions the teacher asked during the corresponding implemented segments. However, no conclusions may be drawn from this data that the more high-press questions the teachers planned the more high-press questions they asked in the debrief. This suggests that simply having a high-press question in mind when implementing a segment has a greater impact on how many high-press questions the teachers asks than the sheer number of high-press questions the teacher has in mind.

Teachers' instructional practices are heavily influenced by their prior experiences as teachers (Superfine, 2009). Given that every teacher has their own unique methods of instruction, the variability in the number of high-press questions asked was not surprising. Based upon the results of this analysis, teachers that wish to engage their students in discussions that are focused on mathematical reasoning and not just reporting out solutions should be encouraged to plan at least one high-press question to be asked during the discussion of a piece of student work. By thinking about what type of high-press question(s) they want to ask their students, in order to help their students to reason mathematically about the task, a teacher is in a better position to move students beyond simply sharing their work on a problem-solving task and begin focusing them on the mathematical nature of their solution strategies. In addition to this move, though, teachers should also think about what types of sociomathematical norms they are developing with their students and cultivate the practice of regularly asking questions that promote mathematical reasoning in their classrooms (Rasmussen, Yackel, & King, 2003; Yackel & Cobb, 1996).

The theoretical framework used for this study of the temporal phases of curriculum implementation (figure 1 in Chapter 1) recognizes that there are many factors that can contribute to the implementation of a lesson beyond the contents of that lesson (in this case, the questions that the teachers plan). It is possible to hypothesize about many other factors that might contribute to the nature of questions asked by the teachers during a problem-solving debrief. These factors may include the social and sociomathematical norms developed by the teacher and students in the classrooms, teachers' beliefs about the teaching and learning of mathematics, the content knowledge of the teachers, the nature of the mathematical learning goals of the ISA and their relationship to the planned questions (Koency & Swanson, 2000; Manouchehri & Goodman, 1998; Rasmussen, Yackel, & King, 2003; Stein, Remillard, & Smith, 2007). Further research would be necessary to identify the manner and extent to which these factors may influence the nature of questions asked by the teachers. However, this is outside of the scope of this research study. In the analysis of my third and final research question, though, I look at the improvisational moves the teachers make as they address the planned questions in their ISAs. In this analysis, I will demonstrate that *how* a teacher addresses a planned question during a problem-solving debrief, as influenced by these external factors, can impact the opportunities for the teachers create for their students to reason mathematically. The analysis and results for research sub-question 3 will be discussed in the following chapter.

Chapter 8. Research Sub-question 3

Research sub-questions 1 and 2 were meant to provide an analysis of certain aspects of the implementation of the Instructional Sequence Analysis (ISA). In research sub-question 1, I analyzed the literal lesson, addressing whether or not the teachers were following their Instructional Sequence Analyses as planned. In research sub-question 2, I analyzed the intended lesson by comparing the frequency of planned high-press questions (i.e. questions intended to create opportunities for students to reason mathematically) to the frequency of high-press questions actually asked by the teachers during implementation of the debrief. In research sub-question 3 ("How do teachers' improvisational moves during whole-class discussions influence the enactment of the questions that were planned by the teacher prior to implementation?") I looked in greater depth at what was actually taking place during the debriefs with an eye to the teachers' moves as he or she implemented the questions planned in the ISA. The goal of this research sub-question was to gain an understanding of how the improvisational moves of the teachers, as they implemented the planned questions, influenced the students' opportunities to reason mathematically. Is it enough to say that the teachers are asking the planned questions, or is there more taking place during the debrief that makes the implementation of the planned questions more or less successful in terms of the opportunities available to the students to engage with the mathematical ideas in the questions?

Method: Grounded Theory

To address research sub-question 3, my analysis was based on the qualitative research methodology of grounded theory as described by Strauss and Corbin (2007) in which systematic and iterative analysis leads to the generation and verification of conceptual theory useful for explaining observed phenomena. The purpose of this analysis was to develop a theory that explains how these teachers' improvisational moves served to influence the discourse that took place in their classrooms with respect to the questions planned by the teachers in their ISA's. To do this, I closely analyzed the transcripts of four teachers whom I selected based upon the findings from research sub-question 2.

Selection of four teachers. The overall goal in my selection of the subset of four teachers was to have a set of teachers who demonstrated a wide range of variability in how they implemented the planned questions with respect to the number of high-press questions planned. For my analysis for research sub-question 3, I chose Teacher 9, Teacher 22, Teacher 23, and Teacher 27. I chose these teachers to represent a varying range of planned questions versus implemented questions as determined in research sub-question 2. In figure 20, these teachers' points have been highlighted on a scatter plot of the planned high-press questions (on the horizontal axis) versus the asked high-press questions (on the vertical axis) from research sub-question 2 to provide a visual representation of how these teachers' high-press questions were implemented. In the chart, each point represents a debrief, placed on the chart according to how many high-press questions the teacher planned for that debrief and how many high-press questions the teacher asked in the debrief. Points representing debriefs implemented by the same teacher have been connected. The solid lines connect the debriefs implemented by the four teachers used for research sub-question 3; the dashed lines connect the points of teachers who were not selected for this study; and the singleton points are from teachers who only had one set of data used in this study. The points are connected from least to most high-press questions planned in a debrief. In the following paragraphs, I describe the relationship between each teacher's planned and implemented high-press questions as well as provide some specific background information about the teachers and their schools.

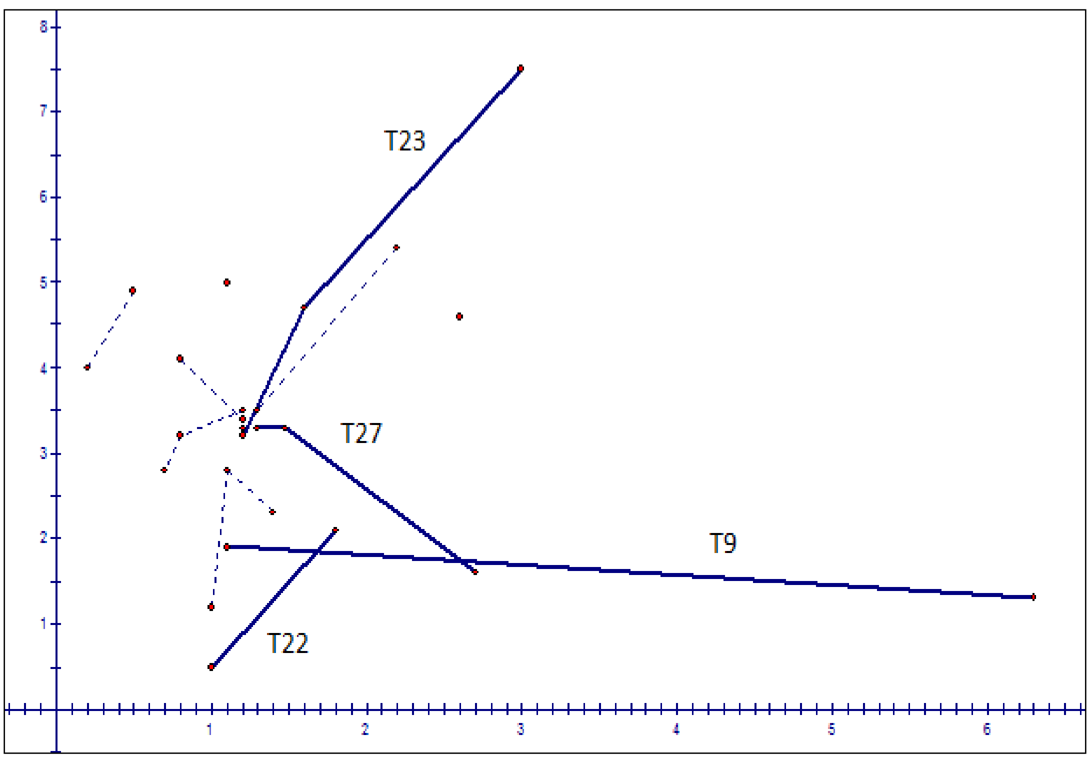


Figure 20. Planned questions vs. Asked questions, four subjects used in Research Sub-question 3 are highlighted.

Teacher 22 represents an example of someone who did not plan for many high-press questions, nor did she ask many high-press questions during her problem-solving debriefs. She had nine years of teaching experience and during the year of data collection, she taught 7th grade math. Teacher 23 taught at the same school as Teacher 22. She had 16 years of experience as a licensed teacher and 5 years of experience as a middle school math teacher. She also taught the 7th grade. Teacher 23 showed the highest number of high-press questions asked in a problem-solving debrief. Also, her three problem-solving debriefs show a very consistent relationship between the number of high-press questions she planned and the number of high-press questions she asked (that is, the more high-press questions she planned, the more high-press questions she asked). The school that teachers 22 and 23 taught at was a public school in a suburban setting. Their school had 21.6% of their students eligible for free and/or reduced lunch. The year prior to data collection and the year of data collection, their school did not meet annual yearly progress requirements. The textbook Teachers 22 and 23 taught with was a traditional textbook.

Teacher 27 was chosen for further analysis because two of his three debriefs fell within the center of the data for overall planned versus asked questions on the scatter plots. In contrast, in his debrief for the Design a Dartboard task, he planned for an unusually large number of high-press questions, but asked an unusually low number of high-press questions in his debrief. Teacher 27 had 13 years of experience as a licensed teacher and nine years of experience teaching middle school math. He taught 6th grade math in a suburban private prep school.

Finally, I chose Teacher 9 for further analysis because his data represented an outlier in the analysis for research sub-question 2. That is, while he had planned more than twice as many high-press questions than any other teacher, his debrief showed a very low number of high-press questions asked. Teacher 9 had three years of experience teaching middle school math. He taught 7th grade math in a rural public school with 10.1% free and/or reduced lunch. He taught using a traditional textbook.

***Descriptions of the teachers*.** In this section, I provide a brief description of how each of the four teachers chosen for this analysis tended to orchestrate their problem-solving debriefs. The information given here is not meant to be part of the analysis. Rather, these are my general impressions of how the teachers were choosing to implement their problem-solving debriefs. As the analysis of research sub-question 3 focuses on the implementation of individual questions from the ISAs, these descriptions here are meant to give the reader a broader perspective of what was happening in their debriefs.

Teacher 27's debriefs tended to be teacher-centered. He was more likely to lead his students through a discussion of the tasks than to orchestrate the discourse for his students to participate in. In Spinner Elimination and Snack Shack, he clearly had a learning goal in mind. He used many leading questions to lead towards what he wanted his students to see. As Teacher 27 focused the discussions on the mathematics he wanted his students to see, his students occasionally chimed in to share their own mathematical reasoning. Teacher 27 was always willing to open the floor to students who wanted to share their thinking, but he rarely initiated opportunities for his students to share their thinking.

An exception to his typical methods of leading a problem-solving debrief was Design a Dartboard. This debrief was not characteristic of his other debriefs. He did not make a significant effort to engage his students with the mathematics. This debrief was characteristic of a low-press debrief in which students shared their work without much additional press for thinking. While it is not clear what caused this change in his usual methods, it is possible that he did not have a clear mathematical learning goal in mind to motivate how he led the discussions, leaving him to focus on student solutions and then move on without attending to the mathematics in the task.

Teacher 9's debriefs tended to take the form of students reporting out with minimal opportunity for reasoning and justification. He rarely pressed his students for thinking. In his ISA, he planned questions that would be considered high-press, but he was less likely to implement a high-press question from his ISA than a low-press question. Surprisingly, despite his overall low-press approach to orchestrating a debrief, in his Snack Shack debrief he demonstrated an example of a well-implemented high-press question. I would reason that it made a difference that he had planned a question which, when asked appropriately, had triggered some good reasoning from his students. His students, in turn, seemed excited to be reasoning about something that was challenging yet accessible.

Teacher 23 was very focused on pressing students' thinking. She planned a lot of high-press questions and asked even more high-press questions. Teacher 23 rarely missed an opportunity to press her students to reason about the mathematics in a task. Sometimes, even when implementing low-press questions from the ISA, she would ask high-press questions as follow-up. However, she struggled to get her students to share their reasoning as her students were sometimes unsure what types of reasoning and explanations they were expected to provide in response to her questions. These challenges were more common with her English language learners. When her students struggled in this way, Teacher 23's high-press questioning did not always result in high-level discourse on the part of her students.

Teacher 22 was attentive to creating positive social norms in her classroom. She did not want to make students uncomfortable, so she typically shared the students' work herself. She asked a lot of open-ended questions, encouraging her students to share their thinking, but she often did not press her students to reason mathematically. Teacher 22 focused on encouraging her students to share in a low-stress environment. She planned very few high-press questions and she, in turn, asked very few high-press questions. She tended to accept all students' thinking without judgment of the validity of their statements. When her students were unable to provide the appropriate type of reasoning to a question she posed, she was likely to provide an answer to the question herself, rather than push her students to think about the question and struggle to give an appropriate answer. While she seemed to succeed in creating an environment where her students felt comfortable, she did not elicit much high-press reasoning from her students.

Data analysis tools. To assist with my analysis for research sub-question 3, I used a second discourse coding scheme (different from the question codes developed for research sub-question 2) designed to analyze teacher utterances. I chose to use this secondary coding scheme because the coding scheme developed for research sub-question 2 was only meant to code questions and did not encompass all types of teacher utterances. For sub-question 3, I was looking to better understand the teachers' moves during the time in which the planned questions were addressed. Since many of the utterances made by a teacher are not questions, it was useful to borrow another coding scheme that addressed all types of utterances made by a teacher. I used the Inquiry Oriented Teaching Analysis (IOTA) codes developed by Rasmussen, Kwon, and Marrongelle (2009) because it was developed to analyze inquiry-oriented classrooms; that is, classrooms in which value is placed on students sharing their thinking about mathematical ideas and providing mathematical argumentation. This view of students sharing their thinking fit well with the values of the MPSM professional development project and, as a result, is both the type of discourse that we expect to see as the teachers are implementing the MPSM problem-solving tasks, but it also represents the aspects of the discourse which are of interest to me in my analysis. I also chose to use this coding scheme because I already had some prior experience using this coding scheme in conjunction with the MPSM research project. Table 23 is a brief overview of the codes (see Appendix E for sub-codes and a description of each code).

Table 23. Inquiry-Oriented Teaching Analysis Codes

Revoicing	**Questioning/Requesting**
Repeating – R1	Evaluating – Q1
Rephrasing – R2	Clarifying – Q2
Expanding – R3	Explaining – Q3
Reporting –R4	Justifying – Q4
Telling	**Managing**
Initiating – T1	Arranging – M1
Facilitating – T2	Directing – M2
Evaluating[8] – T3	Motivating – M3
Summarizing – T4	Checking – M4

[8] Originally, T3 was labeled as Responding, but I changed this to Evaluating since that was the element of this code that is most relevant to my analysis.

When I applied this coding scheme, I was not trying to code every single teacher utterance because some teacher utterances did not contribute significantly to the discourse. For example, I did not code a teacher utterance when they called on a student unless the teacher also asked a question when he or she called on the student. I used the codes to allow me to look for identifiable patterns of behavior on the part of the teachers as they implemented the questions. In particular, my intent was to identify specific patterns of teacher utterances within the context of the implementation of planned questions. My goal was to utilize these patterns to develop categories that described the different ways that teachers implemented questions they planned in their ISAs[9].

Analysis

Amongst the four teachers that were chosen for this analysis, there were a total of 10 transcripts. Teachers 23 and 27 submitted data for all three of the MPSM tasks and teachers 9 and 22 submitted data from two tasks (Design a Dartboard and Snack Shack). In line with grounded theory, analysis of these transcripts took place iteratively, allowing me to look for patterns and continue to verify and develop these patterns as I continued to deepen and expand my analysis. My first step in analyzing these transcripts was to review the transcripts and identify the episodes in which the teachers' planned questions were being addressed. I made physical notes of when these episodes occurred directly in my copies of the transcripts for easy reference as I analyzed the transcripts. The Implementation Fidelity Analysis that I completed for research sub-question 1 was useful for this phase of the analysis as I had already recorded the time stamps in which the

[9] Photocopies of the ISAs for these four teachers may be found in Appendix F. The names of the students have been removed.

questions were addressed. In addition to the questions recognized as being addressed in the Implementation Fidelity Analysis, for this analysis I also included episodes in which the teachers addressed the ideas in the planned questions without eliciting participation from their students. I included these teacher-focused episodes because the purpose of this analysis was to understand what happened to the planned questions as they were being implemented, even if they were not being implemented as questions posed to the students.

Once the episodes of implemented questions were identified in the transcripts, I passed through the transcripts for just one of the tasks. I chose Snack Shack for this phase of the analysis because all four teachers had submitted data for these tasks (as opposed to Spinner Elimination, for which only two of the four teachers had submitted data) and I knew that there was a lot of rich discourse in the Snack Shack tasks. In this analysis, using the IOTA codes, I coded the teachers' utterances for the episodes surrounding the planned questions (that is, the dialogue that occurred immediately before, during, and after when the question emerged). I then compared and contrasted the moves of the teachers in these episodes from the Snack Shack debriefs. In this phase of the analysis, I developed some categories that were useful for describing the improvisational moves of the teachers as they implemented the planned questions. Once these categories were tentatively developed, I moved on to analyze the remaining transcripts, coding each teacher's remaining transcripts together. By conducting my analysis in this fashion, I was allowed to test out my initial categories, verifying their validity, as well as make some modifications to how I defined these categories. Below, I will discuss the four categories that I developed to describe the different ways that these teachers' improvisational moves impacted the implementation of their planned questions, and I will provide examples from the teachers' debriefs to aid in my descriptions and discussions. I will also include discussions of the patterns of teacher talk that occurred within these categories as evidenced by the IOTA codes.

Transformation of planned questions: What happens to planned questions during implementation? While planning questions is an initial step towards bringing up meaningful questions during a problem-solving debrief, the manner in which such questions are addressed by the teacher during the implementation of a debrief can vary significantly from one teacher to the next and also from one question to the next within a single debrief. You will note that I do not say that the teacher asked a planned question. Rather, I say that the teacher addressed a planned question because the teachers rarely asked the questions exactly as they had stated in their ISA's and, sometimes, the questions were never asked at all, but were still addressed when the key ideas in the questions were brought into the discussion. In fact, there were many variations on how the questions were addressed, including how the questions were (or were not) introduced by the teachers, the role that the teacher played in bringing the desired responses into the discussion, and the ways in which the teachers followed up the responses offered by the students.

I identified four different ways that the planned questions were being implemented during the problem-solving debriefs. I named these four different approaches to question implementation drop-in, embedded, telling, and sustained-focus questions. Before moving on to a description of these four categories, I would like to make some things clear to the reader. First, these categories are referred to as questions because they represent how the questions planned in the ISAs were being addressed during the problem-solving debriefs. However, they do not necessarily represent questions posed by the teachers during the implemented problem-solving debriefs. Rather, these categories represent classroom activity that involves the key ideas in these planned questions. Second, these four categories of question implementation are not meant to be exhaustive or exclusive. In fact, later on I will discuss some examples of how two of these categories may become blended together as a question is being addressed in the classroom. As I describe these four categories below, I will provide detailed examples from the data for each category, including a discussion of the types of teacher utterances from the IOTA coding scheme that were common within each category.

A *drop-in* question is one in which the question is asked, student(s) provide(s) responses, the teacher evaluates the responses and then immediately moves on without further discussion. An *embedded* question is one in which the teacher does not actually ask the planned question because the question was essentially addressed by the students without the teacher prompting them. A *telling* question is one in which the teacher never asks the students the planned question but, rather, addresses the question by simply providing the class with the information they would need to know to be able to answer the question themselves (alternatively, the teacher may verbalize the question and then answer the question without waiting for a response from the students). Finally, a *sustained focus* question is one in which the teacher maintains the classroom's focus on the question by engaging the class in further discussion related to the planned question.

Below, I discuss each of these categories in greater detail, providing illustrative examples from the data including some examples of how two categories may occur within the implementation of a single question. I will also describe and discuss four teacher moves that I observed taking place in some of the sustained focus questions from my analysis which I propose contributed to the successful implementation of those particular questions. In my summary and conclusions, I will make some recommendations for how those teacher moves may be used as a tool in professional development to support teachers in the implementation of planned questions.

Throughout my descriptions of each category, I provide examples from the data that I analyzed. Note that these categories that I developed are meant to describe the implementation of planned questions that the teachers recorded in their ISAs. As a result, for each example that I provide below, I will include the question as it was planned in the ISA as well as the transcript of the episode in which the question was implemented. All names used in the transcripts are pseudonyms.

Drop-in. A drop-in question occurs when the teacher addresses a question from the ISA as planned but the question fails to yield any further discussion. These questions are often implemented in an Initiation-Response-Evaluate format (IRE) (Cazden, 1988/2001). That is, the teacher initiates by asking the planned question, a student (or possibly multiple students) provide a response to the question, and the teacher either evaluates the correctness of their response(s), or elaborates on what was said by the student. For a drop-in question, the teacher's responses are typically coded as revoicing (IOTA codes R1, R2, R3) what the student had said or evaluating the correctness of the students' responses (IOTA code T3). In some cases, the teacher may not even respond at all to the students' responses.

Example 1 of a drop-in question. In the following example from Teacher 9's Design a Dartboard debrief, the teacher planned to ask (in reference to the dartboard shown in figure 21) "Why did you use 15 squares?" The question asked in the implemented debrief is very similar; teacher 9 asked "So why did you start with the 15?" (see table 24). Note that in the transcripts, the IOTA codes that I assigned are in the column to the right of the teacher's utterances.

Table 24. Teacher 9 Design a Dartboard Episode (transcript 1)

T9	Clark, do you want to just explain where you started.	**Q3**
Clark	Uh, making the 15% box.	
T9	And how many..?	**Q1**
Clark	15 boxes in there.	
T9	**So why did you start with the 15?**	**Q4**
Clark	**Well, I used Matt's idea. I had, like, 100 boxes and, so, 15% of that is just 15 boxes, or squares**.	
T9	Okay.	T3
T9	So how many are in the middle?	Q1
Clark	25.	
T9	And outer?	Q1
Clark	60.	
T9	And you're positive there's a hundred all together?	Q1
Clark	Yes.	
T9	Are there any questions for Clark? Or comments? Tyler?	M4

In this episode, we see Teacher 9 asking Clark how many squares were in the 15% box, which he uses as a segue into asking why he started with 15 squares. The student offers a response in which he references another student's idea. Teacher 9 offers a simple "okay" in response to the student's explanation and continues asking how many squares are in each of the other sections, checking that the middle and outer sections were, in fact, 25 and 60 squares respectively. Note that this was the first piece of student work presented and took place at the very beginning of the problem-solving debrief. As a result, it is unclear whether or not any of the students understood what Clark meant by 'Matt's idea.' That is, Clark was not referencing something that was said earlier in the debrief, so it is unclear how meaning this reference to Matt's idea would be for the students. Teacher 9 does not ask Clark to clarify what he meant by Matt's idea, although it may have been helpful for making connections between students' strategies.

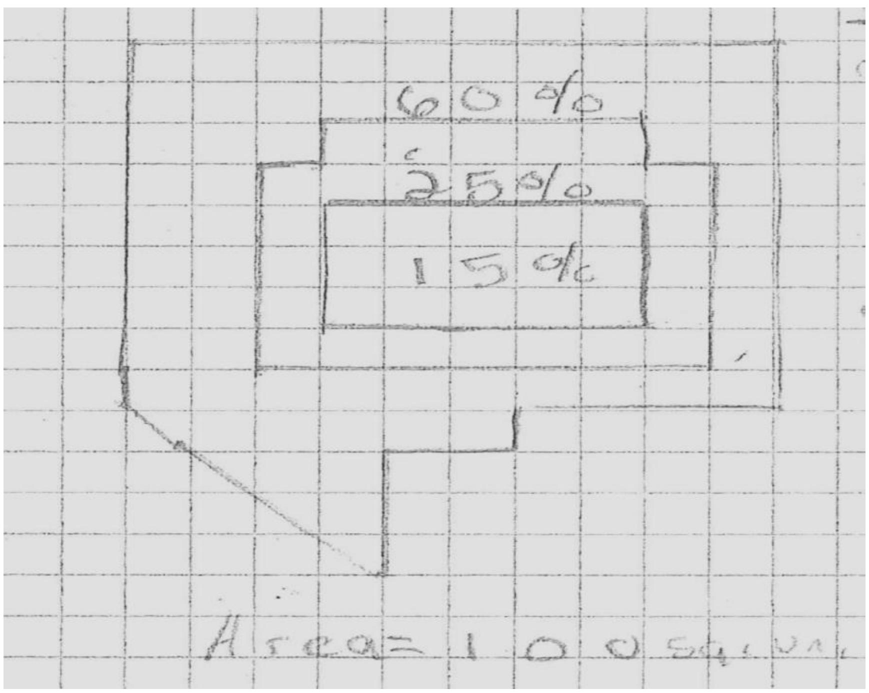


Figure 21. Clark's dartboard

The remaining discourse, in which Teacher 9 checks in about how many squares are in the middle and outer sections, is related to the student's solution, but fails to contribute any useful thinking about the student's strategy of starting with the 15. For example, Teacher 9 could have tried to make a connection between the number of squares in the middle and outer sections and their corresponding percentages as this would have built upon the idea that Clark proposed about using 15 squares to correspond to 15%. We see in this example that Teacher 9 was faithful about implementing this planned question in his debrief, but he failed to follow through with the student's thinking. We see the IRE discourse pattern here when the teacher asks the planned question, the student responds, and then the teacher evaluates when he says "okay". The response of "okay" is an example of a T3, or evaluating comment.

Example 2 of a drop-in. Sometimes, there is a very notable lack of connection between a drop-in question and the surrounding discourse. A drop-in question will likely occur because the teacher is being diligent about implementing the ISA even if there is not a clear connection between the planned questions and, either the ideas to highlight planned by the teacher, and/or what the students are choosing to share as they present their work. In the following example, I demonstrate how a drop-in question can feel somewhat unnatural with respect to the surrounding discourse. The example in table 25 of a drop-in question is from Teacher 22's Design a Dartboard debrief. In this segment, she planned to ask "Does it matter whether or not each section is exactly the same shape?" This question was planned with respect to the fact that the presenting student's dartboard had three different shapes for the inner, middle, and outer shapes. Creating a dartboard with differing shapes would be in contrast to a dartboard in which all three shapes were either similar or all the same type of shape (e.g. all triangles). The episode below picks up where the teacher had just finished helping Shannon, the presenting student, to share her work and explain how she used 100 squares for her dartboard because it was easy to get the percentages since each square took up 1% of the dartboard. The end of this transcript marks the end of the presentation of that student's work. I include the end of this discussion about percentages in the transcript below in order to illustrate the disparity that existed between the dialogue that was already taking place and the dialogue that emerged from the planned question.

Table 25. Teacher 22 Design a Dartboard Episode (transcript 2)

T22	Okay, so you did start on the outside and work in, but you knew how many squares you had to have because you said a hundred squares equals a hundred percent and you just keep redrawing it until it worked.	R4
Shannon	Yeah.	
T22	Ok. Um, anything else you wanted to add to that? What you were thinking when you were working on this?	**Q3**
St.	I was just thinking that 60 squares would be out of 60 and the middle could be 25.	
T22	**Um, did you guys notice that her shapes aren't all exactly the same?**	**M4**
Sts.	yeah. Of course	
T22	**Did we ever say in the directions that all the shapes had to be the same?**	**Q1**
Sts.	no.	
T22	No!	R1
T22	We didn't.	T3
T22	Um, I think that's all I'm going to say on that one for now.	NA

This episode, and the overall segment, were well aligned with what the teacher had planned. In her ISA, the teacher identified her ideas to highlight as follows: “Show that one square can equal 1%. Also, not every square [section] has to be the same shape.” The first half of the discussion of this student’s work successfully addressed the first half of her planned idea to highlight (“Show that one square can equal 1%”). At the very end of the debrief (the episode shown here in table 25), Teacher 22 addressed her planned question of "Does it matter whether or not each section is exactly the same shape?" The questions she asked to address this planned question and the discourse that took place prior to when she addressed this question were not connected. She is making a sudden shift from talking about percentages to talking about the student's choice of shapes. We again see the IRE pattern when she asked the question "Did we ever say in the directions that all the shapes had to be the same?" which was coded as Q1 because there is only one appropriate response to this question. The students responded "No." and she evaluated the response in a way that affirms they were correct (R1, T3). The question that she asked likely did serve its purpose of making the students aware that it was okay for them to create shapes that were not all similar, but it did not afford any opportunities for the students to reason about the task, nor does the question fit with the other ideas that the class had discussed with this piece of student work. She could have pressed the class further to reason about whether or not the condition of similar figures should be a required part of the task, but it is possible she simply wanted her students to recognize that it was okay to make a dartboard in which the inner, middle, and outer shapes were all different.

The IRE discourse pattern was a common occurrence with drop-in questions. While the IRE discourse pattern may have emerged as the teachers were implementing the planned questions in the other ways described below, what made the use of the IRE patterns unique to drop-in questions was that fact that there was little evidence of other discourse taking place surrounding the planned question beyond this IRE pattern. That is, the drop-in questions are characterized by the question being initiated by the teacher, responded to by a student (or multiple students), evaluated by the teacher, and then the discussion moves away from the intended focus of the question to a different topic. Note also, that in the first example of a drop-in question, the planned question was a high-press question (a request for the student to justify a part of their strategy) while the planned question in the second example was low-press (short-answer). However, in both examples the opportunities for students to reason mathematically were very limited.

Embedded. An embedded question is one in which the teacher does not actually ask the planned question because the question was already addressed by the students, without the teacher prompting them. This may be a logical move on the part of the teacher because, if they asked the planned question after a student had already made a statement that would have counted as a desirable response, it might seem odd to the flow of classroom discussion because the answer was already stated earlier, creating a feeling of redundancy. However, the dilemma with this implementation of a planned question is that, if the teacher does not respond in some way to what the student said, the students may not realize that this idea proposed by the student was important to the teacher. The codes for teacher utterances surrounding this type of implementation varied because embedded questions are characterized by the question being 'addressed' in the absence of teacher prompting. Q3 (request for student explanation) may be a likely code if the desired response emerges out of the teachers' requests for students to share their thinking about the problem.

Example 1 of embedded. The following example in table 26 of an embedded question is from teacher 22's Snack Shack debrief. In this segment, the teacher planned several questions: "What do you notice about this paper? What's the Same? Different? Is every part (label) addressed?" We see in the episode shown below that the teacher explicitly asked the first three questions altogether when she said "Let's go with one observation. A similarity, a difference, or just an observation." (The episode below begins with the teacher re-asking these questions, just after she had given them some time to think about the questions). Within this episode in which the students shared their responses to these questions, we see that the students addressed the question "Is every part (label) addressed?" without needing to be prompted by the teacher, as multiple students made comments pertaining to the fact that the presenting student's work was clearly labeled. Since the teacher never had to ask this planned question, but instead the students addressed it without any prompting, this question was *embedded* in the students' discussion. The utterances that support a response to the question of "Is every part (label) addressed?" have been highlighted in bold in the transcript below.

Table 26. Teacher 22 Snack Shack Episode (transcript 3)

T22	Let's go with one observation. A similarity, a difference, or just an observation. Clarence.	Q3
Clarence	I don't know.	
T22	What did you guys talk about?	NA
Clarence	Nothing.	
T22	You all just sat there and nobody said anything?	NA
Clarence	Like one word.	
T22	Does somebody in the group want to help out?	Q3
Student:	Well, what we said **was it looks well thought-out and because the person put the amount of money and the number of cases it looks easier to add** because they have a good rounded number like 10 and 20 cases and they know how much money that is so they can start subtracting one case at a time to find 50 cases and then get 200 dollars.	
T22	All right.	**M3**
Student	They only used 10 cases and 20 cases and they didn't use any other number. ___________[10]	
T22	Okay, so they stuck with ten and twenty cases.	R2
T22	Do you have any idea why they might have done that?	Q3b
Student:	no.	
T22	Could you take a guess?	NA
Student:	Because like 30 and 20 equal 50 easier.	
T22	All right. So they add up tens and twenties to get to 50 easier. Eleanor?	R2
Eleanor	We thought it was confusing to have the lines in the middle.	

[10] Underlined spaces indicate utterances that were inaudible in the recording

T22	That was these lines- the horizontal lines?	Q2
T22	Okay. All right. Megan?	M3
Megan	**I thought it was a lot more organized than the last one; you could read it better. How many cases you can get for how much money**.	
T22	**All right, so it seemed more organized and easier to read**. Tyler?	R2
Tyler	It went way more than 50 cases.	
T22	Oh, if we added up all of the cases it would be way more than fifty.	R3
T22	Okay. What else, ____?	NA
Student	Well, I thought she was adding 20 and 10 first for the candy bars, but then-	
T22	Is that here?	**Q2**
Student	Yeah, but then, but yeah like Megan said **that one side was money and then one side is how many cases they had.**	
Student	**They were more organized**.	
T22	**More organized**.	**R1**
T22	Okay.	**M3**

Students made statements like: "…because the person put the amount of money and the number of cases it looks easier to add…" ; "I thought it was a lot more organized than the last one. You could read it better. How many cases you can get for how much money." and "that one side was money and then one side is how many cases they had." All of these statements alluded to the fact that the students were recognizing that the chart was clearly labeled. However, these comments were all made in passing with minimal follow-up by the teacher. The teacher either repeated (R1) or rephrased (R2) some of these utterances about organizing and labeling the graph but she did not say anything to lead students to recognize that this idea of clearly labeling was a part of her debrief plan, or something that she considered important. Further, she pressed her students to reason about why the presenting student chose to use only 10 and 20 cases even though this was not identified as part of her original plan. It is possible that Teacher 22's desire was for her students to recognize that this student was labeling her chart and when she observed through the discourse that this was something they were all observing, she may have felt there was no need to further emphasize it. On the other hand, it may have been useful for the students if she had engaged them in a more detailed discussion in which they considered what they specifically liked about her labels, or how labeling the graph made the student's work better in contrast to a graph that was unlabeled.

An embedded question may be problematic in terms of clearly accomplishing the goals of the lesson plan because the students may not be aware that these embedded responses put forth by their peers are important to the teacher unless the teacher says something to help emphasize the idea for the students. This type of implementation of a planned question may be risky because the students are not given any cues from the teacher that the presented ideas were considered important, which may cause them to move on without considering the students' utterances. In classes where the teacher shares the mathematical authority with her students, though, this would likely not be an issue. Research has shown that students can be intellectually autonomous in mathematics classes and they know, based upon social norms, that they should be aware of, and draw on, their own intellectual capabilities when making mathematical decisions and judgments as they participate in discourse with their peers (Yackel & Cobb, 1996).

In classrooms where mathematical authority is shared, students may provide input and it will be received as viable input from their peers without it necessarily being identified as such by the teacher. However, this is not a typical norm for most mathematics classes. In order for a teacher to ensure that his or her students are aware that an idea presented by a student (or group of students) is important with respect to the mathematics being addressed in the class, the teacher may need to make some improvisational moves that further draw the class's attention to the planned question that is being implicitly addressed. Later on, I will provide an example that illustrates how another teacher, when one of her planned questions was embedded in the discourse, followed through with the important ideas from the planned question in a way that highlighted the students' comments as important.

Telling. A telling question is when the teacher does not give students an opportunity to answer the question, but the question is clearly addressed by the teacher. This happens when the teacher talks about the important ideas from the planned question without ever posing the question to the class.

Example 1 of a telling question. The episode in table 27 is an example from Teacher 27's Snack Shack task. The teacher planned to ask the class "Why is focusing on the money first more difficult?" This question refers to the process of finding a combination of cases of soda (for $3 per case), chips (for $5 per case), and candy bars (for $8 per case) that add up to a total of $200 without worrying about there being 50 cases with the intention of then adjusting the number of cases to get 50 total. This process would require finding a solution to the equation $3x+5y+8z=200$. An alternative, but similar, strategy would be trying to find a combination of cases of soda, chips, and candy bars that add up to 50 cases, which would require finding a solution to the equation $x+y+z=50$. Instead of posing the question to the class first, T27 just jumps into explaining what makes it more difficult to start with $200. In this example, he doesn't give the presenting student an opportunity to share her own thinking on the problem.

Table 27. Teacher 27 Snack Shack Episode (Transcript 4)

T27	So here was the thing about trying to start with either the money or the cases. For this one, she tried to start with the money. So, you'll notice here you've got a hundred dollars, hundred and ten, thirty, that's too much money. This one, here's finally 200 hundred dollars, but there's only 15, let's see, soda was 3. So 15, 15 and 10 was 40, so not enough cases. Then she tried to change some of the chip ones to soda ones.	**R4**
T27	**The hard part was you had to go through all these calculations. You made it 200 dollars, but then you had to kind of divide to figure out how many cases there were and then it still- this one still didn't get to the right sum.**	**T3**
T27	She got closer, so a few more trades using the one we're talking about and she would have been okay for a solution.	**T4**
T27	What I did like was there was a nice method to this, you could- some of you were trying to erase your mistakes as you went, but by seeing what you tried before, that helps you to decide what to try the next time.	**M3**
T27	Right?	**M4**
T27	**The only thing is, notice this took up the whole page just to get, almost to get-** just one more box and she would have had a solution.	**T3**

In this example above, we see that the teacher addressed the planned question in the midst of reporting out on the student work. The codes in this episode are mostly a combination of reporting out what the student had done (R4) and evaluating the efficiency of her method (T3). Note that, in this example, the statements that addressed the teacher's planned question were all T3. It is worth noting that, in this particular debrief, the teacher had rushed through all of the segments in this manner, telling his students the information they needed to know instead of engaging them in discussions around the relevant ideas. Part of the reason that he chose to do this, was because he was planning on engaging his class in an activity in which the class worked together to develop a new strategy for finding additional solutions. As a result, he may have chosen to address questions like this himself, instead of engaging his students in a lengthier discussion, for the purpose of saving time for the part of his lesson that he considered most important.

Example 2 of a telling question. This next example in table 28 of a telling question is from Teacher 22's Design a Dartboard task. In this episode, Teacher 22 planned to ask "How many squares out of 180 is 15% of the whole?" This question was planned by the teacher to help address the idea to highlight of "Show the use of a proportion to determine percents." In this example, the teacher chooses to describe for the class the process of using proportions to determine what 15% of 180 is.

Table 28. Teacher 22 Design a Dartboard Episode (Transcript 5)

T22	**And so he said how many squares out of a hundred and eighty equal 15% over a hundred? And then he solved the proportion for the number of squares and he found out that, for his picture, 27 squares is going to be the 15 percent** and then he did similar math for the 25% and found that he would need 45 squares for the second portion of his dartboard and 108 squares for the 60%.	**R4**
T22	Do those numbers look reasonable compared to the 180?	Q1
sts	yes.	
sts	No.	
T22	They look pretty good. Don't they?	
sts	Yeah.	
T22	The 15%, the 27 squares is the smallest number, but it's not that much smaller than the 25%, okay? And the 108 squares is more than twice as much as the 45 which makes sense because 60% is more than twice as much as 25%.	T2
T22	So I thought it was cool that he actually used the proportions instead of using a hundred like Shannon did, for the hundred percent.	M3
T22	Jared started with 180 squares as the hundred percent and then found out how many would be 15% or 25% or 60%.	R4

In this second example, Teacher 22 provided an answer to her planned question (that is, that 27 squares is 15% of 180) and also described for the class how the student went about determining this percentage and she verified how one might conclude that 27 is a reasonable solution for determining 15% of 180. In this episode, the teacher involved the class by asking whether or not the solutions seemed reasonable, but she did not acknowledge that some of her students had said 'no', indicating that they did not think the numbers seemed reasonable. By not attending to their thinking, Teacher 22 missed an opportunity to help her students to reason through their errors (Fraivillig, Murphy, & Fuson, 1999). Also, the second question she asked them ("They look pretty good. Don't they?") was a rhetorical question in the sense that she expected agreement from her students. This episode was dominated by the teacher and the only true student input is in the form of the summaries that Teacher 22 is providing of the student's work (R4).

What both of these examples have in common, with respect to teacher utterances, is a large amount of reporting in the form of descriptions of the presenting students' solutions. In both of these cases, we see that the teachers used student work as a vehicle for sharing ideas about the tasks with their students but the students do not get an opportunity to actively engage in reasoning and discussing the planned questions. There are many possible reasons why a teacher might choose to address a planned question in this way. It is possible that they make an on-the-spot decision to address the questions themselves because it seems more time-efficient (Stein, Grover, & Henningsen, 1996). Another possibility is that teachers may occasionally feel that their students need to hear a particular explanation directly from them rather than trying to make sense of what the other students are saying (Baxter & Williams, 2010). In some cases, a teacher may choose to implement a planned question as telling because he or she may think that the students are not capable of successfully addressing the question (Fennema et al., 1996). Later, I will discuss an example in which the teacher attempted to meaningfully address a planned question with her students, but the discussion degenerated into the teacher telling the class the information they needed because she was unable to elicit any worthwhile responses from the students.

Sustained Focus. The final type of question implementation is a sustained focus question. These are planned questions that are posed by the teacher and the teacher continues to focus the class on the ideas connected to the planned question. This particular category will be discussed in greater length than the previous ones, because how the focus is maintained on a question can vary significantly and I will be discussing some key features of a sustained focus question that may positively impact the opportunities for students to reason mathematically. Ideally, the questions teachers ask as they sustain the focus on a planned question should be high-press, encouraging multiple students to think deeply about the question and to provide well thought-out responses (Kazemi & Stipek, 2001). In contrast, the teacher may ask follow-up questions to maintain focus on the planned question, but the questions are primarily evaluative or checking for understanding, resulting in low-press discourse. This next example illustrates how maintaining a high-press discussion around a question can be challenging to accomplish.

Example 1 of a sustained focus question. In the following example in table 29 from Teacher 23's Design a Dartboard, the planned question was "What was your plan?" As identified in Teacher 23's idea to highlight for this segment, she wanted to focus the class on Jolene's strategy to let the total area be 267 and how 267 is the same as 100%. We see in the transcript below, that as the presenting student, Jolene, is attempting to explain her approach (or plan), Teacher 23 is trying to focus her explanation on her use of decimals to determine the percentages of 267.

Table 29. Teacher 23 Design a Dartboard Episode (Transcript 6)

T23	Ok, Jolene. **What was your approach?** What did you start with?	
Jolene	I started by choosing the numbers____________________	
T23	Am I understanding you that you drew kind of like a cross, you drew that first and then counted the boxes?	**Q2**
T23	OK. And then what did you do from there?	**Q3**
Jolene	I multiplied. And then I multiplied by the ______________.	
T23	I can't see here, let's refocus it down on the bottom.	M1
T23	Ok, so down here at the bottom you took your total number, right? Which was 267 and you multiplied that by .15, 15 hundredths.	Q2
T23	Why did you multiply by 15 hundredths?	Q4
Jolene	To make it easier.	
T23	To make it easier for what?	**Q2**
T23	Why .15? Why not .17?	**Q4**
T23	Where did the .15 come from?	**Q2**
Jolene	The yellow paper.	
T23	From the yellow paper, yeah.	**R1**
T23	What is represented?	**Q3**
T23	Who can help her out?	**M4**
T23	I don't think she's understanding what I'm trying to get at.	**M3**
T23	Who can help her out?	**M4**
T23	Where did the .15 come from? Jamie?	**Q3b**
Jamie	15%.	
T23	15%. 15% as a decimal is .15, right?	**R3**
T23	Ok, so she knew that. It's the English that… and my lack of Spanish to ask her well enough what I'm trying to get at.	**M3**
T23	Ok, and then here you multiplied by…by 25 hundredths because that is the same as what, everybody?	**Q1**
sts	25	
T23	25%.	**R2**
T23	And then here multiplied by 60 hundredths. She said that very well.	**R4**
T23	Because that is the decimal form, or the rate, we call it, of the 60%. Right? So she multiplied the total number of the squares by those numbers.	**R4**
T23	Ok. And then what does your answer represent here? 40. What does that tell you? Ok. Help her out. What does the 40 give? 267 multiplied by 15 hundredths and 40 is her product, her answer. What is 40? What does that tell her?	**Q1**
st	40 is 15%	
T23	40 is 15%. So 40 squares would be 15 percent of the total number of squares that she has so that the smaller section of her drawing should be how many squares?	**R1** **Q1**
sts	40	
T23	40 squares.	**R2**

In this example, Teacher 23 pressed Jolene, the presenting student, to provide more information about her choice to multiply by the decimal .15 to obtain the area of the smaller shape. Teacher 23 asked questions like "Why did you multiply by 15 hundredths?" and "Why .15? Why not .17?" These were high-press questions that the teacher asked in an effort to support Jolene in being able to provide a mathematical explanation. Jolene struggled to appropriately answer the teacher's questions, so Teacher 23 began posing the questions to the class as well. Despite Teacher 23's efforts to ask high-press questions, she was, in general, only able to get responses out of the students when she reworded the questions to be low-press such as when she asked "…you multiplied by 25 hundredths because that is the same as what, everybody?" In the end, Teacher 23 resorted to providing all of the explanations and asking only evaluative questions of her students. This is an example of the challenges that teachers can face when trying to sustain the focus on a question. Here, we see that, despite the teacher's efforts to elicit meaningful explanations from her students, she was only able to elicit brief statements of lower-level thinking.

Example 2 of a sustained focus question: Four teacher moves. My second example, in table 30, of a sustained focus question stood out to me as particularly successful based upon the fact that multiple students were able to provide clear, mathematical explanations and reasoning as a result of the manner in which the teacher structured the implementation of the planned question. This episode comes from Teacher 9's Snack Shack task. In this example, the students' work being presented was chosen because they had found a solution that cost 200 dollars, but had only 42 cases instead of the required 50 cases. The planned question was "How can the numbers be adjusted to increase or decrease cases?" The focus of this discussion was on strategies the students came up with to take this incorrect solution and modify it to increase the number of cases without changing the cost and, ultimately, using their strategies to find a solution that worked.

Table 30. Teacher 9 Snack Shack Episode (Transcript 7)

Lucas:	For these one, we didn't exactly get 50 cases but we got 42, we got 12 chips, 10 candy bars, 20 sodas that equaled 200 dollars.	
T9	Ok, one of the reasons I thought this was interesting…[St. because it's like hard enough to get 200] it's hard to get it exactly 200.	M3
T9	So how long did it take you to get 200?	Q1
Lucas:	Only like 10 minutes	
T9	Five or six tries? What I'm curious about right now - Lucas and Tyler came up with this solution. Can we tweak it so that we somehow get more cases, but the same amount of dollars?	T1b
T9	So, take 30 seconds.	M1
T9	Everyone write down that. Write down 12 chips, 10 candy, 20 soda. Yeah.	M1
T9	Write it on your sheet somewhere and, with your partner, see if you can change it somehow, exchange cases for case or something like that so, I don't know, maybe you get 43 cases or 44 cases and 200 dollars.	T1b
	[Students working]	
T9	15 more seconds.	M1
St.	We got one that's exact!	
T9	The question was I want more, I just want either 43 or 44 cases, ok?	T1b
T9	Can you change it so we get 43 or 44 cases?	M4
St.	Yeah.	
T9	This is a new question. Ok. Anyone get it? Jon?	Q3
Jon	Yeah. You can actually take away one of the candy and add two more.	
T9	How do you do that?	Q3
Jon	You take away the candy and you add the 5 dollar chips and 3 dollars sodas and you gain one every time you do that.	
T9	Wait, say that again.	**Q2**
Jon	You need 8 more cases. Every time you take one candy bar from the ten, you can add one chips and one soda.	
St.	I understand that!	

St.	9 candy, 13 chips and 21 soda.	
T9	So if I take one of these away, then I have to….[St.: That means you would have to take 4 away]	**Q2**
T9	So now this is 13 and this is 21?	**Q1**
St.	Yeah. And you're still at 200 dollars.	
T9	We are?	**Q2**
St.	Yeah, cuz…	
St.	The candy bars are 8 dollars!	
T9	No, no. Convince yourselves. Are you still at 200 dollars?	**Q4**
St	5 by 3 is 8.	
St	Yeah, you take away 8	
St	5 times 13 is…	
St	9 times 8 is 72	
T9	65… 9 times 8 is… 72…. 3 times 21, 63.	**R3**
T9	Add all these up we get 200? That equals 200?	**Q1**
St.	You just have to take off the candy for each one and then add two onto each side.	
T9	Ok, so can we do it again?	Q1
[Students go on to find a solution with 43 cases and then a solution with 50 cases.]		

The preceding example showed implementation of a planned question with deliberate moves on the part of the teacher to optimize the meaningfulness of the questions for the students. From this example, I identified four teacher moves that were useful for sustaining focus the focus on the planned question. I call these four teacher moves contextualization, problem-posing, think-time, and follow-up (see table 31 for a summary of these categories). Contextualization refers to discourse that leads up to the question and that ensures the class has the necessary background information needed to successfully address the given question. In this example, the contextualization was as simple as teacher 9 drawing attention to the aspects of the student's work that were relevant to the question about to be posed. We see teacher 9 drawing attention to the fact that it was very difficult for the presenting students to come up with an example in which they had exactly $200, but not enough cases. I coded this utterance from the teacher as

Motivating because the teacher was giving the class reason to value the non-solution that this group had found and to use it to find a better solution. This non-solution of 42 cases worth exactly $200 was used to pose the planned question, providing the class with a mathematical object (that is, the solution with 200 dollars, but not enough cases) to work with as they addressed the planned question.

Research has demonstrated the potential value of providing appropriate contextualization of a question as a way to promote students' successful engagement with the question. In a study that compared three pre-service teachers to three in-service teachers performing an arithmetic problem-solving task with their students, it was found that the in-service teachers spent more time making sure that their students understood the problem before beginning to discuss with their students how to solve the problem. In contrast, the pre-service teachers spent less time guaranteeing that their students understood the problem, initially, but then had to refocus the discussion on understanding the problem as the students struggled to identify the correct operation (Rosales, Orrantia, Vicente, & Chamosa, 2008). In a case study of a teacher who was identified as particularly successful implementing a reform-oriented curriculum, the researchers identified that she supported students in solving problems by reminding them of conceptually similar problems previously solved in the class as well as providing the necessary background knowledge to ensure that the students would be able to solve a problem with understanding (Fraivillig, Murphy, & Fuson, 1999).

Problem-posing refers to the teacher asking the planned question in a way that provides more detail than what was recorded in the ISA. This would be for the purpose of ensuring that everyone understands exactly what is being asked of them. While the question from teacher 9's plan was worded as "How can the numbers be adjusted to increase or decrease cases?" the question was posed to the class as "Can we tweak it so that we somehow get more cases, but the same amount of dollars? So, take 30 seconds. Everyone write down that, including Zac and Jack. Write down 12 chips, 10 candy, 20 soda. Yeah. Write it on your sheet somewhere and with your partner, see if you can change it somehow, exchange cases for case or something like that so, I don't know, maybe you get 43 cases or 44 cases and 200 dollars." He modified the planned question in such a way that he tied the question directly to the presenting students' solution; he gave specific moves for the class to make in order to get started on the problem (writing down 12 chips, 10 candy, and 20 soda), gave specific directions on how to solve the problem (30 seconds, work with your partner). He also provided a hint on how to do it ("exchange case for case or something like that"). It may be argued that he gave too much scaffolding to his students. However, research has shown that some scaffolding is necessary to maintain the cognitive demand of a task (Henningsen & Stein, 1997). Another research study showed that minority students were more likely to respond to a question when the teacher was clear about what mathematical ideas the question was intended to address and how the students were expected to respond to the question (Parks, 2010).

Once the question was clearly stated by the teacher, some think-time was provided so that all students were given an opportunity to consider how they might respond to the posed question. While think-time may be as basic as a few seconds of wait time before asking for students to share their responses, or as extensive as 10 minutes of small-group work, in this case teacher 9 gave just 30 seconds for the class to work on the problem. In this example, the amount of time given seemed sufficient for several students to have come up with a solution. While it is not possible to tell exactly how many students were participating in the discussion of this question, multiple students chimed in with solutions, comments, and explanations. The benefit of think-time is that the teacher provided the class with enough time to think about the question and, as a result, more students felt comfortable sharing their responses. In a research study comparing the discourse of teachers who were asked to allow their students at least 3 seconds of think-time following questions to the discourse of a control group where think-time was not a focus, they found that students in the classes where think-time was provided were more likely to respond and provided longer utterances in response to teacher questions (Tobin, 1986). Another study showed that students were more likely to share their thinking with the class when they were first allowed to respond to the question either individually or in small groups (Parks, 2010).

Finally, we see Teacher 9 follow-up with the class as they provided solutions. He pressed Jon, who offered up his solution strategy, to be more detailed about what his strategy was when he asks “How do we do that?”, a request for explanation (Q3), and then “Wait, say that again”, a request for clarification (Q2). As the rest of the class joined in on the discussion, Teacher 9 continued to ask clarifying questions and asked the class to verify that their new solution was still at 200 dollars “Convince yourselves; are you still at 200 dollars?” He then followed up with extension questions (“can we do it again?” and “can you get to 50 cases doing this? I don’t know if we can…Try it! Try it!” were additional statements made by Teacher 9 following the episode transcript in table 30). While a focus on obtaining a correct solution can lower the cognitive demand of a task like this one, Teacher 9's press for thinking and reasoning from his students was useful for maintaining the cognitive demand of the task (Doyle, 1988).

Table 31. Four Potential Teacher Moves of a Sustained Focus Question

Contextualization	Dialogue preceding the question is deliberately orchestrated to help support students' understanding of the question. This could be as simple as making a statement to draw students' attention to a key element of the upcoming problem or as focused as engaging the class in a discussion of the necessary background information needed to successfully engage in the planned question.
Problem Posing	In addition to just asking the question, there is additional wording from the teacher to make sure all students understand what is being asked of them.
Think time	Teacher allows time for students to consider the question, rather than expecting immediate responses from the students. This could be a few seconds of think time before the teacher calls on students to respond, or several minutes of small group activity, allowing all students time to grapple with the question.
Follow-up	The teacher asks additional pressing questions to sustain the classroom's focus on the posed question. This may include one or more of the following: Encouraging students to clarify their thinking, add additional explanation, justify their reasoning etc. Requesting multiple students to contribute. Asking extension questions to further thinking on the posed problem.

Example 3 of Sustained Focus. These four teacher-moves of a well-implemented, sustained focus question that I described above for Teacher 9's debrief were also evident in many other episodes from all of the teachers. These four moves were not always used together (for example, a teacher may give her students think-time, but not necessarily contextualize the question, or the four teacher moves may all be evident, but the follow-up fell short because the teacher ran out of time). Below, I give another example of a sustained focus question in which we can see these four teacher moves, but they look different from the previous example. This example, shown in table 32 is excerpted from Teacher 27's Spinner Elimination task. Just before this episode, the class had figured out that no more than 15 squares could be crossed off of the 50's chart if your spinner only has the numbers 1, 2, and 3 on it. This discussion that led up to the following example had been based upon the planned question of "How does this [having only 1, 2, and 3 on your spinner] affect the possible results?" The implementation of this previous question had been sustained focus with low-press questions as the students were only prompted to list possible multiples that could be made with 1, 2, and 3. In the following example, the class is addressing the question "Is a 4 necessary?" The goal of the question was for the students to recognize that any multiple of 4 can be crossed off of the 50's chart as long as you have a 2 because 2 times 2 is 4. Notice from the excerpt below that he modified this question to ask, "If I have the same numbers, but now with a 4 on it, what's going to happen here?" This eventually led to the generalization that a 4 is not necessary if you already have a 2 on your spinner.

Table 32. Teacher 27 Spinner Elimination Episode (Transcript 8)

T27	So, if I go back to this. If I just have 1, 2, 3, 4. Let's do what we did before.	M2
T27	Actually, the last time we had this: 1, 2, and 3. What's going to happen if I add a 4 in terms of my possibilities?	T1b
T27	Raise your hand.	M1
T27	Think about this for a second.	M1
T27	Remember, this is Henry's one and we said he had 15 out of 50 possibilities, right? But the most number of squares he could get was 15.	T1c
T27	If I have the same numbers, but now with a 4 on it, what's going to happen here? Peter.	T1b
Peter	You're going to have at least ________ numbers. You'll have 20 at least.	
T27	So you're guessing that with this we might have 20 possibilities out of the 50? Or more maybe?	R2
T27	Any other ideas?	
St 1	I think, because 4 times 3 is 12, so that's one and 4 times 2 is one, well, I guess you could do it again and again. Yeah. I think it would be 20 plus.	
T27	So she thinks it will be 20 plus?	R1

T27	What do the rest of you think?	Q3
St 2	With just the 1, 2, 3 and you add one more you have 4 other possibilities. You have 4 times 3, 4 times 2, 4 times 1, so that would be 20.	
T27	Does anybody think it will be even higher, like 30?	M4
St 3	Maybe. It could be like 25.	
T27	Does anybody think it will be less than 20?	M4
St 4	No, I don't, but when you did the 1, 2, 3 it seems like each one gives you 5 possible entries because 3 times 5 is 15.	
T27	Right, but there's more than one 2. We can use all the 2's and 3's that you want.	T2a
T27	For this sequence of numbers, what is possible? So, Kelsey, what's possible hits on my chart if this is my spinner?	M2
[*Sts list out all of the possible combinations*. *A student realizes that a spinner with 1, 2, 3, and 4 would have the same possible outcomes as a spinner with just 1, 2 and 3*.]		
T27	What are you realizing?	Q3
St 1	You can have 2 and you spun 2 twice on the spinner: times 2 times 2. You get 4 and then if you did any of those other numbers you would get that same sequence of numbers on the board and so…	
T27	As what?	Q2
St 2	You can have 4 times 4. That's true but you could also do that instead because you can do 2 times 2 times 2 times 2.	
T27	So then what should I get in terms of my answers?	Q1
St 1	You should get all the same or less.	
St 2	15 out of 50.	
T27	Do you guys see what he's saying?	M4
Sts	[students respond with some yes's and some no's.]	
T27	He's saying this four can be created from what? The 2's which we already had.	R4
T27	We've already got 2's. We've got all the two's you want. Which means adding this 4 is just going to give you duplicates, right?	T4

In this example, Teacher 27 rephrased the planned question of "Is a 4 necessary?" in a way that allowed the previous example of a Spinner with just the numbers 1, 2, and 3 on it to be a contextualization for this new question. Rather than asking whether or not a 4 was a necessary number on the spinner, generally speaking, he framed the question with respect to comparing a spinner with the numbers 1, 2, and 3 on it to a spinner with the numbers 1, 2, 3, and 4. He does this by posing the question as follows: "The last time we had this: 1, 2, and 3. What's going to happen if I add a 4 in terms of my possibilities?" and he carefully poses the problem by going on to say "Remember, this is Henry's one [with the numbers 1, 2, and 3] and we said he had 15 out of 50 possibilities, right? But the most number of squares he could get was 15. If I have the same numbers, but now with a 4 on it, what's going to happen here?" By connecting the problem to the prior discussion, he is making sure that the class is clear what he is asking them. Also, he gave the class a little think-time by asking the students to raise their hands (as opposed to blurting out their answers) and to "think about this for a second". By carefully repeating the question for the class, he gave the students about an extra 20 seconds to think about the question before eliciting responses from the class. As the students begin to share their thinking, Teacher 27 chose not to correct anyone (even though all of them had answered the question incorrectly). Instead, he encouraged his students to continue to make their predictions until everyone had made a guess. He then followed-up their initial responses by proceeding to lead the class to list out all of the combinations that you can get with a 1, 2, 3, and 4 on the spinner in the exact same manner that they had done earlier with the spinner with the numbers 1, 2, and 3. When two students realized that they were not going to be able to make any new combinations by adding a 4 to the spinner, Teacher 27 encouraged the students to clearly explain their thinking by asking clarifying questions and then summarizing their finding for the class.

In both of these examples of a sustained focus using the four teacher moves described above, there were a wide variety of teacher utterances. In both examples, the planned question was posed with T1b's[11], giving the students something new to reason about. Both posed questions were accompanied by a couple M1 (classroom management) utterances as the teachers kept the students from blurting out responses and prompted them, instead, to spend some time thinking about the problem. As students shared their responses to the posed questions, in both examples, the teachers made requests for their students to provide further explanation (Q3) and clarification (Q2). The IOTA codes on these transcripts allowed us to see the complex range of teacher utterances that can occur during an engaging discussion, as the teacher is managing the discourse.

Meta-Categories. For each of these four categories (drop-in, embedded, telling, and sustained focus), I was able to find multiple examples from the four sample teachers that fit easily into one of these categories. However, not every implemented question that I analyzed fit clearly into one of these four categories. One of the reasons for this was because characteristics from more than one category would be present in the implementation of a question. This would often happen because the nature of the discourse would shift as the question was being implemented. Most commonly, the variations that I observed were derivations of a sustained focus question. Below are two examples of such variations.

Embedded to sustained focus. I had mentioned earlier about an implemented question that had started out as an embedded question, but in order for the teacher to ensure that the students knew that ideas which emerged from the embedded question

[11] T1b is the initiating code defined as: "Telling students what the new problem is that they are to work on next. Must involve some mathematical contextualization of the next task, in contrast to the managing code for students to work on problem x."

were important, she sustained the focus on the question. In this example, shown in table 33, from Teacher 23's Design a Dartboard debrief, Teacher 23 planned to ask the presenting student "Where did you start? How did you use that starting point to find the sizes of the sections?" Rather than Teacher 23 asking this question, following the student's initial explanation of his work, another student in the class asked him to explain the math the he had used. This prompted the presenting student to explain to the class how, after counting the squares of his largest shape, he used proportions to determine how big his three inner sections needed to be.

Table 33. Teacher 23 Design a Dartboard Episode (Transcript 9)

St.	Would you explain, like, what math you used? You said, "yeah, I did math".	
Cade	So I found out, for 15%, I did 15 over 100 equals X over 346. I counted that in this diamond there are 346 squares and then, so I did 15 over 100 equals X over 346 so I did 100 times, uh, 100 times equals 1 - no - 100X equals 5,190. Then I divided by 100 and I figured out that X was 1.9 (sic). I did the same thing for 25% and I figured out that the rest of that had to be 60%.	
Students	Wow.	
T23	So, it sounds like you started with the total number of squares and worked into the middle more.	R3
Cade:	Yeah.	
T23	You found the total and then, from that, how many squares did the middle have to be and you used the number of squares to determine the shape rather than choosing the shape and trying to make it fit the number of squares.	**R3**
T23	And I wanted you to see that because solution-wise it's easier to work it that way. It's just an easier way to go about it, rather than choose, oh, I think I want it to look like a Z and try and figure out how many squares that's going to be and then make it be 15% of the whole thing. It was easier to know how many squares, at least the way Cade went about it, it was easier to count the squares and then make his drawing to fit that.	**T2**
T23	Ok. Turn to your table partner, please, and explain to them how you would do this same thing if the outside diamond was 200 squares instead of Cade's 346 squares, what if the outside was 200 squares? What would you do to find the inside part?	**T1b**

T23	Tell your table partners.	**M1**
[*Students talking amongst themselves*]		

Following this episode, the class spent a few minutes working on the problem posed by Teacher 23 and then multiple students shared their thinking on the problem with the whole class. In this example, we see that the teacher emphasized what the student had shared by telling the class what she thought was useful about his strategy, even commenting that that was why she had picked his work to share. She then emphasized his method further by giving the class a task to work on, based upon the mathematics that the student had shared. In this example, although it could have been treated like an embedded question in which the teacher does not ask the planned question because it was already addressed by a student without being prompted, the teacher clearly emphasized the idea for the class by sustaining a focus on the ideas presented by the student. In fact, we see some of the four elements described above for a sustained focus question. As the teacher is sustaining the focus on the important ideas from her planned question, she poses a related problem, gives the class some think-time, and (although we do not see it in the episode transcript above) she pressed her students for thinking by asking them to explain their thinking (e.g. "explain how you got 30") and justifying questions ("Why would he multiply by 2 in this case?").

Sustained focus to telling. Sometimes, a teacher will try to sustain the focus on a question, but due to a lack of appropriate responses, they just answer the question themselves, causing the implementation of a sustained-focus question to become telling. The following example, shown in table 34, is from Teacher 22's Design a Dartboard

Task. Teacher 22 explained the student's work to the class when she asked the planned question of "Where did the 300 come from?"

Table 34. Teacher 22 Design a Dartboard Episode (Transcript 10)

T22	So what Andrew has here is 3 triangles, one inside another and it looks a lot like the dartboards we're used to with the circles. Just a different shape, right? And what I notice was that he labeled the outer area 'area A', the inner area 'B' and the smallest area 'C' and then off to the side here he said A +B +C has to equal 300.	R4
	Where did you get the 300 from?	Q3a
Andy	It came with the formula.	
T22	It came with the formula on the internet?	Q2
Andy	Not like, it was like a multiple or something on it.	
T22	A multiple of the percents of 15 and 25 and 60?	Q2
	(no audible response)	
T22	Did anybody else find 300 as a multiple of those? Did anybody list multiples?	M4
Sts.	no.	
T22	No? You guys all did it differently?	M4
Sts.	Yeah.	
T22	Yeah. *300 is a multiple of 15, 25, and 60, so we know that all of those* [A, B, and C] *are going to go into there* [the 300].	T1a

In this example, Andy, the presenting student, gave an answer to the asked question that did not fit with what Teacher 22 was expecting to hear, so she prompted Andy a little in the hopes the he would say that he used 300 because it was a multiple of 15, 25, and 60 (the percentages of the three areas in the dartboard design). When he did not respond, she turned to the rest of the class, hoping that someone else would recognize that 300 is a multiple of the three percentages. When no one claimed to have used multiples to determine the areas of their shapes, Teacher 22 gave up prompting the students for an appropriate response to her question and answered it herself. In this example, we see Teacher 22 pressing her students to give a meaningful response to the question, but when she was unable to do so, she took over addressing the question.

Because students are also responsible for creating the norms in the classroom, it is not always easy for teachers to elicit exactly the types of responses they are looking for from the questions that they ask (Heaton, 2000). In this case, when Teacher 22 was unsuccessful in prompting her students to share a reasonable response to her question, she simply chose to provide a suitable explanation herself instead of allowing the question to be unanswered.

A Synthesis

The three research sub-questions that have been presented in this study are intended to work together to address the larger research question: "How do teachers' written plans for orchestrating mathematical discourse around problem-solving tasks influence the opportunities teachers create for students to reason mathematically?" In order to make it clearer to the reader how these three sub-questions can work together to create a robust picture of how the teachers in this study used the questions in their ISAs to create opportunities for students to reason mathematically, I will now individually discuss these four teachers' enactments of their problem-solving debriefs for Snack Shack and Design a Dartboard (I will not be discussing Spinner Elimination in an effort to preserve time and space). The purpose of this section is to provide a more complete picture of how these teachers used their ISAs to implement their debriefs. This is in contrast to the snap-shots provided in the analysis of sub-question 3 in which the implementation of just one question at a time was discussed, isolated from the context of the whole debrief.

In this section, I will describe what took place during each teachers' debriefs, using the tools developed for the three sub-questions to help paint a picture of how these teachers implemented their debriefs, focusing on the unique characteristics of how each of these teachers implemented their debriefs to help explain some of the distinct ways that teachers can plan and implement their ISAs and how these differences may impact (either positively or negatively) the opportunities that the teachers created for their students to reason mathematically. To facilitate this discussion of these teachers' debriefs, I will be referencing several transcripts from this chapter, using both their table numbers and transcript numbers as identifiers.

Teacher 22. Teacher 22 was chosen for the analysis of research sub-question 3 because she had planned very few high-press questions and she also asked very few high-press questions during her implemented debriefs. I included her in the analysis in order to see how her lack of high-press questions may have negatively impacted the opportunities she created for students to reason mathematically. Also, she provided an example of how teachers' improvisational moves may further detract from potential opportunities for students to reason mathematically. Her two documented debriefs demonstrate two distinct examples of how her teacher moves failed to provide opportunities for her students to reason mathematically.

Teacher 22's Snack Shack debrief is an example of an ISA that was faithfully implemented with several questions implemented as sustained focus. However, the planned questions she implemented as sustained focus were low-press with few high-press questions asked as follow-up. Most of her follow-up questions were invitations for more students to share. In the ISA, she planned to ask the class, with respect to each piece of presented work "What do you notice about this paper? Similarities? Differences?" For research sub-question 2, this was coded as two questions. The first question, “What do you notice about this paper?” was coded as L3, non-mathematical. This was because the wording of the question included no criteria for what she wanted her students to notice, leaving an open-ended invitation for students to share anything they would like. The second question was “Similarities? Differences?” This was coded as H4, a request for students to make connections, because it was inviting students to compare and contrast the student’s work to the work previously shown. In the implementation of the Snack Shack debrief, teacher 22 asked both of these questions, but students only responded to the first question, sharing what they noticed about the student’s work, but never connecting it to the other students’ solutions. Even though she sustained the focus on the first question by inviting multiple students to share their observations (see transcript 3 in table 26), she did not press her students to make connections and rarely pressed them for further thinking. The way in which she followed-up on the planned questions were useful for eliciting more responses from her students, but did not support or extend their thinking (Fraivillig, Murphy, & Fuson, 1999). As a result, the episodes of sustained focus questions were not useful for creating opportunities for students to reason mathematically.

While Teacher 22’s debriefs were similar in that, for both of them, she planned few high-press questions and, in turn, asked few high-press questions, her Design a Dartboard debrief was implemented differently from her Snack Shack debrief. In contrast to her Snack Shack debrief, she did not sustain the focus on any questions. All of the questions she addressed were either implemented as telling or drop-in and the two high-press questions she had planned in her ISA were implemented as telling. The nature of her questions (mostly low-press) as well as the manner in which she implemented the questions severely limited the opportunities that Teacher 22 created for her students to reason mathematically

The differences in implementation for these two tasks (the use of sustained focus questions versus the use of only drop-in and telling questions) resulted in differing levels of student engagement for these two problem-solving debriefs. All of the segments in the Snack Shack debrief were assigned a participation code in the IFA (indicating that, within those segments, additional students besides the presenting students participated in the discussion). In contrast, all of the segments in Design a Dartboard were assigned a non-participation code (indicating that the only students to speak during the problem-solving debrief were the ones presenting). While both of these problem-solving debriefs were coded as, overall, low-press, the Snack Shack debrief likely afforded more opportunities for students to reason about the task given that they were frequently prompted by the teacher to verbalize their thinking, even if the teacher was not deliberately pressing their thinking.

In Teacher 22's ISAs, she planned few high-press questions that might prompt her students to reason mathematically. In her implemented problem-solving debriefs she similarly asked a limited number of high-press questions. The limited number of high-press questions she planned certainly contributed to the lack of opportunities she created for her students to reason mathematically. This was particularly true in her Design a Dartboard debrief in which the issue was further compounded by the ways in which she chose to address her planned questions, implementing the high-press questions as telling and treating all other questions as drop-in. In her Snack Shack debrief, although she did promote a greater level of student engagement by addressing some questions as sustained focus, this did little to increase the students' opportunities to reason mathematically because she did not sustain the focus using high-press questions. Instead, she focused on inviting more students to share their thinking without holding them accountable for their level of mathematical reasoning.

Teacher 23. Teacher 23 was chosen for further analysis because she both planned a large number of high-press questions and asked a large number of high-press questions. She actually asked significantly more high-press questions than she had planned. In her Design a Dartboard debrief, Teacher 23 was particularly persistent in asking high-press questions as she addressed the questions planned in her ISA. Teacher 23 implemented half of the questions in her ISA as sustained focus. She sustained focus on these questions by asking several high-press questions as follow-up. These high-press questions included asking students to justify why they used certain strategies (H1) and prompting students to explain other students' reasoning (H5). Teacher 23 also pressed students to provide mathematical reasoning with questions that were originally planned as low-press questions. Transcript 6 in table 29 is an example of a low-press question that was implemented as sustained focus, with the follow-up questions asked as high-press questions. While in her ISA she had simply planned to ask the presenting student to share her plan (L1), she further pressed her students to explain why the strategy used in her plan made sense (H1). We also saw, in this same debrief, the example in which a planned question started out as an embedded question but she successfully turned it into a sustained focus question by drawing everyone's attention to the important comments made by the presenting student and inviting the class to think further about the mathematical strategy the student used.

In contrast to Teacher 22, Teacher 23 planned more high-press questions and asked more high-press questions. She showed evidence of focusing on pressing her students to think about the mathematical reasoning behind their strategies. In her Snack Shack debrief, though, her approach was similar to Teacher 22's Snack Shack debrief with respect to how she tended to sustain focus on low-press questions and the high-press questions in the ISA were less likely to be addressed. The high-press questions that Teacher 23 planned in her Snack Shack debrief were either missing or treated as drop-in. She planned and implemented many low-press questions that prompted her students to share their strategies (L1). When she implemented these low-press questions as sustained focus, though, she did ask some high-press questions, which were mostly H5 (requests to explain another student's thinking). Snack Shack was implemented earlier in the year (while her Design a Dartboard debrief was implemented towards the end of the year) and she may have still been establishing norms in her class, focusing on getting students to listen to one another and understand what the other students in the class were doing and thinking.

We see in Teacher 23's debriefs that her plans to ask high-press questions were useful for creating opportunities for her students to reason mathematically. However, in addition to her plans, her tendency to ask her students high-press questions, in general, also contributed to the opportunities that she created for her students to reason mathematically. We saw that, even when she planned low-press questions, she still used the implementation of those questions as an opportunity to press her students to reason mathematically. It also appeared that, as the school year progressed, she became increasingly comfortable with asking her students high-press questions.

Teacher 9. Teacher 9 was chosen for further analysis because he had implemented a debrief in which he had planned a very large number of high-press questions, but asked very few high-press questions during the implemented debrief. While Teacher 9 planned a large number of high-press questions in his Design a Dartboard ISA, his implemented problem-solving debrief did not reflect this. He had planned several high-press questions (five high-press questions total), but asked only one high-press question during the entire implemented debrief. The one high-press question he asked was a question he had planned in the ISA ("Why did you use 15 squares?"). When he asked this question, the presenting student gave the response of "I had, like 100 boxes and, so, 15% of that is just 15 boxes, or squares." Teacher 9 did not follow up on this response, thus implementing the question as a drop-in (see transcript 1 in table 24), creating no opportunity for further reasoning around the student's justification of his strategy. It is possible that, once hearing the student's response, Teacher 9 did not know how to appropriately follow up his statement because it addressed such a simplistic idea about percentages, leaving limited opportunity for further discussion. The other high-press questions that Teacher 9 had planned to ask were also H1, or requests for justification of a strategy (including "Why did you choose the triangle?" and "Why did you cut some shapes into 2 different sections?"). These high-press questions were all requests for students to explain the choices they had made in their strategies and did not necessarily elicit opportunities for the students to justify why their strategies were mathematically valid. These remaining high-press questions were not implemented in the debrief. In fact, Teacher 9 did not ask any additional questions in this debrief because the low-press questions were addressed as embedded questions. It is possible that, because implementation of the first high-press question failed to elicit useful mathematical reasoning, Teacher 9 may have chosen to avoid addressing the remaining high-press questions in the ISA because they were similarly simplistic.

While his ISA initially appeared to have the potential to create many opportunities to press students to reason mathematically through justification of their strategies, the way in which he implemented the problem-solving debrief failed to create these opportunities. Even though his Design a Dartboard debrief was implemented faithfully according to the Implementation Fidelity Analysis, the level of fidelity that was demonstrated during the debrief was largely due to the presence of embedded questions. That is, as students were explaining their work, they inadvertently addressed all of the low-press questions that Teacher 9 had planned such as "What is the area of the big and middle sections?" and "Does the middle section area include the yellow small section?" These embedded questions led to Teacher 9's Design a Dartboard debrief being, overall, faithfully implemented because the criteria for faithfully implemented questions in an ISA was that the students addressed at least half of the questions within a segment (regardless of whether the teacher had asked the question). However, despite the fact that more than half of the questions planned in the debrief were addressed by the students, only one of the high-press questions Teacher 9 had planned in his ISA was addressed. This provides an example of how the literal lesson in an ISA might appear to be faithfully implemented, according to the objective criteria of the IFA used in research sub-question 1, even though the intended lesson was not faithfully implemented. That is, the number of high-press questions planned in the ISA and the number of high-press questions asked by the teacher in the implemented problem-solving debrief were radically different due to the fact that the teacher chose not to address the majority of the planned high-press questions. This shows how when at least half, but not all, of the planned questions are addressed, drastic differences can occur between what is planned and what actually takes place, especially when the high-press questions are not being addressed.

In Teacher 9's Snack Shack debrief, he once again chose to not ask some of the high-press questions that he had planned. However, there was an exception to Teacher 9's tendency to avoid high-press questions when he implemented the planned high-press question of "How can the numbers be adjusted to increase or decrease cases?" as a sustained focus question (see transcript 7 in table 30). By adequately focusing the class's attention on the question, providing them with time to think about an answer to the question, and then following up their explanations with additional questions, Teacher 9 orchestrated an engaging discussion for his students. During that episode in which he addressed the planned question, he asked 3 high-press questions in addition to the planned high-press question.

In addition to this high-press question from Snack Shack being different from the example in his Design a Dartboard debrief because it was implemented as sustained focus rather than drop-in, the nature of the question was also different. Rather than vaguely requesting the presenting student to explain why they made a certain decision in his strategy, the question asked in the Snack Shack debrief focused the entire class on a particular mathematical idea. This question was well-placed because, before the debrief, many students were close to finding a solution, but were not sure how to use their nearly correct solutions to help them find a correct solution. This example demonstrates how an appropriately planned question that is given adequate attention on the part of the teacher can lead to an engaging discussion in which the students are afforded opportunities to reason mathematically. By sustaining the focus on this high-press question, that particular segment was implemented faithfully with respect to both the literal lesson as well as the intended lesson.

Teacher 27. Teacher 27 was partially faithful for the majority of Snack Shack, not because he wasn't following his plan, but because he was choosing to address the questions himself (telling) rather than giving his students an opportunity to answer the questions he planned in the ISA. His use of the selected pieces of student work was to provide his students with examples of particular features of student solutions that would be helpful for the class to generate a complete solution set together. For example, he showed some work to demonstrate that it would be easier to start with 50 cases rather than $200, he also chose some solutions to demonstrate how they organized their solutions. Finally, he showed some solutions that used strategies of trading certain cases for others. Once teacher 27 shared these solutions with the class, addressing the questions through telling, he proceeded with guiding his class through a discussion of how to use systematic lists to come up with all of the possible solutions. Teacher 27 was only partially faithful in his implementation of the Snack Shack ISA. However, it may be argued that, although he did not use the ISA as intended by the MPSM, he did use all of the ISA to guide his lesson. The difference was that his use of the planned questions was limited to keeping track of the points that he intended to make for his students and not for engaging his students in discourse around the mathematical ideas in the task. I might speculate that, in this debrief, he chose to not ask his students the planned questions because he preferred to move quickly through the demonstration of student work in order to spend more time discussing with his class how to generate new solutions. During the presentation of student work, no high-press questions occurred because Teacher 27 was not creating opportunities for students to share. During the follow-up discussion, after the presentation of student work, he asked the students more high-press questions. These questions were mostly H3, requests to generate new strategies, which opened up opportunities for the students to think about the task in a new way.

Teacher 27's Design a Dartboard debrief was distinct from his other problem-solving debriefs because, while in both of his other debriefs he had asked more high-press questions than he had planned, in his Design a Dartboard debrief he actually asked fewer high-press questions than he had planned, even though the debrief was, overall, faithfully implemented. If Teacher 27 had implemented this debrief faithfully, with all of his questions identified as high-press, then how did he end up with so few high-press questions asked in the problem-solving debrief? One reason for this discrepancy has to do with the way that he implemented the planned questions. His first two planned questions were asked as planned in the ISA, and elicited appropriate responses from the presenting students, but he then followed up the questions he asked by rephrasing his students' responses, rather than engaging the class in further discussion about the ideas presented by the students. He also changed the press of a planned question from high-press to low-press. The question was worded to be asked as an opportunity for the other students to explain the presenting students' strategy (H5), but was redirected to the presenting students rather than the class and became an opportunity for them to share their strategy (L1). Finally, Teacher 27 did not ask the last question in his ISA, which was intended to create an opportunity for his students to make generalizations.

Teacher 27's Design a Dartboard debrief was also distinct from his other two debriefs in that, with the other two problem-solving debriefs, he had some specific learning goals in mind for his students which heavily influenced how he orchestrated the discourse. In contrast, his primary goal for Design a Dartboard was to show his class a variety of solutions with no evidence of any other mathematical learning goal. As a result, his Design a Dartboard Debrief took the form of one solution demonstration after another without much deviation or opportunities for students to reason about the mathematics in the task. This lack of a mathematical focus was also evident in Teacher 9's Design a Dartboard debrief as well as Teacher 22's. This suggests that the Design a Dartboard task may not have been an appropriate task for eliciting opportunities for students to reason mathematically.

In summary, the level of mathematical discourse in Teacher 27's problem-solving debriefs was more dependent upon Teacher 27's lesson plans for the discourse rather than the types of questions he had recorded in the ISA. He demonstrated a higher incidence of high-press questions when he had a clear learning goal in mind (in Snack Shack and Spinner Elimination), as opposed to when he was simply planning on students sharing their solutions (in Design a Dartboard). However, in general, Teacher 27 did not create very many opportunities for students to reason mathematically during the problem-solving debriefs because his teaching style tended to be more teacher-focused rather than focused on generating student discourse.

Summary and Implications

When a teacher plans a question for a classroom discussion, what actually happens when the question is addressed can vary significantly. In this qualitative analysis of four teachers implementing problem-solving debriefs, I identified four variations on how the teachers addressed the questions they had planned in their ISA's. Teachers sometimes would ask a planned question with little attention to how the question is impacting the preceding and following discourse. That is, it appears that they would ask the question because it was part of the ISA, but then they would move on as soon as a suitable response was provided. These drop-in questions typically took the form of IRE discourse in which the teacher would ask the question, a student (or multiple students) would respond, and then the teacher would evaluate or summarize their responses. When a high-press question is implemented in this way, the opportunity to reason mathematically is typically lost. Some of the examples of drop-in questions from this analysis felt disconnected from the regular flow of the classroom discourse, as if the teacher asked the question because it was written in the ISA and not because it was an appropriate question to ask to move forward the classroom discourse. This suggests a possible dilemma with preplanning questions for a whole-class discussion because the questions may not tie in directly with how the students are thinking about the task.

When students addressed the ideas in a planned question without the teacher asking the question, this form of implementation was referred to as embedded, because the relevant ideas were embedded in the discussion rather than being made explicit by the teacher's questions. While I conjecture that the teachers often chose not to ask a question already addressed by a student because they did not want to be redundant, I also argue that this type of implementation of a planned question can be risky because the students are not given any cues from the teacher that the ideas present in the question are important. Research has shown that students can be intellectually autonomous, drawing on their own intellectual capabilities when making mathematical decisions and judgments as they participate in mathematical discourse with their peers and they can share mathematical authority with their teacher (Yackel & Cobb, 1996). In classrooms where such norms are practiced, a student may provide input and its usefulness will be evaluated by his peers without it necessarily being identified as such by the teacher. However, in classes where such norms are not established, teachers may choose to follow up on the independently offered mathematical ideas, guaranteeing that sufficient attention is drawn to the ideas in the planned question. An example of this was demonstrated in transcript 9 in table 33.

A third type of implementation that occurred was when the teacher chose not to ask a planned question but, instead, addressed the important ideas to the class by telling the class the desired responses. While it may be effective for guaranteeing that the class hears an accurate mathematical explanation from the teacher, it is not useful for generating discourse. Teachers may have certain reasons for choosing to tell the class the important ideas in a question rather than posing the question to the class. For example, if the teacher is more concerned with moving on to the latter part of a debrief, because that is where, in the teacher's opinion, the most important ideas emerge, they may 'tell' a response to a planned question in order to move on to other aspects of the debrief. Alternatively, a teacher might choose to 'tell' a question to the class because they suspect their students are not capable of providing a reasonable answer to the question and would prefer to simply allow the class to hear an appropriate response.

The last type of question implementation that I observed was sustained focus. In this form of implementation, the teacher maintains focus on the question in various ways, including eliciting multiple responses from multiple students, asking clarifying questions or asking for more detail in the form of explanation and justification. Also, when a teacher sustains the focus on a question, they may ask additional questions in order to extend the discussion further. While sustained focus on a question may occur in many different ways, I observed four teacher moves that frequently occurred in a sustained focus question that were contributive to allowing the students to respond successfully to the planned question. These four teacher moves were (1) contextualizing the question, that is, providing background information that made the question understandable; (2) problem-posing, or asking the question in a way that was understandable for the class; (3) providing think-time so that everyone has an opportunity to think through the question before responses are elicited; and, finally, (4) the teacher provides follow-up questions that presses students for understanding.

I propose that the sustained focus implementation of a planned question has the potential to be the most productive implementation approach of the four described here. However, sustained focus varies significantly depending upon how the teacher is sustaining the focus. The amount of press that the teacher puts on the students is also going to vary and will affect the quality of the discourse. We have seen for example, a sustained-focus question in which, despite repeated efforts to press for thinking from her students, the teacher was unable to elicit high-level discourse from her students and resorted to asking low-press questions (transcript 6 in table 29). Also, we saw an example in which a teacher attempted to implement a question as a sustained focus, but when her students were not providing the response she was hoping for, she resorted to telling the desired response to her students (transcript 10 in table 34). While these examples demonstrate that implementing a question as sustained focus will not guarantee that students will successfully engage with the ideas in a planned question, I propose that posing a question using contextualization, problem-posing, think-time, and follow-up (the four moves of a sustained-focus question), a teacher is going to have greater success getting students to provide mathematical explanations and arguments. An explicit focus on these four teacher moves may be a useful way for teachers to enhance their questioning strategies in their classrooms.

Future research is necessary to better understand the decisions that teachers are making when they implement planned questions in these various ways. How are these teachers' improvisational moves connected to their beliefs about the teaching of mathematics? What are teachers' motivations behind implementing planned questions in these ways? Future research would be warranted to investigate the value behind the four teacher moves that were observed when teachers implemented sustained focus questions. Are these moves consistently useful for improving the nature of discourse? How would the nature of a planned question (high-press versus low-press) have on attempting to address a question using the four teacher moves? In the following and final chapter I summarize the findings from this research study; I discuss the implications from my research study for researchers and practitioners; and I discussion some of the limitations of this study as well as offer recommendations for future research.

Chapter 9. Conclusion

In this conclusion of my study, I will begin by reviewing the findings of my study, providing an overview of what was learned from the data analysis that was conducted to address my three sub-questions. I then go on to discuss the implications of my study, both with respect to research on mathematics education and implications for practitioners. I conclude this chapter, and my dissertation, with a discussion of some of the limitations of this study and some suggestions for future research that would both help to address these limitations as well as move forward with research on teachers planning for discourse around mathematical problem-solving tasks.

Summary of Findings

Mathematics educators promote the use of problem-solving tasks as an effective medium for teaching students problem-solving skills, mathematical reasoning and argumentation, and mathematical concepts and ideas. To accomplish this, the use of a whole-class discussion following implementation of a cognitively demanding task is promoted as a useful way to bring the class together to discuss students' solution methods and to reason about the important mathematical ideas within the task. By taking the time to plan how such a discussion will be orchestrated, the teacher can more effectively lead the discussion in mathematically productive ways. One way that this can be accomplished is by selecting specific pieces of student work to be shared, sequencing the student work to progress the discussions in a productive way, and planning what questions to ask in order to help make connections to important mathematical ideas. The opportunities that emerge for students to reason mathematically are going to be influenced by the nature of questions that the teacher asks. I assume in this study that questions which promote mathematical reasoning have the potential to create more opportunities for students to reason mathematically than questions that prompt students to share thinking without attention to the mathematical nature of their ideas. In this research study, I analyzed teachers' implementation of their self-written plans for problem-solving debriefs, focusing on the teachers' use of planned questions, with a particular focus on questions intended to prompt students to reason mathematically.

Implementation fidelity. The findings from this study showed that teachers, in general, followed what they had planned in the planning forms for their problem-solving debriefs. Teachers were fairly consistent about sharing the student work they had identified and addressing the questions and mathematical ideas they had planned. For the remainder of my research, it was helpful to know that the teachers were consistent about implementing their ISAs. While there were a few examples in which the teacher ignored most of the ISA, in most cases the teachers implemented their ISAs faithfully, with only a few deviations from the original plan.

For my analysis to the teachers' fidelity to their instructional sequence analyses, I developed the Implementation Fidelity Analysis (IFA) tool. This tool allowed me to objectively compare the steps laid out in the ISA to what actually took place during the debrief. Based upon my analysis of the teachers' implementation of their completed

ISAs, this tool consisted of a series of objective questions to ask with respect to the teachers' lesson plans and clearly defined criteria for the assignment of the levels of fidelity. This tool, or an adaptation of this tool, may be useful in future analyses of teachers implementing plans for discourse around problem-solving tasks.

Implementation of high-press questions. In order to investigate the intended lesson in the ISA, I developed question codes that differentiated between high-press questions (questions that promote mathematical reasoning) and low-press questions (questions which promote communication, but not mathematical thinking). This particular coding scheme was useful for assessing the potential level of reasoning present within the questions that the teachers had planned for discourse around a problem-solving task. I was able to use this coding scheme to assess the press of questions that the teachers were planning in their ISAs and compare it to the frequency with which the teachers were asking high-press questions in their implemented debriefs.

Despite the consistencies present between what the teachers had planned in their ISAs and what they did during the implementation of their ISAs, there was not a clear correlation between the number of high-press questions that were planned in the ISAs and the number of high-press questions that the teachers asked during implementation. My hypothesis for this study was that the more high-press questions the teachers planned in their ISAs, the more high-press questions they would ask in the implemented debrief, creating more opportunities for their students to reason mathematically. However, my analysis of the relationship between the number of planned high-press questions and the number of asked high-press questions showed that the number of high-press questions planned was not a useful predictor for how many high-press questions the teachers would ask. While some teachers seemed to ask more high-press questions during the problem-solving debriefs when they planned more high-press questions in their ISAs, there were also examples in which teachers asked significantly fewer high-press questions during the implemented debriefs in comparison to what they had planned in their ISAs and teachers that planned very few high-press questions, but still asked a lot of high-press questions during the implemented debrief.

While my findings investigating a correlation between the number of high-press questions planned in an ISA and the number of high-press questions asked by the teacher in the implemented debrief did not show any trends, there was statistically significant evidence that teachers were more likely to ask high-press questions when they implemented the segments in their debriefs in which at least one high-press question was planned in contrast to the segments of their debriefs in which no high-press questions were planned. However, planning more than one high-press question in a segment did not necessarily lead to an increase in the number of high-press questions asked during a debrief.

Teachers' improvisational moves. In order to better understand how the teachers in this study were implementing the questions they had planned in their ISAs, I selected a subset of four teachers who had planned and implemented a varying range of high-press questions. I analyzed the transcripts of the problem-solving debriefs for these teachers, focusing on the teachers' utterances during the episodes in the debriefs where the questions from their ISAs were being addressed. The purpose of this analysis was to better understand the improvisational moves of the teachers as they were implementing the questions in their ISAs, with a focus on how those moves might influence the opportunities the teachers created for students to reason mathematically in their ISAs. I identified four different ways that these teachers addressed the questions from the ISAs: drop-in, in which the teachers asked the question and then moved on as soon as a student has offered a response; telling, in which the teachers chose to address the ideas in the question rather than giving the students a chance to reason about the question; embedded, in which the teachers never asked the question but it was inadvertently addressed by a student or students that were sharing their problem-solving strategies; and sustained focus in which the teachers asked the question and then followed up with additional questions to sustain attention on the ideas within the question.

My analysis of the four teachers found that how these teachers implemented their planned questions, using these four different types of question implementation as my focus, influenced the opportunities for students to reason mathematically. For example, when the teachers chose to tell the information pertaining to a planned question to their students rather than addressing the questions to the students, the students were given no opportunity to reason about the question. Similarly, when a question was implemented as drop-in, only the student to whom the question was addressed had an opportunity to express their thinking about the question and, even then, without any additional press, the student who answered the question is afforded no opportunity to think any more deeply about the question than how they initially answered the question. On the other hand, by sustaining the focus on a question, the teachers had the opportunity to press multiple students to think and reason about the ideas in the question and to provide better explanations of their thinking. Franke et al.'s study on the questioning strategies of teachers following up on students' initial explanations (2009) supports the finding that sustaining the focus on a question can beneficial for creating more opportunities for students to reason mathematically. They found that when teachers asked a series of probing questions (as opposed to a single question), it helped students to make connections between their own ideas and the mathematics; helped students to identify errors and clarify misunderstandings; and helped students to connect their own thinking to the thinking of others.

In a closer analysis of some of the sustained focus questions that seemed particularly well-implemented, I identified four steps that the teachers were using as the addressed their planned questions which led to even greater opportunities for their students to reason mathematically. These were *contextualization*, in which the teacher provided sufficient background information, as well as motivation, for students to clearly understand the question about to be addressed; *problem-posing*, in which the teacher carefully explained the planned question to the students to ensure that everyone understands what is being asked of them; *think-time*, in which the students were allowed time to think about the question either on their own or in small groups; finally, the teacher asked *follow-up* questions that further pressed students to clearly explain their solutions and to further reason about the ideas present within the question. These four teacher moves were useful for useful for identifying what caused a planned question to be successfully implemented.

Implications for Researchers

This research study contributed to research on the teachers' role in orchestrating classroom discourse by analyzing the impact of planned questions on the opportunities teachers create for students to reason mathematically. This study shows that, while planning high-press questions may have some influence on the opportunities teachers create for students to reason mathematically, the improvisational moves of the teacher during implementation of the problem-solving debrief can significantly impact the opportunities for students to reason mathematically, regardless of the nature of the planned question. While planning for discourse around a problem-solving task may be helpful for focusing the discourse on particular mathematical ideas and giving teachers an idea of how they would like their students to reason about the task, teachers are still responsible for making on-the-spot decisions with respect to orchestrating the discourse in ways that promote students' mathematical reasoning.

This study also contributes to research on enacted curriculum. Research has shown that when teachers implement curricular materials, such as textbook lessons, they can significantly alter the intended curriculum (Brown, Pitvorec, Ditto, & Kelso, 2009; Remillard, 1999). This study is unique from research on curriculum implementation in that it analyzes how teachers implement their own lesson plans, rather than lessons provided with curricular materials. This study shows that, in a manner similar to how teachers implement curricular materials, teachers also implement their own lesson plans in ways that may vary from their intended lesson. Even when the teachers implemented most of the steps in their ISAs, the questions were sometimes addressed in ways that were not consistent with the intended lesson. This study represents an important reminder to researchers that, as teachers are implementing their own lesson plans, the researcher cannot assume that the lesson is being implemented as intended, just the same as it cannot be assumed that curricular materials (in a textbook, for example) are being implemented as intended by the authors. The quality of a teacher's written lesson plans should be assessed by both the contents of the lesson as well as the manner in which it was implemented.

While professional developers have been encouraging teachers to plan for discourse around problem-solving tasks as a way to orchestrate mathematically productive discourse (Stein, Engle, Smith, & Hughes, 2008; Stein, Smith, Henningsen, & Silver, 2009) no research had been conducted explicitly examining the relationship between the plans that teachers make for orchestrating discourse around problem-solving tasks and the outcomes of implementation of those plans. My research study is intended to open this door to research on planning for discourse around problem-solving tasks. In my study, I specifically examined the impact that teachers' planned questions for promoting mathematical reasoning had on the opportunities the teachers created for students to reason mathematically during the implemented discussions. My hope is that this study will serve as a catalyst for furthering research on the impact that planning for discourse around problem-solving tasks can have on opportunities for students to reason mathematically. Below, I provide some suggestions for further research on teacher planning for discourse around problem-solving tasks.

Implications for Practitioners and Professional Developers

Building an understanding of high-press questions. While this research study showed that there was no clear correlation between the overall number of high-press questions a teacher plans and the number of high-press questions the teacher asks in the implemented debrief, there was evidence that teachers do tend to ask more high-press questions when they have planned at least one high-press question for a piece of student work. This finding is something that may be useful for practitioners and could be applied to professional development. As teachers are learning to prepare for classroom discussions surrounding problem-solving tasks, they should be encouraged to identifying not only a mathematical learning goal but also goals with respect to mathematical practices or goals for developing suitable sociomathematical norms. Once such goals are identified, the teachers should be encouraged to identify at least one high-press question to ask with respect to each piece of student work being shared that would be helpful for moving forward their goals. By doing so, teachers may be more likely to provide opportunities for their students to reason mathematically as they are sharing their problem-solving strategies.

In order for teachers to include high-press questions in their plans for discourse around problem-solving tasks, the teachers will also need to have an understanding of the difference between a high-press question and a low-press question. While the question codes that I developed for this research study were originally created for data analysis purposes, these codes may also serve as a framework to share with teachers, helping them to identify questions that press students to reason mathematically (high-press) and to differentiate such questions from those that merely prompt students to share what they already did or tell information that they already know (low-press). This coding scheme could be used as a tool to help teachers develop questions they could ask their students, evaluate whether or not the questions they have planned to ask will create opportunities for students to reason mathematically, or use it as a tool to evaluate their on-the-spot discourse moves. Teachers also should consider the norms that need to be in place in order for students to feel comfortable with high-press questions (Kazemi & Stipek, 2001).

Recognizing differences in implementation. Another product of this research study that could be helpful for teachers is the framework describing the four different ways that teachers address planned questions. This has potential to be useful for teachers to use, as a tool, to think about how they are addressing the questions they have planned. Even a well-planned question is less likely to create opportunities for students to reason mathematically when insufficient attention is drawn to the question. When a question is "dropped" into a discussion, students lose an opportunity to reason about the ideas potentially present in the question because the discussion moves on too quickly. When the ideas in a question are embedded in the discourse, addressed by students without being prompted, a teacher may feel that they do not need to address the question because it has already been, in a sense, answered by a student. However, when this happens, some of the ideas that the teacher considers important may be missed by other students because they were not highlighted as important by the teacher. Finally, when a teacher chooses to address a planned question by telling the class the information they need to know, the information the teacher wishes to pass on may be presented more clearly than if a student had said it, but, when this happens, students miss out on the opportunity to reason for themselves.

These three approaches to question implementation mentioned above may diminish the opportunities for students to reason mathematically. However, this analysis showed examples from multiple teachers of question implementation in which sustaining the focus on a question with the use of follow-up questions supported students to reason mathematically by giving adequate attention to the ideas present within a planned question. An appropriately planned question with sufficient focus on the ideas in the question has the potential to generate worthwhile discourse in which multiple students are reasoning mathematically. Not all questions implemented as sustained focus resulted in meaningful mathematical discourse, though. At times, a sustained focus question led to little more than students offering surface level responses and meaningful discourse did not emerge. Alternatively, a teacher may be trying to sustain the focus on a question by asking high-press questions as follow-up, but the students were not prepared to provide adequate responses and it resulted in a guessing game in which the students were trying to provide the responses they thought their teacher was looking for.

I identified four teacher moves that were present in the implementation of some questions addressed as sustained focus that were helpful for increasing students' ability to engage meaningfully with the ideas within the planned question. First, by providing contextualization leading up to asking the question, it ensured that students were on the same page with respect to the context of the problem. This involved demonstrating a particular representation or strategy that was used to solve the task, or discussing a problematic element of the task that students were struggling with. Second, by clearly posing the problem to the whole class, the question was set up so that everyone understood both the question they were supposed to address and that they were expected to try to address the question. Third, teachers provided think-time so that everyone had time to think about how they might address the question. Finally, once the teachers had set up the question in this way, it was easier for them to then follow-up students' initial responses with additional pressing questions because the students were better prepared to reason about the mathematical ideas in the question. These four teacher moves could be used in the context of any whole-class discussion, but they have the potential to be particularly useful in the context of a problem-solving debrief because it can create opportunities for students to reason mathematically instead of the class simply listening to the presenting students' strategies and then assuming that the responsibility of addressing questions falls to the presenting student.

I propose that these frameworks for question implementation (drop-in, embedded, telling, and sustained focus) and the moves for successfully sustaining focus on a question (contextualizing, problem-posing, think time, and follow-up) could be used in teacher professional development as a way to help teachers better understand the different ways that they are implementing questions in their classrooms. Giving teachers an awareness of how they are implementing questions, and encouraging them to implement questions in ways that promote student engagement and mathematical reasoning may help teachers to orchestrate discourse that is mathematically productive.

Limitations and Future Research

In this section, I address some aspects of this research study and some features of the professional development that put limitations on the findings in this study. I include some thoughts on how these features may have impacted the findings of this study as well as provide some suggestions for future research that would be helpful for addressing these aspects of the professional development and/or data analysis and how they would be helpful for further contributing to this area of research.

Further analysis of teachers' use of high-press questions. In my analysis of teachers' intended lessons, I compared the frequency with which teachers planned high-press questions in their ISAs to the frequency with which teachers asked high-press questions in their enacted debrief. The findings in this analysis showed that there was no correlation between these two variables. I mentioned that there were many influencing factors that would also impacted the frequency with which teachers planned and asked high-press questions. These included the teachers' knowledge and beliefs, the structure and norms in the classroom, the teacher's professional identity, and features of the curriculum. A multivariable analysis would be useful for taking into account these additional variables to determine whether or not the frequency with which teachers ask high-press questions may be predicted given the number of high-press questions planned as well as some other contributing factor(s). For example, if both the number of high-press questions planned and the teachers' math knowledge scores were taken into account, would we see a higher correlation between the data sets? While this analysis has potential to yield some interesting results, a larger sample size than what I used in this study would likely be necessary to obtain valid results.

Further analysis of four types of question implementation. In this research study, I identified four types of question implementation (drop-in, embedded, telling, and sustained-focused). These were identified through the analysis of the problem-solving debriefs implemented by four of the 12 teachers from the research study. A logical next step would be to categorize questions implemented by other teachers from this study in order to provide a quantitative analysis of how a larger set of teachers were implementing the questions they planned in the debrief. In order to accurately assess how teachers' planned question were implemented, it would be necessary to clearly define each type of implementation, identifying the key features that would place an implemented question into a specific category, as well as define characteristics that would specify when an implemented question would either fit into multiple categories or not fit into any categories. If some of the implemented questions do not fit into any of these categories, it would be necessary to determine if these non-conforming examples of question implementation should form additional categories. Conducting such an analysis would make it possible to determine if, for example, high-press questions are more likely to be implemented as sustained-focus compared to low-press questions, or if low-press questions are more likely to be implemented as drop-in questions. Also, it would be possible to see if teachers are more likely to address their questions one way over another (e.g. addressing most questions as telling, or addressing most questions as drop-in). While this would make for interesting future analysis, as mentioned above, this would be outside of the scope of my research.

Realistic planning practices. One of the limitations of this study was that the data collected for this study was not collected from a typical classroom environment. This research study analyzed the planning forms completed by teachers participating in a professional development research program. The teachers in the program were required to collect the students' work, read through their solutions, and base their plans on the information they gleaned from these solutions. While I believe that these are valuable planning practices and that it would be beneficial for teachers to set aside time for this type of planning, it is not feasible for teachers to plan for problem-solving debriefs in this way on a regular basis, given the limited time for planning that teachers are typically allotted. Rather than completion of the ISA being considered a regular part of their planning practices, the extra time that the teachers in this study spent analyzing student solutions and planning for the problem-solving debrief was considered a part of their professional development experience. By devoting extra time to planning for a problem-solving debrief, the teachers were allowed to reflect upon how they would like to orchestrate these whole-classroom discussions. This was particularly important because implementing a problem-solving debrief was a new experience for these teachers.

This practice of taking time outside of class to plan for a problem-solving debrief is not something that one would expect to see routinely in a typical mathematics class. It was not assumed that the teachers participating in this professional development would necessarily continue to use the ISA to plan for every single problem-solving debrief that they implement in their classroom (although I do believe that planning with time to reflect is ideal). In this study, the ISA was used as both a professional development tool and a data collection tool. A more common approach when selecting and sequencing student work in preparation for a problem-solving debrief is for teachers to navigate around the classroom while their students work on the task, mentally planning whose work will be shared and in what order. In this way, the teacher would be prepared to orchestrate a problem-solving debrief with their students before the end of that class (Smith & Stein, 2011). Teachers that participated in the MPSM professional development program would, ideally, plan for a debrief in a manner similar to how they complete the ISA, whether they completed the form on-the-spot, or made mental plans, noting who would share, what ideas would be highlighted, and what questions would be asked.

The benefit of collecting data from teachers who were completing planning forms was that it was possible to gain access to the planning decisions that the teachers made in preparation for their problem-solving debriefs. While this did make data collection and analysis easier, there were limitations to this approach. In particular, teachers may plan differently for debriefs when they create a written lesson plan outside of class, when they have time to reflect on their decisions, compared to when they prepare for a problem-solving debrief on the spot, without documenting their decisions. When teachers have to bring their class together for a whole-class discussion during the same class period that they implemented a task, they might not explicitly think about what questions they are going to ask until they are actually leading the discussion. Further research on teachers' in-class planning practices would help to clarify what types of questions teachers plan prior to a problem-solving debrief (if any) and how this impacts the outcomes of the discussion. Such an analysis could be done using on-the-spot interviews, or a variation of the ISA could be used by the teachers as a format for recording their on-the-spot decisions.

Training teachers to recognize high-press questions. While my research study focused specifically on the teachers' use of questions that promote mathematical reasoning (high-press questions), the teachers participating in this research study did not receive explicit training on high-press questions. In Chapter 3, Professional Development Description, I included a table of teacher questions from Boaler and Humphrey's Connecting Mathematical Ideas (2005) that was shared with the participating teachers during the summer seminars in the professional development program. The purpose of the table of questions was to show the different types of questions that a teacher might ask for different reasons during a whole-class discussion around a problem-solving task. While some of the questions listed would be useful for creating opportunities for students to reason mathematically (high-press) such as the question type of exploring mathematical meanings and relationships, none of the question types were deliberately highlighted as more or less useful for pressing students to reason mathematically. Also, during the coaching sessions, the professional developer encouraged the teachers to be deliberate and thoughtful about the types of questions they ask.

While asking good questions was emphasized through the professional development, a framework like the one I developed for this study that differentiated between high-press questions and low-press questions was not available for the teachers. It is possible that, if the teachers were more aware of the differences between high-press and low-press questions, as developed for this research study, when they were completing their ISAs, the high-press questions may have had a greater influence on the enactment of the ISA. That is, the teachers possibly would have been more deliberate about implementing the high-press questions in their ISA if they were more aware of their potential influence on creating opportunities for their students to reason mathematically. I recommend future research studying the impact of training teachers to differentiate between high-press and low-press questioning practices for the purpose of creating opportunities for students to reason mathematically. A study analyzing how such training influences their ability to create and implement ISAs would be informative. In particular, do teachers, who receive training in the use of high-press questions as a way to create opportunities for students to reason mathematically, plan and ask more high-press questions during a problem-solving debrief than those who do not?

Mathematical learning goals and planning questions. This research study focused on the nature of questions that teachers planned and asked in their problem-solving debriefs. While I assessed the quality of the teachers' planned questions based upon whether or not the question had the potential to press students to reason mathematically, another possible avenue for assessing teachers' questions in an ISA would be to connect the questions planned in an ISA to the mathematical learning goals identified by the teacher. A teacher may identify mathematical learning goals with respect to the curricular goals at the time of task implementation, the learning goals for the problem-solving task, and the specific learning goals for the problem-solving debrief. The questions in an ISA that clearly addresses any of these learning goals may be more productive and create better learning opportunities for students than a question that is planned out of context.

Analyzing the planned questions with respect to the teachers' learning goals was outside the scope of this research study. I recommend further research on teachers' planned questions, but with a particular focus on teachers' learning goals. I believe this would provide more insight into the opportunities that planned questions can generate for students to reason mathematically. My hypothesis would be that, when those questions are clearly linked to learning goals and those learning goals are clearly connected to the curriculum, greater opportunities will emerge for productive mathematical discourse.

Concluding Remarks

Whole-class discussions around a cognitively demanding problem-solving task have the potential to be a valuable arena for mathematical reasoning to take place. By pre-planning who will share their strategies, what ideas are worth highlighting in their strategies, and what questions to ask to make the mathematics salient, the teacher is well situated to create more opportunities for students to reason mathematically. What we have learned from this research study is that, it not only matters whether or not a teacher plans for a whole-class discussion with questions that prompt students to reason mathematically, or that the teacher implements such plans for a problem-solving debrief faithfully, it also matters how those plans are implemented. Depending upon how a teacher implements a planned question, the opportunities for students to reason mathematically will either be constrained or afforded. This research study is beneficial for providing some insight into how some of the different ways that teachers implemented their planned questions hindered or supported students' opportunities to reason mathematically.

References

Baird, L.S., Schneier, C.E., & Laird, D. (1983). *The training and development sourcebook.* Amherst, MA: Human Resource Development Press.

Ball, D. L., & Bass, H. (2003) Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin & D. E. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics.

Ball, D. L., Lubienski, S. L., & Mewborn, D.S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.) *Handbook of research on teaching* (pp. 433-456). Washington, D.C., American Educational Research Association.

Bangert-Drowns, R. L., Kulik, C. C., Kulik, J. A., & Morgan, M. T. (1991). The instructional effect of feedback in test-like events. *Review of Educational Research*, 61, 213-238.

Baxter, J. & Williams, S. (2010). Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *Journal of Mathematics Teacher Education*, 13, 7-26.

Becker, J. P., & Shimada, S. (Eds.). (1997). *The open-ended approach: A new proposal for teaching mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education*, 29, 41-62.

Boaler, J. & Humphreys, C. (2005). Connecting mathematical ideas: Middle school video cases to support teaching and learning. Portsmouth, NH: Heinemann.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record, 110,* 608-645.

Bonotto, C. (2002). Suspension of sense-making in mathematical word problem solving: A possible remedy. *Proceedings of the 2nd International Conference on the Teaching of Mathematics*. Crete, Greece: Wiley & Sons Publishers.

Boston, M. D., & Smith, M. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education*, 40, 119-156.

Brandt, B. L., Farmer, J. A. Jr., & Bruckmaster, A. (1993). Cognitive apprenticeship approach to helping adults learn. In D. Flannery (Ed.), *Applying cognitive learning theory to adult learning* (pp. 69-78). San Francisco: Jossey-Bass.

Bransford, J. D., Brown, A. L., Cocking, R. R. (Eds.) (2000). *How people learn: Brain, mind, experience, and school.* Washington, DC: National Academy Press.

Brown, D. S. (1988). Twelve middle-school teachers' planning. *The Elementary School Journal*, 89, 69-87.

Carpent, T. P., Ansell, E., & Levi, L. (2001). An alternative conception of teacher for understanding: Case studies of two first-grade mathematics classes. In T. Wood, B. S. Nelson, J. Warfield (Eds.), *Beyond classical pedagogy* (pp. 27-46). Mahwah, NJ: Lawrence Erlbaum Associates.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26, 499-531.

Carpenter, T. P., Fennema, E., Carpenter, T. P., Fennema, E. & Franke Cognitively guided instruction & Franke, M. L. (1997). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97, 3-20.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning* (2nd Ed.). Portsmouth: Heinemann. (Original work published in 1988).

Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Retrieved from: http://www.corestandards.org/ Math/Practice

Chazen, D., & Ball, D. L. (2001). Beyond being told not to tell. *For the Learning of Mathematics, 19*, 2–10.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education*, 28, 258-277.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. A. Forman, N. Minick & C. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91-119). New York, NY, Oxford.

Cobb, P., Wood, T., Yackel, E., Nichols, J., Wheatly, G., Trigatti, B., Perlwitz, M. (1991). Assessment of a problem-centered second-grade mathematics program. *Journal for Research in Mathematics Education* 22, 3-29.

Cohen, J. (2008). *Middle school teachers' use of a formative feedback guide in mathematical problem solving instruction* (Unpublished doctoral dissertation), Oregon State University, Corvallis Oregon.

Cranton, P. (1996). Professional development as transformative learning: New perspectives for teachers of adults. San Francisco, CA: Jossey-Bass Publishers.

Davies, N. & Walker, K. (2005). *Learning to notice: One aspect of teachers' content knowledge in numberacy classrooms*. *Proceedings of the 28th annual conference of the mathematics education research group of Australasia, Australia*, 1, 273-280.

Day, J. D., & Cordon, L. A. (1993). Static and dynamic measures of ability: An experimental comparison. *Journal of Educational Psychology*, 85, 76-82.

Doerr, H. (2006). Examining the tasks of teaching when using students' mathematical thinking. *Educational Studies in Mathematics*, 62, 3-24.

Donovan, M. S., Bransford, J. D., & Pellegrino, J. W. (Eds.). (1999). *How people learn: Bridging research and practice*. Washington, D.C.: National Academy Press.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist*, 23, 167-180.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, M., Jacobs, V., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27, 403-434.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in E*verday Mathematics* classrooms. *Journal for Research in Mathematics Education,* 30, 148-170.

Franke, M. L., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal* 38, 653-689.

Franke, M. L. & Kazemi, E. (2001*).* Teaching as learning within a community of practice: Characterizing generative growth. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy* (pp. 47-74). Mahwah, NJ: Lawrence Erlbaum Associates.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning, volume 1* (pp. 225-256). Reston, VA: National Council of Teachers of Mathematics.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, 60, 380-392.

Fuson, K. C., Kalchman, M., & Bransford, J. (2001). Mathematical understanding: An introduction. In M.S. Donovan & J.D. Bransford (Eds.) *How students learn mathematics in the classroom* (pp. 217-256). Washington, DC: National Academy Press.

Grouws, D.A. (2003). The teacher's role in teaching mathematics through problem solving, In Schoen, H.L. (Ed.) *Teaching mathematics through problem solving: Grades 6-12* (pp. 129-142). Reston, VA: National Council of Teachers of Mathematics.

Groves, S. & Doig B. (2004). Progressive discourse in mathematics classes: The task of the teacher. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 2, 495–502.

Gummer, E. S., Gates, C., Strowbridge, J. (2009). Formative assessment practices that inform both teachers and students. *Presentation at the annual conference of the National Council of Supervisors of Mathematic*, Washington, DC.

Guskey, T. R. & Yoon, K. S. (2009). What works in professional development? *Phi Delta Kappan*, 90, 495-500.

Heaton, R. M. (2000). *Teaching mathematics to the new standards: Relearning the dance*. Teacher's College Press: Williston, VT

Henningsen, M. A. & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 29, 524-549.

Herman, J. L., Osmundson, E. O., Ayala, C., Schneider, S., & Timms, M. J. (2006). *The nature and impact of teachers' formative assessment practices* (Report No. 703), Quebec, Canada: Center for the Study of Evaluation.

Hersch, S. B., Fosnot, C. T., & Cameron, A. (2006). *Fostering children's mathematical development, grades 5-8: The landscape of learning*. Portsmouth, NH: Heinemann.

Hiebert, J. (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 5-24). Reston, VA, National Council of Teachers of Mathematics.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A. & Human, P. (1997). *Making sense: Teaching and learning mathematics.* Portsmouth, NH: Heinemann.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and student learning in second grade. *American Educational Research Journal*, 30, 393-425.

Hiebert, J. & Wearne, D. (2003). Developing understanding through problem solving. In H.L. Schoen (Ed.) *Teaching mathematics through problem solving: Grades 6-12*. (pp. 3-14). Reston, VA: National Council of Teachers of Mathematics.

Ilaria, D. R. (2002). Questions that engage students in mathematical thinking. *Proceedings from the 24th Annual Meeting of the North American Chapter of the international Group for the Psychology of Mathematics Education*, Athens, GA.

Kahan, J. A. & Wyberg, T. R. (2003). Mathematics as sense making. In I. R. Charles (Ed*.). Teaching mathematics through problem solving: Grades 6-12* (pp. 15-26). Reston, VA: National Council of Teachers of Mathematics.

Kalathil, R. R (2006). Using comparisons of alternate strategies to promote discourse in mathematics classrooms. *Proceedings from ICLS 2006*: *The 7th International Conference on Learning Sciences*. Bloomington, IN: The International Society of the Learning Sciences.

Kazemi, E. & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education,* 7, 203-235.

Kazemi, E. & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal* 102, 59-80.

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Koency, G. & Swanson, J. (2000). The special case of mathematics: Insufficient content knowledge a major obstacle to reform. *Paper Presented at the Annual Meeting of the American Educational Research Association*. New Orleans, LA.

Krussel, L., Edwards, B., & Springer, G. T. (2004). The teacher's discourse moves: A framework for analyzing discourse in mathematics classrooms. School Science and Mathematics, 104, 307-312.

Lampert, M. (1989). Choosing and using mathematical tools in classroom research, *Advances in Research on Teaching*, 1, 223-264.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27, 29-63.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven: Yale University Press.

Lampert, M. & P. Cobb (2003). Communication and language. In J. Kilpatrick, W. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics,* (pp. 237-249). Reston, VA: NCTM.

Larsen, S. & Bartlo, J. (2009). The role of tasks in promoting discourse supporting mathematical learning. In L. Knott (Ed.) *The role of mathematics discourse in producing leader of discourse* (pp. 77-98). Charlotte, NC: Information Age Publishing.

Lee, Y. & Takahashi, A. (2011). Lesson plans and the contingency of classroom interactions. *Human Studies*, 34, 209-227.

Leinhardt, G. (1989). Math lessons: A contrast of novice and expert competence. *Journal for Research in Mathematics Education*, 20, 52-75.

Leinhardt, G. (2001). Instructional explanations: A commonplace for teaching and location for contrast. In V. Richardson (Ed.), *Handbook of research on teaching, 4th ed.,* (pp. 333–357). Washington, DC: American Educational Research Association.

Leinhardt, G. & Steele, M.D. (2005). Seeing the complexity of standing to the side: Instructional Dialogues. *Cognition and Instruction*, 23, 87-163.

Lesh, R., K. Cramer, Doerr, H., Post, T., & Zawojewski, J. (2003). Model development sequences. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling* (pp. 35-54). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., Hoover, M., Hole, B., Kelly, A. & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In A. E. Kelly, R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-646), Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning,* v.2 (pp. 763-804). Reston, VA: National Council of Teachers of Mathematics.

Lobato, J., Clark, D. et al. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education*, 36, 101-136.

Marcus, R. & Fey, J. (2003). Selecting quality tasks for problem-based learning. In H.L Schoen (Ed.) *Teaching mathematics through problem solving: Grades 6-12* (pp. 55-68). Reston, VA: National Council of Teachers of Mathematics.

McClain, K. & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education*, 32, 236-266.

Merriam, S. B., Caffarelle, R. S., & Baumgartner, L. M. (2007). *Learning in adulthood: A comprehensive guide, 3rd Ed*. San Francisco: Jossey-Bass.

Merseth, K. K. (1996). Case and case methods in teacher education. In J. Sikula (Ed.), *Handbook of research on teacher education, 2nd ed.* (pp. 722-744), New York, NY: Macmillan.

Mewborn, D. S. (2003). Teaching, teachers' knowledge, and their professional development. In J. Kilpatrick, W. Martin & D. Schifter (Eds.). *A research companion to principles and standards for school mathematics* (pp. 45-52). Reston, VA, National Council of Teachers of Mathematics.

Narciss, S. & Huth, K. (2004). How to design informative tutoring feedback for multimedia learning. In H. M. Niegemmann, D. Leutner, & R. Brunken (Eds.), *Instructional design for multimedia learning* (pp. 181-195). Munster, NY: Waxmann.

Nathan, M. J. & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction* 21, 175-207.

National Council of Teachers of Mathematics (NCTM) (1991). *Professional standards for teaching mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

Nyquist, J. B. (2003). *The benefits of reconstructing feedback as a larger system of formative assessment: a meta-analysis* (Unpublished master's thesis), Vanderbilt University, Nashville Tennessee.

O'Connor, M. C. (1998). Language socialization in the mathematics classroom: Discourse practices and mathematical thinking. In M. Lampert & M. L. Blunk

(Eds.). *Talking mathematics in school: Studies of teaching and learning* (pp. 17-55). Cambridge: Cambridge University Press.

O'Connor, M. C. & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly*, 24, 318-335.

Parks, A. N. (2010). Explicit Versus Implicit Questioning: Inviting All Children to Think Mathematically. *Teachers College Record. 112*, 1871–1896.

Pilling-Cormick, J. (1997). Transformative self-directed learning in practice. *New Directions For Adult and Continuing Education*, *74*, 69-77.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Pridemore, D. R., & Klein, J. D. (1995). Control of practice and level of feedback in computer-based instruction. *Contemporary Educational Psychology*, 20, 444-450.

Rasmussen, C., Kwon, O., & Marrongelle, K. (2009). A framework for interpreting inquiry-oriented teaching: Opportunities for student and teacher learning. Paper presented at the Annual Meeting of the American Educational Research Association, San Diego, CA.

Rasmussen, C. & Marrongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education*, 37, 388-420.

Rasmussen C., Yackel, E., & King, K. (2003). Social and sociomathematical norms in the mathematics classroom. In H.L. Schoen (ed.), *Teaching mathematics through problem solving: Grades 6-12* (pp. 143-154), Reston, VA: NCTM.

Reiser, R. A. (1994). Examining the planning practices of teachers: Reflections on three years of research. *Educational Technology*, 11-16.

Remillard, J. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry*, 29, 315–342.

Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, 75, 211-246.

Rigelman, N. M. (2009). Eliciting high-level student mathematical discourse: Relationships between the intended and enacted curriculum. In L. Knott (Ed.) *The role of mathematics discourse in producing leader of discourse* (pp. 153-172). Charlotte, NC: Information Age Publishing.

Saxe, G. B., Gearhart, M., & Nasir, N. S. (2001). Enhancing students’ understanding of mathematics: A study of three contrasting approaches to professional support. *Journal of Mathematics Teacher Education*, 4, 55-79.

Schifter, D. (1998). Learning mathematics for teaching: From a teachers’ seminar to the classroom. *Journal of mathematics teacher education*, 1, 55-87.

Schifter, D. (2001). Learning to see the invisible: What skills and knowledge are needed to engage with students' mathematics ideas? In T. Wood, B. S. Nelson & J. Warfield (Eds.), *Beyond Classical Pedagogy* (pp. 109-134). Mahwah, NJ: Lawrence Erlbaum Associates.

Schneider, R. M., Krajcik, J., & Blumenfeld, P. (2005). Enacting reform-based science materials: The range of teacher enactments in reform classrooms. *Journal of Research in Science Teaching*, 42, 283-312.

Schoenfeld, A. H. (1979). Explicit heuristic training in problem solving performance. *Journal for Research in Mathematics Education*, 10, 173-187.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-368). New York: Macmillan.

Sherin, M. G. (2002). When teaching becomes learning. *Cognitive and Instruction*, 20, 119-150.

Shute, V. J. (2008). Focus on formative feedback. *Review of Educational Research,* 78, 153-189.

Silver, E. A. & Stein, M. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education*, 30, 476-521.

Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education*, 27, 387-402.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning, volume 1* (pp. 157-224). Reston, VA: National Council of Teachers of Mathematics.

Stanic, G. M. A., & Kilpatrick, J. (1989). Historical perspectives on problem solving in the mathematics curriculum. In R. I. Charles & E. A. Silver (Eds.), *Research agenda for mathematics education: Volume 3. The teaching and assessing of mathematical problem solving* (pp. 1-22). Hillsdale, NJ: Lawrence Erlbaum, & Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10, 313-340.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33, 455-488.

Stein, M. K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, 2, 50-80.

Stein, K., J. Remillard, & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.) *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 319-367). Reston, VA: NCTM.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development, 2nd ed*. Reston, VA: National Council of Teachers of Mathematics.

Stodolsky, S. (1989). Is teaching really by the book? In P. Jackson, & S. Haroutunina-Gordon (Eds.) *From Socrates to software: The teacher as text and the text as teacher*, (pp. 159-184). Chicago: University of Chicago Press.

Strauss, A. & Corbin, J. (2007). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks, CA: Sage Publications.

Superfine, A. C. (2009). The "problem" of experience in mathematics teaching. *School Science and Mathematics*, 109, 7-19.

Tobin, K. (1986). Effects of teacher wait time on discourse characteristics in mathematics and language arts classes. *American Educational Research Journal*, 23, 191-200.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research*, 78, 516-551.

Williams, S. R. & Baxter, J. A. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal* 97, 21-38.

Wiliam, D. (2007). Keeping learning on track: Classroom assessment and the regulation of learning. In F. Lester (Ed.), *Second handbook of research of mathematics teaching and learning, v. 2* (pp. 1051-1098). Reston, VA: National Council of Teachers of Mathematics.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: funneling or focusing? In H. Steinbring, M. G. B. Bussi & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, VA: National Council of Teachers of Mathematics.

Wood, T. & Turner-Vorbeck, T. (2001). Extending the conception of mathematics teaching. *Beyond classical pedagogy* (pp. 185-208). In T. Wood, B. S. Nelson & J. Warfield (Eds.) (2001). *Beyond classical pedagogy: Teaching elementary school mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27, 458-477.

Yackel, E. & Hanna, G. (2003). Reasoning and proof. In J. Kilpatrick, W. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 227-236). Reston, VA, National Council of Teachers of Mathematics.

Young, A. C., Reise, R. A. & Dick, W. (1998). Do superior teachers employ systematic instructional planning procedures? A descriptive study. *Education Technology and Research Development*. 46, 65-78.

Zahorik, J. A. (1970). The effect of planning on teaching. *The Elementary School Journal*, 71, 143-151.

Appendix A Instructional Sequence Analysis

Instructional Sequence Analysis

Teacher Name ______________________ **Task Name**_________________________________ **Date** ____________

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so. *Be sure to indicate whether student work to be shared has misconceptions or mistakes that were common or important and need to be addressed to achieve your mathematical goals. Identify those pieces of student work that incorporate important mathematical concepts, ideas, or procedures that can help you meet your mathematical goals.* Identify the questions you will ask the student or the class during the discussion to expose and/or address any misconceptions or to illuminate the important ideas of the lesson.

Mathematical Learning Goal:____

Order of Sharing	**Student Name**	**Ideas to Highlight**	**Questions to Make the Math Salient**
1			
2			
3			
4			

Please add additional cells to the table above for additional samples of student work you will share in the lesson debrief.

Rationale: For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares.

____
____
____
____
____

Appendix B Results from Implementation Fidelity Analysis Tool

Teacher Number	T22	T22	T23	T23	T23	T24	T24	T25	T25	T26	T26	T3	T3	T3	T27	T27	T27	T31	T31	T31	T42	T50	T57	T9	T9
Task	ss	dd	ss	se	dd	se	dd	dd	se	dd	ss	se	dd	ss	se	dd	ss	ss	dd	se	se	se	dd	ss	dd
segment 1	1+	1-	2b+	1+	1+	1+	2a+	3a+	1+	3a-	3b	1-	1+	1-	1+	1-	2a-	1+	2a+	1+	2a-	1+	1-	2a+	1-
segment 2	1+	2a-	1+	3b	1+	3a+	1+	1+	3b	3b	3b	1-	1-	2a-	1+	1-	2a-	1+	2a-	1-	2a+	2a+	1-	1+	2a+
segment 3	1+	1-	1+	1+	1+	1+	1+	3a+	3b	1+	3b	1-	1+	1-	1+	1+	2a-	1+	NA	1+	1+	1+	1+	2a-	1-
segment 4	1+	NA	3a-	1+	3a-	2a+	1+	1+	3b	2a+	1+	1-	1+	1-	1+	1+	1-	2a+	NA	1+	2a+	2a+	NA	NA	1-
segment 5	NA	NA	NA	NA	NA	2a+	NA	1-	NA	3a+	NA	2a-	NA	1-	NA	2a-	2a-	NA	NA	1-	1+	1+	NA	NA	NA
segment 6	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	2b+	NA	1+	NA	NA	NA

Note: ss = snack shack; dd = Design a Dartboard; and se = Spinner Elimination. These are the three tasks the teachers were required to do.
The number of segments the teachers planned for in the ISAs ranged from 2 to 6 segments. NA indicates that the teacher did not plan more than the segments coded in the table.

Appendix C Question Codes with Descriptions

High-Press Questions

A high-press question is one in which the students are given opportunity to deepen their understanding of mathematical concepts through mathematical inquiry. This happens through mathematical argumentation, analysis of errors, the development of new mathematical ideas, building connections between mathematical ideas, and analysis and interpretation of other students' thinking.

H1. Reasoning and Justification. These questions create an opportunity for students to provide rationale to support their reasoning and/or solution. An answer to this type of question requires that evidence be provided for either a pre-existing idea, an idea proposed by the teacher, or an idea formulated by a student. A question is coded as H1 even if the teacher is asking a student to share reasoning or justification that they already provided in their written work. Some examples of Reasoning and Justification questions that were planned by actual teachers from the MPSM professional development program are listed below.

- What was your reasoning for choosing the smaller numbers?
- What is your rationale for choosing the smaller numbers first? How does this benefit you?
- Explain why this might be a good strategy.
- Why put a 5 or a 7 on your spinner?
- Why are some numbers on the chart impossible?
- How do you know you have all solutions?

H2. Addressing Errors and Verifying Solutions. This type of question includes prompting students to verify whether or not a solution is correct and discussing what makes a solution, or part of a solution, incorrect or problematic. Also, these questions lead students to discuss how to correct an error (if, however, one is correcting the error by completely throwing it out and trying a new approach, it falls under the category of Generalizations, Conjectures, and New Strategies, code H3). Note that a teacher may ask a short-answer question for the purpose of exposing an error. This type of question would be coded as L2 because the students are not being asked to reason about the error. Some examples of Addressing Errors and Verifying Solutions that were planned by actual teachers from the MPSM professional development program are listed below.

- Why doesn't zero work?
- How does [choosing all the same number] affect the possible results?
- What went wrong with your second set of numbers?

- Why did the first table not work?
- What could you do after discovering that your table might not be useful?
- How can we be sure the percents are accurate?

H3. New strategies, Generalizations, and Conjectures. These are questions that prompt students to move forward with their thinking on the task by coming up with new ideas on how to solve the task or exploring the mathematical ideas surrounding the task. These questions do not require a definitive answer and responses should elicit opportunities for elaboration and justification. These questions might actually be short-answer questions as long as the response does not require a definitive answer and there is potential for elaboration and justification following the response. Some examples of New Strategies, Generalizations, and Conjectures questions that were planned by actual teachers from the MPSM professional development program are listed below.

- Focus on the 2 solutions that worked – can you "see" a number for your next guess?
- How can you go forward in an organized way?
- What should be our next step to solve the problem?
- Is it best to have all different numbers?
- Is a 4 necessary?
- Can you think of a number to replace the 4 that would work just as well?
- Could we start with a different size for the inner section and still use this approach?

H4. Making Connections. These are questions that prompt students to make connections between strategies, representations, other mathematics (besides that which is central to solving the task), or real-world context. This may include requests for students to compare and contrast the strategies that have been presented, including discussions of how they are mathematically different as well has what makes on solutions method more efficient than another. Also, connections includes making connections between the strategies the students are developing and the mathematical ideas that are being implicitly used in their strategies. Some examples of Making Connections questions that were planned by actual teachers from the MPSM professional development program are listed below.

- How does the number of squares in each section relate to the percent required?
- How is this different from Baylee's [strategy]?
- Which strategy do you think is more efficient/easier?

H5. Clarifying Other Students' Thinking. These are questions that prompt students to make sense of what other students are saying or thinking. This may mean interpreting the presenting student's written work or restating something they said. H5 is assigned when the teacher asks for explanation of an aspect of the presenting student's work, but the question is not explicitly addressed to the presenting student. Some examples of Clarifying Other Students' Thinking questions that were planned by actual teachers from the MPSM professional development program are listed below.

- Why do you think this person started with the number of cases that they did?
- Would someone else explain what this student did?
- How did they determine base and height?
- Why did Peter add 15+25?
- Where did the 300 come from?

Low-Press Questions

A low-press question is one in which students are *not* led to engage in new thinking. This may be because they are reporting out on something they already did (they may even be able to do this by reading directly from their paper), they are providing answers to questions in which there is only one appropriate response, or because they are regurgitating information that they are already supposed to know (such as text-book knowledge). Also in this category is a code for questions that are non-mathematical in nature and questions that are too ambiguous to be appropriately coded.

L1. Reporting and Explaining. These questions are a request for the presenting student to describe the work that they did, or explain their thinking on the task. This code should only be applied when the wording of the question indicates that the presenting student is being addressed. If the teacher is requesting that a student share a reasoning or justification, it should be assigned as H1. Some examples of Reporting and Explaining questions that were planned by actual teachers from the MPSM professional development program are listed below.

- How did you pick your second set of numbers?
- Did you start with the picture or the math?
- Where did you start when you built your shape?
- What led you to list all of the solutions in the order that you did?
- What is your strategy?
- Tell us how you chose to solve the problem.
- Explain what you did.

- Relay your conclusion about using numbers less than 4.

L2. Short-answer, Recall, and Procedural. These questions are defined such that there is only one appropriate answer and/or these questions would be evaluated as right or wrong based upon text-book knowledge (that is, knowledge learned from a textbook or from a previously covered topic). This includes questions requiring a series of predetermined operational steps to be demonstrated to find the solution (i.e. procedural responses). These types of questions may be leading questions. That is, the appropriate response reveals some information that the teacher considers to be important for the class to know. Some examples of Short-Answer, Recall, and Procedural questions that were planned by actual teachers from the MPSM professional development program are listed below.

- How do we calculate the area of a right triangle?
- How many sides does a hexagon have?
- Does it matter whether or not each section is exactly the same shape? (answer: no)
- Must the number of cases for different items be the same? (answer: no)
- What is the total amount?
- How many squares out of 180 is 15% of the whole?
- Are there numbers you can't reach? How many multiples?
- What other multiples can you get using only the prime numbers on your spinner?

L3. Non-Mathematical. These questions are those that fail to focus on the mathematical content, or questions in which, based upon the wording of the question, it is impossible to tell what the teacher's intention was in asking the question. Non-mathematical questions include questions pertaining to the organization of the paper that are not related to finding a solution. Some examples of Non-Mathematical questions that were planned by actual teachers from the MPSM professional development program are listed below.

- Tell us about the picture you drew in the corner of your paper.
- Describe the color-scheme you chose for your table.
- Did this student do a good job organizing their work?
- Which is your favorite dartboard and why?
- Symmetry of your design? (cannot tell what the intent of the question is).

Appendix D Table of Data Analysis Values used for Research Sub-Question 2

Teacher Number	T22	T22	T23	T23	T23	T24	T24	T25	T25	T26	T26	T3	T3	T3	T27	T27	T27	T31	T31	T31	T42	T50	T57	T9	T9
Task	ss	dd	ss	se	dd	se	dd	dd	se	dd	ss	se	dd	ss	se	dd	ss	ss	dd	se	se	se	dd	ss	dd
(1) # high-press questions planned in ISA	6	2	4	6	5	2	1	5	3	3	2	2	2	2	5	5	4	5	3	4	5	3	3	3	5
(2) Total # high-press questions asked in debrief	7	1	11	15	15	19	18	14	16	8	5	6	8	8	13	3	11	6	5	10	9	14	8	5	1
(3) #minutes in implemented debrief	34	21	34	20	32	39	46	41	39	23	9.3	17	25	29	40	19	27	52	22	36	20	28	25	26	8
(4) # high-press questions planned per 10 minutes of implemented debrief	1.8	1.0	1.2	3.0	1.6	0.5	0.2	1.2	0.8	1.3	2.2	1.2	0.8	0.7	1.3	2.7	1.5	1.0	1.4	1.1	2.6	1.1	1.2	1.1	6.3
(5) # high-press questions asked per 10 minutes of debrief	2.1	0.5	3.2	7.5	4.7	4.9	4.0	3.4	4.1	3.5	5.4	3.5	3.2	2.8	3.3	1.6	4.0	1.2	2.3	2.8	4.6	5.0	3.3	1.9	1.3

ss = Snack Shack; dd = Design a Dartboard; se = Spinner Elimination

Appendix E IOTA Codes use for Research Sub-Question 3

Revoicing (Main code = R)

Revoicing is broadly defined as reuttering – or saying again (could be verbal, symbolic, or gestural) – of someone else's utterances (symbolizing or gesturing). This may be a direct (immediate) restatement or it may involve an adaptation of the original utterance. May or may not include a short follow up question to determine if the revoicing was consistent with what the student said. R1-R3 occurs in the midst of ongoing exchange between the teacher and a student. R1-R3 does NOT have explicit attribution of original speaker(s).

1. Repeating – Teacher repeats a student's utterance using (essentially) the same words or a portion thereof.
2. Rephrasing – Teacher states a student's utterance in a new or different way.
3. Expanding – Teacher adds information to a student's utterance. This typically (but not necessarily) starts as repetition or rephrasing.
4. Reporting – Teacher attributes an idea, claim, argument to a specific student (reporting explicitly attributes ownership of the idea, claim, or argument to a specific student or group, versus implicit attribution of ownership in R1, R2, R3).

Requesting/Questioning (main code = Q)

Requesting/questioning functions to foster an inquiry oriented learning environment. Q codes should be used in cases when there is an expectation that students actually respond.

1. Evaluating. The intention is to check for understanding against what the teacher sees as an expected response. Typically results in teacher affirming or disconfirming student response. Could be in question or request form.
2. Clarifying. Purpose of the request is to seek clarification of detail (either for the teacher or for others) what a student is saying. Typically this occurs in the midst of a student giving an explanation or justification. (suggests that there is some confusion or need to clarify, either for the teacher or for other students).
    a. Request for clarification is directed to the speaker
    b. Request for clarification is direction to someone other than the speaker (including the class)

3. Explaining. (intention is for student(s) to share ideas however tentative). Could be in question or directive form. Results in an immediate voicing.
    a. Requests to explain your thinking or the thinking of your group (could be an open call or a call for a particular student).
    b. Requests to explain or comment on another student's or group's thinking (could be an open call or a call for a particular student).
4. Justifying. Requests to provide warrants or backing for a some conclusion or claim. This could be an open call or a call for a particular student. The intention is that the argument provide reasons for why a particular result has to be the case. In contrast, requests to explain thinking typically result in elaboration of initial, tentative ideas. There is less finality to requests for explanation in comparison to requests for warrants and backing.

Telling (Main code = T)

1. Initiating.
    a) Describing or presenting a *new* concept, representation, procedure, solution method, etc. This discursive move typically brings in new information, extends current topics of discussion, or makes connections to other ideas.
    b) Telling students what the new problem is that they are to work on next. Must involve some mathematical contextualization of the next task, in contrast to the managing code for students to work on problem x. Mathematical contextualization may be in terms of the broader discipline or in terms of the intermediate course goals.
    c) Reminding students of conclusions from a previous problem. This may also include remaining open questions regarding that problem.
2. Facilitating student progress in the midst of a task.
    a) Providing information that students need in order to test their ideas, generate a counterexample, determine if they are correct, offer a way to think about an idea, etc. for a task that students are in the midst of working on.
    b) Reminding students of a conclusion or a way to think about a problem for which there has already been some agreement or public voicing or

reminding students of a line of questioning that was previously pursued. This code is similar to R4, but without the explicit student attribution.

3. Responding to student utterances
   a) Answering a direct student question. This discursive move is typically a short, direct, mathematical answer to a specific mathematical question.
   b) Evaluating a student utterance with a clear indication of correctness or incorrectness. Does not include additional elaboration why the response is correct or incorrect.
   c) Evaluating a student utterance with additional reasoning for why the statement is correct or incorrect.
4. Summarizing/wrapping up. This discursive move typically summarizes (selected) ideas, highlights particular mathematics of importance, and/or points to next steps related to the summary. These utterances typically come at the end of student work on a task.

Managing (Main Code = M)

1. Arranging. Classroom management. This type of utterance tells a student(s) to carry out a physical action without involving particular mathematical concepts. Examples include: "Get in your small groups and work on the next problem and discuss the ideas in your group", "When your group is finished write your results on the assigned area of the blackboard."
2. Directing. Mathematical management. This type of utterance directs a student(s) to carry out a particular mathematical action (e.g., find the x(t) and y(t) equations, graph such and such function). In comparison to T7, this type of utterance is tightly tied to the current problem.
3. Motivating. This type of utterance provides encouragement or motivation for students. For example, the utterance may indicate that a particular student idea is a worthy one (without explicitly stating that it is correct or incorrect as with T5), it may indicate that it is an important idea, it may encourage students to pursue particular line of inquiry, to follow up on their ideas, etc.

4. Checking. Check on student progress (e.g., Have you got to number 2 yet? Do you all have the same question?) or student(s) agreement, disagreement, or understanding on an issue? (e.g., Does anyone have any questions or comments?

Appendix F Teachers' ISAs for Research Sub-Question 3

T22 **Snack Shack - [illegible]structional Sequence Analysis**

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	Average the 50 cases	What do you notice about this paper? Why do you think this person started with the number of cases that they did?
2	[redacted]	Visually organized (bar graph) with $200 and 50 cases, individual cases labeled. ALL DATA SHOWN. ** Missing from paperwork. I forgot to copy it when the student asked to add to it and I didn't see it again.	What do you notice about this paper? What's the same? Different? Is every part (label) addressed?
3	[redacted]	Pre-organized clusters. Focus on 50 cases, then checked the $.	What do you notice? Similarities? Differences? Why do you think this person grouped the data this way?
4	[redacted]	Organized guess and check with valid solutions.	What do you notice? Similarities? Differences? How might this method help you find all solutions?
5			

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. 1) Minimal work shown but starts with the concept of average which is common and accessible. 2) More work shown, organized, visual representation. 3) Work shown in a different way with clusters of information. More developed. 4) Organized guess and check with valid solutions. I chose progressively more organized and complete papers. My goal is to show a variety of strategies that could work and have the students point out how this can be accomplished. Even though I plan on showing the solutions at some point, my extension would be to send the students back to complete the problem.

[redacted] #2

T22

Design A Dartboard Task - In..ructional Sequence Analysis

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	Shelby	100% = whole = 100 squares Count square inside to outside	How does the number of squares in each section relate to the percent required? Where did you start when you built your shape (inside to outside)? Does it matter whether or not each section is exactly the same shape?
2	Jared	100% = whole = 180 squares Percent proportion	Did you start with the picture or the math? What is the total amount? How many squares out of 180 is 15% of the whole?
3	[redacted]	100% = whole = 300 squares Calculating using decimals for percents Common multiple Inner areas subtracted if needed?	Where did the 300 come from? Why is it important? Did you subtract the inner areas when you were calculating the dimensions?
4			
5			

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares.

I want to show different solution methods that increased in complexity. 1) I started with this one to show the simple correlation of one square equals one percent. I also want to show that every section does not have to be exactly the same shape. 2) I chose this one to show that you could have something other than 100 squares represent the whole. I also want to show using a proportion. 3) I chose this one to show using common multiples. I plan to include a discussion on the middle being 40% (the total of the two inner sections) and how you need to subtract it from the total. Also, we will talk about reasonable and unreasonable measurements, parts and pictures.

Snack Shack [redacted] **Instructional Sequence Analysis** T23

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	~~Strategy to start~~ Organizational strategy ~~How did you adjust as you go?~~	1 What is your strategy? 2 How did you adjust as you go? 3 Did this use most of the data? 4 Must the number of cases for dif. items be the same?
~~3~~ 3	[redacted]	Organizational strategy. Running total. Cases + Cost both.	5 What is your strategy? 6 Why did the 1st table not work. 7 What did you adjust in the 2nd table.
4	[redacted]	Organizational strategy	How is this different from Baylee's? What is your strategy?
2 ~~3~~	[redacted]	~~What is your strategy?~~ Adjustment. Communication	What was your starting strategy? What was your adjustment strategy?
5	[redacted]		12 How can you go forward in an organized way?

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. No formal organization → better organization
Show all the solutions. Ask students what patterns they see.

[redacted]

Dartboard

Instructional Sequence Analysis

T23

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	15% + 25% = 40% + 60% = 100% Area of triangle	1 Why did you add 15% + 25%? 2 Why did you add the 40% to the 60%? 3 Why ~~divide~~ multiply by ~~by 2~~ ~~and~~ only 1/2 base length
2	[redacted]	Total = 100% = 267 Areas of rectangles	5 Where will you go from here? 4 What was your plan? 6 Why did you multiply 5×7 + 9×9...
3	[redacted]	Percent is a relationship based on 100 Outside to inside strategy	7 Where did you start? 8 How did you use that starting point to find the ~~size~~ size of the sections?
4	[redacted]	% is a relationship subt. 15% Area vs. linear.	
5			

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. Students really struggled with this task. I had difficulty finding a sequence that flowed. This task ended up being a lot more about area than I had anticipated, & that complicated my sequencing. I ended up sequencing for 2 main ideas, the concept that 100% is the total of something & the ~~difference between~~ effects of ~~changing~~ that 2-D is area not linear. Also [redacted] & [redacted] are both ELLs, and I wanted at least 1 ELL to present.

On the % idea, I sequenced from more concrete to ~~gen~~ abstract. And I added the area points as they came up.

[redacted] Task 4 dartboard

spinner Elim

Instructional Sequence Analysis

Teacher 23
grade 7

Reserve (per 1)

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	Don't use zero	1 Why not?
absent 2	[redacted]	Use 1 so you can get the 1st 8.	2 Would removing 9 make it impossible to get some numbers you could get with a 9?
3	[redacted]	Odds + evens Not high	3 Why does using both odds + evens allow you to cross out more numbers? 4 Why "not high"? 5 What is "not high."
4	[redacted]	Don't repeat because you get the same products.	6 What numbers would you use instead? 7 What effects would this have? 8 Is there any number you might want to repeat? (1 & 2)
5			

Note: [redacted] did not have the "odds + evens" sheet with her. So that was not brought out. [redacted] was absent so I skipped his comments. I planned on stating them, but we ran out of time.

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. Zero is a quite obvious special case.

One is another special case. Including that 1 number can result in an additional 8 cross outs.

Then to more general comments, less obvious. The consequences of these are not as obvious + should draw more discussion.

Follow up: ~~List~~ List multiples of each number 0-9 until over 50 (or stuck on 0). During the numbers 1-9 that are products. ~~Circle the 1st time~~ Tally how many ways you can get each number 1-50. Observe which factors produce more products 1-50. Remember 3+4 spin combinations are possible!

→ Primes + composites.

[illegible] the task and easier.
I pulled two of the work samples from students [illegible] in the original test group.

teacher 27
grade 6
SNACK

Instructional Sequence Analysis

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	This student decided to focus on the money instead of always having 50 cases.	- Why is focusing on the money first more difficult?
2	[redacted]	- Well-organized work sequence - Modified cases to yield $200.	- How could we organize this work more efficiently? - Were other solutions missed?
3	[redacted]	- Use of a Table / List format	- How can we use this method in a more systematic way?
4	[redacted]	- Relationship between soda and chip cases	- What other relationships are there between the cases? (equivalent trades that preserve either the # of cases or dollars.)
5	[redacted]	- Repeated use of a case relationship (but doesn't equal 50 cases)	- Their attempt leads to a systematic list (pattern)

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. This task was difficult for my 6th Graders. Some couldn't decide whether to focus more on the cases (50) or to start with combinations of $200. (Work Sample #1 and #2) Starting with 50 cases and then tweaking them to yield $200 would be easier (Sample #2) The next issue was organizing the work. Some kids wanted to erase errors or their work was disorganized. This impeded their guess and revise strategy. Sample #2 is well-organized, but Sample #3 offers a more compact list. Very few groups found more than one solution. Sample #4 includes thinking about the relationship between the products. (5 cases of soda $15 = 3 cases of chips $15) We will generate other relationships. These equal trades allow for more informed revisions. Sample #5 attempted to repeatedly apply a relationship. This leads to patterns and systematic list strategies.

T27

Instructional Sequence Analysis Design a Dartboard DA[redacted] IGN

task 2

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	- Choosing a shape with 100 units - Relate to percents	-1 Why did Peter add 15 + 25 (inner and middle sections)?
2	[redacted]	- More detailed explanation and verification; extensions	2 What patterns/relationships do you see between their different solutions (Ratios)
3	[redacted]	- Use of triangles	3 How did they determine the Base and Height?
4	[redacted]	- Choosing 60 (not 100) for the area of the board	-4 How did he determine each section's area?
5	[redacted]	- Working inside to outside - 1 square = 5%	5 Could we start with a different size for the inner section and still use this approach?

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. I chose to start with a common solution that uses a dartboard with an area of 100 units. The first one has a limited explanation. [redacted]nd [redacted] contains more detail and verification of the percentages. We will then review a similar approach by [redacted] but they used triangle shapes and correctly calculated the dimensions needed. [redacted] approach used a board with an area of 60. We will discuss how he determined the areas of each section. The last piece of work ([redacted]) demonstrates an alternative method, going from the inside shape to the outer sections by determining how much 5% is.

teacher 27
grade 6
Spinner

Instructional Sequence Analysis

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	Common 1st attempt - Random selection - all different #s	1 - Is it best to have all different numbers? 2 - What about higher numbers (7, 8, 9)?
2	[redacted]	Extreme - choosing the same number	3 - How does this affect the possible results?
3	[redacted]	- Compromise - select 1-4 - Hints at probability	4 - What is the affect of not having a 5 or 7? 5 - Are there numbers you can't reach? 7 - Is a 4 necessary? (Factors/Multiples) 6 - How many multiples?
4	[redacted]	- Included 5, 7; lower #s - Stop when "nervous"	8 - Ian got 20/20, but Is luck involved? 9 When should you stop spinning? - Develop ideas on when to stop (# of spins, over 25, over 50% chance of going over)
5			

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. #1 I started with a common attempt, which was to select all different numbers. Some students found this to work, others went over 50 a lot. In contrast, #2 is an example of choosing the same number. [Those students discovered that this limited the number of values they could reach.] #3 is a compromise, choosing numbers 1-4 twice. It avoids going over, and allows for more options, but could benefit from having a 5 or 7 present (will discuss how many new squares each will allow you to reach). The need for a 4, with two 2's present will also be discussed. #4 The last example used lower numbers including a 5 and 7, and yielded positive results. He also mentioned to stop spinning when you "feel nervous", which we will attempt to define. Others gave values from 13 to 25; no one mentioned assessing the chance of going over 50. (Probability)

T9 Snack Shack

Instructional Sequence Analysis

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	$5+3=8 \rightarrow \frac{200}{8}=50 \rightarrow$	$25 \cdot 5 + 25 \cdot 3 = 200$
2			Does this work w/ other solutions?
3	[redacted]	Got a solution for $200	How can the numbers be adjusted to increase or decrease
4			Cases?
5	[redacted]	4 solutions	Wow! Are there more? Emily mentioned something about it being impossible w/ more than 10 cases candy.

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. Each one has a separate issue.

First, novel approach

② Optimizing on one restraint

③ How did they get it?

Instructional [redacted] nce Analysis T9 design a dartboard

Please complete the following table, identifying the students whose work you plan to share and the order in which you plan to do so.

Order of Sharing	Student Name	Ideas to Highlight	Questions to Ask to Make the Math Salient
1	[redacted]	Used 15 squares for smallest section.	Why did you use 15 squares? Why did you ~~use~~ start with the smallest?
2	[redacted]	Started w/ Triangle of 100 square units	Why did you choose the triangle? Your smallest section not inside middle. Why?
3	[redacted]	Taking 15% out of the middle section.	What is area of big & middle sections? Does middle sh area include the yellow small section?
4	[redacted]	Different shapes repeated.	What is total area? How did you get it? Why did you cut some shapes into 2 different sections? Did you think you'd do that originally?
5			

For the sequence you identified above, please explain your rationale. Include any special reasons for the specific order you indicated, any connections between students' work you want to highlight, and what follow-up questions, extensions, or closure you plan after the final student shares. The first 2 students used similar techniques. One started small and went big. The other started big & went small. Also, one had nested shapes while the other didn't.
The third student took the 15% out of the 25%. Common mistake.
The last used a novel shape repeating it, and was close to having an easy time splitting into sections.

[redacted] Task 2