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AN EMPIRICAL STUDY ON

THE JUMP-DIFFUSION TWO-BETA ASSET PRICING MODEL

by

HONGQING CHEN

A dissertation submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

An abstract of the dissertation of Hongqing Chen for the Doctor of Philosophy in Systems Science: Business Administration presented February 19, 1996

Title: An Empirical Study on the Jump-diffusion Two-beta Asset Pricing Model

This dissertation focuses on testing and exploring the usage of the jump-diffusion two-beta asset pricing model. Daily and monthly security returns from both NYSE and AMEX are employed to form various samples for the empirical study. The maximum likelihood estimation is employed to estimate parameters of the jump-diffusion processes. A thorough study on the existence of jump-diffusion processes is carried out with the likelihood ratio test. The probability of existence of the jump process is introduced as an indicator of "switching" between the diffusion process and the jump process. This new empirical method marks a contribution to future studies on the jump-diffusion process. It also makes the jump-diffusion two-beta asset pricing model operational for financial analyses.

Hypothesis tests focus on the specifications of the new model as well as the distinction between it and the conventional capital asset pricing model. Both parametric and non-parametric tests are carried out in this study.

Comparing with previous models on the risk-return relationship, such as the

capital asset pricing model, the arbitrage pricing theory and various multi-factor models, the jump-diffusion two-beta asset pricing model is simple and intuitive. It possesses more explanatory power when the jump process is dominant. This characteristic makes it a better model in explaining the January effect. Extra effort is put in the study of the January Effect due to the importance of the phenomenon. Empirical findings from this study agree with the model in that the systematic risk of an asset is the weighted average of both jump and diffusion betas. It is also found that the systematic risk of the conventional CAPM does not equal the weighted average of jump and diffusion betas.

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ACKNOWLEDGMENT

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INTRODUCTION

It has been proven empirically that the conventional capital asset pricing model (CAPM) is not working. As early as in 1977, Roll pointed out the impossibility of obtaining a true market portfolio for the use of the model. He also argues the untestablity of the hypothesis of the mean-variance efficiency of the market portfolio. After repeated empirical studies on the CAPM, Fama and French (1992, 1993) and Roll and Ross (1992) concluded that beta has very weak explanatory power.

One of the reasons that may account for the weak explanatory power of the beta is that the CAPM assumes security returns are generated by a normal process, while in fact the empirical observation of the security returns indicates that this assumption is not supported. Studies on stochastic behavior of the security market generally agree that security returns are generated by a mixed process with continuous diffusion and discontinuous jumps. In this sense, the CAPM beta may only capture a part of the mixed-process, the diffusion process. Therefore, the beta of the CAPM is not an accurate risk measure when the process is mixed with jumps. Hsia's (1992) jump-diffusion two-beta asset pricing model provides an alternative to the CAPM. It prices both jump and diffusion risks. Empirical tests of this dissertation show that it is a better asset pricing model than the CAPM, especially for the period when jumps are more intensive and frequent.

Following this introduction, Chapter One presents a review of previous studies on

the asset pricing models. Summaries of the CAPM model, its extensions, and critiques are presented in this chapter, together with reviews of arbitrage pricing theory and option pricing theory. Theoretical and empirical works on the mixed jump and diffusion processes are also reviewed in this chapter.

Chapter 2 presents a brief description of the jump-diffusion model, a discussion on its properties, and the testability of the jump-diffusion model.

Chapter 3 reports the methodologies employed in this research. How to identify a jump has been the focus of empirical studies on the jump-diffusion processes. Starting with a summary of econometrics of structural change, it presents a review of techniques used in previous studies. Benefiting from previous researches, this dissertation employs a maximum likelihood method to estimate the parameters needed to separate the jump process from the diffusion process. The concept of "switching" becomes reality by calculating the probability of the existence of the jump process. Thus the jump process is identified and is physically separated from the diffusion process. The success of this procedure is one of the contributions of this dissertation.

Chapter 4 presents the data analysis of this research. It includes data collection, sampling method and sample statistics, frequency distribution of market returns, estimation of parameters, hypotheses testing and test results.

Chapter 5 presents how the jump-diffusion two beta asset pricing model explains the January effect, a well-documented financial market anomaly. Besides testing the new model with a sample of the turn-of-the-year daily returns, a comparison with the most recent Fama and French's five-factor model is carried out with the exact sample of

monthly returns. At the end of the extensive analysis, the conclusion of the dissertation

research is made and is included in this chapter. So are some thoughts for future

researches.

CHAPTER ONE

A REVIEW OF ASSET PRICING MODELS

Modeling is a tool to simplify and thus to analyze the real world. Models have

been used in mathematics and physics, and to study environmental problems, economic

problems, and social problems. In the fields of social sciences, however, results of

modeling should always be carefully interpreted due to the complexity of problems with

human factors. In the world of finance, modeling processes began with asset pricing

models. Factors contributing to the change of asset values have been the focus of finance

studies. Asset pricing models, as reviewed in this chapter, have proven their usefulness

in analyzing the financial market.

1.1 CAPITAL ASSET PRICING MODEL

1.1.1 Derivation of Capital Asset Pricing Model

Assume all investors are risk-averse. Investors make their decisions based on

maximizing their expected utilities under uncertainty when forming a portfolio.

According to Markowitz (1959), investors' preferences in selecting portfolios are defined

over the mean and variance of the probability distribution of portfolio returns. This

concept leads to a revolution in the theory of finance and lays the foundation for modern

capital market theory (Jensen 1972). The CAPM is developed by Sharpe (1964) and

Lintner (1965) following Markowitz's work on portfolio selection. The mathematical

derivation is illustrated in Appendix A. Figure 1 graphically illustrates the derivation

process:
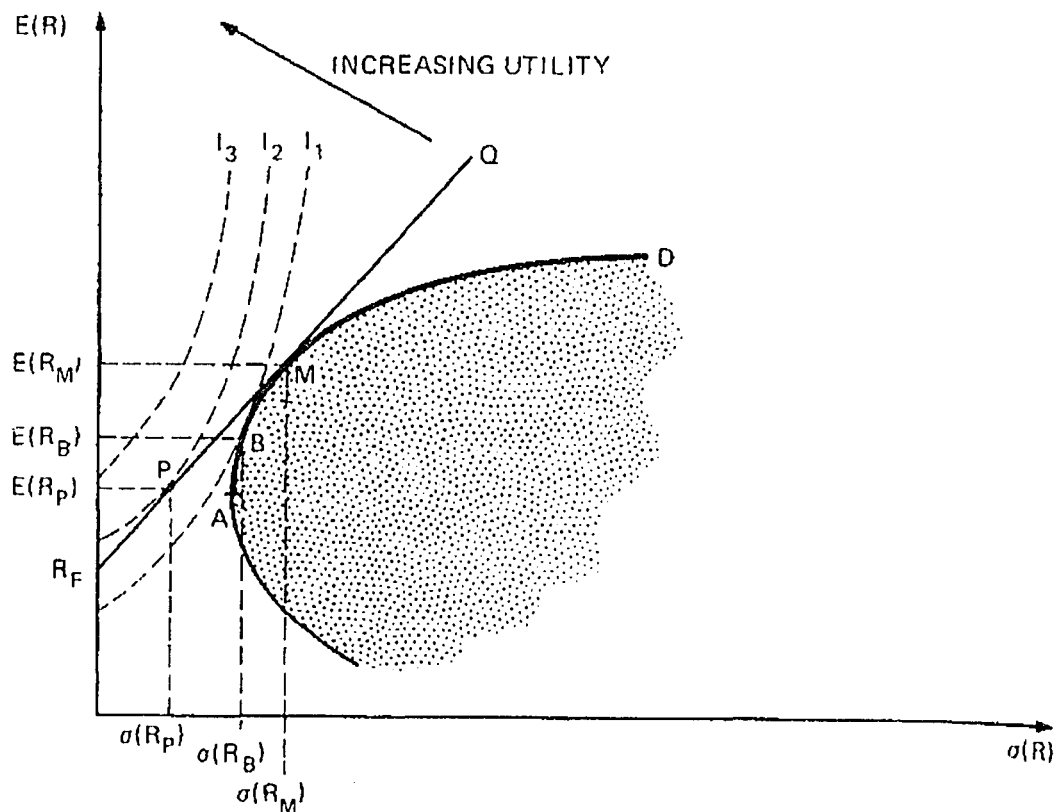
Figure 1:   The Maximization of Investor's Utility Given the Existence of A Riskless
              Asset

Based on investors' utility function, we draw indifference curves on the scale of

"expected return" (mean of return) and "standard deviation" (equivalent alternative of

variance) for every security. Since utility function is concave (Fama and Miller 1972),

investors who want to maximize the expected utility would select securities that place them on the highest indifference curve (represented by I_1, I_2, and I_3). The shaded area in Figure 1 represents all possible combinations of risk and return available from investments in risky securities. It is the opportunity set of risky assets available to investors. The upper boundary ABMD of the opportunity set is called the efficient frontier, which consists of all portfolios with the highest expected returns for a given level of standard deviations. The efficient frontier is also called the minimum variance opportunity set.

An investor limited only to investments in risky assets would choose to invest in portfolio B, which has expected return, $E(R_B)$, and standard deviation of return, $\sigma(R_B)$, to maximize his expected utility I_1. Now assume there is a riskless asset F with a certain return R_F. An investor has the possibility to invest in such a riskless asset as well as in a risky asset (or a portfolio). The R_FMQ line is the so-called capital market line. The tangent point M is the market portfolio. The investor can then construct a combined portfolio which allows him to reach any combination of the risk and return lying along the line. Thus he may distribute his funds between portfolio M and riskless asset F such that his combined portfolio, P, yields him $E(R_P)$ and $\sigma(R_P)$, and maximum utility of $I_2 > I_1$. The CAPM was derived by Sharpe (1964) and Lintner (1965) with the global mean-variance efficiency (i.e., efficient portfolios lie on the frontier of the minimum variance opportunity set) under the following six assumptions:

1. Investors are risk-averse individuals who maximize the expected utility of their end-of-period wealth.
2. Investors have homogeneous expectations about asset returns that have a joint

normal distribution.
3.There is a risk-free asset such that investors may borrow or lend unlimited amounts at the risk-free rate.
4.The quantities of assets are fixed. All assets are marketable and perfectly divisible.
5.Asset markets are frictionless; information is costless and simultaneously available to all investors.
6.There are no market imperfections such as taxes, regulations, or restrictions on short selling. (Copeland, 194)

Given these assumptions, the equilibrium expected return is

$$\overline{R}_j = R_f + (\overline{R}_m - R_f)\beta_j \qquad (1.1)$$

where

$$\overline{R}_j = \text{Expected Rate of Return} \\ \text{of Security (Portfolio) } j$$

$$R_f = \text{Risk-free Rate}$$

$$\overline{R}_m = \text{Expected Rate of Return} \\ \text{of Mean-Variance Efficient Market Portfolio}$$

$$\beta_j = \frac{Cov(R_j, R_m)}{\sigma^2(R_m)} \qquad (1.1a)$$

According to the CAPM, the equilibrium expected return on all risky assets are a function of their covariance with the market portfolio. It implies that the appropriate measure of the risk of any individual asset is its covariance with the market portfolio, $cov(R_j, R_m)$ and not its variance $\sigma^2(R_j)$ (Jensen 1972). This is because that the diversification can eliminate the effects of an asset's own variance on the variance (or the standard deviation)

of a portfolio but cannot eliminate the effects of an asset's covariance with all other assets

in the portfolio. In other words, an asset's own variance (or the standard deviation) is the

nonsystematic risk that can be hedged to zero by forming a portfolio with a number of

selected securities. Its covariance with the market is the systematic risk, β_j that cannot be

diversified away. Therefore as CAPM states, it is the only risk that investors will pay a

premium $(\overline{R}_m - R_f)$ to avoid. This important property of the CAPM, that is, to

quantify the risk of investment, makes its unrealistic assumptions tolerable. In fact, it has

dominated the financial literatures since its invention. Its simple and intuitive structure

makes it popular with the practical financial world (Modigliani, Franco and Pogue 1974,

Rosenberg 1981, Mullins 1982). The risk measure, beta (β), is published by major

investment publications, such as Standard & Poor and Moody's Investors Guide, for every

listed security as well as for evaluated industries.

1.1.2 Modifications of CAPM

Since its invention, the CAPM has been tested empirically by many researchers.

Conclusions are generally unsatisfactory either statistically or instrumentally. Not only

do the test results of the CAPM depend on the length of period[1], but also the explanatory

power of beta is so weak that the CAPM cannot accurately predict the required rates of

return of assets (Black, Jensen and Scholes 1972, Blume and Friend 1973). Many

researches note unexplainable market anomalies when using the CAPM (Banz 1981,

---

1. As stated by Copland, "over long periods of time the rate of return on the
market portfolio is greater than the risk-free rate (215)."

Keim 1986, Keim and Stambaugh 1986, Roll 1983). The common speculation is that the

unrealistic assumptions based on which the CAPM is derived cause those problems.

Therefore, in the past several decades much effort had been concentrated on exploring the

extensions of the CAPM by relaxing some of its assumptions.

A zero-beta CAPM was developed by Black (1972) by assuming there is no

riskless asset for investors to borrow or to lend; instead there exists a zero-beta portfolio.

This means a portfolio orthogonal to the market portfolio, which may be created by short

selling or hedging. The CAPM with the existence of a non-marketable asset was

developed by Mayers (1972). When allowing differential taxes on capital gains and

dividends, Brennan (1970) found that a higher dividend yield requires a higher rate of

return. This effect is later studied by Litzenberger and Ramaswamy (1979). Friend,

Landskroner, and Losq (1976) looked at CAPM under uncertain inflation. Works on

relaxing the assumption of normality are reviewed later in this chapter.

1.1.3 Critiques about CAPM

Modifications on the CAPM are usually related with the criticisms about the

model. Friend and Blume (1970) raised questions about its usefulness in explaining

market behavior. Later in 1973, they argued for the rejection of the CAPM. Despite

these, the CAPM has been considered to be a great model until Roll's critique was

published in 1977. Two fatal points pointed out by Roll are 1) the only testable

hypothesis about CAPM is the mean-variance efficiency of the market portfolio, but the

true market portfolio is unobservable and 2) the market portfolio which should include all

assets of the market is unobtainable (Roll 1977). Upon this ground, Roll concluded that

the CAPM is untestable although it is still valid in theory. In 1980 Roll again pointed out

that benchmark errors exist when using the CAPM to evaluate portfolio performance. He

stated that: "CAPM benchmark error is present whenever the market index is not

'optimized', i.e., whenever the index is not an ex ante mean/variance efficient portfolio.

(12)"

    After almost fifteen years of Roll's critique, the validity of CAPM was brought up

again by several financial economists and later broadcasted by Fama and French (1992) in

their article which questioned the usefulness of beta. Following a series of empirical

research on the correlation between an expected rate of return of an asset and its beta,

Fama and French reported their research in which they found beta is the least important

factor in estimating the expected rate of return. Roll and Ross (1992) then theoretically

proved the little relevance of beta in estimating the expected rate of return by showing

that it is almost impossible to find an accurate market index which representing minimum

variance opportunity set only through which can sensible/relevant beta be obtained.

Needless to say, their opinions shocked the financial world which has adopted beta as the

most important factor in pricing financial assets.

1.2 ARBITRAGE PRICING THEORY

    Before the irrelevance of beta was pointed out, its weak performance in predicting

the expected rate of return urged academicians to find an alternative asset pricing model.

Ross (1976) suggested the arbitrage pricing theory (hereon APT). Naturally this model is embraced by CAPM's critiques because it is not restricted to an all-included market portfolio.

APT is developed based on the notion that security prices adjust as investors form portfolios in search of arbitrage profits. When such profit opportunities have been exhausted, security prices are in equilibrium. In this context a definition of market efficiency is the absence of arbitrage opportunities, having been eliminated by arbitragers. It is an asset pricing model in which the expected return on any risky asset is viewed as a linear combination of various common factors that affect asset returns. The actual return of a security, Rj, is

$$R_j = \overline{R}_j + b_{1j}F_1 + b_{2j}F_2 + \ldots + b_{nj}F_n + e_j \qquad (1.2)$$

where

$$\overline{R}_j = a_0 + a_1 b_{1j} + a_2 b_{2j} + \ldots + a_n b_{nj} \qquad (1.2a)$$

$\overline{R}_j$ = the expected equilibrium
       return of security j

F_i = factor i

b_{ij} = sensitivity coefficients
       associated with factor i

a_i = risk premiums for
       the risk associated with factor i

According to APT, which assumes that investors agree on what the relevant factors are and on the coefficient b's in equation (1.2), two securities with the same array of b's must

provide the same expected return. Otherwise investors would rush to buy the security with the higher expected return and sell the other. Such arbitrage action would cause price adjustments in both securities until their expected returns are the same. Thus the elimination of arbitrage profits with respect to the relevant factors results in equilibrium pricing of all securities.

The difficulty of selecting influential factors has been a problem of implementing the APT since its invention. Roll and Ross (1980) suggested four factors to be systematic sources of security portfolio risks: (1) unanticipated changes in inflation, (2) unanticipated changes in industrial production, (3) unanticipated changes in the yield differential between low- and high-grade bonds (the default-risk premium), and (4) unanticipated changes in the yield differential between long-term and short-term bonds (the term structure of interest rates) (Roll and Ross 1980). According to this model, investors are characterized as having risk preferences along four dimensions. Each investor would formulate a portfolio of securities depending on his or her desired risk exposure to each of the factors. Different investors will have different risk attitudes. For instance, some may want little inflation risk but be willing to tolerate considerable productivity risk and default risk.

When Dybvig and Ross (1985) proved the APT is testable, the superiority of the APT over the CAPM is clear. As proved by Hsia (1981), APT is a more general asset pricing model than the CAPM. The CAPM is a special case of APT: While APT allows more than one factor to explain the expected return on a risky asset, the CAPM only allows one (the mean-variance efficient market portfolio). A number of empirical tests

on the APT show that there are indeed more than one factor, i.e., a market portfolio,

found to influence security returns (Van Horne 1989, p83). Among them, Trzcinka

(1986) concluded his study on the number of factors by stating there is "at least one

(factor)" affecting the returns. Fama and French (1992) found that the CAPM beta is the

least influential on the stock return among five factors. In an effort of searching more

influential factors, Fama and French (1993) identified five common risk factors in returns

of stock and bonds. They are three stock-market factors: a size risk factor, book-to-

market equity risk factor, and market factor; and two bond-market factors: unexpected

changes in the term structure of interest rates, and default risk. According to Fama and

French, "the five factors seem to explain average returns in stocks and bonds" except for

January Effect[2] (43). Nor could Roll (1983) explain the January Effect with his four-

factor APT.

Empirical studies on the comparison between the CAPM and the APT draw

mixed conclusions. Chen (1983) found that empirical tests on the APT are favorable.

Bower, Bower and Logue (1984) found that APT works better than the CAPM in policy-

making because it includes sensitive factors. However it is not clear whether APT

possesses higher explanatory power than the CAPM. Even Roll (1988) himself was

disappointed after comparing the APT with the CAPM. He found little improvement of

the APT over the CAPM on its explanatory power in terms of R^2. The failure of APT in

capturing the January Effect also shows its weakness in the explanatory power.

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2. January Effect is explained in detail in Chapter Five.

1.3 OPTION PRICING THEORY

From a different angle, Black and Scholes (1974) developed the option pricing

theory (Hereon OPT). A term of physics, diffusion was then introduced to the finance

world.

An option is the right to buy (a call option) or to sell (a put option) a share of

stock at a specified price (the exercise price) for a specified period of time (the expiration

date). The value of the option at the expiration date is the value of the stock minus the

exercise price, or it may be zero. It cannot be a negative value. With a stock and an

option on the stock, it is possible to establish a riskless hedged position by buying the

stock and writing options or selling them short, or by buying options and selling the stock

short. In efficient financial markets, the rate of return on a perfectly hedged position

would be the risk-free rate. As a result, the option price will bear a precise relationship to

the stock price. And the value of the option can be determined at the beginning of the

period.

Based on a notion that investors are able to maintain reasonably hedged positions

over time and that arbitrage will drive the return on such positions to the risk-free rate,

the OPT provides an exact formula for determining the value of an option based on the

volatility of the common stock, the price of the common stock, the exercise price of the

option, the time to expiration of the option, and the short-term interest rate. One can also

derive the price of the stock if given the value of its option and other four factors.

The assumptions based on which the model is derived are as following:

1. Only European options are considered, that is, options that can be exercised only at maturity.

2. There are no transaction costs. Options and stocks are infinitely divisible, and information is available to all without cost.

3. No imperfections exist in writing an option or selling a stock short.

4. The short-term interest rate is known and constant throughout the duration of the option contract. Market participants can both borrow and lend at this rate.

5. The stock pays no dividend.

6. Stock prices behave in a manner consistent with a random walk in continuous time.

7. The probability distribution of stock returns over an instant of time is normal.

8. The variance of the return is constant over the life of the option contract and is known to market participants. (Van Horne 1989, p102)

The equilibrium value of an option is

$$V_o = SN(d_1) - \frac{K}{e^{rt}} N(d_2)$$

(1.3)

where

$S =$ current price of the common stock
based on which the option is written

$K =$ the exercise price
of the option

$r =$ the short-term interest rate
continuously compounded

$t =$ the length of time (years)
till the expiration of the option

$N(d) =$ the value of the cumulative
normal density function

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{1}{2}\sigma^2) t}{\sigma\sqrt{t}}$$

(1.3a)

$$d_2 = \frac{\ln(\frac{S}{K}) + (r - \frac{1}{2}\sigma^2) t}{\sigma\sqrt{t}}$$

(1.3b)

σ = *the standard deviation of*
the annual rate of return on
the stock continuously compounded.

The OPT leads a new path for modern financial studies. Not only has OPT been proven to be useful in estimating option prices as well as asset prices, but also it is coherent with CAPM and APT (Hsia 1981). Most important, its introduction of "diffusion process" becomes the base for studies on stochastic processes in the stock market. The early studies of jump and diffusion processes, in fact, are mostly within the context of option pricing theory.

1.4 JUMP AND DIFFUSION STUDIES

When Fama (1965) discovered that the empirical distribution of daily returns on New York Stock Exchange securities are not normally distributed but have "fat tails," the normality assumption of financial theories, including CAPM, APT, and OPT, is questioned. Studies on the stochastic processes start. Merton (1976) concluded the early explanations and laid the foundation for current jump-diffusion studies. He extended the OPT to include the diffusion process and the jump process. Studies on the stochastic processes since Merton is summarized by Bates (1991) who classifies into three major classes (1015):

(1)   *constant elasticity of variance processes*, special cases of which include

arithmetic and geometric Brownian motion, such as in Cox and Ross (1976);

(2)   *stochastic volatility processes*, the benchmark models being those for which

volatility evolves independently of the asset price, such as in Wiggens (1987) and

in Melino and Turnbull (1990);

(3)   *jump-diffusion processes*, the benchmark model being Merton's (1976) and Ball

and Torous'(1983) specification of log-symmetric jumps with zero mean.

Let us focus on the jump-diffusion process studies. The introduction of the notion

of jump is certainly based on empirical observations. Press (1967), Merton (1976), Ball

and Torous (1983) state that the behavior of the (log) returns can be divided into two

independent components:

(1)   the continuous part which is responsible for the usual (or "normal") day-to-day

price movement. The changes in stock prices may be due to variation in

capitalization rates, a temporary imbalance between supply and demand, or the

receipt of any information which only marginally affects stock prices. It is

described by the traditional Brownian motion and modeled as a lognormal

diffusion process;

(2)   a discontinuous part which is due to the receipt of any important information that

causes a more than marginal change (or "abnormal" change) in the price of stock.

The arrival of this kind of information is random. The number of information

items is assumed to be distributed according to a Poisson process. Thus, this part

is modeled as a Poisson jump process.

This description is intuitive. In fact it may be the answer to market anomalies such as the

size effect (small firms' stocks earn higher returns), the January effect or the weekend

effect, holiday effect, and so on. After all, the term "anomaly" means abnormal changes in prices in the financial context (Bhardwaj and Brooks 1992, Keim 1983 and 1989, Keim and Stambaugh 1986, Lakonishok 19, and Roll 1983).

Merton (1976), Cox and Ross (1976), and Ball and Torous (1983) extend the OPT to include the jump process. Their models assume risk-neutral and imply that the distribution is symmetric and slightly positively skewed, as stated by Fama (1965). They assume that jumps in the prices of the underlying asset are uncorrelated with the changes in the price of the market portfolio, so that the jump risk is not priced in equilibrium. While Neither Merton (1976) nor Cox and Ross (1976) conducted any empirical work with their suggested models, Ball and Torous did and found that 78% stocks examined in their study indicate jumps presence.

When important news like wars arrive, however, the market jumps, too. Most recent examples are the crashes of 1987 and the volatile markets during the Persian Gulf War. Naik and Lee (1990) derived an equilibrium model, which includes premiums for both jump and diffusion risks, to price the option on the market portfolio. Their work moves the jump-diffusion studies from focusing on individual assets to the market portfolio. When commenting on Merton's (1976) model, they point out that a "feature of the Merton model is the assumption that the jumps in security prices are uncorrelated with the return on the market portfolio. Clearly, this assumption is violated if the security under consideration is the market portfolio itself (495)." Another merit of their model is its recognition of the jump risk as systematic risks, which is left out by Merton as well as Cox and Ross (1976).

If the return of stocks should be divided into jump part and diffusion part, certainly the risk associated with return of securities should be decomposed into two parts, too. The CAPM states that beta, a diffusion risk, is systematic and nondiversifiable. So is the jump risk when taking both diffusion process and jump process into account. Bates (1991) develop a general equilibrium jump and diffusion option pricing model which has an asymmetric distribution. He also prices both jump and diffusion risks. Based on his contribution, Hsia (1992) obtained a jump-diffusion two-beta asset pricing model. The derivation of the jump-diffusion two-beta asset pricing model is illustrated in Appendix B.

CHAPTER TWO

JUMP-DIFFUSION TWO-BETA ASSET PRICING MODEL

2.1 THE JUMP-DIFFUSION MODEL

Under the same assumptions of the CAPM, except the normality of asset returns, the jump-diffusion model takes two different types of beta into account when pricing the underlying asset. One is the diffusion beta, which measures the systematic risk when no jump occurs. The other is the jump beta, which measures the systematic risk when jumps take place. In a similar form as that of the CAPM, the jump diffusion two-beta asset pricing model is as follows:

$$\overline{R}_j = R_f + (\overline{R}_w - R_f)\,[\,(1 - \phi)\,\beta_j + \phi\beta_{\wedge j}\,] \qquad (2.1)$$

where

$\overline{R}_j$ = *Expected Return of Security j*

$\overline{R}_w$ = *Expected Return of an Optimally Invested Wealth*

R_f = *Risk-free Rate*

β_j = *Diffusion β of Security j*

$$\beta_j = \frac{Cov(R_j, R_w)}{\sigma^2(R_w)} \qquad (2.1a)$$

R_j = Random Return of Security j
Generated by a Diffusion Process

R_w = Random Return of an Optimally Invested
Wealth Generated by a Diffusion Process

$Cov(R_j, R_w)$ = Instantaneous Covariance of R_j, R_w

$\sigma^2(R_w)$ = Variance of R_w

$\beta_{\Lambda j}$ = Jump β of Security j

$$\beta_{\Lambda j} = \frac{Cov(R_{\Lambda j}, R_{\Lambda w})}{\sigma^2(R_{\Lambda w})} \qquad (2.1b)$$

$R_{\Lambda j}$ = Random Return of Security j
Generated by a Jump Process

$R_{\Lambda w}$ = Random Return of an Optimally Invested
Wealth Generated by a Jump Process

$Cov(R_{\Lambda j}, R_{\Lambda w})$ = Covariance of $R_{\Lambda j}$, $R_{\Lambda w}$

$\sigma^2(R_{\Lambda w})$ = Variance of $R_{\Lambda w}$

ϕ = Weight Parameter

$$\phi = \frac{\lambda\sigma^2(R_{\Lambda w})}{\sigma^2(R_w) + \lambda\sigma^2(R_{\Lambda w})} \qquad (2.1c)$$

λ = Mean Rate of Jumps

2.2 PROPERTIES OF THE JUMP-DIFFUSION MODEL

2.2.1 Risk and Return of an asset

The jump-diffusion two-beta asset pricing model shows that the expected return

of a security consists of three components: (i) the risk-free rate, R_f, for compensating the

time value of investing, (ii) the premium for taking diffusion risk $[\overline{R}_w - R_f](1 - \phi)\beta_j$,

and (iii) the premium for taking jump risk $[\overline{R}_w - R_f]\phi\beta_{\Lambda j}$. It clearly shows that jump

risk is a systematic risk and it is priced with premiums.

2.2.2 The Value of ϕ

The weight parameter ϕ is a function of λ, $\sigma^2(R_w)$, and $\sigma^2(R_{\Delta w})$. Since all three

parameters are non-negative, the value of ϕ is bounded between 0 and 1, that is,

$$0 \leq \phi \leq 1$$

which is ideal for ϕ to serve as a weight parameter of the jump-diffusion two-beta asset

pricing model.

2.2.3 Generality of the Jump-Diffusion Model

Equation (2.1) is a more general asset pricing model that takes both the pure

diffusion and pure jump CAPM as two special cases. If there is no jump in the market, λ

= 0 which implies ϕ = 0, equation (2.1) reduces to the conventional CAPM,

$$\overline{R}_j = R_f + (\overline{R}_w - R_f)\beta_j \qquad (2.2)$$

The equation (2.2) differs from equation (1.1) in that it does not use R_m, the market

portfolio, but R_w.

If the intensity of Poisson jumps increases indefinitely (i.e., $\lambda \to \infty$), Poisson distribution converges to Normal and the jump beta converges to the diffusion beta. Consequently, equation (2.1) again reduces to the conventional CAPM, equation (2.2).

On the other hand, if security returns are generated by a pure jump process, $\sigma^2(R_w)$ = 0 which implies $\phi = 1$, then equation (2.1) reduces to a pure jump CAPM,

$$\overline{R}_j = R_f + (\overline{R}_w - R_f) \beta_{\Lambda j} \tag{2.3}$$

Equation (2.2), the pure diffusion CAPM, and equation (2.3), the pure jump CAPM are the two special cases of equation (2.1), the jump-diffusion two-beta asset pricing model.

2.2.4 Expected Return-Risk Relationship

The conventional CAPM implies that securities have same expected returns if they have same betas. The expected-risk relationship of the jump-diffusion model is different. The jump-diffusion model has two different types of beta. It is two-dimensional instead of one. One measures the systematic risk when no jump occurs, and the other measures the systematic risk when jumps occur. These two types of beta are independent by definition. Different securities may have different diffusion and jump risks. As a result, securities will have different expected returns even if they have the same diffusion betas.

The conventional CAPM implies two-fund separation which claims that all investors hold the same two portfolios, a market portfolio and a riskless asset. This is no longer true in the Jump-Diffusion model because investors may have different

preferences to diffusion and jump risks. It is difficult, if not impossible, to find a

portfolio that is optimally invested wealth which requires the same premium on both the

diffusion and jump risks that are carried by different securities.

2.3   TESTABILITY OF THE JUMP-DIFFUSION MODEL

Roll (1977) shows that the conventional CAPM is untestable because the only

testable hypothesis is mean-variance efficiency of the market portfolio, but the market

portfolio is unobservable and cannot be used in the tests. Then the question is: Is the

jump-diffusion two-beta asset pricing model testable? The following reasons suggest its

testability.

First, the concept of mean-variance efficiency is relevant only to equation (2.2), a

special case of the jump-diffusion model. In general, the mean-variance efficiency is not

a testable hypothesis of the new model.

Second, the mean-variance efficiency is equivalent to the two-fund separation.

The mean-variance efficiency is a testable hypothesis of the conventional CAPM, if and

only if the two-fund separation holds, i.e., all investors hold the same two portfolios. As

explained above, the two-fund separation no longer holds in the context of the new

model. Consequently, the mean-variance efficiency of the market portfolio is no longer a

testable hypothesis of the new model.

Third, the concept of an all-inclusive market portfolio is not relevant to the new

model either. The parameter that is closest to the concept of a market portfolio in the new

model is W which is defined as optimally invested wealth. Bates (1991) tested his option pricing model on the futures of the S&P 500 index. From the point of view of component securities, the S&P 500 is optimally invested wealth. The expected return-risk relationship between the component securities and the S&P 500 is described by equation (2), the jump-diffusion two-beta asset pricing model. Thus, the new model is testable and can be tested by a subset of assets against an optimally invested market-wide portfolio.

CHAPTER THREE

METHODOLOGY

3.1 ECONOMETRICS OF STRUCTURAL CHANGE

There have been decades of studies on the structural change in the field of econometrics. The classic example is the structural change between the supply and demand model in economics. Given a sample of time series economic data. Due to changes in policies and other factors, the data may be on the supply curve or on the demand curve during different periods (disequilibrium). If when the change takes place is known, the Chow test is used to identify it (Greene, p218-p224). If it is uncertain when the change takes place, alternative tests are CUSUM (cumulated sum of residuals) or CUSUM-Squared tests (Greene, p224-p227). The null hypothesis used in CUSUM and CUSUM-Squared tests is that the coefficients of the model are the same for all periods. With the CUSUM or CUSUM-Squared method, one can test whether they are the same or not, yet without knowing what they are. The random coefficients model can actually predict them with the assumption that "the parameter heterogeneity . . . as due to stochastic variation (Greene, p476)." As summarized in Greene (p476-p479), the application of this method is with cross-sectional rather than time-series data. For studies

with time series data in disequilibrium, however, a method called switching regression is

most effective to estimate different coefficients. Goldfeld and Quandt (1974)

summarized various usage of the switching regression with detailed explanations. Since

this technique is applied to solve problems with mixed structure where "switching" from

one structure to the other takes place, it can be applied to stochastic problems like the

mixed jump-diffusion process.

3.2 JUMP AND DIFFUSION PROCESS

Since the study of the mixed process of financial markets started in the 1970s,

how to separate jump process from diffusion process has always been a challenge. Early

studies focused on estimating descriptive statistics, and mean and variances of the jump

and diffusion processes without actually separating them. The review of published works

reveals progress of both knowledge and computing technology. Press (1967) limited the

sample size to only ten NYSE listed common stocks over the period 1926 through 1960

in his pioneer study. In order to describe the jump-diffusion process, Press constrains the

instantaneous expected rate of return on the security to be zero. Employing the method of

cumulants, he obtains many negative estimates of the variance parameters for both

diffusion and jump processes. In an effort to modify the procedures of Press, Beckers

(1981) sets the mean jump size to zero. Beckers admits the superiority of the maximum

likelihood method because "the parameter estimates are efficient and the asymptotic

distribution of the estimates is known, the first order condition is highly nonlinear and

contains an infinite sum (Idid: 129)." However, he found it "computationally impractical

. . . because the occurrence of a jump cannot be identified easily (Idid: 129)." Therefore,

Beckers employs the method of cumulants again and obtains many negative estimates of

both diffusion and jump's variance parameters.

Frustration over the difficulty of empirical implementation and verification of the

mixed jump and diffusion processes was overcome when Ball and Torous (1983) reported

their studies. They put forward an implementable model to ascertain the presence of a

jump and employed the maximum likelihood method with their Bernoulli jump process

model:

$$\ln L = \sum_{i=1}^{n} \ln[(1-\lambda)\psi(\alpha, \sigma^2) + \lambda\psi(\alpha, \sigma^2+\delta^2)]$$ (3.1)

where

$$\psi(\alpha, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(\frac{-(x_i - \alpha)^2}{2\sigma^2}) \ .$$ (3.1a)

The λ in equation (3.1) is not the same as that in equation (2.1), the Jump-

Diffusion model. It is only an indicator that is 0 if no jump taking place. By testing

whether $\lambda = 0$ with a form of likelihood ratio test, Ball and Torous found that "over 78%

of stocks indicate presence of jumps at the 1 percent significance level (p60)." They

obtained no negative estimates of variance with this method (p59). While the presence of

jumps is detected, however, the actual jump process is not separated from the diffusion

process.

Jorion (1989) proposes a practical model to identify the jump process in both the

foreign exchange market and stock market. It is implementable and in the form of

maximum likelihood function. The derivation of the equation is illustrated in next

section. Although he notes that "discontinuities are harder to identify in the stock market

index (Idid: 442)," Jorion's model provides a method to empirically study the stock

market. Johnson and Schneeweis (1994) carried out a jump-diffusion study on the impact

of the macroeconomic news based on Jorion's mixed process model. This research is also

based on his method.

Naik and Lee (1990) priced both jump risk and diffusion risk in an extension of

the OPT model. Bates (1991) also prices the jump risk as a systematic risk, but his notion

of the jump risk is not beyond the context of the OPT.

A common feature of the previous discussed works is the complexity of pricing

the jump risk. They estimate jump and diffusion parameters implicitly with the option

price. In contrast, this paper presents an intuitive and effective way to explicitly identify

jump and diffusion processes. This method is not limited by the OPT and can be applied

to all assets.

3.3 MAXIMUM LIKELIHOOD ESTIMATION EQUATION

In his derivation of the maximum likelihood function of the mixed process, Jorion

(1989) illustrates the following. If prices follow a diffusion process with constant drift

parameter $E(\Delta P/P) = \alpha$ and constant variance $V(\Delta P/P) = \sigma^2$, then

$$dP_t/P_t = \alpha \, dt + \sigma \, dz_t \tag{3.2}$$

where P_t is the daily price of stock market index and z is a standard Gauss-Wiener

process (Merton 1976, p128). Let X_t be the logarithm of price relatives, i.e. $X_t = \ln(P_t/P_{t-1})$. The assumption that prices follow the diffusion process implies that $X_t \sim N(\mu, \sigma^2)$, normally distributed with mean $\mu = \alpha - \sigma^2/2$ and variance σ^2. As shown in Oldfield et al (1977, p391-392), the result of solving the differential equation (3.2) is

$$X_t = \ln(P_t/P_{t-1}) = \mu + \sigma z, \tag{3.3}$$

in discrete time. The density function of X_t is a standard Gaussian density:

$$
\begin{aligned}
f(x_t) &= \psi(\mu, \sigma^2) \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x_t - \mu)^2}{2\sigma^2}\right)
\end{aligned}
\tag{3.4}
$$

Now, add the factor of the jump process to equation (3.2), we shall have the mixed jump-diffusion process as modeled by the following differential equation

$$dP_t/P_t = \alpha\, dt + \sigma\, dz_t + dq_t \tag{3.5}$$

where q is the Poisson process with λ as the mean rate of jumps occurring per unit time. The jump size Y has a posited distribution $\ln Y \sim N(\theta, \delta^2)$, normally distributed with mean θ and variance δ^2. q is assumed independent of z. Now X_t can be expressed by the following equation:

$$
\begin{aligned}
x_t &= \ln(P_t/P_{t-1}) \\
&= \mu + \sigma z + \sum_{i=1}^{N_j} \ln Y_i
\end{aligned}
\tag{3.6}
$$

where N_j is the number of jumps during the interval (detailed derivation can be found in Oldfield et al). As derived by Beckers (1981) and reconfirmed by Ball and Torous (1983), in the form of a Poisson mixture of Gaussian densities, the density function of X_t

is:

$$f(x_t) = \sum_{i=0}^{\infty} \frac{e^{-\lambda}\lambda^i}{i!}\, \psi(\mu + \theta i, \; \sigma^2 + \delta^2 i)$$

that is,

$$f(x_t) = \sum_{i=0}^{\sim} \frac{e^{-\lambda}\lambda^i}{i!} \frac{1}{\sqrt{2\pi(\sigma^2 + \delta^2 i)}} \exp\left(\frac{-(x_t - \mu - \theta i)^2}{2(\sigma^2 + \delta^2 i)}\right) \tag{3.7}$$

where i is a random number. It is easy to see that when $\lambda = 0$ and $i = 0$, equation (3.7)

reduces to equation (3.4).

With T independent observations, the likelihood function is

$$L(\psi; x_t) = \prod_{t=1}^{T} \left[\sum_{i=0}^{\sim} \frac{e^{-\lambda}\lambda^i}{i!} \frac{1}{\sqrt{2\pi(\sigma^2 + \delta^2 i)}} \exp\left(\frac{-(x_t - \mu - \theta i)^2}{2(\sigma^2 + \delta^2 i)}\right)\right] \tag{3.8}$$

the logarithm of the likelihood function $L(\psi; x)$ can then be written as

$$\begin{aligned} F(b) &= \ln L(\psi; x_t) \\ &= \sum_{t=1}^{T} \ln\left[\sum_{i=0}^{\sim} \frac{e^{-\lambda}\lambda^i}{i!} \frac{1}{\sqrt{2\pi(\sigma^2 + \delta^2 i)}} \exp\left(\frac{-(x_t - \mu - \theta i)^2}{2(\sigma^2 + \delta^2 i)}\right)\right] \end{aligned} \tag{3.9}$$

where b is the vector of parameters, that is

$$b' = (\lambda, \mu, \sigma, \theta, \delta),$$

as shown in Jorion (p443). Again, λ is the mean rate of jumps, μ mean of diffusion

process, σ the standard deviation of the diffusion process, θ mean of lognormal jump

size, δ the standard deviation of the lognormal jump size. If there is a pure diffusion

process, the logarithm of the likelihood function should be

$$\ln L_{diff} = \sum_{t=1}^{T} \ln\left[\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x_t - \mu)^2}{2\sigma^2}\right)\right] \tag{3.10}$$

where L_{diff} differentiates from $L(\psi; x)$ in equation (3.9). Rename $L(\psi; x)$ as L_{mix}. The equation (3.10) is used to obtain the likelihood ratio:

$$\Lambda = \frac{L_{diff}}{L_{mix}} .$$

(3.11)

The existence of the jump-diffusion process can then be tested by the chi-square statistics:

$$\chi^2 = -2\ln\Lambda = -2[\ln L_{diff} - \ln L_{mix}],$$

(3.11a)

with degrees of freedom equal to the number of parameters between the two models (d.f. = 3 in this case).

3.4 PARAMETERS ESTIMATION

Using Quadratic Hill-Climbing method to numerically maximize the likelihood function, given data X_t, five parameters (λ, μ, σ, θ, and δ) are obtained. The principle is explained in many textbooks. Summarized by Fomby et al. (1984), it can be briefly described as follow.

If F(b) expressed in the equation (3.9) is continuous and has continuous first and second order partial derivatives, the necessary and sufficient conditions for a local maximum are

$$\frac{\partial F}{\partial b}\bigg|_{b\ b^*} = 0$$

(3.12)

and at the point b^* defined by (3.12) that the Hessian matrix

$$\frac{\partial^2 F}{\partial b \partial b'}\bigg|_{b\ b^*} = H^*$$

(3.13)

is negative definite. Hence F(b) is maximized by finding all the solutions to (3.12) such that the Hessian is negative definite and then choosing as the solution the point corresponding to the global maximum (Idid: 604).

In practice, F(b) is maximized numerically because it is extremely difficult to solve (3.12) analytically. The numerical maximization methods are iterative, trial-and-error methods. The quadratic hill climbing method is one of the numerical methods applied to solve maximization problems, suggested by Goldfeld et. al. (1966). Jorion (1989) also estimates these parameters with equation (3.9). This dissertation goes one step further, a critical step to this empirical research: to identify the jump process.

3.5 IDENTIFYING JUMPS AND ESTIMATING PARAMETERS

A concept is borrowed from the switching regression in identifying the jump process. Ball and Torous (1981) employs an indicator (λ) in their Bernoulli mixture of Gaussian densities model, equation (3.1). It identifies whether any relevant information arrival occurs to affect the stock prices. The null hypothesis ($\lambda = 0$) is tested with this model. This coincides with one technique of switching regression, that is, testing the hypothesis that no switch took place (Goldfeld and Quandt, 4). With the hypothesis testing result, Ball and Torous successfully proved the existence of jumps in the stock market.

As stated in section 3.2, the existence of jumps is tested with likelihood ratio obtained by equation (3.11). However, we have to go one step further than Ball and

Torous, i.e., to identify jumps from diffusion. Therefore, a simple test on the occurrence

of jumps is not sufficient although it is important. Let p_t denote the probability of

occurrence of a jump on day t. It is calculated as

$$P_t = 1 - \frac{f(X_t)|_{(3.4)}}{f(X_t)|_{(3.7)}} .$$

(3.14)

As a quantitative method to identify the "switch," probability p presents a state of

adjustment from one "structure" (one "process" in this case) to the other. In the case of

this jump-diffusion study, a higher p_t value indicates the strong presence of a jump while

a lower p_t value shows the dominant diffusion process.

As pointed out previously, equation (3.7) $(f(X_t)|_{(3.7)})$ reduces to a pure diffusion

density function, equation (3.4) $(f(X_t)|_{(3.4)})$ when jump is absent. Therefore, when $\lambda = 0$

and $i = 0$, $p_t = 0$. On the other hand, when there is no diffusion, equation (3.4) equals to

zero. Thus, $p_t = 1$. The Jump-Diffusion model implies that the daily stock prices are

results of the mixed processes. Thus the value of p_t should be within the range [0, 1]. In

the mixed processes, when the diffusion process is dominant, $p_t \rightarrow 0$. When jumps are

more frequent, however, $p_t \rightarrow 1$. Since $0 \leq p_t \leq 1$, it represents the probability of jump

occurrence. This has been proved to be true with our calculation of equation (3.14). ---

For the benefit of future research, the computer program and some outputs are included in

the Appendix D.

The jump process is identified with the estimated value of parameter λ. By its

definition, λ is the ratio of the number of jumps and the number of total observations,

i.e.,

$$\lambda = \#\, jumps\, /\, \#\, observation \tag{3.15}$$

We use the λ obtained from the maximum likelihood estimation equation (3.9) as the standard. As stated before, the equation (3.14) implies that the higher the value of p_i the closer it is to belong to the jump process. Sort the sample first with p_i. Select any number of observations with the highest p_i values. Then divide the number of selected observations by the size of the sample. Compare the solution with λ. If it is higher than λ, select a smaller number of observations with higher pt values. Otherwise select a larger number. Repeat this process until the solution of equation (3.15) equals to the estimated value of λ. Then the final group of selected observations belongs to the jump process and the remaining group belongs to the diffusion process.

The assumption that the jump process is independent from the diffusion process makes it feasible to estimate the rest parameters. The value of the weight parameter is simply calculated by equation

$$\phi = \frac{\lambda \sigma^2 (R_{\Delta w})}{\sigma^2 (R_w) + \lambda \sigma^2 (R_{\Delta w})} \tag{3.16}$$

where $\sigma^2(R_w)$, the diffusion variance, and $\sigma^2(R_{\Delta w})$, the jump variance, are calculated from the separated diffusion and jump samples, R_w and $R_{\Delta w}$ respectively. The value of $\sigma^2(R_w)$ is not necessarily the same as the output of maximum likelihood estimation, σ^2, due to the nature of nonlinear estimates. The same is true for the estimated α via (3.9) and the mean return of R_w. θ and δ^2 are not comparable with the mean and variance estimates of the

jump process because they are the mean and variance of the lognormal jump size rather than the process itself.

The jump beta, $\beta_{\Delta j}$ and diffusion beta, β_j are the ordinary least square estimators. First divide portfolio returns into two separate samples, $R_{\Delta j}$ and R_j, according to $R_{\Delta w}$ and R_w, respectively. $R_{\Delta w}$ and R_w as well as $R_{\Delta j}$ and R_j are vectors of daily returns.

$$R_{\Delta w} = (R_{\Delta w1}, R_{\Delta w2},... R_{\Delta wn\Delta})'$$

$$R_w = (R_{w1}, R_{w2},... R_{wT})'$$

$$R_{\Delta j} = (R_{\Delta j1}, R_{\Delta j2},... R_{\Delta jn\Delta})'$$

$$R_j = (R_{j1}, R_{j2},... R_{jT})'$$

where $n\Delta$ and T are number of observations in jump sample and diffusion sample respectively, and j indicates jth portfolio. $\beta_{\Delta j}$ and β_j are then obtained by

$$\beta_{\Delta j} = (R_{\Delta w}'R_{\Delta w})^{-1}R_{\Delta w}'R_{\Delta j} \qquad (3.17)$$

$$\beta_j = (R_w'R_w)^{-1}R_w'R_j. \qquad (3.18)$$

3.6 ISSUES OF AUTOCORRELATION AND HETEROSCEDASTICITY

Working with time-series data of mixed processes with implied structural changes, issues of autocorrelation and heteroscedasticity should be concerned. Jorion (1989) employed ARCH (AutoRegressive Conditionally Heteroscedastic) model as an additional measure to investigate the stochastic processes in his research. He found that the power of ARCH model for the jump-diffusion process is higher than that without it. Johnson and Schneeweis (1994) point out that a large number of observations are needed

to increase the power of the model (p316). They did not adopt the ARCH model. It is

tempting to adopt the ARCH model in this empirical research. However trials are not

successful. The loglikelihood value does not improve even after 5,000 iterations.

Reviewing Jorion's work, arguments made by Johnson and Schneeweis, it may be

explained that samples used in this study are not fit for ARCH modeling. Unlike Jorion's

sample of 10-year daily data, samples used in this study are jump/event intensified. The

discontinuity of the time-series data disqualifies the assumption of autocorrelation. The

heteroscedasticity is taken care of by the definition of jump variance and diffusion

variance. Thus, adoption of the ARCH model is going to duplicate the effort of adjusting

for heteroscedasticity. That is why the trials in this study with the ARCH model failed to

improve the value of the log likelihood.

3.7 PROCESS OF THIS RESEARCH

The primary goal of this empirical research is to test the specifications of the

jump-diffusion model. Daily security returns and a market index from the CRSP[3]

magnetic tape are employed in this study. The data collection is done by generating 30

portfolios after downloading security returns from the tape. The jump-diffusion

processes are identified through the maximum likelihood estimation. The existence of

the mixed process is confirmed by conducting the likelihood ratio test against the pure

diffusion process. The portfolio jump betas and diffusion betas are obtained for every

3. CRSP --- Center of Research for Stock Prices, University of Chicago.

sample. As a preliminary test on the specifications of the jump-diffusion model, the mean-test is conducted. It tests whether the observed mean returns equal to the expected mean returns calculated by the jump-diffusion model with jump betas and diffusion betas of 30 portfolios. Due to the assumption of a non-normal distribution for the jump-diffusion model, a non-parametric test, the sign test, is employed to check and/or to reinforce results of the mean-test.

Secondly, the comparison between the jump-diffusion model and the conventional CAPM is carried out as follow. CAPM betas for each portfolio are obtained first with the original, mixed or non-separated sample. Then a cross sectional regression is run in the form of the CAPM. The coefficients of the cross sectional regression are tested against the specifications of CAPM. Another cross sectional regression is also run, but in the form created by a hypothesis that the CAPM beta is equivalent to the weighted average of jump beta and diffusion beta. The hypothesis test is carried out on its coefficients.

The well-documented January Effect has now almost become a benchmark test for any new asset pricing model. As mentioned in Chapter One, various models developed so far cannot explain the January Effect or abnormal returns during the turn of the year. After testing the specifications of the Jump-Diffusion model, the new model is applied to the turn of year sample. Since the time span is long, a conventional testing method is employed. A cross sectional regression is run after estimating mean returns, jump betas, and diffusion betas of thirty portfolios. The coefficients of the cross sectional regression are tested against the specifications of the model.

Due to the great influence of Fama and French's five-factor model (1993), the

January Effect study is extended to use another sample of data. In order to make a true comparison with Fama and French's findings, the exact sample of CRSP monthly data is used. Thus Monthly security returns and a market index from the CRSP magnetic tape are used. The data collection is done by generating 20 portfolios after downloading security returns from the tape. Similar procedure as employed by Fama and French is followed in this study.

CHAPTER FOUR

DATA ANALYSIS

4.1 DATA COLLECTION

Three groups of data are required by the jump-diffusion two-beta asset pricing

model: returns of optimally invested wealth, returns of individual securities (portfolios),

and a risk-free rate. The two sets of data are available on the CRSP tape. Value weighted

market index (VW) is employed to represent the daily return of the optimal invested

wealth, R_w. Returns on thirty portfolios, cross-classified by market value of firm's equity

and stock price are obtained each year by the following procedure as R_j:

1. Each year all NYSE and AMEX[4] stocks with the previous year's ending price and

    shares outstanding information available from CRSP are included in the sample.

2. The firm size proxied by equity market value is determined as the share price

    multiplied by the number of shares outstanding. The stocks are ranked in

    ascending order based on the firm size and then divided equally into thirty size

    groups from low to high.

---

4. NYSE --- New York Stock Exchange
   AMEX --- American Exchange

3. Daily returns on stocks within each size group are used to obtain the average return on 30 size-related portfolios.

4. Repeating steps 1 through 3 for each year in the 30-year period, yields R_j, the average daily returns of 30 portfolios for 30 years.

    The risk free rate, R_f, is the daily rate of a treasury bill.

4.2 SAMPLE PERIODS

For the empirical verification of the Jump-Diffusion model, the period around the Persian Gulf Crisis is selected as the sample period. Since August 2, 1990, when Iraqi invaded Kuwait, stock prices dropped (negative jump) precipitously whenever expectations of the war were heightened, and jumped sharply up whenever peace appeared hopeful. More important, the jumps are not just associated with any particular firm, or any particular industry. The jumps are associated with the market as a whole. Thus, the period around the Persian Gulf Crisis is ideal for testing the jump-diffusion two-beta asset pricing model.

The whole sample period is 1990 and 1991, 506 trading days in total. The whole period is divided into three sub-periods. The first period (pre-invasion) is from the beginning of 1990 to the week prior to the Iraqi invasion, i.e., the end of July 1990. The second period (invasion) is from July 30, 1990 to the end of February, about one week after the cease fire agreement was signed (February 18, 1991). The third period (post-invasion) is from then to the end of 1991. Therefore, there are two periods of peace and

one of war.

The results of step 1 through 4 in the section of Data Collection are partially

reported in a set of tables which are presented in Appendix C due to large volume of data.

Table 1:   Average sizes of thirty portfolios during five years;

Table 2:   Statistics of average daily returns of the value weighted market index and

              thirty portfolios during the whole sample period;

Table 3:   A covariance/variance matrix of the average daily returns of thirty

              portfolios and the value weighted market index.

The full report is available upon request. The size of the largest portfolio is

almost a thousand times the size of the smallest portfolio. The average daily return of the

smallest portfolio is almost ten times that of the market index, which is almost the same

as the largest portfolio. On the other hand, the standard deviation of the smallest

portfolio is almost twice as large as that of the largest portfolio. Again, the standard

deviation of the market index is almost identical as that of the largest portfolio. These

facts are also confirmed by the covariance/variance matrix. It shows that the larger the

portfolio is, the bigger the covariance with the market index becomes. That is, the largest

portfolio moves almost identically with the market because it is strongly correlated with

the market. This is significant because the systematic risk, beta of the portfolio is

calculated as dividing the covariance between itself and the market index by the variance

of the market index. Therefore the larger the portfolio, the closer to 1 the beta of the

portfolio.

4.3   BEHAVIOR OF MARKET RETURNS

Since the jump-diffusion asset pricing model is built on the assumption that the underlying probability distribution is a mixture of Normal and Poisson, for testing the model, it is important to examine the frequency distribution of the market returns to see whether the assumption is approximated or not.

In Figure 2, a group of four graphs, heavy black vertical bars are the actual frequency distributions of VW market returns during four sampling period. A theoretical normal curve is superimposed as a line on top of the frequency distribution for ease of comparison. The frequency distribution of the market returns for the whole sample period shows relatively good fit between the actual histogram and the standard normal distribution. However, the following set of four figures, each representing one sampling period, has indeed shown the characteristics of fat tails and leptokurtosis, a phenomenon well documented in the finance literature. As one would suspect, the fat tails and leptokurtosis are more serious in the sampling period of the invasion (figure 2c), when jumps are more frequent than the other two sub-sampling periods of peace (figure 2b and 2d).
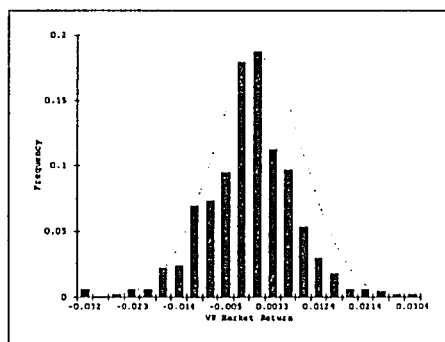
2a: Whole Period
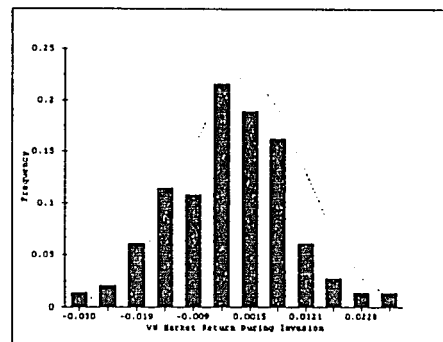
2c: Invasion/War Period
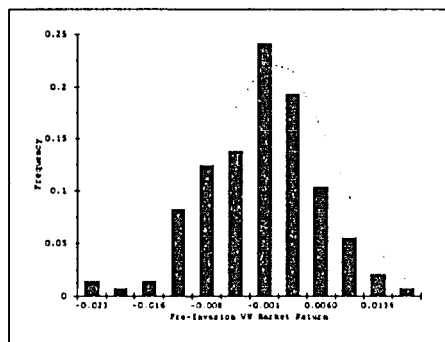
2b: Pre-Invasion Period
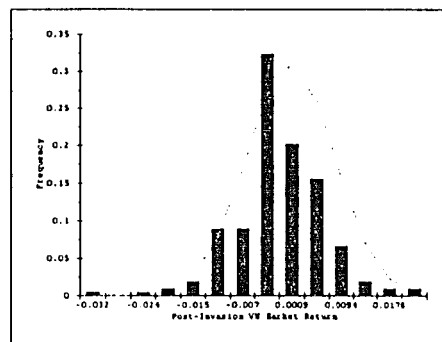
2d: Post-Invasion Period

Figure 2:   Frequency Distribution of Value Weighted Market Index During the Gulf War Sampling Periods
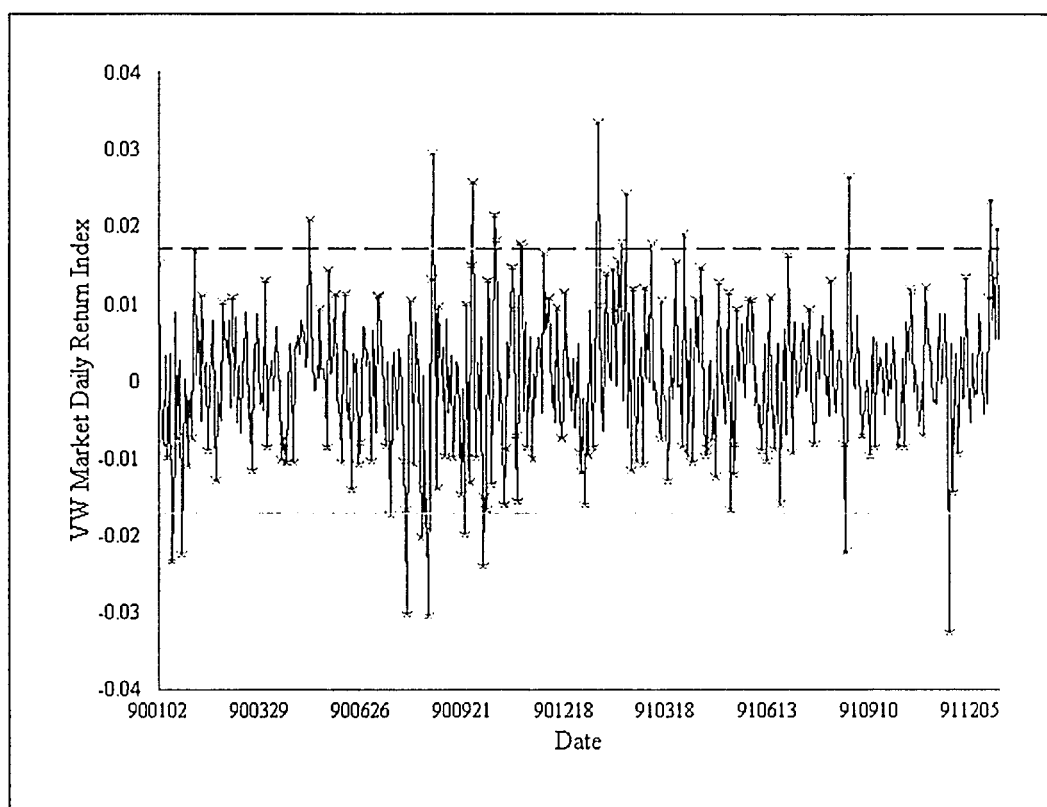
Figure 3: Characteristics of the Daily Value Weighted Market Index During the Gulf War Period, 1990 - 1991

The characteristics of the market returns during the Gulf War period are presented in Figure 3. The existence of jump process is clear from this graphic demonstration. Cross marks are indicators of jumps which are identified via the maximum likelihood estimation. This result is convincing and proves that the identification method is better than any ad hoc method, such as using $\pm 2\sigma$ as a range for diffusion and picking observations beyond that range as jumps. The $+2\sigma$ line and -2σ line in Figure 3 clearly shows that such a method would miss a number of jumps. In fact, it captures about only

16.56% of total jumps with this sample.

Table 4 reports the result of the maximum likelihood estimation and the

likelihood ratio test against the pure diffusion model.

Table 4

Parameter Estimation Results of Maximum Likelihood Estimation

with the Gulf War Period Sample

Process	$\mu(\times10^{-4})$	$\sigma^2(\times10^{-5})$	λ	$\theta(\times10^{-4})$	$\delta^2(\times10^{-4})$	Log Likelihood	Chi-Square
Diffusion	4.3074 (1.1315)	7.3353 (15.873)				1690.6	
Jump-Diffusion	6.7252 (0.4628)	4.0916 (9.145)	0.31 (5.3173)	-7.6966 (-0.544)	1.0528 (4.8707)	1703.9	26.6*

Note: Asymptotic t-statistics in parentheses. Pure diffusion process is modeled by $N(\mu, \sigma^2)$. The jump-diffusion process is the combined Poisson and Normal distribution, with a mean rate of jump as λ and size of jumps modeled by $N(\theta, \delta^2)$. The chi-square statistics (d.f. = 3) tests the hypothesis of a pure diffusion process against a jump-diffusion process. * indicates significant at 1% level.

The null hypothesis of the likelihood ratio test is that the jump-diffusion process is

no different from the pure diffusion process. The significance of the chi-square statistics

shows that the jump-diffusion process describes the price movement of the stock market

better. In other words, jump does exist and the jump-diffusion model should be chosen

over the pure diffusion model.

Table 4 also shows that the estimation of μ and θ are weak. Several steps are

taken to increase the power of the estimation. Then the attempt is given up because it is

not the focus of the research.

4.4 ESTIMATION OF PARAMETERS

4.4.1 Market Parameters

Table 5 lists descriptive statistics of value-weighted market daily returns for different sampling periods as well as for different samples.

Table 5

Descriptive Statistics of Market during the Gulf War Period

	Whole Period	Pre-Invasion	Invasion	Post-Invasion
Mixed Sample				
N	506	145	150	211
$ABS(R_w)$	0.0063925	0.0057199	0.0083559	0.0054730
R_w	0.000467521	0.00007168	0.0004889	0.000760696
	(0.0085727)	(0.0073111)	(0.0108724)	(0.0074862)
Jump Sample				
N_j	157	39	65	53
$ABS(R_{jw})$	0.0130597	0.01155794	0.01450626	0.01239066
R_{jw}	-0.0010284	-0.003049559	-0.00081614	0.00019857
	(0.01411287)	(0.01196162)	(0.01576419)	(0.0135073)
Diffusion Sample				
N_d	349	106	85	158
$ABS(R_{dw})$	0.00339322	0.003571872	0.00365268	0.00313378
R_{dw}	0.00114047	0.001220057	0.0014869	0.000900703
	(0.00398721)	(0.00407123)	(0.0042441)	(0.003793292)

The "Mixed Sample" means a sample without the separation of jump process from

diffusion process. N denotes number of observations within a sample. R_w is the mean of $\mathbf{R}_w$, which is the vector containing daily returns of the value-weighted market index. $R_{\Delta w}$ is the mean of $\mathbf{R}_{\Delta w}$, the jump sample of $\mathbf{R}_w$. R_{dw} is the mean of $\mathbf{R}_{dw}$, the diffusion sample. Data within parentheses is standard deviation of the estimated mean. ABS is the mean of the absolute values.

The absolute values of $R_{\Delta w}$ reveals the magnitude of jump size. It is at least twice as large as the diffusion size, $ABS(R_{dw})$. The standard deviations of R_{dw} and $R_{\Delta w}$ confirm this finding. The negative values of $R_{\Delta w}$ show that most of jumps are downwards. With these statistics, we obtain estimation of ϕ, which are listed in Table 6, together with estimations of λ and R_f, the risk-free rate. R_f is the mean of a daily treasury bill rate. λ is estimated with equation (3.16) for each sampling period, while ϕ is obtained with equation (3.17).

Table 6

Estimates of Jump-Diffusion Model Parameters

with the Gulf War Period Samples

Parameter	Whole	Pre-Invasion	Invasion	Post-Invasion
λ	0.31	0.27	0.43	0.25
ϕ	0.795386	0.698957	0.856704	0.761048
R_f	0.00017943	0.0002152	0.00019103	0.00014628

4.4.2 Portfolio Estimates

The portfolio estimates include portfolio return R_j, jump beta $\beta_{\Delta j}$, diffusion beta β_j, and the CAPM beta β_j'. For easy comparison, these estimated values are listed together in Table 7 which is presented in Appendix C due to the large volume. There are four panels, panel A, B, C, and D in Table 4 for the four sampling periods respectively. While estimating betas, results of the Durbin-Watson test and ρ are included in the output. They show little evidence of autocorrelation.

4.5 HYPOTHESIS TESTING AND TEST RESULTS

4.5.1 Testing Specifications of the Jump-Diffusion Model

The assumption of the jump-diffusion two-beta asset pricing model is the mixed distribution of normal and Poisson. Therefore, one cannot use the ordinary method to test the model specifications. As a start, a mean test is conducted to find out how well the model performs with four samples. T-ratios are computed as follow:

$$ t = \frac{E(\overline{R}_j) - E(R_j)}{\sqrt{\dfrac{\sigma^2(\overline{R}_j) + \sigma^2(R_j)}{(n-1)}}} , \tag{4.1} $$

where $\overline{R}_j = R_f + (R_w - R_f)[(1-\phi)\beta_j + \phi\beta_{\Delta j}]$,   (4.1a)

and

$$ E(\overline{R}_j) = \frac{1}{30}\sum_{j=1}^{30} \overline{R}_j , \tag{4.1b} $$

because the hypothesis is that

$$H_0: \quad \overline{R}_j = R_j \qquad\qquad j=1,\ldots,30 \; .$$

The results are reported in Table 8.

Table 8

Results of Mean-Test

(d.f. = 29)

	Whole	Pre-Invasion	Invasion	Post-Invasion
t-ratio	-1.3009	1.0794	-1.2293	-2.4026*

Note: * indicates the ratio is significant at 0.05 level.

The test results show that the jump-diffusion model functions well. That is, the hypothesis that model estimated portfolio return equals to the observed portfolio return is not rejected in all sample periods except the post-invasion period. Understanding the underlying probability distribution being a mixture of Poisson and normal, however, a strict non-parametric test is conducted to enhance the hypothesis test. The sign test (Hoel, Port, and Stone 1971, p172-174) is chosen because it can be applied to any density function f(x), to test the specification of the jump-diffusion model.

4.5.2 Method of Sign Test

Define the median of f(x) as

$$\int_{-\infty}^{\xi} f(x)\,dx = \int_{\xi}^{\infty} f(x)\,dx = \frac{1}{2} . \tag{4.2}$$

Let the random sample be denoted by $X_1, \ldots, X_n$ which possesses the density function

$f(x)$ and is used to test the hypothesis

$$H_0: \xi = \xi_0 \qquad \text{against} \qquad H_1: \xi > \xi_0.$$

Since $P(X > \xi_0 \mid H_0) = \frac{1}{2}$, then

$$P(X_i - \xi_0 > 0 \mid H_0) = \frac{1}{2}. \; i = 1, \ldots, n \tag{4.3}$$

Let

$$Z_i = \begin{cases} 0, X_i - \xi_0 \le 0 & (4.4a) \\ 1, X_i - \xi_0 > 0 & (4.4b) \end{cases}$$

The Z's are independent, identically distributed random variables. When H_0 is true, $E(Z_i)$

$= \frac{1}{2}$; therefore when H_0 is true, the Z's represent a random sample of size n of a Bernoulli

variable with $p = \frac{1}{2}$. When H_1 is true, $E(Z_i) > \frac{1}{2}$ because the frequency of positive values

of $X_i - \xi_0$ will increase if the true median is larger than ξ_0. Hence, we can choose the

binomial variable

$$Z = \sum_{i=1}^{n} Z_i \tag{4.5}$$

as statistic for constructing the test and use right tail of the distribution of Z as critical

region. Here

$$E(Z) = np = n/2 \tag{4.6a}$$

and

$$Var(Z) = npq = n/4 \tag{4.6b}$$

If n is sufficiently large to justify using the normal approximation to the binomial

distribution, then the test should choose as critical region the region $\tau > \tau_0$, where $P(\tau > \tau_0) = \epsilon$ and where

$$\tau = \frac{Z - (\frac{n}{2})}{\sqrt{\frac{n}{4}}}$$

(4.7)

In this case, n=30 and X_i is denoted by e_j which is the error term of the expected return of stock j from the realized return of stock j, i.e.,

$$e_j = \overline{R}_j - R_j \qquad j=1,\ldots,30$$

(4.8)

where

$$\overline{R}_j = R_f + (R_w - R_f)[(1-\phi)\beta_j + \phi\beta_{Aj}]$$

which is the jump-diffusion two-beta asset pricing model itself, and R_j is estimated mean return of stock j in Table 4. ξ_0 is 0 because our hypothesis should be that the expected returns estimated with the jump-diffusion model equal to the real returns. Referring to the text above,

$$Z_i = \begin{vmatrix} 0, & e_i \leq 0 \\ 1, & e_i > 0 \end{vmatrix}$$

(4.9a)

(4.9b)

and

$$Z = \sum_{i=1}^{n} Z_i$$

(4.10)

The test statistic is

$$\tau = \frac{Z - \mu}{\sqrt{\frac{n}{4}}} = \frac{Z - 15}{\sqrt{\frac{30}{4}}}$$

(4.11)

At the 5% significance level, the hypothesis is rejected if $\tau \geq |1.64|$. Employing data

from table 5 to table 7, the test results for each sampling periods are reported in Table 9.

Table 9

Results of Sign Test

	Whole	Pre-Invasion	· Invasion	Post-Invasion
Z-ratio	1.4606	3.6515*	1.4606	-3.6515*

Note: * indicates the ratio is significant at 0.05 level.

The hypothesis that the expected return of a valued asset by the jump-diffusion model is not significantly different from the average realized return of the asset cannot be rejected for either the whole period or the invasion period. This conclusion agrees with the results of the parametric t-test for all periods except the pre-invasion period. One could argue that the sign test is more strict for the mixed distribution assumption. At least we can conclude that the jump-diffusion model is useful to periods within which jumps are more frequent despite the disagreement between the t-test and the sign test . That the hypothesis is rejected in the post-invasion period, in addition to the pre-invasion period, shows that the jump-diffusion model is not general enough to predict the expected returns during non-eventful time. Logically, one would ask how the conventional CAPM performs with the same sets of data?

4.5.3 Hypothesis Testing on CAPM

The conventional CAPM is based on the assumption of normality. Therefore, we should conduct the parametric t-test on its specifications. Let

$$Y_j = (R_j - R_f)/(R_w - R_f).$$ (4.12)

The CAPM is then rewritten as

$$Y_j = \beta j.$$ (4.13)

The regression equation on which the hypothesis testing is based is

$$Y_j = b_0 + b_1\beta_j' + \epsilon_j.$$ (4.14)

The hypotheses are:

$$H_0: \quad b_0 = 0$$ (4.15a)

$$b_1 = 1.$$ (4.15b)

Table 10

Results of t-Test on the CAPM

Sample Period	b_0	b_1	R^2	$t(b_0=0)$	$t(b_1=1)$
Whole	6.4658 (2.0421)	-7.3417 (2.8704)	0.1894	3.1663*	-2.9061*
Pre-Invasion	0.0472 (2.8819)	2.4561 (4.5128)	0.0105	0.0164	0.3227
Invasion	10.931 (2.6186)	-12.397 (3.3951)	0.3226	4.1743*	-3.9460*
Post-Invasion	3.2026 (0.8307)	-2.9877 (1.2301)	0.1740	3.8552*	-3.2419*

Note: 1. Standard errors of estimations are listed with parentheses.
2. * indicates ratios are significant at 0.05 level.

Table 10 shows that the CAPM is not rejected with only one sample period. It failed with all other three sets of data. During the pre-invasion period when the hypotheses are not rejected, however, R^2 of the cross regression is only 0.0105. Such a small value of R^2 indicates the insignificance of coefficient estimation. Therefore, the regression results generally show that the CAPM is a weaker model in explaining the return of assets.

4.5.4 Distinction between the Jump-Diffusion Model and the CAPM

How distinct is the jump-diffusion model from the conventional CAPM? Tests on model specifications are in favor of the jump-diffusion model. However, it is necessary to test whether the jump-diffusion model is related to the CAPM. Since the jump-diffusion model can be written as

$$Y_j = (1-\phi)\beta_j + \phi\beta_{\Delta j} \tag{4.16}$$

with equation (4.13), such a test should then be based on whether the beta in the conventional CAPM is the weighted average of the jump beta and diffusion beta in the Jump-Diffusion model. The hypothesis therefore is

$$H_0: \quad \beta_j^{'} = (1-\phi)\beta_j + \phi\beta_{\Delta j} \quad j = 1, \ldots, 30 \tag{4.17}$$

The hypothesis can be tested with the following regression model

$$\beta_j^{'} = c_0 + c_1\beta_j + c_2\beta_{\Delta j} + \mu_j. \quad j = 1, \ldots, 30 \tag{4.18}$$

The testable hypotheses are

$$c_0 = 0, \tag{4.19a}$$

$$c_1 + c_2 = 1, \qquad\qquad\qquad\qquad\qquad\qquad\qquad (4.19b)$$

$$c_2 = \phi. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4.19c)$$

The results of testing are reported in Table 11.

Note that the hypothesis $c_1 + c_2 = 1$ and $c_2 = \phi$ are really joint hypotheses. The

rejection of either one implies the rejection of the hypothesis that the systematic risk is a

weighted average of diffusion and jump betas. Hypothesis $c_2 = \phi$ is rejected by the t-test

for all sample periods except the post-invasion samples. For the post-invasion period, the

hypothesis testing results on the specifications of both jump-diffusion model and the

CAPM suggest that there is no significant difference between specifications of the jump-

diffusion model and those of the CAPM.

Table 11

Testing Results on Distinction

between Jump-Diffusion Model and the CAPM

Sample Period	c_0	c_1	c_2	R^2	$t(c_0=0)$	$t(c_1+c_2=1)$	$t(c_2=\phi)$
Whole	0.0067 (0.0056)	0.1540 (0.0095)	0.8412 (0.0146)	0.9996	1.1878	-0.8119	3.1628*
Pre-Invasion	0.0035 (0.0077)	0.3460 (0.0152)	0.6553 (0.0187)	0.9964	0.4563	0.1147	-2.3362*
Invasion	0.0087 (0.0046)	0.0836 (0.0036)	0.9079 (0.0073)	0.9996	1.8826	-1.7391	6.9977*
Post-Invasion	0.0110 (0.0068)	0.2057 (0.0156)	0.7791 (0.0221)	0.9983	1.6146	-1.5816	0.8392

Note: 1. Standard errors of estimations are listed with parentheses.

2. * indicates ratios are significant at 0.05 level.

CHAPTER FIVE

JANUARY EFFECT ANALYSIS

5.1   REVIEW OF PREVIOUS STUDIES

The January effect has been well documented. In a series of works, Keim (1983,
1986, and 1989) documented the January effect in various perspectives. Lakonishok and
Smidt found the existence of the January effect over ninety years of the stock market
history. Roll (1983) found that the January effect is size-related, i.e. small firm stocks
tend to be more volatile at the turn of the year. Keim and Stambaugh (1986) further
explored size effect/January effect relationship and concluded the same. Bhardwaj and
Brooks (1992) argue, however, that it would be more accurate to state that the January
effect is "a low-price phenomenon rather than a small firm effect (p553)."

Finance academicians agree that the January effect exists, but disagree on the
cause(s). Some well known hypotheses are tax selling to create capital loss in the end of
the year, information shortage for small firms, etc. None of these hypotheses are
confirmed. In fact, the January effect is such a classical problem (unexplainable so far)
that it has become a standard application to test any new asset pricing models (Fama and
French 1993, p43). The CAPM has been widely criticized for it (Banz 1981 and Keim

1986). The APT cannot explain it either (Roll 1983). The most recent development by

Fama and French (1993), a five-common-risk-factor model also failed to explain it

because their risk factors do not contain January seasonals (p46). How would the jump-

diffusion model perform this task?

5.2   SAMPLE COLLECTION AND ANALYSIS

The complete recorded thirty years are selected. The last trading day of a year and

the first three trading days of the following year, totaling 4 daily returns, are sampled for

"the turn of the year," while the rest days are grouped together as another sample. Due to

the limitation of the computing device, only the smaller size sample, i.e., the turn-of-the-

year 4 trading days from 1962 to 1991, is used for the jump-diffusion study. However, in

order to get a close-to-reality data set to estimate jump process and diffusion process, the

whole two months of the turn of the year, i.e., December and January data are used for

maximum likelihood estimation. The distribution of the sample is shown in figure 6 as

an evidence of abnormality. The results of the maximum likelihood estimation are shown

in figure 7 and table 12.
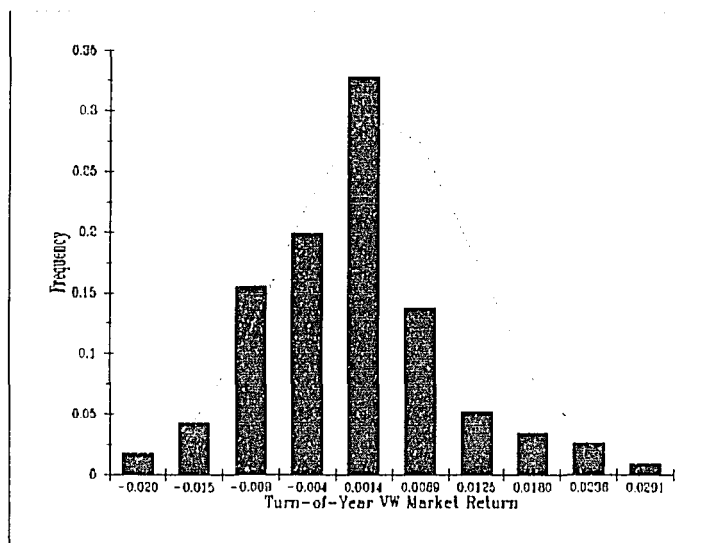
Figure 6: Distribution of Value-Weighted Market Index During Turn-of-Year from 1962 - 1991
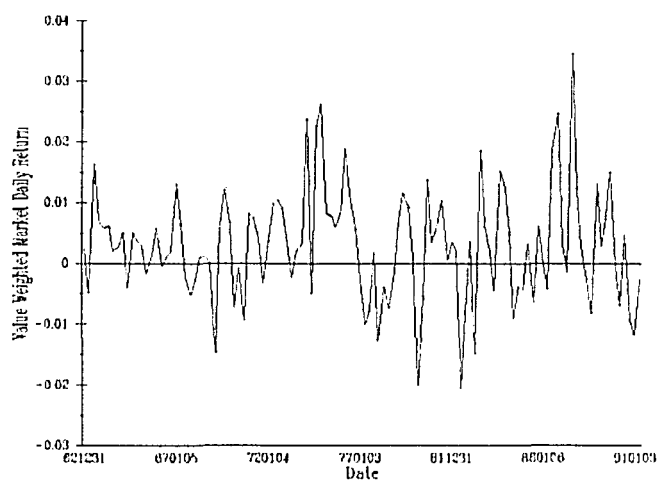
Figure 7: Characteristics of The Value Weighted Market Return at Turn of The Year from 1962 to 1991

As a reminder, note that table 12 reports estimation results based on two sampling months' data from 1962 to 1991, while figure 7 plots the estimation results with turn-of-

year four trading days only which are based on Table 12 estimations. Again the cross

marks indicate jumps. The skewness to the left and fat tails in figure 6 as well as the

intensity of jumps in figure 7 are clear signs of "January Effect." An interesting point,

which actually assures the process of identifying the jump-diffusion process via the

maximum likelihood estimation, is that the jumps identified independently with the Gulf

War period sample are confirmed with the turn-of-the-year sample. However, due to

different data sets and the nature of nonlinear estimation, not every jump matches in both

samples. Detailed output of the estimation can be obtained upon request.

Table 12

Results of Maximum Likelihood Estimation

with Turn-of-the-Year Sample

Process	$\mu(x10^{-4})$	$\sigma^2(x10^{-5})$	λ	$\theta(x10^{-4})$	$\delta^2(x10^{-4})$	Log Likelihood	Chi-Square
Diffusion	8.5153 (3.6901)	0.6524 (24.674)				4164.8	
Jump-Diffusion	8.2665 (4.4549)	1.9780 (11.842)	0.49558 (8.1466)	0.4998 (0.1125)	0.9048 (10.966)	4257.5	185.4*

Note: Asymptotic t-statistics in parentheses. Pure diffusion process is modeled by N(μ, σ^2). The jump-diffusion process is the combined Poisson and Normal distribution, with mean rate of jump as λ and size of jumps modeled by N(θ, δ^2). The chi-square statistic (d.f. = 3) tests the hypothesis of a pure diffusion process against a jump-diffusion process. * indicates significant at 1% level.

The mean rate of jump, λ in Table 12 is almost 50 percent --- for whole two months

around the turn of the year, comparing with 31 percent during the Gulf War sampling

period. In addition, the chi-square statistic resulted from the likelihood ratio test shows

that the jump-diffusion process is far superior to the pure diffusion assumption.

Following the same procedure as described in the previous chapter, the whole

sample is divided into a jump data set and a diffusion data set. Estimated market

parameters are reported in Table 13.

Table 13

Estimated Market Parameters

with Turn-of-Year Sample

	Whole Sample	Jump	Diffusion
N	116	73	43
ABS(R_w)	0.0073783	0.010406	0.0022387
R_w	0.002945	0.004056	0.001059
	(0.009267)	(0.011432)	(0.002306)
λ	0.6293		
ϕ	0.93927		
R_f	0.00018563		

This λ (almost 63 percent) is the mean rate of jump during the turn of year four

trading days from 1962 to 1991. It is logical that this λ is much higher than the λ in

Table 12 because it represents a concentrated period of the turn of the year. Comparing

with the statistics of market returns listed in Table 5, R_w is almost seven times higher

during the turn of the year than during the whole sample period of 1991 and 1992. There are more positive jumps than negative jumps during the turn of the year, resulting in an even higher value of R_w in the jump set. These statistics agree with empirical observations and documentation on the January effect.

Table 14 in Appendix C reports estimations on portfolio returns, jump betas, diffusion betas, and CAPM betas. It shows that the smaller the portfolio size, the higher the average returns. This is so-called January/size effect as Roll (1983) pointed out.

5.3 HYPOTHESIS TESTING

Due to the long time span (30 years) of the turn-of-year sample, the ordinary parametric testing methods are employed in this section. The specifications of the jump-diffusion model are tested with the turn of the year sample as follow.

Rewrite the jump-diffusion model as

$$Y_j = a_0 + a_1\beta_j + a_2\beta_{\Lambda j} + e_j \qquad j=1, \ldots, 30. \qquad (5.1)$$

where $\qquad Y_j = \dfrac{\overline{R}_j - R_f}{R_w - R_f}$. $\qquad\qquad\qquad (5.1a)$

The hypotheses to be tested are then

$$H_0: \qquad a_0 = 0 \qquad\qquad\qquad (5.2a)$$

$$a_1 + a_2 = 1 \qquad\qquad\qquad (5.2b)$$

$$a_2 = \phi. \qquad\qquad\qquad (5.2c)$$

Other tests are the same as described in Chapter Four. The purpose is to find out how this

model explains the January effect, compared with the conventional CAPM.

Table 15

Hypotheses Testing & Regression Results

with Turn-of-Year Sample

Model	a_0	a_1	a_2	R^2	$t(a_0=0)$	$t(a_1+a_2=1)$	$t(a_2=\phi)$
Jump-Diffusion	-1.8684 (4.1465)	-2.7366 (0.7293)	7.1246 (4.0296)	0.3645	-0.4506	0.8515	1.5350
	b_0	b_1		R^2	$t(b_0=0)$	$t(b_1=1)$	
CAPM	-3.7008 (4.8048)	6.5020 (4.5815)		0.0671	-0.7702	1.2009	
	c_0	c_1	c_2	R^2	$t(c_0=0)$	$t(c_1+c_2=1)$	$t(c_2=\phi)$
$\beta_i = c_0 + c_1\beta_i + c_2\beta_{\Delta i} + \mu_i$	-0.0163 (0.0296)	0.0064 (0.0052)	1.0163 (0.0288)	0.9796	-0.5512	0.7966	2.6772*

Note: 1. Standard errors of estimations are listed with parentheses.

2. * indicates ratios are significant at 0.05 level.

The results of hypotheses test show that the jump-diffusion model is a better model in explaining the January or turn-of-year effect than the CAPM based on R^2. The jump-diffusion model is significantly different from the CAPM in terms of the risk measure specifications.

5.4 COMPARISON BETWEEN THE JUMP-DIFFUSION MODEL

AND THE FAMA AND FRENCH'S FIVE FACTOR MODEL

Due to the great influence of Fama and French's five-factor model (1993), this study is bound to compare it with the Jump-Diffusion model. In order to do so, an exact sample (CRSP monthly data from July 1963 to December 1991) of 342 observations is used. Instead of forming 30 portfolios, only 20 portfolios are formed with the monthly sample. The monthly value-weighted market return index is used as R_w.

Fama and French use the following model to examine the January Effect:

$$R(t) = a + b*JAN(t) + e \tag{5.3}$$

where JAN(t) is a dummy variable that is 1 if month t is January and 0 otherwise and R(t) is the explanatory return of month t given by each of five factors. For easy comparison, table 16 is built based on Fama and French's result of the five risk factors model applying on the January effect. One can hardly conclude that the reported results by Fama and French are satisfactory.

Table 16

Results of Fama and French's Five-Factor Model

Fama and French's "Five-Factor Explanatory Returns" [5]				
R^2(RM-RF)	R^2(SMB)	R^2(HML)	R^2(TERM)	R^2(DEF)
0.00	0.06	0.06	-0.00	0.03

where

RM   = Value-weighted monthly stock market return
RF   = one-month Treasury bill rate
SMB = returns on the mimicking portfolios for the size factor in stock returns
HML = returns on the mimicking portfolios for the book-to-market factor in stock returns
TERM = LTG - RF where LTG is the long-term government bond return
DEF = CB - LTG where CB is the return on a proxy for the market portfolio of corporate bonds

In order to examine the January Effect with the Jump-Diffusion model with monthly data, a complete process of the estimation is run again. Results are summarized in Table 17 and Table 18. Since the goal of this part of research is to compare the Jump-Diffusion model with the Fama and French's model, details of estimation are not reported.

5. Source: Fama and French, "Common Stock Factors in the Returns on Stocks and Bonds," Journal of Financial Economics, 1993, Table 10, p44.

Table 17

Results of Maximum Likelihood Estimation

with Monthly Data

Process	$\mu(\times 10^{-2})$	$\sigma^2(\times 10^{-3})$	λ	$\theta(\times 10^{-2})$	$\delta^2(\times 10^{-3})$	Log Likelihood	Chi-Square
Diffusion	0.8525 (3.5127)	2.0223 (13.145)				575.53	
Jump-Diffusion	1.2737 (3.6866)	1.0601 (3.6851)	0.3506 (1.2476)	-1.2010 (-1.2856)	2.5075 (1.7145)	588.53	26*

Note: Asymptotic t-statistics in parentheses. Pure diffusion process is modeled by $N(\mu, \sigma^2)$. The jump-diffusion process is the combined Poisson and Normal distribution, with mean rate of jump as λ and size of jumps modeled by $N(\theta, \delta^2)$. The chi-square statistic (d.f. = 3) tests the hypothesis of a pure diffusion process against a jump-diffusion process. * indicates significant at 1% level.

Table 18

Estimated Market Parameters

with Monthly Data Sample

	Whole Sample	Jump	Diffusion
N	342	120	222
ABS(R_w)	0.0349278	0.0592705	0.0217696
R_w	0.0095734	-0.004698	0.0172876
	(0.0449385)	(0.0682296)	(0.0209512)
λ	0.35		
ϕ	0.7882		
R_f	0.00018563		

Once betas are estimated, run the following regression:

$$R_j(t) = a + b*\overline{R}_j(t) + c*JAN(t) + e_j \qquad j=1,\ldots,30. \qquad (5.4)$$

where $R_j(t)$ is monthly portfolio return. $\overline{R}_j(t)$ is monthly estimated portfolio return

using the Jump-Diffusion model. JAN(t) is a dummy variable as defined by Fama and

French. Comparing with the following model:

$$R_j'(t) = a' + b'*\overline{R}_j(t) + e_j' \qquad j=1,\ldots,30. \qquad (5.5)$$

where the dummy variable JAN(t) is added to capture the January Effect. R'j(t) in

equation (5.5) is also the monthly portfolio return. In their study on the jump-diffusion

processes in the foreign exchange markets and the release of macroeconomic news,

Johnson and Schneeweis (1994) incorporated such a variable to examine the impact of

the release of macroeconomic news (p312). If January Effect does exist, equation (5.4)

would have better fit than equation (5.5).

The regression results of equation (5.4) and (5.5) are reported in Table 19. The

higher value of R^2 and better estimates of parameters show that taking account of the

January Effect certainly improves fitness of the regression model. Therefore, we

conclude that the January Effect does exist.

Comparing values of R^2 in Table 19 with those in Table 16, the results of Fama

and French, it is not difficult to conclude that the Jump Diffusion model is a better model

to explain the January Effect.

Table 19

Regression Results of Monthly Sample
342 Months: from July 1963 to December 1991

	a	b	c	R²	a'	b'	R'²
P 1	0.001	0.962	0.105	0.496	0.009	1.012	0.3847
	(0.289)	(15.232)	(8.653)		(2.423)	(14.579)	
P 2	-0.004	0.972	0.091	0.586	0.003	1.015	0.4841
	(-1.481)	(19.005)	(9.153)		(0.874)	(17.863)	
P 3	-0.004	0.981	0.081	0.618	0.002	1.019	0.5290
	(-1.59)	(20.757)	(8.858)		(0.705)	(19.54)	
P 4	-0.004	0.979	0.069	0.657	0.008	1.011	0.5873
	(-1.862)	(23.238)	(8.299)		(0.316)	(21.997)	
P 5	-0.004	0.976	0.065	0.679	0.0006	1.005	0.6143
	(-1.917)	(24.607)	(8.245)		(0.25)	(23.273)	
P 6	-0.002	0.986	0.058	0.699	0.002	1.013	0.6435
	(-0.951)	(26.104)	(7.937)		(1.095)	(24.773)	
P 7	-0.003	0.986	0.048	0.722	0.0004	1.009	0.6808
	(-1.688)	(28.042)	(7.065)		(0.175)	(26.931)	
P 8	-0.03	0.987	0.044	0.731	0.0004	1.008	0.6969
	(-1.545)	(28.902)	(6.521)		(0.176)	(27.96)	
P 9	-0.003	0.970	0.043	0.755	0.0002	1.01	0.7216
	(-1.713)	(30.851)	(6.804)		(0.085)	(29.686)	
P 10	-0.002	0.985	0.038	0.801	0.0005	1.002	0.7789
	(-1.503)	(36.073)	(6.813)		(0.292)	(34.606)	
P 11	-0.002	0.987	0.031	0.810	0.0004	1.002	0.7899
	(-1.289)	(36.795)	(5.908)		(0.27)	(35.748)	
P 12	-0.0016	0.9998	0.024	0.826	0.0002	1.011	0.8135
	(-1.15)	(39.169)	(4.869)		(0.139)	(38.511)	
P 13	-0.0003	0.996	0.023	0.843	0.001	1.007	0.8309
	(-0.252)	(41.698)	(5.067)		(1.09)	(40.877)	
P 14	-0.0017	0.992	0.022	0.861	-0.0001	1.002	0.8492
	(-1.455)	(44.788)	(5.29)		(-0.06)	43.755	
P 15	-0.0003	0.991	0.016	0.891	0.0009	0.9986	0.8850
	(-0.321)	(51.938)	(4.481)		(0.872)	(51.148)	
P 16	-0.0005	0.992	0.011	0.909	0.0003	0.9977	0.9057
	(-0.589)	(57.506)	(3.445)		(0.329)	(57.154)	
P 17	-0.0003	0.9917	0.009	0.921	0.0004	0.9961	0.9188
	(-0.367)	(62.244)	(3.044)		(0.449)	(62.031)	
P 18	0.0001	0.9922	0.004	0.940	0.0004	0.994	0.9394
	(0.163)	(72.322)	(1.567)		(0.598)	(72.611)	
P 19	-0.0002	0.9889	0.008	0.953	0.0004	0.993	0.9504
	(-0.385)	(81.737)	(3.968)		(0.674)	(80.707)	
P 20	0.0002	0.999	-0.001	0.959	0.0002	0.998	0.9592
	(0.455)	(88.966)	(-0.485)		(0.339)	(89.392)	

Note: t-statistics in parentheses. a, b, and c are coefficients estimated with equation (5.4). a' and b' are coefficients estimated with equation (5.5).

CHAPTER SIX

CONCLUSION

This empirical study is based on the jump-diffusion asset pricing model developed by Hsia (1992). Employing all valid trading stocks in the NYSE and AMEX during the Gulf War period, the specifications of the jump-diffusion model are tested. The comparison between the jump-diffusion model and the conventional CAPM is carried out from two perspectives: on the specification of the CAPM and on the comparison of risk measures of two models. Then the application of the jump-diffusion model on January effect studies is executed in the same degree of strict manner as in the specification testing section. Findings generally favor the jump-diffusion model, especially in the January effect study. However, it is noticed that the jump-diffusion model is not as general as it is theoretically proved. It works much better than the conventional CAPM in studying period that is more eventful or when jumps are more frequent. In the period when diffusion is dominant, the jump-diffusion model does not work as well as the CAPM which is a diffusion model. That the jump-diffusion model does not converge to the diffusion CAPM should be a subject of future studies.

The major contribution of this empirical study is that it undertakes the challenge to identify the jump process from the diffusion process. It employs an innovative method

which combines the efficient maximum likelihood estimation and concept of stochastic switching. The graphical results show that this is a reliable, efficient and sensible method. It makes the future jump-diffusion study easier and enhances the possibility of applying the jump-diffusion asset pricing model to financial practices.

6.1 POTENTIAL APPLICATIONS

Despite its limitations, the jump-diffusion model is an improvement over current asset pricing tools. The concept of mixed jump-diffusion processes should be useful in fields other than finance, including any discipline in the social sciences concerning the adoption of technology or social change. In the world of investment, two betas should make more sense than one beta. Current rating services such as Moody's, Standard and Poors, publish one beta as a risk factor. It would be to the users' advantage if they could publish two betas instead. Every industry has a different kind of business. External factors, such as policy change, have different focuses. Therefore, different securities should have different jump risks, associated with external factors, and different diffusion risks, associated with normal demand and supply movement. For example, policy regulating high-tech industry would affect Sears much less than Microsoft. Defense industry stocks may perform well during a war period but not be exciting during peace. Even a common factor like interest rate change would have a diferent impact on a highly leveraged industry than on one that is not highly leveraged. Stocks of industries that are sensitive to external influences would certainly have higher jump beta than other stocks.

While diffusion beta measures the security's relative risk to the market during normal times, jump beta measures its relative risk to the market during extraordinary times. This fact should be of interest to portfolio managers and sophisticated investors.

6.2   REMARKS ON FOLLOW-UP STUDIES

The jump-diffusion model has the potential to directly utilize deterministic switching (Goldfeld and Quandt, 10) . Rewrite the model as

$$\overline{R}_j = R_f + (1 - \phi) (\overline{R}_w - R_f) \beta_j + \phi (\overline{R}_w - R_f) \beta_{\wedge j} \tag{6.1}$$

This coincides with the combined two-structure switching regression equation. It combines the following two equations:

$$\overline{R}_j = R_f + (\overline{R}_w - R_f) \beta_j + \kappa_j \quad j \in [\text{diffusion}] \tag{6.2}$$

and

$$\overline{R}_{\wedge j} = R_f + (\overline{R}_{\wedge w} - R_f) \beta_{\wedge j} + \kappa_{\wedge j} \quad \wedge j \in [\text{jump}] \tag{6.3}$$

Multiplying (6.2) with $(1 - \phi)$ and (6.3) with ϕ, we get the switching regression function

$$\overline{R}_j = R_f + (1 - \phi) (\overline{R}_w - R_f) \beta_j + \phi (\overline{R}_w - R_f) \beta_{\wedge j} + (1 - \phi) \kappa_j + \phi \kappa_{\wedge j} \tag{6.4}$$

in which same number of parameters must be estimated.

According to equation (6.4), the expected return is the combination of the excessive return associated with the diffusion risk and that with the jump risk. Two structures (processes) are clearly defined in equation (6.4). The challenge is to approximate ϕ (or λ) with a continuous function if adopting Goldfeld and Quandt's method.

The potential of equation (6.4) is great. Since all needed parameters, including ϕ, β_j, and $\beta_{\Delta j}$ can be estimated with one equation, problems associated with error-in-variable caused by several steps of estimation are eliminated. Therefore, results are more reliable. Unfortunately, the theoretical work on this suggestion is beyond the scope of this dissertation. Thus, it is pointed out for the future research. Until then, the method suggested in this dissertation works the best with jump-diffusion processes.

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APPENDIX A:

DERIVATION OF THE MEAN-VARIANCE EFFICIENT FRONTIER

I. Objective Function

$$\text{Minimize} \quad \sigma^2_p = X'VX \tag{A1}$$

$$\text{s.t.} \quad X'E = E_p \tag{A1a}$$

$$X'L = 1. \tag{A1b}$$

II. The Lagrangian Function

$$L = X'VX - a1(X'E - Ep) - a2(X'L - 1). \tag{A2}$$

Let $\quad \partial L/\partial X = 2VX - a1E - a2L = 0$,

we then get

$$VX = \frac{1}{2}(E \quad L)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} ,$$

which implies that

$$X = \frac{1}{2}V^{-1}(E \quad L)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} . \tag{A3}$$

Pre-multiply (A3) by (E L)', we obtain

$$(E \quad L)'X = \frac{1}{2}(E \quad L)'V^{-1}(E \quad L)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} .$$

Let $\quad G = (E \quad L)'V^{-1}(E \quad L)$,

then $\quad (E \quad L)'X = \frac{1}{2}G\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}. \tag{A4}$

Therefore, substitute (A1a) and (A1b) to obtain

$$\frac{1}{2}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = G^{-1}(E \quad L)'X$$

$$= G^{-1}\begin{pmatrix} E_p \\ 1 \end{pmatrix} \, . \tag{A5}$$

Now substitute (A5) to (A3), it yields

$$X = V^{-1}(E \quad L)G^{-1}\begin{pmatrix} E_p \\ 1 \end{pmatrix} \, . \tag{A6}$$

Substitute (A6) to (A1), the objective function becomes

$$\sigma_p^2 = X'VX = X'(E \quad L)G^{-1}\begin{pmatrix} E_p \\ 1 \end{pmatrix}$$

$$= X'(E \quad L)G^{-1}\begin{pmatrix} E_p \\ 1 \end{pmatrix} \, . \tag{A7}$$

$$= (E_p \quad 1)G^{-1}\begin{pmatrix} E_p \\ 1 \end{pmatrix}$$

III. The Value of G^{-1}

Recall that

$$G = (E \quad L)'V^{-1}(E \quad L)$$

$$= \begin{bmatrix} E'V^{-1}E & L'V^{-1}E \\ E'V^{-1}L & L'V^{-1}L \end{bmatrix}$$

$$= \begin{bmatrix} B & A \\ A & C \end{bmatrix}$$

Thus

$$G^{-1} = \frac{1}{D}\begin{bmatrix} C & -A \\ -A & B \end{bmatrix} \tag{A8}$$

where

$$D = BC - A^2 > 0 \tag{A8a}$$

IV. The Parabolic Function of A Mean-Variance Efficient Frontier:

Substitute (A8) back to (A7), we can obtain

$$
\begin{aligned}
\sigma_p^2 &= (E_p \quad 1) \, (\frac{1}{D}) \begin{bmatrix} C & -A \\ -A & B \end{bmatrix} \begin{pmatrix} E_p \\ 1 \end{pmatrix} , \\
&= \frac{1}{D} \, (CE_p^2 - 2AE_p + B)
\end{aligned}
\tag{A9}
$$

the minimum variance frontier as a parabola in the (E, σ^2) space.

V. The Global Minimum Variance Portfolio (GMVP):

It is easy to derive that GMVP is

$$E_0 = A/C \tag{A10a}$$

$$\sigma_0^2 = 1/C. \tag{A10b}$$

VI. The Market Portfolio with the Existence of R_f:

$$E_m = \frac{B - AR_f}{A - CR_f} \tag{A11a}$$

$$\sigma_m^2 = \frac{E_m - R_f}{A - CR_f} \tag{A11b}$$

VII. The Composition of Efficient Portfolios

Substitute (A8) back to (A6), we get

$$X = V^{-1}(\tau_1 E + \tau_2 L) \tag{A12}$$

where

$$\tau_1 = (CE_p - A)/D$$

$$\tau_2 = (-AE_p + B)/D.$$

APPENDIX B:

DERIVATION OF THE JUMP-DIFFUSION TWO-BETA ASSET PRICING MODEL

In his Proposition A1, Bates (1991, p1039) shows that when asset prices follow
jump-diffusion processes the equilibrium cum-dividend excess return is

$$\mu - r = R(W,t)E_{dq=0}[(dS/S)(dW/W) - \lambda E_{dq=1}[(\Delta J_w/J_w)(\Delta S/S)],\qquad (B1)$$

where

$$dS/S = (\mu - \lambda K - d_t)dt + \sigma dz + Kdq \qquad (B1a)$$

which is equation (4) of Bates (p1023), and

$$dW/W = (\mu_w - \lambda K_w - C/W)dt + \sigma_w dz_w + K_w dq \qquad (B1b)$$

which is equation (8) of Bates (p1024), and

$$\Delta J_w/J_w = (1 + K_w)^{-R} - 1 \qquad (B1c)$$

which appears in page 1039 of Bates, and

$$\Delta S/S = K = \text{the percent discrete rate of return on the asset}$$

$$\text{when jump occurring.} \qquad (B1d)$$

The notations are defined as follow:

$\mu =$ instantaneous cum-dividend expected return on the asset, and μ_w is

       similarly defined for optimally invested wealth

$r =$ instantaneous riskless rate, a constant, which is endogenously derived

       from the model (p1039)

$E =$ expectations operator

$S =$ cum-dividend stock price

$W =$ optimally invested wealth

$R(W,t) =$ coefficient of proportional risk aversion which is a constant

$q =$ a Poisson counter

$(dq=0) =$ no jump occurring

$(dq=1) =$ 1 jump occurring

$\lambda =$ frequency of Poisson events

$K =$ random percentage jump conditional upon a Poisson-distributed event occurring, and K_w is similarly defined for optimally invested wealth

$d_t =$ flow rate of dividend yield

$\sigma^2 =$ instantaneous variance conditional on no jumps, and σ_w^2 is similarly defined for optimally invested wealth

$z =$ standard Wiener Process, and z_w is similarly defined for optimally invested wealth

$C =$ aggregate consumption

$J_w =$ marginal utility of wealth

$\Delta j_w=$ incremental marginal utility of wealth conditional on jumps occurring.

Instead of further developing equation (B1), Bates continues immediately to develop his option pricing model on financial futures. Equation (B1) is interesting and innovative, but it needs further development because it has unobservable parameters. This Appendix is devoted to continue the development of the jump-diffusion two-beta asset pricing model on the basis of Bates foundation.

Note that the left hand side (LHS) of equation (B1), $\mu - r$, is the excess return on a risky asset, while the right hand side (RHS) of (B1) is composed of two risk premiums,

RP^D and RP^J, where

$$RP^D \text{ (premium for diffusion risk)} = Re_{dq=0}[(dS/S)(dW/W)] \qquad \text{(B2a)}$$

and $\quad RP^J$ (premium for jump risk) $= \lambda E_{dq=1}[(\Delta J_w/J_w)(\Delta S/S)]. \qquad$ (B2b)

These two risk premiums are developed respectively in the following:

A. Premium for Diffusion Risk

Equation (B2a) looks familiar to finance readers. It can be written as

$$RP^D = RCov(r_w, r_s) \qquad \text{(B3)}$$

because (dW/W) is a zero-mean increment in wealth, and $dW/W = r_w$, $dS/S = r_s$,

respectively, where r_w and r_s are instantaneous returns on the optimally invested wealth

and the asset, respectively, conditional on no jumps occurring. The $Cov(r_w, r_s)$ is the

instantaneous covariance per unit time between the asset and wealth returns conditional

on no jumps. Not surprisingly, Bates also write $RP^D = R(W, t)\sigma_{sw}$ in his equation (5) on

page 1039, which is same as Equation (B3) presented above.

B. Premium for Jump Risk

The premium for jump risk is more complicated because it involves an

unobservable term, $\Delta J_w/J_w$. However, as a first step, equation (B2b) can also be written as

a covariance,

$$RP^J = \lambda Cov(\Delta J_w/J_w, \Delta S/S) \qquad \text{(B4)}$$

because $\Delta J_w/J_w$ is also a zero-mean increment. Now we treat the term $\Delta J_w/J_w$. Using

(B1c) to obtain $[1 + (\Delta J_w/J_w)] = [(1 + K_w)^{-R}]$ which by taking logarithm on both sides is

$$\ln(1 + \Delta J_w/J_w) = -R\ln(1 + K_w). \tag{B5}$$

Expanding the LHS and RHS of (B5) respectively, and collecting the second and higher

order terms, and because $K_w = \Delta W/W$, we have

$$(\Delta J_w/J_w) = -R(\Delta W/W) + \xi \tag{B6}$$

where

$$\xi = (\tfrac{1}{2})[(\Delta J_w/J_w)^2 + R(\Delta W/W)^2]$$

$$- (\tfrac{1}{3})[(\Delta J_w/J_w)^3 + R(\Delta W/W)^3] + ... \tag{B6a}$$

Substituting (B6) back to (B4) and since $\text{Cov}(\xi, \Delta S/S) = 0$, we have

$$RP^J = -\lambda R\text{Cov}(\Delta W/W, \Delta S/S). \tag{B7}$$

Defining $\Delta r_w = \Delta W/W$ and $\Delta r_s = \Delta S/S$ as the discrete rate of return on the optimally

invested wealth and the asset, respectively, conditional on jumps occurring, (B7) is

rewritten as

$$RP^J = -\lambda R\text{Cov}(\Delta r_w, \Delta r_s). \tag{B8}$$

C. The Jump-Diffusion Asset Pricing Model

Substituting (B3) and (B8) back to (B2a) and (B2b), respectively, which in turn

are substituted back to (B1), we have

$$\mu - r = R\text{Cov}(r_s, r_w) + \lambda R\text{Cov}(\Delta r_s, \Delta r_w)$$

$$= R[\text{Cov}(r_s, r_w) + \lambda\text{Cov}(\Delta r_s, \Delta r_w)]. \tag{B9}$$

Since μ_w, the instantaneous expected return on the optimally invested wealth, must also

satisfy (B9), we have

$$\mu_w - r = R[\sigma^2(r_w) + \lambda\sigma^2(\Delta r_w)]$$

which implies

$$R = \frac{\mu_w - r}{\sigma^2(r_w) + \sigma^2(\Delta r_w)} \ .$$

(B10)

Substitute (B10) in (B9), it yields

$$\mu - r = (\mu_w - r)\frac{Cov(r_s, r_w) + \lambda Cov(\Delta r_s, \Delta r_w)}{\sigma^2(r_w) + \lambda \sigma 2(\Delta r_w)}$$

(B11)

and simplified to be

$$\mu - r = (\mu_w - r)[(1-\phi)\beta_s + \phi\beta_{\Delta s}]$$

(B12)

where

$$\beta_s = \frac{Cov(r_s, r_w)}{\sigma^2(r_w)} = diffusion \ \beta$$

(B12a)

$$\beta_{\Delta s} = \frac{Cov(\Delta r_s, \Delta r_w)}{\sigma^2(\Delta r_w)} = jump \ \beta$$

(B12b)

$$\phi = \frac{\lambda\sigma^2(\Delta r_w)}{\sigma^2(r_w) + \lambda\sigma^2(\Delta r_w)}$$

(B12c)

Equation (B12) is the jump-diffusion two-beta asset pricing model used in this research. It indicates that under the condition that asset returns are generated by a mixture of jump and diffusion processes, the systematic risk of a risky asset is a weighted average of diffusion beta and jump beta, where the weight factor, ϕ, is a market parameter as defined by (B12c).

APPENDIX C: TABLES

Table 1

Average Sizes of 30 Portfolios for 5 Years

(in thousand dollars)

	1987	1988	1989	1990	1991
P 1	5179	4010	4382	3578	2439
P 2	9997	8413	9336	7688	5930
P 3	14940	12107	13779	12722	9574
P 4	19561	16528	19446	17866	14173
P 5	26069	21651	24595	24759	19518
P 6	34080	27672	31424	32357	25439
P 7	42391	35266	40571	40845	32339
P 8	51550	42999	49617	48938	40358
P 9	61705	50755	59774	59863	50068
P 10	72860	59906	70290	71475	59912
P 11	85497	70305	81837	84206	71584
P 12	101295	82144	94600	97533	86677
P 13	121507	96238	110037	113447	103046
P 14	144852	111469	130414	134673	123003
P 15	169158	134819	154140	159766	148537
P 16	199181	163286	187759	189856	175331
P 17	238948	199254	225582	234123	205641
P 18	292189	238423	276765	278961	254997
P 19	363176	298495	352333	334014	320098
P 20	459293	378882	438661	410887	410734
P 21	572884	472088	539969	530661	505157
P 22	709815	586045	650741	659334	628460
P 23	895901	758901	830721	833974	792802
P 24	1118397	968761	1069331	1053733	1017444
P 25	1404878	1224342	1359764	1400342	1308129
P 26	1815192	1646024	1850324	1894753	1819058
P 27	2463188	2213584	2475313	2547907	2523307
P 28	3426640	3208499	3606521	3763844	3917712
P 29	5438555	5440840	5985875	6511852	6967539
P 30	15174136	31900480	45152080	48041520	45885696

Table 2

Statistics of Average Daily Returns

NAME	N	MEAN	ST.DEV	VARIANCE	MINIMUM	MAXIMUM
Rw	506	4.68E-04	8.57E-03	7.35E-05	-3.27E-02	3.35E-02
R1	506	4.55E-03	1.53E-02	2.35E-04	-4.29E-02	6.49E-02
R2	506	2.46E-03	1.21E-02	1.47E-04	-4.66E-02	7.11E-02
R3	506	1.07E-03	9.81E-03	9.62E-05	-3.21E-02	3.52E-02
R4	506	6.70E-04	9.28E-03	8.61E-05	-2.97E-02	3.62E-02
R5	506	5.22E-04	8.36E-03	7.00E-05	-3.06E-02	2.86E-02
R6	506	3.12E-04	9.03E-03	8.16E-05	-3.30E-02	5.24E-02
R7	506	7.03E-05	8.73E-03	7.62E-05	-3.36E-02	3.41E-02
R8	506	1.83E-04	7.63E-03	5.82E-05	-3.21E-02	3.56E-02
R9	506	2.87E-04	7.06E-03	4.98E-05	-3.24E-02	3.44E-02
R10	506	2.09E-04	8.87E-03	7.87E-05	-4.55E-02	4.30E-02
R11	506	2.30E-04	6.76E-03	4.56E-05	-3.11E-02	2.77E-02
R12	506	5.98E-04	7.13E-03	5.08E-05	-3.45E-02	3.81E-02
R13	506	9.11E-05	7.31E-03	5.35E-05	-3.44E-02	2.88E-02
R14	506	1.35E-04	7.61E-03	5.80E-05	-3.26E-02	3.28E-02
R15	506	4.30E-04	7.86E-03	6.18E-05	-4.12E-02	3.73E-02
R16	506	2.36E-04	6.75E-03	4.56E-05	-2.98E-02	1.97E-02
R17	506	2.20E-04	7.37E-03	5.44E-05	-3.36E-02	2.91E-02
R18	506	3.46E-04	7.21E-03	5.20E-05	-3.59E-02	3.04E-02
R19	506	4.01E-04	7.87E-03	6.19E-05	-3.78E-02	2.87E-02
R20	506	3.38E-04	7.49E-03	5.61E-05	-3.16E-02	3.00E-02
R21	506	2.78E-04	6.80E-03	4.62E-05	-3.42E-02	2.42E-02
R22	506	3.87E-04	7.48E-03	5.59E-05	-3.42E-02	2.90E-02
R23	506	3.01E-04	7.40E-03	5.48E-05	-3.64E-02	2.79E-02
R24	506	4.61E-04	8.32E-03	6.92E-05	-3.14E-02	3.44E-02
R25	506	5.87E-04	7.78E-03	6.05E-05	-2.85E-02	3.22E-02
R26	506	3.73E-04	8.69E-03	7.55E-05	-2.98E-02	3.18E-02
R27	506	4.19E-04	8.70E-03	7.56E-05	-3.04E-02	3.26E-02
R28	506	4.35E-04	8.84E-03	7.82E-05	-3.25E-02	3.49E-02
R29	506	4.17E-04	8.90E-03	7.92E-05	-3.21E-02	3.49E-02
R30	506	4.87E-04	8.78E-03	7.71E-05	-3.22E-02	4.16E-02

Table 3

Covariance/Variance Matrix

	Rw	R1	R2	R3	R4	R5	R6	R7
Rw	7.35E-05							
R1	2.59E-05	2.35E-04						
R2	3.19E-05	5.60E-05	1.47E-04					
R3	3.25E-05	4.91E-05	4.40E-05	9.62E-05				
R4	4.22E-05	5.34E-05	4.93E-05	4.77E-05	8.61E-05			
R5	3.77E-05	4.62E-05	3.99E-05	4.03E-05	4.84E-05	7.00E-05		
R6	3.92E-05	3.65E-05	4.11E-05	4.07E-05	4.56E-05	4.01E-05	8.16E-05	
R7	4.32E-05	3.89E-05	4.92E-05	4.20E-05	5.00E-05	4.14E-05	4.18E-05	7.62E-05
R8	4.13E-05	3.64E-05	4.22E-05	3.76E-05	4.24E-05	3.95E-05	4.10E-05	4.55E-05
R9	4.12E-05	2.86E-05	3.54E-05	3.48E-05	4.00E-05	3.50E-05	3.91E-05	4.25E-05
R10	5.24E-05	4.12E-05	4.84E-05	4.20E-05	4.94E-05	4.68E-05	4.81E-05	5.75E-05
R11	3.99E-05	3.22E-05	3.35E-05	3.25E-05	3.75E-05	3.40E-05	3.61E-05	4.14E-05
R12	4.48E-05	3.02E-05	3.58E-05	3.46E-05	4.15E-05	3.63E-05	3.91E-05	4.43E-05
R13	4.54E-05	3.60E-05	3.72E-05	3.62E-05	4.25E-05	3.78E-05	3.97E-05	4.47E-05
R14	4.81E-05	3.33E-05	3.98E-05	3.69E-05	4.25E-05	3.95E-05	4.24E-05	4.84E-05
R15	5.32E-05	3.51E-05	4.18E-05	4.06E-05	4.81E-05	4.27E-05	4.44E-05	4.96E-05
R16	4.35E-05	2.74E-05	3.40E-05	3.11E-05	3.76E-05	3.32E-05	3.61E-05	4.12E-05
R17	4.89E-05	3.23E-05	3.62E-05	3.26E-05	4.09E-05	3.81E-05	3.97E-05	4.48E-05
R18	4.79E-05	3.07E-05	3.34E-05	3.24E-05	3.90E-05	3.50E-05	3.93E-05	4.08E-05
R19	5.57E-05	3.09E-05	3.84E-05	3.70E-05	4.41E-05	4.06E-05	4.22E-05	4.69E-05
R20	5.28E-05	3.43E-05	3.58E-05	3.47E-05	4.07E-05	3.76E-05	3.98E-05	4.38E-05
R21	5.04E-05	2.79E-05	3.11E-05	3.12E-05	3.76E-05	3.49E-05	3.64E-05	4.01E-05
R22	5.70E-05	2.57E-05	3.41E-05	3.49E-05	4.08E-05	3.70E-05	3.99E-05	4.26E-05
R23	5.77E-05	2.84E-05	3.29E-05	3.34E-05	3.88E-05	3.71E-05	3.72E-05	4.14E-05

R24	6.59E-05	3.22E-05	3.81E-05	3.65E-05	4.44E-05	4.15E-05	4.24E-05	4.65E-05
R25	6.26E-05	2.63E-05	3.19E-05	3.32E-05	4.07E-05	3.70E-05	3.91E-05	4.27E-05
R26	6.95E-05	2.88E-05	3.43E-05	3.57E-05	4.32E-05	4.04E-05	4.21E-05	4.60E-05
R27	7.14E-05	2.99E-05	3.25E-05	3.53E-05	4.41E-05	3.94E-05	4.11E-05	4.51E-05
R28	7.33E-05	2.80E-05	3.11E-05	3.44E-05	4.39E-05	3.85E-05	4.10E-05	4.55E-05
R29	7.47E-05	2.64E-05	3.19E-05	3.30E-05	4.18E-05	3.75E-05	3.89E-05	4.32E-05
R30	7.06E-05	2.37E-05	3.11E-05	3.16E-05	4.11E-05	3.61E-05	3.72E-05	4.24E-05

	R8	R9	R10	R11	R12	R13	R14	R15
R8	5.82E-05							
R9	3.88E-05	4.98E-05						
R10	5.00E-05	4.74E-05	7.87E-05					
R11	3.83E-05	3.54E-05	4.77E-05	4.56E-05				
R12	3.86E-05	3.95E-05	4.97E-05	3.78E-05	5.08E-05			
R13	4.18E-05	3.91E-05	5.22E-05	4.05E-05	4.13E-05	5.35E-05		
R14	4.20E-05	4.14E-05	5.31E-05	4.06E-05	4.39E-05	4.49E-05	5.80E-05	
R15	4.46E-05	4.39E-05	5.76E-05	4.27E-05	4.63E-05	4.79E-05	5.06E-05	6.18E-05
R16	3.62E-05	3.56E-05	4.64E-05	3.57E-05	3.80E-05	3.91E-05	4.16E-05	4.35E-05
R17	4.13E-05	3.88E-05	5.13E-05	3.96E-05	4.17E-05	4.32E-05	4.56E-05	4.87E-05
R18	3.87E-05	3.82E-05	4.85E-05	3.81E-05	4.09E-05	4.19E-05	4.39E-05	4.69E-05
R19	4.39E-05	4.17E-05	5.28E-05	4.14E-05	4.50E-05	4.60E-05	4.83E-05	5.15E-05
R20	4.12E-05	3.98E-05	5.21E-05	3.94E-05	4.16E-05	4.37E-05	4.57E-05	4.94E-05
R21	3.73E-05	3.63E-05	4.67E-05	3.59E-05	3.88E-05	4.03E-05	4.20E-05	4.57E-05
R22	3.99E-05	3.94E-05	5.03E-05	3.82E-05	4.28E-05	4.28E-05	4.62E-05	4.95E-05
R23	3.90E-05	3.94E-05	5.03E-05	3.76E-05	4.16E-05	4.31E-05	4.50E-05	4.91E-05
R24	4.40E-05	4.30E-05	5.62E-05	4.25E-05	4.66E-05	4.74E-05	4.98E-05	5.50E-05
R25	4.14E-05	3.94E-05	5.08E-05	3.88E-05	4.24E-05	4.38E-05	4.56E-05	5.03E-05
R26	4.29E-05	4.30E-05	5.49E-05	4.17E-05	4.61E-05	4.78E-05	4.93E-05	5.51E-05
R27	4.21E-05	4.28E-05	5.46E-05	4.11E-05	4.67E-05	4.73E-05	5.00E-05	5.49E-05
R28	4.24E-05	4.27E-05	5.30E-05	4.11E-05	4.55E-05	4.65E-05	4.97E-05	5.52E-05

R29	4.13E-05	4.11E-05	5.22E-05	4.01E-05	4.49E-05	4.58E-05	4.80E-05	5.30E-05
R30	3.97E-05	3.95E-05	5.04E-05	3.85E-05	4.39E-05	4.33E-05	4.58E-05	5.15E-05

	R16	R17	R18	R19	R20	R21	R22	R23
R16	4.56E-05							
R17	4.02E-05	5.44E-05						
R18	3.92E-05	4.38E-05	5.20E-05					
R19	4.27E-05	4.88E-05	4.73E-05	6.19E-05				
R20	4.06E-05	4.66E-05	4.46E-05	5.14E-05	5.61E-05			
R21	3.76E-05	4.31E-05	4.09E-05	4.72E-05	4.45E-05	4.62E-05		
R22	4.10E-05	4.67E-05	4.52E-05	5.16E-05	4.89E-05	4.56E-05	5.59E-05	
R23	4.02E-05	4.57E-05	4.43E-05	5.06E-05	4.83E-05	4.57E-05	5.07E-05	5.48E-05
R24	4.51E-05	5.13E-05	4.92E-05	5.72E-05	5.37E-05	5.09E-05	5.64E-05	5.72E-05
R25	4.12E-05	4.69E-05	4.55E-05	5.27E-05	5.00E-05	4.76E-05	5.31E-05	5.30E-05
R26	4.55E-05	5.08E-05	4.97E-05	5.73E-05	5.52E-05	5.18E-05	5.69E-05	5.84E-05
R27	4.50E-05	5.09E-05	5.02E-05	5.70E-05	5.56E-05	5.22E-05	5.81E-05	5.93E-05
R28	4.52E-05	5.10E-05	4.92E-05	5.79E-05	5.55E-05	5.25E-05	5.87E-05	5.98E-05
R29	4.31E-05	4.97E-05	4.84E-05	5.68E-05	5.42E-05	5.12E-05	5.82E-05	5.88E-05
R30	4.23E-05	4.70E-05	4.62E-05	5.37E-05	5.09E-05	4.81E-05	5.38E-05	5.47E-05

	R24	R25	R26	R27	R28	R29	R30
R24	6.92E-05						
R25	6.00E-05	6.05E-05					
R26	6.64E-05	6.31E-05	7.55E-05				
R27	6.73E-05	6.36E-05	7.08E-05	7.56E-05			
R28	6.78E-05	6.43E-05	7.21E-05	7.33E-05	7.82E-05		
R29	6.72E-05	6.39E-05	7.11E-05	7.30E-05	7.52E-05	7.92E-05	
R30	6.27E-05	5.92E-05	6.49E-05	6.72E-05	6.94E-05	7.18E-05	7.71E-05

Table 7

Portfolio Estimates with Gulf War Period Samples

	N	R_j	$\sigma(R_j)$	$\beta_j{}^*$	$\beta_{\Delta j}$	β_j
R_1	506	0.0045478	0.0153432	0.35181	0.41734	0.01722
R_2	506	0.0024577	0.0121144	0.43339	0.48651	0.13181
R_3	506	0.0010740	0.0098104	0.44171	0.48668	0.21047
R_4	506	0.000669660	0.0092781	0.57374	0.58094	0.53808
R_5	506	0.000522056	0.0083636	0.51321	0.53479	0.35937
R_6	506	0.000312450	0.0090312	0.53320	0.55808	0.33898
R_7	506	0.000070261	0.0087287	0.58835	0.62875	0.34041
R_8	506	0.000183328	0.0076302	0.56197	0.59348	0.37755
R_9	506	0.000286576	0.0070592	0.56112	0.58607	0.38756
R_{10}	506	0.000208624	0.0088739	0.71361	0.75627	0.44743
R_{11}	506	0.000230119	0.0067559	0.54265	0.57240	0.35348
R_{12}	506	0.000597908	0.0071259	0.60922	0.62588	0.46481
R_{13}	506	0.000091132	0.0073121	0.61825	0.64331	0.43807
R_{14}	506	0.000135447	0.0076130	0.65409	0.68384	0.45133
R_{15}	506	0.000429806	0.0078598	0.72373	0.75454	0.53517
R_{16}	506	0.000235631	0.0067521	0.59139	0.60296	0.47121
R_{17}	506	0.000220159	0.0073746	0.66475	0.69345	0.47238
R_{18}	506	0.000346241	0.0072133	0.65225	0.67472	0.49437
R_{19}	506	0.000400690	0.0078681	0.75727	0.78013	0.57842
R_{20}	506	0.000337710	0.0074895	0.71818	0.73842	0.59182
R_{21}	506	0.000278132	0.0068006	0.68531	0.70558	0.54647
R_{22}	506	0.000386554	0.0074753	0.77577	0.78405	0.69995
R_{23}	506	0.000301208	0.0074028	0.78515	0.80306	0.66387
R_{24}	506	0.000460598	0.0083211	0.89674	0.91554	0.77445
R_{25}	506	0.000586969	0.0077808	0.85234	0.85866	0.81205
R_{26}	506	0.000372755	0.0086903	0.94606	0.94207	0.95335
R_{27}	506	0.000419243	0.0086963	0.97148	0.97997	0.93097
R_{28}	506	0.000434601	0.0088419	0.99798	1.00081	1.00224
R_{29}	506	0.000417271	0.0089001	1.01645	1.01406	1.03373
R_{30}	506	0.000487035	0.0087817	0.96005	0.97452	0.88214

Panel A: For the whole period

	N	R_j	$\sigma(R_j)$	$\beta_j{}'$	$\beta_{\Delta j}$	β_j
R_1	145	0.0031251	0.0130067	0.42227	0.67882	-0.00953
R_2	145	0.000888382	0.0085315	0.47643	0.40659	0.69379
R_3	145	0.000129495	0.0070481	0.33045	0.35935	0.27725
R_4	145	0.000236594	0.0068465	0.42178	0.47802	0.37785
R_5	145	-0.000303728	0.0062561	0.37799	0.39465	0.36414
R_6	145	-0.000703143	0.0075596	0.57016	0.62702	0.45890
R_7	145	-0.000498368	0.0057149	0.42417	0.42995	0.37476
R_8	145	-0.000087907	0.0050464	0.41290	0.48503	0.22436
R_9	145	-0.000259828	0.0060657	0.50926	0.53047	0.42263
R_{10}	145	-0.000418257	0.0066893	0.56126	0.59615	0.48338
R_{11}	145	-0.000232966	0.0045915	0.39209	0.41189	0.34544
R_{12}	145	0.000276755	0.0052494	0.49773	0.49314	0.45226
R_{13}	145	-0.000581247	0.0051357	0.47644	0.49417	0.3925
R_{14}	145	-0.000450624	0.0057756	0.52083	0.51318	0.49436
R_{15}	145	-0.000152318	0.0061462	0.63842	0.67276	0.54709
R_{16}	145	7.55658E-6	0.0051090	0.53580	0.52697	0.51163
R_{17}	145	-0.000069809	0.0050034	0.52731	0.50783	0.54527
R_{18}	145	-0.000052294	0.0057743	0.62067	0.62844	0.59568
R_{19}	145	-0.000022921	0.0058378	0.61036	0.59978	0.58599
R_{20}	145	-0.000236234	0.0053704	0.56997	0.55355	0.58745
R_{21}	145	-0.000424400	0.0051319	0.58950	0.59296	0.57588
R_{22}	145	-0.000146408	0.0057312	0.68094	0.66700	0.68336
R_{23}	145	-0.000207912	0.0063156	0.76740	0.78702	0.70919
R_{24}	145	-0.000026812	0.0063849	0.79461	0.81128	0.71481
R_{25}	145	-0.000053024	0.0062413	0.80191	0.82856	0.72779
R_{26}	145	-0.000242032	0.0070528	0.88583	0.89085	0.87526
R_{27}	145	-0.000172465	0.0072453	0.95860	0.96685	0.93492
R_{28}	145	0.000055298	0.0070521	0.93320	0.95743	0.87846
R_{29}	145	-1.420412E-6	0.0074502	1.00012	0.99159	1.00693
R_{30}	145	0.000433793	0.0075754	0.97192	0.96860	0.99183

Panel B: For Pre-Invasion Period

	N	R_j	$\sigma(R_j)$	$\beta_j{}'$	$\beta_{\Delta j}$	β_j
R_1	150	0.0054097	0.0178690	0.35399	0.37487	0.07575
R_2	150	0.0040496	0.0163955	0.36541	0.45254	-0.62921
R_3	150	0.0016458	0.0125292	0.54662	0.59881	0.02483
R_4	150	0.0008065	0.0120802	0.63246	0.62786	0.67583
R_5	150	0.0006182	0.0111331	0.58477	0.61546	0.16948
R_6	150	0.0001420	0.0115426	0.53668	0.56302	0.09549
R_7	150	6.18067e-05	0.0124038	0.72959	0.78377	0.20247
R_8	150	0.0002787	0.0097641	0.56408	0.60463	0.11122
R_9	150	-0.0001172	0.0095632	0.65365	0.69135	0.25967
R_{10}	150	9.13665e-05	0.0125275	0.89238	0.94421	0.30013
R_{11}	150	0.0003681	0.0090461	0.64467	0.68288	0.23100
R_{12}	150	0.0009514	0.0098977	0.72796	0.74107	0.54131
R_{13}	150	4.26089e-05	0.0097802	0.70822	0.73796	0.33683
R_{14}	150	-3.24566e-05	0.0105034	0.73738	0.77084	0.30804
R_{15}	150	0.0003241	0.0110217	0.84667	0.88202	0.45133
R_{16}	150	0.0001612	0.0088424	0.64136	0.66267	0.28145
R_{17}	150	4.69865e-05	0.0099769	0.74577	0.78538	0.29186
R_{18}	150	0.0003428	0.0098698	0.72938	0.75251	0.42610
R_{19}	150	0.0001579	0.0108641	0.83818	0.87511	0.34951
R_{20}	150	4.87588e-05	0.0102602	0.79581	0.81938	0.53659
R_{21}	150	0.0002665	0.0090915	0.74769	0.77489	0.40665
R_{22}	150	0.0005681	0.0098212	0.81261	0.83363	0.52989
R_{23}	150	0.0003458	0.0099579	0.85890	0.88348	0.54445
R_{24}	150	0.0006691	0.0111455	0.96460	0.99307	0.63485
R_{25}	150	0.0007332	0.0097697	0.85481	0.86678	0.72256
R_{26}	150	0.0001999	0.0112779	0.97874	0.97011	1.03996
R_{27}	150	0.0005911	0.0113237	1.00558	1.02386	0.84044
R_{28}	150	0.0004630	0.0114894	1.02897	1.03268	1.01628
R_{29}	150	0.0005877	0.0111636	1.00733	1.00749	1.02263
R_{30}	150	0.0004647	0.0112979	0.99044	1.00930	0.81144

Panel C: For Invasion Period

	N	R_j	$\sigma(R_j)$	$\beta_j{}'$	$\beta_{\Delta j}$	β_j
	----	----------	----------	---------	--------	--------
R_1	211	0.0049126	0.0148828	0.29471	0.36265	-0.00297
R_2	211	0.0024044	0.0104291	0.50052	0.57721	0.18011
R_3	211	0.0013166	0.0092244	0.35296	0.37018	0.28100
R_4	211	0.0008700	0.0084346	0.58396	0.58182	0.57606
R_5	211	0.0010212	0.0072599	0.48981	0.48728	0.48496
R_6	211	0.0011316	0.0078111	0.49680	0.51303	0.41389
R_7	211	0.0004670	0.0071610	0.48157	0.49032	0.42047
R_8	211	0.0003020	0.0074046	0.65620	0.64959	0.67333
R_9	211	0.0009490	0.0053808	0.45270	0.44455	0.46582
R_{10}	211	0.0007228	0.0068399	0.54250	0.54304	0.52373
R_{11}	211	0.0004502	0.0060852	0.48682	0.49142	0.44613
R_{12}	211	0.0005673	0.0057916	0.50479	0.51913	0.41697
R_{13}	211	0.0005877	0.0064854	0.57271	0.57459	0.54665
R_{14}	211	0.0006576	0.0061396	0.61341	0.63501	0.51526
R_{15}	211	0.0009050	0.0059995	0.59277	0.59181	0.58742
R_{16}	211	0.0004452	0.0060146	0.55275	0.54949	0.55658
R_{17}	211	0.0005425	0.0065427	0.63260	0.65144	0.54012
R_{18}	211	0.0006226	0.0057407	0.55627	0.57616	0.46027
R_{19}	211	0.0008644	0.0064324	0.73049	0.78350	0.72586
R_{20}	211	0.0009375	0.0063064	0.69569	0.70731	0.64249
R_{21}	211	0.0007692	0.0058246	0.65108	0.65593	0.62078
R_{22}	211	0.0006238	0.0065681	0.78135	0.76958	0.82156
R_{23}	211	0.0006194	0.0058134	0.68485	0.67882	0.70514
R_{24}	211	0.0006473	0.0070840	0.86147	0.84848	0.90861
R_{25}	211	0.0009228	0.0071233	0.87958	0.86379	0.93980
R_{26}	211	0.0009181	0.0075283	0.93403	0.92560	0.96310
R_{27}	211	0.0007037	0.0073852	0.92766	0.91247	0.99023
R_{28}	211	0.0006750	0.0077511	0.99412	0.97313	1.08851
R_{29}	211	0.0005838	0.0079893	1.04108	1.03661	1.06010
R_{30}	211	0.0005395	0.0074519	0.90931	0.92486	0.84241

Panel D: For Post-Invasion Period

Table 14

Portfolio Estimates with Turn-of-the-Year Sample

	R_j	$\sigma(R_j)$	$\beta_j{}'$	$\beta_{\Delta j}$	β_j
R_1	0.0224082	0.0189570	1.012122	0.9978	-0.2981
R_2	0.0185801	0.0179088	1.058303	1.02669	0.62322
R_3	0.0153708	0.0146008	1.006402	1.01639	-0.1723
R_4	0.0144954	0.0129956	0.930818	0.9038	1.27434
R_5	0.0138876	0.0144181	1.065084	1.05818	0.94643
R_6	0.0129085	0.0139093	1.098238	1.08941	0.88958
R_7	0.0120597	0.0143230	1.111219	1.09318	0.91875
R_8	0.0115253	0.0137148	1.141785	1.12329	0.98419
R_9	0.0107713	0.0137029	1.174707	1.15149	1.29907
R_{10}	0.0105937	0.0130334	1.138424	1.13354	0.75325
R_{11}	0.0097603	0.0136043	1.10133	1.10104	0.64932
R_{12}	0.0088547	0.0121757	1.052405	1.04662	0.78382
R_{13}	0.0092436	0.0139359	1.245749	1.23874	1.19228
R_{14}	0.00814	0.0120518	1.043669	1.0377	0.96678
R_{15}	0.0081496	0.0123757	1.119904	1.11372	1.2585
R_{16}	0.0076694	0.0114132	1.057456	1.05192	0.96944
R_{17}	0.0067321	0.0114158	1.045925	1.04138	1.18771
R_{18}	0.0065637	0.0106772	0.9832133	0.98786	0.71551
R_{19}	0.0066445	0.0107311	0.9969362	0.99359	0.99662
R_{20}	0.0064564	0.0109896	1.027927	1.01848	0.91243
R_{21}	0.0057077	0.0103320	0.9807668	0.96968	1.13092
R_{22}	0.005188	0.0106267	1.014363	1.00218	1.35165
R_{23}	0.004622	0.0102475	1.012212	1.02007	0.89479
R_{24}	0.0048745	0.0103861	1.027704	1.03836	0.79764
R_{25}	0.0043455	0.0094244	0.95825	0.96347	0.57834
R_{26}	0.0043657	0.0095300	0.9690998	0.97321	0.8104
R_{27}	0.0040211	0.0097021	1.001781	1.00208	0.90163
R_{28}	0.0037508	0.0097376	1.015797	1.00881	1.20026
R_{29}	0.0028753	0.0096244	1.007809	1.00787	1.24703
R_{30}	0.0019264	0.0094893	0.995836	0.99651	0.97606

APPENDIX D: GAUSS® PROGRAM AND SAMPLE OUTPUTS

```
***********************PROGRAM********************************
use nlopt;

output file=vwest2.out reset;

n=342;
load data[n,5]=c:\store\tape\mkt.mth;        @var: caldt, date, ew, vw, s&p 500 @
x=data[1:n,4];                    @use VW @
clear data;
print meanc(x)~stdc(x)~stdc(x)^2;

z=ln(x+1);     @ x=(P-P(-1)/P(-1); z=ln(P/P(-1)) @
wait;

@ estimating the structure of z @
@ (ln) z ~ poisson mixture of normals @

_method=5;                        @use quadratic hill climbing method@
_iter=2000;
_step=2;
_conv=2;
let c=0.3 0.0003 -0.0008 0.00005 0.0001;
{c,vc}=maxlik(&llfp,z,c);
wait;

l=prob(z,c);
@ probability of a jump process occuring @
@ numerically the number of jumps can be estimated too @
lambda=meanc(l .> 0.25);            @calculate λ by guessing@

@separate samples into jump and diffusion sets@
xj=selif(x,l.>0.25);
x0=delif(x,l.>0.25);
varxj=stdc(xj)^2;
varx0=stdc(x0)^2;
```

```
xi = 1 .> 0.25;

print x~l~xi;
print meanc(x)~meanc(xj)~meanc(x0);
print stdc(x)^2~varxj~varx0;
phi=lambda./(varx0./varxj+lambda);

print lambda;
print phi;

end;


proc pdfp(l,x);
@ poisson probability distribution @
@ l = mean rate of an event's occurance @
@ x = number of event = 0,1,2,... @
  retp(exp(-l)*(l^x)./(x!));
endp;


proc llfp(x,b);
@ log-likelihood function of poisson mixture of normals @
@ b = [lamda,mu,theta,sigma2,delta2]' @
  local p,j,z,s,f;
  p=1; f=0; j=0;
  do until p<_tol;
    s=maxc(real(sqrt(b[4]+b[5]*j))|1.0e-13);
    z=(x-b[2]-b[3]*j)./s;
    p=pdfp(b[1],j).*pdfn(z)./s;
    f=f+p;
    j=j+1;
  endo;
  retp(sumc(ln(f)));
endp;


proc prob(x,b);
@ sample separation information (probability) @
```

```
@ return probability of lambda>0 (jump) @
@ b = [lamda,mu,theta,sigma2,delta2]' @
 local p,j,z,s,f,f0;
 p=1; f=0; j=0;
 do until p<_tol;
   s=sqrt(b[4]+b[5]*j);
   z=(x-b[2]-b[3]*j)./s;
   p=pdfp(b[1],j).*pdfn(z)./s;
   if j==0; f0=p; endif;
   f=f+p;
   j=j+1;
 endo;
 retp(1-(f0./f));
endp;
```

.

****************OUTPUT***

	mean(vw)	σ(vw)	σ^2(vw)
	0.0095733	0.044938	0.0020194

NLOPT/GAUSS Version 3.1C: Applied Data Associates.(1996/01/16/08:05:57)

Maximum Likelihood Estimation

Model: Total Log-Likelihood Function
Number of Parameters = 5

Maximum Number of Iterations = 5000
Step Size Search Method = 2
Convergence Criterion = 2
Tolerance = 0.001

Initial Result:
Log Likelihood =   -681.30
Parameters =
    0.30000   0.00030000 -0.00080000 5.0000e-05 0.00010000

Using Quadratic Hill-Climbing Algorithm
Iteration = 1 Step Size = 1.2000 Log Likelihood =   154.21
Parameters =   0.29992   0.0019863 -0.0019549 0.00038185 3.9674e-05
Iteration = 2 Step Size = 2.5859 Log Likelihood =   391.44
Parameters =   0.29992   0.0019907 -0.0019478 0.00016001 0.00085953
Iteration = 3 Step Size = 2.9869 Log Likelihood =   577.14
Parameters =   0.30058   0.013119 -0.0041669 0.00082172 0.0015546
Iteration = 4 Step Size = 1.9248 Log Likelihood =   587.91
Parameters =   0.30147   0.010747 -0.0081215 0.0012237 0.0023090
Iteration = 5 Step Size = 0.9578 Log Likelihood =   588.49
Parameters =   0.30198   0.012300 -0.011931 0.0010857 0.0027571
Iteration = 6 Step Size = 1.0515 Log Likelihood =   588.51
Parameters =   0.30256   0.012414 -0.012820 0.0011057 0.0027630
Iteration = 7 Step Size = 2.2555 Log Likelihood =   588.53
Parameters =   0.34185   0.012703 -0.012154 0.0010650 0.0025272
Iteration = 8 Step Size = 1.0229 Log Likelihood =   588.53
Parameters =   0.34227   0.012684 -0.012146 0.0010677 0.0025476
Iteration = 9 Step Size = 0.9771 Log Likelihood =   588.53
Parameters =   0.35057   0.012737 -0.012009 0.0010601 0.0025069
Iteration = 10 Step Size = 0.9979 Log Likelihood =   588.53
Parameters =   0.35064   0.012737 -0.012011 0.0010601 0.0025075
Iteration = 11 Step Size = 0.7972 Log Likelihood =   588.53
Parameters =   0.35064   0.012737 -0.012010 0.0010601 0.0025075
Iteration = 12 Step Size = 0.0032 Log Likelihood =   588.53
Parameters =   0.35064   0.012737 -0.012010 0.0010601 0.0025075

Final Result:
Log Likelihood =   588.53
Gradient of Log Likelihood =
  0.00012969 -0.0062481 -0.0037863 -0.0022737 0.0045475

	Asymptotic	
Parameter	Std. Error	t-ratio
0.35064	0.27989	1.2528
0.012737	0.0034507	3.6910
-0.012010	0.0093349	-1.2866

0.0010601	0.00028679	3.6965
0.0025075	0.0014575	1.7203

-0.0016906	0.19468	0.0000
0.052682	0.23730	0.0000
-0.011931	0.22316	0.0000
0.027738	0.17975	0.0000
-0.0055536	0.20367	0.0000
0.021740	0.17591	0.0000
0.025793	0.17812	0.0000
0.017190	0.17527	0.0000
0.017681	0.17524	0.0000
0.0045900	0.18399	0.0000
0.017415	0.17525	0.0000
0.015097	0.17563	0.0000
0.020132	0.17546	0.0000
-0.011283	0.22089	0.0000
0.030477	0.18266	0.0000
0.0088496	0.17925	0.0000
0.0030871	0.18613	0.0000
0.0037412	0.18517	0.0000
0.038696	0.19604	0.0000
0.0069412	0.18113	0.0000
-0.0097050	0.21566	0.0000
0.033636	0.18696	0.0000
-0.0044042	0.20079	0.0000
-0.051854	0.54211	1.0000
0.016743	0.17531	0.0000
0.030898	0.18317	0.0000
0.031955	0.18454	0.0000
0.029308	0.18133	0.0000
0.0030976	0.18611	0.0000
0.013539	0.17617	0.0000
0.012131	0.17687	0.0000
-0.0086405	0.21234	0.0000

-0.020897	0.26220	1.0000
0.024831	0.17746	0.0000
-0.052508	0.55045	1.0000
-0.010300	0.21759	0.0000
-0.012869	0.22657	0.0000
-0.075355	0.83291	1.0000
-0.0069725	0.20749	0.0000
0.042359	0.20442	0.0000
0.017490	0.17525	0.0000
0.0062021	0.18197	0.0000
0.085504	0.46344	1.0000
0.010937	0.17761	0.0000
0.043419	0.20715	0.0000
0.041599	0.20255	0.0000
-0.039347	0.39948	1.0000
0.026871	0.17898	0.0000
0.049173	0.22447	0.0000
-0.0062388	0.20548	0.0000
0.034287	0.18797	0.0000
-0.027401	0.30089	1.0000
0.0079032	0.18013	0.0000
0.033697	0.18705	0.0000
-0.036356	0.37121	1.0000
-0.033621	0.34749	1.0000
0.0050746	0.18335	0.0000
0.094102	0.55491	1.0000
0.027025	0.17911	0.0000
0.011526	0.17722	0.0000
-0.022046	0.26835	1.0000
0.018047	0.17524	0.0000
0.044444	0.20991	0.0000
0.0090438	0.17907	0.0000
0.058523	0.26286	1.0000
-0.033886	0.34970	1.0000
-0.0067120	0.20676	0.0000
-0.053628	0.56486	1.0000

0.030493	0.18268	0.0000
0.020560	0.17555	0.0000
0.0050643	0.18336	0.0000
-0.067296	0.74257	1.0000
-0.065078	0.71482	1.0000
0.051498	0.23277	0.0000
-0.022584	0.27133	1.0000
0.055630	0.24951	0.0000
-0.032164	0.33568	1.0000
-0.019671	0.25596	1.0000
-0.073256	0.81127	1.0000
0.056722	0.25438	1.0000
-0.0047206	0.20156	0.0000
-0.10532	0.98486	1.0000
-0.064342	0.70541	1.0000
-0.051130	0.53295	1.0000
0.074206	0.36194	1.0000
0.050049	0.22750	0.0000
0.047477	0.21890	0.0000
-0.018171	0.24872	0.0000
0.050456	0.22895	0.0000
0.060708	0.27389	1.0000
0.052021	0.23474	0.0000
0.016886	0.17529	0.0000
0.044794	0.21089	0.0000
0.033328	0.18649	0.0000
-0.036399	0.37160	1.0000
0.0031291	0.18607	0.0000
-0.040284	0.40883	1.0000
0.042502	0.20478	0.0000
-0.0049980	0.20226	0.0000
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0.031343	0.18374	0.0000

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0.0055040	0.18281	0.0000
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-0.0042341	0.20038	0.0000
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0.0086963	0.17938	0.0000
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-0.044431	0.45302	1.0000
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-0.086125	0.91907	1.0000
-0.11008	0.99069	1.0000
0.16529	0.99129	1.0000
-0.042136	0.42801	1.0000
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